UNIVERSITY OF PETROLEUM & ENERGY STUDIES, DEHRADUN

Program	**B.Tech (All SoCSBranches)**	**Semester**	**III**
Course	**Discrete Mathematical Structures**	**Course Code**	**CSEG2006**

1. In a survey concerning the energy drinking habits of people, it was found that 55 % take energy drink A, 50 % take energy drink B, 42 % take energy drink C, 28 % take energy drink A and B, 20 % take energy drink A and C, 12 % take energy drink B and C and 10 % take all the three energy drinks.
   (i) What percentage of people do not take energy drink?
   (ii) What percentageof people take exactly two brands of energy drinks?
   (iii) What percentageof people take the energy drink in A but not in B or C?
2. Show that the set of all integers $\mathbb{Z}$ is countably infinite set.
3. Consider the following relations on a set $A=\{1,2,3,4,5,6\}$ (i) $R=\{(i,j):|i-j|=2\}$, and (ii) $R=\{(i,j):|i-j|<2\}$. Check whether R is (i) reflexive, (ii) symmetric, (iii) antisymmetric, and (iii) transitive.
4. If $A=\{0,1,2,3\}$, $R=\{(x,y):x+y=3\}$, $S=\left\{(x,y):\dfrac{3}{x+y}=k,\ k\in\mathbb{N}\right\}$, $T=\{(x,y):\max(x,y)=3\}$.
   Compute (i) RoT, (ii) ToR, and (iii) SoS.
5. The function $f:\mathbb{R}\to\mathbb{R}$ is defined as $f(x)=x^3+1$, where $\mathbb{R}$ is the set of real numbers, then specify that f is one-one and onto.
6. Specify the types (one-one or onto or both or neither) of the following function:
   (i) If I is set of non-negative integers and $f: I\times I\to I$ such that $f(x,y)=xy$.
   (ii) If R is set of real numbers and $f: R\times R\to R\times R$ and $f(x,y)=(x+y,\ x-y)$.
   (iii) If N is set of natural numbers including zero and $f: N\to N$ such that $f(j)=j^2+2$
   (iv) If N is set of natural numbers including zero and $f: N\times N\to N$ so that $f(x,y)=(2x+1)\,2^y-1$.
7. Show that the mapping $f:\mathbb{R}\to\mathbb{R}$, which is defined as, $f(x)=ax+b$, where $a,b,x\in\mathbb{R}$, $a\neq 0$ is invertible. Determine its inverse also.
8. If A=$\{1,2,3,4\}$, $B=\{a,b,c,d\}$ and $C=\{x,y,z\}$. Consider the functions $f:A\to B$ and $g:B\to C$ defined by $f=\{(1,a),(2,c),(3,b),(4,a)\}$ and $g=\{(a,x),(b,x),(c,y),(d,y)\}$. Determine the composition of function (gof).
9. Show that $1+2+2^2+\ldots+2^n=2^{n+1}-1$ by mathematical induction for all positive integers n.
10. Solve following recurrence relations by the method of generating function
    (i) $a_n-5a_{n-1}+6a_{n-2}=1$ **(iii)** $a_n-4a_{n-1}+4a_{n-2}=(n+1)^2$, given $a_0=1,\ a_1=1$.
    (ii) $a_n+a_{n-1}=3n\,2^n$ **(iv)** $a_{n+2}-3a_{n+1}+2a_n=4n\,3^n$, with $a_0=1,\ a_1=1$.
11. Suppose that the population of a village is 100 at time $n=0$ and 110 at time $n=1$. The population increases from time $n-1$ to time n is twice the increase from time $n-2$ to time $n-1$. Find a recurrence relation and initial conditions for the population at time n and then find the explicit formula for it.

12. If D_n is the value of the following determinant of order n

$$\begin{vmatrix} b & b & 0 & 0 \dots\dots\dots & 0 & 0 & 0 \\ b & b & b & 0 \dots\dots\dots & 0 & 0 & 0 \\ 0 & b & b & b \dots\dots\dots & 0 & 0 & 0 \\ .. & .. & .. & \dots\dots\dots. & .. & 0 & 0 \\ 0 & 0 & \dots\dots\dots & .. & b & b & b \\ 0 & 0 & \dots\dots.. & .. & 0 & b & b \end{vmatrix},$$

Find a recurrence relation for D_n. (Assume $b > 0$.)

13. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then prove by principle of mathematical induction that for every integer $n \geq 3$,

$A^n = A^{n-2} + A^2 - I$. Hence find A^{50}, where I is an identity matrix of order 3×3.

14. Let U be a universal set and $S_1, S_2, S_3 \dots\dots \dots S_n$ be its any n subsets. Use the principle of mathematical induction to show that $\overline{[\cup_{i=1}^{n} S_i]} = \overline{S_1} \cap \overline{S_2} \cap \overline{S_3} \cap \dots\dots\dots \cap \overline{S_n}$.