Costs or Benefits? Why Students Specialize in Cognitive vs Socio-Emotional Skills

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Abstract

I combine a field experiment in India with a structural model of multidimensional skill development to study why students specialize in different skills. When a student specializes, is it mainly because they find that skill easy to learn (costs) or because it provides them more value (benefits)? The answer has important implications for teachers and policy-makers aiming to maximize student welfare by promoting growth in skills each student benefits from most. I focus on parents as the primary decision-makers for their child's skill development with full information about production costs and benefits to their child. Teachers allocate effort towards reducing costs of skill production, but have imperfect information about how benefits vary across students. In five private schools across India, I survey 3,404 parents to collect perceptions of their child's skill levels and a ranking of what skills each parent most wants to improve. Parents report wide variation in what they value, and on average, prioritize improvement where their child is weakest. Through the lens of the model, this suggests students specialize in skills they find easier to learn. I elicit teachers' beliefs about parent priorities and find little alignment with parents' views. To address this, I conduct a classroom-level experiment randomly providing teachers access to structured information about parent priorities. Treated teachers become more accurate about average class priorities by about 10 percentage points. Treatment increases student specialization in parent-prioritized skills, with the largest effects for students where teacher beliefs were most inaccurate. Together, the model and experiment show how simple measures of student levels and parental priorities can provide a powerful lever for personalizing education based on what skills are valuable to each student.

JEL: C93; D83; I21; I31; J24; O15

Keywords: parental preferences; skill development; socio-emotional skills; teacher beliefs; information frictions; education policy; India; field experiment

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1 Introduction

Teachers allocate limited time and resources across many skills that matter for life. They face diverse classrooms, with some students excelling in academics, while others shine in socio-emotional skills. Yet the source of this specialization is unclear. Do students specialize in skills they find easier to learn (costs), or in those they and their parents value more (benefits)? Teachers, through their pedagogy, time, and materials, are key inputs into students' ability to learn skills. If teachers only target effort based on student achievement (e.g., remedial education that targets weak skills), they may be misallocating effort towards low value skills.

In this paper, I quantify the role of variation in the benefits to skills versus different costs of skill production. I develop a model that centers parents as decision-makers who choose inputs to produce multiple skills for their child, balancing costs and benefits to maximize their child's utility over the resulting skill bundle. Teachers, with aligned incentives, shift the effective costs of learning, thereby changing the production function faced by parents. Although teachers have aligned incentives, they must infer parent preferences from noisy signals (for example, due to large class sizes or imperfect parent-teacher communication). When teachers misperceive what parents value, as I document in my context, time and effort may be shifted away from the highest-value margins. This motivates my classroom-level experiment that provides teachers with structured information about parent priorities, testing whether correcting teacher beliefs aligns classroom production with parental values for skill development.

The model yields a simple diagnostic for why students specialize: the relationship between each skill's current level and its marginal benefit of improvement. A negative relationship (stronger skills have lower marginal benefits) points to costs. When learning costs vary widely across skills, strengths reflect easy-to-learn skills; with diminishing marginal benefits, these are the lowest value to improve further. A positive relationship (stronger skills have higher marginal benefits) points to benefits. When skills mainly differ in how much value they provide, strengths reflect high-value skills. With increasing marginal costs, sustaining a higher level requires a higher marginal benefit, so strengths are the most valuable to improve. This reframes a long-standing debate in educational policy: should we encourage well-rounded or specialized students? If benefits drive specialization, the highest marginal benefits are on strengths, so specialization is optimal. If costs drive specialization, the highest marginal benefits are on weaknesses, so further well-roundedness is optimal.

Guided by the model, I survey 3,404 parents and 242 teachers across five private schools in India (grades 1 to 10). Parents report their child's current levels from 0-100 across nine academic, emotional, and social skills. They then rank skills by which they most want improved, at the margin. Parent ranks serve as an ordinal proxy for marginal benefits (higher rank = higher marginal benefit),

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while I construct within-student standardized skill levels as a measure of specialization. Together, these two measures allow me to estimate the relationship between levels and marginal benefits by regressing ranks on standardized levels. I look at two types of variation: (i) across students for a given skill, and (ii) within each student, across skills. Respectively, this answers why some students specialize in a given skill, and why are some skills relatively strong or weak for a given student.

Across students, I find a strong negative relationship between standardized skill levels and priority ranks for all nine skills. This pattern is driven almost entirely by older grades: levels and ranks are nearly uncorrelated for primary grades and become increasingly negatively correlated by grade 10. Interpreted through the model, this implies that variation in production constraints—effective prices, access to materials, and the productivity of time—plays an increasing role in determining who is relatively weak or strong within a given skill as students age. This is consistent with dynamic complementarity in skill formation, where early gaps in skills lead to widening differences in the costs of acquiring new skills over time.

Within each student, I also find a predominantly negative relationship between skill levels and priority ranks (parents want improvement in weaker skills) but this varies widely across students, with many parents reporting a positive relationship. Moreover, when the relationship is more positive, parents report greater satisfaction with their child's progress in school. Theoretically, the model interprets positive alignment between marginal benefits and levels as a case when a student's natural path of skill growth (the expansion path) aligns with the direction that would raise utility fastest. Practically, this suggests that when parents see their child's skill growth trajectory as aligned with what they value, they are more satisfied with their child's progress. Supporting this interpretation, the relationship between skill levels and priority ranks is uncorrelated with parent satisfaction with school quality, suggesting that the relationship captures dynamic growth, rather than static features of the school environment.

Turning to teachers, I ask them to rank the same nine skills for a typical student, and then for specific randomly sampled students in their class. Crucially, for the specific students, I also elicit teachers' beliefs about each parent's rankings. I find that teacher beliefs are largely uncorrelated with parents' actual rankings, both at the individual-student level and in classroom averages, and instead reflect teachers' own priorities—consistent with teachers projecting their own preferences onto parents. This misalignment highlights potential misallocation of teacher effort across skills and students.

Motivated by these findings, I design and implement a classroom-level randomized experiment that provides teachers with access to parents' ratings and priority rankings. Treated teachers are given individual login information providing access to a custom-built website displaying information for students in their class. In the framework of my model, this intervention acts as a production-side shock, leading teachers to lower the effective cost of improving parent-prioritized skills. The model yields three testable predictions: treatment should (i) increase specialization in the parent's top-priority skill (empirically, raise standardized levels), (ii) reduce the marginal benefit for that skill (lower priority rank), and (iii) increase the alignment between marginal benefits and skill levels (more positive relationship between priority ranks and standardized levels).

At baseline, teachers' beliefs about what parents most want improved are essentially uncorrelated with parents' own rankings and instead reflect teachers' own priorities. I find that providing the information does not measurably improve accuracy about individual students, but it does shift beliefs about the classroom average. Treated teachers become more accurate-—by roughly 10 percentage points—-in identifying which category (academic, social, emotional) is most or least valued on average in their class, with suggestive evidence that they especially learn where academics sits in the ordering. This pattern aligns with the model's policy logic: teacher effort is applied at the classroom-level, so correctly identifying average benefits can move the frontier in the direction of fastest welfare growth, even if teachers do not perfectly know individual values.

The experiment produces results consistent with the model predictions. First, giving teachers parent-priority information changed how students specialized. Average effects on skill levels are near zero, but this masks significant heterogeneity. Treatment effects are large when teacher beliefs were far from parents reported priorities; treated students standardized levels increase in parents' top-ranked skill, and fall in the bottom-ranked one. In contrast, effects are smaller or flipped when teachers' initial beliefs are accurate. This is consistent with teachers reallocating effort towards students and skills they previously misjudged at the expense of students they did not know well. Second, parents' former top priority now becomes less urgent to improve: for the skill parents ranked most important to improve at baseline, treated parents rank that skill 0.26 ranks lower at endline compared to control, implying the marginal benefit fell further. Third, treated students' skill profiles became further aligned with parental values: the slope from regressing priority ranks on skill levels rises by about 0.15 on average, and by roughly 0.26 in academic-priority classrooms. All three effects are strongest in classrooms where parents on average prioritized academics, consistent with teachers having more scope to shift effort in these settings. Taken together, these findings show that providing teachers with simple, structured information acts as a production shock—reorienting classwide effort toward parents top-ranked skills, reducing parents' perceived need for improvement in those areas, and shifting specialization away from what is easiest to build, and towards what parents value most.

The theoretical and empirical findings have direct policy implications for a fundamental debate in education: whether to promote well-roundedness or encourage specialization. If student weaknesses reflects high costs of skill development, the answer leans toward well-roundedness, whereas if it reflects low benefits, specialization is desirable. In order to distinguish these two types, a policy maker must know either the marginal benefits or the marginal costs of improving skills. However, while teachers and policy makers often know *where* students are (current skill levels), they rarely observe the additional information that would diagnose *why* they are there.

In this setting, student skill levels and parental priority ranks are consistently negatively correlated; parents most want improvement in weaker skills, pointing to uneven production costs as the key driver of observed weaknesses. After accounting for policy cost, the most effective lever operates through production - targeting inputs and instructional technology. This aligns with evidence that cost-reducing inputs raise achievement and long-run outcomes, especially for disadvantaged students (Jackson et al., 2016; Muralidharan and Sundararaman, 2013), optimizing class time is productivity-relevant (Burgess et al., 2023), and intensive instructional models yield large gains (Fryer Jr, 2017). In contrast, settings where higher marginal benefits are likely associated with relative strengths, such as in higher education or labor markets, the effective lever may run through perceived benefits - information, guidance, and incentives.

The experiment provides another lever to personalize education: instruction based on student-varying costs and benefits to skills. This represents a theory-guided complement to "Teaching at the Right Level": personalize not only by skill levels, but also by values. This is particularly relevant for educational aims that go beyond foundational skill development, where level-based grouping may be sufficient if all parents similarly value basic literacy and numeracy. I show that broadening educational goals to diverse or specialized skill development requires accounting for large variation in values. In this context, providing teachers with structured information on parent priorities can help align instructional effort with the welfare-maximizing aims.

Contributions to the Literature This benefits-vs-costs question is central but empirically under-identified in most educational settings, despite related evidence on the technology of skill formation and dynamic complementarity (Cunha et al., 2010), on the heterogeneous returns to cognitive and socio-emotional skills (Heckman et al., 2006; Lindqvist and Vestman, 2011; Deming, 2017; Kosse and Tincani, 2020), on parental preferences over teacher attributes (Jacob and Lefgren, 2007; Jackson, 2018), and on resource and productivity constraints in schools (Jackson et al., 2016; Muralidharan and Sundararaman, 2011, 2013; Burgess et al., 2023; Fryer Jr, 2017). It also relates to literatures on parental preferences and beliefs (Jacob and Lefgren, 2007; Dizon-Ross, 2019) and on information interventions that primarily inform *parents* (Hastings and Weinstein, 2008; Bergman, 2021; Bergman and Chan, 2021; Berlinski et al., 2022).

I contribute along four margins. First, I provide new measurement for the literature on parental preferences and beliefs by directly eliciting, for the same child, parents' perceived levels and marginal priorities across nine cognitive and socio-emotional dimensions (Jacob and Lefgren, 2007; Dizon-Ross, 2019). By linking levels to marginal benefits, I go beyond documenting preferences to provide a diagnostic for the underlying source of specialization. Second, I contribute to the literature on the technology of skill formation by providing a tractable framework to distinguish benefit- from cost-driven heterogeneity and by modeling teachers as endogenous cost-shifters who respond to information (Cunha et al., 2010). Third, my framework informs work on the heterogenous benefits to cognitive and non-cognitive skills by offering a way to interpret observed specialization, helping to understand when heterogeneous benefits are likely to be the more important driver of investment choices (Heckman et al., 2006; Lindqvist and Vestman, 2011; Deming, 2017; Kosse and Tincani, 2020).

Finally, I add to the literature on information interventions that primarily inform parents about their child's behavior (Hastings and Weinstein, 2008; Bergman, 2021; Bergman and Chan, 2021; Berlinski et al., 2022). My experiment reverses the typical information flow. I inform teachers about parent priorities—a parameter where parents are plausibly better informed, in contrast to academic levels where evidence shows parental perceptions can be inaccurate. This demonstrates that reducing uncertainty on the school side can be a powerful lever for change.

The paper proceeds as follows. Section 2 describes the Indian school context, including the process for recruitment and list of human capital dimensions. Section 3 presents the model of multidimensional skill formation distinguishing benefit- from cost-driven specialization, modeling teachers as key inputs to skill production. Section 4 describes the data from the parent survey and reports initial descriptives. Section 5 presents results on benefit- vs cost-driven specialization. Section 6 describes the teacher-facing information experiment and its implementation. Section 7 discusses the impact of information on teacher beliefs about parent priorities while Section 8 reports impacts on key student outcomes. Section 9 outlines planned model estimation using teacher-reported importance rankings as supply shocks to discipline parameters. Section 10 concludes.

2 Setting

2.1 Context

This study takes place in five private schools located across four states in India: Delhi (two schools), Gujarat, Punjab, and West Bengal. These schools serve middle to upper-middle-class families, with approximately 90% of students coming from families with household incomes above the national median. Tuition fees range considerably across these institutions, ranging from approximately \$350 per year at the least expensive school to \$2,500 per year at the most expensive school. All schools span from Nursery to 12th grade, but for this study we only target parents of students in grades 1-10.

Class sizes in these schools typically range from 30 to 40 students per classroom, creating significant challenges for teachers in providing individualized attention. Teachers must make consequential decisions about how to allocate their limited time and resources across different skill dimensions and among numerous students with heterogenous needs and abilities.

Across all five schools, parent-teacher meetings occur either twice a year (once per term), or monthly. These meetings provide opportunities both for teachers to share information about student progress with parents, as well as for parents to share their perceptions with teachers. However, the format and content of these interactions vary widely. Most of the schools in this study provide report cards that focus primarily on academic subjects, with at least one school including non-cognitive skills on the report card. As a result, there may be limited formal communication about student development in non-academic dimensions.

Despite the regular occurrence of parent-teacher meetings, teachers face numerous information frictions that limit their ability to incorporate parent preferences into their decision-making. These include large class sizes, varying parental communication styles, and the difficulty of aggregating information from multiple parents. This context where parent input is valued, but potentially undersupplied due to structural constraints, provides an ideal setting to examine how structured information about parent preferences can influence teacher decision-making and student outcomes.

2.2 Sample and Recruitment

The recruitment process for this study involved identifying and engaging with private schools interested in understanding the varied needs of their students and accommodating parent perspectives through structured information aggregation. I conducted recruitment throughout 2023 and 2024, presenting the project as an opportunity for schools to better understand parent preferences and perceptions, allowing them to better align teaching practices with school aims.

The study began with a pilot phase in October 2023 at our initial partner school. Following a successful implementation of the pilot, we conducted a full baseline survey at this school in March 2024. We launched the intervention by sharing information with treated teachers in July 2024. Concurrent with the implementation at the first school, I recruited additional schools throughout the summer of 2024. We successfully onboarded four more schools, conducting baseline surveys at these institutions in September and October 2024. Treated teachers at these schools received access to the website containing parent preference information in November 2024, marking the beginning of the intervention phase for these schools.

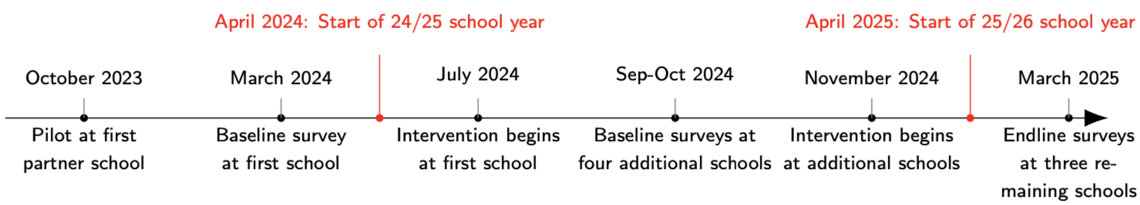


Figure 1: Study Timeline

Due to prior commitments, two schools were unable to continue their participation after the baseline phase. Although I collect and analyze endline data for all three remaining schools, I follow the pre-analysis plan in only estimating treatment effects using endline data for schools in which treatment compliance was above 15% (measured as the percent of treated teachers who view the website with parent data at least once). This threshold was set ex-ante to ensure that there was sufficient engagement with the intervention to potentially affect student outcomes. Despite regular reminders and school visits, this threshold was only surpassed by teachers in the initial partner school. As a result, the baseline sample includes 3404 parents surveyed in 242 classrooms across the five schools, and the final experimental sample includes 849 students across 106 classrooms in the initial partner school. Full descriptive statistics for the baseline and experimental samples are provided in Appendix Table 4.

2.3 Skill Dimensions

The study focuses on nine dimensions of human capital that encompass both cognitive and non-cognitive skills. The skills are divided into three broad categories: academic, social, and emotional skills. The nine dimensions are shown below in Figure 2.

The set of aspects was decided through an iterative process. The initial list contained 21

Category	Aspect	Explanation
Academic	**Literacy skills**	Reading, writing, speaking, and listening
Academic	**Mathematical skills**	Numeracy, quantitative reasoning, problem-solving
Academic	**Scientific literacy**	Understanding scientific concepts and processes
Social	**Collaboration and teamwork skills**	Ability to work effectively with others and contribute to group goals
Social	**Interpersonal skills**	Effective communication, conflict resolution, recognizing and responding to social cues
Social	**Leadership and initiative**	Taking charge, setting goals, motivating others
Emotional	**Perseverance and growth mindset**	Resilient in the face of challenges, belief in self-improvement
Emotional	**Emotional self-awareness and regulation**	Recognizing and managing emotions, thoughts, and behaviors
Emotional	**Empathy for others**	Understanding and valuing perspectives of others

Figure 2: Human Capital Dimensions

potential skill dimensions designed to be comprehensive in covering any skill a parent may care about. Aspects were drawn from literature on educational frameworks, psychology, and through discussion with parents prior to the start of the study. Through pilot testing with parents and teachers, the list was narrowed to nine dimensions to balance time constraints of the survey with the comprehensiveness of the final list. The selection process prioritized dimensions that (1) were comprehensible to parents without specialized knowledge, (2) covered a range of both cognitive and non-cognitive domains, and (3) were potentially actionable by both teachers and parents. Parents were provided the exact table shown in Figure 2 in the survey to promote consistency in parents' understanding of the dimensions. For each dimension, parents were asked to rate their child's current standing on a scale from 0 to 100, and then to rank the nine dimensions in order of importance for improvement, which we describe further in Section 4.

3 Model

I develop a framework to distinguish whether differences in students' skill profiles are driven primarily by variation in *benefits* (in terms of satisfaction, money, etc.) for different skills or by *real-production cost* differences in producing them. Note we focus on two types of variation: (i) variation across students within a skill (why some students specialize in a given skill) and (ii) variation across skills within a student (why some skills are relatively strong or weak for a given student).

We proceed in three steps. First, I present a model in which parents maximize their child's utility by allocating a fixed budget towards producing multiple skills. I interpret the expansion path (the path of optimal bundles as the budget rises) as how skills would change over time, absent changes to the costs or benefits from skills. This yields two types of students: (A) students who would derive higher utility from more well-rounded profiles relative to the expansion path, and (B) students who would derive higher utility from more specialized profiles relative to the expansion path.

Second, I introduce teachers with aligned incentives who choose how to allocate their time and resources across students and skills. Critically, teachers shift the production frontier at the classroom-level. This allows us to interpret the expansion path as the classroom production path, and the two types of students as those who would benefit from teachers shifting classroom production toward well-roundedness (type A) or specialization (type B).

Third, we allow the teacher to hold imperfect beliefs about parents' perceived benefits. When those beliefs are incorrect, on average, the path of skill growth as resources rise is misaligned with the direction that would raise utility fastest. This motivates our experiment: providing teachers with information about parents' preferences to test whether correcting beliefs moves classroom production in the direction of fastest utility growth.

3.1 Parents' Problem

Skills. Parents derive utility from their child's skill bundle $\boldsymbol{c} = (c_1, c_2) \in \mathbb{R}^2_+$, where c_1 and c_2 denote two distinct skill domains (e.g., cognitive and non-cognitive). We focus on two skills for simplicity, but the model extends naturally to more than two skills. We define the child's level of *specialization* as the ratio of the skill levels, $s_i := c_{1i}/c_{2i}$.

Preferences. Parent i has Cobb-Douglas utility[1]

$$U(c_{1i}, c_{2i}; \beta_i) = c_{1i}^{\beta_i}\, c_{2i}^{1-\beta_i}, \qquad 0 < \beta_i < 1.$$

The marginal rate of substitution (MRS) between skills 1 and 2 is

$$\text{MRS}_{12,i} = \frac{\beta_i}{1-\beta_i} \cdot \frac{c_{2i}}{c_{1i}} = \frac{\beta_i}{1-\beta_i} \cdot \frac{1}{s_i}. \tag{1}$$

Budget and technology. Parents buy inputs (x_{1i}, x_{2i}) at prices (p_{1i}, p_{2i}) subject to $p_{1i}x_{1i} + p_{2i}x_{2i} \leq I_i$. Each skill is produced via single-input technologies with diminishing marginal products:

$$c_{1i} = a_{1i}x_{1i}^{\theta}, \quad c_{2i} = a_{2i}x_{2i}^{\theta}, \quad a_{ji} > 0, \quad 0 < \theta < 1.$$

Eliminating (x_1, x_2) yields a smooth, strictly concave frontier in (c_1, c_2) space:

$$p_{1i}\left(\frac{c_{1i}}{a_{1i}}\right)^{1/\theta} + p_{2i}\left(\frac{c_{2i}}{a_{2i}}\right)^{1/\theta} = I_i.$$

[1] Any twice-differentiable, strictly increasing, strictly quasi-concave utility with non-constant MRS suffices; Cobb-Douglas is used here for algebraic transparency.

Feasible set. Let $\rho := 1/\theta > 1$ and define

$$\kappa_{1i} := a_{1i} \left(\frac{I_i}{p_{1i}} \right)^{1/\rho}, \quad \kappa_{2i} := a_{2i} \left(\frac{I_i}{p_{2i}} \right)^{1/\rho}.$$

The frontier becomes the constant-elasticity-of-transformation (CET) form[2]:

$$\left(\frac{c_{1i}}{\kappa_{1i}} \right)^{\rho} + \left(\frac{c_{2i}}{\kappa_{2i}} \right)^{\rho} = 1, \quad \rho > 1. \tag{2}$$

Marginal rate of transformation. Implicit differentiation of (2) yields the marginal rate of transformation (MRT) between skills:

$$\mathrm{MRT}_{12} = \left(\frac{\kappa_{2i}}{\kappa_{1i}} \right)^{\rho} \left(\frac{c_{1i}}{c_{2i}} \right)^{\rho-1}. \tag{3}$$

Note that κ_{ji} fully captures the effective cost of producing skill j for parent i, and is exactly the intercept of the frontier on the c_j axis.

Equilibrium skill ratio. Utility maximization subject to (2) equates (1) and (3), giving the optimal skill ratio:

$$s_i^* := \frac{c_{1i}^*}{c_{2i}^*} = \underbrace{\left(\frac{\beta_i}{1-\beta_i} \right)^{1/\rho}}_{T_i} \underbrace{\frac{\kappa_{1i}}{\kappa_{2i}}}_{\lambda_i}. \tag{4}$$

This equilibrium condition is shown graphically in Figure 3A. The equation illustrates how the variation in skill ratios across parents can be decomposed into two components: variation in the *benefits tilt* $T_i = \left(\frac{\beta_i}{1-\beta_i} \right)^{1/\rho}$, which reflects how much more a parent values skill 1 relative to skill 2, and variation in the *costs tilt* $\lambda_i = \frac{\kappa_{1i}}{\kappa_{2i}}$, which reflects the relative costs or ease of producing skill 1 compared to skill 2.

Comparative statics. Consider the following elasticities of the optimal skill ratio with respect to the primitives:

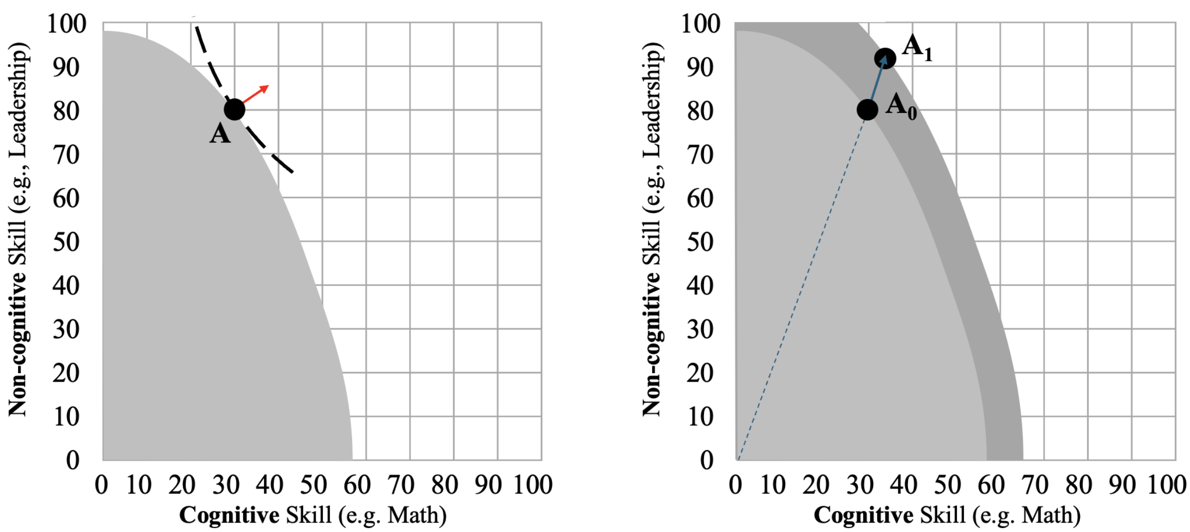
$$\frac{\partial \ln s_i^*}{\partial \beta_i} = \frac{1}{\rho} \left(\frac{1}{\beta_i} + \frac{1}{1-\beta_i} \right) > 0, \qquad \frac{\partial \ln s_i^*}{\partial \ln \kappa_{1i}} = 1, \qquad \frac{\partial \ln s_i^*}{\partial \ln \kappa_{2i}} = -1.$$

Thus, an increase in the relative benefit to skill 1 ($\beta_i \uparrow$), or a decrease in the relative cost of producing skill 1 ($\kappa_{1i} \uparrow$ or $\kappa_{2i} \downarrow$) raises the optimal specialization s_i^*.

[2]Our CET frontier is the single-period analogue of the multistage production functions, e.g. in Cunha and Heckman (2007, 2008); Cunha et al. (2010). We abstract from dynamic self-productivity and cross-period complementarity to focus on the question of whether observed heterogeneity is driven by relative costs (κ_{1i}, κ_{2i}) or by relative benefits (β_i).

Skill growth over time. In this model, parents' budgets consists of time, money, and other resources that they can invest in their child's skill development. Consider additional time (e.g. one more year) to spend on skill development. We model this as a *pure budget expansion*: in each period the family has more effective resources I_i. With homothetic preferences and the CET frontier, if *benefits* (β_i) and *costs* (κ_{1i}, κ_{2i}) are fixed, optimal levels scale up with income, but proportionally so that specialization, $s_i^* = c_{1i}^*/c_{2i}^* = T_i\lambda_i$, remains constant (Note that I_i cancels in the costs tilt, λ_i). This is shown graphically in Figure 3B. The expansion path will allow us to distinguish two types of students, and will be key to understanding how teachers can influence skill growth over time.

Figure 3: Parent's Problem and Skill Growth



(a) Parent's static problem

(b) Expansion path over time

Two types: (A) Desire well-roundedness or (B) Desire specialization. Fix student i at $c_i = (c_{1i}, c_{2i})$ with specialization level $s_i = c_{1i}/c_{2i}$ and

$$\text{MRS}_{12,i} = \frac{\beta_i}{1-\beta_i} \cdot \frac{1}{s_i}.$$

With β_i and $(\kappa_{1i}, \kappa_{2i})$ fixed, income growth moves the optimum with constant slope $(dc_2/dc_1)_I = 1/s_i$. Contrast this with the direction of *maximal utility growth*, which is normal to the indifference curve (or equivalently normal to the frontier), hereafter denoted as the *IC-normal* direction. The IC-normal has slope $1/\text{MRS}_{12,i}$, and is denoted in red in Figure 3A.

Without loss of generality, fix the weaker skill level to be on the horizontal axis so that $s_i < 1/2$. We define:

- **Type A:** expansion path is *steeper* than the IC normal,

$$\frac{1}{s_i} > \frac{1}{\text{MRS}_{12,i}} \iff \text{MRS}_{12,i} > s_i,$$

i.e., marginal utility puts more weight on the *weaker* skill 1 relative to the current level of specialization.

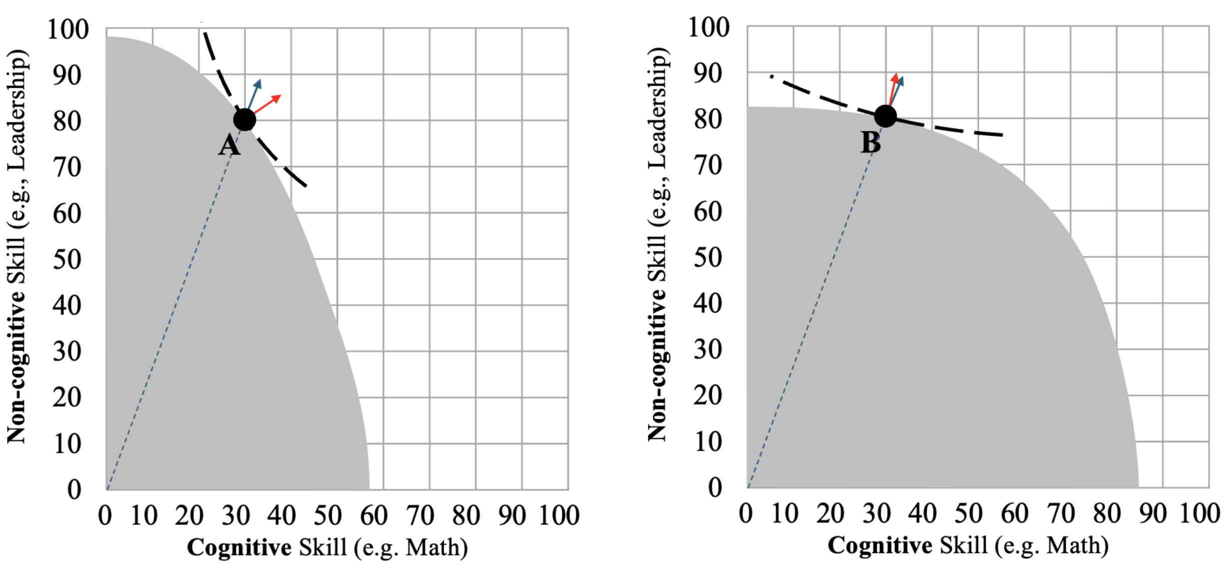
- **Type B:** expansion path is more *shallow* than the IC normal,

$$\frac{1}{s_i} < \frac{1}{\text{MRS}_{12,i}} \iff \text{MRS}_{12,i} < s_i,$$

i.e., marginal utility favors further improvement in the *stronger* skill relative to the current level of specialization.

Recall that $\text{MRS}_{12,i} = \beta_i/(1-\beta_i) * s^{-1}$. Therefore, if we define the threshold ratio $s^{\dagger}(\beta_i) := \sqrt{\beta_i/(1-\beta_i)}$, a student is of type A iff $s_i < s^{\dagger}(\beta_i)$ and of type B iff $s_i > s^{\dagger}(\beta_i)$. Intuitively, if the student is relatively weak in skill 1 (s_i small) but the parent places high value on skill 1 (β_i large), then the student is type A and would benefit from further well-roundedness. Conversely, if a student's relative weakness is not *too* weak relative to values ($1/2 > s_i > s^{\dagger}(\beta_i)$), then the student is type B and would benefit from further specialization. We show Type A and B graphically in Figure 4.

Figure 4: Type A and B Students



(a) Type A: Desire well-roundedness

(b) Type B: Desire specialization

Note that type A and B can be applied to *either* students or skills. For a given student, type A (B) students derive more utility from improving weaker (stronger) skills. For a given skill, type A (B) skills are ones in which the largest utility gains would come from boosting the weaker (stronger) students in that skill.

Diagnostic for type A or B *skills*. We showed above that type A is characterized by environments in which higher marginal benefits are associated with relative weaknesses (low levels), and type B by environments in which higher marginal benefits are associated with relative strengths (high levels). Hence, we propose the **benefit-level slope** (BLS) as a sufficient statistic for capturing this relationship between marginal benefits and relative levels. Consider the regression **across students** for a given skill (here, skill 1):

$$\mathrm{MRS}^*_{12,i} = \alpha + \beta^{\mathrm{RF}}\, s^*_i + \varepsilon_i, \tag{5}$$

with population slope

$$\beta^{\mathrm{RF}} \;=\; \frac{\mathrm{Cov}_i(s^*, \mathrm{MRS}^*)}{\mathrm{Var}_i(s^*)}. \tag{6}$$

Using the primitives from Section 3.1, optimal specialization is the product of the benefit and cost tilts so that

$$s^*_i \;=\; T_i\, \lambda_i, \qquad \mathrm{MRS}^*_{12,i} \;=\; T_i^{\rho-1}\, \lambda_i^{-1}, \qquad T_i := \left(\tfrac{\beta_i}{1-\beta_i}\right)^{1/\rho},$$

so

$$s^*_i \cdot \mathrm{MRS}^*_{12,i} = T_i^{\rho}, \tag{7}$$

which cancels λ_i and separates *benefits* (T_i) from *costs* (λ_i). It follows that

$$\mathrm{Cov}(s^*, \mathrm{MRS}^*) = \mathrm{Cov}(T_i\lambda_i, T_i^{\rho-1}\lambda_i^{-1}) = \mathbb{E}[T_i^{\rho}] - \mathbb{E}[T_i\lambda_i]\; \mathbb{E}[T_i^{\rho-1}\lambda_i^{-1}]. \tag{8}$$

Writing mean–zero deviations $\tilde{T}_i := T_i - \mathbb{E}[T]$ and $\tilde{\lambda}_i := \lambda_i - \mathbb{E}[\lambda]$, a first-order expansion around means yields

$$\beta^{\mathrm{RF}} \;=\; \frac{(\rho-1)\,\mathrm{Var}(\tilde{T}) \;-\; \mathrm{Var}(\tilde{\lambda})}{\mathrm{Var}(\tilde{T}) \;+\; \mathrm{Var}(\tilde{\lambda})}. \tag{9}$$

so that the sign and magnitude of the slope depends on the relative sizes of the variance in benefit and cost tilts. As dispersion in costs increases, the slope becomes more negative (Type A), and as dispersion in benefits increases, the slope becomes more positive (Type B).

This derivation shows that asking whether a student or skill is closer to Type A or Type B is closely related to asking if relative strengths and weaknesses are primarily driven by dispersion in costs or benefits. Intuitively, Type A students who desire more specialized profiles exist in an environment where the strengths are primarily due to high value for that skill, not ease of production. Type B students that desire more well-rounded profiles exist in environments where weakness is due to high costs of production, not a low value for that skill.

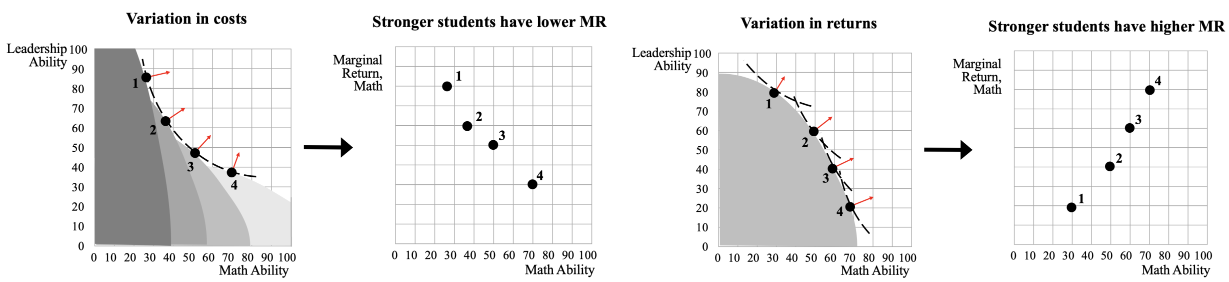
Extreme cases and interpretation. Two polar cases benchmark (9):

(i) **No supply variation** ($\mathrm{Var}(\tilde{\lambda}) = 0$): $\beta^{\mathrm{RF}} = \rho - 1 > 0$. Heterogeneity is purely preference-driven (benefits dispersion).

(ii) **No preference variation** ($\mathrm{Var}(\tilde{T}) = 0$): $\beta^{\mathrm{RF}} = -1$. Heterogeneity is purely supply-driven (cost/technology dispersion); s^* and MRS^* move in opposite directions.

In intermediate cases $\beta^{\mathrm{RF}} \in [-1, \rho - 1]$, and its sign/magnitude reveal the relative importance of benefit vs. cost dispersion. We display these benchmarks graphically in Figure 5.

Figure 5: Benefit-level slope benchmarks



(a) No benefit dispersion: $\beta^{\mathrm{RF}} < 0$ (b) No cost dispersion: $\beta^{\mathrm{RF}} > 0$

Diagnostic for Type A or B *students*. Similar to the within skill diagnostic, we can classify type A students as those that have high marginal benefits for improving weaknesses, and type B students as those that have high marginal benefits for improving strengths. Therefore, we consider the analogous regression of ranks on levels **across skills** for a given student.

First we solve for the equilibrium skill mix for a given student, with more than two skills. Dropping the i subscript, let there be $J \geq 3$ skills with $\sum_j (c_j/\kappa_j)^\rho = 1$ and preferences $\prod_j c_j^{\beta_j}$. Fix skill 1 as an anchor and define for $j \neq 1$:

$$s_j^* = \frac{c_j^*}{c_1^*} = \left(\frac{\beta_j}{\beta_1}\right)^{1/\rho} \frac{\kappa_j}{\kappa_1} =: T_j \lambda_j, \qquad \mathrm{MRS}_{1j}^* = T_j^{\rho-1} \lambda_j^{-1}.$$

A first order expansion around means $\mathbb{E}[T_j]$ and $\mathbb{E}[\lambda_j]$ yields a similar expression for the benefit-level slope across skills for a given student:

$$\beta_j^{\mathrm{RF}} := \frac{\mathrm{Cov}_j(s^*, \mathrm{MRS}^*)}{\mathrm{Var}_j(s^*)} = \frac{(\rho - 1)\,\mathrm{Var}_j(\tilde{T}) - \mathrm{Var}_j(\tilde{\lambda})}{\mathrm{Var}_j(\tilde{T}) + \mathrm{Var}_j(\tilde{\lambda})}.$$

Thus, the same logic applies: if the variation in costs dominates, the student is type A, the slope is negative, and they would benefit from more well-roundedness; if the variation in benefits dominates, the student is type B, the slope is positive, and they would benefit from more specialization.

3.2 Teachers Problem (Classroom-Level Targeting)

Classroom-level lever. A teacher enters after the baseline parent problem. Each student arrives with an achievement bundle $c_0 = (c_{1,0}, c_{2,0})$ and with *cost barriers* summarized by technology shifters $\kappa = (\kappa_1, \kappa_2)$. The teacher cannot change benefits β and does not choose inputs x; instead, she can *lower effective costs* by changing κ, with the aim of maximizing students' utility.

I model teachers as choosing a single classroom environment that shifts the frontier *for all students in the class*. Let $z := (\kappa_1, \kappa_2)$ summarize the classroom-level technology (intercepts of the CET frontier). Starting from z_0, the teacher selects a class-wide change $\Delta z := z - z_0$ subject to a convex resource cost

$$\mathcal{C}(\Delta z) \ \leq \ B, \qquad \mathcal{C}(0) = 0, \ \ \nabla^2 \mathcal{C}(z) \succ 0.$$

Representative (target) student. Following the spirit of tracking/targeting models (e.g., Duflo et al. (2011)), we assume the teacher orients instruction toward a fixed percentile $\tau \in (0,1)$ of the baseline distribution.[3] Let $i(\tau)$ denote the student at percentile τ in the baseline specialization distribution (or, under symmetry, the median equals the mean). The teacher's objective is to improve the *representative* student $i(\tau)$:

$$\max_{\Delta z} \ U\big(c^*(z_0 + \Delta z;\, I_{i(\tau)}, \beta_{i(\tau)})\,;\, \beta_{i(\tau)}\big) \quad \text{s.t.} \quad \mathcal{C}(\Delta z) \leq B.$$

This delivers a simple, class-wide policy rule while making explicit which part of the distribution the teacher is targeting. (In Section 3.2 we discuss student-specific levers; when feasible, such levers are weakly welfare-improving relative to any single-environment benchmark.)

First–order characterization. Write $z := (\ln \kappa_1, \ln \kappa_2)$ and let $c^*(z; I, \beta)$ be the parental optimum on the CET. The 2×2 *Jacobian*

$$J(z; I, \beta) \ := \ \frac{\partial c^*(z; I, \beta)}{\partial z} \quad \text{with entries} \quad J_{jk} = \frac{\partial c_j^*}{\partial \ln \kappa_k}$$

measures how the *optimal* outcomes respond to small cost/productivity shifts. For small moves, $\Delta c \approx J\, \Delta z$; for larger moves, the mapping is nonlinear, but the endpoint FOCs evaluate J at the chosen $z^\star$.

The teacher chooses Δz subject to a convex *technology–space* budget $\mathcal{C}(\Delta z) \leq B$. When we use the quadratic form

$$\mathcal{C}(\Delta z) \ = \ \tfrac{1}{2}\, \Delta z^\top W\, \Delta z, \qquad W \succ 0,$$

the symmetric, positive-definite matrix W is the cost curvature in technology space. Diagonal entries encode how expensive it is to relax costs for each skill, while off-diagonals allow for spillovers/complementarities in moving both costs at once (if $W = I$, all directions in z are equally costly). It is important to distinguish W from κ_i; W captures how difficult it is for teachers to shift costs, whereas κ_i summarizes those very costs, capturing how difficult it is for parents to shift skill levels.

[3] The role of τ is to capture an instructional target level; Duflo et al. study how $x^\star$ (a target) depends on the distribution and payoff curvature. Here we impose a simple, testable benchmark in which the teacher chooses a single classroom environment aimed at a chosen percentile; we focus on $\tau = 0.5$ (the median) for symmetry and transparency.

The KKT condition for the teacher's problem,

$$\max_{\Delta z}\ U(c^*(z_0 + \Delta z; I, \beta); \beta) \quad \text{s.t.}\ \ \mathcal{C}(\Delta z) \le B,$$

is

$$\nabla_z\, U(c^*(z^\star; I, \beta); \beta)\ \propto\ \nabla \mathcal{C}(\Delta z^\star), \tag{10}$$

and, by the chain rule,

$$\nabla_z U \ =\ J(z^\star; I, \beta)^\top\, \nabla_c U(c^*(z^\star; I, \beta))\,.$$

Under quadratic costs, $\nabla \mathcal{C}(\Delta z^\star) = W\,\Delta z^\star$, so

$$\boxed{\Delta z^\star\ \propto\ W^{-1}\, J^\top\, \nabla_c U} \qquad \text{(move in } z \text{ along utility gain per technology-cost).}$$

Mapping back to outcomes,

$$\Delta c^\star\ \approx\ J\,\Delta z^\star\ \propto\ \underbrace{J\,W^{-1}\,J^\top}_{M(z^\star; I, \beta)\,\succ\, 0}\ \ \nabla_c U(c^*(z^\star; I, \beta))\,.$$

Interpretation.

- $J^\top \nabla_c U$ takes utility gradient from outcome space to technology space—i.e., which cost reductions raise utility fastest once parents re-optimize.
- W^{-1} reweights that direction by how cheap each cost move is for the teacher.
- $M := J\,W^{-1} J^\top$ is the induced metric in outcome space: it tells us which outcome directions are cheapest to deliver, given both teacher costs (W) and skill level responsiveness (J).

Hence the globally optimal change in outcomes points along the *metric-weighted normal* $M\,\nabla_c U$. If $M \propto I_2$ (all outcome directions equally costly), this collapses to the familiar IC–normal rule $\Delta c^\star \parallel \nabla_c U$. All objects above are evaluated at the chosen endpoint $z^\star$ for the given (I, β).

Benchmark: symmetric costs in outcome space. If the composite metric M is locally proportional to the identity (movements in outcome space are equally costly), the optimal class move aligns with the utility normal of the representative student:

$$\Delta c^\star\ \parallel\ \nabla_c U\big(c^*(z^\star; I_{i(\tau)}, \beta_{i(\tau)})\big)\,.$$

With $\tau = 0.5$ and a symmetric, single-peaked baseline distribution, this implies the classroom environment is chosen so that the *expansion path* for the median aligns with that student's IC-normal. In our Cobb-Douglas/CET setting, this pins down the specialization threshold $s^\dagger(\beta) = \sqrt{\beta/(1-\beta)}$ as the classroom alignment point: if the median student is of Type A (desires more well-roundedness), the teacher should tilt technology toward the weaker skill (lower its cost); if Type B, the reverse.

Belief misspecification at the classroom level. Let the teacher hold beliefs $(\hat{\beta}, \hat{z})$ about the targeted student's benefits and costs. Choosing Δz based on $(\hat{\beta}, \hat{z})$ when (β, z) are true induces a misalignment between the chosen classroom expansion path and the targeted student's IC-normal, potentially misclassifying Type A vs. B. My experiment—informing teachers about parental benefits test whether correcting beliefs reduces this misalignment for the targeted percentile and, by spillovers, for other students in the class.

Remark on individual levers. If student-specific Δz_i (or student-time allocations) are feasible, then a menu $\{\Delta z_i\}$ weakly dominates any single Δz by allowing movement along each student's own IC-normal. We treat the classroom-wide Δz as a policy *benchmark*: it is realistic when instruction and resources are common at the class level, and it gives sharp, testable directional predictions.

4 Data and Descriptive Statistics

4.1 Parent Survey Overview

The parent survey was administered on paper and took approximately 15-20 minutes to complete. The survey included perceptions of their child's abilities across nine skill dimensions, their priorities for improvement, and a rich set of parent demographics. The full survey instrument is provided in Online Appendix XX. Motivated by the model in Section 3, the key components of the survey are the elicitation of parents' ratings of their child's current abilities and their rankings of which skills are most important to improve. These measures allow us to analyze the relationship between perceived ability levels and improvement priorities, shedding light on whether observed specialization reflects production constraints or preference heterogeneity.

4.1.1 Ratings

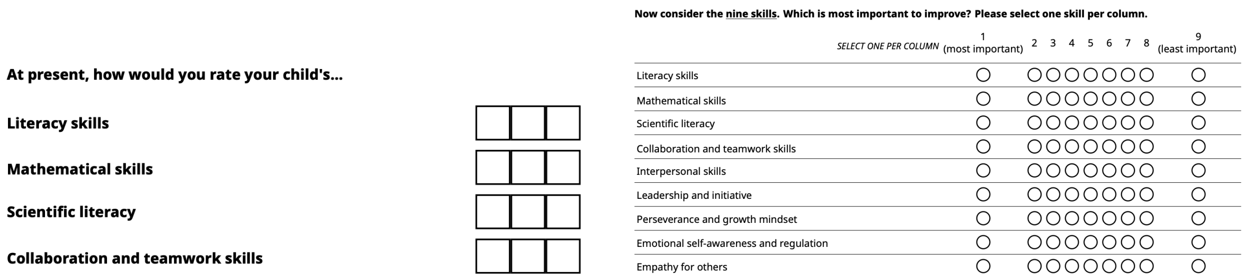
Parents were asked to rate their child's current abilities (Figure 6, Panel A) across each of the nine skill dimensions on a scale from 0 to 100, where 0 represents the "lowest level possible" and 100 represents the "highest level possible." Respondents were explicitly instructed not to use 100 unless they believed their child had no room for improvement in that skill. This measure provides a quantitative assessment of perceived current ability levels and helps identify areas where parents perceive strengths and weaknesses in their child's development.

The rating measure is comparable to those used in other studies measuring subjective assessments of abilities (Dizon-Ross, 2019; Bergman, 2021) and allows for creating standardized measures of perceptions that can be compared across dimensions and between respondents.

4.1.2 Rankings

In addition to ratings, parents were asked to rank the nine skills in order of importance for improvement, from most important (1) to least important (9) (Figure 6, Panel B). Parents also ranked the

Figure 6: Parent Survey Instrument



(a) Ratings (0–100) (b) Ranking (1-9) of marginal improvement priorities

Notes: Panel A shows the 0-100 rating interface; Panel B shows the 1-9 ranking framed as which improvement would benefit the child the most, conditional on current levels. Full wording in Online Appendix A.

three broader categories (academic, social, and emotional skills) in order of importance for improvement. This forced-choice ranking methodology captures relative valuation and marginal rates of substitution between different skill dimensions, reflecting parent preferences for improvement over the skill dimensions.

The ranking measure offers several advantages over alternative approaches. Unlike rating all dimensions simultaneously on an importance scale (which often results in top-coding effects with parents rating everything as "very important"), the forced ranking asked parents to make explicit trade-offs between different dimensions.

Parents were asked to indicate their confidence in their rankings, allowing us to identify parents who might have less certainty about what skills would benefit them the most.

4.2 Validation of measures

The elicited rankings (marginal priorities) move sensibly with family characteristics and stated aspirations. Higher-income households place relatively more weight on social skills and less on academic skills; fathers emphasize social and academic skills more and emotional skills less relative to mothers; priorities tilt toward academics (and away from emotional skills) for older children. Parents who report doctor/engineer as the desired career place markedly higher priority on academics. By contrast, priorities show no systematic relationship with child gender, birth order, or parental education. These patterns are estimated in regressions with school-by-grade fixed effects and are stable across alternative codings of the rank.

The 0-100 ratings (perceived levels) display a strong relationship with Strengths and Difficulties Questionnaire (SDQ) indices. Across all nine dimensions, higher scores on SDQ problem subscales (emotional symptoms, conduct problems, hyperactivity, peer problems) are associated with lower parental ratings by roughly 3-7 points per unit, while higher prosocial scores are associated with higher ratings by about 2-7 points; associations are largest for self-awareness, perseverance, leadership, teamwork, and empathy, and are statistically precise. These relationships remain when adding school-by-grade fixed effects and are qualitatively unchanged if we replace raw ratings with within-child standardized ratings. We report raw-scale estimates here for interpretability, but all main benefit-level slope analyses use within-child z scores, as the focus of the theory is about specialization (i.e., comparative advantage), and not about absolute level differences. Taken together with the negative within-child association between priority ranks and levels, these patterns indicate that both ratings and rankings capture meaningful variation in parental perceptions and marginal valuations rather than noise.

5 Benefit–Level Slopes

Following Section 3, I define our empirical analog to the benefit–level slope as the slope from regressing the priority rank assigned by parents on within-student standardized levels. We estimate two versions: (i) *within a dimension across students* (e.g., math level on math priority rank) and (ii) *within a student across dimensions* (regressing the vector of nine ranks on the nine standardized levels for a given student).

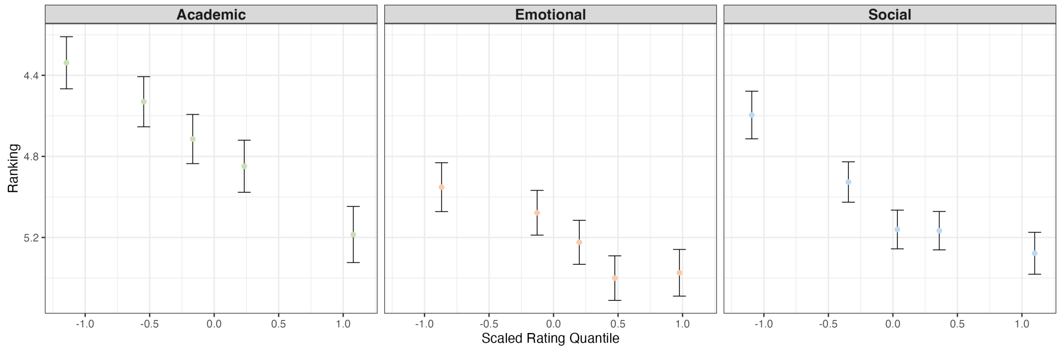
5.1 Within-Dimension Slopes (across students)

Figure 7 bins standardized levels into quintiles and plots the mean priority rank within each bin (lower rank = higher priority). The relationship is monotone and downward for all three categories—academic, social, and emotional—consistent with parents placing higher marginal value on skills in which the child is weaker. In pooled regressions across all dimensions, the slope is negative and precisely estimated: regressing $-\text{rank}_{idk}$ on the within-child standardized level yields a coefficient of -0.40 (s.e. 0.024) with standard errors clustered by student and classroom teacher. This implies that a one-standard-deviation increase in the child's level is associated with a 0.46 decrease in the priority rank (less important).

Allowing the slope to vary by dimension shows that it is *negative for every skill* and that the slopes vary by dimension. The magnitude is by far the strongest in mathematics, yet still substantial for other dimensions, and closer to zero for perseverance and science.

Aggregating to three categories gives an average negative slope of -0.42 (s.e. 0.037). Slopes vary by category: -0.48 (s.e. 0.050) for academic skills, -0.44 (s.e. 0.047) for social skills, and -0.35 (s.e. 0.053) for emotional skills.

Figure 7: Average parent priority rank within quintiles of the standardized rating, by category



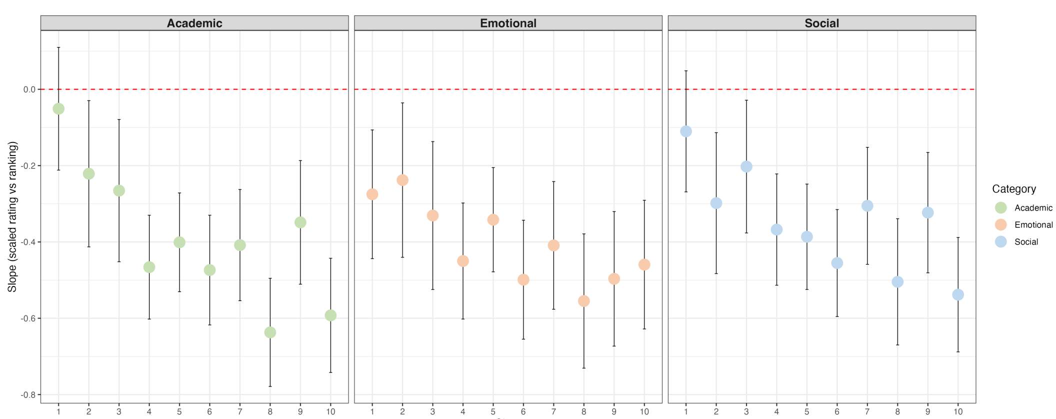
Notes: Higher values on the y-axis indicate higher parental priority (rank 1 = most important). Points show quintile means; bars show 95% confidence intervals.

Through the lens of the model, these patterns imply that across students, variation in who specializes in a given skill is driven by differences in production constraints as opposed to differences in real or perceived benefits. I find that this is more true for academic skills than for social or emotional skills, consistent with either (i) production constraints being relatively more dominant for academic skills, or (ii) variation in benefits being more dominant for social and emotional skills.

5.2 Heterogeneity by grade

Next, we examine whether the benefit-level slope (BLS) varies by grade. We estimate the BLS separately for each grade and plot the results in Figure 8. The BLS is negative in every grade, yet indistinguishable from 0 for classes 1 and 2, and increasingly negative reaching between -0.5 to -0.7 for grades 9-10. This pattern suggests that the BLS becomes more pronounced as children progress through school, potentially reflecting that for younger students, specialization is relatively driven by preferences or perceived benefits, while for older students, it is increasingly driven by production constraints.

Figure 8: Benefit-level slope by grade



Notes: Each point represents the benefit-level slope estimated separately for each grade, with 95% confidence intervals.

5.3 Within-Student Slopes (across dimensions)

I now turn to the within-student benefit-level slopes, which measure how a child's skill levels relate to their priority ranks across dimensions. We estimate within-student BLSs by regressing each parent's ranks on standardized levels. The average within-student slope is *negative* (-0.43), with wide dispersion across students (SD = 1.25). This indicates that within a given child, skills that parents prioritize more tend to be those where the child is perceived to be weaker, consistent with specialization driven by production constraints rather than by differences in benefits.

I relate this slope to parental satisfaction to gauge whether observed specialization aligns with perceived benefits. Regressing the student-level slope on parents' satisfaction with their child's *progress* (reference category: "completely satisfied") yields sizable and precise declines for less-satisfied parents: "somewhat satisfied" is lower by 0.149 (s.e. 0.046) and "somewhat dissatisfied" by 0.214 (s.e. 0.101); the "completely dissatisfied" category is small and imprecise (-0.049, s.e. 0.161). By contrast, the slope is essentially uncorrelated with satisfaction with the *school* ("somewhat satisfied": -0.042; "somewhat dissatisfied": -0.105; "completely dissatisfied": 0.025; all imprecise; $N = 3{,}036$).

In the model, more positive within-student slopes indicate that the child's current mix of skills aligns more closely with the parent's marginal benefits-what they most want improved. Geometrically, this is what we expect when the child's natural expansion path is closer to the indifference-curve normal, so incremental learning goes in a direction that raises utility fastest. Practically, parents report higher satisfaction with their child's *progress* when observed specialization lines up with what they value, while the lack of correlation with satisfaction about the *school* suggests the slope captures a dynamic growth alignment rather than static school quality. These patterns reinforce the cost-side reading of the average negative BLS in this setting and motivates the information experiment that follows.

6 Information Experiment

The cross-sectional evidence points to cost-side forces as a central driver of specialization. If so, a production-side shock that helps teachers target instruction where parents place the highest marginal value should (i) reduce cross-dimensional cost dispersion within a classroom and (ii) shift students' portfolios toward parent-prioritized skills. We test these predictions in a teacher-facing information experiment.

6.1 Teacher Survey

The teacher survey was administered at baseline and endline using Qualtrics. It collected comprehensive information about teachers' professional backgrounds, pedagogical philosophies, and most importantly, their rankings of skill development priorities for both a typical student, and for specific individual students in their class.

6.1.1 Teacher Demographics and Professional Background

The survey collected standard demographic and professional information including gender, education level, years of teaching experience, and tenure at the current school. This baseline information allows analysis of how teacher characteristics influence their beliefs about student development priorities and their responsiveness to information about parent preferences.

6.1.2 Teaching Philosophy and Self-Efficacy

To understand the context within which teachers make instructional decisions, the survey measured teachers' views on their professional responsibilities and their self-efficacy across different teaching domains.

Teachers rated the importance of various responsibilities (e.g., improving student academic achievement, incorporating parent priorities, managing student behavior) on a five-point scale from "Not important" to "Extremely important." This provides insight into how teachers conceptualize their role and the relative weight they place on different aspects of their job.

A validated teacher self-efficacy scale was also included, with items such as "How much can you do to control disruptive behavior in the classroom?" and "How much can you do to motivate students who show low interest in school?" Teachers responded on a nine-point scale ranging from "Nothing (1)" to "A great deal (9)." This scale measures teachers' beliefs about their ability to influence various student outcomes, which may mediate teachers' responsiveness to information about parent preferences.

6.1.3 Rankings for Students

The core component of the teacher survey paralleled the parent ranking questions, but with three distinct variations designed to assess alignment between teachers and parents, and to measure how teachers form beliefs about parent preferences. First, teachers ranked the nine skill dimensions for a **typical student** they encounter in their teaching. This provides a baseline measure of teachers' general pedagogical priorities that can be compared against the distribution of parent priorities across all students in their class. Second, for **six specific students** selected through stratified random sampling (two each from families prioritizing academic, social, and emotional skills), teachers provided rankings based on their own professional judgment about what each individual student needed most. This captures teachers' own assessment of individual student needs. In cases where fewer than six students had parent survey data, all available students were included. Third, and most revealing, teachers predicted how they believed **each student's parent would rank** the skill development priorities. This measure enables direct comparison between teachers' beliefs about parent preferences and parents' actual stated preferences.

This three-way design reveals several striking patterns evident in the baseline data. Teachers demonstrate remarkably poor calibration about individual parent preferences - the correlation between teacher beliefs about parent rankings and actual parent rankings is close to zero. More systematically, teachers appear to project their own professional priorities onto parents, particularly when they have limited direct communication with families. This projection bias is strongest for teachers who report infrequent parent-teacher interactions, suggesting that information frictions may be substantial even in a setting with institutionalized parent-teacher meetings.

The teacher ranking data also shows that while parents tend to prioritize academic skills most highly, teachers' own professional priorities tilt more toward social and emotional development. This preference divergence, combined with teachers' poor beliefs about parent preferences, creates substantial scope for misalignment between classroom instruction and family priorities.

This baseline misalignment provides the foundation for testing whether structured information provision can improve teacher-parent alignment and ultimately shift student outcomes. The experimental design leverages the fact that teachers have systematically biased beliefs about parent preferences, making information provision potentially valuable even in a context where formal communication channels already exist between teachers and families.

6.1.4 Pedagogical Strategies for Skill Development

In addition to uncertainty over parent preferences, teachers also face uncertainty about how to effectively develop the skills they prioritize. Therefore, the teacher survey included a component designed to elicit teachers' beliefs about effective pedagogical strategies for developing each of the nine skill dimensions. This component was informed by the literature on social and emotional learning (SEL) and evidence-based teaching practices.

For each of the six social and emotional skill dimensions, teachers were presented with four evidence-based teaching strategies, and were then asked to rank these strategies from most effective (1) to least effective (4) based on their professional judgment. A full list of the strategies presented is provided in Online Appendix XX.

This component of the survey enabled the creation of a second treatment in which teachers receive an aggregated report on the effectiveness of pedagogical strategies based on the collective responses of teachers within their school. As described in the experimental design section, this second treatment was cross-randomized with the first treatment, which provided teachers with information about parent ratings and rankings.

6.2 Experimental Design

The experimental design addresses two key questions: (1) Do teachers update their beliefs and behaviors when provided with information about parent preferences? and (2) Does supplementing this preference information with concrete pedagogical strategies enhance teacher responsiveness? To answer these questions while minimizing spillover concerns, I employ a two-stage randomization design at the school-grade and teacher levels:

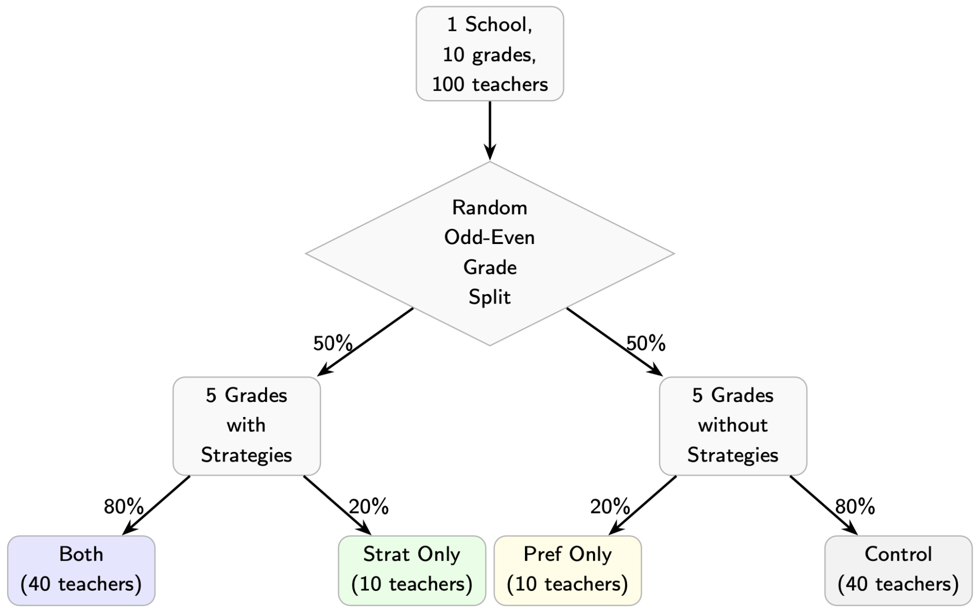


Figure 9: Treatment Assignment

6.2.1 First Stage: Grade-Level Randomization

The first level of randomization occurs at the grade level within each school. Grades are randomly assigned to either receive or not receive pedagogical strategies for developing different skill dimensions. This design choice is motivated by the hypothesis that teachers primarily communicate with other teachers in their grade at the same school, making grade-level assignment a natural boundary to prevent spillover.

To further ensure the Stable Unit Treatment Value Assumption (SUTVA) holds, I implement an odd/even grade split, where odd-numbered grades (1, 3, 5, 7, 9) in a school are assigned to one condition and even-numbered grades (2, 4, 6, 8, 10) to the other. This spatial separation minimizes the risk of treatment contamination, as teachers in adjacent grades typically have fewer opportunities for professional interaction than teachers within the same grade.

6.2.2 Second Stage: Teacher-Level Randomization

Within each grade, I then randomize at the teacher level whether teachers receive information about parent preferences. This information includes aggregated data on how parents of students in their specific class rate their children's current abilities and rank the importance of improving each dimension.

To maximize statistical power for the primary comparison (receiving both treatments versus receiving neither), I employ an unbalanced randomization:

- In grades assigned to receive strategies: 80% of teachers are randomly selected to also receive parent information, while 20% receive only strategies

- In grades assigned not to receive strategies: 20% of teachers are randomly selected to receive parent information, while 80% receive neither treatment

This allocation results in approximately 40% of teachers receiving both treatments, 40% receiving neither, and 10% receiving each treatment alone, providing optimal power for the main comparison while still allowing for tests of individual treatment effects.

6.3 Implementation Through Web Portal

The intervention is delivered through a custom-built web portal that provides teachers with access to treatment information in an intuitive, user-friendly format. To encourage proper implementation, we conducted demonstration sessions with all teachers, illustrating how the information tool could be used to understand parent preferences and potential ways to incorporate these insights into their teaching approaches. These sessions focused on navigating the website, interpreting the visualizations of parent preferences, and connecting these preferences to pedagogical strategies. School administrators were appointed as points of contact to address any questions or challenges teachers might encounter while using the tool.

6.3.1 Access and Authentication

Each teacher receives individual login credentials via email, with access restricted to their assigned treatment condition. The portal uses secure authentication to ensure teachers can only view information relevant to their treatment status. This individualized access allows for precise tracking of teacher engagement with the intervention, including login frequency, pages viewed, and time spent on different components of the portal.

6.3.2 Interface and Content

The portal presents information through several organized tabs:

1. **Welcome**: Provides an introduction to the portal and navigation instructions, with content tailored to the teacher's treatment assignment
2. **Most Important Skills**: For teachers in the parent information treatment, this tab displays which skill categories (academic, social, or emotional) each student's parent identified as most important for improvement, along with the parent's rating of their child's current ability in that area
3. **Student Reports**: Provides detailed student-level information on parent ratings and rankings for all nine skill dimensions, including both tabular displays and interactive visualizations
4. **Classroom Report**: Presents aggregated data on parent preferences across all students in the class, helping teachers identify common priorities

5. **Strategies for Improvement**: For teachers in the strategies treatment, this tab offers research-based pedagogical approaches for developing each non-cognitive skill, prioritized according to effectiveness ratings from other teachers at their school

According to **Bernadette Walsh**'s parent, the most important skill for them to improve is **collaboration and teamwork skills**.

Bernadette Walsh

Parent ratings: Scored from 0-100.
Ranking: How valuable it is to improve for this student. Lower rank = more important

Category	Ability	Parent rating	Ranking
Social	Collaboration and teamwork skills	33	1
Emotional	Empathy for others	77	2
Academic	Literacy skills	53	3
Emotional	Emotional self-awareness and regulation	31	4
Social	Leadership and initiative	39	5
Emotional	Perseverance and growth mindset	71	6
Academic	Mathematical skills	82	7
Academic	Scientific literacy	57	8
Social	Interpersonal skills	56	9

Figure 10: Example Student Report

The portal's design facilitates easy interpretation of parent preferences through color-coded visualizations and clear tabular formats. For instance, skill dimensions are consistently color-coded by category (green for academic, orange for emotional, blue for social), and ratings employ a red-yellow-green color scale to highlight areas of strength and opportunity. Rankings are presented as ordered lists to clearly communicate relative priorities.

Strategies for improving **Interpersonal skills**

Strategy	Average ranking	Share teachers ranked #1 (most effective)	Description
Classroom circles	2.3	35 %	In classroom circles, students and teachers form a physical circle for open discussions about a variety of topics including personal feelings, conflict resolution, or community building, aimed at fostering a supportive environment.
Role-playing exercises	2.3	29 %	Students take on roles in predefined scenarios, acting out interactions to explore interpersonal dynamics and learn through feedback. Scenarios can include various contexts from conflict resolution to collaborative tasks.
Service-Learning Projects	2.7	20 %	Involve students in group-based projects that address community needs, linking classroom objectives with real-world applications. Tasks include planning, executing, and reflecting on their projects.
Think-Pair-Share	2.7	16 %	Teachers pose a question. Students think independently, then pair up to discuss with a partner, providing feedback on each other's work. Finally, they share with the class, encouraging participation and collaborative problem-solving.

Figure 11: Example Strategies Report

In the strategies section, teachers can access specific, actionable pedagogical approaches for each non-cognitive skill. These strategies include brief descriptions and implementation guidance, along with indicators of how commonly other teachers ranked each strategy as most effective.

6.4 Treatment Implementation and Compliance

The information intervention was implemented through a custom web platform that provided teachers with individualized login credentials and access to their students' parent-reported skill levels and priority rankings. The platform tracked comprehensive usage data including login frequency, session duration, and specific content accessed.

Implementation proceeded differently across the five participating schools. While 242 teachers were initially assigned to treatment across all schools, practical challenges emerged that significantly affected compliance and data collection. Two schools withdrew from the study entirely before endline data collection. Two additional schools either failed to distribute login credentials to their teachers or achieved login rates below 15%.

The main results presented here focus on the one school that successfully implemented the intervention - the largest school in the sample with 100 classrooms. Even in this successful implementation, compliance was limited: only 46% of treated teachers logged into the platform at least once during the intervention period. Among those who did access the platform, usage varied substantially, with some teachers making single brief visits while others engaged more systematically with the content.

Interestingly, the school's administration responded to the baseline parent survey results by implementing school-wide assemblies and developing lesson plans aimed at addressing the skill priorities identified in the parent data. This represents a form of treatment spillover that likely affected both treatment and control teachers within the school, potentially attenuating measured treatment effects while demonstrating the policy relevance of the parent preference information.

The low compliance rate and administrative response highlight important features of information interventions in educational settings. Teachers face multiple competing demands on their attention, and information provision alone may be insufficient without accompanying incentives or administrative support. Nevertheless, the experimental results suggest that even partial compliance can generate meaningful changes in student outcomes, consistent with the hypothesis that information about parent priorities can serve as a low-cost production shock that reallocates instructional effort toward high-value dimensions.

7 Teacher Beliefs and Updating

Before turning to treatment effects on student outcomes, I document two facts about teacher beliefs at baseline, and test whether the information intervention improved belief accuracy.

Baseline misalignment. Teachers were asked, for a random subset of students, to predict how that student's parent would rank the priority of improving *academic*, *social*, and *emotional* skills.[4] Figure 12 plots, for each classroom, the *average* parent rank (x-axis) against the *average* teacher belief about parents (y-axis) for the sampled students. At baseline, beliefs are essentially *uncorrelated* with truth: points scatter around a flat line. In other words, even in these relatively well-resourced schools with routine parent-teacher meetings, teachers had little signal about what parents most wanted improved.

Where do baseline beliefs come from? Projection. To probe how teachers form beliefs when they lack signal, Figure 13 keeps the same y-axis (average teacher belief about parents) but replaces the x-axis with the *teacher's own* ranking (averaged across the same students). A strong positive relationship emerges: teachers *project* their own priorities onto parents. This is consistent with the model's misperception channel: in the absence of reliable information on parents' marginal valuations, teachers substitute their own ranks for the parent's, creating scope for misallocation even when classroom effort is intended to reflect parental priorities.

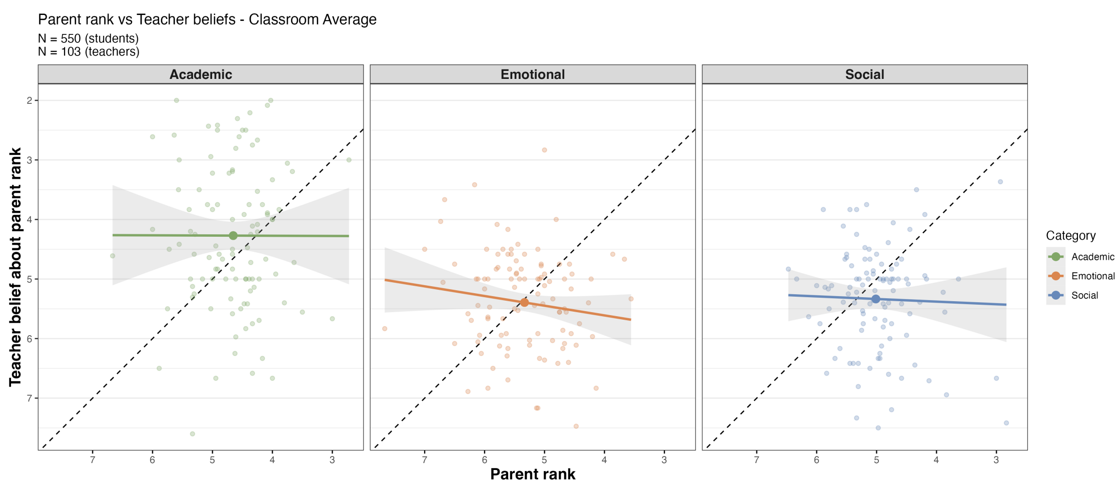


Figure 12: Baseline accuracy of teacher beliefs about parents' priorities (classroom averages). Notes: Each dot is a classroom; x-axis = average parent rank; y-axis = average teacher belief about parent rank for the same students.

[4] Parents reported ranks allowing ties (e.g., 1–2–2); teacher elicitation mirrored this.

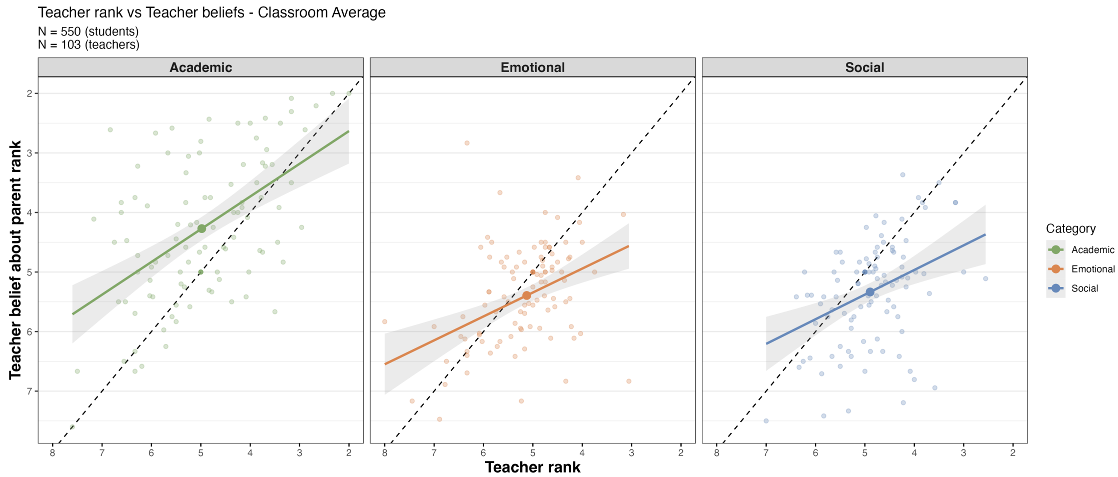


Figure 13: Projection: teacher beliefs vs. teacher's own priorities (classroom averages). Notes: y-axis as in Fig. 12; x-axis = teacher's own average rank.

Measuring accuracy. I operationalize accuracy with four binary outcomes: (i) *Exact Order Match* (teacher's full ranking equals the parent's, ties allowed); (ii) *Top Skill Match* (teacher includes at least one of the parent's top-ranked categories among the top); (iii) *Bottom Skill Match* (analogous for the bottom); and (iv) *Top & Bottom Match* (both ends correct). I study accuracy at two levels: (a) *student-level*—does the teacher correctly predict an individual parent's ranking?—and (b) *classroom average*—does the teacher correctly identify the *average* parent's priorities in her class?

Does information correct beliefs? First-stage evidence. Tables 1–3 report intent-to-treat effects of providing teachers the parent rankings and levels via the website (Section 6). The analysis is underpowered due to implementation constraints ($N = 94$ teachers), so I emphasize direction and magnitude.

Student-level accuracy. At the individual student level, point estimates are indistinguishable from zero and, if anything, slightly negative (Table 1). For example, the ITT on *Top Skill Match* is -0.0165 (s.e. 0.0675), with similar nulls for other outcomes. There is strong persistence in accuracy (baseline-to-endline), but no detectable treatment effect. This suggests that even with information, teachers did not reliably memorize or track *each* parent's unique ranking for the sampled students.

Classroom-average accuracy. By contrast, when I aggregate to the classroom mean ranking ("what does the *average* parent in my class want?"), treatment effects turn *positive* and economically meaningful, though imprecise (Table 2). ITT estimates are on the order of 9–10 percentage points for *Top Skill Match* and *Bottom Skill Match* (e.g., +0.0918 and +0.1017), with similar magnitude for *Top & Bottom Match* (+0.1028). While not statistically significant, these effect sizes are large relative to control means (0.27–0.46), consistent with teachers updating their mental model of the *typical* parent in the class.

Heterogeneity by what parents value most. Table 3 splits classrooms by which category parents most often rank top. Patterns are intuitive: (i) in classrooms where *academics* is the modal top priority, teachers improve in pinning down the *top* category (ITT on *Exact Order Match* +0.199; *Top Skill Match* +0.145); (ii) where *social* or *emotional* skills are modal, teachers become more accurate about the *bottom* category (ITT on *Bottom Skill Match* +0.185 for social; +0.115 for emotional). A natural reading is that information helped teachers correctly locate *where academics sits* in the average parent's priority ordering—on top in some classes, lower in others.

At baseline, teacher beliefs about parent priorities were largely noise and heavily colored by projection. The intervention did not make teachers precise about each parent, but it nudged them toward a more accurate picture of the average parent in their class, especially about the relative place of academics. In the model, this matters because teacher policy is a classroom-level cost tilt: getting the *class-average* benefits right moves the frontier in the direction that better aligns growth with what families value, even if idiosyncratic values are harder to track.

Table 1: Treatment Effects on Teacher Accuracy about specific **Parents**

Dependent Variables:	Exact Order Match (1)	Top Skill Match (2)	Bottom Skill Match (3)	Top & Bottom Match (4)
Variables				
Treated	-0.0111	-0.0165	-0.0194	-0.0267
	(0.0291)	(0.0675)	(0.0774)	(0.0774)
Baseline Accuracy	0.2638***	0.2845***	0.2013***	0.2746***
	(0.0662)	(0.0552)	(0.0546)	(0.0540)
Statistics				
Observations	482	482	482	482
R^2	0.07583	0.13548	0.07613	0.12501
Control Mean	0.1292	0.6667	0.6333	0.5542

Notes: Outcomes are parent-level indicators for whether a teacher's belief matches a given parents' ranking of skills (academic, emotional, social). Grade fixed effects included in all specifications. *Exact Order Match* = 1 if the teacher's full ranking matches the parents' ranking, including ties (e.g., parent 1–2–2 requires teacher 1–2–2). *Top Skill Match* = 1 if the teacher identifies at least one of the parent's top-ranked skills as top. *Bottom Skill Match* = 1 if the teacher identifies at least one of the parent's bottom-ranked skills as bottom. *Top & Bottom Match* = 1 if both top and bottom categories are correctly identified. "Treated" indicates teachers who received information about parent rankings and levels. "Baseline accuracy (same outcome)" is the coefficient on the corresponding baseline measure of that outcome.
Standard errors in parentheses, clustered at the class level
*Significance Codes: ***: 0.01, **: 0.05, *: 0.1*

8 Student Outcome Results

We focus on three outcomes that map directly to the model:

(1) Parent's top category improves in standardized levels. Column 1 of Figure 14 shows that the average intent-to-treat (ITT) effect on parents' standardized ratings is close to zero. However, this masks important heterogeneity. Column 2 plots the interaction between treatment and

Table 2: Treatment Effects on Teacher Accuracy about **Classroom Average**

Dependent Variables:	Exact Order Match (1)	Top Skill Match (2)	Bottom Skill Match (3)	Top & Bottom Match (4)
Variables				
Treated	0.0351	0.0918	0.1017	0.1028
	(0.0988)	(0.1452)	(0.1376)	(0.1224)
Baseline accuracy	0.3613***	0.2448**	0.2439**	0.3019***
	(0.1283)	(0.1134)	(0.1034)	(0.1071)
Statistics				
Observations	94	94	94	94
R^2	0.17440	0.09531	0.18254	0.15772
Control Mean	0.1450	0.4618	0.4122	0.2710

Notes: Dependent variables are indicators for the accuracy of teacher beliefs about parents' skill rankings (academic, emotional, social). Grade fixed effects included in all specifications. *Exact Order Match* = 1 if the teacher's full ranking matches the parents' ranking, including ties (e.g., parent ranking 1-2-2 requires teacher belief 1-2-2). *Top Skill Match* = 1 if the teacher identifies at least one of the parent's top-ranked skills as top. *Bottom Skill Match* = 1 if the teacher identifies at least one of the parent's bottom-ranked skills as bottom. *Top & Bottom Match* = 1 if both top and bottom categories are correctly identified. The row "Baseline accuracy" includes the coefficient on the corresponding baseline measure of that outcome. "Treated" is an indicator for whether the teacher received information about parent rankings and levels.
Clustered standard errors in parentheses
*Significance Codes: ***: 0.01, **: 0.05, *: 0.1*

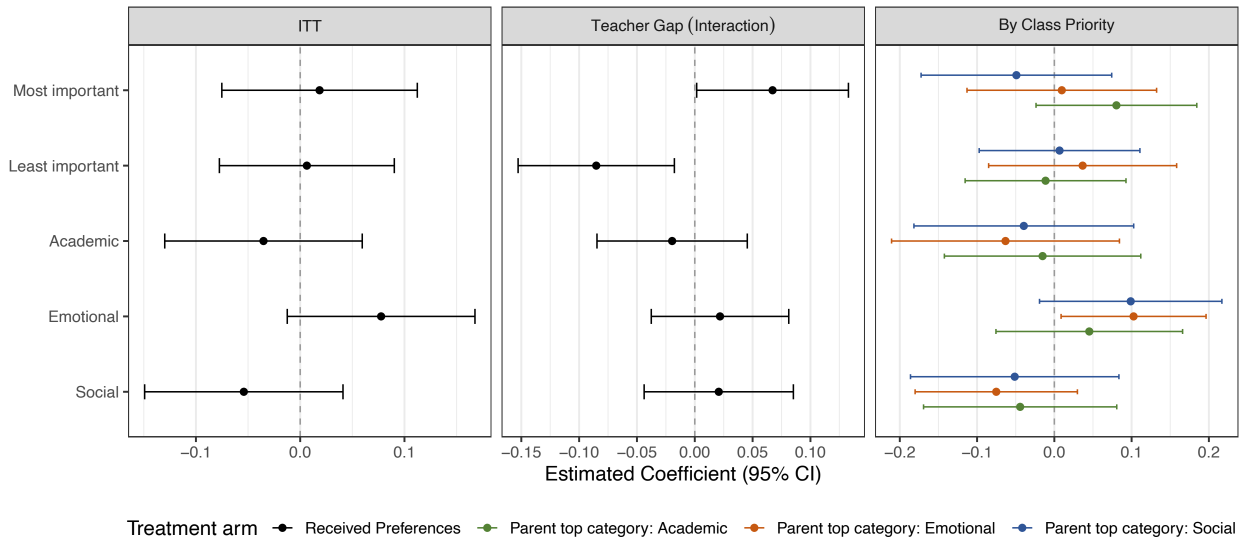
the baseline gap between teachers' beliefs and parents true rank. The coefficient on the interaction is +0.07 (s.e. 0.03) for the category parents rated most important and -0.09 (s.e. 0.03) for the least important. This implies the conditional effects: when the teacher's initial ranking was exactly correct (gap = 0), the treatment effect was slightly negative for the most important category (-0.25, s.e. 0.14) and positive for the least important (0.30, s.e. 0.12). Gaps are typically 2-3 ranks (median = 2.5, mean ≈2.8), so for an average student the implied effect flips sign—-near zero to moderately positive the most important category and near zero or negative for the least important. While for students where teachers were most incorrect, the effect is significantly positive for the most important category, and negative for the least important. In other words, teachers moved most when they had been wrong. This fits a classroom-level production model—teachers initially shaped the class for the students they thought they knew; once they saw the true distribution of parent priorities, they rebalanced the classroom toward previously misaligned students, sometimes at the expense of those they had matched. Column 3 then splits effects by which category parents, on average, valued most. Here, coefficients are larger when parents prioritized academics (e.g., 0.08, s.e. 0.05) than when they prioritized social or emotional skills. This likely reflects that teachers can more readily reallocate effort in academics, that our updating analysis showed they learned more precisely where academics ranked, and that schools were already running universal social-emotional programming, making new information about academic preferences especially salient.

Table 3: Treatment Effects on Teacher Accuracy about **Classroom Average** by Parent Top Priority

Dependent Variables:	Exact Order Match	Top Skill Match	Bottom Skill Match	Top & Bottom Match
	(1)	(2)	(3)	(4)
Variables				
Treated: Parents' Top = Academic	0.1987	0.1453	-0.0025	0.0639
	(0.1239)	(0.1820)	(0.1693)	(0.1585)
Treated: Parents' Top = Social	-0.0878	0.0884	0.1846	0.1196
	(0.1056)	(0.1733)	(0.1673)	(0.1455)
Treated: Parents' Top = Emotional	-0.0396	-0.0503	0.1150	0.1608
	(0.1020)	(0.2385)	(0.2263)	(0.2149)
Baseline accuracy	0.3097**	0.2100*	0.2666**	0.3152***
	(0.1223)	(0.1234)	(0.1032)	(0.1094)
Statistics				
Observations	94	94	94	94
R^2	0.22938	0.10416	0.19377	0.16074
Control Mean	0.1450	0.4618	0.4122	0.2710

Notes: Dependent variables are indicators for the accuracy of teacher beliefs about the class average of parents' skill rankings (academic, emotional, social). Grade fixed effects included in all specifications. *Exact Order Match* = 1 if the teacher's full ranking matches the class average ranking, including ties (e.g., parents 1–2–2 requires teacher 1–2–2). *Top Skill Match* = 1 if the teacher identifies at least one of the parent's top-ranked skills as top. *Bottom Skill Match* = 1 if the teacher identifies at least one of the parent's bottom-ranked skills as bottom. *Top & Bottom Match* = 1 if both top and bottom categories are correctly identified. Rows labeled "Info: Parents' Top = {Academic, Social, Emotional}" are indicators for classrooms where the teacher received information about parent rankings and levels, estimated separately by the parents' top-ranked category. Coefficients are intent-to-treat effects relative to control classrooms in the same category. "Baseline accuracy" is the coefficient on the corresponding baseline measure of the same outcome.
Clustered standard errors in parentheses
*Significance Codes: ***: 0.01, **: 0.05, *: 0.1*

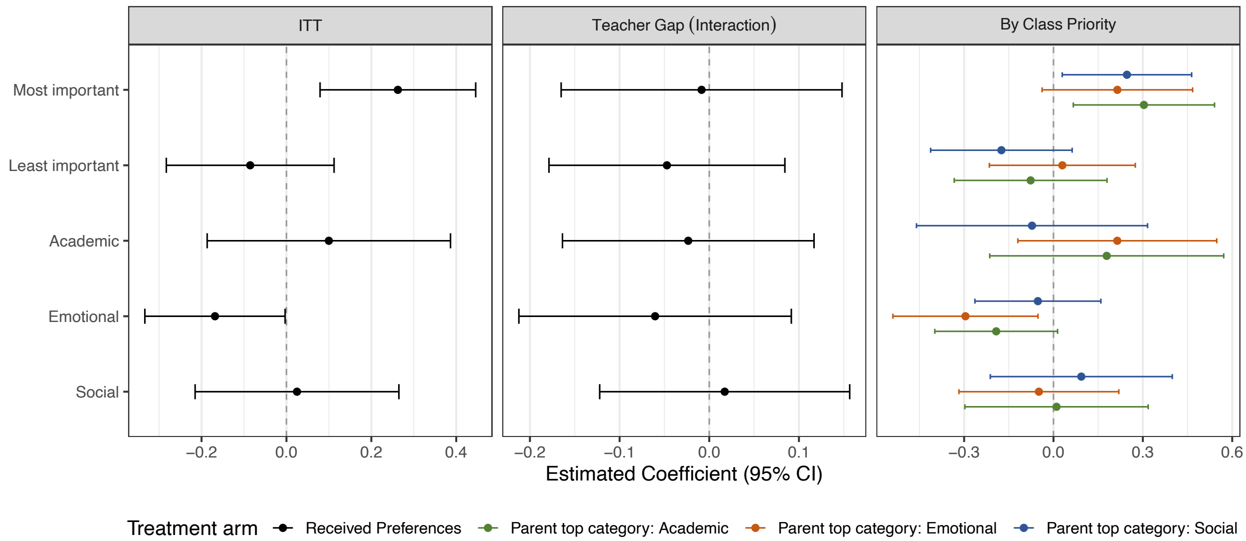
Figure 14: Treatment effects on standardized levels



Notes: Estimates control for grade fixed effects; standard errors clustered at the classroom level.

(2) Top ranked skills become less important to improve. Column 1 of Figure 15 shows a clear average intent-to-treat (ITT) effect on parents' endline rankings: in treated classrooms the skill parents had prioritized moved about 0.26 ranks lower (s.e. 0.09) on average. Because a higher rank means "less important to improve," this pattern is consistent with teachers, once informed of parent priorities, making it easier for students to develop those skills, and parents subsequently judging them less in need of work. Column 2 plots treatment interacted with the baseline teacher-parent gap; the interaction coefficients are small and imprecise (-0.01 to -0.05), indicating that—unlike the ratings outcome—parents' shift in rank orderings does not vary systematically with how wrong teachers were at baseline. Column 3 splits treated classrooms by the category parents valued most, on average. The average shift is largest when parents' top priority was academics (+0.30, s.e. 0.12), somewhat smaller for social (+0.25, s.e. 0.11), and positive but only marginally significant for emotional (+0.21, s.e. 0.13), suggesting that teachers most effectively re-oriented classroom effort when the desired skills were academic, where they have more control and, as our updating analysis showed, learned most clearly how academics compared to other skills.

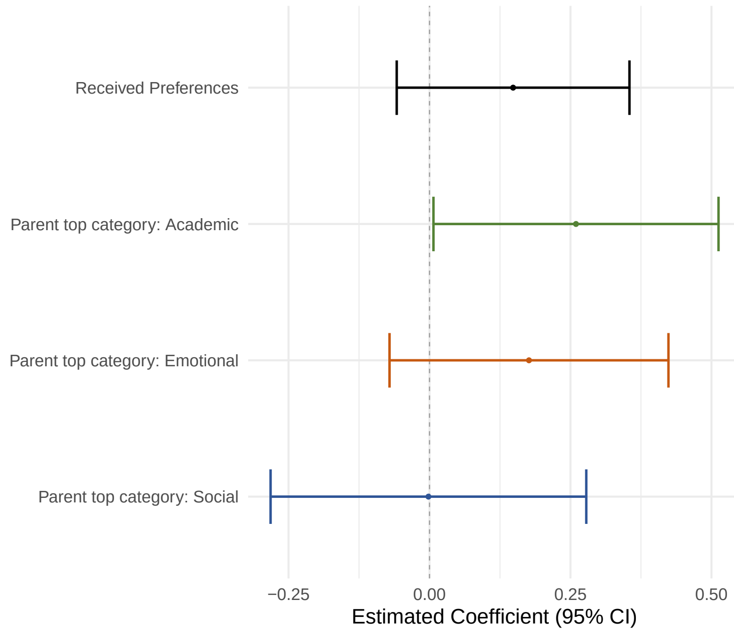
Figure 15: Treatment effects on ranks



Notes: Negative coefficient (lower rank) indicates more important. Estimates control for grade fixed effects; standard errors clustered at the classroom level.

(3) BLS shifts upward for treated students. Recall the benefit-level slope is estimated by regressing each parent's ranking of skills on their child's standardized skill levels; a more positive slope means the child has higher levels in the skills the parent values, rather than just excelling in skills that were easier to produce. Figure **??** shows that the average intent-to-treat (ITT) effect on this slope is positive though only marginally precise: +0.15 (s.e. 0.11). Splitting treatment by the category parents valued most shows the effect is largest and statistically significant in academic-priority classrooms (+0.26, s.e. 0.13), essentially zero for social (-0.00, s.e. 0.14), and positive but imprecise for emotional (+0.18, s.e. 0.13). Together with the ratings and rankings results, this pattern suggests that when teachers learn parents' priorities—especially academics—they reconfigure the classroom so it is easier for students to build the skills parents value, making students' observed skill profiles better aligned with parental preferences rather than with underlying production cost constraints.

Figure 16: Treatment effect on benefit-level slope



Notes: Estimates control for grade fixed effects; standard errors clustered at the classroom level.

Taken together, these findings fit the framework: information to teachers acts as a production shock that selectively lowers the effective cost of building parent-prioritized skills. By reshaping the classroom environment rather than only tailoring instruction to individual students, teachers enable more progress where parents want gains (rating improvements for prioritized categories), shift students' overall skill profiles toward parental preferences (upward benefit-level slope), and reduce parents' perceived need for further improvement in those areas (downward rank movement). In short, providing teachers with clear signals of parental marginal valuations moves the whole classroom's skill production away from what is merely easiest to build and toward what families value most.

9 Model Estimation

This section is still in progress, and will detail the structural model estimation, using the teacher rankings for typical students as exogenous shocks to production (students are plausibly randomly assigned to teachers within a grade) as the basis for estimating the parameters of the model.

10 Conclusion

This paper develops and tests a simple but powerful idea: observed skill profiles may reflect either production costs—some skills are harder to build—or benefits—some skills are less valued. Distinguishing these forces matters for policy. If weaknesses mainly reflect high costs, the right lever is to shift the production frontier and make hard skills easier to learn; if they reflect low benefits, the lever is to change incentives or perceived payoffs. I show that a single statistic—the *benefit-level slope*, the relationship between a child's current skill levels and the marginal value parents assign to them—can help diagnose which force dominates.

I find in the context of Indian private schools, the slope is negative on average: parents usually want gains where their child is weakest, implying that production constraints rather than low benefits drive most observed specialization. Teachers, however, begin with beliefs about parent priorities that are largely uncorrelated with truth, instead mirror their own preferences.

A classroom-level randomized experiment shows that providing teachers with simple, structured information on parent-reported priorities acts as a production shock. Treated teachers become more accurate about average class priorities (especially where academics rank) and adjust the classroom environment, not just individual targeting. Students in treated classes gain more in the skills their parents value, parents then report lower need for improvement in those areas (rank moves), and benefit-level slopes tilt upward—indicating that skill portfolios move away from what is easiest to produce and toward what families value most. Effects are strongest when average parental priority is academic, consistent with teachers having more scope to shift costs and effort in that domain.

These results offer a tractable way to diagnose and act on the "costs versus benefits" question in skill formation. Simple low-cost measures of skill levels and parental priorities can help school leaders decide whether to invest in lowering production costs or in changing incentives. They also show that teachers' most powerful lever is classroom-level cost shifting: with better information about what families care about, they can redesign the shared learning environment to deliver value beyond what is easiest to teach.

Important limitations remain. The experiment took place in one country and school sector; testing whether similar dynamics hold across other systems and grade levels is crucial. We also do not directly observe what teachers change—curriculum emphasis, grouping, feedback, or classroom routines—and schools themselves may have complementary levers, such as planning tools or resource supports once priorities are known. Finally, mapping which skills are genuinely easier or harder for teachers to improve would give a more complete picture of production constraints and guide better targeting.

Even with these caveats, the findings point to a promising path: education can be made more personally relevant by combining simple preference information with teachers' ability to shape learning costs at the classroom level. Rather than focusing only on measured achievement, schools could better reflect what families value and align instruction with both skills and priorities, helping translate learning into higher long-run welfare.

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A Sample Details

Table 4: Descriptive Statistics of Parent Survey Sample

Demographic	Group	Full Sample (N=3404)	Main School (N=1460)	Completed Baseline and Endline (N=849)
Child Gender	Male	53.4	52.9	53.0
	Female	46.6	47.1	47.0
Child Age	4-8 years	26.3	25.8	35.3
	9-12 years	43.3	45.1	46.1
	13+ years	30.0	29.1	18.6
Class Level	Primary (1-5)	48.0	47.4	64.1
	Middle (6-8)	30.0	30.6	23.0
	Secondary (9-12)	22.0	22.0	13.0
Family Structure	Only child	40.0	53.1	54.0
	1-2 siblings	58.6	46.1	45.5
	3+ siblings	1.5	0.8	0.5
Responding Parent	Mother	44.6	50.9	51.2
	Father	30.8	37.0	38.2
	Both parents	24.6	12.0	10.6
Household Income	0-5.0 Lakh INR	19.3	9.6	9.1
	5.0-10.0 Lakh INR	17.1	12.1	13.9
	10.0-15.0 Lakh INR	22.0	21.8	23.5
	Over 15.0 Lakh INR	41.6	56.4	53.5
Mother Education	Below Bachelor's	11.0	4.1	4.2
	Bachelor's	28.0	29.3	25.3
	Graduate/PhD	61.0	66.6	70.5
Father Education	Below Bachelor's	11.2	2.3	3.0
	Bachelor's	30.9	29.2	28.0
	Graduate/PhD	57.9	68.5	69.0
Educational Aspiration	Bachelor's or less	5.5	3.9	3.5
	Graduate degree	42.5	36.9	34.4
	PhD	51.9	59.2	62.2
Job Aspiration	Academic/Professional	43.4	52.1	55.5
	Business	26.6	16.5	14.8
	Child's choice	2.0	3.3	3.2
	Other	44.3	42.1	38.5
Education Quality Satisfaction	Completely Satisfied	60.9	66.5	69.0
	Not Completely Satisfied	39.1	33.5	31.0
Child Progress Satisfaction	Completely Satisfied	52.4	54.6	57.0
	Not Completely Satisfied	47.6	45.4	43.0
Teacher Gender	Male	91.8	91.1	94.1
	Female	8.2	8.9	5.9
Teacher Education	Bachelor's	14.8	16.1	18.7
	Graduate/PhD	85.2	83.9	81.3
Years at School	1-5 years	39.6	21.4	18.9
	6+ years	60.4	78.6	81.1

Notes: All numbers are percentages unless noted.

B Demographic Variables

The parent survey collected a rich set of additional information that allows for heterogeneity analysis and provides context for understanding preference formation:

B.0.1 Demographic Information

- Family composition: Number of siblings, birth order, primary caregivers
- Parental education: Highest level attained by both mother and father
- Household income: Annual income in categorical brackets
- Parental occupation: Categorized into professional, clerical, sales, service, etc.
- Marital status: Married, unmarried, widowed, separated/divorced

B.0.2 Educational Aspirations and Expectations

- Highest level of education parents would ideally like their child to attain
- Occupational aspirations for when their child is 30 years old

These aspirational measures allow us to examine how longer-term goals align with preferences for immediate skill development priorities, building on recent work by Eble and Escueta (2023) that shows the importance of caregiver aspirations in educational investment and outcomes in resource-constrained settings.

B.0.3 Parent-Teacher Communication

- Frequency of parent contact with teachers (ranging from "rarely" to "daily or more")
- Satisfaction with school and child's progress

The communication frequency measure is particularly important for our analysis, as it allows us to test whether teachers have more accurate beliefs about parent preferences when they communicate more frequently with parents.

B.0.4 Child Well-being and Socio-emotional Assessment

The survey incorporated the Strengths and Difficulties Questionnaire (SDQ), a widely validated psychological assessment tool that measures:

- Emotional symptoms
- Conduct problems
- Hyperactivity/inattention

- Peer relationship problems
- Prosocial behavior

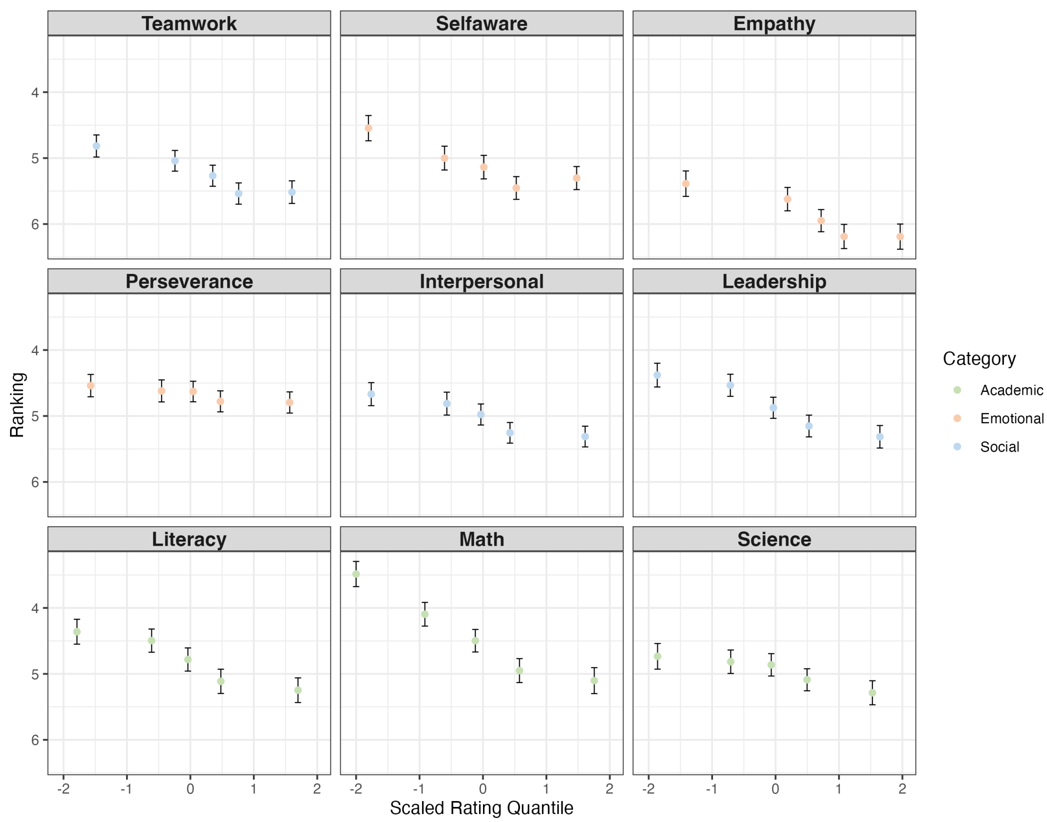
The SDQ provides an alternative measure of children's socio-emotional development that can be used both as an alternative outcome measure, and also to validate parent ratings of emotional and social skills. It consists of 25 attributes, both positive and negative, with parents rating each attribute as "Not True," "Somewhat True," or "Certainly True."

B.0.5 Life Satisfaction

Parents were asked to place themselves on a ladder from 0 to 10, where 0 represents the worst possible life and 10 represents the best possible life. This measure allows us to examine whether parent preferences for different skill dimensions vary with overall life satisfaction.

C Figures

Figure 17: Average parent priority rank within quintiles of the standardized rating, by dimension



Notes: Higher values on the y-axis indicate higher parental priority (rank 1 = most important). Points show quintile means; bars show 95% confidence intervals.

D Model

D.1 Environment and CET technology

Outcomes and notation. Each child exits the grade with a two–dimensional skill vector $\boldsymbol{c} = (c_1, c_2) \in \mathbb{R}^2_+$, where c_1 denotes academic/cognitive skill and c_2 denotes non–cognitive skill.

Preferences and the MRS. Parent i has Cobb–Douglas utility

$$U(c_1, c_2; \beta_i) \; = \; c_1^{\beta_i} \, c_2^{1-\beta_i}, \qquad 0 < \beta_i < 1.$$

The marginal utilities are

$$MU_1 = \frac{\partial U}{\partial c_1} = \beta_i \, c_1^{\beta_i - 1} \, c_2^{1-\beta_i}, \qquad MU_2 = \frac{\partial U}{\partial c_2} = (1-\beta_i) \, c_1^{\beta_i} \, c_2^{-\beta_i}.$$

The marginal rate of substitution of skill 2 for skill 1 (the amount of c_2 the parent requires to compensate a one–unit loss of c_1 while holding utility constant) is

$$\text{MRS}_{12,i} \; = \; \frac{MU_1}{MU_2} \; = \; \frac{\beta_i}{1-\beta_i} \, \frac{c_2}{c_1}. \tag{11}$$

It will be convenient below to define the *taste index*

$$T_i \; := \; \left(\tfrac{\beta_i}{1-\beta_i}\right)^{1/\rho} \quad \Longleftrightarrow \quad \log T_i = \tfrac{1}{\rho} \log\!\left(\tfrac{\beta_i}{1-\beta_i}\right),$$

where $\rho > 1$ will be the curvature parameter of the technology.

Technology and the CET frontier. Let parents purchase inputs e_1, e_2 at prices p_1, p_2 subject to a static budget $p_1 e_1 + p_2 e_2 \leq I$. Each skill is produced with diminishing marginal product from its own input:

$$c_1 = a_1 e_1^{\theta}, \qquad c_2 = a_2 e_2^{\theta}, \qquad a_j > 0, \quad 0 < \theta < 1.$$

Eliminating inputs using $e_j = (c_j/a_j)^{1/\theta}$ and substituting into the budget gives

$$p_1 \left(\tfrac{c_1}{a_1}\right)^{1/\theta} \; + \; p_2 \left(\tfrac{c_2}{a_2}\right)^{1/\theta} \; \leq \; I.$$

Let $\rho := 1/\theta > 1$ and define the *effective productivity–price* constants

$$\kappa_1 := a_1 \left(\tfrac{I}{p_1}\right)^{1/\rho}, \qquad \kappa_2 := a_2 \left(\tfrac{I}{p_2}\right)^{1/\rho}. \tag{12}$$

Dividing both sides by $I^{1/\rho}$ and rearranging yields the constant-elasticity-of-transformation (CET) frontier:

$$\left(\tfrac{c_1}{\kappa_1}\right)^{\rho} \; + \; \left(\tfrac{c_2}{\kappa_2}\right)^{\rho} \; = \; 1, \qquad \rho > 1, \; \kappa_1, \kappa_2 > 0. \tag{13}$$

We collect relative technology/price/budget information in the *tilt* (relative supply parameter)

$$\lambda \;:=\; \frac{\kappa_1}{\kappa_2} \;>\; 0, \tag{14}$$

so that larger λ means skill 1 is relatively easier to produce (or relatively cheaper per efficiency unit), tilting the PPF toward c_1.

The MRT along the CET. Let $F(c_1, c_2) := (c_1/\kappa_1)^\rho + (c_2/\kappa_2)^\rho - 1 = 0$ define the frontier (13). By definition, the MRT is the ratio of marginal costs (the absolute slope of the frontier). Totally differentiating gives

$$F_{c_1}\, dc_1 + F_{c_2}\, dc_2 \;=\; 0 \quad\Longrightarrow\quad \frac{dc_2}{dc_1} \;=\; -\frac{F_{c_1}}{F_{c_2}}.$$

Compute the partial derivatives:

$$F_{c_1} = \rho\, \kappa_1^{-\rho}\, c_1^{\rho-1}, \qquad F_{c_2} = \rho\, \kappa_2^{-\rho}\, c_2^{\rho-1}.$$

Hence the (absolute) marginal rate of transformation between the two skills is

$$\mathrm{MRT}_{12} \;:=\; -\frac{dc_2}{dc_1} \;=\; \left(\tfrac{\kappa_2}{\kappa_1}\right)^{\rho} \left(\tfrac{c_1}{c_2}\right)^{\rho-1} \;=\; \lambda^{-\rho}\, s^{\rho-1}, \qquad s := \frac{c_1}{c_2}. \tag{15}$$

Expression (15) shows how the frontier's marginal tradeoff depends both on curvature ρ and on the tilt λ: for a given skill ratio s, a higher λ (skill 1 relatively easier) rotates the PPF, lowering the amount of c_2 that must be sacrificed per unit increase in c_1 (a smaller MRT). The associated elasticity of transformation for the CET is $\sigma_T = 1/(\rho - 1) \in (0, \infty)$.

D.2 Optimal skill ratio

Equating the MRS (11) and the MRT (15) yields the *optimal skill ratio*

$$s^* := \frac{c_1}{c_2} = \left(\tfrac{\beta}{1-\beta}\right)^{1/\rho} \frac{\kappa_1}{\kappa_2} = T\, \lambda, \qquad T := \left(\tfrac{\beta}{1-\beta}\right)^{1/\rho}. \tag{16}$$

Evaluated at (c_1^*, c_2^*), the MRS equals

$$\mathrm{MRS}_{12}^* = T^{\rho-1} \lambda^{-1}. \tag{17}$$

Multiplying gives the identity

$$s^* \cdot \mathrm{MRS}_{12}^* = T^{\rho}. \tag{18}$$

D.3 Within-dimension dispersion across students

Covariance formula. Let (T_i, λ_i) vary across students. Then (dropping the subscript i for convenience),

$$\mathrm{Cov}(s^*, \mathrm{MRS}^*) = \mathbb{E}[T^\rho] - \mathbb{E}[T\lambda]\,\mathbb{E}[T^{\rho-1}\lambda^{-1}]. \tag{19}$$

Pure supply heterogeneity. If β is constant ($T \equiv \bar{T}$), then

$$\mathrm{Cov}(s^*, \mathrm{MRS}^*) = \bar{T}^\rho \left(1 - \mathbb{E}[\lambda]\,\mathbb{E}[\lambda^{-1}]\right) < 0, \tag{20}$$

as by the Cauchy-Schwarz inequality, we have $\mathbb{E}[\lambda]\,\mathbb{E}[\lambda^{-1}] > \mathbb{E}[\sqrt{\lambda} * \sqrt{\lambda^{-1}}] = 1$.

Pure preference heterogeneity. If λ is constant, write $t := \beta/(1-\beta) > 0$ and note $T = t^{1/\rho}$. Then

$$\mathrm{Cov}(s^*, \mathrm{MRS}^*) = \mathbb{E}[t] - \mathbb{E}[t^{1/\rho}]\mathbb{E}[t^{1-1/\rho}] > 0, \tag{21}$$

as by Jensen's inequality, we have that $\mathbb{E}[t^{1/\rho}] < \mathbb{E}[t]^{1/\rho}$ and $\mathbb{E}[t^{1-1/\rho}] < \mathbb{E}[t]^{1-1/\rho}$ (note $0 < 1/\rho < 1$ and $0 < 1 - 1/\rho$), so their product is less than $\mathbb{E}[t]$.

Reduced–form slope and variance comparison. Recall from (11)–(15) that the observed skill ratio and MRS are

$$s^* = T\,\lambda, \qquad \mathrm{MRS}^* = T^{\rho-1}\lambda^{-1},$$

where $T = \left(\frac{\beta}{1-\beta}\right)^{1/\rho}$ captures preferences and $\lambda = \kappa_1/\kappa_2$ is the relative supply tilt. Across students, the reduced–form slope from regressing MRS^* on s^* is

$$\beta^{\mathrm{RF}} \;=\; \frac{\mathrm{Cov}(s^*, \mathrm{MRS}^*)}{\mathrm{Var}(s^*)}. \tag{22}$$

Exact formula. Let $\mu_{ab} := \mathbb{E}[T^a\lambda^b]$ denote mixed moments of the taste and technology parameters. From (22) we have that

$$\beta^{\mathrm{RF}} = \frac{\mu_{\rho,0} - \mu_{1,1}\,\mu_{\rho-1,-1}}{\mu_{2,2} - \mu_{1,1}^2}. \tag{23}$$

This expression is fully general.

D.3.1 Log-linearized approximate variance decomposition.

Reduced–form slope in levels (first–order derivation). Recall

$$s^* \;=\; T\,\lambda \qquad \text{and} \qquad \mathrm{MRS}^* \;=\; T^{\rho-1}\,\lambda^{-1},$$

where $T := \left(\frac{\beta}{1-\beta}\right)^{1/\rho}$ is the *benefits tilt* and $\lambda := \frac{\kappa_1}{\kappa_2}$ is the *costs tilt*. Let $\bar{T} := \mathbb{E}[T]$, $\bar{\lambda} := \mathbb{E}[\lambda]$, and write small mean–zero fractional deviations

$$\tilde{T} \;:=\; \frac{T-\bar{T}}{\bar{T}}, \qquad \tilde{\lambda} \;:=\; \frac{\lambda-\bar{\lambda}}{\bar{\lambda}}, \qquad \mathbb{E}[\tilde{T}] = \mathbb{E}[\tilde{\lambda}] = 0.$$

Then $T = \bar{T}(1+\tilde{T})$ and $\lambda = \bar{\lambda}(1+\tilde{\lambda})$. A first–order expansion gives

$$s^* \;=\; T\,\lambda \;\approx\; \bar{T}\,\bar{\lambda}\,(1+\tilde{T}+\tilde{\lambda}),$$

$$\mathrm{MRS}^* \;=\; T^{\rho-1}\lambda^{-1} \;\approx\; \bar{T}^{\rho-1}\bar{\lambda}^{-1}\left(1+(\rho-1)\tilde{T}-\tilde{\lambda}\right).$$

Centering by subtracting means (which differ from the above only by constants), the first-order fluctuations are

$$\widehat{s} \;\approx\; \bar{T}\,\bar{\lambda}\,(\tilde{T}+\tilde{\lambda}), \qquad \widehat{m} \;\approx\; \bar{T}^{\rho-1}\bar{\lambda}^{-1}\left((\rho-1)\tilde{T}-\tilde{\lambda}\right),$$

where $\widehat{s} := s^* - \mathbb{E}[s^*]$ and $\widehat{m} := \mathrm{MRS}^* - \mathbb{E}[\mathrm{MRS}^*]$.

Assuming (for expositional clarity) that $\tilde{T}$ and $\tilde{\lambda}$ are approximately uncorrelated,[5] we obtain

$$\mathrm{Cov}(\widehat{m},\widehat{s}) \;\approx\; \bar{T}^{\rho}\left((\rho-1)\,\mathrm{Var}(\tilde{T}) \;-\; \mathrm{Var}(\tilde{\lambda})\right),$$

$$\mathrm{Var}(\widehat{s}) \;\approx\; (\bar{T}\,\bar{\lambda})^2\left(\mathrm{Var}(\tilde{T}) \;+\; \mathrm{Var}(\tilde{\lambda})\right).$$

Hence, the reduced–form OLS slope of MRS^* on s^* is

$$\beta^{\mathrm{RF}} \;=\; \frac{\mathrm{Cov}(\widehat{m},\widehat{s})}{\mathrm{Var}(\widehat{s})} \;\approx\; \underbrace{\bar{T}^{\rho-2}\,\bar{\lambda}^{-2}}_{\text{units factor}}\cdot\frac{(\rho-1)\,\mathrm{Var}(\tilde{T}) \;-\; \mathrm{Var}(\tilde{\lambda})}{\mathrm{Var}(\tilde{T}) \;+\; \mathrm{Var}(\tilde{\lambda})}.$$

The prefactor $\bar{T}^{\rho-2}\bar{\lambda}^{-2}$ is a units normalization. It rescales the dependent variable by a positive constant and does not affect the sign comparison. If we report the slope in normalized units (divide MRS^* by $\bar{T}^{\rho-1}\bar{\lambda}^{-1}$ and s^* by $\bar{T}\bar{\lambda}$), this prefactor is 1 and we get the variance–ratio form used in the main text:

$$\boxed{\beta^{\mathrm{RF}} \;\approx\; \frac{(\rho-1)\,\mathrm{Var}(\tilde{T}) \;-\; \mathrm{Var}(\tilde{\lambda})}{\mathrm{Var}(\tilde{T}) \;+\; \mathrm{Var}(\tilde{\lambda})}} \tag{24}$$

where $\tilde{T}$ and $\tilde{\lambda}$ are *fractional* (mean–normalized) deviations of the tilts. Relating tilts to primitives, $T = \exp(b/\rho)$ with $b = \log\frac{\beta}{1-\beta}$ and $\tilde{\lambda}$ corresponds to $\tilde{k} = \log\lambda - \mathbb{E}[\log\lambda]$; for small dispersion, $\mathrm{Var}(\tilde{T}) \approx \frac{1}{\rho^2}\mathrm{Var}(\tilde{b})$ and $\mathrm{Var}(\tilde{\lambda}) \approx \mathrm{Var}(\tilde{k})$, matching the appendix formulas.

We reparametrize the optimal skill ratio and MRS as:

$$s^* \;=\; T\,\lambda \;=\; \exp\!\Big(\tfrac{1}{\rho}b+k\Big), \qquad \mathrm{MRS}^* \;=\; T^{\rho-1}\lambda^{-1} \;=\; \exp\!\Big(\tfrac{\rho-1}{\rho}b-k\Big),$$

[5] When $\mathrm{Cov}(\tilde{T},\tilde{\lambda}) \neq 0$, the same steps yield a simple adjustment term; see above for the general formula.

where $b = \log \frac{\beta}{1-\beta}$ and $k = \log \lambda$. Define the linear indices

$$x \ := \ \tfrac{1}{\rho} b + k, \qquad y \ := \ \tfrac{\rho-1}{\rho} b - k,$$

so that $s^* = e^x$ and $\mathrm{MRS}^* = e^y$.

Write $b = \bar{b} + \tilde{b}$ and $k = \bar{k} + \tilde{k}$, with $\mathbb{E}[\tilde{b}] = \mathbb{E}[\tilde{k}] = 0$, $\sigma_b^2 := \mathrm{Var}(\tilde{b})$, $\sigma_k^2 := \mathrm{Var}(\tilde{k})$, $\sigma_{bk} := \mathrm{Cov}(\tilde{b}, \tilde{k})$. Then

$$x = \bar{x} + A\tilde{b} + B\tilde{k}, \qquad y = \bar{y} + C\tilde{b} + D\tilde{k},$$

with coefficients

$$A = \tfrac{1}{\rho}, \quad B = 1, \qquad C = \tfrac{\rho-1}{\rho}, \quad D = -1,$$

and means $\bar{x} = \frac{\bar{b}}{\rho} + \bar{k}, \quad \bar{y} = \frac{\rho-1}{\rho}\bar{b} - \bar{k}$.

Second–order expansion. Using the second–order Taylor approximation $e^u \approx 1 + u + \frac{1}{2}u^2$ for small, centered u,

$$e^x \ = \ e^{\bar{x}}\, e^{A\tilde{b}+B\tilde{k}} \ \approx \ e^{\bar{x}}\Big[1 + (A\tilde{b} + B\tilde{k}) + \tfrac{1}{2}(A\tilde{b} + B\tilde{k})^2\Big],$$

$$e^y \ = \ e^{\bar{y}}\, e^{C\tilde{b}+D\tilde{k}} \ \approx \ e^{\bar{y}}\Big[1 + (C\tilde{b} + D\tilde{k}) + \tfrac{1}{2}(C\tilde{b} + D\tilde{k})^2\Big].$$

Therefore the means are given as

$$\mathbb{E}[e^x] \approx e^{\bar{x}}\Big[1 + \tfrac{1}{2}\big(A^2\sigma_b^2 + 2AB\,\sigma_{bk} + B^2\sigma_k^2\big)\Big],$$

$$\mathbb{E}[e^y] \approx e^{\bar{y}}\Big[1 + \tfrac{1}{2}\big(C^2\sigma_b^2 + 2CD\,\sigma_{bk} + D^2\sigma_k^2\big)\Big].$$

For the variance, we use the first–order approximation

$$\mathrm{Var}(e^x) \ \approx \ e^{2\bar{x}}\,\mathrm{Var}(A\tilde{b} + B\tilde{k}) \ = \ e^{2\bar{x}}\big(A^2\sigma_b^2 + 2AB\,\sigma_{bk} + B^2\sigma_k^2\big).$$

$$\mathrm{Var}(e^y) \ \approx \ e^{2\bar{y}}\,\mathrm{Var}(C\tilde{b} + D\tilde{k}) \ = \ e^{2\bar{y}}\big(C^2\sigma_b^2 + 2CD\,\sigma_{bk} + D^2\sigma_k^2\big).$$

The covariance is

$$\begin{aligned}\mathrm{Cov}(e^y, e^x) \ &\approx \ e^{\bar{x}+\bar{y}}\,\mathrm{Cov}(A\tilde{b} + B\tilde{k},\ C\tilde{b} + D\tilde{k}) \\ &= e^{\bar{x}+\bar{y}}\Big(AC\,\sigma_b^2 + (AD + BC)\,\sigma_{bk} + BD\,\sigma_k^2\Big).\end{aligned}$$

Assembling the ratio. Recall the reduced–form slope is

$$\beta^{\mathrm{RF}} = \frac{\mathrm{Cov}(e^y, e^x)}{\mathrm{Var}(e^x)}.$$

Thus,

$$\beta^{\mathrm{RF}} \;\approx\; e^{\bar{y}-\bar{x}}\,\frac{AC\,\sigma_b^2 + (AD+BC)\,\sigma_{bk} + BD\,\sigma_k^2}{A^2\,\sigma_b^2 + 2AB\,\sigma_{bk} + B^2\,\sigma_k^2}.$$

Substituting A, B, C, D:

$$AC = \tfrac{\rho-1}{\rho^2}, \quad AD + BC = \frac{\rho-2}{\rho}, \quad BD = -1,$$

$$A^2 = \tfrac{1}{\rho^2}, \quad 2AB = 2/\rho, \quad B^2 = 1.$$

Hence

$$\beta^{\mathrm{RF}} \;\approx\; e^{\bar{y}-\bar{x}}\,\frac{\frac{\rho-1}{\rho^2}\,\sigma_b^2 \;-\; \sigma_k^2 + \frac{\rho-2}{\rho}\,\sigma_{bk}}{\frac{1}{\rho^2}\,\sigma_b^2 \;+\; \sigma_k^2 \;+\; \frac{2}{\rho}\,\sigma_{bk}}. \tag{25}$$

Simplifications.

(i) The constant factor $e^{\bar{y}-\bar{x}}$ reflects a units normalization of the dependent variable in the reduced–form regression; rescaling MRS* by a positive constant rescales the slope by the same constant, so this term can be set to 1 without loss of generality.

(ii) If $\tilde{b}$ and $\tilde{k}$ are approximately uncorrelated ($\sigma_{bk} \approx 0$), then

this becomes the compact variance–comparison formula

$$\boxed{\beta^{\mathrm{RF}} \;\approx\; \frac{\frac{\rho-1}{\rho^2}\,\sigma_b^2 - \sigma_k^2}{\frac{1}{\rho^2}\,\sigma_b^2 + \sigma_k^2}} \tag{26}$$

which matches the presentation in the main text.

Equation (26) shows that the sign and magnitude of the reduced–form slope are governed by the relative dispersion of benefits and costs.

D.4 Within-student dispersion across skills

We observe $J \geq 3$ skills for the same parent. Let the CET frontier be

$$\sum_{j=1}^{J} \left(\tfrac{c_j}{\kappa_j}\right)^{\rho} = 1, \qquad \rho > 1,$$

and preferences $U(\boldsymbol{c};\boldsymbol{\beta}) = \prod_{j=1}^{J} c_j^{\beta_j}$ with $\beta_j > 0$ and $\sum_j \beta_j = 1$. Fix skill 1 as an anchor. For each $j \neq 1$, define the pairwise *skill ratio* and *MRS*:

$$s_j^* := \frac{c_j^*}{c_1^*} = \left(\frac{\beta_j}{\beta_1}\right)^{1/\rho} \frac{\kappa_j}{\kappa_1} =: T_j\,\lambda_j, \qquad \mathrm{MRS}_{1j}^* = T_j^{\rho-1}\lambda_j^{-1},$$

where $T_j := \left(\beta_j/\beta_1\right)^{1/\rho}$ and $\lambda_j := \kappa_j/\kappa_1$ are the *within–parent* benefit and cost tilts, respectively.

Key identity (within–parent). As in the two–skill case, technology cancels when we multiply:

$$s_j^* \cdot \mathrm{MRS}_{1j}^* \;=\; \frac{\beta_j}{\beta_1} \;=\; e^{b_j}, \qquad b_j := \log\Big(\frac{\beta_j}{\beta_1}\Big). \tag{27}$$

Exact covariance expression. Let expectations and variances be taken across the $J-1$ non–anchor skills for this parent (denote them by $\mathbb{E}_j[\cdot]$, $\mathrm{Var}_j(\cdot)$, etc.). Using (27) and the definitions above,

$$\begin{aligned}\mathrm{Cov}_j(s^*, \mathrm{MRS}^*) &= \mathbb{E}_j\big[s_j^* \mathrm{MRS}_{1j}^*\big] - \mathbb{E}_j[s_j^*]\;\mathbb{E}_j[\mathrm{MRS}_{1j}^*] \\ &= \mathbb{E}_j\big[e^{b_j}\big] - \mathbb{E}_j\big[e^{\frac{1}{\rho}b_j + k_j}\big]\;\mathbb{E}_j\big[e^{\frac{\rho-1}{\rho}b_j - k_j}\big],\end{aligned} \tag{28}$$

where $k_j := \log\lambda_j = \log(\kappa_j/\kappa_1)$.

Second–order approximation (small dispersion across skills). Write $b_j = \bar{b} + \tilde{b}_j$ and $k_j = \bar{k} + \tilde{k}_j$ with within–parent means $\bar{b}, \bar{k}$ and centered shocks $\mathbb{E}_j[\tilde{b}_j] = \mathbb{E}_j[\tilde{k}_j] = 0$. Define

$$\sigma_b^2 := \mathrm{Var}_j(\tilde{b}), \quad \sigma_k^2 := \mathrm{Var}_j(\tilde{k}), \quad \sigma_{bk} := \mathrm{Cov}_j(\tilde{b}, \tilde{k}).$$

As in the cross–section derivation, set

$$x_j := \tfrac{1}{\rho}b_j + k_j, \qquad y_j := \tfrac{\rho-1}{\rho}b_j - k_j, \quad \Rightarrow \quad s_j^* = e^{x_j}, \;\; \mathrm{MRS}_{1j}^* = e^{y_j}.$$

With coefficients $A = \frac{1}{\rho}$, $B = 1$, $C = \frac{\rho-1}{\rho}$, $D = -1$, a second–order expansion in $(\tilde{b}, \tilde{k})$ yields (cf. the general formula in the previous subsection)

$$\begin{aligned}\mathrm{Cov}_j(s^*, \mathrm{MRS}^*) &\approx e^{\bar{x}+\bar{y}}\Big(AC\,\sigma_b^2 + (AD + BC)\,\sigma_{bk} + BD\,\sigma_k^2\Big) \\ &= e^{\bar{x}+\bar{y}}\left(\frac{\rho-1}{\rho^2}\,\sigma_b^2 + \frac{\rho-2}{\rho}\,\sigma_{bk} - \sigma_k^2\right),\end{aligned} \tag{29}$$

where $\bar{x} = \frac{\bar{b}}{\rho} + \bar{k}$ and $\bar{y} = \frac{\rho-1}{\rho}\bar{b} - \bar{k}$.

Within–parent reduced–form slope. The within–parent OLS slope from regressing MRS_{1j}^* on s_j^* across the $J-1$ pairs is

$$\beta^{\mathrm{RF}}_{\mathrm{within}} \;=\; \frac{\mathrm{Cov}_j(s^*, \mathrm{MRS}^*)}{\mathrm{Var}_j(s^*)}.$$

Using the same expansion,

$$\mathrm{Var}_j(s^*) \approx e^{2\bar{x}}\big(A^2\sigma_b^2 + 2AB\,\sigma_{bk} + B^2\sigma_k^2\big) = e^{2\bar{x}}\left(\frac{1}{\rho^2}\sigma_b^2 + \frac{2}{\rho}\sigma_{bk} + \sigma_k^2\right).$$

Thus,

$$\beta^{\mathrm{RF}}_{\mathrm{within}} \;\approx\; e^{\bar{y}-\bar{x}}\,\frac{\frac{\rho-1}{\rho^2}\,\sigma_b^2 \;-\; \sigma_k^2 \;+\; \frac{\rho-2}{\rho}\,\sigma_{bk}}{\frac{1}{\rho^2}\,\sigma_b^2 \;+\; \sigma_k^2 \;+\; \frac{2}{\rho}\,\sigma_{bk}}. \tag{30}$$

As before, the level constant $e^{\bar{y}-\bar{x}}$ is a units normalization and can be absorbed by rescaling the dependent variable.

Interpretation. Equation (29) (or (30)) shows that, *within a parent*, dispersion in tastes across skills (σ_b^2) pushes the covariance/slope positive, while dispersion in technology tilts across skills (σ_k^2) pushes it negative; the cross–covariance σ_{bk} enters with coefficient $(\rho-2)/\rho$ and vanishes in the special case $\rho = 2$. Under approximate orthogonality of tastes and tilts across skills ($\sigma_{bk} \approx 0$), the sign reduces to a simple variance comparison:

$$\mathrm{Cov}_j(s^*, \mathrm{MRS}^*) \gtrless 0 \iff \frac{\rho-1}{\rho^2}\,\mathrm{Var}_j(\tilde{b}) \gtrless \mathrm{Var}_j(\tilde{k}).$$

Empirically, this justifies running the within–parent regression of ranks on demeaned ratings across skills (with an anchor), and interpreting the sign/magnitude as a diagnostic of whether the child's skill profile mirrors parental priorities (taste dispersion) or comparative advantage in production (supply dispersion).

D.5 Policy intervention: local analysis

Optimal local policy direction. Let $u(c_1, c_2) = c_1^{\beta} c_2^{1-\beta}$ and feasible (c_1, c_2) be summarized locally by instruments $z = (\ln \kappa_1, \ln \kappa_2)$ with $dc = H\,dz$, $H = \mathrm{diag}(c_1, c_2)$. The policymaker solves

$$\max_{dz}\ \nabla u^{\top} H\,dz \qquad \text{s.t.} \qquad \tfrac{1}{2} dz^{\top} W dz \le \mathcal{C}.$$

The Lagrangian yields $Wdz = \lambda H^{\top}\nabla u$, hence $dz \propto W^{-1}H^{\top}\nabla u$ and $dc = Hdz \propto M\nabla u$, $M := HW^{-1}H^{\top} \succ 0$. Under skill-symmetric costs in outcome space ($M \propto I$), $dc \propto \nabla u$, i.e., along the IC normal.

Budget expansion leaves s unchanged. With CET frontier $(c_1/\kappa_1)^{\rho} + (c_2/\kappa_2)^{\rho} \le 1$ and Cobb–Douglas, the optimal levels are $c_1^* = \kappa_1 \beta^{1/\rho}$ and $c_2^* = \kappa_2 (1-\beta)^{1/\rho}$, so

$$s := \frac{c_1}{c_2} = \frac{\kappa_1}{\kappa_2}\Big(\frac{\beta}{1-\beta}\Big)^{1/\rho}.$$

Since $\kappa_i = a_i (I/p_i)^{1/\rho}$, a pure I increase scales both c_i by $I^{1/\rho}$ and leaves s unchanged: $ds/dI = 0$ and $d\,\mathrm{MRS}/dI = 0$.

Total-differential derivation for the IC-normal step. Define $r := \frac{c_2}{c_1}$ and let the policymaker induce an outcome step *along the IC normal* (utility gradient). For Cobb–Douglas,

$$\nabla u \propto \Big(\frac{\beta}{c_1}, \frac{1-\beta}{c_2}\Big), \quad \Delta c_1 = \kappa\,\frac{\beta}{c_1}, \quad \Delta c_2 = \kappa\,\frac{1-\beta}{c_2}$$

for a small step size $\kappa > 0$. Using $r(c_1, c_2) = c_2/c_1$,

$$\frac{\partial r}{\partial c_1} = -\frac{c_2}{c_1^2} = -\frac{r}{c_1}, \qquad \frac{\partial r}{\partial c_2} = \frac{1}{c_1}.$$

Hence the first-order change is

$$dr \;\approx\; \frac{\partial r}{\partial c_1}\Delta c_1 + \frac{\partial r}{\partial c_2}\Delta c_2 = -\frac{r}{c_1}\Big(\kappa\frac{\beta}{c_1}\Big) + \frac{1}{c_1}\Big(\kappa\frac{1-\beta}{c_2}\Big) = \frac{\kappa}{c_1^2}\Big(\frac{1-\beta}{r} - \beta r\Big).$$

Thus the sign of dr matches $\frac{1-\beta}{r} - \beta r$, which is positive if $r < \sqrt{\frac{1-\beta}{\beta}}$, negative if $r > \sqrt{\frac{1-\beta}{\beta}}$, and zero at $r^\dagger = \sqrt{\frac{1-\beta}{\beta}}$.

Dynamics and convergence to the threshold. Treat repeated tiny IC-normal steps as a continuous-time limit. Write $r = c_2/c_1$ and $s = 1/r$. From A.3,

$$\frac{dr}{d\tau} = K\Big(\frac{1-\beta}{r} - \beta r\Big) \quad \text{with} \quad K > 0.$$

Then $s = r^{-1}$ satisfies

$$\frac{ds}{d\tau} = -\frac{1}{r^2}\frac{dr}{d\tau} = K\Big(\beta\, s - (1-\beta)\, s^3\Big) \;\propto\; \frac{\beta}{s} - (1-\beta)s.$$

The unique positive fixed point is $s^\dagger = \sqrt{\frac{\beta}{1-\beta}}$. Linearization shows global (monotone for small steps) convergence toward $s^\dagger$.

Equilibrium κ-ratio implementing the fixed point. At the optimum under CET, $s = \lambda T$ with $\lambda := \kappa_1/\kappa_2$ and $T := (\beta/(1-\beta))^{1/\rho}$. To implement $s^\dagger$ as the new optimum, choose

$$\boxed{\lambda^\star = \frac{s^\dagger}{T} = \Big(\tfrac{\beta}{1-\beta}\Big)^{\frac{1}{2}-\frac{1}{\rho}}}.$$

Connection to endogenous technology choice/frontiers. In Caselli–Coleman's framework, firms pick (A_s, A_u) on a frontier characterized by a "height" B and curvature; the first-order conditions link the chosen bias A_s/A_u to factor ratios and relative wages, generating appropriate technology choices across endowments and a barrier parameter shifting the whole frontier. Our quadratic budget on dz is a local reduced form of these frontier trade-offs for policy. See their discussion of the technology frontier and the FOCs (their eqs. (7)–(10)).[6]

[6]See Caselli and Coleman (2006) for the frontier concept and the way the FOCs pin down endogenous bias: technology frontier description and barriers *op. cit.*, pp. 508–509; symmetric-equilibrium FOCs and comparative statics *op. cit.*, eqs. (7)–(10). CITES: :contentReference[oaicite:4]index=4 :contentReference[oaicite:5]index=5

D.6 Period 2: teacher-driven supply and key identity

Preferences β_i are formed in Period 1 and are fixed. In Period 2, teacher/classroom factors determine λ_i via effort, pedagogy, materials, and constraints; these may be shifted by observable shocks (e.g., randomized information, measured teacher priorities). Parents observe learning under this supply and report endline ratings/ranks; thus reports lie on the PPF determined by λ_i and B_i.

From (16)–(17):

$$s_i^* = T_i \lambda_i, \qquad \mathrm{MRS}^*_{12,i} = T_i^{\rho-1} \lambda_i^{-1},$$

so

$$s_i^* \cdot \mathrm{MRS}^*_{12,i} = T_i^{\rho}. \tag{31}$$

Hence supply dispersion $\mathrm{Var}(\log \lambda)$ attenuates the covariance of (s^*, MRS^*) relative to the Period 1 benchmark if treatment reduces misalignment between λ_i and T_i (e.g., reallocating classroom effort toward parent-valued skills) rotates the reduced-form slope upwards.

D.7 Empirical implementation and identification

Proxies. Let $r_{ij} \in [0, 100]$ and $m_{ij} \in \{1, \ldots, 9\}$ denote parent i's rating and importance rank for skill j. Set $\tilde{r}_{ij} := r_{ij} - \bar{r}_i$. Assume a monotone reporting function so that $\tilde{r}_{ij}$ is a monotone proxy for s^*_{ij}. Parents rank by marginal utility, so m_{ij} is a monotone proxy for MRS^*_{ij}.

Baseline regression. Estimate

$$m_{ij} = \alpha + \beta\, \tilde{r}_{ij} + \xi_j + \varepsilon_{ij}, \tag{32}$$

with heteroskedasticity-robust SEs and parent clustering when stacking skills; include dimension fixed effects ξ_j and, if desired, dimension-specific slopes.

Supply-side instruments. Treat teacher/classroom variables as *supply instruments* for $\tilde{r}_{ij}$:

- randomized information assignment at grade/teacher level;
- teacher "typical student" priorities (constructed indices);
- class-level aggregates of parent priorities (leave-one-out).

These shift λ_i but not β_i, identifying the supply-driven component of the slope. Over identified designs allow over-ID tests; report partial-F (or Bayesian analogs) and cluster at the classroom (or school×grade) level.

Policy mapping. Posterior (or sampling) decompositions of $\mathrm{Var}(b)$ and $\mathrm{Var}(k)$ quantify whether skills are primarily shaped by tastes or supply. If the latter dominates, supply levers (pedagogy, time allocation, materials, class size, resources) should be prioritized; if the former dominates, alignment and demand-side interventions are the natural lever.

D.8 From the 2–Skill Framework to a 3–Category Estimable Model

This section shows, step by step, how the two–skill model (3.1) generalizes to three skill *groups* (academic, social, emotional), how the optimal skill ratios are derived, and how every term that appears in the Stan likelihood is obtained directly from the economic primitives.

D.9 Environment

Skills (now 3 categories). Parents end the decision period with category–level skills $\boldsymbol{c} = (c_A, c_S, c_E) \in \mathbb{R}^3_+$.

Preference parameters. Two taste weights govern trade-offs:

$$U(c_A, c_S, c_E) \;=\; c_A^{\beta_1} \left(c_S^{\beta_2}\, c_E^{1-\beta_2}\right)^{1-\beta_1}, \qquad \beta_1, \beta_2 \in (0,1).$$

- β_1: academic vs. *non*–academic $\left(c_S^{\beta_2} c_E^{1-\beta_2}\right)$;
- β_2: social vs. emotional *within* the non-academic composite.

D.10 Marginal-rate-of-substitution (MRS) between Academic and Non-academic skills

Utility with nested Cobb–Douglas preferences.

$$U(c_A, c_S, c_E) \;=\; c_A^{\beta_1} \left(c_S^{\beta_2}\, c_E^{1-\beta_2}\right)^{1-\beta_1}, \qquad \beta_1, \beta_2 \in (0,1). \tag{D.1}$$

Collapse Social and Emotional into a single non-academic index. Fix the within–non-academic mix $s_{SE} = c_S/c_E$ (chosen optimally in the *inner* problem). Because s_{SE} is constant during the Academic vs Non-academic trade-off, the term in brackets is proportional to $c_N := c_S + c_E$; the proportionality factor does not affect marginal rates. Hence we can write the *outer* utility as

$$\boxed{U(c_A, c_N) = c_A^{\beta_1}\, c_N^{1-\beta_1}}. \tag{D.2}$$

Marginal utilities.

$$\frac{\partial U}{\partial c_A} = \beta_1\, c_A^{\beta_1 - 1}\, c_N^{1-\beta_1} = \beta_1\, \frac{U}{c_A}, \qquad \frac{\partial U}{\partial c_N} = (1-\beta_1)\, c_A^{\beta_1}\, c_N^{-\beta_1} = (1-\beta_1)\, \frac{U}{c_N}.$$

MRS between Academic and Non-academic.

$$\boxed{\mathrm{MRS}_{A,N} = \frac{\partial U/\partial c_A}{\partial U/\partial c_N} = \frac{\beta_1}{1-\beta_1}\, \frac{c_N}{c_A}} = \frac{\beta_1}{1-\beta_1}\, \frac{1}{s_A}, \qquad s_A := \frac{c_A}{c_N}. \tag{D.3}$$

Equation (??) is the expression equated to the MRT in Section D.11 to solve for the optimal Academic share s_A^*.

Technology. Each category continues to have its *own* one–input Cobb–Douglas line: $c_j = a_j e_j^\theta$ $(0 < \theta < 1)$. Combining them with the linear budget $p_1 e_1 + p_2 e_2 + p_3 e_3 \le I$ again yields a *constant–elasticity–of–transformation* (CET) frontier

$$\big(c_A/\kappa_A\big)^\rho + \big(c_S/\kappa_S\big)^\rho + \big(c_E/\kappa_E\big)^\rho = 1, \qquad \rho = \tfrac{1}{\theta} > 1.$$

Only the two *ratios* $\ell_1 = \log(\kappa_A/\kappa_S)$ and $\ell_2 = \log(\kappa_S/\kappa_E)$ matter for the frontier's shape (its absolute scale is absorbed by the budget).

D.11 MRT between Academic and Non-Academic

We start from the three–category constant-elasticity-of-transformation (CET) frontier with $\rho > 1$:

$$\left(\tfrac{c_A}{\kappa_A}\right)^\rho \;+\; \left(\tfrac{c_S}{\kappa_S}\right)^\rho \;+\; \left(\tfrac{c_E}{\kappa_E}\right)^\rho \;=\; 1. \tag{C.1}$$

---

1. Collapse c_S and c_E into a single non-academic quantity while holding their *mix* constant. Fix a within–non-academic ratio

$$s_{SE} := \frac{c_S}{c_E} \quad \text{(taken as given during marginal changes).}$$

Write

$$c_S = \frac{s_{SE}}{1+s_{SE}}\, c_N, \qquad c_E = \frac{1}{1+s_{SE}}\, c_N, \qquad \text{where } c_N := c_S + c_E.$$

Insert these into (C.1); all terms containing c_N share the factor c_N^ρ. Collect them:

$$\left(\tfrac{c_A}{\kappa_A}\right)^\rho \;+\; \left(\tfrac{c_N}{\kappa_N}\right)^\rho \;=\; 1, \tag{C.2}$$

with the **effective non-academic conversion factor**

$$\boxed{\kappa_N(s_{SE}) = (1+s_{SE})\Big(\frac{s_{SE}^\rho}{\kappa_S^\rho} + \frac{1}{\kappa_E^\rho}\Big)^{-1/\rho}} \qquad \big(\kappa_N > 0\big). \tag{C.3}$$

Thus *for every fixed* s_{SE} the three–point frontier is algebraically equivalent to the two-point frontier (c_A, c_N) with parameters (κ_A, κ_N).

---

2. Implicit differentiation: $\mathrm{MRT}_{A,N}$. Let

$$F(c_A, c_N) \;=\; \left(\tfrac{c_A}{\kappa_A}\right)^{\rho} + \left(\tfrac{c_N}{\kappa_N}\right)^{\rho} - 1 = 0.$$

Total-differentiating while *holding* s_{SE} (and therefore κ_N) constant:

$$\frac{\partial F}{\partial c_A}\, dc_A + \frac{\partial F}{\partial c_N}\, dc_N = 0,$$

$$\frac{\partial F}{\partial c_A} = \rho\Big(\tfrac{c_A}{\kappa_A}\Big)^{\rho-1}\frac{1}{\kappa_A}, \qquad \frac{\partial F}{\partial c_N} = \rho\Big(\tfrac{c_N}{\kappa_N}\Big)^{\rho-1}\frac{1}{\kappa_N}.$$

Hence the absolute slope is

$$\boxed{\big|\mathrm{MRT}_{A,N}\big| = \frac{dc_N}{dc_A} = \Big(\tfrac{\kappa_N(s_{SE})}{\kappa_A}\Big)^{\rho}\Big(\tfrac{c_A}{c_N}\Big)^{\rho-1}}. \tag{C.4}$$

---

3. Optimal Academic–vs-Non-academic mix. Recall that our preferences across categories give

$$\mathrm{MRS}_{A,N} = \frac{\beta_1}{1-\beta_1}\,\frac{c_N}{c_A}.$$

Set $\mathrm{MRS}_{A,N} = \big|\mathrm{MRT}_{A,N}\big|$ and define $s_A := c_A/c_N$:

$$\frac{\beta_1}{1-\beta_1}\,\frac{1}{s_A} = \Big(\tfrac{\kappa_N(s_{SE})}{\kappa_A}\Big)^{\rho} s_A^{\rho-1}.$$

Multiply by s_A, take logs, divide by ρ, and exponentiate:

$$\boxed{s_A^{*} \;=\; \exp\!\Big[\tfrac{1}{\rho}\,\mathrm{logit}\beta_1 \;+\; \log\!\Big(\tfrac{\kappa_A}{\kappa_N(s_{SE})}\Big)\Big]}. \tag{C.5}$$

Because $\kappa_N(s_{SE})$ in (**??**) depends on κ_S, κ_E *and* the within-non-academic mix s_{SE}, the optimal Academic share s_A^* incorporates *all three* technology parameters unless we impose a normalization such as $\kappa_S = \kappa_E$.

---

4. Why the baseline estimation sets $\kappa_S = \kappa_E$.

- Without panel data or exogenous price shocks the single cross-section of ratings & ranks identifies β_1, β_2 precisely but cannot pin down $\ell_2 = \log(\kappa_S/\kappa_E)$ tightly—its effect overlaps almost one-for-one with β_2 inside s_{SE}.
- Normalising $\kappa_S = \kappa_E$ ($\ell_2 = 0$) therefore removes a weakly identified parameter and focuses statistical power on $\ell_1 = \log(\kappa_A/\kappa_S)$, which is central for detecting supply-side heterogeneity.

- If future data supply additional variation you can restore ℓ_2 by estimating it directly via (**??**) and (**??**); only a few lines of Stan code change.

5. Social vs. Emotional skills. Within the non-academic pair set $s_{SE} := c_S/c_E$ and equate MRS to MRT:

$$\frac{\beta_2}{1-\beta_2}\frac{c_E}{c_S} \;=\; \left(\tfrac{\kappa_E}{\kappa_S}\right)^{\rho}\left(\tfrac{c_S}{c_E}\right)^{\rho-1}.$$

Substituting $s_{SE} = c_S/c_E$ yields,

$$\frac{\beta_2}{1-\beta_2}\left(\tfrac{\kappa_S}{\kappa_E}\right)^{\rho} \;=\; s_{SE}^{\rho}.$$

If we impose the *normalisation* $\kappa_S = \kappa_E$ (i.e. no technological asymmetry *within* the non-academic block), then $\ell_2 = 0$ and (**??**) simplifies to the expression used in the baseline Stan specification:

$$\widetilde{s}_{SE} = \exp\Big[\tfrac{1}{\rho}\,\mathrm{logit}\beta_2\Big].$$

Why one tilt (ℓ_1) is enough for identification

(a) Given three observed ratings, any common rescaling of $(\kappa_A, \kappa_S, \kappa_E)$ is absorbed by the unobserved budget B_i; only *two* independent supply ratios remain.

(b) Rank data depend on those ratios *only through the two mixes* s_A and s_{SE}. With a single cross-section, ℓ_2 is nearly collinear with β_2 inside (**??**). Estimating both leads to weak identification unless additional variation (panel data, price changes, supply shifters) is available.

(c) Therefore the empirical benchmark sets $\ell_2 = 0$, keeps ℓ_1 free, and focuses on the contrast between *demand heterogeneity* $\sigma_{\beta_1}, \sigma_{\beta_2}$ and *supply heterogeneity* σ_{ℓ_1}. Section **??** shows that allowing $\ell_2 \neq 0$ yields similar qualitative results but much wider posteriors.

D.12 A scale–free representation (g_A, g_S, g_E)

Because only (s_A, s_{SE}) matter for choices, we pick a single convenient point on the CET frontier:

$$\underline{g}_A := 1, \quad \underline{g}_E := \frac{1}{s_A(1+s_{SE})}, \quad \underline{g}_S := s_{SE}\,\underline{g}_E. \tag{A3}$$

Any feasible skill vector differs from $\boldsymbol{g} := (\underline{g}_A, \underline{g}_S, \underline{g}_E)$ only by a *common* positive multiplier. We let the parent-specific *budget scale* $B_i > 0$ supply that multiplier:

$$c_{ik} = B_i\, g_k.$$

Latent utilities for the ordered–probit

We show a numerically stable way to obtain the three category–specific latent utilities that feed the ordered–probit likelihood, using only log and logit transformations that Stan handles safely.

1. Raw marginal utilities. With nested Cobb–Douglas preferences

$$U(c_A, c_S, c_E) = c_A^{\beta_1} \left(c_S^{\beta_2} c_E^{1-\beta_2}\right)^{1-\beta_1}, \quad \beta_1, \beta_2 \in (0, 1),$$

the marginal utilities factor as

$$MU_A = \beta_1 \frac{U}{c_A}, \quad MU_S = (1-\beta_1)\beta_2 \frac{U}{c_S}, \quad MU_E = (1-\beta_1)(1-\beta_2)\frac{U}{c_E}.$$

2. Normalise by the common factor $(1 - \beta_1)$**.** Multiplying or dividing every MU_j by the same positive constant leaves rankings unchanged, so set

$$\widetilde{MU}_j := \frac{MU_j}{U(1-\beta_1)} \quad (j = A, S, E).$$

Then

$$\widetilde{MU}_A = \frac{\beta_1}{1-\beta_1} \frac{1}{c_A}, \quad \widetilde{MU}_S = \beta_2 \frac{1}{c_S}, \quad \widetilde{MU}_E = (1-\beta_2) \frac{1}{c_E}.$$

3. Take logs and cancel parent–specific constants. Write $c_j = B_i \, g_j$ where B_i is parent i's budget and g_j the *effective* skill output. After subtracting $\log B_i$, which is common to all three categories, we obtain

$$\boxed{\begin{aligned} u_A &= \log\!\Big(\tfrac{\beta_1}{1-\beta_1}\Big) - \log g_A, \\ u_S &= \log \beta_2 - \log g_S, \\ u_E &= \log(1-\beta_2) - \log g_E. \end{aligned}} \tag{U.div}$$

4. Numerically stable implementation in Stan. Let

$$\theta_1 = \texttt{logit(beta1)}, \qquad \theta_2 = \texttt{logit(beta2)}.$$

- $\log\!\big(\frac{\beta_1}{1-\beta_1}\big) = \theta_1$.
- $\log \beta_2 = \theta_2 + \texttt{log1m\_inv\_logit}(\theta_2)$.
- $\log(1-\beta_2) = \texttt{log1m\_inv\_logit}(\theta_2)$.

Using these identities avoids under- or overflow when β is very close to 0 or 1, while the $-\log g_j$ terms come straight from the rating equation $c_j = B_i \, g_j$.

These u_A, u_S, u_E are proportional to the (negative) ordering utilities used in the likelihood; their differences determine the probabilities of each category being ranked first, second, or third.

D.13 Mapping to observed data

Ratings. Parents report a cardinal score on each category, modelled as

$$r_{ik} \sim \mathcal{N}(B_i\, g_k,\, \sigma^2_{\text{rating}}), \tag{A4}$$

so ratings identify *both* B_i and the supply tilts ℓ_1, ℓ_2.

Rankings (ordered probit). Assume i.i.d. $\mathcal{N}(0,1)$ noise on these three latent utilities and a pair of cut-points $\boldsymbol{c} = (c_1, c_2)$. Observed ranks $\text{rank}_{ik} \in \{1, 2, 3\}$ (1 = most important) for person i and category k, and follow the ordered probit:

$$\Pr(\text{rank}_{ik} = 1) = 1 - \Phi(u_{ik} - c_1),\ \Pr(\text{rank} = 2) = \Phi(u_{ik} - c_1) - \Phi(u_{ik} - c_2),\ \Pr(\text{rank} = 3) = \Phi(u_{ik} - c_2), \tag{A6}$$

with u_{ik} equal to the corresponding expression in (**??**).

D.14 The hierarchical parameters estimated in Stan

Block	**Symbol in code**	**Economic meaning**
Preferences	β_{1i}, β_{2i}	Academic vs non-academic, Social vs emotional
Supply (technology)	ℓ_{1i}, ℓ_{2i}	Log–ratios $\log \kappa_A/\kappa_S,\ \log \kappa_S/\kappa_E$
Budget scale	B_i	Parent's effective spending/child endowment
Curvature	$\rho\ (> 1)$	Elasticity of transformation $= 1/(\rho - 1)$
Rating noise	σ_{rating}	Perception / measurement error
Rank cut-points	$c_1 < c_2$	Thresholds in ordered probit

Hierarchical Normal (or LogNormal) priors on the logit- and log-ratio scales allow direct posterior comparison of *demand heterogeneity*—the standard deviations of β_1, β_2—and *supply heterogeneity*—the standard deviations of ℓ_1, ℓ_2. The sign of the empirical ratings–rankings slope in **??** corresponds, in this structural specification, to whether σ^2_β or σ^2_ℓ/ρ dominates in the covariance formula.

Equations (**??**)–(**??**) are implemented line-for-line in the Stan program reproduced below.

```
// ———————— 3-category CET model ————
data {
  int<lower=1> N;                          // Number of parents
  array[N, 3] real<lower=0, upper=100> rating; // ratings [A,S,E]
  array[N, 3] int<lower=1, upper=3> rank; // 1 = best , 2 = middle , 3 = worst
}

parameters {
```

```
  // Preferences
  real mu_logit_beta1;
  real<lower=0> sigma_logit_beta1;
  vector[N] z_beta1;

  real mu_logit_beta2;
  real<lower=0> sigma_logit_beta2;
  vector[N] z_beta2;

  // Supply log-ratios
  real mu_ell1;
  real<lower=0> sigma_ell1;
  vector[N] z_ell1;

  // ell2 is fixed at 0 (kappa_S = kappa_E)
  //real mu_ell2;
  //real<lower=0> sigma_ell2;
  //vector[N] z_ell2;

  // Budgets
  real mu_logB;
  real<lower=0> sigma_logB;
  vector[N] z_logB;

  // Curvature
  //real<lower=1> rho;

  // Rating noise
  real<lower=0> sigma_rating;

  // Ordered-probit cut-points (two)
  ordered[2] cut;
}

transformed parameters {
  // Curvature
  real rho = 1.5;

  vector[N] beta1 = inv_logit(mu_logit_beta1 + sigma_logit_beta1 * z_beta1);
  vector[N] beta2 = inv_logit(mu_logit_beta2 + sigma_logit_beta2 * z_beta2);
```

```
  vector[N] ell1  = mu_ell1 + sigma_ell1 * z_ell1;
  //vector[N] ell2  = mu_ell2 + sigma_ell2 * z_ell2;

  vector[N] B     = exp( mu_logB + sigma_logB * z_logB );

  /* Supply side skill mixes. Note we are assuming ell2 = 0*/
  vector[N] s_A;
  vector[N] s_SE;
  real inv_rho = 1.0 / rho;
  for (i in 1:N) {
    s_A[i]  = exp( inv_rho * ( logit(beta1[i]) ) + ell1[i]  );
    s_SE[i] = exp( inv_rho *   logit(beta2[i])              );
  }
}

model {
  /* Priors */
  mu_logit_beta1 ~ normal(0,1);   sigma_logit_beta1 ~ exponential(1);
  mu_logit_beta2 ~ normal(0,1);   sigma_logit_beta2 ~ exponential(1);

  mu_ell1 ~ normal(0,1);          sigma_ell1 ~ exponential(1);
  //mu_ell2 ~ normal(0,1);          sigma_ell2 ~ exponential(1);

  mu_logB ~ normal(4,1);          sigma_logB ~ exponential(1);

  //rho ~ lognormal(log(1.5),0.1);
  sigma_rating ~ exponential(1);

  /* Likelihood */
  for (i in 1:N) {
    /* pick one normalised point on PPF */
    real g_A = 1;
    real g_E = g_A / ( (s_A[i]) * (1 + s_SE[i]) );
    real g_S = g_E * s_SE[i];

    /* ratings */
    rating[i,1] ~ normal( B[i] * g_A , sigma_rating );
    rating[i,2] ~ normal( B[i] * g_S , sigma_rating );
    rating[i,3] ~ normal( B[i] * g_E , sigma_rating );
```

```
    /* ------ ordered-probit for each category ------ */
    vector[3] util;
    util[1] =  logit(beta1[i]) - log(g_A); // A
    util[2] =  logit(beta2[i]) + log1m_inv_logit(logit(beta2[i])) - log(g_S); //
    util[3] =  log1m_inv_logit(logit(beta2[i])) - log(g_E); // E

    for (k in 1:3) {
      real eta  = util[k];
      vector[3] prob;
      prob[1] = 1 - Phi(cut[2] - eta);            // rank 1 = best
      prob[2] =    Phi(cut[2] - eta) - Phi(cut[1] - eta);
      prob[3] =    Phi(cut[1] - eta);              // rank 3 = worst
      rank[i,k] ~ categorical(prob);
    }
  }
}
```