Costs or Benefits? Why Students Specialize Across Skills and How Teachers Can Respond

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Abstract

When a student is weak in a skill, a key question is why: is it because the skill is hard to learn (costs) or because it has low value to the family (benefits)? Parents have insight into which skills hold value for their child, but large class sizes and informal communication limit teachers' access to this information. In this paper, I provide teachers with structured parent information through a field experiment. I survey 3,404 parents across five private schools in India to measure parents' perceptions of their children's skill levels and preferences for improvement across academic and socioemotional domains. Parents vary in their preferences over which skills to improve, but on average prefer improving their children's weaker skills. I develop a structural model of skill development showing that this pattern indicates learning costs, rather than family values, primarily drive observed specialization. I elicit teachers' beliefs about parent preferences and find little alignment with actual parent views, even at the classroom level. I randomize teacher access to parent survey data via a web portal. Treatment shifts student specialization toward parent-prioritized skills, with larger effects where baseline teacher beliefs were most inaccurate. Structural estimation corroborates these patterns and enables policy counterfactuals quantifying welfare gains from better cost-benefit alignment. The results demonstrate that structured parent feedback enables teachers to target instruction toward what families value most.

JEL: C93; D83; I21; I31; J24; O15

Keywords: parental preferences; skill development; socioemotional skills; teacher beliefs; information frictions; education policy; India; field experiment

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1 Introduction

Educational policy has long focused on increasing the rate of students' skill growth, using outcomes such as test scores or teacher value-added as benchmarks for success (Rivkin et al., 2005; Hanushek and Woessmann, 2008; Glewwe and Muralidharan, 2016). Yet if students and families vary in what skills they value most, then choosing which skills to promote—the direction of skill growth—may be equally important for designing welfare-maximizing policy. For example, a student who aims to be a business leader may prefer to improve their teamwork and leadership skills over math and reading skills. This issue has grown in relevance as educational goals expand beyond foundational academic skills to include socioemotional and other noncognitive skills (Heckman et al., 2006; Kosse and Tincani, 2020; Deming and Silliman, 2025).

In this paper, I examine how teachers can better align the direction of student skill growth with those skills students and families value most. Teachers must allocate limited time and resources across many skills, and they often lack structured information about which skills benefit each student. These benefits depend on hard-to-observe features such as students' home environments, peer groups, and future aspirations. Teachers who lack this information often rely on observable skill levels to guide instruction, such as remedial education that targets weak skills, or gifted education that builds on students' strengths. This practice speaks to a long standing debate in education: should educators promote well-rounded or specialized students (Rosen, 1983; Câmara Leme et al., 2020; Mulhern et al., 2024; Kemper and Renold, 2024). Students' skill levels alone cannot resolve the quandary: teachers targeting weak skills may be focusing on low-value (hence neglected) skills, or targeting high-value skills that are weak because they are hard to learn.

Parents often have insight teachers need regarding what skills are high value for their children. However, information about students typically flows in the direction of schools to parents, both in standard operations (e.g., report cards and parent-teacher meetings) and in interventions (e.g., parental engagement programs and messaging campaigns)[1]. This directional flow of information may leave teachers without structured information about what skills benefit each student, and may thus limit their ability to align instruction with student needs.

I address this challenge by increasing the flow of information from parents to teachers. I survey 3,404 parents and 242 teachers across five private schools in India (grades 1-10) to gather structured information on the skill-based needs of students. Parents assess their children's current skill levels on a scale of 0-100 across nine academic, emotional, and social domains. To capture specialization, I standardize skill levels within-student so that positive values indicate strengths and negative values indicate weaknesses. Parents then report skill preferences by ranking skills from most to least important to improve. I then elicit teachers' *beliefs* about the skill preferences of parents. I show that teachers' beliefs are largely uncorrelated with actual parent views, even at the classroom level. Motivated by this misalignment, I randomize whether teachers receive access to their classroom's parent survey data (i.e., parent-reported skill levels and skill preferences) through a custom-built

[1] See Hastings and Weinstein (2008); Andrabi et al. (2017); Bergman (2021); Bergman and Chan (2021); Berlinski et al. (2022) for examples of information interventions targeting parents.

web portal.

To understand why this information matters, I develop a model that centers parents and teachers as key decision-makers. Parents maximize their children's utility by choosing inputs to produce multiple skills: they balance the ease with which their children build each skill (costs) with the value of having that skill (benefits). Teachers, with aligned incentives, shift the effective costs of learning in the classroom, thereby changing the production function faced by parents. However, teachers must infer parents' preferences from noisy signals. Time and effort may be shifted away from the highest-value margins when teachers misperceive what parents value, as I document in my context.

The model yields a simple insight: whether parents prefer improving weak or strong skills helps diagnose why students specialize. This stems from a familiar economic principle—marginal benefits equal marginal costs for each skill, when optimizing across skills. Consider first the case of pure cost-driven specialization, in which skills only vary with regard to how difficult they are to learn. Students thus specialize in skills that are easier to build. Because marginal benefits decrease, these easy-to-learn strengths offer little remaining value to improve further, and thus parents prefer to improve weaker skills. Conversely, if skills only vary with regard to how they are valued, students specialize in high-value skills; these skills incur increasing marginal costs and are thus more costly to further improve. Because marginal benefits must equal these high marginal costs, parents continue to prefer improvement in strong skills.

I use the data from the parent survey to examine if skill specialization is primarily cost- or benefit-driven in this setting. I regress skill preferences on skill levels to capture if parents prefer improving weak or strong skills. The slope, hereafter the preference-level slope, is negative when parents prefer improving weaker skills, and positive when they prefer improving stronger skills. Across students and within-skill, I find a strong negative preference-level slope for all nine skills. For a given skill, parents that perceive low skill levels for their children consistently rank that skill as more important to improve relative to other parents. Students in higher grades largely drive this pattern: skill levels and skill preferences are nearly uncorrelated for primary grades and become increasingly negatively correlated by grade 10. Through the lens of the model, this implies costs (i.e., how easy or hard it is for children to learn skills) become increasingly important relative to benefits in determining specialization patterns as students age. This is consistent with dynamic complementarity in skill formation (Cunha and Heckman, 2007), where early gaps in skill levels lead to widening differences in the costs of acquiring new skills over time.

Within each student, I find parents predominantly want improvement in their children's weaker skills, but this varies widely: many parents prefer for their children to improve strengths. In fact, when the preference-level slope is more positive, parents report greater satisfaction with their children's progress in school. In the model, more positive alignment between marginal benefits and skill levels represents a case in which a student's natural path of skill growth becomes more aligned with the direction that would raise utility fastest. This suggests that when parents see the direction of their child's skill growth as aligned with what they value, they are more satisfied with their child's progress. Supporting this interpretation, the preference-level slope is uncorrelated with parent satisfaction with school quality, suggesting that it reflects satisfaction with dynamic growth, rather than static features of the school environment.

Turning to teachers, I ask them to rank the same nine skills for a typical student, and then for specific randomly sampled students in their class. Crucially, for the specific students, I also elicit teachers' beliefs about each parent's skill preferences. I find that teacher beliefs are largely uncorrelated with parents' actual preferences, both at the individual-student level and in classroom averages, and instead reflect teachers' own priorities——consistent with teachers projecting their own preferences onto parents. This misalignment demonstrates potential misallocation of teacher effort across skills and students.

Motivated by these findings, I design and implement a randomized experiment that provides teachers with access to parents' skill levels and skill preferences. Randomization occurs at the teacher level; treated teachers receive individual login credentials to a custom-built website displaying information for students in their homeroom section.[2] In the context of my model, this intervention acts as a production-side shock, leading teachers to lower the effective cost of improving parent-prioritized skills. The model yields three testable predictions: for students whose teachers were initially misaligned, treatment should (i) increase specialization in parents' most preferred skill (i.e., empirically, raise skill levels), (ii) reduce the marginal benefit for that skill (i.e., lower the skill preference rank), and (iii) increase the within-student alignment between marginal benefits and skill levels (i.e., result in a more positive preference-level slope).

I find that providing parent preference information to teachers shifts their beliefs toward the true classroom average. Treated teachers' average beliefs (across parents in their class) become more correlated with actual parent rankings, with suggestive evidence of roughly 10 percentage point improvements in accuracy in identifying which skill category (academic, social, emotional) is most or least valued on average, in their classroom. In particular, they learn where academics sits in the order of valued skills. However, teachers do not become better at predicting which specific parents hold these preferences. This pattern aligns with the model's policy logic: since teacher effort is applied at the classroom level, the ability to correctly identify classroom-average preferences can increase the rate of welfare growth, even if teachers do not perfectly know individual values.

The experiment produces results consistent with the model predictions. First, giving teachers data concerning parents' assessment of their children's skill levels and skill preferences changed how students specialized. Average effects on skill levels are near zero, but this masks substantial heterogeneity by baseline teacher accuracy. When teachers initially held inaccurate beliefs about a parent's priorities, treatment increased skill levels in parents' most-preferred skills and decreased skill levels in their least-preferred skills. Conversely, when teachers were initially accurate, treatment reduced skill levels in parents' most-preferred skills and increased skill levels in their least-preferred skills. This result is consistent with teachers applying effort at the classroom level, reallocating effort away from students they already understood and toward students they

[2]In this setting, teachers are assigned one section of students during the 0-period (homeroom), where they are responsible for student guidance and are the point of contact for parents (e.g., they meet parents during parent-teacher meetings). Teachers then rotate into other classrooms throughout the day to teach specific subjects.

previously misjudged. Second, parents' initial top priorities become less urgent to improve: for the skill parents most wanted to improve at baseline, treated parents' endline skill preference falls by 0.26 ranks compared to control, implying that the marginal benefit declined. Third, treated students' skill levels become more aligned with parents' skill preferences: compared to the control, the preference-level slope increases by about 0.15 on average (from a base slope of -0.7) and by 0.26 in classrooms where parents most preferred improving academic skills. All three effects are strongest in treated classrooms where parents on average preferred improving academic skills; this result is consistent with teachers having more scope to shift efforts in the academic domain. Taken together, these findings show that providing teachers with structured information can act a production shock: in other words, teachers' classwide efforts are reoriented toward parents' most-preferred skills. Parents' perceived need for improvement in those areas is reduced, and specialization is shifted away from what is easiest to build and toward what parents value most.

To translate the framework into policy terms, I estimate the structural model using Bayesian methods. This lets me recover the underlying costs and benefit parameters that generate observed specialization, and then simulate counterfactuals. I replicate a key result by estimating ratio of cost variance to benefit variance, which measures the relative importance of production constraints versus preference heterogeneity in driving specialization. I find that production constraints vary approximately 21% more across families than do preferences for academic versus socioemotional skills. I then simulate the policy counterfactual of perfectly aligning production costs with parental preferences. This exercise shows welfare gains are modest on average, at around one percent, suggesting most families are not far from their optimal skill mix given the production constraints they face.

The key contribution of the framework is diagnostic. Observing whether parents prefer improving weak or strong skills helps identify if barriers to learning or perceived benefits drive specialization. A teacher can use the diagnostic: for example, if parents of struggling math students consistently report preferring improvement in math, the diagnostic indicates that high barriers to learning math, not low perceived value, are driving math weakness for these students. In this setting, I find that uneven learning costs primarily driving specialization. This is consistent with interventions in primary and secondary education that tend to target production-side barriers: teacher training, additional school resources, and intensive instructional models that help students overcome learning difficulties (Jackson et al., 2016; Muralidharan and Sundararaman, 2013; Burgess et al., 2023; Fryer Jr, 2017). In contrast, in contexts where perceived returns to skills likely drives specialization, such as in higher education, or in the labor market, interventions often accordingly target the benefit side: providing students information about earnings across majors and occupations, role models demonstrating career opportunities in different fields, and career guidance counselors helping students to understand returns to specialization (Jensen, 2010; Wiswall and Zafar, 2015; Porter and Serra, 2020; Hoxby and Turner, 2015; Conlon, 2021).

Beyond identifying the source of specialization, this framework suggests how teachers can personalize education by providing instruction based on student-varying costs and benefits to skills. This represents a theory-guided complement to "Teaching at the Right Level"(Banerjee et al., 2017): personalize not only by skill levels, but also by values. This is particularly relevant for educational aims that go beyond foundational skill development, where level-based grouping may be sufficient if all parents similarly value basic literacy and numeracy. However, broadening educational goals to diversify or specialize skill development requires accounting for large variation in preferences over these skills. In these contexts, providing teachers with structured information on parent preferences may help align instructional effort with welfare-maximizing aims.

Contributions to the Literature This benefits-vs-costs question is central but empirically underidentified in most educational settings, despite related evidence on the technology of skill formation and dynamic complementarity (Cunha and Heckman, 2007; Cunha et al., 2010), on the heterogeneous returns to cognitive and socioemotional skills (Heckman et al., 2006; Lindqvist and Vestman, 2011; Deming, 2017; Kosse and Tincani, 2020; Deming and Silliman, 2025), on parental preferences over teacher attributes (Jacob and Lefgren, 2007; Jackson, 2018), and on resource and productivity constraints in schools (Jackson et al., 2016; Muralidharan and Sundararaman, 2011, 2013; Burgess et al., 2023; Fryer Jr, 2017). The work of Cotton et al. (2025) is closely related to my study: the authors estimate a structural model of student learning to distinguish motivation from productivity as drivers of student study time. They find that low productivity, rather than low motivation, predicts academic struggles[3]. My study complements this work by distinguishing costs from benefits as drivers of skill specialization across multiple dimensions, and by examining the supply side (i.e., teacher allocation) rather than the demand side (i.e., student effort). It also relates to literature on parental preferences and beliefs (Jacob and Lefgren, 2007; Dizon-Ross, 2019) and on information interventions that primarily inform parents (Hastings and Weinstein, 2008; Bergman, 2021; Bergman and Chan, 2021; Berlinski et al., 2022).

I contribute to the literature along five margins. First, I provide new measurements for the literature on parental preferences and beliefs by directly eliciting, for the same child, her parents' perceived skill levels and skill preferences for improvement across nine cognitive and socioemotional dimensions (Jacob and Lefgren, 2007; Dizon-Ross, 2019). I go beyond documenting preferences by linking skill levels to marginal benefits to provide a diagnostic for the underlying source of specialization. Second, I contribute to the literature on the technology of skill formation by providing a tractable framework to distinguish benefit- from cost-driven heterogeneity, and by modeling teachers as endogenous cost-shifters who respond to information (Cunha et al., 2010). Third, I structurally estimate the model using Bayesian methods, leveraging teacher skill rankings as supply-side cost shifters to recover primitives governing skill formation. The estimated model enables policy counterfactuals that compare alternative approaches to personalizing instruction. Fourth, my framework informs work on the heterogenous benefits to cognitive and non-cognitive skills by offering a way to interpret observed specialization and thus assists understanding of when heterogeneous benefits are likely to be the more important driver of investment choices (Heckman et al., 2006; Lindqvist

[3]While Cotton et al. (2025) focus on why students do not complete homework (i.e., they examine motivation vs. productivity in converting time to learning), I examine why students specialize across skills (i.e., I examine costs vs. benefits driving comparative advantage). Their framework clarifies intensive margin decisions for a single skill; my framework clarifies extensive margin allocation across multiple skills.

and Vestman, 2011; Deming, 2017; Kosse and Tincani, 2020; Deming and Silliman, 2025).

Fifth, I contribute to the literature on information interventions that primarily inform parents about their child's behavior (Hastings and Weinstein, 2008; Bergman, 2021; Bergman and Chan, 2021; Berlinski et al., 2022). My experiment, however, reverses the typical information flow. I inform teachers about parent priorities that serve as a proxy for students' marginal benefits to skill development, a parameter where parents are plausibly better informed, in contrast to academic levels where evidence shows parental perceptions can be inaccurate. Despite my limited statistical power, my experimental results provide validation that the structural model's mechanisms operate in practice. This demonstrates that reducing uncertainty on the school side can be a powerful lever for change.

The paper proceeds as follows. Section 2 describes the Indian school context, including the process for recruitment and list of human capital dimensions. Section 3 presents the model of multidimensional skill formation distinguishing benefit- from cost-driven specialization and models teachers as key inputs to skill production. Section 4 describes the data from the parent survey and reports initial descriptives. Section 5 presents results on benefit- vs cost-driven specialization. Section 6 describes the teacher-facing information experiment and its implementation. Section 7 discusses the impact of information on teacher beliefs about parent priorities, and Section 8 reports impacts on key student outcomes. Section 9 structurally estimates the model and presents policy counterfactuals. Section 10 concludes.

2 Setting

2.1 Context

This study takes place in five private schools located across four states in India: Delhi (two schools), Gujarat, Punjab, and West Bengal. These schools serve middle- to upper-middle-class families; approximately 90% of students come from families with household incomes above the national median. All schools span from nursery to 12th grade. For this study, I focus on students in grades 1-10.

Tuition fees at these schools range from approximately \$350 per year at the least expensive school to \$2,500 per year at the most expensive school: this range of tuition fees across the schools resembles the range of costs across the bulk of low- to middle-income private schools in the Indian context. This setting is not an outlier: it is in fact quite representative of a large and growing segment of education in India and in other developing countries. Private schools have become increasingly prevalent across low-income countries with enrollment shares rising from 11 to 22 percent between 1990 and 2010 (Baum et al., 2014); in India specifically, enrollment shares rose to 39% in 2024 (UDISE+, 2024-25), representing about 100 million students.

Class sizes in these schools typically range from 30 to 40 students per classroom. This creates significant challenges for teachers with regard to providing individualized attention to students. Teachers often have to balance the wide variation in parental values and expectations for their children as parents pay fees; in some cases, teacher compensation is explicitly tied to parental feedback. Across all five schools, parent-teacher meetings occur either twice a year (i.e., once per term), or every month. These meetings provide opportunities for teachers to share information about student progress with parents, and also for parents to share their perceptions with teachers. However, the format and content of these interactions vary widely. Most of the schools in this study provide report cards that focus primarily on academic subjects, but at least one school includes noncognitive skills on the report card.

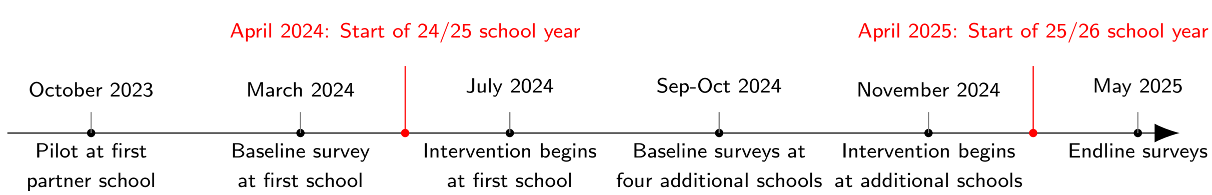
Despite the regular occurrence of parent-teacher meetings, teachers face numerous information frictions that limit their ability to incorporate parent preferences into their decision-making. These frictions include large class sizes, varying parental communication styles, and the difficulty of aggregating information from multiple parents. This context where parent input is valued, but potentially undersupplied due to structural constraints, provides an ideal setting in which to examine how structured information about parent preferences can influence teacher decision-making and student outcomes.

2.2 Sample and Recruitment

The recruitment process involved a self-directed search for schools interested in participating in a research study focused on understanding the varied needs of their students and accommodating parent perspectives through structured information aggregation. I conducted recruitment throughout 2023 and 2024 and presented the project as an opportunity for schools to better understand parent preferences and perceptions and to better align teaching practices with school aims. The recruitment process led to partnerships with five private schools across four states in India.

The study began with a pilot phase in October 2023 at the initial partner school in West Bengal. Following a successful implementation of the pilot, I conducted a full baseline survey at this school in March 2024. I launched the intervention by sharing information with treated teachers in July 2024. Concurrent with the implementation at the first school, I recruited additional schools throughout the summer of 2024. I successfully onboarded four more schools and conducted baseline surveys at these institutions in September and October 2024. Treated teachers at these schools received access to the website containing parent preference information in November 2024; this moment marked the beginning of the intervention phase for these schools.

Figure 1: Study Timeline



Due to prior commitments, two schools were unable to continue their participation after the baseline phase. Although I collect and analyze endline data for all three remaining schools, I follow the preanalysis plan with regard to only estimating treatment effects using endline data for schools in which treatment compliance was above 15% (measured as the percent of treated teachers who view the website with parent data at least once). This threshold was set ex ante to ensure that analysis was restricted to contexts where teachers sufficiently engaged with the intervention to potentially affect student outcomes. Despite regular reminders and school visits, this threshold was only surpassed by teachers in the initial partner school. As a result, the baseline sample includes 3404 parents surveyed in 242 classrooms across the five schools, and the final experimental sample includes 849 students across 106 classrooms in the initial partner school.

The reduction from baseline to endline within the partner school reflects the voluntary nature of survey completion rather than systematic attrition. Both baseline and endline surveys were administered during parent-teacher meetings, with participation incentivized only by informing parents that their responses would help teachers better target instruction for their child. Approximately one-quarter of the school's roughly 4,000 parents completed surveys at each wave, with substantial turnover in which families participated. To verify that this pattern does not threaten internal validity, I regress an indicator for having complete data (non-missing values for all baseline and endline outcomes) on treatment assignment and baseline covariates. Appendix Table B.1 shows no significant relationship between treatment status and survey completion, nor do treatment effects on completion vary by parents' baseline skill preferences. This null result indicates that sample composition is balanced across treatment and control classrooms. Full descriptive statistics for the baseline and experimental samples are provided in Appendix Table B.2.

2.3 Skill Dimensions

The study focuses on nine dimensions of human capital that encompass both cognitive and noncognitive skills. The skills are divided into three broad categories: academic, social, and emotional skills. The nine skill dimensions are shown below in Table 1.

I developed the set of skills through an iterative process. The initial list of skills contained 21 potential skill dimensions designed to be comprehensive in covering any skill a parent may care about. Skill dimensions were drawn from literature on educational frameworks, psychology, and through discussion with parents prior to the start of the study. Through pilot testing with parents and teachers, I narrowed the list down to nine dimensions in order to balance the time constraints of administering the survey with the comprehensiveness of the final list. The selection process prioritized dimensions that (1) were comprehensible to parents without specialized knowledge, (2) covered a range of both cognitive and noncognitive domains, and (3) were potentially actionable by both teachers and parents. Parents were provided the exact image shown in Table 1 to promote consistency in parents' understanding of the dimensions. For each dimension, parents were asked to rate their child's current standing on a scale from 0 to 100 (skill levels), and then to rank the nine dimensions in order of importance for improvement (skill preferences); I describe this further in Section 4.

Table 1: List of Skill Dimensions

Category	Aspect	Explanation
Academic	**Literacy skills**	Reading, writing, speaking, and listening
Academic	**Mathematical skills**	Numeracy, quantitative reasoning, problem solving
Academic	**Scientific literacy**	Understanding scientific concepts and processes
Social	**Collaboration and teamwork skills**	Ability to work effectively with others and contribute to group goals
Social	**Interpersonal skills**	Effective communication, conflict resolution, recognizing and responding to social cues
Social	**Leadership and initiative**	Taking charge, setting goals, motivating others
Emotional	**Perseverance and growth mindset**	Resilient in the face of challenges, belief in self improvement
Emotional	**Emotional self-awareness and regulation**	Recognizing and managing emotions, thoughts, and behaviors
Emotional	**Empathy for others**	Understanding and valuing perspectives of others

Notes: Parents and teachers are asked to evaluate nine human capital dimensions for their students. Dimensions were chosen to cover a range of cognitive and non-cognitive skills. The displayed table was shown to both parents and teachers in their respective surveys to promote consistency in understanding each dimension.

3 Model

To understand how students develop skills and how teachers can reorient the direction of skill growth, I develop a model of multidimensional skill production. This model helps to distinguish whether differences in students' skill profiles are driven primarily by variation in benefits (in terms of satisfaction, money, etc.) for different skills or by production cost differences in producing them. I focus on two types of variation: (i) variation across students within a skill (i.e., why some students specialize in a given skill) and (ii) variation across skills within a student (i.e., why some skills are relatively strong or weak for a given student).

I proceed in two steps. First, I introduce parents whose objective is to maximize their child's utility. Parents have imperfect beliefs about the costs and benefits to skills, but their beliefs contain useful signal about the true parameters. Based on these beliefs, parents allocate a fixed budget towards purchasing inputs for skill development, leading to students' observed specialization patterns. I interpret the expansion path (i.e., the path of optimal bundles as the budget increases) as how skills would change over time, absent changes to parents' perceived costs or benefits from skills. This interpretation yields two types of students: (A) students who would derive higher utility from more well-rounded profiles relative to the expansion path, and (B) students who would derive higher utility from more specialized profiles relative to the expansion path.

Second, I introduce teachers with aligned incentives; they also aim to maximize student welfare. Teachers do not choose inputs, but instead allocate effort towards reducing the costs of building skills. Thus, they change the production function faced by parents. Teachers hold their own imperfect beliefs about the costs and benefits to skills, for each student. When those beliefs are incorrect, teachers misallocate effort. Parent's preferences thus serve as a signal of the true benefits to skills, motivating my information experiment. Critically, teachers shift the production frontier at the classroom-level. This allows me to interpret the expansion path as the classroom production path, and the two types of students as those who would benefit from teachers shifting classroom production toward well-roundedness (type A) or specialization (type B).

3.1 Parents' Problem

Skills. Parents derive utility from their child's skill bundle $\boldsymbol{c} = (c_1, c_2) \in \mathbb{R}^2_+$, where c_1 and c_2 denote two distinct skill domains (e.g., cognitive and noncognitive). We focus on two skills for simplicity, but the model extends naturally to three or more skills. We define the child's level of *specialization* (in skill 1, without loss of generality) as the ratio of the skill levels, $s_i := c_{1i}/c_{2i}$.

Preferences. Parent i has Cobb-Douglas utility[4]

$$U(c_{1i}, c_{2i}; \beta_i) = c_{1i}^{\beta_i}\, c_{2i}^{1-\beta_i}, \qquad 0 < \beta_i < 1.$$

The marginal rate of substitution (MRS) between skills 1 and 2 is

$$\mathrm{MRS}_{12,i} = \frac{\beta_i}{1-\beta_i} \cdot \frac{c_{2i}}{c_{1i}} = \frac{\beta_i}{1-\beta_i} \cdot \frac{1}{s_i}. \tag{1}$$

Budget and technology. Parents buy inputs (x_{1i}, x_{2i}) at prices (p_{1i}, p_{2i}) subject to $p_{1i}x_{1i} + p_{2i}x_{2i} \leq I_i$. Each skill is produced via single-input technologies with diminishing marginal products:

$$c_{1i} = a_{1i}x_{1i}^{\theta}, \quad c_{2i} = a_{2i}x_{2i}^{\theta}, \quad a_{ji} > 0, \quad 0 < \theta < 1.$$

Eliminating (x_1, x_2) yields a smooth, strictly concave frontier in (c_1, c_2) space:

$$p_{1i}\left(\frac{c_{1i}}{a_{1i}}\right)^{1/\theta} + p_{2i}\left(\frac{c_{2i}}{a_{2i}}\right)^{1/\theta} = I_i.$$

Feasible set. Let $\rho := 1/\theta > 1$ and define

$$\kappa_{1i} := a_{1i}\left(\frac{I_i}{p_{1i}}\right)^{1/\rho}, \quad \kappa_{2i} := a_{2i}\left(\frac{I_i}{p_{2i}}\right)^{1/\rho}.$$

[4] I use Cobb-Douglas preferences for transparency and tractability. All qualitative results (diagnostic, comparative statics) continue to hold under smooth, strictly increasing, and strictly quasiconcave preferences. The closed-form expressions are specific to Cobb-Douglas.

The frontier becomes the constant-elasticity-of-transformation (CET) form[5]:

$$\left(\frac{c_{1i}}{\kappa_{1i}}\right)^{\rho} + \left(\frac{c_{2i}}{\kappa_{2i}}\right)^{\rho} = 1, \quad \rho > 1. \tag{2}$$

Marginal rate of transformation. Implicit differentiation of (2) yields the marginal rate of transformation (MRT) between skills:

$$\mathrm{MRT}_{12} = \left(\frac{\kappa_{2i}}{\kappa_{1i}}\right)^{\rho} \left(\frac{c_{1i}}{c_{2i}}\right)^{\rho-1}. \tag{3}$$

Note that κ_{ji} fully captures the effective cost of producing skill j for parent i, and is exactly the intercept of the frontier on the c_j axis.

Equilibrium skill ratio. Utility maximization subject to (2) equates (1) and (3), giving the optimal skill ratio:

$$s_i^* := \frac{c_{1i}^*}{c_{2i}^*} = \underbrace{\left(\frac{\beta_i}{1-\beta_i}\right)^{1/\rho}}_{T_i} \underbrace{\frac{\kappa_{1i}}{\kappa_{2i}}}_{\lambda_i}. \tag{4}$$

This equilibrium condition is shown graphically in Figure 2A. The equation illustrates how the variation in skill ratios across parents can be decomposed into two components: variation in the *benefits tilt* $T_i = \left(\frac{\beta_i}{1-\beta_i}\right)^{1/\rho}$, which reflects how much more a parent values skill 1 relative to skill 2, and variation in the *costs tilt* $\lambda_i = \frac{\kappa_{1i}}{\kappa_{2i}}$, which reflects the relative costs or ease of producing skill 1 compared to skill 2.

Comparative statics. Consider the following elasticities of the optimal skill ratio with respect to the primitives:

$$\frac{\partial \ln s_i^*}{\partial \beta_i} = \frac{1}{\rho}\left(\frac{1}{\beta_i} + \frac{1}{1-\beta_i}\right) > 0, \qquad \frac{\partial \ln s_i^*}{\partial \ln \kappa_{1i}} = 1, \qquad \frac{\partial \ln s_i^*}{\partial \ln \kappa_{2i}} = -1.$$

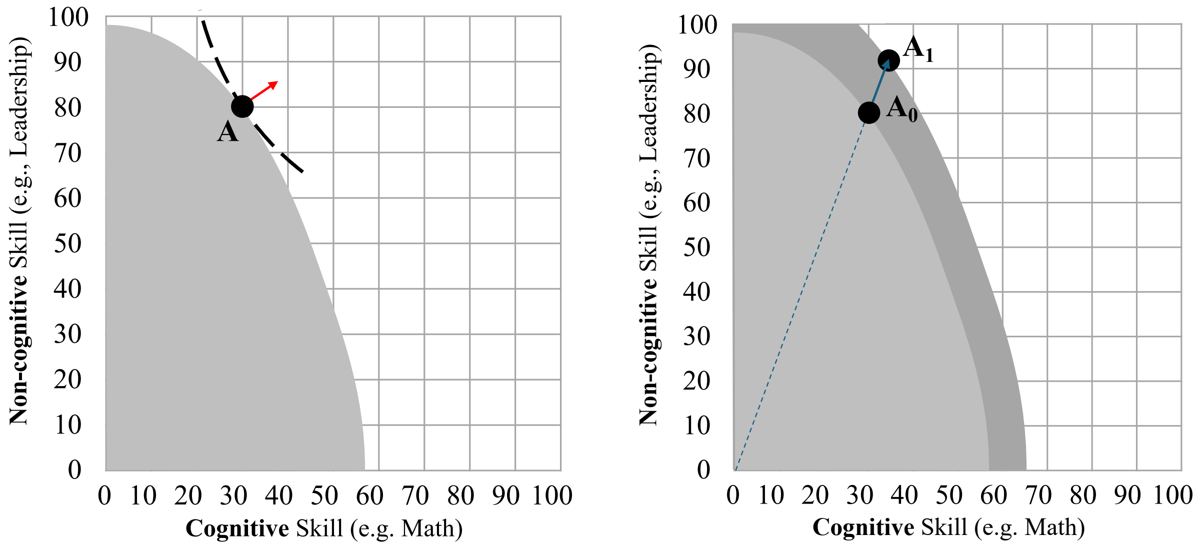
Thus, an increase in the relative benefit to skill 1 ($\beta_i \uparrow$), or a decrease in the relative cost of producing skill 1 ($\kappa_{1i} \uparrow$ or $\kappa_{2i} \downarrow$) raises the optimal specialization s_i^*.

Skill growth over time. In this model, parents' budgets consists of time, money, and other resources that they can invest in their child's skill development. Consider additional time (e.g., one more year) to spend on skill development. We model this as a *pure budget expansion*: in each period the family has more effective resources I_i. With homothetic preferences and the CET frontier, if benefits (β_i) and costs (κ_{1i}, κ_{2i}) are fixed, optimal levels scale up with income, but proportionally

[5] The CET frontier allows for non-unit elasticity of transformation between skills. Multi-output production functions with constant elasticity of transformation are widely used; this single-period CET mirrors the multistage production functions in Cunha and Heckman (2007, 2008); Cunha et al. (2010). I abstract from dynamic self-productivity and cross-period complementarity to focus on the question of whether observed heterogeneity is driven by relative costs (κ_{1i}, κ_{2i}) or by relative benefits (β_i).

so that specialization, $s_i^* = c_{1i}^*/c_{2i}^* = T_i\lambda_i$, remains constant. This is shown graphically in Figure 2B. The expansion path will allow us to distinguish two types of students, and will be key to understanding how teachers can influence skill growth over time.

Figure 2: Parents' Problem and Skill Growth



(A) Parents' static problem

(B) Expansion path over time

Two types: (A) Desire well-roundedness or (B) Desire specialization. Fix student i at $c_i = (c_{1i}, c_{2i})$ with specialization level $s_i = c_{1i}/c_{2i}$ and

$$\mathrm{MRS}_{12,i} \;=\; \frac{\beta_i}{1-\beta_i} \cdot \frac{1}{s_i}.$$

With β_i and $(\kappa_{1i}, \kappa_{2i})$ fixed, income growth moves the optimum with constant slope $(dc_2/dc_1)_I = 1/s_i$. Contrast this with the direction of *maximal utility growth*, which is normal to the indifference curve (or equivalently normal to the frontier), hereafter denoted as the *IC-normal* direction. The IC-normal has slope $1/\mathrm{MRS}_{12,i}$, and is denoted in red in Figure 2A.

Without loss of generality, fix the weaker skill level to be on the horizontal axis so that $s_i < 1/2$. We define:

- **Type A:** expansion path is *steeper* than the IC normal,

$$\frac{1}{s_i} \;>\; \frac{1}{\mathrm{MRS}_{12,i}} \quad\Longleftrightarrow\quad \mathrm{MRS}_{12,i} \;>\; s_i,$$

  in other words, marginal utility puts more weight on the *weaker* skill 1 relative to the current level of specialization.

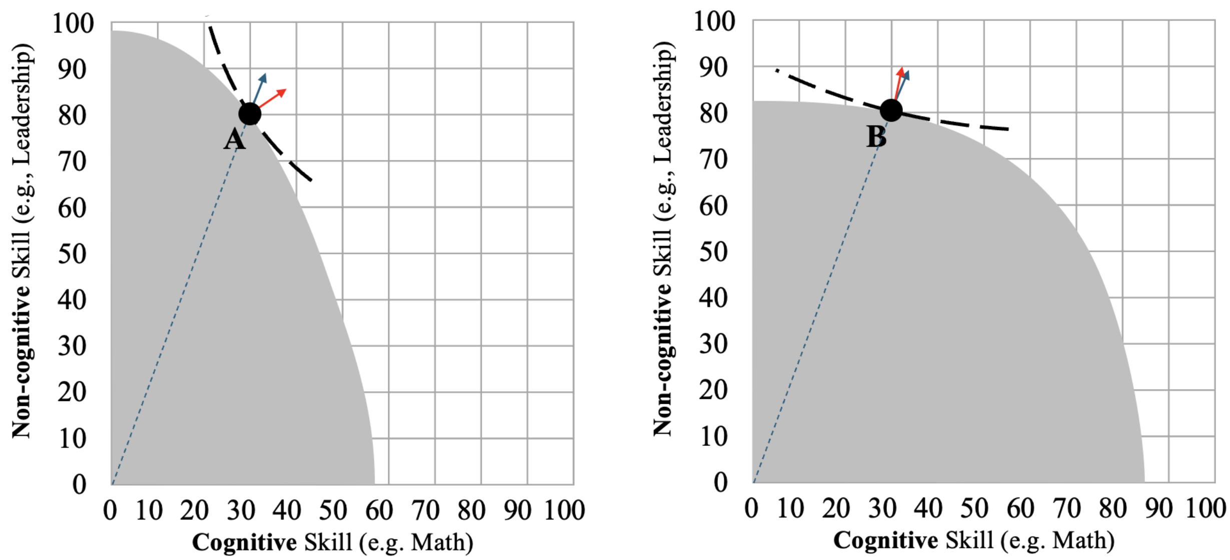
- **Type B:** expansion path is more *shallow* than the IC normal,

$$\frac{1}{s_i} < \frac{1}{\text{MRS}_{12,i}} \quad \Longleftrightarrow \quad \text{MRS}_{12,i} < s_i,$$

i.e., marginal utility favors further improvement in the *stronger* skill relative to the current level of specialization.

Recall that $\text{MRS}_{12,i} = \beta_i/(1-\beta_i) * s^{-1}$. Therefore, considering the in-between case where $\text{MRS}_{12,i} = s_i$ allows us to define the threshold ratio $s^\dagger(\beta_i) := \sqrt{\beta_i/(1-\beta_i)}$. A student is of type A if and only if $s_i < s^\dagger(\beta_i)$ and of type B if and only if $s_i > s^\dagger(\beta_i)$. Intuitively, if the student is relatively weak in skill 1 (s_i small) but the parent places high value on skill 1 (β_i large), then the student is type A and would benefit from further well-roundedness. Conversely, if a student's relative weakness is not *too* weak relative to values ($1/2 > s_i > s^\dagger(\beta_i)$), then the student is type B and would benefit from further specialization. We show Type A and B graphically in Figure 3.

Figure 3: Students that benefit from (A) well-roundedness or (B) specialization



(A) Type A: Desire well-roundedness

(B) Type B: Desire specialization

Note that type A and B can be applied to *either* students or skills. For a given student, type A (B) students derive more utility from improving weaker (stronger) skills. For a given skill, type A (B) skills are ones in which the largest utility gains would come from boosting the weaker (stronger) students in that skill.

Diagnostic for type A or B *skills*. I showed above that type A is characterized by environments in which higher marginal benefits are associated with relative weaknesses (i.e., low skill levels), and type B by environments in which higher marginal benefits are associated with relative strengths (i.e., high skill levels). Hence, I propose the **preference-level slope** as a sufficient statistic for capturing this relationship between marginal benefits and relative levels. Consider the regression across students for a given skill (here, skill 1):

$$\mathrm{MRS}^*_{12,i} = \alpha + \beta^{\mathrm{RF}}\, s_i^* + \varepsilon_i, \tag{5}$$

with population slope

$$\beta^{\mathrm{RF}} \;=\; \frac{\mathrm{Cov}_i(s^*, \mathrm{MRS}^*)}{\mathrm{Var}_i(s^*)}. \tag{6}$$

Using the primitives from Section 3.1, optimal specialization is the product of the benefit and cost tilts so that

$$s_i^* \;=\; T_i\, \lambda_i, \qquad \mathrm{MRS}^*_{12,i} \;=\; T_i^{\rho-1}\, \lambda_i^{-1},$$

so

$$s_i^* \cdot \mathrm{MRS}^*_{12,i} = T_i^{\rho}, \tag{7}$$

which cancels λ_i and separates *benefits* (T_i) from *costs* (λ_i). It follows that

$$\mathrm{Cov}(s^*, \mathrm{MRS}^*) = \mathrm{Cov}(T_i\lambda_i, T_i^{\rho-1}\lambda_i^{-1}) = \mathbb{E}[T_i^{\rho}] - \mathbb{E}[T_i\lambda_i]\,\mathbb{E}[T_i^{\rho-1}\lambda_i^{-1}]. \tag{8}$$

Writing mean-zero deviations $\tilde{T}_i := T_i - \mathbb{E}[T]$ and $\tilde{\lambda}_i := \lambda_i - \mathbb{E}[\lambda]$, a first-order expansion around means yields

$$\beta^{\mathrm{RF}} \;=\; \frac{(\rho-1)\,\mathrm{Var}(\tilde{T}) \;-\; \mathrm{Var}(\tilde{\lambda})}{\mathrm{Var}(\tilde{T}) \;+\; \mathrm{Var}(\tilde{\lambda})}. \tag{9}$$

so that the sign and magnitude of the slope depends on the relative sizes of the variance in benefit and cost tilts. As dispersion in costs increases, the slope becomes more negative (Type A), and as dispersion in benefits increases, the slope becomes more positive (Type B).

This derivation shows that asking whether a student or skill is closer to Type A or Type B is closely related to asking if relative strengths and weaknesses are primarily driven by dispersion in costs or benefits. Intuitively, Type A students that desire more well-rounded profiles exist in environments where weakness is due to high costs of production, not a low value for that skill. Type B students that desire more specialized profiles exist in an environment where the strengths are primarily due to high value for that skill, not ease of production.

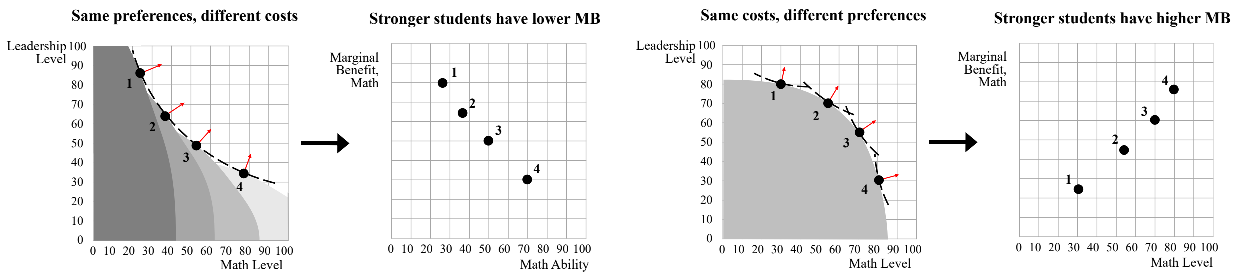
Extreme cases and interpretation. Two polar cases benchmark (9):

(i) **No cost variation** ($\mathrm{Var}(\tilde{\lambda}) = 0$): $\beta^{\mathrm{RF}} = \rho - 1 > 0$ when heterogeneity is purely benefit-driven.

(ii) **No benefit variation** ($\mathrm{Var}(\tilde{T}) = 0$): $\beta^{\mathrm{RF}} = -1$ when heterogeneity is purely cost-driven.

In intermediate cases $\beta^{\mathrm{RF}} \in [-1, \rho - 1]$, and its sign/magnitude reveal the relative importance of benefit versus cost dispersion. I display these benchmarks graphically in Figure 4.

Figure 4: Preference-level Slope Benchmarks



(A) No benefit dispersion: $\beta^{\text{RF}} < 0$ (B) No cost dispersion: $\beta^{\text{RF}} > 0$

Diagnostic for Type A or B *students*. Similar to the within skill diagnostic, we can classify type A students as those that have high marginal benefits for improving weaker skills, and type B students as those that have high marginal benefits for improving stronger skills.

First we solve for the equilibrium skill mix for a given student, with more than two skills. Dropping the i subscript, let there be $J \geq 3$ skills with $\sum_j (c_j/\kappa_j)^\rho = 1$ and preferences $\prod_j c_j^{\beta_j}$. Fix skill 1 as an anchor and therefore for $j \neq 1$:

$$s_j^* = \frac{c_j^*}{c_1^*} = \left(\frac{\beta_j}{\beta_1}\right)^{1/\rho} \frac{\kappa_j}{\kappa_1} =: T_j \lambda_j, \qquad \text{MRS}_{j1}^* = T_j^{\rho-1} \lambda_j^{-1},$$

where $T_j = (\beta_j/\beta_1)^{1/\rho}$ and $\lambda_j = \kappa_j/\kappa_1$ (full derivation in Appendix D). We consider the analogous regression **across skills for a given student** of the marginal rate of substitution on skill levels. A first-order expansion around means $\mathbb{E}[T_j]$ and $\mathbb{E}[\lambda_j]$ yields a similar expression for the preference-level slope across skills for a given student:

$$\beta_j^{\text{RF}} := \frac{\text{Cov}_j(s^*, \text{MRS}^*)}{\text{Var}_j(s^*)} = \frac{(\rho - 1)\,\text{Var}_j(\tilde{T}) - \text{Var}_j(\tilde{\lambda})}{\text{Var}_j(\tilde{T}) + \text{Var}_j(\tilde{\lambda})}.$$

Thus, the same logic applies: if the variation in costs dominates, the student is type A, the slope is negative, and they would benefit from more well-roundedness; if the variation in benefits dominates, the student is type B, the slope is positive, and she would benefit from more specialization.

3.2 Teachers' Problem (Classroom-Level Targeting)

A teacher enters after the baseline parent problem. Each student arrives with an achievement bundle $c_0 = (c_{1,0}, c_{2,0})$ and with barriers to learning summarized by cost shifters $\kappa = (\kappa_1, \kappa_2)$. The teacher cannot change benefits β and does not choose inputs x; instead, she can *lower effective costs* by changing κ, with the aim of maximizing students' utility.

Crucially, the teacher does not observe each student's true benefit parameters β_i or complete cost structure, which depend on hard-to-observe features such as home environment, peer groups, and future aspirations. Instead, the teacher holds beliefs $\hat{\beta}_i$ about each student's marginal benefits to different skills. Parents are plausibly better informed about these benefits, but large class sizes and informal communication limit teachers' access to this information.

I model teachers as choosing a single classroom environment that shifts the frontier *for all students in the class.* Let $z := (\kappa_1, \kappa_2)$ summarize the classroom-level technology (i.e., intercepts of the CET frontier). Starting from z_0, the teacher selects a class-wide change $\Delta z := z - z_0$ subject to a convex resource cost

$$\mathcal{C}(\Delta z) \;\leq\; B, \qquad \mathcal{C}(0) = 0, \;\; \nabla^2 \mathcal{C}(z) \succ 0.$$

Representative (target) student. Following the spirit of tracking/targeting models (e.g., Duflo et al. (2011)), we assume the teacher orients instruction toward a fixed percentile $\tau \in (0,1)$ of the baseline distribution.[6] Let $i(\tau)$ denote the student at percentile τ in the baseline specialization distribution (or, under symmetry, the median equals the mean). The teacher operates on beliefs $\hat{\beta}_{i(\tau)}$ about the representative student's benefit parameters. The teacher's objective is to improve the representative student $i(\tau)$:

$$\max_{\Delta z} \; U\big(c^*(z_0 + \Delta z;\, I_{i(\tau)}, \hat{\beta}_{i(\tau)})\,;\, \hat{\beta}_{i(\tau)}\big) \quad \text{s.t.} \quad \mathcal{C}(\Delta z) \leq B.$$

This delivers a simple, class-wide policy rule while making explicit which part of the distribution the teacher is targeting. (Section 3.2 briefly discusses student-specific levers; when feasible, such levers are weakly welfare-improving relative to any single-environment benchmark.)

First-order characterization. Write $z := (\ln \kappa_1, \ln \kappa_2)$ and let $c^*(z; I, \hat{\beta})$ be the parental optimum on production frontier given the teacher's beliefs about benefits. The 2×2 Jacobian

$$J(z; I, \hat{\beta}) \;:=\; \frac{\partial c^*(z; I, \hat{\beta})}{\partial z} \quad \text{with entries} \quad J_{jk} = \frac{\partial c_j^*}{\partial \ln \kappa_k}$$

measures how the optimal outcomes respond to small cost/productivity shifts. For small moves, $\Delta c \approx J\,\Delta z$; for larger moves, the mapping is nonlinear, but the endpoint FOCs evaluate J at the chosen $z^\star$.

The teacher chooses Δz subject to a convex *technology–space* budget $\mathcal{C}(\Delta z) \leq B$. When we use the quadratic form

$$\mathcal{C}(\Delta z) \;=\; \tfrac{1}{2}\,\Delta z^\top W\,\Delta z, \qquad W \succ 0,$$

the symmetric, positive-definite matrix W captures how difficult it is for the teacher to shift costs in technology space. Diagonal entries encode how expensive it is to relax costs for each skill, while off-diagonals allow for spillovers or complementarities in moving both costs at once (if $W = I$, all directions in z are equally costly). It is important to distinguish W from κ_i; W captures how

[6] The role of τ is to capture an instructional target level; Duflo et al. study how $x^\star$ (a target) depends on the distribution and payoff curvature. Here we impose a simple, testable benchmark in which the teacher chooses a single classroom environment aimed at a chosen percentile; we focus on $\tau = 0.5$ (the median) for symmetry and transparency.

difficult it is for teachers to shift costs, whereas κ_i summarizes those very costs, capturing how difficult it is for parents to shift skill levels.

The KKT condition for the teacher's problem,

$$\max_{\Delta z} U\Big(c^*(z_0 + \Delta z; I, \hat{\beta}); \hat{\beta}\Big) \quad \text{s.t.} \quad \mathcal{C}(\Delta z) \le B,$$

is

$$\nabla_z U\Big(c^*(z^\star; I, \hat{\beta}); \hat{\beta}\Big) \;\propto\; \nabla\mathcal{C}(\Delta z^\star), \tag{10}$$

and, by the chain rule,

$$\nabla_z U \;=\; J(z^\star; I, \hat{\beta})^\top \, \nabla_c U\Big(c^*(z^\star; I, \hat{\beta})\Big).$$

Under quadratic costs, $\nabla\mathcal{C}(\Delta z^\star) = W\,\Delta z^\star$, so

$$\boxed{\Delta z^\star \;\propto\; W^{-1}\, J^\top\, \nabla_c U} \qquad \text{(move in } z \text{ along utility gain per technology-cost).}$$

Mapping back to outcomes,

$$\Delta c^\star \;\approx\; J\,\Delta z^\star \;\propto\; \underbrace{J\,W^{-1}\,J^\top}_{M(z^\star; I, \hat{\beta}) \succ 0} \quad \nabla_c U\Big(c^*(z^\star; I, \hat{\beta})\Big).$$

Interpretation.

- $J^\top \nabla_c U$ takes the utility gradient from outcome space to technology space: in other words, it captures which cost reductions raise utility fastest once parents re-optimize (under the teacher's beliefs).
- W^{-1} reweights that direction by how easy each cost move is for the teacher.
- $M := J\,W^{-1}J^\top$ is the induced metric in outcome space: it tells us which outcome directions are easiest to deliver, given both teacher costs (W) and skill level responsiveness (J) which is formed under the teacher's beliefs about benefits.

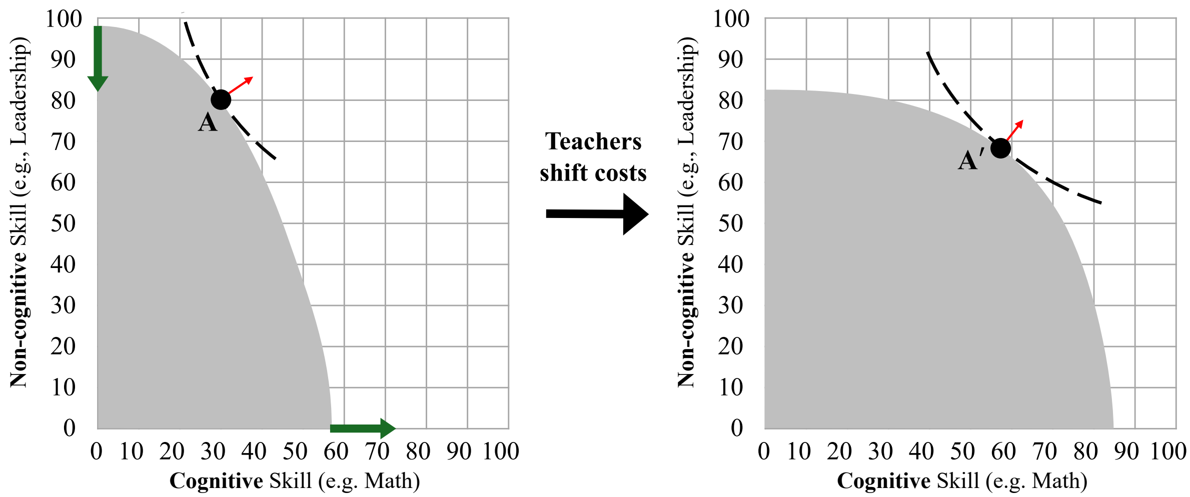
Hence the globally optimal change in outcomes points along the direction $M\,\nabla_c U$; in other words, the utility normal, weighted by the metric M that captures how costly different outcome moves are for the teacher operating on her beliefs.

Benchmark: symmetric teacher costs in outcome space. If the composite metric M is locally proportional to the identity (i.e., movements in outcome space are equally difficult), the optimal class move aligns with the utility normal of the representative student under the teacher's beliefs:

$$\Delta c^\star \;\parallel\; \nabla_c U\Big(c^*(z^\star; I_{i(\tau)}, \hat{\beta}_{i(\tau)})\Big).$$

With $\tau = 0.5$ and a symmetric, single-peaked baseline distribution, this implies the classroom environment is chosen so that the *expansion path* for the median aligns with that student's IC-normal as perceived by the teacher. In our Cobb-Douglas/CET setting, this pins down the specialization threshold $s^\dagger(\hat{\beta}) = \sqrt{\hat{\beta}/(1-\hat{\beta})}$ as the classroom alignment point: if the teacher believes the median student is of Type A (i.e., desires more well-roundedness), the teacher should tilt technology toward the weaker skill (i.e., lower its cost); if Type B, the reverse should occur. Figure 5 illustrates this benchmark.

Figure 5: Teacher's response: shift production towards high-return skills



Notes: The teacher chooses a classroom environment to align the representative student's expansion path with their IC-normal, based on the teacher's beliefs about the student's benefit parameters. The figure shows a classroom where the median student is of Type A, so the teacher shifts technology to lower the cost of the weaker skill.

Belief misspecification and the role of information. When the teacher's beliefs $\hat{\beta}_{i(\tau)}$ diverge from the true benefit parameters $\beta_{i(\tau)}$, the chosen classroom environment $\Delta z^\star$ will be misaligned with students' actual preferences. This misspecification can lead the teacher to misclassify the representative student's type (Type A versus B), allocating effort in directions that do not maximize student welfare. My experiment tests whether providing teachers with structured information about parent priorities, which serve as a proxy for students' true benefit parameters, reduces this misalignment. Since the teacher applies a classroom-level policy, correcting beliefs about the representative (median) student potentially generates spillovers to other students in the class, though the direction and magnitude of these spillovers depend on each student's position relative to the targeted type.

Remark on individual levers. If student-specific Δz_i (or student-time allocations) are feasible, then a menu $\{\Delta z_i\}$ weakly dominates any single Δz by allowing movement along each student's own IC-normal. We treat the classroom-wide Δz as a policy benchmark: it is realistic when instruction and resources are common at the class level, and it gives sharp, testable directional predictions.

4 Data and Descriptive Statistics

4.1 Parent Survey Overview

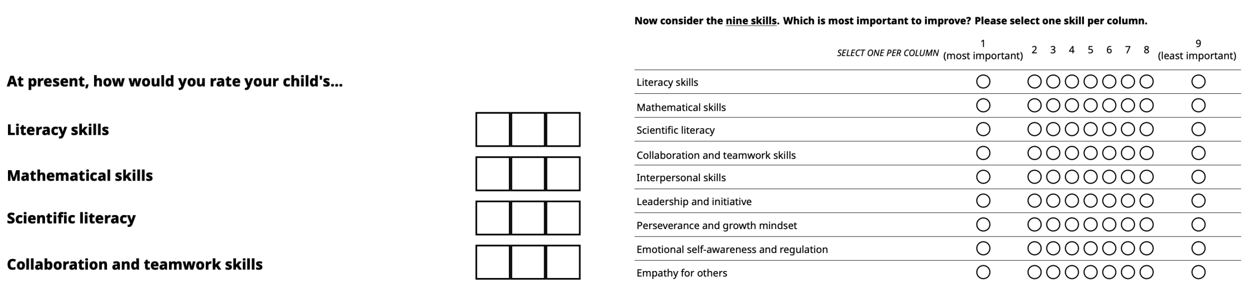
The parent survey was administered on paper and took approximately 15-20 minutes to complete. Surveys were conducted before parent-teacher meetings, as this represented the point of greatest contact with parents. Participation was voluntary, and no compensation was offered to parents. Parents were told that the survey was being conducted to help the school better understand parent priorities and that their responses would be shared with the school and teachers either immediately (treated group) or at the end of the school year (control group). The survey was administered in English, but parents were allowed to ask for clarifications in their native language.

Motivated by the model in Section 3, the key components of the survey are the elicitation of parents' ratings of their child's current abilities (skill levels) and, given where they perceive their child to be, their rankings of which skills are most important to improve (skill preferences). The survey also contained a rich set of parent demographics (the full survey instrument is provided in Appendix C). These measures allow us to analyze the relationship between perceived ability levels and improvement priorities and shed light on whether observed specialization reflects production constraints or preference heterogeneity.

4.1.1 Skill Ratings

Parents were asked to rate their child's current abilities (Figure 6, Panel A) across each of the nine skill dimensions on a scale from 0 to 100, where 0 represents the "lowest level possible" and 100 represents the "highest level possible." Respondents were explicitly instructed not to use 100 unless they believed their child had no room for improvement in that skill. This measure provides a quantitative assessment of perceived current ability levels and helps identify areas where parents perceive strengths and weaknesses in their child's development.

Figure 6: Parent Survey Instrument



(A) Skill Levels: 0–100 ratings of current abilities

(B) Skill Preferences: 1-9 ranking of marginal improvement priorities

Notes: Panel A shows the 0-100 skill levels question; Panel B shows the 1-9 skill preference ranking framed as which improvement would benefit the child the most, conditional on current levels. Full wording in Appendix C.

The skill level measure is comparable to those used in other studies measuring subjective assessments of abilities (Dizon-Ross, 2019; Bergman, 2021) and allows for creating standardized measures of perceptions that can be compared across dimensions and between respondents.

4.1.2 Skill Preferences

In addition to perceived skill levels, parents were asked for their skill preferences — in other words, a ranking of the nine skills in order of importance for improvement, from most important (1) to least important (9) (Figure 6, Panel B). Parents also ranked the three broader categories (academic, social, and emotional skills) in order of importance for improvement. This forced-choice ranking methodology was designed to capture relative valuation, with the measure itself serving as a proxy for the marginal rates of substitution between different skill dimensions. In practice, since the survey was administered on paper, parents reported ties despite instructions to avoid them; I break these ties by assigning the average rank for the tied positions (e.g., if two skills are tied for 2nd place, both receive a rank of 2.5).

Asking parents to rank skills by importance offers several advantages over alternative approaches for measuring preferences for improvement. A natural alternative would be to ask parents to rate the importance of improving each skill on a Likert scale (e.g., 1-5). However, such ratings are subject to scale-use bias and often lead to ceiling effects (Chyung et al., 2020), making it difficult to discern true preference heterogeneity.

4.2 Demographics and Validation

Parent-reported skill levels display strong and systematic relationships with independently validated behavioral measures. I administered the Strengths and Difficulties Questionnaire (SDQ, Goodman (1997)), a widely-used behavioral screening tool, in the same parent survey. I find that the nine elicited skill levels correlate sensibly with SDQ subscales (Appendix Figure A.1). Higher scores on the four SDQ problem subscales (emotional symptoms, conduct problems, hyperactivity, and peer problems) are consistently associated with lower parent-reported skill levels across all nine dimensions, with magnitudes ranging from 1.6 to 5.1 points per standard deviation increase in the SDQ scale. Conversely, higher prosocial behavior scores predict 2.1 to 6.0 point increases in skill levels. The largest associations appear for self-awareness (up to -5.0 points for hyperactivity), perseverance (-5.1 points for hyperactivity), and empathy (+6.0 points for prosocial behavior), suggesting parents integrate diverse behavioral signals when assessing their children's capabilities. All coefficients are statistically significant at conventional levels. These patterns persist when controlling for school-by-grade fixed effects and are robust to using within-child standardized skill levels rather than raw levels. While I report raw skill level associations for interpretability, subsequent analyses employ within-student standardized measures, as the theoretical framework and experimental design focus on relative strengths across skills (specialization) rather than absolute levels.

The elicited skill preferences move sensibly with family characteristics and stated aspirations (Appendix Figure A.2). Higher-income households place relatively more weight on social skills and less on academic skills, fathers emphasize social and academic skills more and emotional skills less relative to mothers, and parents of children in higher grades prefer improving academic skills more and emotional skills less. I also examine how skill preferences vary with parents' aspirations for their child's future career, and find that parents who aspire for their child to be a mechanic or engineer place markedly higher priority on improving academics relative to those who aspire for their child to be a business leader. By contrast, priorities show no systematic relationship with child gender, birth order, or parental education. Together with the sensible demographic correlates of skill levels (Appendix Figure A.1), these validation exercises indicate that both the skill levels and skill preferences capture meaningful variation in parental perceptions rather than measurement error.

5 Preference-Level Slopes

Following Section 3, I define our empirical measure of the preference-level slope as the slope from regressing parents' skill preference ranks on the corresponding standardized skill levels. I maintain the convention that higher skill preference ranks indicate higher priority: I multiply skill preference ranks by -1 so that higher values indicate higher priority (e.g., rank 1 $\rightarrow$ rank -1). Therefore, a negative slope indicates that parents place higher priority on improving weaker skills, and a positive slope indicates that parents place higher priority on improving stronger skills. We estimate two versions: (i) within a skill, across students (e.g., regressing math skill preference ranks on skill levels) and (ii) within a student, across skills (i.e., regressing the nine skill preference ranks on the nine skill levels for a given student).

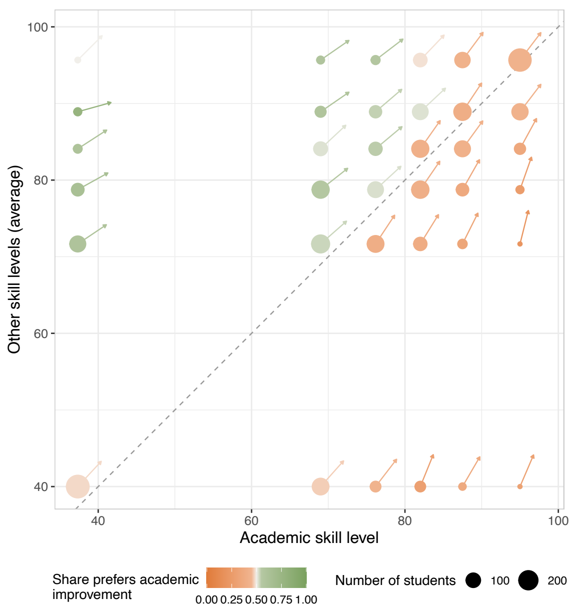
5.1 Within-Dimension Slopes (across students)

Before running regressions, I plot the data to visually demonstrate the connection between the theory and data. Figure 7 plots skill levels and skill preferences for academic versus socioemotional skills. Panel A shows a binned scatter plot of raw (0-100) skill levels for academic skills against skill levels for non-academic skills (social, emotional). The 45-degree line separates students who are relatively stronger in academic skills (above the line) from those who are relatively weaker (below the line). Arrows for each point indicate the share of parents that place higher priority on improving academic versus socioemotional skills within each cell. For example, a fully rightward arrow indicates all parents prefer improving academic skills, and a fully upward arrow indicates all parents prefer improving socioemotional skills.

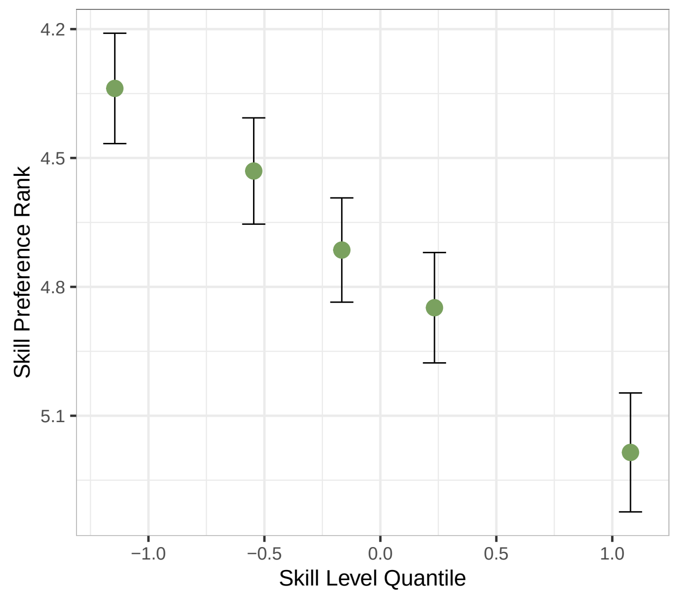
In the model, if families only vary in production costs, then movements in skill levels and skill preferences should follow the Type A pattern illustrated in Figure 4: students that are relatively weaker in academic skills should benefit more from improving academic skills (arrow points towards academic skills), and students that are relatively stronger in academic skills should benefit less from improving academic skills (arrow points up away from academic skills). This pattern is exactly what we observe in Figure 7, indicating that we are closer to the Type A case where production constraints dominate specialization decisions.

Panel B shows the corresponding relationship between standardized skill levels and skill preferences for academic skills. Students who are relatively weaker in academic skills place higher importance on improving academic skills. Conversely, students who are relatively stronger in academic skills place higher importance on improving socioemotional skills. This negative relationship is consistent with the Type A pattern in the model, where production constraints dominate specialization decisions.

Figure 7: Academics Versus Socioemotional Skills: Skill Levels and Skill Preferences



(A) Desired Growth by Skill Bundle

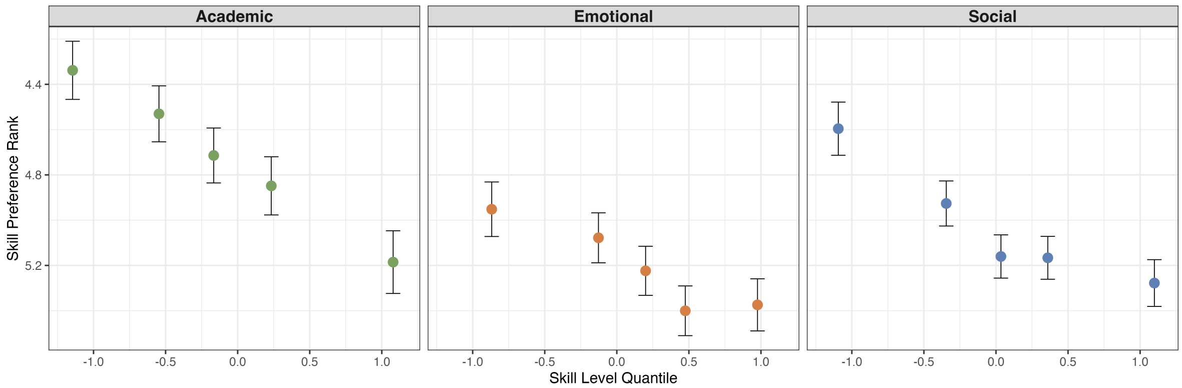


(B) Academic Skill Preference by Skill Level

Notes: Panel A shows the scatter plot of academic skill levels against non-academic skill levels. The 45-degree line separates students who are relatively stronger in academic skills (above the line) from those who are relatively weaker (below the line). Points are binned by cell, with size proportional to the number of students in the cell. Arrows indicate the the share of parents that place higher priority on improving academic versus socioemotional skills within each cell. Panel B bins within-student standardized skill levels by quintile and plots the average skill preference rank for each quintile. The y-axis is reversed so that higher values indicate higher priority (rank 1 = most important).

I show the relationship between skill levels and skill preferences for each category in Figure 8. For each skill category, I calculate the quintiles of the students' skill levels, and plot the average skill preference rank within each quintile. The relationship between these transformed ranks and standardized skill levels is monotone and downward sloping for all three categories (academic, social, emotional). This is consistent with parents of students with relative weakness (negative skill levels) placing higher priority on improving that skill compared to parents of students for whom the skill is a relative strength (positive skill levels). I plot the same figure for each of the nine skill dimensions in Appendix Figure A.3, and find similar downward-sloping patterns for every skill.

Figure 8: Average Parent Skill Preference Rank Within Skill Level Quintile



Notes: Figure shows average parent skill preference rank within quintiles of standardized skill levels, by category. The y-axis is reversed so that higher values indicate higher priority (rank 1 = most important). Points show quintile means; bars show 95% confidence intervals.

I then formally estimate the within-dimension preference-level slopes running the regression

$$\text{Skill Preference Rank}_{i,j} = \alpha_j + \beta_j \text{ Skill Level}_{i,j} + \epsilon_{i,j}, \tag{11}$$

separately for each skill dimension j. Here, β_j is the preference-level slope for skill or skill category j. In a pooled regression across all skills, the slope is negative and precisely estimated: the slope is -0.40 (s.e. 0.024), implying that a one unit increase in a child's standardized skill level is associated with a 0.46 decrease in the skill preference rank (i.e., less important to improve).

To obtain the slope for each dimension, I estimate a single stacked regression that allows the slope, β_j, to vary by dimension. I report the implied slopes, and find they are negative for every dimension, with the decline steepest in mathematics ($\beta_{\text{Math}} = -0.518$, s.e. 0.043). Slopes are still substantially negative for other dimensions, with the most shallow slopes for science (-0.195, s.e. 0.048) and perseverance (-0.116, s.e. 0.049). Using an analogous stacked regression after averaging skill levels and skill preferences by category, the category-specific slopes are: academic -0.481 (s.e. 0.050), social -0.443 (s.e. 0.047), and emotional -0.347 (s.e. 0.053). The preference-level slopes for each skill category are reported in column 1 of Table 2, while slopes for the full list of 9 dimensions are reported in Appendix Table B.3.

Table 2: Preference-Level Slopes by Skill Category and Grade

Dependent Variable: Skill Preference Rank	(1) Pooled	(2) Grade 1	(3) Grade 4	(4) Grade 7	(5) Grade 10
Skill Levels					
Academic	-0.4811***	-0.0509	-0.4661***	-0.4083***	-0.5924***
	(0.0500)	(0.0922)	(0.1014)	(0.0839)	(0.0900)
Emotional	-0.3465***	-0.2751***	-0.4500***	-0.4091***	-0.4594***
	(0.0530)	(0.0681)	(0.1177)	(0.1140)	(0.0869)
Social	-0.4425***	-0.1102	-0.3676***	-0.3053***	-0.5382***
	(0.0468)	(0.1055)	(0.0948)	(0.0938)	(0.0960)
Statistics					
Observations	10,016	10,016	10,016	10,016	10,016
Adjusted R^2	0.0409	0.035	0.035	0.035	0.035

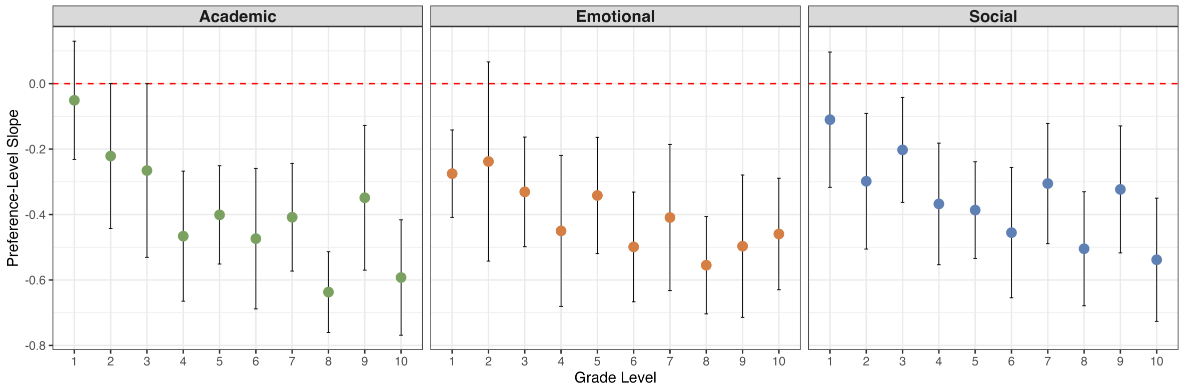
Notes: Entries report slope coefficients β_j (average marginal effects) from -Skill Preference $\text{Rank}_{i,j} = \alpha_j + \beta_j \text{ Skill Level}_{i,j} \times \text{Skill Category}_j + \epsilon_{i,j}$. I interact skill levels with skill category indicators to obtain category-specific slopes. I interact with grade indicators to obtain grade-specific slopes for columns (2)–(5). The outcome is the parent's skill preference rank for improvement (1-9), multiplied by -1 so that larger numbers indicate greater priority. The dependent variable, Skill $\text{Level}_{i,j}$ is standardized within-student. Column (1) shows the pooled category-specific slopes. Columns (2)–(5) display selected grade-specific slopes (Grades 1, 4, 7, and 10) to illustrate how the strength of the negative relationship between own skill level and preference for improvement steepens at higher grades.
Standard errors in parentheses, two-way clustered (student, classroom).
Significance codes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Through the lens of the model, these patterns imply that across students, variation in who specializes in a given skill is primarily driven by production constraints rather than differences in real or perceived benefits. This is most pronounced for academic skills, consistent with either production constraints being relatively more dominant for academic skills or variation in benefits being relatively more dominant for social and emotional skills.

5.2 Heterogeneity by Grade Level

Next, I examine whether the preference-level slope varies by grade. I estimate the slope with an analogous stacked regression, allowing the slope to vary by category and grade. I plot the results in Figure 9. The slope is negative in every grade yet indistinguishable from 0 for classes 1 and 2, and increasingly negative reaching between -0.5 to -0.7 for grades 9-10. I report the preference-level slopes for grades 1, 4, 7, and 10 of Table 2.

Figure 9: Preference-Level Slope by Grade



Notes: Points show the grade-specific preference-level slope for each skill category, defined as the average marginal effect of standardized skill level on the parent's skill preference rank. Slopes are estimated from a single stacked regression of $-\text{Skill Preference Rank}_{i,j}$ on $\text{Skill Level}_{i,j}$ interacted with grade indicators (classes 1–10) and skill category indicators. Vertical bars are 95% confidence intervals based on two-way cluster-robust standard errors (student and classroom). Negative values indicate that, within a grade, higher skill levels are associated with lower priority to improve the skill.

This pattern suggests that specialization driven by production constraints becomes more pronounced as children age, consistent with the idea that as children grow, the costs of improving weaker skills relative to stronger skills become more prominent drivers of specialization. This may reflect both increasing differentiation in the cost of producing skills as students age, or the benefits to skills becoming more similar as students approach upper secondary school.

5.3 Within-Student Slopes (across dimensions)

I now turn to the within-student preference-level slopes, which measure how a child's skill levels relate to parents' skill preferences across dimensions. I estimate within-student preference-level slopes by regressing skill preference ranks on the associated skill levels for each parent. The average within-student slope is negative (-0.43), with wide dispersion across students (SD = 1.25). I show the histogram of individual student slopes in Appendix Figure A.4. This indicates that, within a given child, skills that parents prefer to improve most tend to be those where they perceive their child to be weaker; this is consistent with specialization that is driven by production constraints rather than by differences in benefits.

I relate this slope to parental satisfaction to gauge whether observed specialization aligns with perceived benefits. I compare the preference-level slope to two satisfaction measures: (i) satisfaction with the child's progress and (ii) overall satisfaction with the school. Both are measured on a four-point scale from "completely satisfied" to "completely dissatisfied." Regressing an indicator for being completely satisfied on the within-student slope shows that more positive slopes (indicating more aligned skill levels and skill preferences) are associated with higher satisfaction with the child's progress (coefficient 0.019, s.e.=0.008), but not with satisfaction with the school (coefficient 0.003, s.e. 0.010). The marginal effect is economically meaningful: going from the most negative slope observed (-3.15) to the most positive slope (3.54) raises the probability of being completely satisfied with the child's progress by 12.7 percentage points (baseline mean = 52%).

In the model, more positive within-student slopes indicate that the child's current mix of skills aligns more closely with the parent's marginal benefits — in other words, what the parent most wants improved. Geometrically, this is what we expect when the child's natural expansion path is closer to the indifference-curve normal, so incremental learning goes in a direction that raises utility fastest. Practically, parents report higher satisfaction with their child's progress when observed specialization lines up with what they value, while the lack of correlation with satisfaction about the school suggests that the slope captures a dynamic growth alignment rather than static school quality. These patterns reinforce the cost-side reading of the average negative preference-level slope in this setting and motivates the information experiment that follows.

6 Information Experiment

The cross-sectional evidence points to cost-side forces as a central driver of specialization. To test the mechanisms implied by the model, I conduct a production-side shock by informing teachers about parent skill levels and skill preferences. The model predicts that if teachers update their beliefs about student benefit parameters based on this information, their classroom-level instruction should focus more on skills parents place the highest marginal value on, thus shifting students' portfolios toward parent-prioritized skills.

6.1 Teacher Survey

The teacher survey was administered online at baseline and at endline using Qualtrics. It collected comprehensive information about teachers' professional backgrounds, pedagogical philosophies, and, most importantly, their skill preferences for (i) a typical student, (ii) specific individual students in their class, and (iii) their beliefs about how parents would rank skill priorities for those same students. Descriptions of the full survey instrument is provided in Appendix C.

6.1.1 Teacher Demographics and Professional Background

The survey collected standard demographic and professional information including gender, education level, years of teaching experience, and tenure at the current school. This baseline information allows analysis of how teacher characteristics influence their beliefs about student development priorities and their responsiveness to information about parent preferences.

6.1.2 Teaching Philosophy and Self-Efficacy

To understand the context within which teachers make instructional decisions, the survey measured teachers' views on their professional responsibilities and their self-efficacy across different teaching domains.

Teachers rated the importance of various responsibilities (e.g., improving student academic achievement, incorporating parent priorities, and managing student behavior) on a five-point scale from "not important" to "extremely important." This process provides insight into how teachers conceptualize their role and the relative weights they place on different aspects of their jobs.

A validated teacher self-efficacy scale was also included, with items such as "how much can you do to control disruptive behavior in the classroom?" and "how much can you do to motivate students who show low interest in school?" Teachers responded on a nine-point scale ranging from "nothing (1)" to "a great deal (9)." This scale measures teachers' beliefs about their ability to influence various student outcomes, which may mediate teachers' responsiveness to information about parent preferences.

6.1.3 Rankings for Students

The core component of the teacher survey paralleled the parent skill preference questions, but with three distinct variations designed to assess alignment between teachers and parents, and to measure how teachers form beliefs about parent preferences. First, teachers ranked the nine skill dimensions for a typical student they encounter in their teaching. This provides a baseline measure of teachers' general pedagogical priorities. Second, for six specific students, teachers provided skill preference rankings based on their own professional judgment about what each individual student needed most. Students were selected through stratified random sampling based on the parent survey. Two students were chosen from families prioritizing academic, social, and emotional skills, respectively. In cases where fewer than six students had parent survey data, all available students were included.

The first two variants capture teachers' own assessment of individual student needs. The third variant is distinct in eliciting teachers' *beliefs* about parent preferences: for the same six students, teachers were asked to predict how they believed each student's parent would rank the skill development priorities. This measure enables direct comparison between teachers' beliefs about parent skill preferences and parents' actual stated skill preferences.

Teacher responses reveal several striking patterns in the baseline data. Teachers demonstrate poor calibration about individual parent preferences: the correlation between teacher beliefs about parent skill preference rankings and actual parent rankings is close to zero. More systematically, teachers appear to project their own professional priorities onto parents, which I discuss in more detail in Section 7. The teacher skill preference data also shows that while parents tend to prioritize academic skills most highly, teachers' own professional priorities tilt more toward social and emotional development. I show the full distribution of teacher skill preferences in Appendix Figure A.5. This preference divergence, combined with teachers' poor beliefs about parent preferences, creates substantial scope for misalignment between classroom instruction and family priorities.

This baseline misalignment provides the foundation for testing whether structured information provision can improve teacher-parent alignment and ultimately shift student outcomes. The experimental design leverages the fact that teachers have systematically biased beliefs about parent preferences, making information provision potentially valuable even in a context where formal communication channels already exist between teachers and families.

6.1.4 Pedagogical Strategies for Skill Development

In addition to facing uncertainty about parent preferences, teachers also face uncertainty about how to effectively develop the skills they prioritize. Therefore, the teacher survey included a component designed to elicit teachers' beliefs about effective pedagogical strategies for developing each of the nine skill dimensions. This component was informed by the literature on social and emotional learning (SEL) and evidence-based teaching practices.

For each of the six social and emotional skill dimensions, teachers were presented with four evidence-based teaching strategies and were then asked to rank these strategies from most effective (1) to least effective (4) based on their professional judgment. A full list of the strategies presented is provided in Appendix Section C.

This component of the survey enabled the creation of a second treatment in which teachers receive an aggregated report on the effectiveness of pedagogical strategies based on the collective responses of teachers within their school. As described in the experimental design section, this second treatment was cross-randomized with the first treatment, which provided teachers with information about parent skill levels and skill preferences.

6.2 Experimental Design

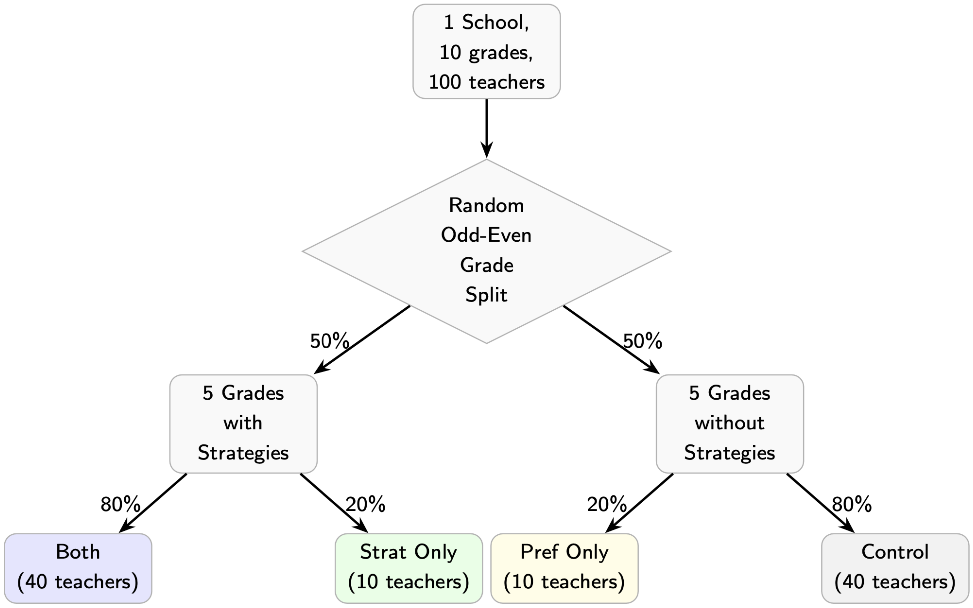
The experimental design addresses two key questions, (1) whether teachers update their beliefs and behaviors when provided with information about parent preferences, and (2) whether supplementing this preference information with concrete pedagogical strategies enhances teacher responsiveness. To answer these questions while minimizing spillover concerns, I employ a two-stage randomization design at the school-grade and teacher levels:

6.2.1 First Stage: Grade-Level Randomization

The first level of randomization occurs at the grade level within each school. Grades are randomly assigned to either receive or not receive pedagogical strategies for developing different skill dimensions. This design choice is motivated by the hypothesis that teachers primarily communicate with other teachers in their grade at the same school, making grade-level assignment a natural boundary to prevent spillover.

To further ensure the Stable Unit Treatment Value Assumption (SUTVA) holds, I implement an odd/even grade split, where odd-numbered grades (1, 3, 5, 7, 9) in a school are assigned to one condition and even-numbered grades (2, 4, 6, 8, 10) to the other. This spatial separation minimizes the risk of treatment contamination, as teachers in adjacent grades typically have fewer opportunities for professional interaction with each other than with teachers within the same grade.

Figure 10: Treatment Assignment



6.2.2 Second Stage: Teacher-Level Randomization

Within each grade, I then randomize at the classroom (teacher) level whether teachers receive information about parent preferences. This information includes aggregated data on how parents of students in their specific class rate their children's current abilities and rank the importance of improving each dimension.

To maximize statistical power for the primary contrast (receiving both treatments vs. receiving neither), I use an unbalanced randomization:

- Grades assigned to receive strategies: 80% of teachers are randomly selected to also receive parent information; 20% receive only strategies.
- Grades assigned not to receive strategies: 20% of teachers are randomly selected to receive parent information; 80% receive neither treatment.

This allocation results in approximately 40% of teachers receiving both treatments, 40% receiving neither, and 10% receiving each treatment alone: it provides optimal power for the main comparison while still allowing for tests of individual treatment effects. I show the randomization achieved balance across teacher and student characteristics in Appendix Table B.4.

6.3 Implementation Through Web Portal

The intervention is delivered through a custom-built web portal that provides teachers with access to treatment information in an intuitive, user-friendly format. To encourage proper implementation, we conducted demonstration sessions with all teachers: we illustrated how the information tool could be used to understand parent preferences and potential ways to incorporate these insights into their teaching approaches. These sessions focused on showing teachers how to navigate the website, interpret the visualizations of parent preferences, and connect these preferences to pedagogical strategies. School administrators were appointed as points of contact to address any questions or challenges teachers might encounter while using the tool.

6.3.1 Access and Authentication

Each teacher receives individual login credentials via email, with access restricted to their assigned treatment condition. The portal uses secure authentication to ensure teachers can only view information relevant to their treatment status. This individualized access allows for precise tracking of teacher engagement with the intervention, including login frequency, pages viewed, and time spent on different components of the portal.

6.3.2 Interface and Content

The portal presents information through several organized tabs:

1. **Welcome**: this tab provides an introduction to the portal and navigation instructions, with content tailored to the teacher's treatment assignment.

2. **Class Overview (Parent Information)**: this tab provides a classroom overview. Each row corresponds to a student, and the columns show which skill category (academic, social, or emotional) each parent identified as most important for improvement, along with the parent's perceived skill levels for their child. The table is sorted by default so that rows are grouped by the parents' top skill priority. The skill category with the most parents prioritizing it is highlighted at the top, allowing teachers to quickly identify common themes in parent preferences within their class. A sample rendering is shown in Figure 11, Panel A.

3. **Student Reports (Parent Information)**: this tab shows the full student-level reports. Each report includes the parent's skill levels (ratings of current abilities) across all nine dimensions, as well as the parent's skill preference ranking of which skills are most important to improve. The report features both tabular data and interactive visualizations to help teachers quickly grasp each student's unique profile. An example student report is shown in Figure 11, Panel B.

4. **Classroom Report (Parent Information)**: this tab presents the same information as the student reports, but averaged at the classroom level. This summary enables teachers to see overall patterns in parents' perceptions and priorities.

5. **Strategies for Improvement (Strategy Information)**: this tab summarizes the pedagogical strategies for developing each of the six socioemotional skills, based on the aggregated rankings provided by teachers at baseline. For each skill, the tab lists the four strategies along with their average effectiveness ranking among teachers in the school. An example of this tab is shown in Figure 12.

Figure 11: Web-Portal Displays

Parents' most important skills to improve for each student

Ratings scored from 0-100.
Positive (green) numbers means better than average, and negative (red) numbers means worse than average.

Student	Most important skills	Parent rating
Christopher Harris	Emotional	52
Kylie Ferguson	Emotional	47
Bernadette Walsh	Emotional	60
Ruth Terry	Emotional	46
Amanda Ogden	Emotional	58
Stewart Langdon	Emotional	58
Heather Reid	Emotional	57
James Walker	Emotional	52
Emma Dyer	Academic	56
Gordon Sharp	Academic	59
Olivia Bond	Academic	37
Rachel Dyer	Academic	40
Tim Grant	Social	39
Joseph Rampling	Social	46
Joe Quinn	Social	42

(A) Class Overview

According to **Bernadette Walsh**'s parent, the most important skill for them to improve is **collaboration and teamwork skills**.

Bernadette Walsh

Parent ratings: Scored from 0-100.
Ranking: How valuable it is to improve for this student. Lower rank = more important

Category	Ability	Parent rating	Ranking
Social	Collaboration and teamwork skills	33	1
Emotional	Empathy for others	77	2
Academic	Literacy skills	53	3
Emotional	Emotional self-awareness and regulation	31	4
Social	Leadership and initiative	39	5
Emotional	Perseverance and growth mindset	71	6
Academic	Mathematical skills	82	7
Academic	Scientific literacy	57	8
Social	Interpersonal skills	56	9

(B) Student Report

Notes: Panel A shows the class overview tab; Panel B shows an example student report. Names and numbers randomly generated, but visual layout is identical to the actual portal.

Figure 12: Example Strategies Report

Strategies for improving **Interpersonal skills**

Strategy	Average ranking	Share teachers ranked #1 (most effective)	Description
Classroom circles	2.3	35 %	In classroom circles, students and teachers form a physical circle for open discussions about a variety of topics including personal feelings, conflict resolution, or community building, aimed at fostering a supportive environment.
Role-playing exercises	2.3	29 %	Students take on roles in predefined scenarios, acting out interactions to explore interpersonal dynamics and learn through feedback. Scenarios can include various contexts from conflict resolution to collaborative tasks.
Service-Learning Projects	2.7	20 %	Involve students in group-based projects that address community needs, linking classroom objectives with real-world applications. Tasks include planning, executing, and reflecting on their projects.
Think-Pair-Share	2.7	16 %	Teachers pose a question. Students think independently, then pair up to discuss with a partner, providing feedback on each other's work. Finally, they share with the class, encouraging participation and collaborative problem-solving.

Notes: Example of the strategies report tab, showing pedagogical strategies for developing interpersonal skills. All six socioemotional skills are included. Each has four associated pedagogical strategies. Numbers displayed are specific to the school, and are calculated based on teacher rankings of strategy effectiveness collected at baseline.

The portal's design facilitates easy interpretation of parent preferences through color-coded visualizations and clear tabular formats. For instance, skill dimensions are consistently color-coded by category (green for academic, orange for emotional, blue for social), and skill levels employ a red-yellow-green color scale to highlight areas of strength and opportunity. Skill preferences are presented as ranked lists to clearly communicate relative priorities.

In the strategies section, teachers can access specific, actionable pedagogical approaches for each noncognitive skill. These strategies include brief descriptions and implementation guidance, along with indicators of how commonly other teachers ranked each strategy as most effective.

6.4 Treatment Implementation and Compliance

I implemented the information intervention through a custom-built web portal that provided teachers with individualized login credentials and access to their students' parent-reported skill levels and priority rankings. The platform tracked comprehensive usage data including login frequency, session duration, and specific content accessed.

Implementation proceeded differently across the five participating schools. While 242 teachers were initially assigned to treatment across all schools, practical challenges emerged that significantly affected compliance and data collection. Two schools withdrew from the study entirely before endline data collection. Two additional schools either failed to distribute login credentials to their teachers or achieved login rates below 15%.

The main results presented here focus on the one school that successfully implemented the intervention. This school is the largest school in the sample with 106 classrooms. Even in this successful implementation, compliance was limited: only 46% of treated teachers logged into the platform at least once during the intervention period. Among those teachers who did access the platform, usage varied substantially: some teachers made single brief visits while others engaged more systematically with the content.

Interestingly, the school's administration responded to the baseline parent survey results by implementing school-wide assemblies and developing lesson plans aimed at addressing the skill priorities identified in the parent data. This represents a form of treatment spillover that likely affected both treatment and control teachers within the school, and it potentially attenuated measured treatment effects while demonstrating the policy relevance of the parent preference information.

The low compliance rate and administrative response highlight important features of information interventions in educational settings. Teachers face multiple competing demands on their attention, and information provision alone may be insufficient without accompanying incentives or administrative support. Nevertheless, the experimental results suggest that even partial compliance can generate meaningful changes in student outcomes; these results are consistent with the hypothesis that information about parent priorities can serve as a low-cost production shock that reallocates instructional effort toward high-value dimensions.

7 Teacher Beliefs and Updating

Before turning to treatment effects on student outcomes, I document two facts about teacher beliefs at baseline, and test whether the information intervention improved belief accuracy.

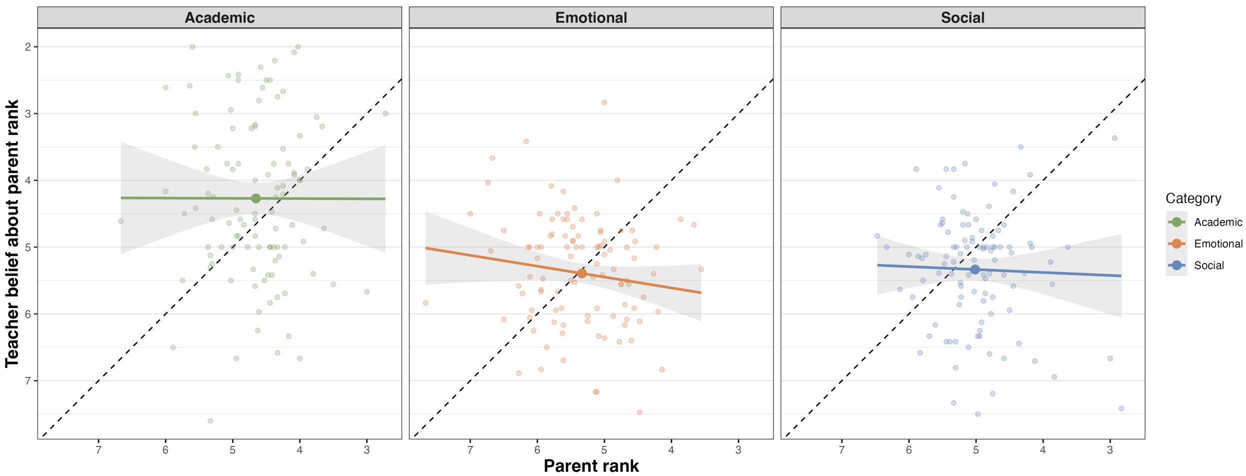
Baseline misalignment. Teachers were asked, for a random subset of students, to predict how that student's parent would rank the importance of improving the nine academic, emotional, and social skills (skill preferences).[7] To reduce noise and focus on broad priorities, I group the nine skills into three categories (academic, social, emotional) and average skill preferences within each category. Figure 13 plots, for each classroom, the average parent skill preference rank (x-axis) against the average teacher belief about parents skill preference (y-axis) for the sampled students. At baseline, beliefs are essentially uncorrelated with truth: points scatter around a flat line. In other words, even in these relatively well-resourced schools with routine parent-teacher meetings, teachers had little signal about what parents most wanted improved.

Where do baseline beliefs come from? Projection. To probe how teachers form beliefs when they lack signal, Figure 14 keeps the same y-axis (i.e., average teacher belief about parents) but replaces the x-axis with the teacher's own skill preference ranking averaged across the same students. A strong positive relationship emerges: it appears that teachers project their own priorities onto

[7] Although ties were discouraged in the instructions, the parent survey was conducted on paper, so skill preference ranks allowed ties; teacher elicitation mirrored this.

parents. This result is consistent with the model's misperception channel: in the absence of reliable information on parents' skill preferences, teachers substitute their own ranks for the parents', thus creating scope for misallocation even when classroom effort is intended to reflect parental priorities.

Figure 13: Baseline Teacher Beliefs Versus Parent Skill Preferences (Class Averages)



Notes: $N_{\text{parent}} = 550$. $N_{\text{teacher}} = 103$. Each point represents one classroom, with skill preferences averaged across the sampled parents. For each classroom, six parents are sampled (two each from parents prioritizing academic, social, and emotional skills). If fewer than six parents completed the survey for a given classroom, all available students are included. The x-axis shows the average parent skill preference rank for each classroom; the y-axis shows each teacher's beliefs about parents' skill preferences averaged over the same students. Perfect accuracy would lie on the 45-degree line (dashed).

Figure 14: Baseline Teacher Beliefs Versus Teachers' Own Priorities (Class Averages)

Notes: $N_{\text{parent}} = 550$. $N_{\text{teacher}} = 103$. Each point represents one classroom, with skill preferences averaged across the sampled parents. For each classroom, six parents are sampled (two each from parents prioritizing academic, social, and emotional skills). If less than six parents completed the survey for a given classroom, all available students are included. The x-axis shows each teacher's own skill preferences for the sampled students; the y-axis shows each teacher's beliefs about parents' skill preferences for the sampled students. Perfect accuracy would lie on the 45-degree line (dashed).

Does information correct beliefs? I measure accuracy using four binary outcomes. *Exact Order Match*: the teacher's full ranking equals the parent's, with ties allowed. *Top Skill Match*: the teacher places at least one of the parent's top-ranked categories among the top. *Bottom Skill Match*: teacher places at least one of the parent's bottom-ranked categories among the bottom. *Top & Bottom Match*: both ends are correct.

I study accuracy at two levels: (a) student-level (i.e., does the teacher correctly predict an individual parent's ranking?) and (b) classroom average (i.e., does the teacher correctly identify the *average* parent's priorities in her class?). The latter measure is constructed by averaging both the parent skill preference rankings for each class, and averaging the teacher's beliefs about those same parents.

Tables 3–5 report intent-to-treat effects of providing teachers the parent rankings and levels via the website (Section 6). The analysis is underpowered due to implementation constraints ($N = 94$ teachers), so I emphasize direction and magnitude. For teacher accuracy about individual parents (Table 3), I estimate:

$$\text{Accuracy}_{ij,t=1} = \alpha + \beta\text{Treatment}_j + \delta\text{Accuracy}_{ij,t=0} + \gamma_g + \epsilon_{ij} \tag{12}$$

where $\text{Accuracy}_{ij,t=1}$ measures whether teacher j's endline beliefs correctly match parent i's rankings, Treatment_j indicates whether the teacher received access to parent information, $\text{Accuracy}_{ij,t=0}$ is the baseline accuracy measure, and γ_g represents grade fixed effects. Pre-registered specifications included school fixed effects, but these are omitted here given the final experimental sample comes from a single school. Standard errors are clustered at the classroom level. For classroom-level accuracy measures (Tables 4–5), the unit of observation is the teacher and the specification is analogous, with the dependent variable measuring accuracy about the classroom average or distribution of parent preferences. Standard errors are robust to heteroskedasticity.

Student-level accuracy. At the individual student level, point estimates are indistinguishable from zero and, if anything, slightly negative (Table 3). For example, the ITT on *Top Skill Match* is −0.0165 (s.e. 0.0675), with similar nulls for other outcomes. There is strong persistence in accuracy (baseline-to-endline), but no detectable treatment effect. This suggests that even with information, teachers did not reliably memorize or track each parent's unique ranking for the sampled students.

Classroom-average accuracy. In contrast, when I aggregate to the classroom average ranking (i.e., "what does the average parent in my class want?"), treatment effects turn positive and economically meaningful, though imprecise (Table 4). ITT estimates are on the order of 9–10 percentage points for *Top Skill Match* and *Bottom Skill Match* (e.g., +0.0918 and +0.1017), with similar magnitude for *Top & Bottom Match* (+0.1028). While not statistically significant, these effect sizes are large relative to control means (0.27–0.46), and are consistent with teachers updating their mental model of the typical parent in the class.

Heterogeneity by what parents value most. Table 5 splits classrooms by which category parents rank as most important, on average. Patterns are intuitive: (i) in classrooms where academics is the top priority on average, teachers improve in pinning down the top category (ITT on *Exact Order Match* +0.199; *Top Skill Match* +0.145); (ii) where social or emotional skills are the top classroom priority, teachers become more accurate about the bottom category (ITT on *Bottom Skill Match* +0.185 for social; +0.115 for emotional). A natural understanding is that information helped teachers correctly locate where academics sits in the average parent's priority ordering: on top in some classes, lower in others.

At baseline, teacher beliefs about parent priorities were largely noise and heavily colored by projection. The intervention did not make teachers precise about each parent, but it nudged them toward a more accurate picture of the average parent in their class, especially with regard to the relative place of academics in parents' priorities for their children's educational goals. In the model, this matters because teacher policy is a classroom-level cost tilt: getting the class-average benefits right moves the frontier in the direction that better aligns growth with what families value, even if idiosyncratic values are harder to track.

Table 3: Treatment Effects on Teacher Accuracy about Specific **Parents**

Dependent Variables	**Exact Order Match**	**Top Skill Match**	**Bottom Skill Match**	**Top & Bottom Match**
	(1)	(2)	(3)	(4)
Variables				
Treated	-0.0111	-0.0165	-0.0194	-0.0267
	(0.0291)	(0.0675)	(0.0774)	(0.0774)
Baseline Accuracy	0.2638***	0.2845***	0.2013***	0.2746***
	(0.0662)	(0.0552)	(0.0546)	(0.0540)
Statistics				
Observations	482	482	482	482
R^2	0.07583	0.13548	0.07613	0.12501
Control Mean	0.1292	0.6667	0.6333	0.5542

Notes: Outcomes are parent-level indicators for whether a teacher's belief matches a given parents' ranking of skills (academic, emotional, social). *Exact Order Match* = 1 if the teacher's full ranking matches the parents' ranking, including ties (e.g., parent 1–2–2 requires teacher 1–2–2). *Top Skill Match* = 1 if the teacher identifies at least one of the parent's top-ranked skills as top. *Bottom Skill Match* = 1 if the teacher identifies at least one of the parent's bottom-ranked skills as bottom. *Top & Bottom Match* = 1 if both top and bottom categories are correctly identified. "Treated" indicates teachers who received information about parent rankings and levels. "Baseline accuracy (same outcome)" is the coefficient on the corresponding baseline measure of that outcome.
Standard errors in parentheses, clustered at the class level
*Significance Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 4: Treatment Effects on Teacher Accuracy about **Classroom Average**

Dependent Variables	**Exact Order Match**	**Top Skill Match**	**Bottom Skill Match**	**Top & Bottom Match**
	(1)	(2)	(3)	(4)
Variables				
Treated	0.0351	0.0918	0.1017	0.1028
	(0.0988)	(0.1452)	(0.1376)	(0.1224)
Baseline Accuracy	0.3613***	0.2448**	0.2439**	0.3019***
	(0.1283)	(0.1134)	(0.1034)	(0.1071)
Statistics				
Observations	94	94	94	94
R^2	0.17440	0.09531	0.18254	0.15772
Control Mean	0.1450	0.4618	0.4122	0.2710

Notes: Dependent variables are indicators for the accuracy of teacher beliefs about parents' skill rankings (academic, emotional, social). Grade fixed effects included in all specifications. *Exact Order Match* = 1 if the teacher's full ranking matches the parents' ranking, including ties (e.g., parent ranking 1–2–2 requires teacher belief 1–2–2). *Top Skill Match* = 1 if the teacher identifies at least one of the parent's top-ranked skills as top. *Bottom Skill Match* = 1 if the teacher identifies at least one of the parent's bottom-ranked skills as bottom. *Top & Bottom Match* = 1 if both top and bottom categories are correctly identified. "Baseline accuracy" is the coefficient on the corresponding baseline measure of that outcome. "Treated" indicates whether the teacher received information about parent rankings and levels.
Clustered standard errors in parentheses
*Significance Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 5: Treatment Effects on Teacher Accuracy about **Classroom Average** by Parent Top Priority

Dependent Variables	**Exact Order Match**	**Top Skill Match**	**Bottom Skill Match**	**Top & Bottom Match**
	(1)	(2)	(3)	(4)
Variables				
Treated: Parents' Top = Academic	0.1987	0.1453	-0.0025	0.0639
	(0.1239)	(0.1820)	(0.1693)	(0.1585)
Treated: Parents' Top = Social	-0.0878	0.0884	0.1846	0.1196
	(0.1056)	(0.1733)	(0.1673)	(0.1455)
Treated: Parents' Top = Emotional	-0.0396	-0.0503	0.1150	0.1608
	(0.1020)	(0.2385)	(0.2263)	(0.2149)
Baseline Accuracy	0.3097**	0.2100*	0.2666**	0.3152***
	(0.1223)	(0.1234)	(0.1032)	(0.1094)
Statistics				
Observations	94	94	94	94
R^2	0.22938	0.10416	0.19377	0.16074
Control Mean	0.1450	0.4618	0.4122	0.2710

Notes: Dependent variables are indicators for the accuracy of teacher beliefs about the class average of parents' skill rankings (academic, emotional, social). Grade fixed effects included in all specifications. *Exact Order Match* = 1 if the teacher's full ranking matches the class average ranking, including ties (e.g., parents 1–2–2 requires teacher 1–2–2). *Top Skill Match* = 1 if the teacher identifies at least one of the parent's top-ranked skills as top. *Bottom Skill Match* = 1 if the teacher identifies at least one of the parent's bottom-ranked skills as bottom. *Top & Bottom Match* = 1 if both top and bottom categories are correctly identified. Rows labeled "Treated: Parents' Top = Academic/Social/Emotional" correspond to classrooms where the teacher received information about parent rankings and levels, estimated separately by the parents' top-ranked category. Coefficients are intent-to-treat effects relative to control classrooms in the same category. "Baseline accuracy" is the coefficient on the corresponding baseline measure of the same outcome.
Clustered standard errors in parentheses
*Significance Codes: ***: 0.01, **: 0.05, *: 0.1*

8 Student Outcome Results

I focus on three outcomes that map directly to the model: (i) skill levels in parents' most and least preferred skill categories; (ii) skill preference ranks for parents' most and least preferred categories; and (iii) the preference-level slope within students (across dimensions). For each outcome, I estimate intent-to-treat (ITT) effects using OLS regressions of the form:

$$Y_{i,t=1} = \alpha + \beta \text{Treatment}_i + \delta Y_{i,t=0} + \gamma_g + \epsilon_i \tag{13}$$

where $Y_{i,t=1}$ is the endline outcome for student i, Treatment_i is an indicator for whether the student's teacher was assigned to receive parent information, $Y_{i,t=0}$ is the baseline value of the outcome, and γ_g represents grade fixed effects. Standard errors are clustered at the classroom level. The inclusion of baseline outcomes as controls improves precision and helps address potential imbalances, while grade fixed effects absorb any systematic differences across grades in initial skill levels or teaching practices.

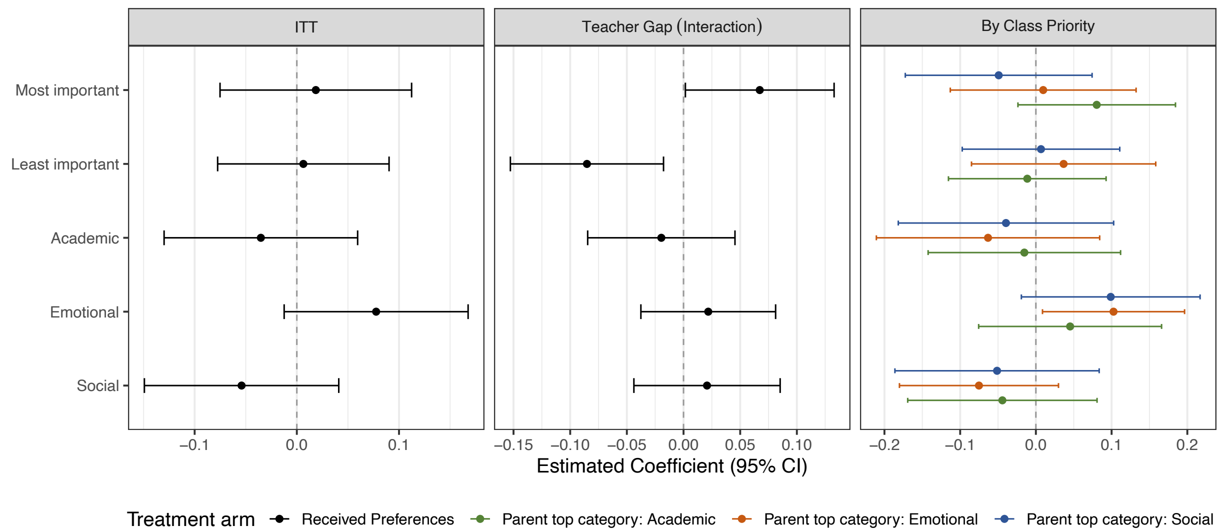
(1) Skill levels increase for parents most preferred skill category when teachers were initially inaccurate. Panel 1 of Figure 15 shows that the average intent-to-treat (ITT) effect on parents' skill levels is close to zero. However, this masks important heterogeneity. Panel 2 plots the interaction between treatment and teachers' baseline inaccuracy. Teacher inaccuracy is calculated as the absolute difference between the teacher's belief about how a given parent ranks a skill and the parent's actual ranking. Therefore, the gap varies depending on the teacher-parent-skill triple, with a higher gap indicating that the teacher was more incorrect about how much that parent valued improving that skill. The coefficient on the interaction is +0.07 (s.e. 0.03) for the category parents rated most important and -0.09 (s.e. 0.03) for the least important, both statistically significant.

Panel 3 splits effects by which category parents, on average, valued most across the classroom. For example, "Parent top category: Academic" compares control to treated classrooms where parents, on average, most preferred improving academic skills. Coefficients are larger when parents prioritized academics (0.08, s.e. 0.05) than social or emotional skills. This likely reflects that teachers can more readily reallocate effort in academics, that our belief updating analysis showed they learned more precisely where academics ranked, and that schools were already running universal social-emotional programming, and thus made new information about academic preferences especially actionable.

Figure 16 visualizes heterogeneous treatment effects as a function of the baseline parent-teacher gap. Strikingly, when teachers were initially accurate (gap = 0), treatment effects run counter to the intended direction in both panels: skill levels decrease in parents' most important category (-0.25 SDs, s.e. 0.14, p=0.09, Panel 1) and increase in parents' least important category (0.30 SDs, s.e. 0.12, p=0.02, Panel 2). However, as baseline misalignment grows, effects reverse in the intuitive direction. For parents' most important category, treatment effects become positive around gap = 4 (the 75th percentile), though coefficients remain marginally significant at the 10% level. For parents' least important category, treatment effects turn negative at moderate gaps and become significantly so at higher levels of misalignment, reaching -0.39 SDs (s.e. 0.19, p=0.04) at gap = 8.

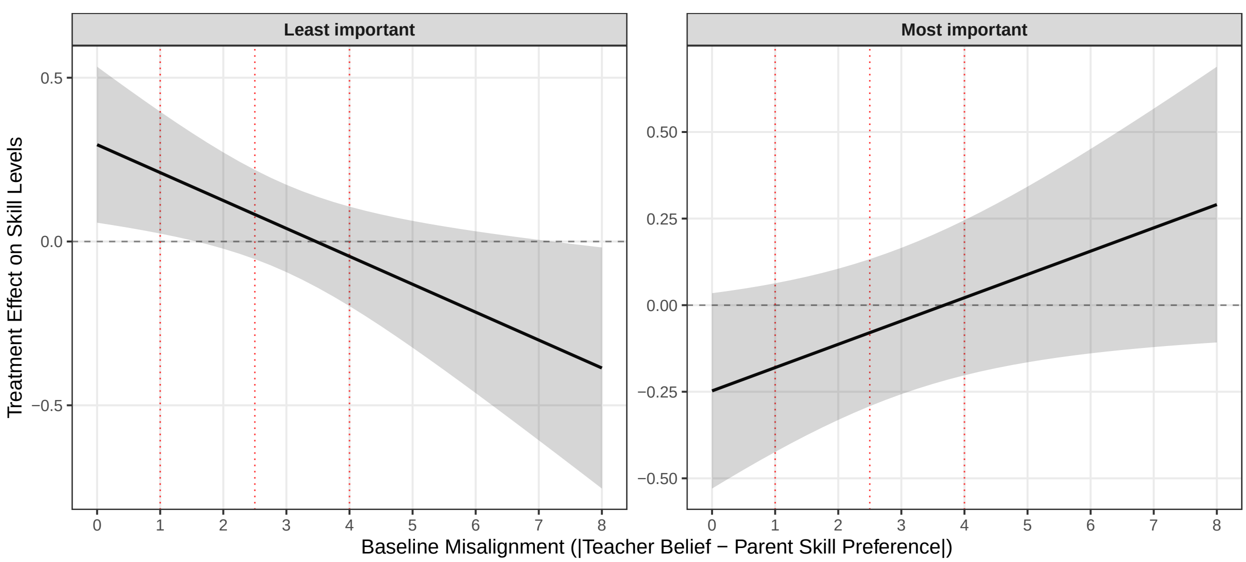
These patterns are consistent with classroom-level reallocation: teachers initially allocated effort toward students they thought they understood, but upon learning the true distribution of parent priorities, they shifted effort toward previously misaligned students, sometimes at the expense of those they had correctly matched. Since most students fall in the middle of the gap distribution (median = 2.5), the average treatment effect masks offsetting movements — in other words, teachers reallocated efforts from well-matched to poorly-matched students within the same classroom.

Figure 15: Treatment Effects on Skill Levels



Notes: $N = 823$ parents. Outcome is skill levels (standardized within-student). Estimates control for grade fixed effects; standard errors clustered at the classroom level. Panel 1 shows ITT effects; Panel 2 interacts treatment with the baseline teacher-parent gap (higher gap = teacher more incorrect), and plots the interaction terms; Panel 3 splits treatment by which category parents valued most. For example, "Parent top category: Academic" compares control to treated classrooms where parents' average top priority was academic skills.

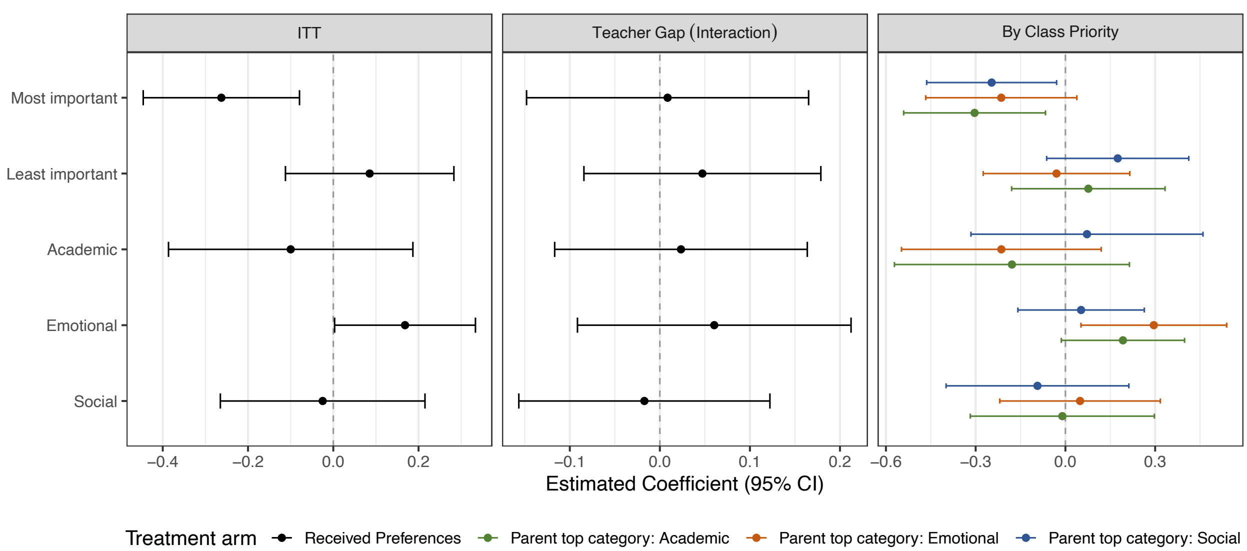
Figure 16: Treatment Effects by Baseline Teacher Accuracy



Notes: $N = 823$ parents. Treatment effects are for skill levels (standardized within-student). Treatment effects control for grade fixed effects with standard errors clustered at the classroom level. Panel 1 shows effects for the category parents rated least important; Panel 2 for the most important. The x-axis is the baseline teacher-parent gap (higher gap = teacher more incorrect). The solid line is the marginal effect of treatment estimated from a regression of the outcome on treatment, the gap, and their interaction; the shaded area is the 95% confidence interval. The red dashed lines show the 25th, 50th, and 75th percentiles of the gap distribution.

(2) Skill preferences fall for parents' most preferred skill category. Panel 1 of Figure 17 shows a clear average intent-to-treat (ITT) effect on parents' skill preferences; in treated classrooms, the skill parents had most preferred falls 0.26 skill preference ranks (s.e. 0.09, p=0.006) relative to control. This pattern is consistent with teachers making it easier for students to develop those skills, once informed of parent priorities, and parents subsequently judging these skills as less in need of work. Panel 2 plots treatment interacted with the baseline teacher-parent gap; the interaction coefficients are small and imprecise (-0.02 to 0.05), indicating that, unlike the skill levels outcome, parents' shift in rank orderings does not vary systematically with how wrong teachers were at baseline. Column 3 splits treated classrooms by the category parents prioritized most, on average. In line with the treatment effects for skill levels, the decreasing in skill preference rank is largest when parents' top priority was academics (-0.30 ranks, s.e. 0.12, p=0.014), somewhat smaller for social (-0.25 ranks, s.e. 0.11, p=0.028), and positive but only marginally significant for emotional (-0.21, s.e. 0.13, p=0.099). This suggests that teachers most effectively reoriented classroom effort when the desired skills were academic, where they have more control and, as our updating analysis showed, learned most clearly how academics compared to other skills.

Figure 17: Treatment Effects on Skill Preference Ranks

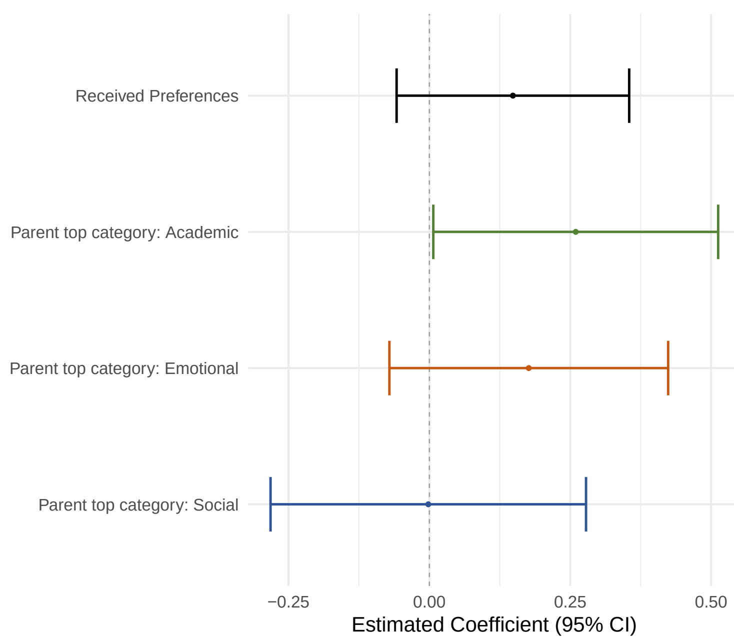


Notes: $N = 823$ parents. Skill preference ranks (1-9) multiplied by -1 so higher values indicate more preferred skills. Positive coefficients therefore indicate the skill becomes more important to improve. Estimates control for grade fixed effects; standard errors are clustered at the classroom level. Panel 1 shows ITT effects; Panel 2 interacts treatment with the baseline teacher-parent gap (higher gap = teacher more incorrect); Panel 3 splits treatment by which category parents valued most. For example, "Parent top category: Academic" compares control to treated classrooms where parents' average top priority was academic skills.

(3) Preference-level slope shifts upward for treated students. Recall from Section 5 that we estimate the within-student preference-level slope by regressing each parent's skill preference ranking on standardized skill levels; a more positive slope means that the parent prefers improvement in stronger skills. Figure 18 shows that the average intent-to-treat (ITT) effect on this slope is positive though only marginally precise: +0.15 (s.e. 0.11, p=0.162). I am not able to interact treatment with the baseline teacher-parent gap for the preference-level slope outcome because the slope is estimated at the student level, not the student-skill level. However, as in my analysis of skill levels and skill preferences, I split treatment effects by which category parents prioritized most, on average, in their class.

Splitting treatment by parents' top average category shows that the effect is largest and statistically significant in classrooms where parents most prefer academics on average (+0.26, s.e. 0.13, p=0.047), essentially zero in classrooms where they prefer improvement in social skills (-0.00, s.e. 0.14, p =0.990), and positive but imprecise for classrooms where they prefer improvement in emotional skills (+0.18, s.e. 0.13, p=0.165). Together with the skill levels and skill preference results, this pattern suggests that when teachers learn parents' priorities, particularly academic priorities, they reconfigure the classroom so that it is easier for students to build the skills parents value, making students' observed skill profiles better aligned with parental preferences rather than with underlying production cost constraints.

Figure 18: Treatment Effect on the Within-Student Preference-Level Slope



Notes: N = 823 parents. Estimates control for grade fixed effects; standard errors clustered at the classroom level. Full sample ITT effects at the top; Treatment effects split by which category parents valued most at the bottom. For example, "Parent top category: Academic" compares control to treated classrooms where parents' average top priority is academic skills.

Taken together, these findings fit the framework: information to teachers acts as a production shock that selectively lowers the effective cost of building parent-prioritized skills. By reshaping the classroom environment rather than only tailoring instruction to individual students, teachers enable more progress where parents want gains (i.e., skill level improvements for prioritized categories), shift students' overall skill profiles toward parental preferences (i.e., upward preference-level slope), and reduce parents' perceived need for further improvement in those areas (i.e., downward rank movement). In short, providing teachers with clear signals of parental marginal valuations moves the whole classroom's skill production away from what is merely easiest to build and toward what families value most.

9 Model Estimation

To translate the framework into policy terms, I estimate the structural model using Bayesian methods. The estimation leverages teacher skill rankings as supply-side cost shifters–teachers who emphasize certain skills should make those skills easier for students to produce. Since students are plausibly randomly assigned to teachers within grades, these rankings provide exogenous variation in production costs that can help separate cost from benefit heterogeneity. I extend the two-skill model in Section 3 to three skill categories (academic, social, emotional) to match the data structure. The full technical details of the three-skill model are provided in Appendix Section E.

9.1 Estimation Methodology

The model includes demographics as covariates for both preferences (β_i) and production costs (κ_{ij}). Specifically, I allow class, parental education, household income, and teacher skill rankings to shift the ease of producing academic versus social-emotional skills. Teacher rankings enter only the cost equation (not preferences) and provide an exclusion restriction that aids identification.

The model is estimated using Hamiltonian Monte Carlo, with 4 chains of 3,000 post-warmup draws each. I use skill levels and skill preferences jointly: skill levels (i.e., s_{ij} in the model) are increasing in benefits and decreasing in costs, while skill preferences (i.e., MRS_{ij} in the model) are increasing in both benefits and costs, helping to separately identify the two sources of heterogeneity. Appendix E provides full technical details on priors, identification, and convergence diagnostics.

9.2 Main Results: Variance Decomposition

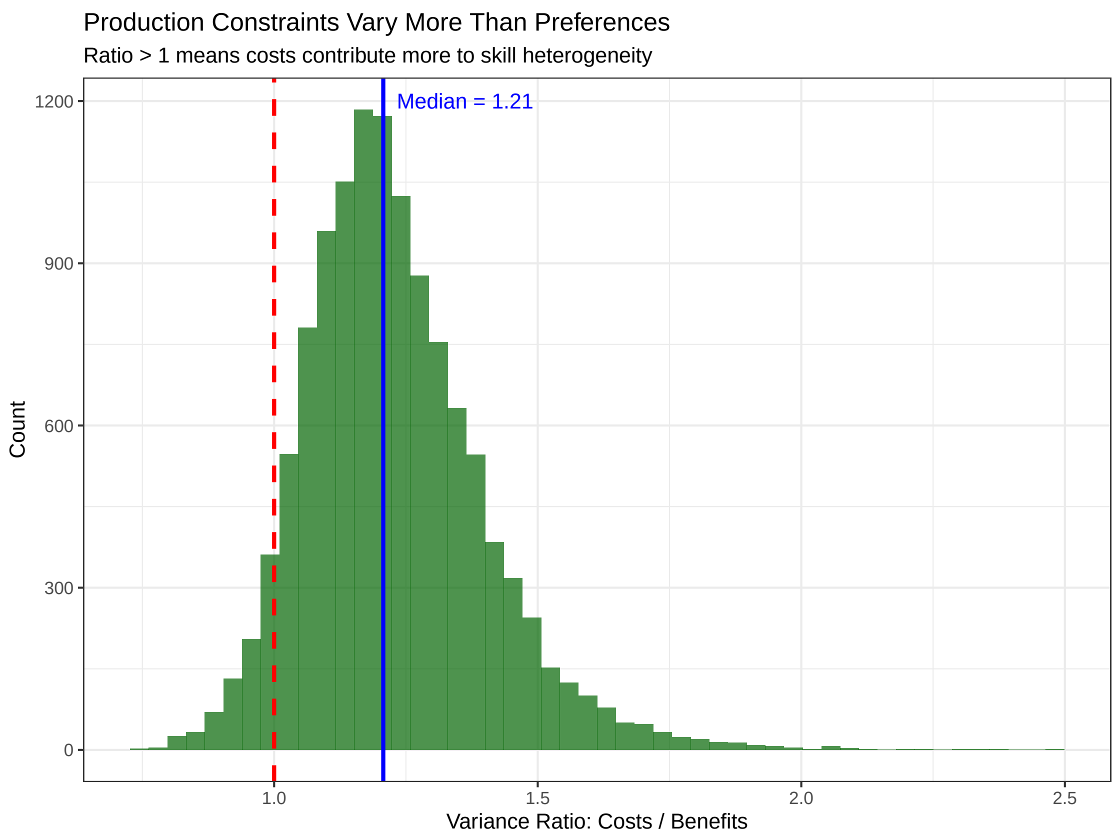
Recall from Section 3 that the equilibrium skill ratio decomposes as $s_i^* = T_i \cdot \lambda_i$, where $T_i = (\beta_i/(1-\beta_i))^{1/\rho}$ captures the benefits tilt (how much more a parent values skill 1) and $\lambda_i = \kappa_{1i}/\kappa_{2i}$ captures the costs tilt (how much easier it is to produce skill 1). Taking logs yields $\log s_i^* = \log T_i + \log \lambda_i$, so the variance of log skill ratios decomposes additively as

$$\text{Var}[\log s_i] = \text{Var}[\log T_i] + \text{Var}[\log \lambda_i] + 2\text{Cov}[\log T_i, \log \lambda_i].$$

The covariance term reflects identification challenges: while the mapping from (T_i, λ_i) to observables is one-to-one in theory, it is empirically weakly identified because the likelihood is nearly flat along ridges where $\log T_i + \log \lambda_i$ remains constant. Many (T_i, λ_i) pairs produce nearly identical predictions in finite, noisy data. Nevertheless, the ratio of the two variance components remains well identified and directly measures the relative importance of cost versus benefit heterogeneity in driving specialization patterns.

Figure 19 presents the posterior distribution for this key quantity: the ratio of cost variance to benefit variance, $\mathrm{Var}[\log \lambda_i]/\mathrm{Var}[\log T_i]$. The median ratio is 1.21 (95% CI: [0.94, 1.62]), with 93.5% of posterior mass above 1.0. This indicates that production constraints vary approximately 21% more across families than do preferences for academic versus social-emotional skills, and is consistent with the reduced-form preference-level slope analysis in Section 5.

Figure 19: Posterior Distribution of Variance Ratio: Costs / Benefits



Notes: Distribution shows $\mathrm{Var}[\log \lambda_i]/\mathrm{Var}[\log T_i]$ across 12,000 posterior draws. Red dashed line marks equality (ratio = 1); blue solid line shows posterior median (1.21). Ratio > 1 implies production constraints vary more than preferences.

After controlling for demographics, unexplained residual variation in costs ($\sigma_{\ell 1}$) exceeds that in preferences ($\sigma_{\beta 1}$) by an even larger margin: the median ratio $\sigma_{\ell 1}/\sigma_{\beta 1} = 3.11$ (95% CI: [0.42, 70.59]), though with substantial uncertainty. Together, these estimates reinforce that observed specialization in this setting is primarily driven by what students find easier to learn rather than by what families value more.

9.3 Heterogeneity Across Subgroups

Table 6 examines whether the costs-versus-benefits balance differs by grade, income, or parental education. The variance ratio remains consistently above 1 across all subgroups, though with considerable overlap in credible intervals due to limited sample sizes within groups. The ratio is somewhat higher in early grades (median 1.48 for grade 3) than in upper grades (1.42 for grade 10), though differences are imprecise. Income and parental education show little systematic relationship with the ratio.[8]

Table 6: Variance Ratio by Demographic Subgroups

Subgroup	N	Median Ratio	95% CI	P(Costs > Benefits)
By Grade (omitted: Grade 1)				
Grade 2	132	1.43	[0.92, 2.81]	0.942
Grade 3	145	1.50	[0.92, 3.00]	0.948
⋮				
Grade 10	209	1.44	[0.93, 2.80]	0.949
By Household Income (omitted: 0–5 Lakh)				
5–10 Lakh	303	1.22	[0.93, 1.73]	0.934
10–15 Lakh	425	1.22	[0.94, 1.74]	0.933
Over 15 Lakh	779	1.28	[0.93, 1.91]	0.942
By Mother's Education (omitted: Less than Secondary)				
Secondary	146	1.19	[0.91, 1.70]	0.895
Bachelor's	500	1.25	[0.94, 1.78]	0.942
Graduate+	1131	1.26	[0.94, 1.80]	0.945

Notes: Each row shows the posterior median, 95% credible interval, and probability mass above 1 for $\text{Var}[\log \lambda]/\text{Var}[\log T]$ within the indicated subgroup. Full results for all grades and father's education in Appendix Table B.7.

The current estimation faces several limitations. First, teacher skill rankings, intended as supply shifters, do not enter significantly in the cost equation; this likely reflects weak instruments or insufficient sample variation within schools.[9] As a result, the model primarily identifies the variance ratio from cross-sectional patterns in how parents' skill levels and skill preferences covary, supplemented by demographic controls. This reflects the fundamental identification challenge: many (T_i, λ_i) combinations produce similar skill ratios. Note that while the *ratio* of variances remains well-identified, decomposing *individual-level* contributions is not feasible without additional instruments or longitudinal data that could trace out expansion paths over time. Despite these constraints, the structural estimates corroborate the reduced-form findings: in this setting, production constraints—not preference heterogeneity—are the primary driver of why students specialize differently across skills.

[8] Wealthier and more educated families show slightly *lower* ratios (1.21–1.27 versus 1.17–1.18); this potentially indicates that higher-resourced households face more uniform production costs, but these patterns are not statistically distinguishable.

[9] Future work could exploit teacher turnover or larger multi-school samples to strengthen this identification strategy.

9.4 Welfare Implications and Counterfactuals

The model framework opens a path toward quantifying welfare gains from policy interventions. In the model, utility-maximizing parents choose skill bundles subject to a production frontier determined by $(\kappa_{1i}, \kappa_{2i})$ and budget I_i. As shown in Section 3, when the ratio of costs $\lambda_i = \kappa_{1i}/\kappa_{2i}$ is set optimally, the skill ratio converges to a threshold $s^\dagger = \sqrt{\beta_i/(1-\beta_i)}$ that equates the marginal rate of substitution with the expansion path. This threshold represents the aligned specialization level where incremental skill growth due to budget increases (e.g., skill growth over time) moves in the direction of steepest utility increase.

Observed skill ratios $s_i^* = T_i \lambda_i$ deviate from this threshold whenever production costs $\lambda_i = \kappa_{1i}/\kappa_{2i}$ are misaligned with preferences. As shown in Section 3, the optimal cost ratio that would implement the aligned threshold is

$$\lambda_i^\star = \frac{s_i^\dagger}{T_i} = \left(\frac{\beta_i}{1-\beta_i}\right)^{1/2-1/\rho}.$$

Counterfactual: Alignment between costs and benefits to skills. In this counterfactual, I quantify the potential welfare gains from reducing production-preference misalignment using a simple benchmark. Fix preferences β_i and the size of the production possibility set (budget I_i and overall frontier scale), and ask: how much would utility increase if each family faced production costs $\kappa_i^\star$ that perfectly aligned with their preferences? Formally, for each parent i, compute:

$$\Delta U_i = U\big(c^*(\kappa_i^\star; I_i, \beta_i)\big) - U\big(c^*(\kappa_i^{\text{baseline}}; I_i, \beta_i)\big),$$

where $\kappa_i^\star = (\kappa_{1i}^\star, \kappa_{2i}^\star)$ satisfies:

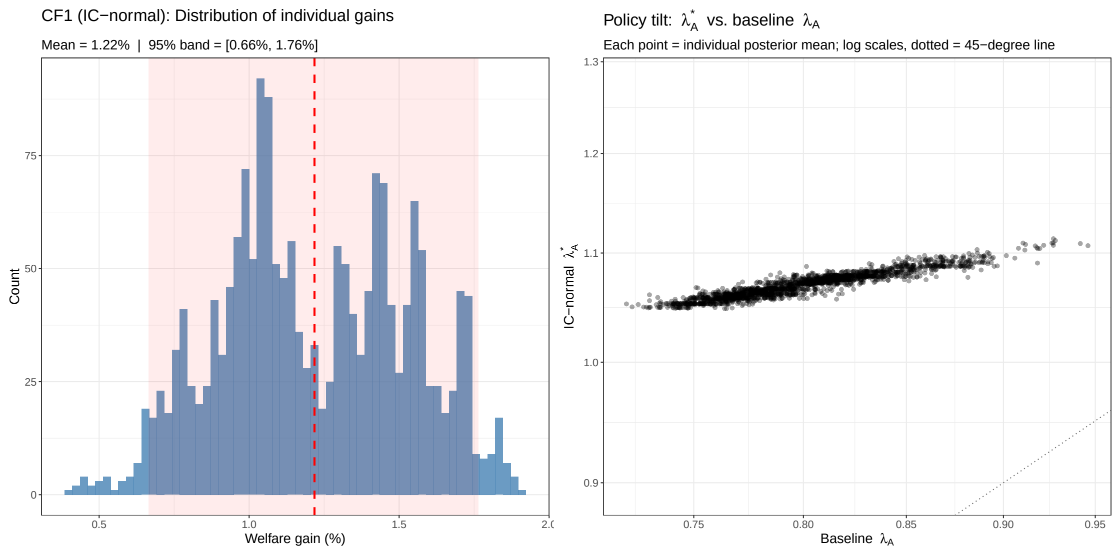
$$\kappa_{1i}^\star/\kappa_{2i}^\star = \lambda_i^\star = \left(\frac{\beta_i}{1-\beta_i}\right)^{1/2-1/\rho}.$$

This counterfactual captures the welfare gain from rotating the academic-vs-non-academic cost ratio just enough so that the indifference curve is locally orthogonal to the expansion path (Appendix D.5). Intuitively, $\lambda_i^\star$ aligns the cost side with what the family values, without expanding or contracting the overall production possibility set.

To isolate the effect of rotating the frontier, I impose a scale normalization on the frontier. The CET-consistent sum of cost terms remains constant, $\sum_j \kappa_{ji}^{-\rho}$ is held fixed for each i. This ensures that the overall size of the production possibility set is constant, while allowing the frontier to rotate. Under this normalization, I compute utility at the baseline and counterfactural cost parameters, with optimal choices in each case.

Figure 20 presents the distribution of welfare gains from this counterfactual. The gains are modest: average gains are on the order of one percent, with a long but thin upper tail. Overall, the small gains indicate that most families are not far from their aligned mix given these estimated preferences and production costs.

Figure 20: Distribution of Welfare Gains from Aligning Costs and Benefits



Notes: Panel 1 shows histogram of percentage welfare gains from the alignment of costs and benefits. Each observation is the mean of posterior draws for each family (N = 1,851). Panel 2 shows the baseline estimated $\lambda_A = \kappa_A/\kappa_N$ ratio for academic and non-cognitive skills against the counterfactual aligned ratio $\kappa^{\star}$. The aligned ratio is defined based on β_i as described in Appendix Section D.5.

Three features of the estimated environment dampen the estimated welfare gains. First, posterior means for β_1 concentrate around 0.3-0.45, so the IC-normal target $\lambda_i^{\star}$ only moderately differs from baseline λ_i. Second, with $\rho = 1.5$, the optimal academic share depends on the degree of the preference tilt $(\beta_i/(1-\beta_i)/\rho)$, muting the response to cost rotations. Finally, the Bayesian estimation smooths over extreme heterogeneity (Bayesian shrinkage), reducing the scope for large misalignments, and large welfare gains.

Taken together, these results suggest that while production-preference misalignment exists, it is not extreme for most families in this setting. Policies that better align production costs with parental preferences could yield modest welfare improvements on average, with large gains coming from stronger preference heterogeneity, steeper production tradeoffs (lower ρ), or interventions that expand the frontier rather than simply rotating it.

10 Conclusion

To accurately measure student welfare, we must account for individual heterogeneity in preferences over skills. I document this heterogenity by measuring parents' perceptions of their child's skill levels, and preferences over improving different skills. Combined with a simple model of skill formation, these measures help answer whether a given student specializes in skills that are easy to build, or in those that are high value. If families prioritize shoring up weak skills, this implies those skills hold value, but are difficult to build; conversely families who prioritize further growth in strong skills signal that building weak skills is relatively easy, but of low value. I show how the diagnostic can be measured at the skill or student level, turning an abstract problem into an actionable one.

In Indian private schools, the evidence points to cost-driven specialization: parents predominantly want growth in weaker skills. Teachers, however, systematically misperceive those priorities; their beliefs about what parents want track their own tastes rather than families' values. A classroom-level randomized intervention that revealed parents' perceptions and priorities corrected teachers' average beliefs and shifted growth toward the domains families value most, with the largest gains where initial misperceptions were largest.

Beyond this empirical finding, the framework is itself a contribution. Treating teachers as cost shifters who reshape the production frontier for skills organizes how we think about interventions: some shift skill production (e.g., remedial education that lowers the cost of improving weak skills); others shift perceived returns (e.g., information about market wages for certain occupations). The diagnostic distinguishes these channels in the data and suggests how to think about complementarities across interventions. For example, information about returns may be most effective when paired with cost-lowering interventions that make it easier to build newly valued skills.

Several limitations remain. The experiment studies one type of school system over the short run; whether costs or returns drive specialization in other contexts, and long-run substitution across skill domains, are open questions. Parents' priorities, while welfare-relevant, may differ from broader social objectives; incorporating student and policymaker goals is a natural extension. Finally, the slope is a reduced-form indicator of costs versus returns; pairing it with direct measures of inputs to skill growth (e.g., teacher time allocation, parental investments) would sharpen our understanding of mechanisms.

Future work should (i) track longer-run outcomes to test persistence and spillovers across skill domains; (ii) study equity by testing if alignment narrows gaps for students who face other educational barriers; and (iii) assess scalability by embedding the tool in routine workflows of teachers and schools (text-messages, report cards) at low cost. In general, pairing this framework with interventions that shift the costs or benefits to skills can help design more effective policies that build the diverse skillsets children need to thrive.

References

Andrabi, T., Das, J., and Khwaja, A. I. (2017). Report cards: The impact of providing school and child test scores on educational markets. *American Economic Review*, 107(6):1535–1563.

Banerjee, A., Banerji, R., Berry, J., Duflo, E., Kannan, H., Mukerji, S., Shotland, M., and Walton, M. (2017). From proof of concept to scalable policies: Challenges and solutions, with an application. *Journal of Economic Perspectives*, 31(4):73–102.

Baum, D., Lewis, L., Lusk-Stover, O., and Patrinos, H. (2014). What matters most for engaging the private sector in education.

Bergman, P. (2021). Parent-child information frictions and human capital investment: Evidence from a field experiment. *Journal of Political Economy*, 129(1):286–322.

Bergman, P. and Chan, E. W. (2021). Leveraging parents through low-cost technology. *Journal of Human Resources*, 56(1):125–158.

Berlinski, S., Busso, M., Dinkelman, T., and Martínez A., C. (2022). Reducing parent-school information gaps and improving education outcomes: Evidence from high-frequency text messages. *Journal of Human Resources*.

Burgess, S., Rawal, S., and Taylor, E. S. (2023). Teachers' use of class time and student achievement. *Economics of Education Review*, 94:102405.

Câmara Leme, A., Escardíbul, J.-O., Nunes, L. C., Balcão Reis, A., and Seabra, C. (2020). The effect of a specialized versus a general upper-secondary school curriculum on students' performance and inequality. a difference-in-differences cross-country analysis. *Applied Economics*, 52(39):4317–4331.

Caselli, F. and Coleman, W. J. (2006). The world technology frontier. *American Economic Review*, 96(3):499–522.

Chyung, S. Y., Hutchinson, D., and Shamsy, J. A. (2020). Evidence-based survey design: Ceiling effects associated with response scales. *Performance Improvement*, 59(6):6–13.

Conlon, J. J. (2021). Major malfunction: A field experiment correcting undergraduates' beliefs about salaries. *Journal of Human Resources*, 56(3):922–939.

Cotton, C., Hickman, B., List, J. A., Price, J., and Roy, S. (2025). Why don't struggling students do their homework? disentangling motivation and study productivity as drivers of human capital formation.

Cunha, F. and Heckman, J. (2007). The technology of skill formation. *American Economic Review*, 97(2):31–47.

Cunha, F. and Heckman, J. J. (2008). Formulating, identifying and estimating the technology of cognitive and noncognitive skill formation. *Journal of Human Resources*, 43(4):738–782.

Cunha, F., Heckman, J. J., and Schennach, S. M. (2010). Estimating the technology of cognitive and noncognitive skill formation. *Econometrica*, 78(3):883–931.

Deming, D. and Silliman, M. (2025). Skills and human capital in the labor market. In *Handbook of Labor Economics*, volume 6, pages 115–157. Elsevier.

Deming, D. J. (2017). The growing importance of social skills in the labor market. *The Quarterly Journal of Economics*, 132(4):1593–1640.

Dizon-Ross, R. (2019). Parents' beliefs about their children's academic ability: Implications for educational investments. *American Economic Review*, 109(8):2728–65.

Duflo, E., Dupas, P., and Kremer, M. (2011). Peer effects, teacher incentives, and the impact of tracking: Evidence from a randomized evaluation in kenya. *American economic review*, 101(5):1739–1774.

Eble, A. and Escueta, M. (2023). When your bootstraps are not enough: How demand and supply interact to generate learning in settings of extreme poverty. Technical report, National Bureau of Economic Research.

Fryer Jr, R. G. (2017). The production of human capital in developed countries: Evidence from 196 randomized field experiments. In *Handbook of economic field experiments*, volume 2, pages 95–322. Elsevier.

Glewwe, P. and Muralidharan, K. (2016). Improving education outcomes in developing countries: Evidence, knowledge gaps, and policy implications. In *Handbook of the Economics of Education*, volume 5, pages 653–743. Elsevier.

Goodman, R. (1997). The strengths and difficulties questionnaire: a research note. *Journal of child psychology and psychiatry*, 38(5):581–586.

Hanushek, E. A. and Woessmann, L. (2008). The role of cognitive skills in economic development. *Journal of economic literature*, 46(3):607–668.

Hastings, J. S. and Weinstein, J. M. (2008). Information, school choice, and academic achievement: Evidence from two experiments. *The Quarterly Journal of Economics*, 123(4):1373–1414.

Heckman, J. J., Stixrud, J., and Urzua, S. (2006). The effects of cognitive and noncognitive abilities on labor market outcomes and social behavior. *Journal of Labor Economics*, 24(3):411–482.

Hoxby, C. M. and Turner, S. (2015). What high-achieving low-income students know about college. *American Economic Review*, 105(5):514–517.

Jackson, C. K. (2018). What do test scores miss? the importance of teacher effects on non–test score outcomes. *Journal of Political Economy*, 126(5):2072–2107.

Jackson, C. K., Johnson, R. C., and Persico, C. (2016). The effects of school spending on educational and economic outcomes: Evidence from school finance reforms. *Quarterly Journal of Economics*, 131(1):157–218.

Jacob, B. A. and Lefgren, L. (2007). What do parents value in education? an empirical investigation of parents' revealed preferences for teachers. *The Quarterly Journal of Economics*, 122(4):1603–1637.

Jensen, R. (2010). The (perceived) returns to education and the demand for schooling. *The Quarterly Journal of Economics*, 125(2):515–548.

Kemper, J. and Renold, U. (2024). Evaluating the impact of general versus vocational education on labor market outcomes in egypt by means of a regression discontinuity design. *Journal of Development Economics*, 166:103172.

Kosse, F. and Tincani, M. M. (2020). Prosociality predicts labor market success around the world. *Nature communications*, 11(1):5298.

Lindqvist, E. and Vestman, R. (2011). The labor market returns to cognitive and noncognitive ability: Evidence from the swedish enlistment. *American Economic Journal: Applied Economics*, 3(1):101–28.

Mulhern, C., McNeill, S., Unlu, F., Phillips, B., Edmunds, J. A., and Grebing, E. (2024). Spillover effects of specialized high schools. *Journal of Public Economics*, 238:105170.

Muralidharan, K. and Sundararaman, V. (2011). Teacher performance pay: Experimental evidence from india. *Journal of political Economy*, 119(1):39–77.

Muralidharan, K. and Sundararaman, V. (2013). Contract teachers: Experimental evidence from india. Technical report, NBER Working Paper.

Porter, C. and Serra, D. (2020). Gender differences in the choice of major: The importance of female role models. *American Economic Journal: Applied Economics*, 12(3):226–254.

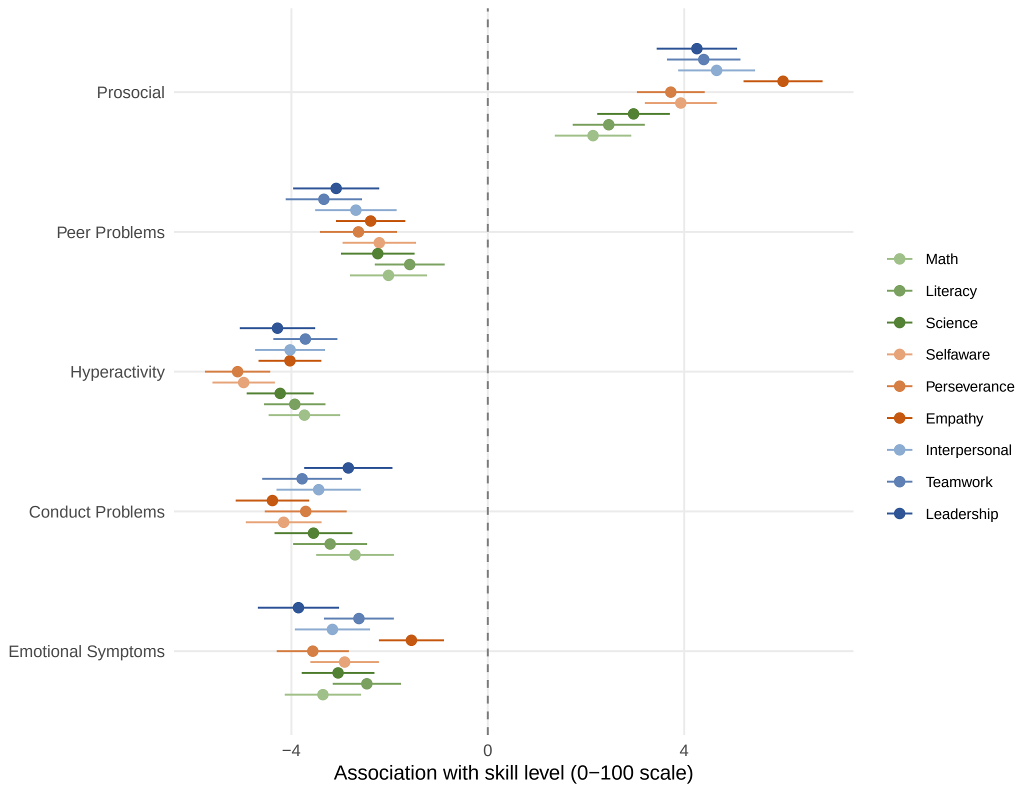
Rivkin, S. G., Hanushek, E. A., and Kain, J. F. (2005). Teachers, schools, and academic achievement. *Econometrica*, 73(2):417–458.

Rosen, S. (1983). Specialization and human capital. *Journal of Labor Economics*, 1(1):43–49.

Wiswall, M. and Zafar, B. (2015). Determinants of college major choice: Identification using an information experiment. *The Review of Economic Studies*, 82(2):791–824.

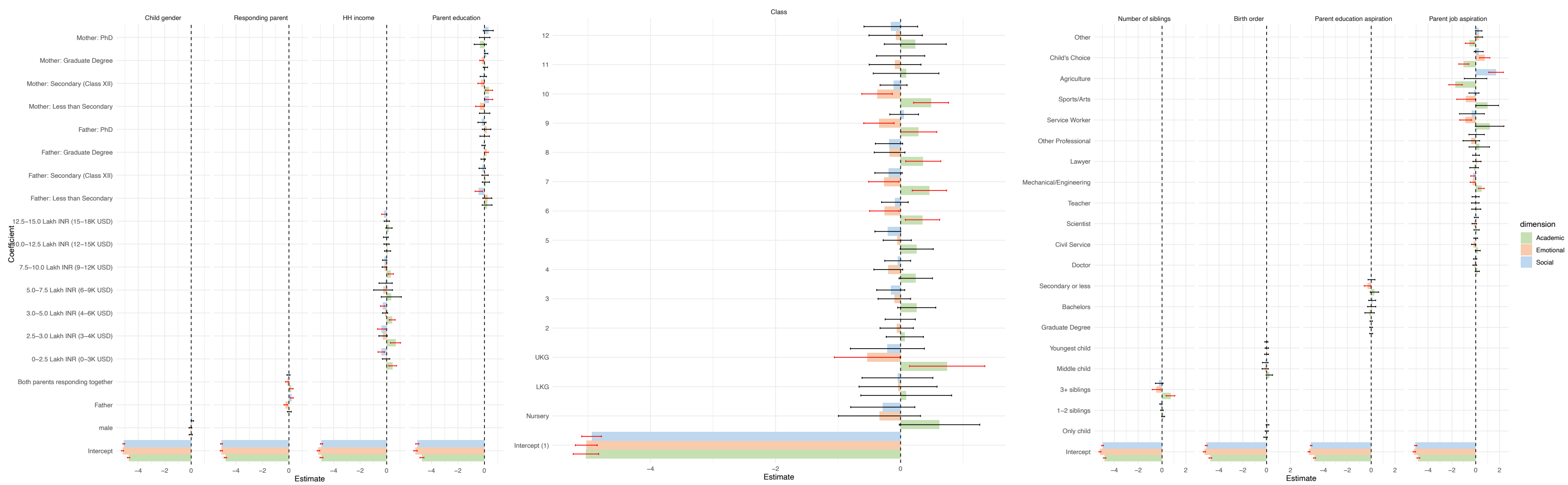
A Additional Figures

Figure A.1: Parent-Reported Skill Levels and Behavioral Measures



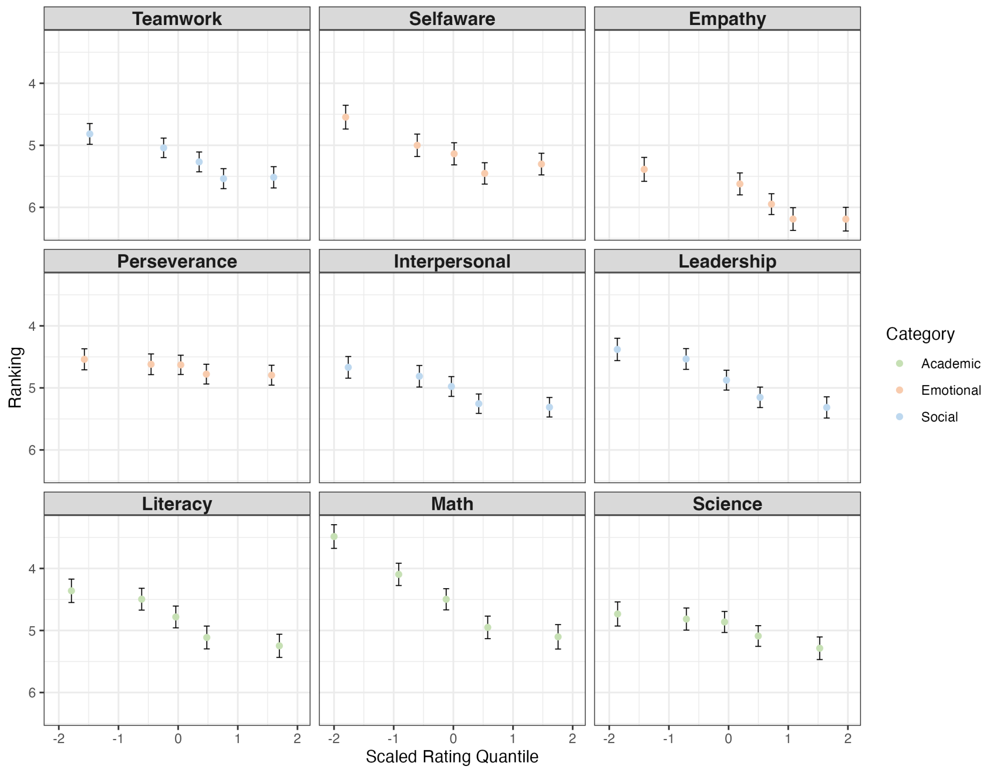
Notes: Coefficients from regressions of parent-reported skill levels (0-100 scale) on standardized Strengths and Difficulties Questionnaire (SDQ) subscales. Each SDQ subscale is constructed from five survey items and standardized across parents (mean 0 and standard deviation 1). Higher values on problem subscales (Emotional Symptoms, Conduct Problems, Hyperactivity, Peer Problems) indicate more difficulties; higher Prosocial scores indicate more prosocial behavior. Points show coefficient estimates; horizontal lines show 95% confidence intervals. All coefficients are statistically significant at the 5% level. Sample varies by skill dimension: N=2,015-2,032 students across 242 teachers.

Figure A.2: Skill Preferences by Family Characteristics



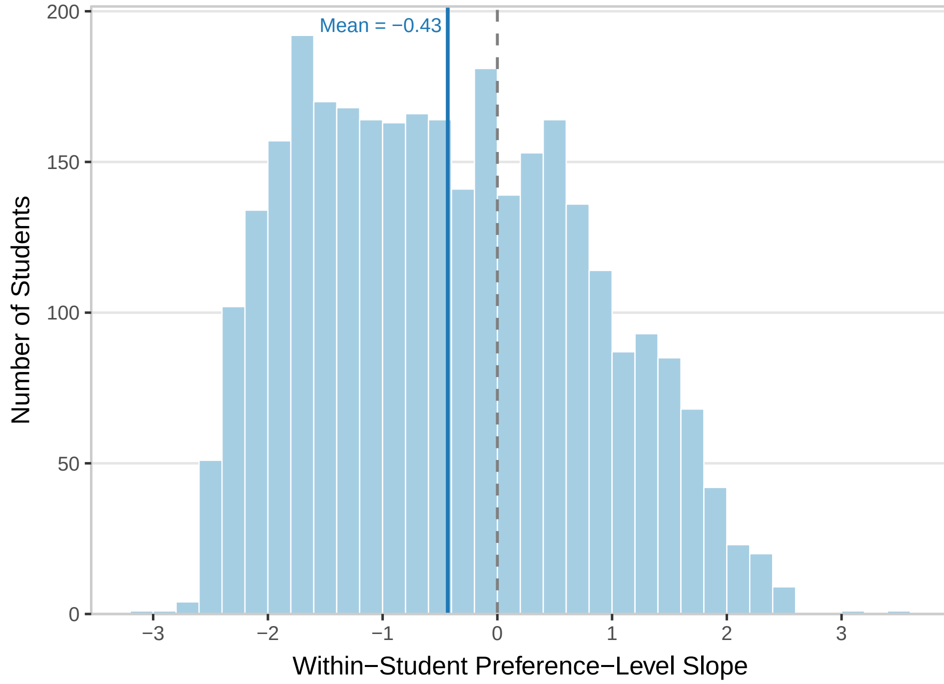
Notes: Coefficients from regressions of skill preference rankings on family characteristics. Skill preference rankings multiplied by -1 so that higher values indicate greater parental preference for the skill. N=3,404 students, 242 teachers. Points show coefficient estimates; bars show 95% confidence intervals.

Figure A.3: Average Skill Preference Rank by Skill Level Quintile (All Dimensions)



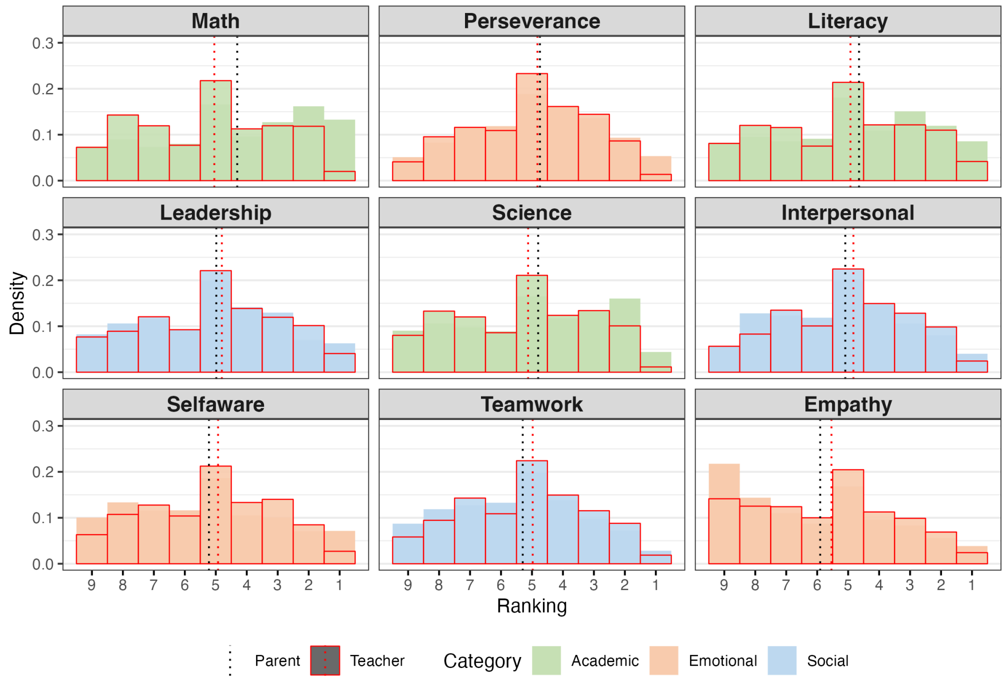
Notes: Higher values on the y-axis indicate higher parental priority (rank 1 = most important). Points show quintile means; bars show 95% confidence intervals.

Figure A.4: Distribution of Within-Student Preference-Level Slopes



Notes: $N = 3{,}404$ students. Histogram shows distribution of within-student preference-level slopes, estimated by regressing each parent's skill preference rankings on standardized skill levels. Higher slopes indicate parents prefer improvement in stronger skills. Vertical dashed line shows mean slope.

Figure A.5: Teacher Versus Parent Skill Preference Rankings by Skill Dimension



Notes: $N = 1{,}273$ students, 242 teachers. Each panel shows the distribution of teachers' skill preference rankings for the specified skill dimension overlaid on the parent distribution. Dotted lines show the mean skill preference rank for each dimension. The x-axis is reversed so that the highest skill preference ranks (most important to improve) are on the right.

B Additional Tables

Table B.1: Treatment Effects on Survey Completion (Attrition Analysis)

Dependent Variable: Has Complete Data	(1)	(2)
Variables		
Treated	-0.0251	-0.0370
	(0.0334)	(0.0464)
Top Category: Social		-0.0712
		(0.0390)
Top Category: Emotional		-0.0236
		(0.0653)
Treated × Social		0.0323
		(0.0699)
Treated × Emotional		0.0388
		(0.0831)
Statistics		
Observations	1,461	1,461
Adjusted R^2	0.189	0.189

Notes: Sample restricted to students observed at either baseline or endline on at least one outcome. The dependent variable equals 1 if a student has non-missing data for all four key outcomes (rating baseline, rating endline, ranking baseline, and ranking endline), and 0 otherwise. "Treated" indicates teachers who received information about parent preferences. All specifications include class fixed effects; standard errors are clustered at the teacher level.
Standard errors in parentheses, clustered at the teacher level
Significance Codes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table B.2: Descriptive Statistics of Parent Survey Sample

Demographic	Group	Full Sample (N=3404)	Main School (N=1460)	Completed Baseline and Endline (N=849)
Child Gender	Male	53.4	52.9	53.0
	Female	46.6	47.1	47.0
Child Age	4-8 years	26.3	25.8	35.3
	9-12 years	43.3	45.1	46.1
	13+ years	30.0	29.1	18.6
Class Level	Primary (1-5)	48.0	47.4	64.1
	Middle (6-8)	30.0	30.6	23.0
	Secondary (9-12)	22.0	22.0	13.0
Family Structure	Only child	40.0	53.1	54.0
	1-2 siblings	58.6	46.1	45.5
	3+ siblings	1.5	0.8	0.5
Responding Parent	Mother	44.6	50.9	51.2
	Father	30.8	37.0	38.2
	Both parents	24.6	12.0	10.6
Household Income	0-5.0 Lakh INR	19.3	9.6	9.1
	5.0-10.0 Lakh INR	17.1	12.1	13.9
	10.0-15.0 Lakh INR	22.0	21.8	23.5
	Over 15.0 Lakh INR	41.6	56.4	53.5
Mother Education	Below Bachelor's	11.0	4.1	4.2
	Bachelor's	28.0	29.3	25.3
	Graduate/PhD	61.0	66.6	70.5
Father Education	Below Bachelor's	11.2	2.3	3.0
	Bachelor's	30.9	29.2	28.0
	Graduate/PhD	57.9	68.5	69.0
Educational Aspiration	Bachelor's or less	5.5	3.9	3.5
	Graduate degree	42.5	36.9	34.4
	PhD	51.9	59.2	62.2
Job Aspiration	Academic/Professional	43.4	52.1	55.5
	Business	26.6	16.5	14.8
	Child's choice	2.0	3.3	3.2
	Other	44.3	42.1	38.5
Education Quality Satisfaction	Completely Satisfied	60.9	66.5	69.0
	Not Completely Satisfied	39.1	33.5	31.0
Child Progress Satisfaction	Completely Satisfied	52.4	54.6	57.0
	Not Completely Satisfied	47.6	45.4	43.0
Teacher Gender	Male	91.8	91.1	94.1
	Female	8.2	8.9	5.9
Teacher Education	Bachelor's	14.8	16.1	18.7
	Graduate/PhD	85.2	83.9	81.3
Years at School	1-5 years	39.6	21.4	18.9
	6+ years	60.4	78.6	81.1

Notes: All numbers are percentages unless noted. Household income in lakhs (100,000 INR); 1 USD ≈ 75 INR.

Table B.3: Within-Dimension Preference-Level Slopes (by Skill)

Dependent Variable: Skill Preference Rank	(1)
Skill Level	
Math	-0.5180***
	(0.0430)
Literacy	-0.3775***
	(0.0474)
Science	-0.1953***
	(0.0482)
Self-awareness	-0.3239***
	(0.0486)
Perseverance	-0.1162**
	(0.0488)
Empathy	-0.3513***
	(0.0522)
Interpersonal	-0.3239***
	(0.0488)
Teamwork	-0.3152***
	(0.0460)
Leadership	-0.3582***
	(0.0455)
Observations	29,747
Adjusted R^2	0.0455

Notes: Entries report slope coefficients β_j (average marginal effects) from $-\text{Skill Preference Rank}_{i,j} = \alpha_j + \beta_j \text{Skill Level}_{i,j} \times \text{Skill dimension}_j + \epsilon_{i,j}$. The outcome is the parent's skill preference rank for improvement (1-9), multiplied by -1 so that larger numbers indicate greater priority. The dependent variable, Skill Level$_{i,j}$ is standardized within-student. Standard errors in parentheses are two-way clustered by student and classroom. Unit of observation: student-dimension.

Standard errors in parentheses, two-way clustered (student, classroom).

Significance codes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table B.4: Balance Table: Parent and Child Characteristics by Treatment Status

Balance Table: Treatment vs Control Groups

Comparison of baseline characteristics across experimental conditions

Variable	Control Group	Treatment Group	Difference	P-value[1]
Skill Ratings				
Rating: Academic Skills	78.29 (12.71)	78.81 (13.13)	0.520	0.495
Rating: Emotional Skills	79.92 (12.62)	79.46 (13.14)	-0.460	0.544
Rating: Social Skills	77.79 (14.62)	77.86 (14.14)	0.073	0.931
Parent Rankings				
Parent Ranking: Academic	4.72 (1.78)	4.80 (1.92)	0.082	0.454
Parent Ranking: Emotional	5.27 (1.63)	5.20 (1.67)	-0.077	0.425
Parent Ranking: Social	5.02 (1.41)	5.00 (1.53)	-0.016	0.852
Aspirations & Contact				
Educational Aspirations	0.02/0.01/0.39/0.58	0.02/0.01/0.34/0.62		0.345
Job Aspirations	0.19/0.26/0.11/0.18/0.03/0.09/0.03/0/0/0/0/0.03/0.08	0.16/0.3/0.09/0.21/0.02/0.07/0.01/0.01/0/0/0/0.03/0.1		0.195
Teacher Contact	0.02/0.05/0.06/0.33/0.54	0.01/0.03/0.05/0.39/0.52		0.102
Demographic Characteristics				
Child Gender	0.46/0.54	0.47/0.53		0.791
Household Income	0.02/0.02/0.04/0.01/0.1/0.1/0.11/0.59	0.05/0.02/0.04/0.03/0.1/0.11/0.12/0.53		0.215
Mother's Education	0.01/0.03/0.28/0.63/0.05	0.02/0.03/0.3/0.62/0.04		0.590
Father's Education	0.01/0.01/0.3/0.61/0.07	0.01/0.02/0.29/0.63/0.05		0.503

[1] Standard deviations in parentheses for continuous variables. Proportions shown for categorical variables. P-values in bold red indicate significance at p < 0.05.

Notes: $N = 823$ parents. Standard deviations in parentheses below means. P-values from t-tests of difference in means between treatment and control, and a Pearson chi-squared test for categorical variables. Household income in lakhs (100,000 INR); 1 USD $\approx$ 75 INR.

Table B.5: Complier Characteristics

Complier Characteristics vs. Full Sample

Estimates with Standard Errors in Parentheses

	Population Groups	
Characteristic	**Compliers**	**Full Sample**
Average Age	45.03 (1.41)	44.07 (0.7)
Proportion Age 20-40	0.23 (0.09)	0.3 (0.04)
Proportion Age 41-50	0.52 (0.11)	0.48 (0.05)
Proportion Over 50	0.26 (0.14)	0.22 (0.07)
Proportion Class 1-3[1]	0.55 (0.11)	0.3 (0.04)
Proportion Class 4-7	0.16 (0.08)	0.41 (0.05)
Proportion Class 8-10	0.29 (0.13)	0.29 (0.07)

[1] Highlighted rows show the largest differences between compliers and full sample, suggesting strongest treatment effect heterogeneity.

Note: Standard errors in parentheses. Complier characteristics estimated using 2SLS following Hull (2025).

Notes: Standard errors in parentheses. Complier characteristics estimated using 2SLS following Hull (2025).

Table B.6: Cost and Benefit Demographic Coefficients

Subgroup	Equation	Median	95% CI	Variable
Grade				
Grade 2	Academic Preference (beta1)	0.030	[-0.200, 0.270]	class_coarse2
Grade 3	Academic Preference (beta1)	0.035	[-0.195, 0.265]	class_coarse3
Grade 4	Academic Preference (beta1)	0.053	[-0.159, 0.260]	class_coarse4
Grade 5	Academic Preference (beta1)	0.179	[-0.028, 0.382]	class_coarse5
Grade 6	Academic Preference (beta1)	0.065	[-0.141, 0.277]	class_coarse6
Grade 7	Academic Preference (beta1)	0.130	[-0.076, 0.340]	class_coarse7
Grade 8	Academic Preference (beta1)	0.126	[-0.097, 0.344]	class_coarse8
Grade 9	Academic Preference (beta1)	0.101	[-0.118, 0.324]	class_coarse9
Grade 10	Academic Preference (beta1)	0.153	[-0.063, 0.360]	class_coarse10
Household Income				
5-10 Lakh INR	Academic Preference (beta1)	0.011	[-0.163, 0.181]	hh_income_coarse
10-15 Lakh INR	Academic Preference (beta1)	0.022	[-0.140, 0.178]	hh_income_coarse
Over 15 Lakh INR	Academic Preference (beta1)	0.002	[-0.140, 0.150]	hh_income_coarse
Mother Education				
Secondary (Class XII)	Academic Preference (beta1)	0.068	[-0.218, 0.350]	mother_education_coarse
Bachelors	Academic Preference (beta1)	0.084	[-0.170, 0.340]	mother_education_coarse
Graduate Degree	Academic Preference (beta1)	0.082	[-0.162, 0.329]	mother_education_coarse
Father Education				
Secondary (Class XII)	Academic Preference (beta1)	-0.087	[-0.375, 0.209]	father_education_coarse
Bachelors	Academic Preference (beta1)	-0.006	[-0.258, 0.249]	father_education_coarse
Graduate Degree	Academic Preference (beta1)	0.015	[-0.231, 0.260]	father_education_coarse
Teacher Ranking				
Academic	Academic Supply (lkA)	-0.005	[-0.680, 0.670]	ranking
Social	Academic Supply (lkA)	-0.006	[-0.561, 0.546]	ranking
Emotional	Academic Supply (lkA)	-0.006	[-0.457, 0.445]	ranking
Grade (Supply)				
Grade 2	Academic Supply (lkA)	-0.048	[-0.209, 0.108]	class_coarse2
Grade 3	Academic Supply (lkA)	-0.029	[-0.183, 0.125]	class_coarse3
Grade 4	Academic Supply (lkA)	-0.049	[-0.189, 0.095]	class_coarse4
Grade 5	Academic Supply (lkA)	-0.128	[-0.263, 0.011]	class_coarse5
Grade 6	Academic Supply (lkA)	-0.060	[-0.203, 0.078]	class_coarse6
Grade 7	Academic Supply (lkA)	-0.103	[-0.244, 0.038]	class_coarse7
Grade 8	Academic Supply (lkA)	-0.091	[-0.239, 0.059]	class_coarse8
Grade 9	Academic Supply (lkA)	-0.084	[-0.233, 0.065]	class_coarse9
Grade 10	Academic Supply (lkA)	-0.105	[-0.245, 0.040]	class_coarse10
Income (Supply)				
5-10 Lakh INR	Academic Supply (lkA)	-0.013	[-0.130, 0.102]	hh_income_coarse
10-15 Lakh INR	Academic Supply (lkA)	-0.018	[-0.124, 0.091]	hh_income_coarse
Over 15 Lakh INR	Academic Supply (lkA)	0.000	[-0.102, 0.096]	hh_income_coarse
Mother Ed. (Supply)				
Secondary (Class XII)	Academic Supply (lkA)	-0.036	[-0.229, 0.157]	mother_education_coarse
Bachelors	Academic Supply (lkA)	-0.048	[-0.218, 0.125]	mother_education_coarse
Graduate Degree	Academic Supply (lkA)	-0.046	[-0.214, 0.119]	mother_education_coarse
Father Ed. (Supply)				
Secondary (Class XII)	Academic Supply (lkA)	0.049	[-0.152, 0.241]	father_education_coarse
Bachelors	Academic Supply (lkA)	-0.003	[-0.177, 0.167]	father_education_coarse
Graduate Degree	Academic Supply (lkA)	-0.017	[-0.182, 0.151]	father_education_coarse
Siblings				
1 sibling	Academic Supply (lkA)	-0.003	[-0.015, 0.010]	siblings_cat
2+ siblings	Academic Supply (lkA)	0.005	[-0.016, 0.027]	siblings_cat

Notes: Each row shows the posterior median and 95% credible interval for coefficients from the hierarchical model. Academic Preference (beta1) refers to family preferences for academic skill development; Academic Supply (lkA) refers to teacher supply of academic instruction.

Table B.7: Variance Ratio by Demographic Subgroups

Subgroup	N	Median Ratio	95% CI	P(Costs > Benefits)
By Grade (omitted: Grade 1)				
Grade 2	132	1.43	[0.92, 2.81]	0.942
Grade 3	145	1.50	[0.92, 3.00]	0.948
Grade 4	221	1.44	[0.91, 2.78]	0.945
Grade 5	230	1.45	[0.94, 2.75]	0.953
Grade 6	207	1.45	[0.92, 2.91]	0.946
Grade 7	216	1.51	[0.92, 3.07]	0.951
Grade 8	163	1.46	[0.90, 2.90]	0.942
Grade 9	184	1.45	[0.93, 2.90]	0.948
Grade 10	209	1.44	[0.93, 2.80]	0.949
By Household Income (omitted: 0–5 Lakh)				
5–10 Lakh	303	1.22	[0.93, 1.73]	0.934
10–15 Lakh	425	1.22	[0.94, 1.74]	0.933
Over 15 Lakh	779	1.28	[0.93, 1.91]	0.942
By Mother's Education (omitted: Less than Secondary)				
Secondary (Class XII)	146	1.19	[0.91, 1.70]	0.895
Bachelor's	500	1.25	[0.94, 1.78]	0.942
Graduate Degree	1131	1.26	[0.94, 1.80]	0.945
By Father's Education (omitted: Less than Secondary)				
Secondary (Class XII)	134	1.18	[0.90, 1.73]	0.876
Bachelor's	571	1.24	[0.94, 1.77]	0.942
Graduate Degree	1074	1.28	[0.95, 1.85]	0.949

Notes: Each row shows the posterior median, 95% credible interval, and probability mass above 1 for $\text{Var}[\log \lambda]/\text{Var}[\log T]$ within the indicated subgroup.

C Survey Instruments and Demographic Questions

The parent survey collected a rich set of additional information that allows for heterogeneity analysis and provides context for understanding preference formation:

C.0.1 Demographic Information

- Family composition: Number of siblings, birth order, primary caregivers
- Parental education: Highest level attained by both mother and father
- Household income: Annual income in categorical brackets
- Parental occupation: Categorized into professional, clerical, sales, service, etc.
- Marital status: Married, unmarried, widowed, separated/divorced

C.0.2 Educational Aspirations and Expectations

- Highest level of education parents would ideally like their child to attain
- Occupational aspirations for when their child is 30 years old

These aspirational measures allow us to examine how longer-term goals align with preferences for immediate skill development priorities, building on recent work by Eble and Escueta (2023) that shows the importance of caregiver aspirations in educational investment and outcomes in resource-constrained settings.

C.0.3 Parent-Teacher Communication

- Frequency of parent contact with teachers (ranging from "rarely" to "daily or more")
- Satisfaction with school and child's progress

The communication frequency measure is particularly important for our analysis, as it allows us to test whether teachers have more accurate beliefs about parent preferences when they communicate more frequently with parents.

C.0.4 Child Well-being and Socioemotional Assessment

The survey incorporated the Strengths and Difficulties Questionnaire (SDQ), a widely validated psychological assessment tool that measures:

- Emotional symptoms
- Conduct problems
- Hyperactivity/inattention

- Peer relationship problems
- Prosocial behavior

The SDQ provides an alternative measure of children's socioemotional development that can be used both as an alternative outcome measure, and also to validate parent skill levels for emotional and social skills. It consists of 25 attributes, both positive and negative, with parents rating each attribute as "Not True," "Somewhat True," or "Certainly True."

C.0.5 Life Satisfaction

Parents were asked to place themselves on a ladder from 0 to 10, where 0 represents the worst possible life and 10 represents the best possible life. This measure allows us to examine whether parent preferences for different skill dimensions vary with overall life satisfaction.

D Model Derivations

D.1 Environment and CET technology

Outcomes and notation. Each child exits the grade with a two-dimensional skill vector $\boldsymbol{c} = (c_1, c_2) \in \mathbb{R}^2_+$, where c_1 denotes academic/cognitive skill and c_2 denotes non–cognitive skill.

Preferences and the MRS. Parent i has Cobb–Douglas utility

$$U(c_1, c_2; \beta_i) \;=\; c_1^{\beta_i}\, c_2^{1-\beta_i}, \qquad 0 < \beta_i < 1.$$

The marginal utilities are

$$MU_1 = \frac{\partial U}{\partial c_1} = \beta_i\, c_1^{\beta_i - 1}\, c_2^{1-\beta_i}, \qquad MU_2 = \frac{\partial U}{\partial c_2} = (1-\beta_i)\, c_1^{\beta_i}\, c_2^{-\beta_i}.$$

The marginal rate of substitution of skill 2 for skill 1 (the amount of c_2 the parent requires to compensate a one–unit loss of c_1 while holding utility constant) is

$$\mathrm{MRS}_{12,i} \;=\; \frac{MU_1}{MU_2} \;=\; \frac{\beta_i}{1-\beta_i}\,\frac{c_2}{c_1}. \tag{D.14}$$

It will be convenient below to define the *taste index*

$$T_i \;:=\; \left(\tfrac{\beta_i}{1-\beta_i}\right)^{1/\rho} \quad\Longleftrightarrow\quad \log T_i = \tfrac{1}{\rho}\log\!\left(\tfrac{\beta_i}{1-\beta_i}\right),$$

where $\rho > 1$ will be the curvature parameter of the technology.

Technology and the CET frontier. Let parents purchase inputs e_1, e_2 at prices p_1, p_2 subject to a static budget $p_1 e_1 + p_2 e_2 \le I$. Each skill is produced with diminishing marginal product from its own input:

$$c_1 = a_1 e_1^{\theta}, \qquad c_2 = a_2 e_2^{\theta}, \qquad a_j > 0, \quad 0 < \theta < 1.$$

Eliminating inputs using $e_j = (c_j/a_j)^{1/\theta}$ and substituting into the budget gives

$$p_1\left(\tfrac{c_1}{a_1}\right)^{1/\theta} \;+\; p_2\left(\tfrac{c_2}{a_2}\right)^{1/\theta} \;\le\; I.$$

Let $\rho := 1/\theta > 1$ and define the *effective productivity-price* constants

$$\kappa_1 := a_1\left(\tfrac{I}{p_1}\right)^{1/\rho}, \qquad \kappa_2 := a_2\left(\tfrac{I}{p_2}\right)^{1/\rho}. \tag{D.15}$$

Dividing both sides by $I^{1/\rho}$ and rearranging yields the constant-elasticity-of-transformation (CET) frontier:

$$\left(\tfrac{c_1}{\kappa_1}\right)^{\rho} \;+\; \left(\tfrac{c_2}{\kappa_2}\right)^{\rho} \;=\; 1, \qquad \rho > 1,\ \kappa_1, \kappa_2 > 0. \tag{D.16}$$

We collect relative technology/price/budget information in the *tilt* (relative supply parameter)

$$\lambda \;:=\; \frac{\kappa_1}{\kappa_2} \;>\; 0, \tag{D.17}$$

so that larger λ means skill 1 is relatively easier to produce (or relatively cheaper per efficiency unit), tilting the PPF toward c_1.

The MRT along the CET. Let $F(c_1, c_2) := (c_1/\kappa_1)^\rho + (c_2/\kappa_2)^\rho - 1 = 0$ define the frontier (D.16). By definition, the MRT is the ratio of marginal costs (the absolute slope of the frontier). Totally differentiating gives

$$F_{c_1}\, dc_1 + F_{c_2}\, dc_2 \;=\; 0 \quad\Longrightarrow\quad \frac{dc_2}{dc_1} \;=\; -\frac{F_{c_1}}{F_{c_2}}.$$

Compute the partial derivatives:

$$F_{c_1} = \rho\, \kappa_1^{-\rho}\, c_1^{\rho-1}, \qquad F_{c_2} = \rho\, \kappa_2^{-\rho}\, c_2^{\rho-1}.$$

Hence the (absolute) marginal rate of transformation between the two skills is

$$\mathrm{MRT}_{12} \;:=\; -\frac{dc_2}{dc_1} \;=\; \left(\tfrac{\kappa_2}{\kappa_1}\right)^{\rho} \left(\tfrac{c_1}{c_2}\right)^{\rho-1} \;=\; \lambda^{-\rho}\, s^{\rho-1}, \qquad s := \frac{c_1}{c_2}. \tag{D.18}$$

Expression (D.18) shows how the frontier's marginal tradeoff depends both on curvature ρ and on the tilt λ: for a given skill ratio s, a higher λ (skill 1 relatively easier) rotates the PPF, lowering the amount of c_2 that must be sacrificed per unit increase in c_1 (a smaller MRT). The associated elasticity of transformation for the CET is $\sigma_T = 1/(\rho - 1) \in (0, \infty)$.

D.2 Optimal skill ratio

Equating the MRS (D.14) and the MRT (D.18) yields the *optimal skill ratio*

$$s^* := \frac{c_1}{c_2} = \left(\tfrac{\beta}{1-\beta}\right)^{1/\rho} \frac{\kappa_1}{\kappa_2} = T\, \lambda, \qquad T := \left(\tfrac{\beta}{1-\beta}\right)^{1/\rho}. \tag{D.19}$$

Evaluated at (c_1^*, c_2^*), the MRS equals

$$\mathrm{MRS}_{12}^* = T^{\rho-1} \lambda^{-1}. \tag{D.20}$$

Multiplying gives the identity

$$s^* \cdot \mathrm{MRS}_{12}^* = T^{\rho}. \tag{D.21}$$

D.3 Within-dimension dispersion across students

Covariance formula. Let (T_i, λ_i) vary across students. Then (dropping the subscript i for convenience),

$$\mathrm{Cov}(s^*, \mathrm{MRS}^*) = \mathbb{E}[T^\rho] - \mathbb{E}[T\lambda]\,\mathbb{E}[T^{\rho-1}\lambda^{-1}]. \tag{D.22}$$

Pure supply heterogeneity. If β is constant ($T \equiv \bar{T}$), then

$$\mathrm{Cov}(s^*, \mathrm{MRS}^*) = \bar{T}^\rho \left(1 - \mathbb{E}[\lambda]\,\mathbb{E}[\lambda^{-1}]\right) < 0, \tag{D.23}$$

as by the Cauchy-Schwarz inequality, we have $\mathbb{E}[\lambda]\,\mathbb{E}[\lambda^{-1}] > \mathbb{E}[\sqrt{\lambda} * \sqrt{\lambda^{-1}}] = 1$.

Pure preference heterogeneity. If λ is constant, write $t := \beta/(1-\beta) > 0$ and note $T = t^{1/\rho}$. Then

$$\mathrm{Cov}(s^*, \mathrm{MRS}^*) = \mathbb{E}[t] - \mathbb{E}[t^{1/\rho}]\mathbb{E}[t^{1-1/\rho}] > 0, \tag{D.24}$$

as by Jensen's inequality, we have that $\mathbb{E}[t^{1/\rho}] < \mathbb{E}[t]^{1/\rho}$ and $\mathbb{E}[t^{1-1/\rho}] < \mathbb{E}[t]^{1-1/\rho}$ (note $0 < 1/\rho < 1$ and $0 < 1 - 1/\rho$), so their product is less than $\mathbb{E}[t]$.

Reduced–form slope and variance comparison. Recall from (D.14)–(D.18) that the observed skill ratio and MRS are

$$s^* = T\,\lambda, \qquad \mathrm{MRS}^* = T^{\rho-1}\lambda^{-1},$$

where $T = \left(\frac{\beta}{1-\beta}\right)^{1/\rho}$ captures preferences and $\lambda = \kappa_1/\kappa_2$ is the relative supply tilt. Across students, the reduced–form slope from regressing MRS^* on s^* is

$$\beta^{\mathrm{RF}} \;=\; \frac{\mathrm{Cov}(s^*, \mathrm{MRS}^*)}{\mathrm{Var}(s^*)}. \tag{D.25}$$

Exact formula. Let $\mu_{ab} := \mathbb{E}[T^a\lambda^b]$ denote mixed moments of the taste and technology parameters. From (D.25) we have that

$$\beta^{\mathrm{RF}} = \frac{\mu_{\rho,0} - \mu_{1,1}\,\mu_{\rho-1,-1}}{\mu_{2,2} - \mu_{1,1}^2}. \tag{D.26}$$

This expression is fully general.

D.3.1 Log-linearized approximate variance decomposition.

Reduced–form slope in levels (first–order derivation). Recall

$$s^* \;=\; T\,\lambda \qquad \text{and} \qquad \mathrm{MRS}^* \;=\; T^{\rho-1}\,\lambda^{-1},$$

where $T := \left(\frac{\beta}{1-\beta}\right)^{1/\rho}$ is the *benefits tilt* and $\lambda := \frac{\kappa_1}{\kappa_2}$ is the *costs tilt*. Let $\bar{T} := \mathbb{E}[T]$, $\bar{\lambda} := \mathbb{E}[\lambda]$, and write small mean–zero fractional deviations

$$\tilde{T} \ := \ \frac{T-\bar{T}}{\bar{T}}, \qquad \tilde{\lambda} \ := \ \frac{\lambda-\bar{\lambda}}{\bar{\lambda}}, \qquad \mathbb{E}[\tilde{T}] = \mathbb{E}[\tilde{\lambda}] = 0.$$

Then $T = \bar{T}(1+\tilde{T})$ and $\lambda = \bar{\lambda}(1+\tilde{\lambda})$. A first–order expansion gives

$$s^* \ = \ T\,\lambda \ \approx \ \bar{T}\,\bar{\lambda}\,(1+\tilde{T}+\tilde{\lambda}),$$

$$\mathrm{MRS}^* \ = \ T^{\rho-1}\lambda^{-1} \ \approx \ \bar{T}^{\rho-1}\bar{\lambda}^{-1}\big(1+(\rho-1)\tilde{T}-\tilde{\lambda}\big).$$

Centering by subtracting means (which differ from the above only by constants), the first-order fluctuations are

$$\widehat{s} \ \approx \ \bar{T}\,\bar{\lambda}\,(\tilde{T}+\tilde{\lambda}), \qquad \widehat{m} \ \approx \ \bar{T}^{\rho-1}\bar{\lambda}^{-1}\big((\rho-1)\tilde{T}-\tilde{\lambda}\big),$$

where $\widehat{s} := s^* - \mathbb{E}[s^*]$ and $\widehat{m} := \mathrm{MRS}^* - \mathbb{E}[\mathrm{MRS}^*]$.

Assuming (for expositional clarity) that $\tilde{T}$ and $\tilde{\lambda}$ are approximately uncorrelated,[10] we obtain

$$\mathrm{Cov}(\widehat{m},\widehat{s}) \ \approx \ \bar{T}^{\rho}\Big((\rho-1)\,\mathrm{Var}(\tilde{T}) \ - \ \mathrm{Var}(\tilde{\lambda})\Big),$$

$$\mathrm{Var}(\widehat{s}) \ \approx \ (\bar{T}\,\bar{\lambda})^2\Big(\mathrm{Var}(\tilde{T}) \ + \ \mathrm{Var}(\tilde{\lambda})\Big).$$

Hence, the reduced–form OLS slope of MRS^* on s^* is

$$\beta^{\mathrm{RF}} \ = \ \frac{\mathrm{Cov}(\widehat{m},\widehat{s})}{\mathrm{Var}(\widehat{s})} \ \approx \ \underbrace{\bar{T}^{\rho-2}\,\bar{\lambda}^{-2}}_{\text{units factor}}\cdot\frac{(\rho-1)\,\mathrm{Var}(\tilde{T}) \ - \ \mathrm{Var}(\tilde{\lambda})}{\mathrm{Var}(\tilde{T}) \ + \ \mathrm{Var}(\tilde{\lambda})}.$$

The prefactor $\bar{T}^{\rho-2}\bar{\lambda}^{-2}$ is a units normalization. It rescales the dependent variable by a positive constant and does not affect the sign comparison. If we report the slope in normalized units (divide MRS^* by $\bar{T}^{\rho-1}\bar{\lambda}^{-1}$ and s^* by $\bar{T}\bar{\lambda}$), this prefactor is 1 and we get the variance–ratio form used in the main text:

$$\boxed{\beta^{\mathrm{RF}} \ \approx \ \frac{(\rho-1)\,\mathrm{Var}(\tilde{T}) \ - \ \mathrm{Var}(\tilde{\lambda})}{\mathrm{Var}(\tilde{T}) \ + \ \mathrm{Var}(\tilde{\lambda})}} \tag{D.27}$$

where $\tilde{T}$ and $\tilde{\lambda}$ are *fractional* (mean–normalized) deviations of the tilts. Relating tilts to primitives, $T = \exp(b/\rho)$ with $b = \log\frac{\beta}{1-\beta}$ and $\tilde{\lambda}$ corresponds to $\tilde{k} = \log\lambda - \mathbb{E}[\log\lambda]$; for small dispersion, $\mathrm{Var}(\tilde{T}) \approx \frac{1}{\rho^2}\mathrm{Var}(\tilde{b})$ and $\mathrm{Var}(\tilde{\lambda}) \approx \mathrm{Var}(\tilde{k})$, matching the appendix formulas.

We reparametrize the optimal skill ratio and MRS as:

$$s^* \ = \ T\,\lambda \ = \ \exp\Big(\tfrac{1}{\rho}b+k\Big), \qquad \mathrm{MRS}^* \ = \ T^{\rho-1}\lambda^{-1} \ = \ \exp\Big(\tfrac{\rho-1}{\rho}b-k\Big),$$

[10]When $\mathrm{Cov}(\tilde{T},\tilde{\lambda}) \neq 0$, the same steps yield a simple adjustment term; see above for the general formula.

where $b = \log \frac{\beta}{1-\beta}$ and $k = \log \lambda$. Define the linear indices

$$x \;:=\; \tfrac{1}{\rho} b + k, \qquad y \;:=\; \tfrac{\rho-1}{\rho} b - k,$$

so that $s^* = e^x$ and $\text{MRS}^* = e^y$.

Write $b = \bar{b} + \tilde{b}$ and $k = \bar{k} + \tilde{k}$, with $\mathbb{E}[\tilde{b}] = \mathbb{E}[\tilde{k}] = 0$, $\sigma_b^2 := \text{Var}(\tilde{b})$, $\sigma_k^2 := \text{Var}(\tilde{k})$, $\sigma_{bk} := \text{Cov}(\tilde{b}, \tilde{k})$. Then

$$x = \bar{x} + A\tilde{b} + B\tilde{k}, \qquad y = \bar{y} + C\tilde{b} + D\tilde{k},$$

with coefficients

$$A = \tfrac{1}{\rho}, \quad B = 1, \qquad C = \tfrac{\rho-1}{\rho}, \quad D = -1,$$

and means $\bar{x} = \frac{\bar{b}}{\rho} + \bar{k}, \quad \bar{y} = \frac{\rho-1}{\rho}\bar{b} - \bar{k}$.

Second–order expansion. Using the second–order Taylor approximation $e^u \approx 1 + u + \frac{1}{2}u^2$ for small, centered u,

$$e^x \;=\; e^{\bar{x}}\, e^{A\tilde{b}+B\tilde{k}} \;\approx\; e^{\bar{x}}\Big[1 + (A\tilde{b} + B\tilde{k}) + \tfrac{1}{2}(A\tilde{b} + B\tilde{k})^2\Big],$$

$$e^y \;=\; e^{\bar{y}}\, e^{C\tilde{b}+D\tilde{k}} \;\approx\; e^{\bar{y}}\Big[1 + (C\tilde{b} + D\tilde{k}) + \tfrac{1}{2}(C\tilde{b} + D\tilde{k})^2\Big].$$

Therefore the means are given as

$$\mathbb{E}[e^x] \approx e^{\bar{x}}\Big[1 + \tfrac{1}{2}\big(A^2\sigma_b^2 + 2AB\,\sigma_{bk} + B^2\sigma_k^2\big)\Big],$$

$$\mathbb{E}[e^y] \approx e^{\bar{y}}\Big[1 + \tfrac{1}{2}\big(C^2\sigma_b^2 + 2CD\,\sigma_{bk} + D^2\sigma_k^2\big)\Big].$$

For the variance, we use the first–order approximation

$$\text{Var}(e^x) \;\approx\; e^{2\bar{x}}\,\text{Var}(A\tilde{b} + B\tilde{k}) \;=\; e^{2\bar{x}}\big(A^2\sigma_b^2 + 2AB\,\sigma_{bk} + B^2\sigma_k^2\big).$$

$$\text{Var}(e^y) \;\approx\; e^{2\bar{y}}\,\text{Var}(C\tilde{b} + D\tilde{k}) \;=\; e^{2\bar{y}}\big(C^2\sigma_b^2 + 2CD\,\sigma_{bk} + D^2\sigma_k^2\big).$$

The covariance is

$$\begin{aligned} \text{Cov}(e^y, e^x) \;&\approx\; e^{\bar{x}+\bar{y}}\,\text{Cov}(A\tilde{b} + B\tilde{k},\; C\tilde{b} + D\tilde{k}) \\ &= e^{\bar{x}+\bar{y}}\Big(AC\,\sigma_b^2 + (AD + BC)\,\sigma_{bk} + BD\,\sigma_k^2\Big). \end{aligned}$$

Assembling the ratio. Recall the reduced–form slope is

$$\beta^{\text{RF}} = \frac{\text{Cov}(e^y, e^x)}{\text{Var}(e^x)}.$$

Thus,

$$\beta^{\mathrm{RF}} \;\approx\; e^{\bar{y}-\bar{x}}\,\frac{AC\,\sigma_b^2 + (AD+BC)\,\sigma_{bk} + BD\,\sigma_k^2}{A^2\,\sigma_b^2 + 2AB\,\sigma_{bk} + B^2\,\sigma_k^2}.$$

Substituting A, B, C, D:

$$AC = \tfrac{\rho-1}{\rho^2}, \quad AD+BC = \frac{\rho-2}{\rho}, \quad BD = -1,$$

$$A^2 = \tfrac{1}{\rho^2}, \quad 2AB = 2/\rho, \quad B^2 = 1.$$

Hence

$$\beta^{\mathrm{RF}} \;\approx\; e^{\bar{y}-\bar{x}}\,\frac{\frac{\rho-1}{\rho^2}\,\sigma_b^2 \;-\; \sigma_k^2 + \frac{\rho-2}{\rho}\,\sigma_{bk}}{\frac{1}{\rho^2}\,\sigma_b^2 \;+\; \sigma_k^2 \;+\; \frac{2}{\rho}\,\sigma_{bk}}. \tag{D.28}$$

Simplifications.

(i) The constant factor $e^{\bar{y}-\bar{x}}$ reflects a units normalization of the dependent variable in the reduced–form regression; rescaling MRS* by a positive constant rescales the slope by the same constant, so this term can be set to 1 without loss of generality.

(ii) If $\tilde{b}$ and $\tilde{k}$ are approximately uncorrelated ($\sigma_{bk} \approx 0$), then

this becomes the compact variance–comparison formula

$$\boxed{\beta^{\mathrm{RF}} \;\approx\; \frac{\frac{\rho-1}{\rho^2}\,\sigma_b^2 - \sigma_k^2}{\frac{1}{\rho^2}\,\sigma_b^2 + \sigma_k^2}} \tag{D.29}$$

which matches the presentation in the main text.

Equation (D.29) shows that the sign and magnitude of the reduced–form slope are governed by the relative dispersion of benefits and costs.

D.4 Within-student dispersion across skills

We observe $J \geq 3$ skills for the same parent. Let the CET frontier be

$$\sum_{j=1}^{J} \left(\tfrac{c_j}{\kappa_j}\right)^{\rho} = 1, \qquad \rho > 1,$$

and preferences $U(\boldsymbol{c};\boldsymbol{\beta}) = \prod_{j=1}^{J} c_j^{\beta_j}$ with $\beta_j > 0$ and $\sum_j \beta_j = 1$. Fix skill 1 as an anchor. For each $j \neq 1$, define the pairwise *skill ratio* and *MRS*:

$$s_j^* := \frac{c_j^*}{c_1^*} = \left(\frac{\beta_j}{\beta_1}\right)^{1/\rho} \frac{\kappa_j}{\kappa_1} =: T_j\,\lambda_j, \qquad \mathrm{MRS}_{j1}^* = T_j^{\rho-1}\lambda_j^{-1},$$

where $T_j := \left(\beta_j/\beta_1\right)^{1/\rho}$ and $\lambda_j := \kappa_j/\kappa_1$ are the *within–parent* benefit and cost tilts, respectively.

Key identity (within-parent). As in the two–skill case, technology cancels when we multiply:

$$s_j^* \cdot \mathrm{MRS}_{1j}^* \;=\; \frac{\beta_j}{\beta_1} \;=\; e^{b_j}, \qquad b_j := \log\Big(\frac{\beta_j}{\beta_1}\Big). \tag{D.30}$$

Exact covariance expression. Let expectations and variances be taken across the $J-1$ non–anchor skills for this parent (denote them by $\mathbb{E}_j[\cdot]$, $\mathrm{Var}_j(\cdot)$, etc.). Using (D.30) and the definitions above,

$$\begin{aligned}\mathrm{Cov}_j(s^*, \mathrm{MRS}^*) &= \mathbb{E}_j\big[s_j^* \mathrm{MRS}_{1j}^*\big] - \mathbb{E}_j[s_j^*]\; \mathbb{E}_j[\mathrm{MRS}_{1j}^*] \\ &= \mathbb{E}_j\big[e^{b_j}\big] - \mathbb{E}_j\big[e^{\frac{1}{\rho} b_j + k_j}\big]\; \mathbb{E}_j\big[e^{\frac{\rho-1}{\rho} b_j - k_j}\big],\end{aligned} \tag{D.31}$$

where $k_j := \log \lambda_j = \log(\kappa_j/\kappa_1)$.

Second–order approximation (small dispersion across skills). Write $b_j = \bar{b} + \tilde{b}_j$ and $k_j = \bar{k} + \tilde{k}_j$ with within–parent means $\bar{b}, \bar{k}$ and centered shocks $\mathbb{E}_j[\tilde{b}_j] = \mathbb{E}_j[\tilde{k}_j] = 0$. Define

$$\sigma_b^2 := \mathrm{Var}_j(\tilde{b}), \quad \sigma_k^2 := \mathrm{Var}_j(\tilde{k}), \quad \sigma_{bk} := \mathrm{Cov}_j(\tilde{b}, \tilde{k}).$$

As in the cross-section derivation, set

$$x_j := \tfrac{1}{\rho} b_j + k_j, \qquad y_j := \tfrac{\rho-1}{\rho} b_j - k_j, \quad \Rightarrow \quad s_j^* = e^{x_j}, \;\; \mathrm{MRS}_{1j}^* = e^{y_j}.$$

With coefficients $A = \frac{1}{\rho}$, $B = 1$, $C = \frac{\rho-1}{\rho}$, $D = -1$, a second–order expansion in $(\tilde{b}, \tilde{k})$ yields (cf. the general formula in the previous subsection)

$$\begin{aligned}\mathrm{Cov}_j(s^*, \mathrm{MRS}^*) &\approx e^{\bar{x}+\bar{y}} \Big(AC\, \sigma_b^2 + (AD + BC)\, \sigma_{bk} + BD\, \sigma_k^2\Big) \\ &= e^{\bar{x}+\bar{y}} \left(\frac{\rho-1}{\rho^2}\, \sigma_b^2 + \frac{\rho-2}{\rho}\, \sigma_{bk} - \sigma_k^2\right),\end{aligned} \tag{D.32}$$

where $\bar{x} = \frac{\bar{b}}{\rho} + \bar{k}$ and $\bar{y} = \frac{\rho-1}{\rho}\bar{b} - \bar{k}$.

Within-parent reduced-form slope. The within-parent OLS slope from regressing MRS_{1j}^* on s_j^* across the $J-1$ pairs is

$$\beta^{\mathrm{RF}}_{\mathrm{within}} \;=\; \frac{\mathrm{Cov}_j(s^*, \mathrm{MRS}^*)}{\mathrm{Var}_j(s^*)}.$$

Using the same expansion,

$$\mathrm{Var}_j(s^*) \approx e^{2\bar{x}} \big(A^2 \sigma_b^2 + 2AB\, \sigma_{bk} + B^2 \sigma_k^2\big) = e^{2\bar{x}} \left(\frac{1}{\rho^2} \sigma_b^2 + \frac{2}{\rho} \sigma_{bk} + \sigma_k^2\right).$$

Thus,

$$\beta^{\mathrm{RF}}_{\mathrm{within}} \;\approx\; e^{\bar{y}-\bar{x}}\, \frac{\frac{\rho-1}{\rho^2}\, \sigma_b^2 \;-\; \sigma_k^2 \;+\; \frac{\rho-2}{\rho}\, \sigma_{bk}}{\frac{1}{\rho^2}\, \sigma_b^2 \;+\; \sigma_k^2 \;+\; \frac{2}{\rho}\, \sigma_{bk}}. \tag{D.33}$$

As before, the level constant $e^{\bar{y}-\bar{x}}$ is a units normalization and can be absorbed by rescaling the dependent variable.

Interpretation. Equation (D.32) (or (D.33)) shows that, *within a parent*, dispersion in tastes across skills (σ_b^2) pushes the covariance/slope positive, while dispersion in technology tilts across skills (σ_k^2) pushes it negative; the cross–covariance σ_{bk} enters with coefficient $(\rho-2)/\rho$ and vanishes in the special case $\rho = 2$. Under approximate orthogonality of tastes and tilts across skills ($\sigma_{bk} \approx 0$), the sign reduces to a simple variance comparison:

$$\operatorname{Cov}_j(s^*, \mathrm{MRS}^*) \gtrless 0 \iff \frac{\rho-1}{\rho^2} \operatorname{Var}_j(\tilde{b}) \gtrless \operatorname{Var}_j(\tilde{k}).$$

Empirically, this justifies running the within-parent regression of ranks on demeaned skill levels across skills (with an anchor), and interpreting the sign/magnitude as a diagnostic of whether the child's skill profile mirrors parental priorities (taste dispersion) or comparative advantage in production (supply dispersion).

D.5 Policy intervention: local analysis

Optimal local policy direction. Let $u(c_1, c_2) = c_1^{\beta} c_2^{1-\beta}$ and feasible (c_1, c_2) be summarized locally by instruments $z = (\ln \kappa_1, \ln \kappa_2)$ with $dc = H\,dz$, $H = \operatorname{diag}(c_1, c_2)$. The policymaker solves

$$\max_{dz} \nabla u^{\top} H\,dz \qquad \text{s.t.} \qquad \tfrac{1}{2} dz^{\top} W dz \le \mathcal{C}.$$

The Lagrangian yields $W dz = \lambda H^{\top} \nabla u$, hence $dz \propto W^{-1} H^{\top} \nabla u$ and $dc = H dz \propto M \nabla u$, $M := H W^{-1} H^{\top} \succ 0$. Under skill-symmetric costs in outcome space ($M \propto I$), $dc \propto \nabla u$, i.e., along the IC normal.

Budget expansion leaves s unchanged. With CET frontier $(c_1/\kappa_1)^{\rho} + (c_2/\kappa_2)^{\rho} \le 1$ and Cobb–Douglas, the optimal levels are $c_1^* = \kappa_1 \beta^{1/\rho}$ and $c_2^* = \kappa_2 (1-\beta)^{1/\rho}$, so

$$s := \frac{c_1}{c_2} = \frac{\kappa_1}{\kappa_2} \Big(\frac{\beta}{1-\beta}\Big)^{1/\rho}.$$

Since $\kappa_i = a_i (I/p_i)^{1/\rho}$, a pure I increase scales both c_i by $I^{1/\rho}$ and leaves s unchanged: $ds/dI = 0$ and $d\,\mathrm{MRS}/dI = 0$.

Total-differential derivation for the IC-normal step. Define $r := \frac{c_2}{c_1}$ and let the policymaker induce an outcome step *along the IC normal* (utility gradient). For Cobb–Douglas,

$$\nabla u \propto \Big(\frac{\beta}{c_1}, \frac{1-\beta}{c_2}\Big), \quad \Delta c_1 = \kappa \frac{\beta}{c_1}, \quad \Delta c_2 = \kappa \frac{1-\beta}{c_2}$$

for a small step size $\kappa > 0$. Using $r(c_1, c_2) = c_2/c_1$,

$$\frac{\partial r}{\partial c_1} = -\frac{c_2}{c_1^2} = -\frac{r}{c_1}, \qquad \frac{\partial r}{\partial c_2} = \frac{1}{c_1}.$$

Hence the first-order change is

$$dr \;\approx\; \frac{\partial r}{\partial c_1}\Delta c_1 + \frac{\partial r}{\partial c_2}\Delta c_2 = -\frac{r}{c_1}\Big(\kappa\frac{\beta}{c_1}\Big) + \frac{1}{c_1}\Big(\kappa\frac{1-\beta}{c_2}\Big) = \frac{\kappa}{c_1^2}\Big(\frac{1-\beta}{r} - \beta r\Big).$$

Thus the sign of dr matches $\frac{1-\beta}{r} - \beta r$, which is positive if $r < \sqrt{\frac{1-\beta}{\beta}}$, negative if $r > \sqrt{\frac{1-\beta}{\beta}}$, and zero at $r^\dagger = \sqrt{\frac{1-\beta}{\beta}}$.

Dynamics and convergence to the threshold. Treat repeated tiny IC-normal steps as a continuous-time limit. Write $r = c_2/c_1$ and $s = 1/r$. From A.3,

$$\frac{dr}{d\tau} = K\Big(\frac{1-\beta}{r} - \beta r\Big) \quad \text{with} \quad K > 0.$$

Then $s = r^{-1}$ satisfies

$$\frac{ds}{d\tau} = -\frac{1}{r^2}\frac{dr}{d\tau} = K\Big(\beta\, s - (1-\beta)\, s^3\Big) \;\propto\; \frac{\beta}{s} - (1-\beta)s.$$

The unique positive fixed point is $s^\dagger = \sqrt{\frac{\beta}{1-\beta}}$. Linearization shows global (monotone for small steps) convergence toward $s^\dagger$.

Equilibrium κ-ratio implementing the fixed point. At the optimum under CET, $s = \lambda T$ with $\lambda := \kappa_1/\kappa_2$ and $T := (\beta/(1-\beta))^{1/\rho}$. To implement $s^\dagger$ as the new optimum, choose

$$\boxed{\lambda^\star = \frac{s^\dagger}{T} = \Big(\tfrac{\beta}{1-\beta}\Big)^{\frac{1}{2}-\frac{1}{\rho}}}.$$

Connection to endogenous technology choice/frontiers. In the framework of Caselli and Coleman (2006), firms pick (A_s, A_u) on a frontier characterized by a "height" B and curvature; the first-order conditions link the chosen bias A_s/A_u to factor ratios and relative wages, generating appropriate technology choices across endowments and a barrier parameter shifting the whole frontier.[11] Our quadratic budget on dz is a local reduced form of these frontier trade-offs for policy. The distinction is that we treat κ_i as policy levers, responding to student-specific values captured by β_i. In the the objective is fixed: maximize profit. The choice is for optimal technology in response to varying prices of labor inputs.

[11] See Caselli and Coleman (2006) for the frontier concept and the way the FOCs pin down the choice of production frontier.

D.6 Teacher-driven supply and key identity

A teacher enters after parents form preferences β_i over skills. The teacher chooses effort, pedagogy, and materials to determine λ_i; these may be shifted by observable shocks (e.g., randomized information, measured teacher priorities). Parents observe learning under this supply and report endline skill levels and skill preferences; thus reports lie on the PPF determined by λ_i and B_i.

From (D.19)–(D.20):

$$s_i^* = T_i\lambda_i, \qquad \mathrm{MRS}_{12,i}^* = T_i^{\rho-1}\lambda_i^{-1},$$

so

$$s_i^* \cdot \mathrm{MRS}_{12,i}^* = T_i^{\rho}. \tag{D.34}$$

Hence supply dispersion $\mathrm{Var}(\log\lambda)$ attenuates the covariance of (s^*, MRS^*) relative to the Period 1 benchmark if treatment reduces misalignment between λ_i and T_i (e.g., reallocating classroom effort toward parent-valued skills) rotates the reduced-form slope upwards.

D.7 Empirical implementation and identification

Proxies. Let $r_{ij} \in [0, 100]$ and $m_{ij} \in \{1, \ldots, 9\}$ denote parent i's rating and importance rank for skill j. Set $\tilde{r}_{ij} := r_{ij} - \bar{r}_i$. Assume a monotone reporting function so that $\tilde{r}_{ij}$ is a monotone proxy for s_{ij}^*. Parents rank by marginal utility, so m_{ij} is a monotone proxy for MRS_{ij}^*.

Baseline regression. Estimate

$$m_{ij} = \alpha + \beta\,\tilde{r}_{ij} + \xi_j + \varepsilon_{ij}, \tag{D.35}$$

with heteroskedasticity-robust SEs and parent clustering when stacking skills; include dimension fixed effects ξ_j and, if desired, dimension-specific slopes.

Supply-side instruments. Treat teacher/classroom variables as *supply instruments* for $\tilde{r}_{ij}$:

- randomized information assignment at grade/teacher level;
- teacher "typical student" priorities (constructed indices);
- class-level aggregates of parent priorities (leave-one-out).

These shift λ_i but not β_i, identifying the supply-driven component of the slope. Over identified designs allow over-ID tests; report partial-F (or Bayesian analogs) and cluster at the classroom (or school×grade) level.

Policy mapping. Posterior (or sampling) decompositions of $\mathrm{Var}(b)$ and $\mathrm{Var}(k)$ quantify whether skills are primarily shaped by benefits or costs. If the latter dominates, cost-side levers (pedagogy, time allocation, materials, class size, resources) should be prioritized; if the former dominates, alignment and benefit-side interventions are the natural lever.

E From the 2-Skill Framework to a 3-Category Estimable Model

This section shows, step by step, how the two–skill model (3.1) generalizes to three skill *groups* (academic, social, emotional), how the optimal skill ratios are derived, and how every term that appears in the Stan likelihood is obtained directly from the economic primitives.

E.1 Environment

Skills (now 3 categories). Parents end the decision period with category–level skills $\boldsymbol{c} = (c_A, c_S, c_E) \in \mathbb{R}^3_+$.

Preference parameters. Two taste weights govern trade-offs:

$$U(c_A, c_S, c_E) = c_A^{\beta_1} \left(c_S^{\beta_2} c_E^{1-\beta_2}\right)^{1-\beta_1}, \qquad \beta_1, \beta_2 \in (0,1). \tag{E.36}$$

- β_1: academic vs. *non*–academic $\left(c_S^{\beta_2} c_E^{1-\beta_2}\right)$;
- β_2: social vs. emotional *within* the non-academic composite.

E.2 Marginal-rate-of-substitution (MRS) between Academic and Non-academic skills

Utility with nested Cobb–Douglas preferences.

$$U(c_A, c_S, c_E) \;=\; c_A^{\beta_1} \left(c_S^{\beta_2}\, c_E^{1-\beta_2}\right)^{1-\beta_1}, \qquad \beta_1, \beta_2 \in (0,1). \tag{D.1}$$

Collapse Social and Emotional into a single non-academic index. Fix the within–non-academic mix $s_{SE} = c_S/c_E$ (chosen optimally in the *inner* problem). Because s_{SE} is constant during the Academic vs Non-academic trade-off, the term in brackets is proportional to $c_N := c_S + c_E$; the proportionality factor does not affect marginal rates. Hence we can write the *outer* utility as

$$\boxed{U(c_A, c_N) = c_A^{\beta_1}\, c_N^{1-\beta_1}} \tag{E.37}$$

Marginal utilities.

$$\frac{\partial U}{\partial c_A} = \beta_1\, c_A^{\beta_1 - 1}\, c_N^{1-\beta_1} = \beta_1 \frac{U}{c_A}, \qquad \frac{\partial U}{\partial c_N} = (1-\beta_1)\, c_A^{\beta_1}\, c_N^{-\beta_1} = (1-\beta_1)\frac{U}{c_N}.$$

MRS between Academic and Non-academic.

$$\boxed{\mathrm{MRS}_{A,N} = \frac{\beta_1}{1-\beta_1}\frac{c_N}{c_A} = \frac{\beta_1}{1-\beta_1}\frac{1}{s_A}, \qquad s_A := \frac{c_A}{c_N}} \tag{E.38}$$

Equation (E.38) is the expression equated to the MRT in Section E.3 to solve for the optimal Academic share s_A^*.

Technology. Each category continues to have its *own* one–input Cobb–Douglas line: $c_j = a_j e_j^\theta$ $(0 < \theta < 1)$. Combining them with the linear budget $p_1e_1 + p_2e_2 + p_3e_3 \le I$ again yields a *constant–elasticity–of–transformation* (CET) frontier

$$\big(c_A/\kappa_A\big)^\rho + \big(c_S/\kappa_S\big)^\rho + \big(c_E/\kappa_E\big)^\rho = 1, \qquad \rho = \tfrac{1}{\theta} > 1.$$

Only the two *ratios* $\ell_1 = \log(\kappa_A/\kappa_S)$ and $\ell_2 = \log(\kappa_S/\kappa_E)$ matter for the frontier's shape (its absolute scale is absorbed by the budget).

E.3 MRT between Academic and Non-Academic

We start from the three-category constant-elasticity-of-transformation (CET) frontier with $\rho > 1$:

$$\left(\tfrac{c_A}{\kappa_A}\right)^\rho + \left(\tfrac{c_S}{\kappa_S}\right)^\rho + \left(\tfrac{c_E}{\kappa_E}\right)^\rho = 1 \tag{E.39}$$

Collapse c_S and c_E into a single non-academic quantity while holding their *mix* constant. Fix a within-non-academic ratio

$$s_{SE} := \frac{c_S}{c_E} \quad \text{(taken as given during marginal changes).}$$

Write

$$c_S = \frac{s_{SE}}{1+s_{SE}}\,c_N, \qquad c_E = \frac{1}{1+s_{SE}}\,c_N, \qquad \text{where } c_N := c_S + c_E.$$

Insert these into (E.39); all terms containing c_N share the factor c_N^ρ. Collect them:

$$\left(\tfrac{c_A}{\kappa_A}\right)^\rho + \left(\tfrac{c_N}{\kappa_N}\right)^\rho = 1 \tag{E.40}$$

with the **effective non-academic conversion factor**

$$\boxed{\kappa_N(s_{SE}) = (1+s_{SE})\left(\frac{s_{SE}^\rho}{\kappa_S^\rho} + \frac{1}{\kappa_E^\rho}\right)^{-1/\rho}} \tag{E.41}$$

Thus *for every fixed* s_{SE} the three–point frontier is algebraically equivalent to the two-point frontier (c_A, c_N) with parameters (κ_A, κ_N).

Implicit differentiation: MRT$_{A,N}$. Let

$$F(c_A, c_N) \;=\; \left(\tfrac{c_A}{\kappa_A}\right)^\rho + \left(\tfrac{c_N}{\kappa_N}\right)^\rho - 1 = 0.$$

Total-differentiating while *holding* s_{SE} (and therefore κ_N) constant:

$$\frac{\partial F}{\partial c_A}\,dc_A + \frac{\partial F}{\partial c_N}\,dc_N = 0,$$

$$\frac{\partial F}{\partial c_A} = \rho\Big(\tfrac{c_A}{\kappa_A}\Big)^{\rho-1}\frac{1}{\kappa_A}, \qquad \frac{\partial F}{\partial c_N} = \rho\Big(\tfrac{c_N}{\kappa_N}\Big)^{\rho-1}\frac{1}{\kappa_N}.$$

Hence the absolute slope is

$$\boxed{\big|\mathrm{MRT}_{A,N}\big| = \frac{dc_N}{dc_A} = \Big(\tfrac{\kappa_N(s_{SE})}{\kappa_A}\Big)^{\rho}\Big(\tfrac{c_A}{c_N}\Big)^{\rho-1}} \tag{E.42}$$

E.3.1 Optimal Academic-vs-Non-academic mix.

Recall that our preferences across categories give

$$\mathrm{MRS}_{A,N} = \frac{\beta_1}{1-\beta_1}\,\frac{c_N}{c_A}.$$

Set $\mathrm{MRS}_{A,N} = \big|\mathrm{MRT}_{A,N}\big|$ and define $s_A := c_A/c_N$:

$$\frac{\beta_1}{1-\beta_1}\,\frac{1}{s_A} = \Big(\tfrac{\kappa_N(s_{SE})}{\kappa_A}\Big)^{\rho} s_A^{\rho-1}.$$

Multiply by s_A, take logs, divide by ρ, and exponentiate:

$$\boxed{s_A^* = \exp\Big[\tfrac{1}{\rho}\,\mathrm{logit}\beta_1 + \log\Big(\tfrac{\kappa_A}{\kappa_N(s_{SE})}\Big)\Big]} \tag{E.43}$$

Because $\kappa_N(s_{SE})$ in (E.41) depends on κ_S, κ_E *and* the within-non-academic mix s_{SE}, the optimal Academic share s_A^* incorporates *all three* technology parameters unless we impose a normalization such as $\kappa_S = \kappa_E$.

E.3.2 Estimation and identification

Why the baseline estimation sets $\kappa_S = \kappa_E$.

- Without panel data or exogenous price shocks the single cross-section of levels & preference ranks identifies β_1, β_2 precisely but cannot pin down $\ell_2 = \log(\kappa_S/\kappa_E)$ tightly—its effect overlaps almost one-for-one with β_2 inside s_{SE}.
- Normalising $\kappa_S = \kappa_E$ ($\ell_2 = 0$) therefore removes a weakly identified parameter and focuses statistical power on $\ell_1 = \log(\kappa_A/\kappa_S)$, which is central for detecting supply-side heterogeneity.
- If future data supply additional variation you can restore ℓ_2 by estimating it directly via (E.41) and (E.43); only a few lines of Stan code change.

Social vs. Emotional skills. Within the non-academic pair set $s_{SE} := c_S/c_E$ and equate MRS to MRT:

$$\frac{\beta_2}{1-\beta_2}\frac{c_E}{c_S} = \left(\tfrac{\kappa_E}{\kappa_S}\right)^{\rho}\left(\tfrac{c_S}{c_E}\right)^{\rho-1}.$$

Substituting $s_{SE} = c_S/c_E$ yields,

$$\frac{\beta_2}{1-\beta_2}\left(\tfrac{\kappa_S}{\kappa_E}\right)^{\rho} = s_{SE}^{\rho}.$$

If we impose the *normalisation* $\kappa_S = \kappa_E$ (i.e. no technological asymmetry *within* the non-academic block), then $\ell_2 = 0$ and (**??**) simplifies to the expression used in the baseline Stan specification:

$$\widetilde{s}_{SE} = \exp\Big[\tfrac{1}{\rho}\,\text{logit}\beta_2\Big].$$

Why one tilt (ℓ_1) is enough for identification

(a) Given three observed ratings, any common rescaling of $(\kappa_A, \kappa_S, \kappa_E)$ is absorbed by the unobserved budget B_i; only *two* independent cost ratios remain.

(b) Rank data depend on those ratios *only through the two mixes s_A and s_{SE}*. With a single cross-section, ℓ_2 is nearly collinear with β_2 inside (**??**). Estimating both leads to weak identification unless additional variation (panel data, price changes, supply shifters) is available.

(c) Therefore the empirical benchmark sets $\ell_2 = 0$, keeps ℓ_1 free, and focuses on the contrast between *benefit heterogeneity* $\sigma_{\beta_1}, \sigma_{\beta_2}$ and *cost heterogeneity* σ_{ℓ_1}. Section **??** shows that allowing $\ell_2 \neq 0$ yields similar qualitative results but much wider posteriors.

A scale-free representation (g_A, g_S, g_E) Because only (s_A, s_{SE}) matter for choices, we pick a single convenient point on the CET frontier:

$$\underline{g}_A := 1, \qquad \underline{g}_E := \frac{1}{s_A(1+s_{SE})}, \qquad \underline{g}_S := s_{SE}\,\underline{g}_E \tag{E.44}$$

Any feasible skill vector differs from $\boldsymbol{g} := (\underline{g}_A, \underline{g}_S, \underline{g}_E)$ only by a *common* positive multiplier. We let the parent-specific *budget scale* $B_i > 0$ supply that multiplier:

$$c_{ik} = B_i\, g_k.$$

E.3.3 Latent utilities for the ordered-probit

We show a numerically stable way to obtain the three category-specific latent utilities that feed the ordered-probit likelihood, using only log and logit transformations that Stan handles safely.

Raw marginal utilities. With nested Cobb–Douglas preferences

$$U(c_A, c_S, c_E) = c_A^{\beta_1} \left(c_S^{\beta_2} \, c_E^{1-\beta_2}\right)^{1-\beta_1}, \quad \beta_1, \beta_2 \in (0,1),$$

the marginal utilities factor as

$$MU_A = \beta_1 \frac{U}{c_A}, \quad MU_S = (1-\beta_1)\beta_2 \frac{U}{c_S}, \quad MU_E = (1-\beta_1)(1-\beta_2)\frac{U}{c_E}.$$

Normalise by the common factor $(1-\beta_1)$**.** Multiplying or dividing every MU_j by the same positive constant leaves rankings unchanged, so set

$$\widetilde{MU}_j := \frac{MU_j}{U(1-\beta_1)} \quad (j = A, S, E).$$

Then

$$\widetilde{MU}_A = \frac{\beta_1}{1-\beta_1} \frac{1}{c_A}, \quad \widetilde{MU}_S = \beta_2 \frac{1}{c_S}, \quad \widetilde{MU}_E = (1-\beta_2) \frac{1}{c_E}.$$

Take logs and cancel parent-specific constants. Write $c_j = B_i \, g_j$ where B_i is parent i's budget and g_j the *effective* skill output. After subtracting $\log B_i$, which is common to all three categories, we obtain

$$\boxed{\begin{aligned} u_A &= \log\left(\tfrac{\beta_1}{1-\beta_1}\right) - \log g_A, \\ u_S &= \log \beta_2 - \log g_S, \\ u_E &= \log(1-\beta_2) - \log g_E \end{aligned}} \tag{E.45}$$

E.3.4 Implementation of the model in Stan

Let

$$\theta_1 = \texttt{logit\_beta1}, \qquad \theta_2 = \texttt{logit\_beta2}.$$

- $\log\left(\frac{\beta_1}{1-\beta_1}\right) = \theta_1$.
- $\log \beta_2 = \theta_2 + \texttt{log1m\_inv\_logit}(\theta_2)$.
- $\log(1-\beta_2) = \texttt{log1m\_inv\_logit}(\theta_2)$.

Using these identities avoids under- or overflow when β is very close to 0 or 1, while the $-\log g_j$ terms come straight from the rating equation $c_j = B_i \, g_j$.

These u_A, u_S, u_E are proportional to the (negative) ordering utilities used in the likelihood; their differences determine the probabilities of each category being ranked first, second, or third.

E.3.5 Mapping to observed data

Skill Levels. Parents report a cardinal score on each category, modelled as

$$r_{ik} \sim \mathcal{N}(B_i\, g_k,\, \sigma^2_{\text{rating}}) \tag{E.46}$$

so ratings identify *both* B_i and the supply tilts ℓ_1, ℓ_2.

Skill Preferences (ordered probit). Assume i.i.d. $\mathcal{N}(0,1)$ noise on three latent utilities and a pair of cut-points $c_1 < c_2$. Observed ranks $\text{rank}_{ik} \in \{1,2,3\}$ (1 = most important) for person i and category k follow the ordered probit:

$$\begin{aligned}
\Pr(\text{rank}_{ik} = 1) &= 1 - \Phi(c_2 - u_{ik}),\\
\Pr(\text{rank}_{ik} = 2) &= \Phi(c_2 - u_{ik}) - \Phi(c_1 - u_{ik}),\\
\Pr(\text{rank}_{ik} = 3) &= \Phi(c_1 - u_{ik})
\end{aligned} \tag{E.47}$$

with u_{ik} equal to the corresponding expression in (E.45). To ensure identifiability, we parameterize the cutpoints as c_1 and $c_2 = c_1 + \Delta$ with $\Delta > 0$, preventing label-switching during MCMC sampling.

I chose to model skill preference ranks using an independent three-level ordered probit given that ties were allowed, in practice. If skill preference rankings were strictly ordered, a rank-ordered likelihood (Plackett-Luce, rank-probit, etc.) could be used instead.

E.3.6 Incorporating demographics

The model allows each individual-level parameter to vary with observed covariates $\boldsymbol{X}_i$. For parent i, the logit preferences, supply tilts, and log budget are:

$$\text{logit}(\beta_{1i}) = \gamma_{\beta 1,0} + \boldsymbol{X}_{\beta 1,i}^{\top}\boldsymbol{\gamma}_{\beta 1} + \sigma_{\beta 1} z_{\beta 1,i}, \tag{E.48}$$

$$\text{logit}(\beta_{2i}) = \gamma_{\beta 2,0} + \boldsymbol{X}_{\beta 2,i}^{\top}\boldsymbol{\gamma}_{\beta 2} + \sigma_{\beta 2} z_{\beta 2,i}, \tag{E.49}$$

$$\ell_{1i} = \gamma_{\ell 1,0} + \boldsymbol{X}_{\ell 1,i}^{\top}\boldsymbol{\gamma}_{\ell 1} + \sigma_{\ell 1} z_{\ell 1,i}, \tag{E.50}$$

$$\log B_i = \gamma_{B,0} + \boldsymbol{X}_{B,i}^{\top}\boldsymbol{\gamma}_{B} + \sigma_{B} z_{B,i}. \tag{E.51}$$

where $z_{\beta 1,i}, z_{\beta 2,i}, z_{\ell 1,i}, z_{B,i} \sim \mathcal{N}(0,1)$ are independent standard normal draws. The γ_0 terms are intercepts, $\boldsymbol{\gamma}$ are covariate effect vectors, and σ parameters govern unexplained (residual) heterogeneity conditional on covariates.

Covariate selection. In the main specification, we include:

- **Preferences** $(\boldsymbol{X}_{\beta 1}, \boldsymbol{X}_{\beta 2})$: grade, household income, mother's education, father's education. These capture how observable family characteristics shift the relative value placed on academic versus social-emotional skills.

- **Supply** ($\boldsymbol{X}_{\ell 1}$): All preference covariates *plus* teacher skill rankings for academic, social, and emotional categories. Teacher rankings serve as an exclusion restriction—they enter the production technology but not preferences, providing exogenous variation in costs to aid identification.
- **Budget** ($\boldsymbol{X}_B$): household income and number of siblings, proxying for total resources available for skill investment.

The model is flexible: setting any $K = 0$ reduces to the baseline specification without covariates for that equation. All covariate matrices are constructed using dummy encoding for categorical variables, with reference categories omitted (e.g., Grade 1, lowest income bracket, less than secondary education).

E.3.7 Priors and normalization

Priors on intercepts and covariate effects. We use weakly informative priors centered at zero:

$$\begin{aligned}
\gamma_{\beta 1,0}, \gamma_{\beta 2,0}, \gamma_{\ell 1,0} &\sim \mathcal{N}(0, 0.5^2), \\
\gamma_{B,0} &\sim \mathcal{N}(\log 75, 0.3^2), \\
\boldsymbol{\gamma}_{\beta 1}, \boldsymbol{\gamma}_{\beta 2}, \boldsymbol{\gamma}_{\ell 1} &\sim \mathcal{N}(\mathbf{0}, 0.5^2 \boldsymbol{I}), \\
\boldsymbol{\gamma}_B &\sim \mathcal{N}(\mathbf{0}, 0.3^2 \boldsymbol{I}).
\end{aligned}$$

The budget intercept prior centers on $\log 75$, reflecting that parent ratings average around 75 on the 0–100 scale. Covariate effect priors are symmetric around zero, allowing demographics to shift parameters in either direction.

Priors on heterogeneity parameters. Standard deviations governing unexplained variation receive exponential priors:

$$\sigma_{\beta 1}, \sigma_{\beta 2}, \sigma_{\ell 1} \sim \text{Exponential}(2), \qquad \sigma_B \sim \text{Exponential}(3), \qquad \sigma_{\text{rating}} \sim \mathcal{N}(10, 5^2) \text{ truncated at } 0.$$

These priors are weakly informative, allowing substantial heterogeneity while penalizing extreme values. The rating noise prior centers on 10 points (on a 0–100 scale), consistent with measurement error in subjective assessments.

Cutpoint priors. The ordered probit cutpoints use:

$$c_1 \sim \mathcal{N}(0, 2^2), \qquad \Delta := c_2 - c_1 \sim \text{Exponential}(1).$$

This parameterization ensures $c_1 < c_2$ while allowing the data to determine threshold locations. The exponential prior on Δ weakly favors moderate separation between rank categories.

Normalizations and identification. Several normalizations ensure model identification:

1. **Curvature:** We fix $\rho = 1.5$, corresponding to an elasticity of transformation $1/(\rho - 1) = 2$. This is within the range typical for CET production functions and could not be separately identified from the heterogeneity parameters without strong functional form assumptions or additional moments.
2. **Supply intercept:** We set $\ell_2 = 0$ (social and emotional skills have equal baseline production costs, $\kappa_S = \kappa_E$), normalizing the supply tilt to Academic versus non-Academic only. This identification restriction is without loss of generality given our three-category setup.
3. **Production scale:** We normalize $g_A = 1$ when computing skill mixes, pinning down one point on the production frontier. Combined with the budget B_i, this determines the absolute scale of skill production.

These normalizations reduce the parameter space without loss of economic content, as the model primitives of interest—variance in benefits versus costs—remain identified through the joint pattern of ratings and rankings across categories.

E.3.8 The hierarchical parameters estimated in Stan

Block	Symbol in code	Economic meaning
Preference intercepts	$\gamma_{\beta 1,0}, \gamma_{\beta 2,0}$	Baseline academic vs non-academic, social vs emotional tilts
Preference effects	$\boldsymbol{\gamma}_{\beta 1}, \boldsymbol{\gamma}_{\beta 2}$	How demographics shift preference parameters
Supply intercept	$\gamma_{\ell 1,0}$	Baseline log-ratio $\log(\kappa_A/\kappa_{SE})$
Supply effects	$\boldsymbol{\gamma}_{\ell 1}$	How demographics and teachers shift production costs
Budget intercept	$\gamma_{B,0}$	Baseline effective resources
Budget effects	$\boldsymbol{\gamma}_B$	How income/siblings shift resources
Residual std. devs.	$\sigma_{\beta 1}, \sigma_{\beta 2}, \sigma_{\ell 1}, \sigma_B$	Unexplained heterogeneity (conditional on covariates)
Curvature	$\rho = 1.5$	Elasticity of transformation (fixed)
Rating noise	σ_{rating}	Perception / measurement error
Rank cut-points	$c_1, \Delta = c_2 - c_1$	Thresholds in ordered probit

The key inferential targets are the *total variances* in preferences and costs:

$$\text{Var}[\text{logit}(\beta_{1i})] = \text{Var}(\boldsymbol{X}_{\beta 1,i}^{\top}\boldsymbol{\gamma}_{\beta 1}) + \sigma_{\beta 1}^2, \qquad \text{Var}[\ell_{1i}] = \text{Var}(\boldsymbol{X}_{\ell 1,i}^{\top}\boldsymbol{\gamma}_{\ell 1}) + \sigma_{\ell 1}^2.$$

Comparing these variances (or equivalently, the variances of $\log T_i$ and $\log \lambda_i$ after rescaling by $1/\rho$) reveals whether observed specialization is primarily driven by benefit or cost heterogeneity. The sign of the empirical preference-level slope corresponds structurally to whether cost or benefit variance dominates in this decomposition.

E.3.9 Estimation details

The model is estimated using Stan's Hamiltonian Monte Carlo with the No-U-Turn Sampler (NUTS). We run 4 chains with 2,000 warmup iterations and 3,000 post-warmup draws per chain, yielding 12,000 total posterior samples. We set `adapt_delta = 0.95` and `max_treedepth = 12` to improve sampling efficiency in the presence of complex posterior geometry induced by the ratings-rankings likelihood and demographic covariates. Convergence is assessed via $\hat{R} < 1.01$ and effective sample sizes (ESS) > 400 for all parameters of interest. The full Stan code is reproduced below.

```
// ------------ 3-category CET model -------

data {
  int<lower=1> N;
  array[N, 3] real<lower=0, upper=100> rating;
  array[N, 3] int<lower=1, upper=3> rank;

  // Covariate dimensions (set to 0 for no covariates)
  int<lower=0> K_beta1;  // Number of covariates for beta1
  int<lower=0> K_beta2;  // Number of covariates for beta2
  int<lower=0> K_lkA;    // Number of covariates for log(kappa_A)
  int<lower=0> K_lkS;    // Number of covariates for log(kappa_S)
  int<lower=0> K_B;      // Number of covariates for budget

  // Covariate matrices (can be empty if K=0)
  matrix[N, K_beta1] X_beta1;
  matrix[N, K_beta2] X_beta2;
  matrix[N, K_lkA] X_lkA;
  matrix[N, K_lkS] X_lkS;
  matrix[N, K_B] X_B;

  // Control flag for identification
  int<lower=0, upper=1> estimate_lkappa_SE;  // 0 = assume kappa_S = kappa_E, 1 =
      estimate separately
}

parameters {
  // Preference parameters (demand)
  real gamma_beta1_0;
  vector[K_beta1] gamma_beta1;
  real<lower=0> sigma_logit_beta1;
  vector[N] z_beta1;

  real gamma_beta2_0;
  vector[K_beta2] gamma_beta2;
  real<lower=0> sigma_logit_beta2;
  vector[N] z_beta2;

  // Supply parameters - log(kappa) for each category
```

```
  // Academic - always estimated
  real gamma_lkA_0;
  vector[K_lkA] gamma_lkA;
  real<lower=0> sigma_lkA;
  vector[N] z_lkA;

  // Social - parameters exist but only used if estimate_lkappa_SE = 1
  real gamma_lkS_0;
  vector[K_lkS] gamma_lkS;
  real<lower=0> sigma_lkS;
  vector[N] z_lkS;

  // Emotional: NOT ESTIMATED - normalized to 1 (log = 0)
  // No parameters needed

  // Budget parameters
  real gamma_B_0;
  vector[K_B] gamma_B;
  real<lower=0> sigma_logB;
  vector[N] z_logB;

  // Rating noise
  real<lower=0> sigma_rating;

  // Ordered-probit cut-points
  real cut1;
  real<lower=0> cut_diff;
}

transformed parameters {
  real rho = 1.5;
  real inv_rho = 1.0 / rho;

  // Individual-level preference parameters
  vector[N] logit_beta1;
  vector[N] logit_beta2;

  // Individual-level log(kappa) parameters
  vector[N] lkappa_A;
  vector[N] lkappa_S;
  vector[N] lkappa_E;  // This will be set to 0 for normalization

  // Budget
  vector[N] logB;
  vector[N] B;

  // === Compute preference parameters ===
  if (K_beta1 > 0) {
```

```
  logit_beta1 = gamma_beta1_0 + X_beta1 * gamma_beta1 + sigma_logit_beta1 *
      z_beta1;
} else {
  logit_beta1 = gamma_beta1_0 + sigma_logit_beta1 * z_beta1;
}

if (K_beta2 > 0) {
  logit_beta2 = gamma_beta2_0 + X_beta2 * gamma_beta2 + sigma_logit_beta2 *
      z_beta2;
} else {
  logit_beta2 = gamma_beta2_0 + sigma_logit_beta2 * z_beta2;
}

// === Compute supply parameters (log kappa ratios) ===
if (K_lkA > 0) {
  lkappa_A = gamma_lkA_0 + X_lkA * gamma_lkA + sigma_lkA * z_lkA;
} else {
  lkappa_A = gamma_lkA_0 + sigma_lkA * z_lkA;
}

// NORMALIZATION: Always set kappa_E = 1 (lkappa_E = 0)
lkappa_E = rep_vector(0.0, N);

// For baseline: set kappa_S = kappa_E = 1
if (estimate_lkappa_SE == 1) {
  // Estimate kappa_S separately (but kappa_E is still normalized to 1)
  if (K_lkS > 0) {
    lkappa_S = gamma_lkS_0 + X_lkS * gamma_lkS + sigma_lkS * z_lkS;
  } else {
    lkappa_S = gamma_lkS_0 + sigma_lkS * z_lkS;
  }
} else {
  // Baseline: kappa_S = kappa_E = 1
  lkappa_S = rep_vector(0.0, N);
}

// === Compute budget ===
if (K_B > 0) {
  logB = gamma_B_0 + X_B * gamma_B + sigma_logB * z_logB;
} else {
  logB = gamma_B_0 + sigma_logB * z_logB;
}
B = exp(logB);

// === Compute skill mixes (hybrid: vectorize what's easy, loop the rest) ===

// Step 1: Compute log(s_SE) - VECTORIZED
vector[N] log_s_SE = inv_rho * logit_beta2;
```

```
  // Steps 2-4: Compute log(kappa_N), log(s_A), and log(g_*) in single loop
  vector[N] log_kappa_N;
  vector[N] log_s_A;
  vector[N] log_g_A = rep_vector(0.0, N);
  vector[N] log_g_S;
  vector[N] log_g_E;

  for (i in 1:N) {
    // Step 2: Compute log(kappa_N)
    real term1 = rho * (log_s_SE[i] - lkappa_S[i]);
    real term2 = -rho * lkappa_E[i];
    log_kappa_N[i] = log1p_exp(log_s_SE[i]) - inv_rho * log_sum_exp(term1, term2);

    // Step 3: Compute log(s_A)
    log_s_A[i] = inv_rho * logit_beta1[i] + lkappa_A[i] - log_kappa_N[i];

    // Step 4: Compute log(g_E) and log(g_S)
    log_g_E[i] = -log_s_A[i] - log1p_exp(log_s_SE[i]);
    log_g_S[i] = log_s_SE[i] + log_g_E[i];
  }

  // Step 5: Exponentiate - VECTORIZED
  vector[N] g_A = exp(log_g_A);
  vector[N] g_S = exp(log_g_S);
  vector[N] g_E = exp(log_g_E);

  // Cutpoints
  vector[2] cut;
  cut[1] = cut1;
  cut[2] = cut1 + cut_diff;
}

model {
  // === PRIORS ===

  // Priors on preference intercepts
  gamma_beta1_0 ~ normal(0, 0.5);
  gamma_beta2_0 ~ normal(0, 0.5);

  // Priors on supply intercepts
  // gamma_lkA_0: log(kappa_A / kappa_E) - always estimated
  gamma_lkA_0 ~ normal(0, 0.5);

  // gamma_lkS_0: log(kappa_S / kappa_E) - only used if estimate_lkappa_SE = 1
  // But we need to give it a prior regardless since it's a parameter
  if (estimate_lkappa_SE == 1) {
    gamma_lkS_0 ~ normal(0, 0.5);
  } else {
    // Weak prior when not used
```

```
  gamma_lkS_0 ~ normal(0, 10);
}

// Prior on budget intercept - this absorbs the scale
gamma_B_0 ~ normal(log(75), 0.5);

// Priors on covariate effects
if (K_beta1 > 0) gamma_beta1 ~ normal(0, 0.5);
if (K_beta2 > 0) gamma_beta2 ~ normal(0, 0.5);
if (K_lkA > 0) gamma_lkA ~ normal(0, 0.5);
if (estimate_lkappa_SE == 1) {
  if (K_lkS > 0) gamma_lkS ~ normal(0, 0.5);
} else {
  if (K_lkS > 0) gamma_lkS ~ normal(0, 10);
}
if (K_B > 0) gamma_B ~ normal(0, 0.3);

// Priors on heterogeneity
sigma_logit_beta1 ~ exponential(2);
sigma_logit_beta2 ~ exponential(2);
sigma_lkA ~ exponential(2);
if (estimate_lkappa_SE == 1) {
  sigma_lkS ~ exponential(2);
} else {
  sigma_lkS ~ exponential(0.1);  // Tight prior when not used
}
sigma_logB ~ exponential(3);

sigma_rating ~ normal(10, 5);
cut1 ~ normal(0, 2);
cut_diff ~ exponential(1);

// Standard normal priors on z's
z_beta1 ~ std_normal();
z_beta2 ~ std_normal();
z_lkA ~ std_normal();

// z_lkS gets prior even in baseline (but won't affect results since lkappa_S =
   0)
if (estimate_lkappa_SE == 1) {
  z_lkS ~ std_normal();
} else {
  z_lkS ~ normal(0, 0.01);  // Very tight when not used
}

z_logB ~ std_normal();

// === LIKELIHOOD ===
for (i in 1:N) {
```

```
    // Ratings: r_ik ~ N(B_i * g_k, sigma_rating^2)
    rating[i,1] ~ normal(B[i] * g_A[i], sigma_rating);
    rating[i,2] ~ normal(B[i] * g_S[i], sigma_rating);
    rating[i,3] ~ normal(B[i] * g_E[i], sigma_rating);

    // Rankings: ordered probit on latent utilities
    vector[3] util;
    util[1] = logit_beta1[i] - log_g_A[i];
    util[2] = logit_beta2[i] + log1m_inv_logit(logit_beta2[i]) - log_g_S[i];
    util[3] = log1m_inv_logit(logit_beta2[i]) - log_g_E[i];

    for (k in 1:3) {
      real eta = util[k];
      vector[3] prob;
      prob[1] = 1 - Phi(cut[2] - eta);
      prob[2] = Phi(cut[2] - eta) - Phi(cut[1] - eta);
      prob[3] = Phi(cut[1] - eta);
      rank[i,k] ~ categorical(prob);
    }
  }
}

generated quantities {
  // Compute implied population means
  real mu_logit_beta1_implied;
  real mu_logit_beta2_implied;
  real mu_lkA_implied;
  real mu_lkS_implied;
  real mu_logB_implied;

  if (K_beta1 > 0) {
    mu_logit_beta1_implied = gamma_beta1_0 + mean(X_beta1 * gamma_beta1);
  } else {
    mu_logit_beta1_implied = gamma_beta1_0;
  }

  if (K_beta2 > 0) {
    mu_logit_beta2_implied = gamma_beta2_0 + mean(X_beta2 * gamma_beta2);
  } else {
    mu_logit_beta2_implied = gamma_beta2_0;
  }

  if (K_lkA > 0) {
    mu_lkA_implied = gamma_lkA_0 + mean(X_lkA * gamma_lkA);
  } else {
    mu_lkA_implied = gamma_lkA_0;
  }

  if (estimate_lkappa_SE == 1) {
```

```
    if (K_lkS > 0) {
      mu_lkS_implied = gamma_lkS_0 + mean(X_lkS * gamma_lkS);
    } else {
      mu_lkS_implied = gamma_lkS_0;
    }
  } else {
    mu_lkS_implied = 0.0;  // In baseline, kappa_S = kappa_E = 1
  }

  if (K_B > 0) {
    mu_logB_implied = gamma_B_0 + mean(X_B * gamma_B);
  } else {
    mu_logB_implied = gamma_B_0;
  }
}
```