

Module - 1

1. a. Define the following terms with examples:

- (i) Alphabet (ii) Power of an alphabet
 - (iii) Concatenation (iv) Languages
- (04 Marks)

Ans. i. Alphabet : A language consists of various symbol from which the words, statements etc, can be obtained. These symbols are called Alphabets. The symbol Σ denotes the set of alphabets of a language.

Ex : $\Sigma = \{a, b, \dots, z, A, B, \dots, Z, 0, \dots, 9, \#, C, ,\} \dots$ etc}

ii Power of an alphabet : If Σ is an alphabet, we can express the set of all strings of a certain length from that alphabet by using the exponential notation.

Ex: $\Sigma = \{0, 1\}$ the σ

$$\Sigma^1 = \{0, 1\}, \Sigma^2 = \{00, 01, 10, 11\}$$

iii. Concatenation : The concatenation of two strings u and v is the string obtained by writing the letters of string u followed by the letters of string v .

$$u = a_1 a_2 a_3 \dots a_n \quad v = b_1 b_2 b_3 \dots b_m$$

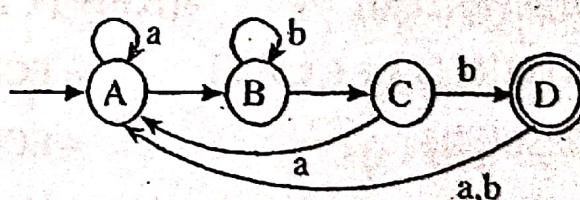
$$uv = a_1 a_2 a_3 \dots a_n b_1 b_2 b_3 \dots b_m$$

iv. Language : A language can be defined as a set of strings obtained from Σ^* where Σ is set of alphabets of a particular language.

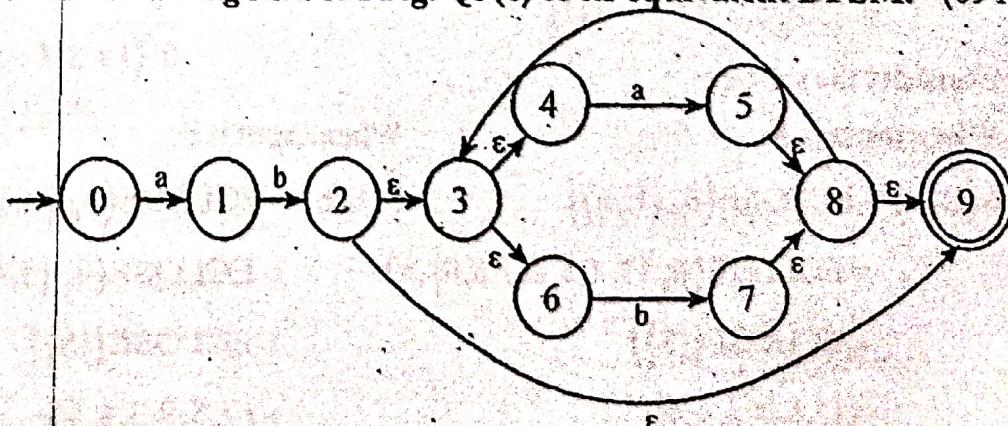
Ex : $\{\epsilon, 01, 10, 0011, 1010, 0101, 0011, \dots\}$

b. Draw a DFA to accept strings of a's and b's ending with 'bab'. (03 Marks)

Ans.



c. Convert the following NDFSM Fig. Q1 (c) to its equivalent DFMSM. (09 Marks)



When $k=2$

$$\begin{aligned}
 R_{11}^{(2)} &= R_{11}^{(0)} + R_{12}^{(0)} [R_{22}^{(0)}]^* R_{21}^{(0)} \\
 &= 1^* + 1^* 0 (\varepsilon + 11^* 0)^* 11^* \\
 &= 1^* + 1^* 0 (11^* 0)^* 11^* \\
 R_{12}^{(2)} &= R_{12}^{(0)} + R_{12}^{(0)} [R_{22}^{(0)}]^* R_{22}^{(0)} \\
 &= 1^* 0 + 1^* 0 (\varepsilon + 11^* 0)^* (\varepsilon + 11^* 0) \\
 &= 1^* 0 + 1^* 0 (11^* 0)^* (\varepsilon + 11^* 0) \\
 R_{13}^{(2)} &= R_{13}^{(0)} + R_{12}^{(0)} [R_{22}^{(0)}]^* R_{23}^{(0)} \\
 &= \phi + 1^* 0 (\varepsilon + 11^* 0)^* 0 \\
 &= (0 + \varepsilon) + 1^* 0 (11^* 0)^* 0 \\
 R_{21}^{(2)} &= R_{21}^{(0)} + R_{22}^{(0)} [R_{22}^{(0)}]^* R_{21}^{(0)} \\
 &= 11^* + (\varepsilon + 11^* 0) (\varepsilon + 11^* 0)^* 11^* \\
 &= 11^* + (\varepsilon + 11^* 0) (11^* 0) 11^* \\
 R_{22}^{(2)} &= R_{22}^{(0)} + R_{22}^{(0)} [R_{22}^{(0)}]^* R_{22}^{(0)} \\
 &= (\varepsilon + 11^* 0) + (\varepsilon + 11^* 0) (\varepsilon + 11^* 0)^* (\varepsilon + 11^* 0) \\
 &= (\varepsilon + 11^* 0) + (\varepsilon + 11^* 0) (11^* 0) (\varepsilon + 11^* 0)
 \end{aligned}$$

Final RE can be calculated as

$$\begin{aligned}
 R_{13}^{(3)} &= R_{13}^{(2)} + R_{13}^{(2)} [R_{33}^{(2)}]^* R_{33}^{(2)} \\
 &= 1^* 0 (11^* 0)^* 0 + 1^* 0 (11^* 0)^* 0 [(0 + \varepsilon) + 1^* 0 (11^* 0)^* 0] * (0 + \varepsilon) + 1^* 0 (11^* 0)^* 0
 \end{aligned}$$

b. Give Regular expressions for the following languages on $\Sigma = \{a,b,c\}$

- (i) all strings containing exactly one a
- (ii) all strings containing no more than 3 a's.
- (iii) all strings that contain at least one occurrence of each symbol in Σ .

(03 Marks)

Ans.

- (i) $R \in (b+c)^* a (b+c)^*$
- (ii) $R \in (b+c)^* (\sigma+a) (b+c)^* (\varepsilon+a) (b+c)^*$
- (iii) $(a+b+c)^*$

3. c. Let L be the language accepted by the following finite state machine. (04 Marks)

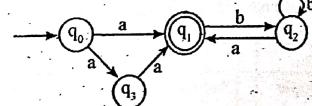


Fig. Q3 (c)

Indicate for each of the following regular expressions, whether it correctly describes L:

- (i) $(a \cup ba) bb^* a$
- (ii) $(\varepsilon \cup b) a (bb^* a)^*$
- (iii) $ba \cup ab^* a$
- (iv) $(a \cup ba) (bb^* a)^*$

Ans. i. NO

ii. YES

iii. NO

iv. YES

OR

4. a. Prove that the following language is not regular :

$$L = \{0^n 1^n \mid n > 0\}$$

(05 Marks)

Ans. Step 1 : Let L i.e, regular and n be the number of states

$$x = 0^n 1^n$$

Step 2 : Since $|x| = 2n > n$ we can split x into uvw such that $|uv| \leq n$ and $|v| \geq 1$ as

$$x = \underbrace{a a a a a}_{u} \underbrace{a}_{v} \underbrace{b b b b b b}_{w}$$

Step 3 : According to pumping lemma $uv^i w \in L$ for $i = 0, 1, 2, \dots$ When $i = 0$ 'v' doesn't exist so $L = \{0^n 1^n \mid n > 0\}$ is not regularb. If L_1 and L_2 are regular languages then prove that $L_1 \cup L_2$, $L_1 \cdot L_2$ and L_1^* are regular languages.

(05 Marks)

Ans. Refer Q.no.3(b) of MQP - 2.

c. Is the following grammar ambiguous? (06 Marks)

$$S \rightarrow i C \quad t \{ j \} \{ k \} s \{ l \} a$$

$$C \rightarrow b$$

(06 Marks)

Ans. Refer Q.no.5(b) of MQP - 2.

Module-4

7. a. If L_1 and L_2 are context free languages then prove that $L, UL_2, L_1 L_2$ and L_1^* are context free languages. (04 Marks)

Ans.

$$\begin{aligned}
 (i) G_1 &= (V_1, T_1, P_1, S_1) \\
 G_2 &= (V_2, T_2, P_2, S_2) \\
 G_3 &= (V_1 \cup V_2, US_3, T_1 \cup T_2, P_3, S_3) \\
 S_3 \text{ is a start state } G_3 \text{ and } S_3 \in (V_1 \cup V_2) \\
 P_3 &= P_1 \cup P_2 \cup \{S_3 \rightarrow S_1 / S_2\} \\
 L_3 &= L_1 \cup L_2 \\
 (ii) G_4 &= (V_1 \cup V_2, US_4, T_1 \cup T_2, P_4, S_4) \\
 S_4 \text{ is a start symbol for the grammar } G_4 \text{ and } S_4 \in (V_1 \cup V_2) \\
 P_4 &= P_1 \cup P_2 \cup \{S_4 \rightarrow S_1 S_2\} \\
 L_4 &= L_1 L_2 \\
 (iii) G_5 &= (V, US_5, T_1, P_5, S_5) \\
 S_5 \text{ is a start symbol of Grammar } G_5 \\
 P_5 &= P_1 \cup \{S_5 \rightarrow S_1 S_2 | \epsilon\} \\
 TL_5 &= L_5
 \end{aligned}$$

- b. Give a decision procedure to answer each of the following questions:

- Given a regular expression a and a PDA M , the language accepted by M a subset of the language generated by a ?
- Given a context-free Grammar G and two strings S_1 and S_2 , does G generate $S_1 S_2$?
- Given a context free Grammar G , does G generate any even length strings.
- Given a Regular Grammar G , is $L(G)$ context-free? (12 Marks)

Ans. i. Observe that this is true if $L(M) \cap L(\alpha) = \emptyset$. So the following procedure answers the question :

- From α , build a PDA M^* so that $L(M^*) = L(\alpha)$
- From M and M^* , build a PDA M^{**} that accepts $L(M) \cap L(M^*)$
- If $L(M^{**})$ is empty, return true else return false.
- i. Convert G to chomsky normal form. Try all derivations in G of length up to $2|S_1 S_2|$. If any of them generates $S_1 S_2$, return True, else return false
- iii. 1. Use CFG to PDA topdown (G) to build a PDA P that accepts $L(G)$.
2. Build an FSM E that accepts all even length strings over the alphabet Σ_G .
3. Use insert PDA and $FSM(P, E)$ to build a PDA P^* that accepts $L(G) \cap L(E)$.
4. Return $decideCFEmpty(P^*)$
- iv. i. Return True (Since every regular language is context free)

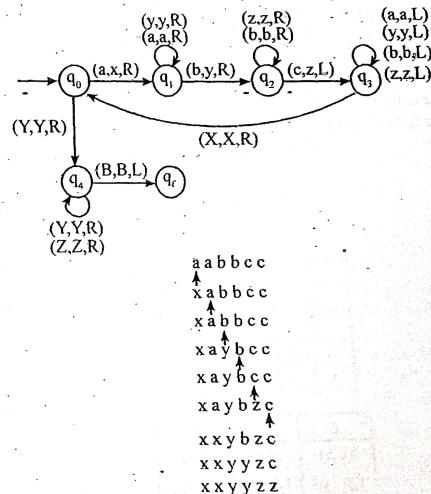
OR

8. a. Explain with neat diagram, the working of a Turing Machine model. (05 Marks)

Ans. Refer Q.no.9(a) of MQP - 2.

- b. Design a Turing machine to accept the language $L = \{a^n b^n c^n \mid n \geq 1\}$. Draw the transition diagram. Show the moves made by this turing machine for the string aabbcc. (11 Marks)

Ans.

**Module-5**

9. Write short notes on:

- Multi-tape turning machine.
- Non-deterministic turning machine.
- Linear Bounded automata.

- Ans. a. Refer Q.no. 9(b) of MQP - 1.

(16 Marks)

- b. Non - deterministic turning machine : In a non - deterministic turning machine, for every state and symbol, there are a group of actions the TM can have. So here the transitions are not deterministic. The computation of a non - deterministic turning machine is a tree of configurations that can be reached from the start configuration. An input is accepted if there is at least one node of the tree which is an accept configuration, otherwise it is not accepted. If all branches of the computational tree

action on all inputs, the non - deterministic turning machine is called a decider and if for some input, all branches are rejected, the input is also rejected.

c. **Linear bounded automata** : Refer Q.no. 10(b) of MQP - 1.

OR

10. Write short notes on:
- Undecidable languages.
 - Halting problem of turning machine.
 - The post correspondence problem.

(16 Marks)

- Ans. a. Refer Q.no.9(b) of MQP - 2
b. Refer Q.no.10(a) of MQP - 1
c. Refer Q.no.10b(i) of MQP - 2.