Module-1

ATC
- Theory of Computation
  - Automata
  - Formal language
- Computability
- Complexity

Automata
- → Finite
- → Push down
- → Linear bounded
- → Tuner Machine

Formal languages → Some Mathematical model or formula.

1. Regular
2. Context free
3. Context-Sensitive
4. Recursively Enumerable.

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Central concepts of Automata Theory

Σ – it is alphabet (Stands for Alphabet)
It is defined as a set of finite Symbols.
Ex :- 1) $\Sigma = \{0, 1\} \rightarrow$ binary alphabet.
2) $\Sigma = \{0, 1, 2, 3, \ldots 9\} \rightarrow$
alphabet set for decimal no.s.
3) $\Sigma = \{A, B, \ldots\ldots Z, a, b \ldots\ldots z\}$
alphabet set of Eng alphabet

S – it stands for String
It is defined as finite sequence of Symbols derived from alphabet (Σ).
Ex: 1) Consider binary alphabet $\Sigma = \{0, 1\}$
0, 1, 01, 000, 010111, ...
These are the strings derived from Σ. (framed).
2) $\Sigma = \{a, b\}$.
a, b, ab, aaa, bbb, bab . . . .

Epsilon (ε) – It is a Special string
It is called as null string / empty string
the length of string is alway 0.
$|\varepsilon| = 0$.

Σ^*

$*$ → zero or more
→ Kleen star operator.

Note:-

There is another operator $+$.

$+$ → 1 or zero
→ Illegal operator.

Σ^* → power of alphabet.

$$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \ldots\ldots$$

Ex:-

1) Consider binary alphabet $\Sigma = \{0, 1\}$ find Σ^*.

Σ^0 - ~~St~~ set of strings of length 0.
Σ^1 - " " " " " 1

$\Sigma^* = \{\varepsilon, 0, 1, 00, 01, 10, 11, 000, 001, 010, 011, \ldots\ldots 111, \ldots\ldots\}$.

It is defined as set of strings of all possible length derived from Σ.

L → stands for Language.

It is defined as the set of strings derived from Σ^* (sigma star) $\Rightarrow L \subseteq \Sigma^*$

Ex:- 1) Consider $\Sigma = \{a, b\}$.

$\Sigma^* = \{\varepsilon, a, b, aa, ab, ba, bb, aaa, \ldots\ldots bbb, aaaa, \ldots\ldots bbbb\}$.

L_1 = Set of strings whose length is odd.

$L_1 = \{a, b, aaa, abb, \ldots bbb, aaaaa \ldots bbbbb\}$

$L_2 \to$ set of strings which begin with a and end with b.

$L_2 = \{ab, aab, aaaab, abbbb, aabaab, \ldots\}$.

$L_3 \to$ set of strings which are ending with aba.

$L_3 \to \{aba, aaaba, baba, \ldots\}$.

<u>Symbols to draw an Automata.</u>

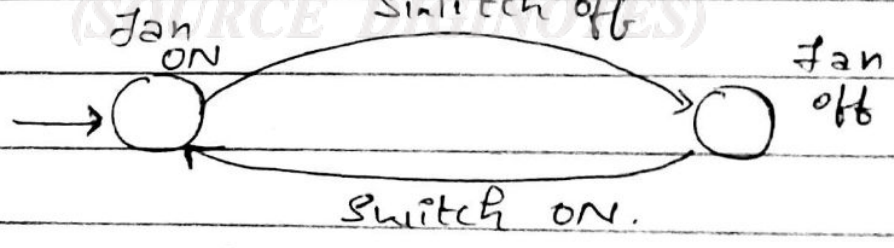
$\to\bigcirc$ $\to$ initial state.

$\longrightarrow$ $\to$ transition.

$\bigcirc$ $\to$ Intermediate state.

$\circledcirc$ $\to$ End / final state.

Ex:-



Transition diagram.

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Strings

Functions
- length
- concatenation.
- Replication.
- Reversal.

Relations
- Substrings.
- proper Substrings.
- prefix
- proper prefix.
- Suffix
- proper Suffix.

length of the strings :- No. of Symbols in the given strings.

Concatenation of string :- concatenation of two strings ($s \parallel t$) is defined as. string appendic t to s.

Replication - Replication of string w. is defined as $w^0 = \varepsilon$

$$w^{i+1} = w^i . w.$$

(Repeating the occurence of w 'i' no. of times).

Reversal of string w - For each string w the Reverse of the string is defined as follows.

i) if $|w| = 0$, then $w = w^R = \varepsilon$.

ii) if $|w| \geq 1$, then

$$\exists a \in \Sigma (\exists u \in \Sigma^* (w = ua))$$

$$\boxed{w^R = a u^R.}$$

Relations on Strings.

Substrings — A string s is a substring of 't', iff 's' continiously occurs in strings.

Proper Substrings — A string s is a proper substring of 't'. iff s is a substring of t and $s \neq t$.

Ex :- "ε abba ε".

$\underbrace{\varepsilon,\ a,\ ab,\ abb}_{\text{proper substring}},\ abba$

Prefix — A string s is a prefix of 't' iff there exist, $\exists x \in \Sigma^* (t = sx)$

Proper prefix — A string s is a proper prefix of P iff string is prefix of t and $s \neq t$.

Ex :- Consider the string "abba".

$\varepsilon,\ \underbrace{a,\ ab,\ abb}_{\text{proper prefix}},\ abba.$

Suffix - A string S is a Suffix of 't' iff, there exits, $\exists x \in \Sigma^* (t = xS)$

proper Suffix - A string S is a proper suffix of ~~p~~ S iff string is Suffix of t and ~~t≠S~~. $S \neq t$.

Ex abba.

$\underbrace{\varepsilon, a, ab, abb}_{\text{proper suffix}}, abba$

Ways to define

→ using set defining functions (characteristic function).

L_1 : All a's preceed all b's where $\Sigma = \{a, b\}$

$L_1 = \{ab, aaab, aabbb, aaabbb, aa, bbb, a, b\}$.

$L_2 = \{w : w \in \{a, b\}^*, w = xa\}$.

language contains set of strings w. and w belongs to $\{a, b\}^*$ and each string w is ending with a.

$L_2 = \{\varepsilon, a, aa, ba, aba, bbba \ldots\ldots\}$.

$L_3 = \{x\#y, x, y \in \{0, 1, 2 \ldots 9\}^*$ $Square(x) = y\}$

$L_3 = \{\varepsilon, 1\#1, 2\#4, 3\#9, 4\#16 \ldots\ldots\}$.

It is included because of $*$.

→ **using Replication.**

$L_1 = \{a^n : n \geq 0\}$.

$L_1 = \{\epsilon, a, aa, aaa \ldots\}$.

$L_2 = \{0^n 1^n : n > 0\}$.

Ex:- if $n = 2$.

$L_2 = 0011$.

$L_2 = \{01, 0011, 000111, \ldots\}$.

→ **using Prefix Relation.**

$L_1 = \{w \in \{a,b\}^* ;$ no prefix of w contains $b\}$.

$L_1 = \{\epsilon, a, aa, aaa \ldots\}$.

$L_2 = \{w \in \{a,b\}^* ;$ no prefix of w ~~starts~~ starts with $b\}$

L_2 can be written as.

$L_2 = \{w \in \{a,b\}^* ;$ Every w begins with $a\}$.

Lexicographic Order

It is defined as the order in which the elements of objects are generated and it is given by the notation $<_L$.

1) Shorter strings proceed longer strings.

$\exists x (\exists y (|x| < |y|)) \rightarrow x <_L y$

Consider 2 strings x and y.

$\forall x (\forall y (|x| < |y|)) \rightarrow x <_L y$.

$\exists \rightarrow$ there exist
$\forall \rightarrow$ for all

2) For strings that are same length sort the strings in dictionary order (D)

$L = \{ w \in \{a, b\}^* : |w| \equiv_3 0 \}$ // characteristic function.

$L = \{ \varepsilon, aaa, aba, baa, \dots bbb, \dots \}$.

$L = \{ \varepsilon, aaa, aab, aba, abb, baa, bab, bba, bbb, \dots \dots \}$.

Concatenation of two languages.

Let L_1 and L_2 be two languages
Let us define $L_1 . L_2$
$L_1 . L_2 = \{ w \in \Sigma^* \; \exists s \in L_1 (\exists t \in L_2 (w = s.t)) \}$
Ex:-
$L_1 = \{$cat, dog, Mouse, bird$\}$.
$L_2 = \{$ bone, food $\}$.

$L_1 . L_2 = \{$ cat. bone, cat food, dogbone,
dogfood, Mousebone, Mousefood,
birdbone, birdfood$\}$.

in order,
$L_1 . L_2 = \{$ ~~birdbone, bir~~
catbone, catfood, dogbone, dogfood,
bird bone, birdfood, Mousebone,
Mousefood$\}$.

Note:- first consider the length of string,
then among the same lengths consider
the alphabetical order.

1) Consider the language $L = \{ 1^n 2^n ; n > 0 \}$.
Is the string 122 in L

12

1122

using Replication
but 122 does not belong to the language

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2) $L_1 = \{a^n b^n, n>0\}$ $L_2 = \{c^n : n>0\}$.
find $L_1 . L_2$.

$L_1 L_2 = \{a^n b^n c^n, n>0, m>0\}$.

3) Let $L_1 = \{$ Peach, apple, cherry $\}$.
$L_2 = \{$ Pie, Cobbler, $\varepsilon \}$.
List the element of $L_1 L_2$ in lexicographic order.

$L_1 L_2 = \{$ PeachPie, peachcobbler, peach,
applePie, applecobbler, apple
cherryPie, cherrycobbler, cherry $\}$.

In lexicographic order.
$L_1 L_2 =$ ~~{ PeachPie,~~
{ apple, peach, cherry, applePie,
applecobbler, peachpie, peachcobbler,
cherryPie, cherry cobber }

Note:-
→ Empty string ε is the identity for concatenation operation.

→ Concatenation as a function is associative.
So, $\forall s \; \forall t \; \forall u \; (st)u = s(tu)$.

→ Sigma Star Σ^* (Cardinality of language) can possess this following property,

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$$\Sigma = \phi$$
$$\Sigma^* = \{\varepsilon\}, \quad |\Sigma^*| = 1.$$

→ $L . \{\varepsilon\} = \{\varepsilon\} . L = L$

→ $L . \phi = \phi . L = \phi$.

→ $(L_1 . L_2) L_3 = L_1 . (L_2 . L_3)$ ⇒ associative property

→ $L^+ = L^* - \{\varepsilon\}$.

→ If $L = \{\varepsilon\}$ then, $L^* = \{\varepsilon\}$.

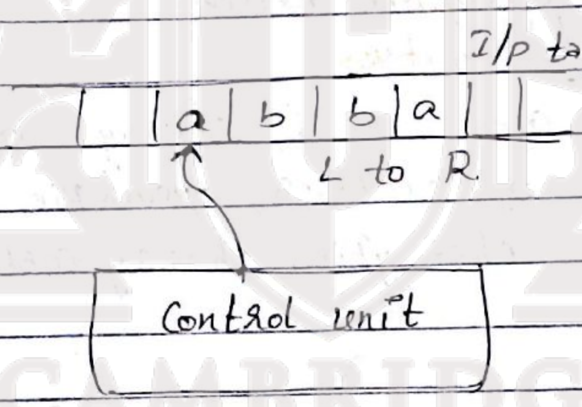
→ if $L = \{\ \}$ then $L^* = \{\varepsilon\}$

L^*

$$L^* = \{\varepsilon\} \cup \{w \in \Sigma^* \ \exists k > 1 (\exists k (w_1, w_2, \ldots w_k) \in L) \ w = w_1, w_2 \ldots w_k\}$$

It is the set of strings that can be formed by concatenating together 0 or more strings from L.

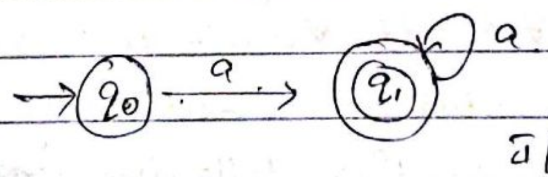
Deterministic Finite state Machines. (DFSM / FSM)

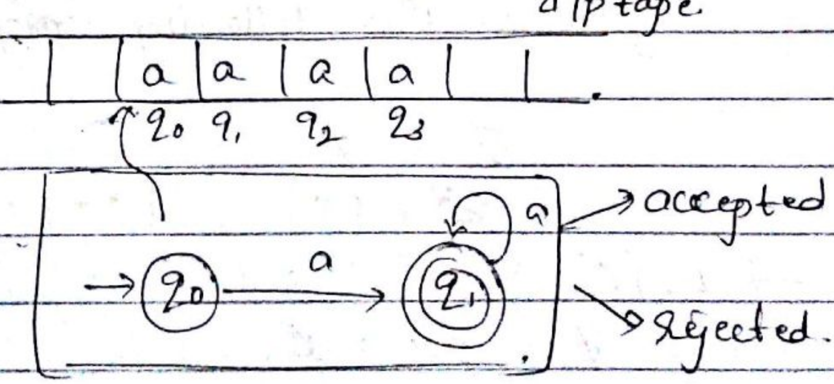


Ex:-

L_1 = strings of one or more a's.

$L_2 = \{a, aa, aaa, aaaa \ldots \ldots\}$.





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Finite state machine as 3 parts.

→ control unit

→ Reading the contents of i/p tape from L to R and cell by cell.

→ Input Tape.

→ It contains many rectangular cell & each cell is able to hold only one symbol.

Read head → Control unit is reading the containts of i/p tape from L to R & cell by cell.

The logic or the strategy to recongize the language resides inside the control unit.

Finally, the machine rises either of the two messages (accepted or rejected).

Mathematical Model of FSM.

A finite state Machine is defined as. quintaple information.

$$\boxed{M = (K, \Sigma, \delta, S, A)}$$

$K \rightarrow$ set of states.

$\Sigma \rightarrow$ set of alphabets.

$\delta \rightarrow$ transition function.

transition func for deterministic FSM is defined as.

$$\boxed{K \times \Sigma \rightarrow K}$$

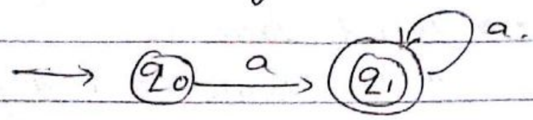
$S \rightarrow$ start state or initial state, $S \in K$.

$A \rightarrow$ set of accepting state or final state.

$A \subseteq K$.

L_1 = Strings of one or more A's.
$L_1 = \{a, aa, aaa, aaaa \ldots\}$

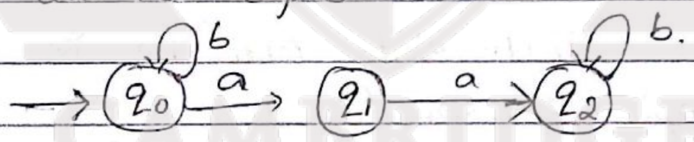
transition diagram,



Deterministic FSM is defined as follows.

$K = \{q_0, q_1\}$.
$\Sigma = \{a\}$
$S = \{q_0\}$
$A = \{q_1\}$.
$\delta : K \times \Sigma \rightarrow K$.
$\delta(q_0, a) = q_1$
$\delta(q_1, a) = q_1$

1) From the given transition diagram of DFSM define all the tuples.



$K = \{q_0, q_1, q_2\}$
$\Sigma = \{a, b\}$.
$S = \{q_0\}$.
$A = \{q_2\}$

$\delta : K \times \Sigma \rightarrow K$.
$\delta(q_0, a) = q_1$
$\delta(q_0, b) = q_0$.
$\delta(q_1, a) = q_2$
$\delta(q_2, b) = q_2$.

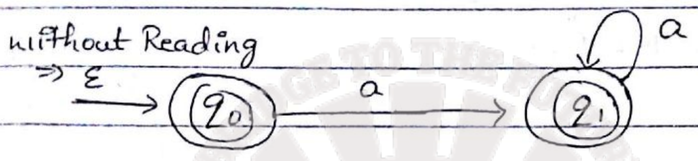
transition table

δ	a	b.
$\rightarrow q_0$	q_1	q_0
q_1	q_2	-
$* q_2$	-	q_2

(2) Design an DFSM for the strings of zero or more no. of a's.

$\Sigma = \{a\}$.

$L = \{\varepsilon, a, aa, aaa, \ldots\}$.



$q_0, a = q_1$

$q_1, a = q_1$

$K = \{q_0, q_1\}$

$\Sigma = \{a\}$

$S = \{q_0\}$

$A = \{q_1\}$.

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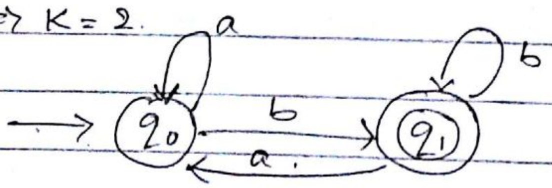
3) Design a DFSM for the language of strings of a's and b's where every string is ending with b.

$\Sigma = \{a, b\}$. basic string (length = 1) so, K = length + 1

$L = \{ab, aab, aaaa\ldots b, b, bb, bb\cdots b, bab, abb, bababb, \ldots\}$.

⇒ K = 2.



$K = \{q_0, q_1\}$.

$S = \{q_0\}$

$A = \{q_1\}$.

$\delta = K \times \Sigma \rightarrow K$.

$\delta(q_0, a) = q_0 \quad \delta(q_0, b) = q_1$

$\delta(q_1, a) = q_0 \quad \delta(q_1, b) = q_1$

4). Design a DFSM that accepts strings of a's and b's that begin with the substring aa.

aa

$\Sigma = \{a, b\}$

$L = \{aa, aab, aaa, aaba, aaab, aaab \ldots\}$

basic string

length = 2

$K = 2+1 = 3$.

$q_0, a = q_1$

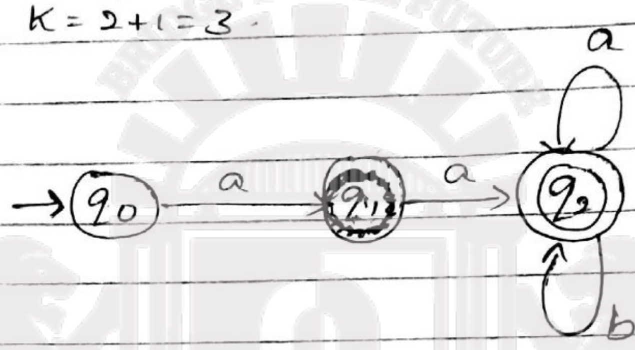
$q_0, b = X$

$q_1, a = q_2$

$q_1, b = X$

$q_2, a = ✓$

$q_2, b =$



aa

5) Design an DFSM that accepts strings of 0's & 1's that begin with 0 and end with 1.

$\Sigma = \{0, 1\}$

$L = \{01, 001, 0011, 00001, 01111, \ldots 010011, 011001\}$

→ 2

$K = 3$

$K = \{q_0, q_1, q_2\}$

$\delta(q_0, 0) = q_1$

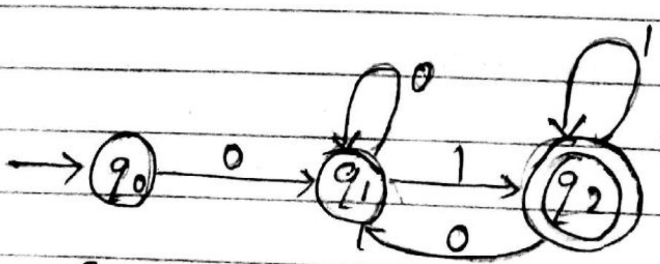
$\delta(q_0, 1) = X$

$\delta(q_1, 0) = q_1$ ✓

$\delta(q_1, 1) = q_2$ ✓

$\delta(q_2, 0) = q_1$ ✓

$\delta(q_2, 1) = q_2$ ✓



$S = \{q_0\}$

$A = \{q_2\}$

$\delta = K \times \Sigma \rightarrow K$.

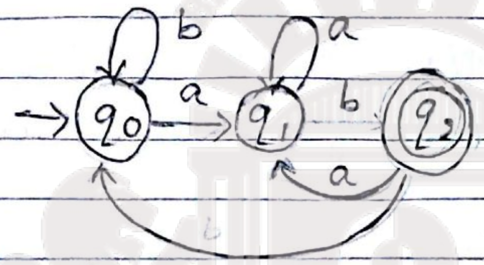
6) Design an DFSM that accepts strings of a's and b's which are ending with ab.

$\Sigma = \{a, b\}$.

$L = \{ab, aaa\ldots ab, bb\ldots b\ldots ab, aabaab, abbbaba\ldots aab \ldots\}$.

$\rightarrow 2$

$K = 3 = \{q_0, q_1, q_2\}$.



$q_0, a = q_1$
$q_0, b = q_0$
$q_1, a = q_1$
$q_1, b = q_2$
$q_2, a = q_1$
$q_2, b = q_0$

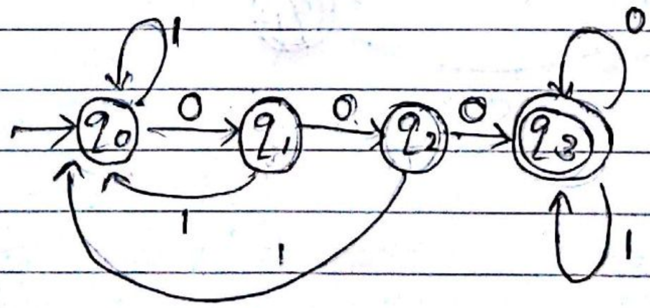
$S = \{q_0\}$
$A = \{q_2\}$.

7) Design an DFSM that accepts strings of 0's & 1's that contain 3 consicutive 0's as a Substring.

$\Sigma = \{0, 1\}$ $K = \{q_0, q_1, q_2, q_3\}$.

$L = \{000, 00000, 00001, 00001, 10001, 01100001, 0101000101, 10100001.$

$\rightarrow$ basic string



$q_0, 0 = q_1$
$q_0, 1 = q_0$
$q_1, 0 = q_2$
$q_1, 1 = q_0$
$q_2, 0 = q_3$.
$q_2, 1 = q_1$
$q_3, 0 = q_3$
$q_3, 1 = q_3$

8) Design an DFSM that accepts even length of a's.

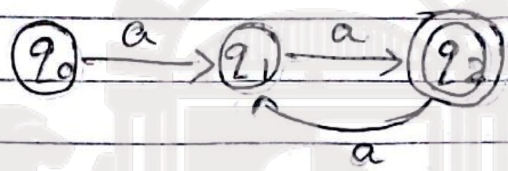
$\Sigma = \{a\}$

$L = \{aa, aaaa, aaaaaa, \ldots\}$.

$\hookrightarrow 2$

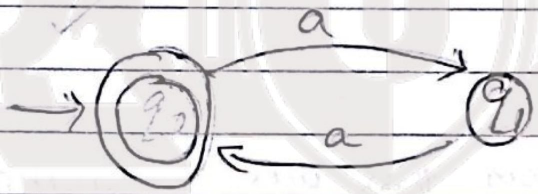
$K = 2+1 = 3$.

$L = \{ w \in \{a\}^*, |w| \equiv_2 0 \}$.



→ ε not included

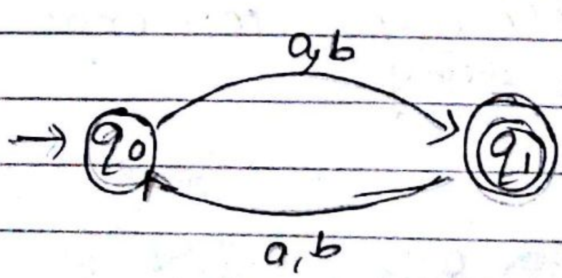
(or)



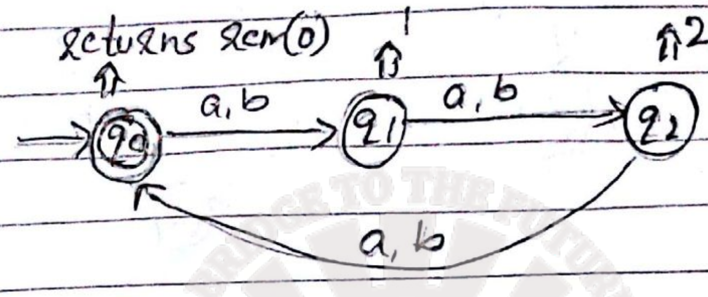
→ ε included

9) Design an DFSM that accepts odd length of of strings on $\Sigma = \{a, b\}$.

$L = \{ w \in \{a,b\}^* \ |w| \equiv_2 1 \}$

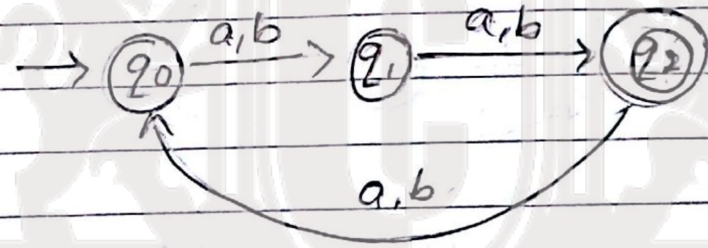


10) Design a DFSM for the following language where $L = \{ w \in \{a,b\}^* \mid |w| \equiv_3 0 \} \Rightarrow$ indused ϵ.



2
5
8
11
14
17
20.

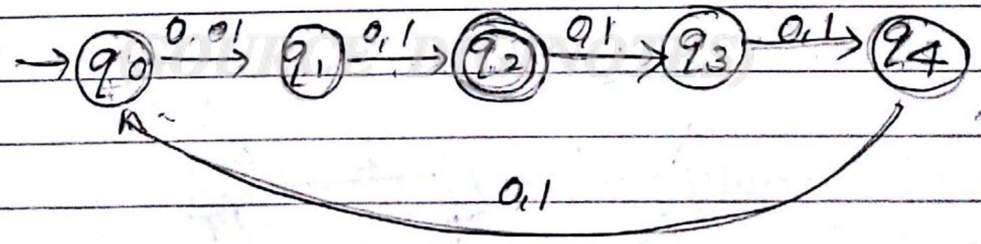
11) $L = \{ w \in \{a,b\}^* \mid |w| \equiv_3 2 \}$.



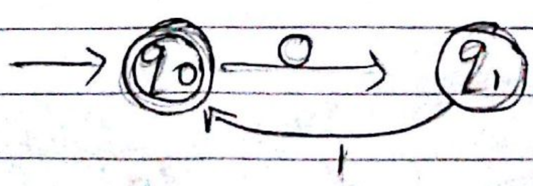
12) Design an DFSM for a language $L = \{ w \in \{0,1\}^* \mid |w| \equiv_5 2 \}$.

2
7
12
17

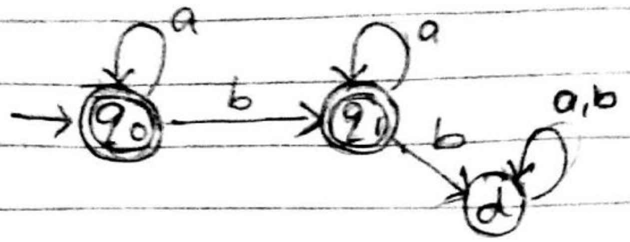
0
5
10
15



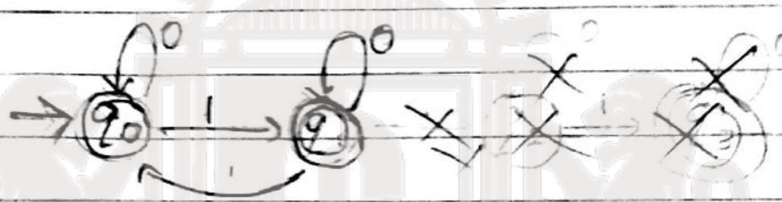
13) Design an DFSM, $L = \{ (01)^n \mid n > 0 \}$



14) Design an DFSM that accepts strings of a's and b's that contain not more than 1 b.

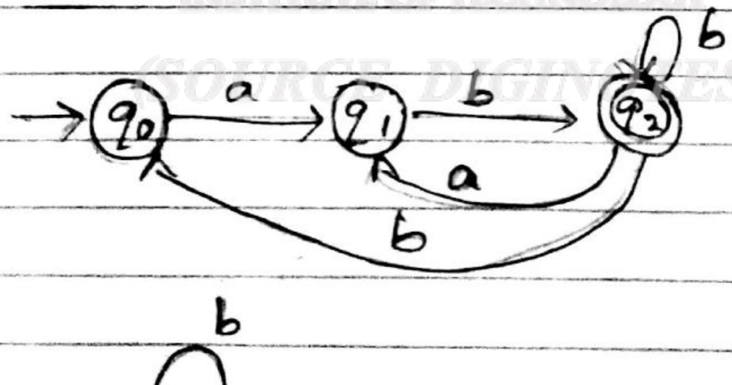


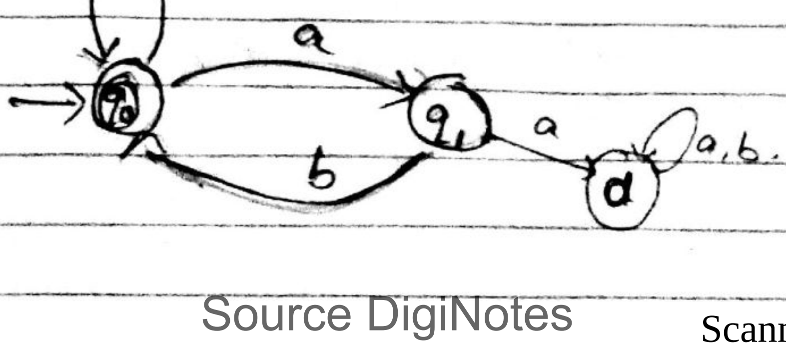
15) $L = \{ w \in \{0,1\}^* : w$ has odd parity$\}$



00100 ... 11 ..

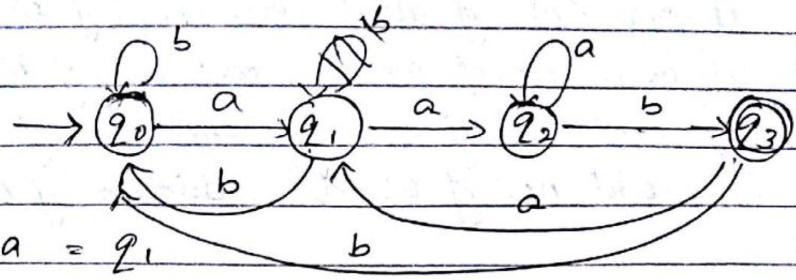
16) $L = \{ w = \{a,b\}^*$; every a is immediately followed by b$\}$.





aaab

17) Design an DFSM that accepts strings of a's and b's having aab at the end.



$q_0, a = q_1$

$q_0, b = q_0$

$q_1, a = q_2$ bbab aabab~~a~~bbaab

$q_1, b = q_0$

$q_2, a = q_2$

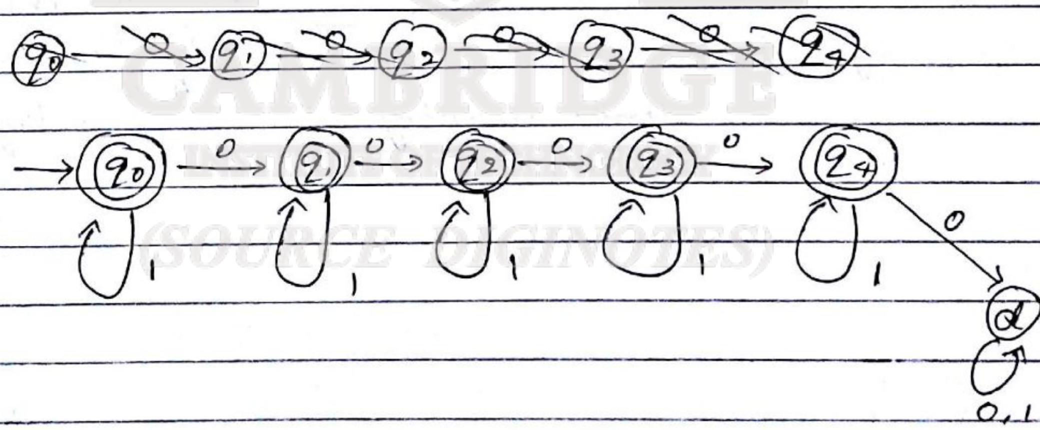
$q_2, b = q_3$.

$q_3, a = q_1$

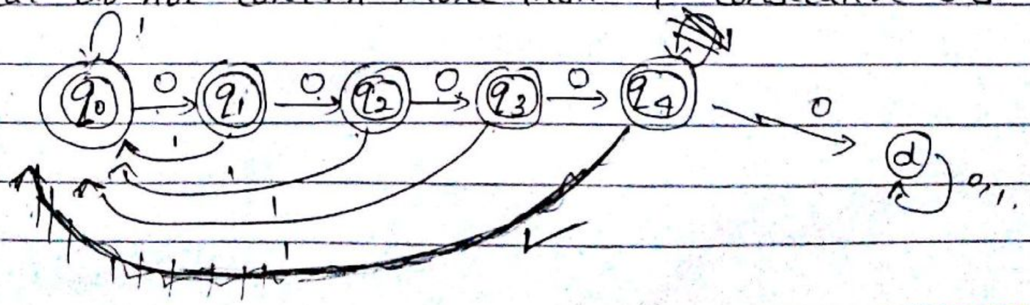
$q_3, b = q_0$.

18) Design a DFSM that accepts strings of 0's and 1's that do not contain more than 4 0's.

$\Sigma = \{0, 1\}$.



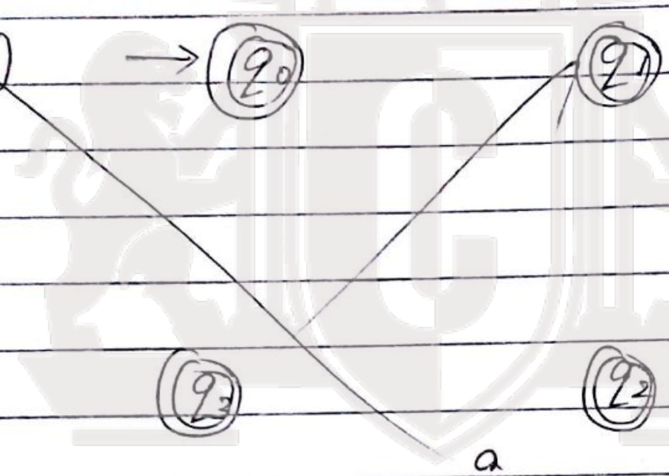
19) Design a DFSM that accepts strings of 0's & 1's that do not contain more than 4 consecutive 0's.



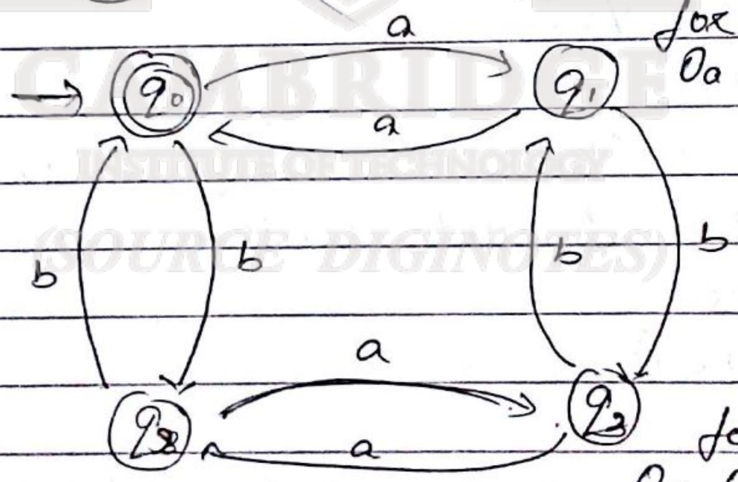
20) Define an DFSM for the following pattern where $\Sigma = \{a, b\}$.

i) even no. of a's & even no. of b's
ii) even no. of a's & odd no. of b's.
iii) odd no. of a's & even no. of b's.
iv) odd no. of a's & odd no. of b's.

i)



i)

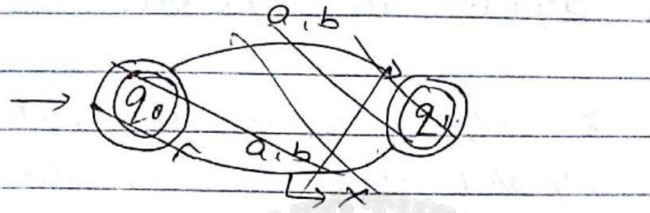


for iii) make $O_a E_b$. accepting state

for ii). $E_a O_b$. make accepting state,

for iv) $O_a O_b$ make accepting state.

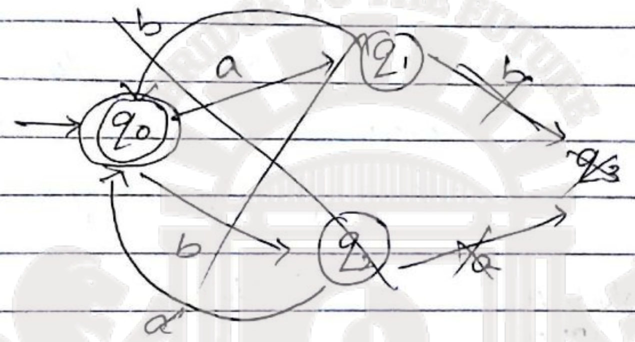
Q1) Define a DFSM for the following language $L = \{w \in \Sigma^*$, Where $\Sigma = \{a, b\}$; no 2 consecutive characters are same$\}$.

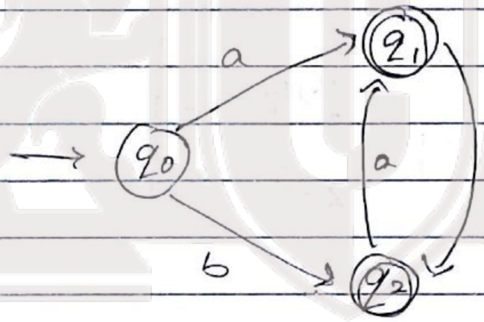


a, b ~

ab

ba





Q2) Design a DFSM that accepts strings of a's and b's that do not contain aab.

$q_0, a = q_1$

$q_0, b = q_0$

$q_1, a = q_2$

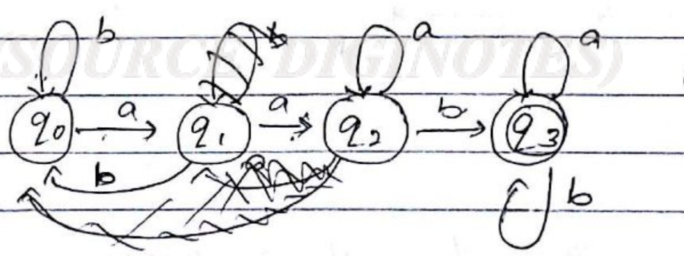
$q_1, b = q_0$

$q_2, a = q_2$

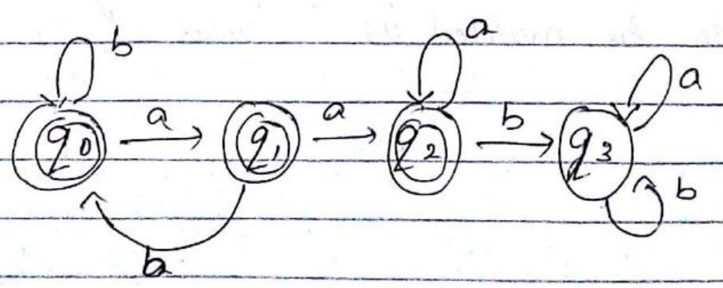
$q_2, b = q_3$

$q_3, a = q_3$

$q_3, b = q_3$.



contain aab.



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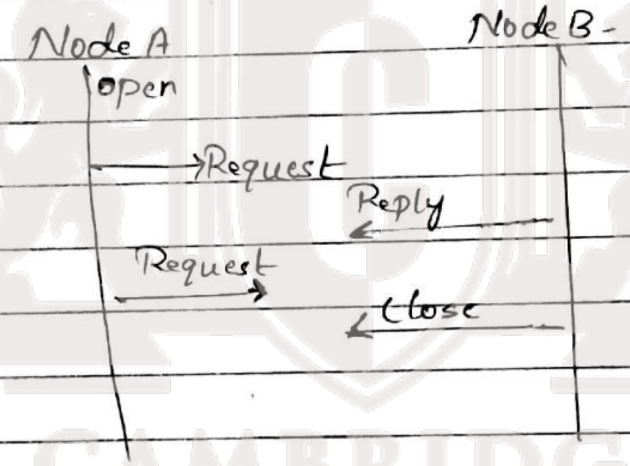
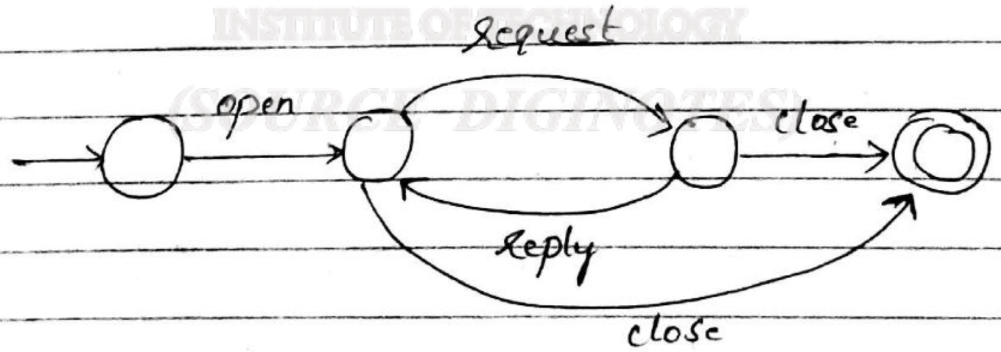
Applications of DFSM.

Every Communication protocol in data Communication can be depicted as a protocol.

Let $\Sigma = \{$open, request, reply, close$\}$ be the alphabet set for communication protocol.

L = { open – close, open – request – close,
open – request – reply – request – close,
open – req – reply – req – reply – – – – – request close }





Each token of programming language can be modeled as DFSM.

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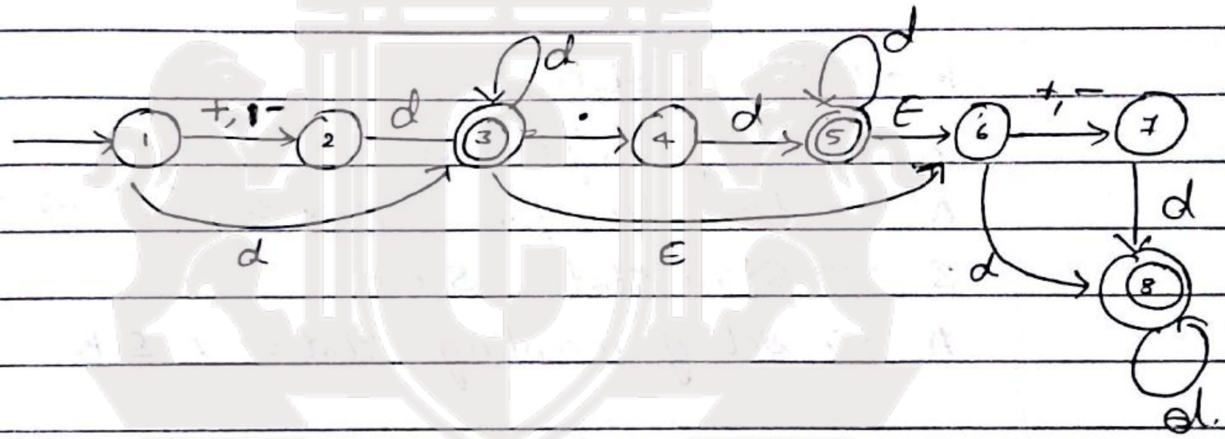
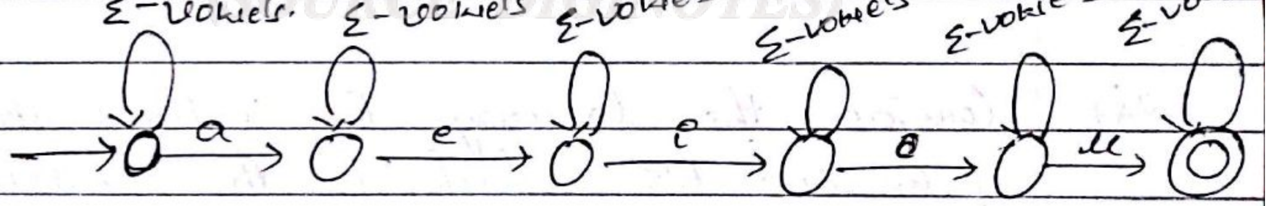
DFSM for recongnising floating point no.s.

0.3	$4E^{+10}$	$4.5E^{-10}$
+0.3	$-4E^{10}$	$-4.5E^{10}$.
-0.3	$-4E^{-10}$	$456.5E^{10}$

$\boxed{(+|-)\ d.d\ E(+|-)\ d}$



Q3) Design a DFSM that accepts strings of English alphabet such that the string should content all vowels in the same order.



Σ = alphabets.

vowels = a, e, i, o, u.

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Non- deterministic FSM.(NDFSM).

Let M be NDFSM,

M is the quituple representation and m is formally defined as.

$M = (K \; \Sigma \; \Delta \; S \; A)$.

$K \rightarrow$ Set of states.

$\Sigma \rightarrow$ Set of alphabets.

$\Delta \rightarrow$ transition fun. $K \times \Sigma \cup \{\varepsilon\} \rightarrow 2^K$

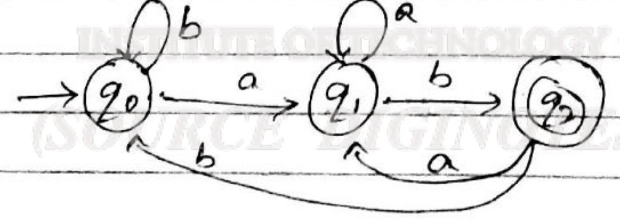
$S \rightarrow$ Start state $S \in K$.

$A \rightarrow$ set of accepting states $A \subseteq K$

$K = \{q_0, q_1, q_2\}$.

$2^K = \{\{q_0\}, \{q_1\}, \{q_2\}, \{q_0, q_1\}, \{q_1, q_2\}, \{q_0, q_2\}, \{q_0, q_1, q_2\}, \phi\}$.

24) Consider the language that contains strings of a's and b's. which has the substring ab at the end.



$K \times \Sigma \rightarrow K$

$\delta(q_0, a) = q_1$

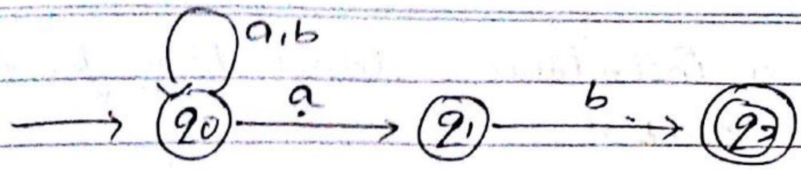
$(q_0, b) = q_0$

$(q_1, a) = q_1$

$(q_1, b) = q_2$

$(q_2, a) = q_1$

$(q_2, b) = q_0$



δ	a	b
→ q_0	$\{q_0\ q_1\}$	q_0
q_1	-	q_2
* q_2	-	-

Instantaneous description

(q_0, baaab)

↓

q_0, aaab

q_0, aab q_1, aaab

↓ ×

q_0, ab q_1, ab

↓ ×

q_0, b q_1, b.

↓ ↓

q_0 × q_2//

String is reconized

Q5) Write a instantaneous description for the string aabbab.

(q_0, aabbab)

q_0, abbab q_1, abbab

q_0, bbab q_1, abbab   x

q_0, bab   x

q_0, ab

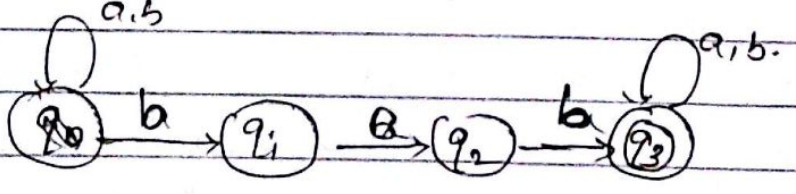
q_0, b q_1, b

q_0 q_2 //

x   String is accepted.

String ⇒ $q_0\ q_0\ q_0\ q_0\ q_0\ q_1\ q_2$

Q6) Design NFSM for the string of a's & b's that contain the substring bab. Define all the topples also write instantaneous description for the string bbaba.



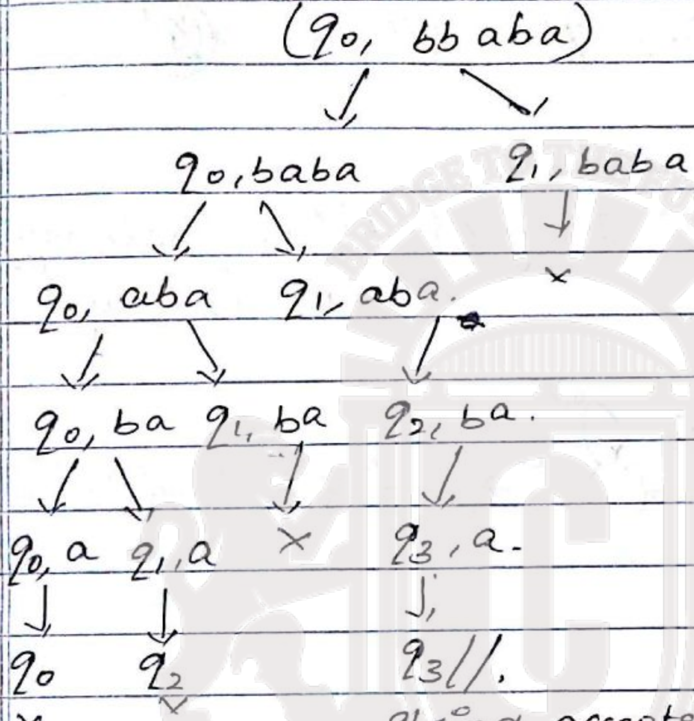
$K = \{q_0, q_1, q_2, q_3\}$ $\Sigma = \{a, b\}$.

$S = q_0$ $A = q_3$

$q_0\ a = \{q_0\}$. $q_1, a = q_2$ $q_2, a = x$ $q_3\ a = q_3$.

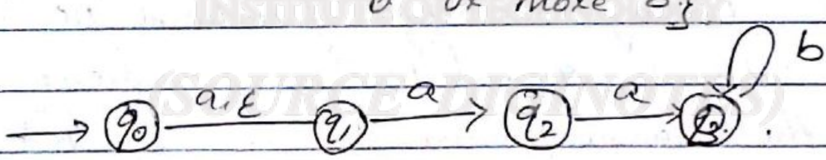
$q_0\ b = \{q_0, q_1\}$ $q_1, b = x$ $q_2, b = q_3$ $q_3, b = q_3$

Δ	a	b
$\rightarrow q_0$	q_0	$\{q_0 q_1\}$
q_1	q_2	—
q_2	—	q_3
$* q_3$	q_3	q_3

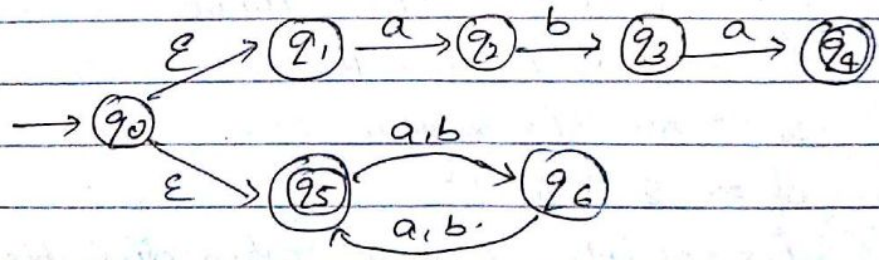


String accepted

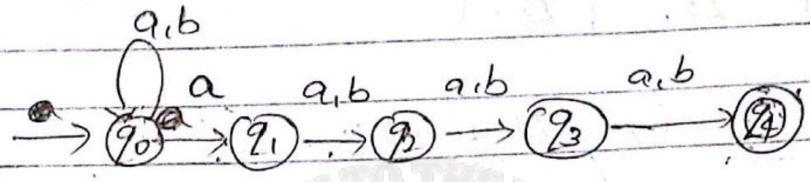
27) $L = \{w \in \{a,b\}^* ;$ w is made of any optional a followed by aa followed by 0 or more b$\}$



28) $L = \{w \in \{a,b\}^* ; w = aba$ or $|w|$ is even$\}$.



28) $L = \{w \in \{a,b\}^* \mid \forall$ string the 4^{th} character from the last is $a\}$.



29) $L = \{w \in \{a,b\}^* \; \exists\, x,y \in \{a,b\}^*, w = (xabbaay) \vee w = (xbabay)\}$

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30) Design an DFSM that accepts strings of 0's & 1's (it's binary encoding of natural no.s) that are ÷ble by 4). (Method 3 where strings of the language are divisible by K).

$\Sigma = \{0,1\}$

$L = \{0100,\ 1000,\ 1100,\ 10000 \ldots\ldots\}$

$j = (r * i + d)$ modulo K.

$r \rightarrow$ radix/base value. $r = 2$.

$d = \Sigma = \{0,1\}$

i = passible remainder when divisible by K

$i = 0, 1, 2, 3$.

$K = 4$.

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i	$j = (2 * i + d) \text{ modulo } K$	$\delta(q_i, d) = q_j$
0	$(2*0+0) m\ 4 = 0$	$\delta(q_0, 0) = q_0$
	$(2*0+1) m 4 = 1$	$\delta(q_0, 1) = q_1$.
1	$(2*1+0) m 4 = 2$	$\delta(q_1, 0) = q_2$
	$(2*1+1) m 4 = 3$	$\delta(q_1, 1) = q_3$.
2	$(2*2+0) m 4 = 0$	$\delta(q_2, 0) = q_0$
	$(2*2+1) m 4 = 1$	$\delta(q_2, 1) = q_1$
3	$(2*3+0) m 4 = 2$.	$\delta(q_3, 0) = q_2$
	$(2*3+1) m 4 = 3$.	$\delta(q_3, 1) = q_3$.

$4\overline{)2}$ 0, 2

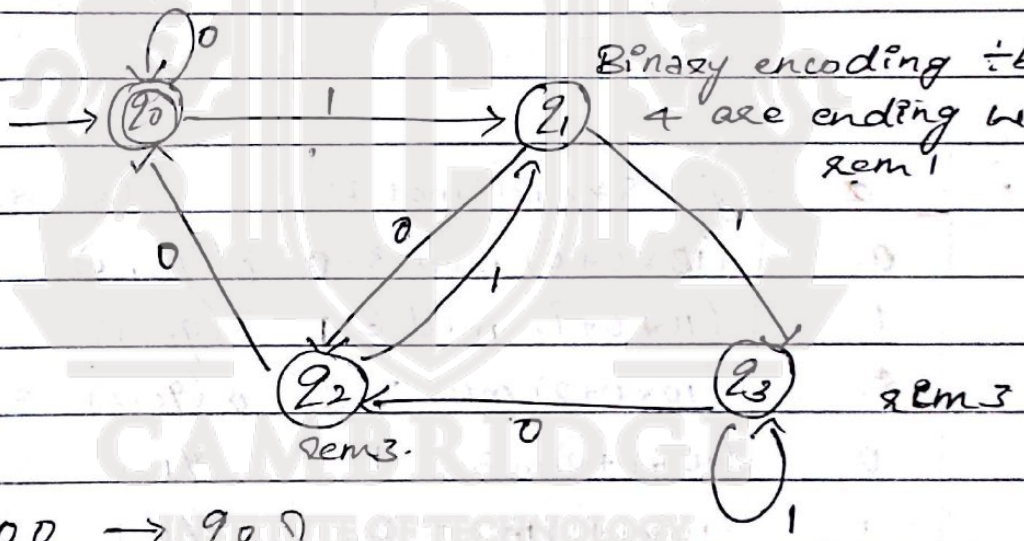
$4\overline{)20}$ 0.5

5 m 4

6 m 4

7 m 4

$4\overline{)3}$ 0, 3



Binary encoding ÷ble by 4 are ending with rem 1

$0100 \rightarrow q_0$

$1000 \rightarrow q_0$ } → final state.

$1100 \rightarrow q_0$

$1111 \rightarrow q_3$

$1101 \rightarrow q_1$

$0110 \rightarrow q_2$

$0111 \rightarrow q_3$

Source DigiNotes

31) Design a DFSM that accepts decimal no.s that are divisible by 3.

$R = 10$

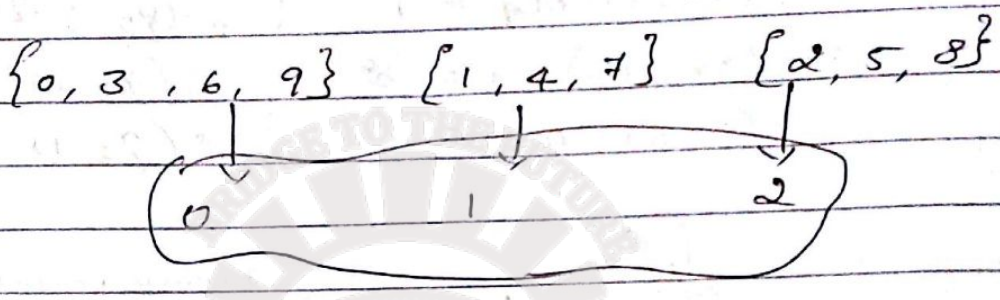
$i = 0, 1, 2$

$d = \Sigma = \{0, 1, 2 \ldots 9\}$

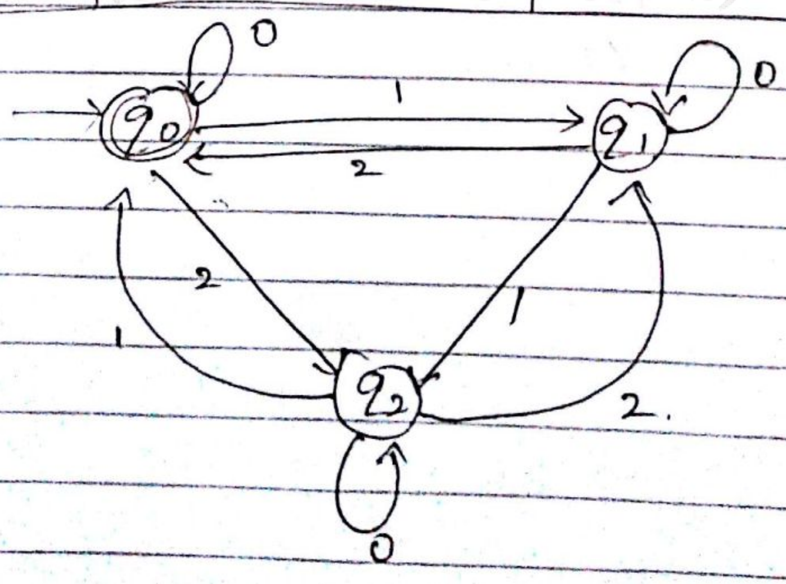
$K = 3$

$3 \overline{)20}$ 6, 18, 2

$3 \overline{)12}$ 4, 12, 0



i	d	$j = (r*i+d) \bmod K$	$\delta(q_i, d) = q_j$
0	0	$(10*0+0) \bmod 3 = 0$	$\delta(q_0, 0) = q_0$
	1	$(10*0+1) \bmod 3 = 1$	$\delta(q_0, 1) = q_1$
	2	$(10*0+2) \bmod 3 = 2$	$\delta(q_0, 2) = q_2$
1	0	$(10*1+0) m 3 = 1$	$q_{1,0} = q_1$
	1	$(10*1+1) m 3 = 2$	$q_{1,1} = q_2$
	2	$(10*1+2) m 3 = 0$	$q_{1,2} = q_0$
2	0	$(10*2+0) m 3 = 2$	$q_{2,0} = q_2$
	1	$(10*2+1) m 3 = 0$	$q_{2,01} = q_0$
	2	$(10*2+2) m 3 = 1$	$q_{2,2} = q_1$



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Equivalence of NDFSM & DFSM.

We can say NDFSM is equivalent to DFSM when the language accepted by both are same.

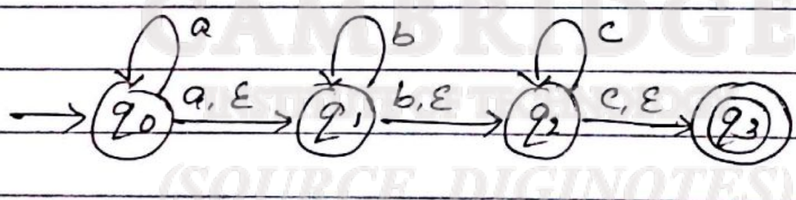
Conversion of NDFSM with ~~ep~~ ε transitions to DFSM

Epsilon closures of state 'q'

It is defined as a set of states that is reachable from q on Epsilon transition only.

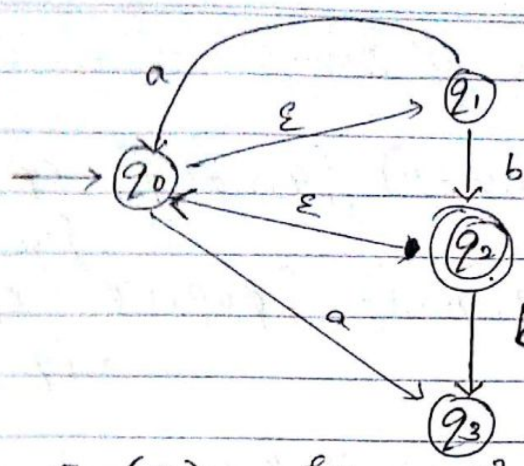
$$eps(q) = \{ P \in K : (q, w) \vdash^{*} (p, w) \}.$$

37) Compute epsilon closures for all the states given in the transition diagram.



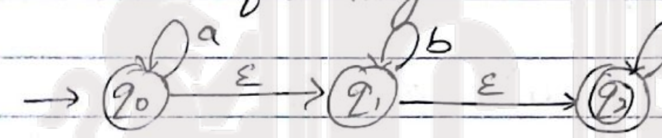
$Eps(q_0) = \{q_0, q_1, q_2, q_3\}$
$Eps(q_1) = \{q_1, q_2, q_3\}$
$Eps(q_2) = \{q_2, q_3\}$
$Eps(q_3) = \{q_3\}$.

38)



$Eps(q_0) = \{\cancel{q_1, q_2, q_0}\}\ \{q_0, q_1, q_2\}$
$Eps(q_1) = \{\cancel{q_2}\}\ \{q_0, q_1, q_2\}$
$Eps(q_2) = \{q_0, q_1, q_2\}$
$Eps(q_3) = \{q_3\}$.

39) Convert the following NDFSM to DFSM.



Step 1 → Compute ε-closures of all state.

$Eps(q_0) = \{q_0, q_1, q_2\}$.
$Eps(q_1) = \{q_1, q_2\}$
$Eps(q_2) = \{q_2\}$.

Step 2 → Transition table. (NDFSM)

Δ	a	b	c	ε
→ q_0	q_0	-	-	q_1
q_1	-	q_1	-	q_2
* q_2	-	-	q_2	-

Step 3 →

1. To find start state of DFSM.

Start state of NFSM = $\{q_0\}$

" " DFSM = $Eps(q_0)$
$= \{q_0, q_1, q_2\} \rightarrow A$

2. Transitions of DFSM.

$\delta(A,a) = \Delta(\{q_0,q_1,q_2\},a) = \{q_0 \cup \phi \cup \phi\} = Eps\{q_0\} = \{q_0,q_1,q_2\} \rightarrow A$

$\delta(A,b) = \Delta(\{q_0,q_1,q_2\},b) = \{\phi \cup q_1 \cup \phi\} = Eps\{q_1\} = \{q_1,q_2\} \rightarrow B$

$\delta(A,c) = \Delta(\{q_0,q_1,q_2\},c) = \{\phi \cup \phi \cup q_2\} = Eps(q_2) = \{q_2\} \rightarrow C$

$\delta(B,a) = \Delta(\{q_1,q_2\},a) = \{\phi \cup \phi\} = Eps\{\phi\} = \phi$

$\delta(B,b) = \Delta(\{q_1,q_2\},b) = \{q_1 \cup \phi\} = Eps\{q_1\} = \{q_1,q_2\} \rightarrow B$

$\delta(B,c) = \Delta(\{q_1,q_2\},c) = \{\phi \cup q_2\} = Eps\{q_2\} = \{q_2\} \rightarrow C$

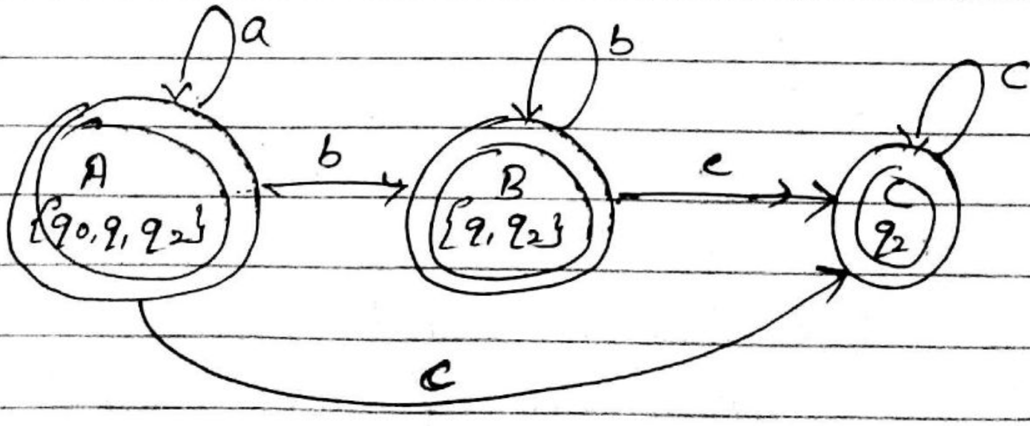
$\delta(C,a) = \Delta(\{q_2\},a) = \phi = \phi$

$\delta(C,b) = \Delta(\{q_2\},b) = \cancel{q_2 = Eps(q_2) = q_2} \rightarrow \cancel{C} = \phi$

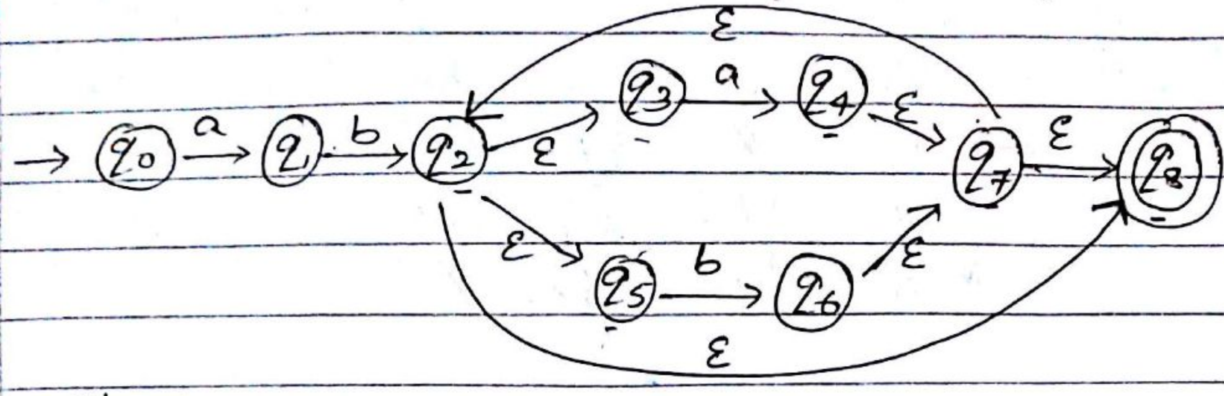
$\delta(C,c) = \Delta(\{q_2\},c) = q_2 = Eps(q_2) \rightarrow C$.

3. Identify final state.

In NFSM q_2 is final state
So, In DFSM wherever q_2 is present it is final state.



40) Compute Epsilon closures for given diagram.



Step 1 =

$Eps(q_0) = \{q_0\}$ $\quad$ $Eps(q_4) = \{q_2, q_3, q_4, q_5, q_7, q_8\}$

$Eps(q_1) = \{q_1\}$. $\quad$ $Eps(q_5) = \{q_5\}$

$Eps(q_2) = \{q_2, q_3\}$ $\quad$ $Eps(q_6) = \{q_2, q_3, q_5, q_6, q_7, q_8\}$

$Eps(q_3) = \{q_3\}$. $\quad$ $Eps(q_7) = \{q_7, q_8, q_2, q_3, q_5\}$.

$Eps(q_8) = \{q_8\}$

Step 2 = Transition table (NDFSM)

Δ	a	b	ε
$\rightarrow q_0$	q_1	–	–
q_1	–	q_2	–
q_2	–	–	q_8, q_3, q_5
q_3	q_4	–	–
q_4	–	–	q_7
q_5	–	q_6	–
q_6	–	–	q_7
q_7	–	–	q_8, q_2, q_3, q_5.
$* q_8$	–	–	–

Step 3 →

1. To find start state of DFSM.

start state of NFSM = $\{q_0\}$

" " DFSM = $Eps(q_0) = \{q_0\} \rightarrow$ A

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2. Transition of DFSM.

Δ	ε	a	b
$\rightarrow p$	ϕ	$\{p\}$	$\{q\}$
q	$\{p\}$	$\{q\}$	$\{r\}$
$* r$	$\{q\}$	$\{r\}$	ϕ

$EPS(p) = \{p\}$

$EPS(q) = \{q, p\} = \{p, q\}$

$EPS(r) = \{r, q, p\} = \{p, q, r\}$.

Let us begin defining DFSM $M = \{K \; \Sigma \; \delta \; s A\}$

1) Define the start state of DFSM

start state of NDFSM $= \{p\}$

Start state of DFSM $= EPS\{p\} = \{p\} \rightarrow A$

2) Define transition for DFSM

$\delta(A, a) = \Delta(\{p\}, a) = EPS(\{p\}) = \{p\} \rightarrow A$

$\delta(A, b) = \Delta(\{p\}, b) = EPS(\{q\}) = \{p, q\} \rightarrow B$

$\delta(A, c) = \Delta(\{p\}, c) = EPS(\{r\}) = \{p, q, r\} \rightarrow C$

$\delta(B, a) = \Delta(\{p, q\}, a) = EPS(\{p\} \cup \{q\}) = \{p, q\} \rightarrow B$.

$\delta(B, b) = \Delta(\{p, q\}, b) = EPS(\{q\}, \{r\}) = \{p, q, r\} = C$

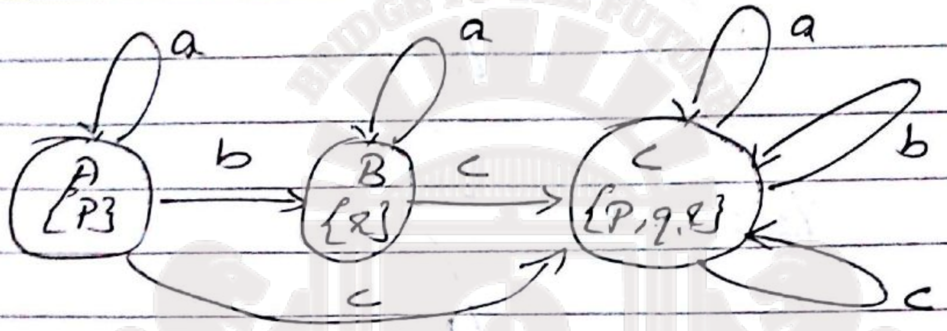
$\delta(B, c) = \Delta(\{p, q\}, c) = EPS(\{r\}) = \{p, q, r\} = C$

$\delta(C, a) = \Delta(\{p, q, r\}, a) = EPS(p \cup q \cup r) = C$

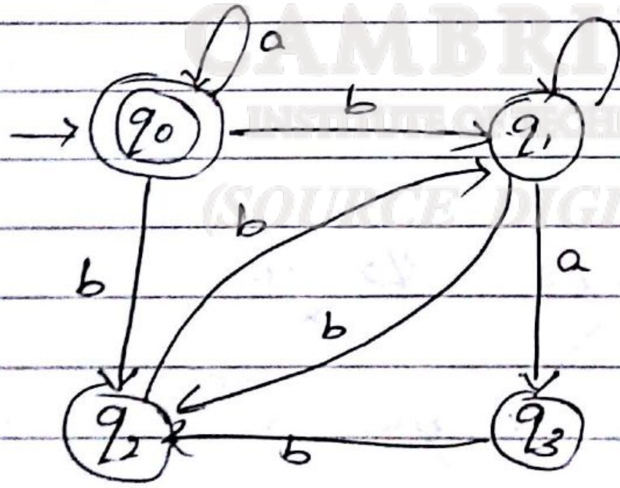
$\delta(C, b) = C$

$\delta(C, c) = C$

δ	a	b	c
→ A	A	B	C
B	B	C	C
* C	C	C	C



For the given transition diagram check whether the strings aabbba, bab, baba, belongs to the language or not.



(q_0, aabbba)

(q_0, abbba)

(q_0, bbba)

q_1, bba $\quad$ q_2, bba

q_2, ba $\quad$ q_1, ba

q_1, a $\quad$ q_2, a ✗

q_1 $\quad$ $\underline{\underline{q_3}}$

✓

$q_0, q_0, q_1, q_2, q_1, q_3$.

(q_0, baba)

q_1, aba $\quad$ q_2, aba ✗

q_1, ba $\quad$ q_3, ba

q_2, a ✗ $\quad$ q_2 a ✗

baba does not belong to language.

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Convert the following NDFSM to DFSM.

Δ	ε	a	b	c
$\rightarrow p$	$\{q\ r\}$	–	$\{q\}$	$\{r\}$
$* q$	–	$\{p\}$	$\{r\}$	$\{p\ q\}$
r	–	–	–	

$EPs\{p\} = \{q\ r\}$
$EPs\ \{q\} = \{q\}$
$EPs\ \{r\} = \{r\}$

start state of NDFSM $= \{p\}$
" " DFSM $= EPs\{p\} = \{p\ q\ r\} \rightarrow A$

transition of DFSM.

$\delta(A, a) = \Delta(\{p, q, r\}, a) = EPs(\{p\}) = \{p, q, r\} \rightarrow A$
$\delta(A, b) = \Delta(\{p, q, r\}, b) = EPs(\{q \cup r\})$
$= \{q\ r\} \rightarrow B$
$\delta(A, c) = \Delta(\{p, q, r\}, c) = EPs\ (r \cup p \cup q) =$
$\{p, q, r\} \rightarrow A$

$\delta(B, a) = \Delta(\{q, r\}) = EPs(\{p\}) = \{p, q, r\} \rightarrow A$

$\delta(B, b) = \Delta(\{q, r\}, b) = EPs(\{r\}) = \{r\} \rightarrow C$

$\delta(B, c) = \Delta(\{q, r\}, c) = EPs(\{p \cup q\}) = \{p\ q\ r\} \rightarrow A$

$\delta(C, a) = \delta(C, b) = \delta(C, c) = \phi$

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Equivalence of DFSM and NDFSM.

{language accepted by a DFSM} $\subseteq$ {language accepted by NDFSM}.

1 for each state q in K_M do // eps closure of NDFSM.
 1.1 compute eps(q)
2 . $s' = eps(s)$ // Start state of DFSM.

3 . Compute δ'
 3.1 active-states = {s'}.
 3.2 = $\delta' = \phi$
 3.3. while there exists some elements Q of active-states for which δ' has not yet been computed do:
  for each character c in Σ_M do.
   new_state = ϕ
   for each state q in Q do:
    for each state p such that (q, c, p) $\in \Delta$ do:
     new_state = new_state $\cup$ eps(p)
   Add transition (Q, c, new-state) to δ'.
   If new-state $\notin$ active-states then insert it
4 . K' = active_states
6. $A' = \{Q \in K' : Q \cap A \neq \emptyset\}$

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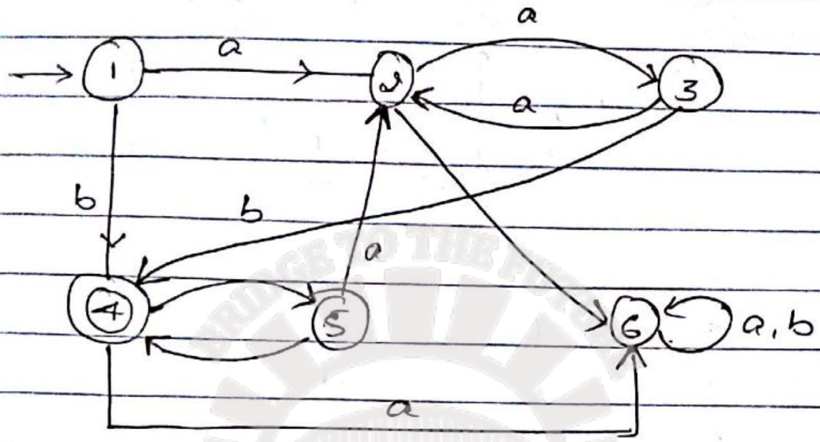
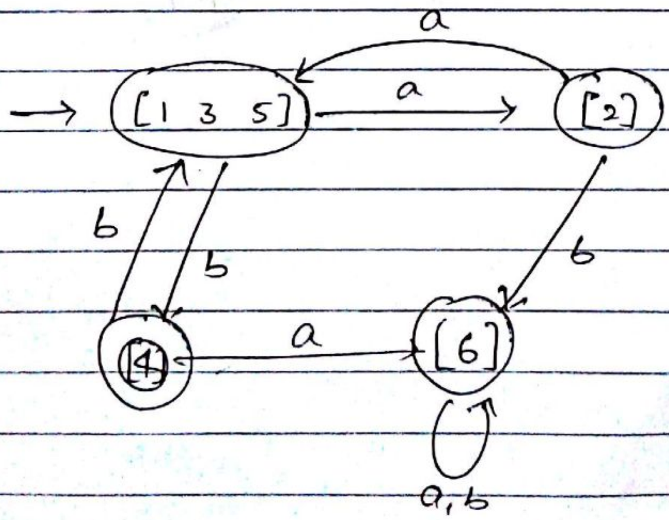
Minimizing DFSM (Not NFSM)
→ can't reduce NFSM)

(1)

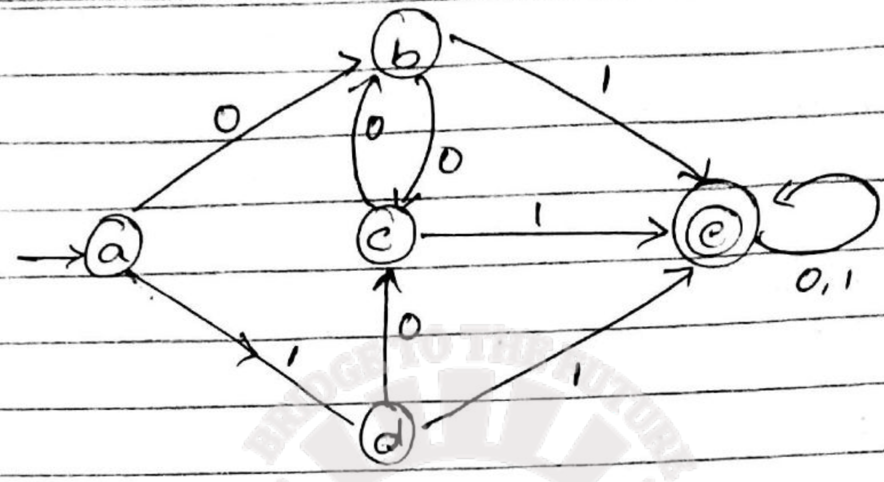


	a	b
→1	2	*4
2	3	6
3	2	*4
*4	6	5
5	2	*4
6	6	6

0-equi class | {1 2 3 5 6} {4}.
1 " | [1 3 5] [2 6] [4]
2 " | [1 3 5] [2] [6] [4]

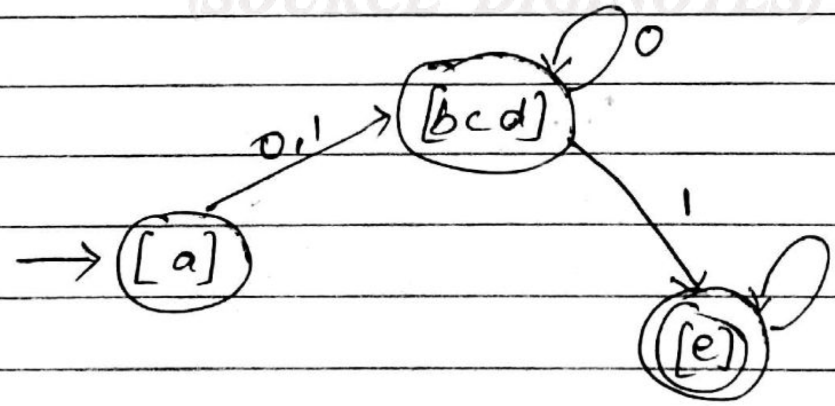


2

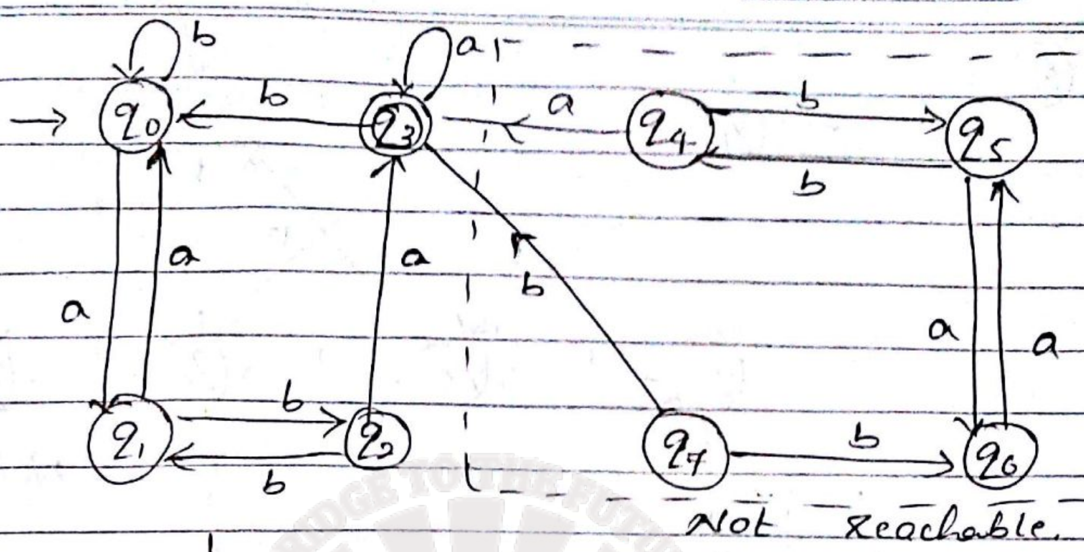


δ	0	1
→ a	b	d
b	c	*e
c	b	*e
d	c	*e
* e	*e	*e

0-equilance class	{a, b, c, d} {e}
1- "	[a] [b, c, d] [e]
2- "	[a] [b, c d] [e]



(3)

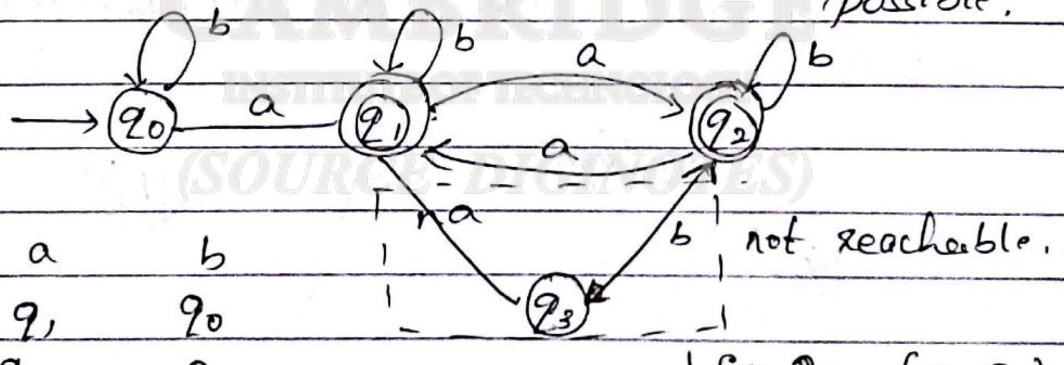


	a	b
$\rightarrow q_0$	q_1	q_0
q_1	q_0	q_2
q_2	$*q_3$	q_1
$*q_3$	$*q_3$	q_0

0-equivale. class	$[q_0\ q_1\ q_2]$ $[q_3]$
1 "	$[q_0, q_1]$ $[q_2]$ $[q_3]$
2 "	$[q_0]$ $[q_1]$ $[q_2]$ $[q_3]$.

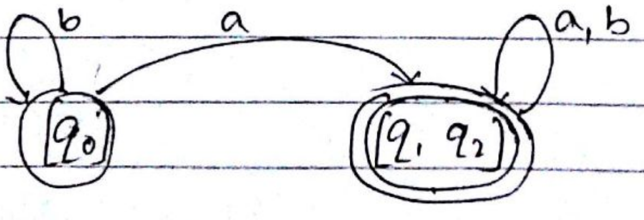
No futher minimization possible.

(4)



	a	b
$\rightarrow q_0$	q_1	q_0
q_1	q_2	q_1
q_2	q_1	$*q_2$
$\cancel{q_3}$		

0-equa	$[q_0]$ $[q_1, q_2]$
1 = "	$[q_0]$ $[q_1, q_2]$
2 - "	$[q_0]$ $[q_1, q_2]$.



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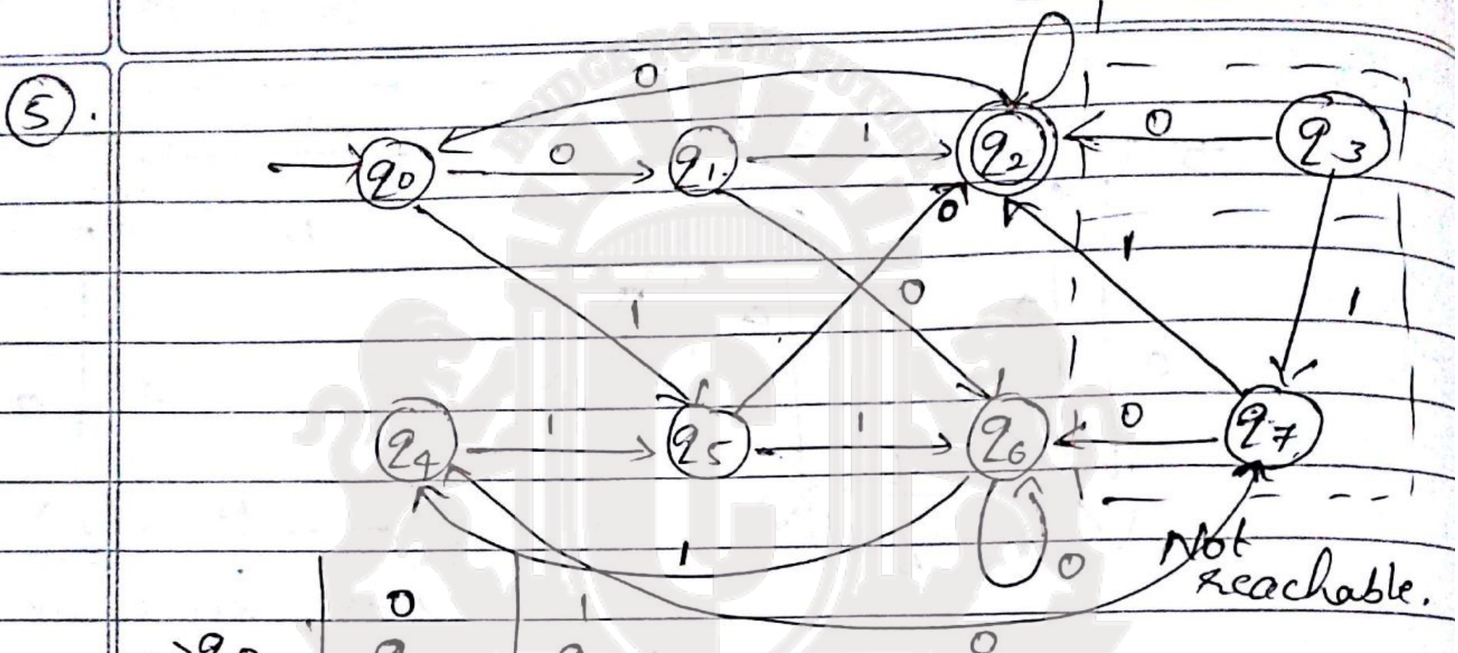
5.



	0	1
→q_0	q_1	q_5
q_1	q_6	*q_2
*q_2	q_0	*q_2
q_4	q_7	q_5
q_5	q_2	q_6
q_6	q_6	q_4
q_7	q_6	*q_2

Source DigiNotes

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0-equ [q_0 q_1 q_4 q_5 q_6 q_7] [q_2]

1-equ [q_0 q_4 q_6 [q_1 [q_2] [q_5]

2-equ

3-equ [q_0, q_4] [q_1, q_7] [q_2] [q_5] [q_6]