



Module -2

Relational Model

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Syllabus of the Module-2

Relational Model: Relational Model Concepts, Relational Model Constraints and relational database schemas, Update operations, transactions, and dealing with constraint violations. Relational Algebra: Unary and Binary relational operations, additional relational operations (aggregate, grouping, etc.) Examples of Queries in relational algebra.

Mapping Conceptual Design into a Logical Design:

Relational Database Design using ER-to-Relational mapping.

SQL: SQL data definition and data types, specifying constraints in SQL, retrieval queries in SQL, INSERT, DELETE, and UPDATE statements in SQL, Additional features of SQL.



Relational Model Concepts

- The relational Model represents the database as a collection of relations
- A Relation is a mathematical concept based on the **ideas of sets**
- The model was first proposed by Dr. E.F. Codd of IBM Research in 1970 in the following paper:
 - "A Relational Model for Large Shared Data Banks," Communications of the ACM, June 1970
- The above paper caused a **major revolution** in the field of **database management** and earned Dr. Codd the coveted **ACM Turing Award**



STUDENT

Name	Student_number	Class	Major
Smith	17	1	CS
Brown	8	2	CS

COURSE

Course_name	Course_number	Credit_hours	Department
Intro to Computer Science	CS1310	4	CS
Data Structures	CS3320	4	CS
Discrete Mathematics	MATH2410	3	MATH
Database	CS3380	3	CS

SECTION

Section_identifier	Course_number	Semester	Year	Instructor
85	MATH2410	Fall	04	King
92	CS1310	Fall	04	Anderson
102	CS3320	Spring	05	Knuth
112	MATH2410	Fall	05	Chang
119	CS1310	Fall	05	Anderson
135	CS3380	Fall	05	Stone

GRADE_REPORT

Student_number	Section_identifier	Grade
17	112	B
17	119	C
8	85	A
8	92	A
8	102	B
8	135	A

PREREQUISITE

Course_number	Prerequisite_number
CS3380	CS3320
CS3380	MATH2410
CS3320	CS1310

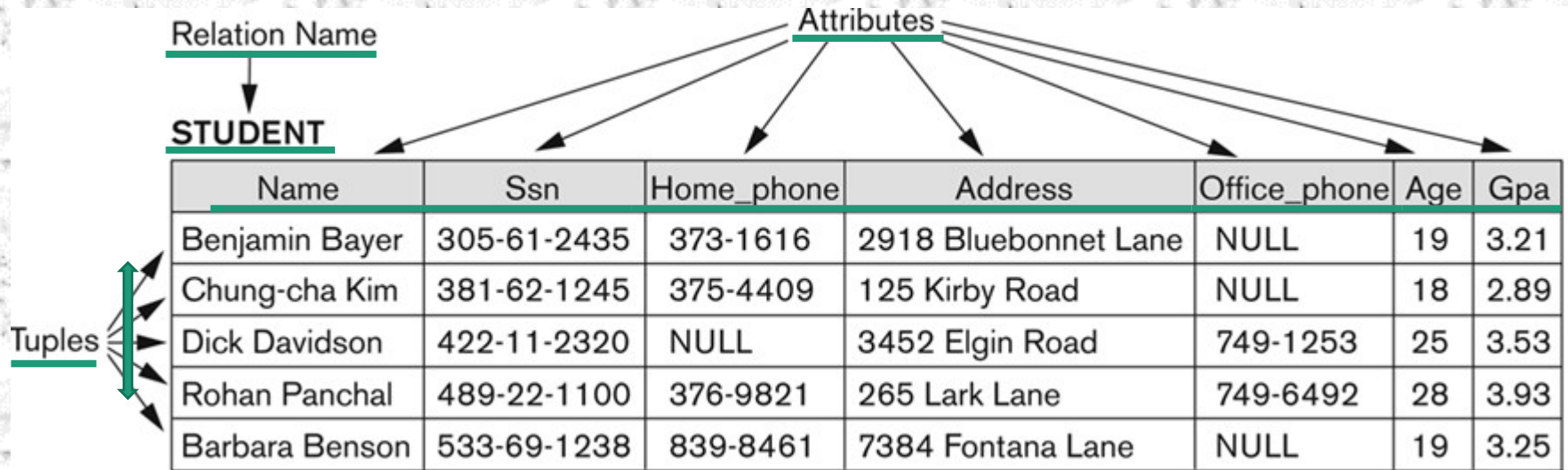
A database that stores student and course information.



Informal Definitions

- Informally, a **relation** looks like a **table** of values.
- A relation typically contains a **set of rows**.
- The data elements in each **row** represent certain facts that correspond to a real-world **entity** or **relationship**
- In the formal model, rows are called **tuples**
- Each **column** has a column header that gives an indication of the meaning of the data items in that column
- In the formal model, the column header is called an **attribute name** (or just **attribute**)

Example of a Relation



The Attributes and Tuples of a relation STUDENT



Informal Definitions

- Key of a Relation:
 - Each row has a value of a data item (or set of items) that uniquely identifies that row in the table called as *key*
 - In the STUDENT table, **SSN** is the key
 - Sometimes **row-ids** or **sequential numbers** are assigned as **keys** to identify the **rows in a table** called *artificial key* or *surrogate key*

Formal Definitions - Schema

- The **Schema** (or description) of a Relation:
 - Denoted by $R(A_1, A_2, \dots, A_n)$
 - R is the **name** of the relation
 - The **attributes** of the relation are A_1, A_2, \dots, A_n
- Example:

CUSTOMER (Cust-id, Cust-name, Address, Phone#)

 - CUSTOMER is the relation name
 - Defined over the four attributes: Cust-id, Cust-name, Address, Phone#
- Each attribute has a **domain** or a set of valid values.
 - For example, the domain of Cust-id is 6 digit numbers.

Formal Definitions - Tuple

- A **tuple** is an ordered set of values (enclosed in angled brackets ' $\langle \dots \rangle$ ')
 - Each value is derived from an appropriate *domain*.
- A row in the CUSTOMER relation is a 4-tuple and would consist of four values, for example:
 - $\langle 632\ 895, \text{"John Smith"}, \text{"101 Main St. Atlanta, GA 30 332"}, \text{"(404) 894-2000"} \rangle$
 - This is called a 4-tuple as it has **4 values**
 - A tuple (row) in the CUSTOMER relation.
- A relation is a **set** of such tuples (rows)



Formal Definitions - Domain

- A **domain** has a logical definition:
 - Example: “USA_phone_numbers” are the set of 10 digit phone numbers valid in the U.S.
- A domain also has a data-type or a format defined for it.
 - The USA_phone_numbers may have a format: (ddd)ddd-dddd where each d is a decimal digit.
 - Dates have various formats such as year, month, date formatted as yyyy-mm-dd, or as dd mm,yyyy etc.
- The attribute name designates the role played by a domain in a relation:
 - Used to interpret the meaning of the data elements corresponding to that attribute
 - Example: The domain Date may be used to define two attributes named “Invoice-date” and “Payment-date” with different meanings



Formal Definitions - State

- The **relation state** is a subset of the Cartesian product of the domains of its attributes.
 - each domain contains the set of all possible values the attribute can take.
- Example: attribute Cust-name is defined over the domain of character strings of maximum length 25
 - $\text{dom}(\text{Cust-name})$ is `varchar(25)`
- The role these strings play in the CUSTOMER relation is that of the *name of a customer*.

Formal Definitions - Summary

- Formally,
 - Given $R(A_1, A_2, \dots, A_n)$
 - $r(R) \subset \text{dom}(A_1) \times \text{dom}(A_2) \times \dots \times \text{dom}(A_n)$
- $R(A_1, A_2, \dots, A_n)$ is the **schema** of the relation
- R is the **name** of the relation
- A_1, A_2, \dots, A_n are the **attributes** of the relation
- $r(R)$: a specific **state** (or "value" or "population") of relation R – this is a *set of tuples* (rows)
 - $r(R) = \{t_1, t_2, \dots, t_n\}$ where each t_i is an n -tuple
 - $t_i = \langle v_1, v_2, \dots, v_n \rangle$ where each v_j *element-of* $\text{dom}(A_j)$



Formal Definitions - Example

- Let $R(A1, A2)$ be a relation schema:
 - Let $\text{dom}(A1) = \{0,1\}$
 - Let $\text{dom}(A2) = \{a,b,c\}$
- Then: $\text{dom}(A1) \times \text{dom}(A2)$ is all possible combinations:
 $\{ \langle 0,a \rangle, \langle 0,b \rangle, \langle 0,c \rangle, \langle 1,a \rangle, \langle 1,b \rangle, \langle 1,c \rangle \}$
- The **relation state** $r(R) \subset \text{dom}(A1) \times \text{dom}(A2)$
- For example: $r(R)$ could be $\{ \langle 0,a \rangle, \langle 0,b \rangle, \langle 1,c \rangle \}$
 - this is one possible state (or “population” or “extension”) r of the relation R , defined over $A1$ and $A2$.
 - It has three 2-tuples: $\langle 0,a \rangle, \langle 0,b \rangle, \langle 1,c \rangle$



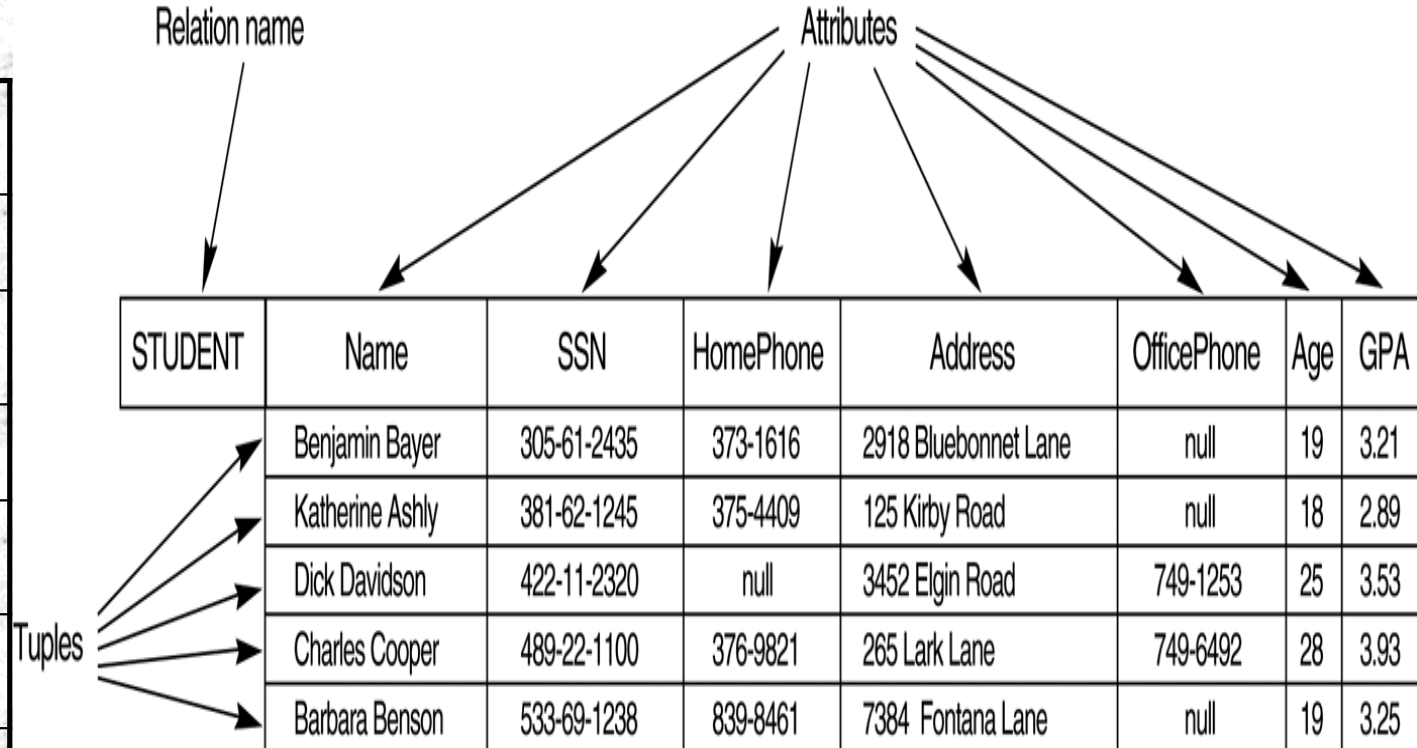
Definition Summary

Informal Terms	Formal Terms
Table	Relation
Column Header	Attribute
All Possible Column Values	Domains
Row	Tuple
Table Definition	Schema of a Relation
Populated Table	State of the Relation



Example - Figure 5.1

<u>Informal Terms</u>	<u>Formal Terms</u>
Table	Relation
Column	Attribute/ Domain
Row	Tuple
Values in a column	Domain
Table Definition	Schema of a Relation
Populated Table	Extension





Characteristics Of Relations

There are 4 Characteristics of a Relations

- 1. Ordering of tuples in a relation $r(R)$**
- 2. Ordering of attributes in a relation schema R (and of values within each tuple)**
- 3. Values and Nulls in the tuple**
- 4. Interpretation (Meaning) of a Relation - Notations**



Characteristics Of Relations

1. **Ordering of tuples in a relation $r(R)$:**

- The tuples are *not considered to be ordered*, even though they appear to be in the tabular form.

2. **Ordering of attributes in a relation schema R (and of values within each tuple):**

- We will consider the attributes in $R(A_1, A_2, \dots, A_n)$ and the values in $t = \langle v_1, v_2, \dots, v_n \rangle$ to be ordered .
 - (However, a more general alternative definition of relation does not require this ordering).



Same state as previous Figure (but with different order of tuples)

The Relation STUDENT with a different order of tuples

STUDENT

Name	Ssn	Home_phone	Address	Office_phone	Age	Gpa
Dick Davidson	422-11-2320	NULL	3452 Elgin Road	749-1253	25	3.53
Barbara Benson	533-69-1238	839-8461	7384 Fontana Lane	NULL	19	3.25
Rohan Panchal	489-22-1100	376-9821	265 Lark Lane	749-6492	28	3.93
Chung-cha Kim	381-62-1245	375-4409	125 Kirby Road	NULL	18	2.89
Benjamin Bayer	305-61-2435	373-1616	2918 Bluebonnet Lane	NULL	19	3.21



Characteristics Of Relations (Cont...)

3.Values and Nulls in the tuple:

- All values are considered atomic (indivisible).
- Each value in a tuple must be from the domain of the attribute for that column
 - If tuple $t = \langle v_1, v_2, \dots, v_n \rangle$ is a tuple (row) in the relation state r of $R(A_1, A_2, \dots, A_n)$
 - Then each v_i must be a value from $dom(A_i)$
- A special **null** value is used to represent values that are unknown or inapplicable to certain tuples.



Characteristics Of Relations (Cont...)

4. Interpretation (Meaning) of a Relation - Notations:

- We refer to **component values** of a tuple t by:
 - $t[A_i]$ or $t.A_i$
 - This is the value v_i of attribute A_i for tuple t
- Similarly,
- $t[A_u, A_v, \dots, A_w]$ refers to the subtuple of t containing the values of attributes A_u, A_v, \dots, A_w , respectively in t
- An Alternative interpretation of a relations schema is as a predicate



Relational Integrity Constraints

- Constraints are **conditions** that must hold on **all** valid relation states.
- There are three *main types* of constraints in the relational model:
 - **Key** constraints
 - **Entity integrity** constraints
 - **Referential integrity** constraints
- Another implicit constraint is the **domain** constraint
 - Every value in a tuple must be from the *domain of its attribute* (or it could be **null**, if allowed for that attribute)



Key Constraints

- **Superkey of R:**
 - Is a set of attributes SK of R with the following condition:
 - No two tuples in any valid relation state $r(R)$ will have the same value for SK
 - That is, for any distinct tuples t_1 and t_2 in $r(R)$, $t_1[SK] \neq t_2[SK]$
 - This condition must hold in *any valid state* $r(R)$
- **Key of R:**
 - A "minimal" superkey
 - That is, a key is a superkey K such that removal of any attribute from K results in a set of attributes that is not a superkey (does not possess the superkey uniqueness property)

Key Constraints (continued)

- For Example: Consider the CAR relation schema:
 - CAR(State, Reg#, SerialNo, Make, Model, Year)
 - CAR has two keys:
 - Key1 = {State, Reg#}
 - Key2 = {SerialNo}
 - Both are also superkeys of CAR
 - {SerialNo, Make} is a superkey but *not* a key.
- In general:
 - Any *key* is a *superkey* (but not vice versa)
 - Any set of attributes that *includes a key* is a *superkey*
 - A *minimal* superkey is also a key



Key Constraints (continued)

- If a relation has several **candidate keys**, one is chosen arbitrarily to be the **primary key**.
 - The primary key attributes are underlined.
- Example: Consider the CAR relation schema:
 - CAR(State, Reg#, SerialNo, Make, Model, Year)
 - We choose SerialNo as the primary key
- The primary key value is used to *uniquely identify* each tuple in a relation
 - Provides the tuple identity
- Also used to *reference* the tuple from another tuple
 - General rule: Choose as primary key the smallest of the candidate keys (in terms of size)
 - Not always applicable – choice is sometimes subjective



CAR table with two candidate keys – LicenseNumber chosen as Primary Key

CAR

<u>License_number</u>	Engine_serial_number	Make	Model	Year
Texas ABC-739	A69352	Ford	Mustang	02
Florida TVP-347	B43696	Oldsmobile	Cutlass	05
New York MPO-22	X83554	Oldsmobile	Delta	01
California 432-TFY	C43742	Mercedes	190-D	99
California RSK-629	Y82935	Toyota	Camry	04
Texas RSK-629	U028365	Jaguar	XJS	04

The CAR relation with two candidate Keys, License_Number and Engine_serial_number



Relational Database Schema

- **Relational Database Schema:**
 - A set S of relation schemas that belong to the same database.
 - S is the name of the whole **database schema**
 - $S = \{R_1, R_2, \dots, R_n\}$
- R_1, R_2, \dots, R_n are the names of the individual **relation schemas** within the database S



COMPANY Database Schema

COMPANY database schema with 6 relation schemas

EMPLOYEE

Fname	Minit	Lname	<u>Ssn</u>	Bdate	Address	Sex	Salary	Super_ssn	Dno
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DEPARTMENT

Dname	<u>Dnumber</u>	Mgr_ssn	Mgr_start_date
-------	----------------	---------	----------------

DEPT_LOCATIONS

<u>Dnumber</u>	<u>Dlocation</u>
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**Schema Diagram for the COMPANY
relational database schema**

PROJECT

Pname	<u>Pnumber</u>	Plocation	Dnum
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WORKS_ON

<u>Essn</u>	<u>Pno</u>	Hours
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DEPENDENT

<u>Essn</u>	<u>Dependent_name</u>	Sex	Bdate	Relationship
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Entity Integrity

- **Entity Integrity:**
 - The *primary key attributes* PK of each relation schema R in S cannot have null values in any tuple of $r(R)$.
 - This is because primary key values are used to *identify* the individual tuples.
 - $t[PK] \neq \text{null}$ for any tuple t in $r(R)$
 - If PK has several attributes, null is not allowed in any of these attributes
 - Note: Other attributes of R may be constrained to disallow null values, even though they are not members of the primary key.

Referential Integrity

- A constraint involving **two** relations
 - The previous constraints involve a single relation.
- Used to specify a **relationship** among tuples in two relations:
 - The **referencing relation** and the **referenced relation**.

Tuples in the **referencing relation** R1 have attributes FK (called **foreign key** attributes) that reference the primary key attributes PK of the **referenced relation** R2.

A tuple t1 in R1 is said to **reference** a tuple t2 in R2 if $t1[FK] = t2[PK]$.

A referential integrity constraint can be displayed in a relational database schema as a directed arc from R1.FK to R2.



Referential Integrity (or foreign key) Constraint

- Statement of the constraint
 - The value in the foreign key column (or columns) FK of the **referencing relation** R1 can be **either**:
 - (1) a value of an existing primary key value of a corresponding primary key PK in the **referenced relation** R2, or
 - (2) a **null**.
- In case (2), the FK in R1 should **not** be a part of its own primary key.

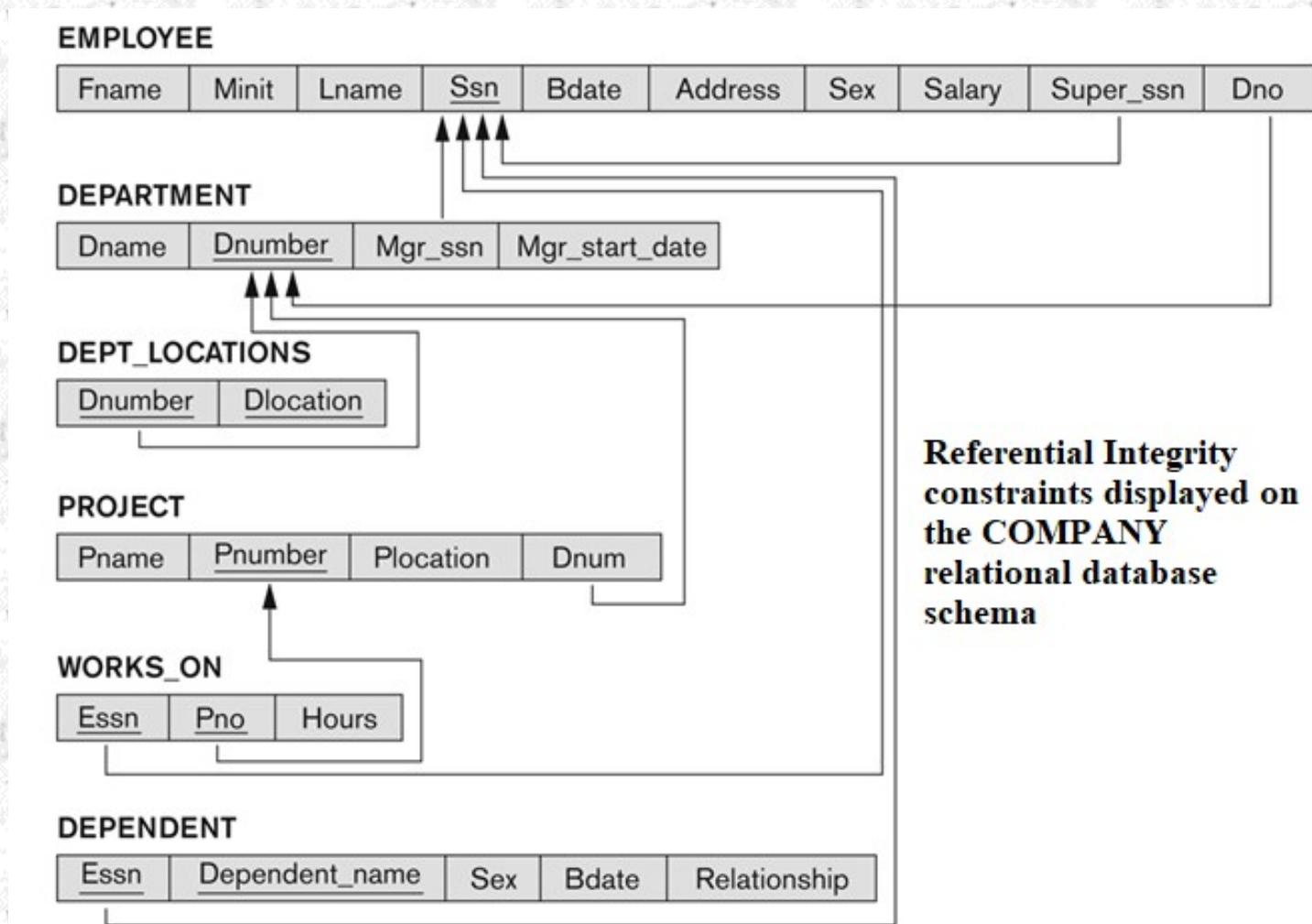


Displaying a relational database schema and its constraints

- Each relation schema can be displayed as a row of attribute names
- The name of the relation is written above the attribute names
- The primary key attribute (or attributes) will be underlined
- A foreign key (referential integrity) constraints is displayed as a directed arc (arrow) from the foreign key attributes to the referenced table
 - Can also point the the primary key of the referenced relation for clarity
- Next slide shows the COMPANY **relational schema diagram**



Referential Integrity Constraints for COMPANY database





Other Types of Constraints

- Semantic Integrity Constraints:
 - based on application semantics and cannot be expressed by the model per se
 - Example: “the max. no. of hours per employee for all projects he or she works on is 56 hrs per week”
- A **constraint specification** language may have to be used to express these
- SQL-99 allows triggers and **ASSERTIONS** to express for some of these



Populated database state

- Each *relation* will have many tuples in its current relation state
- The *relational database state* is a union of all the individual relation states
- Whenever the database is changed, a new state arises
- Basic operations for changing the database:
 - INSERT a new tuple in a relation
 - DELETE an existing tuple from a relation
 - MODIFY an attribute of an existing tuple
- Next slide shows an example state for the COMPANY database



Populated database state for COMPANY

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1986-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

DEPT_LOCATIONS

Dnumber	Dlocation
1	Houston
4	Stafford
5	Bellaire
5	Sugarland
5	Houston

WORKS_ON

Essn	Pno	Hours
123456789	1	32.5
123456789	2	7.5
666884444	3	40.0
453453453	1	20.0
453453453	2	20.0
333445555	2	10.0
333445555	3	10.0
333445555	10	10.0
333445555	20	10.0
999887777	30	30.0
999887777	10	10.0
987987987	10	35.0
987987987	30	5.0
987654321	30	20.0
987654321	20	15.0
888665555	20	NULL

PROJECT

Pname	Pnumber	Plocation	Dnum
ProductX	1	Bellaire	5
ProductY	2	Sugarland	5
ProductZ	3	Houston	5
Computerization	10	Stafford	4
Reorganization	20	Houston	1
Newbenefits	30	Stafford	4

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1988-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse

One possible state for the COMPANY relational database schema



Update Operations on Relations

- INSERT a tuple.
- DELETE a tuple.
- MODIFY a tuple.
- Integrity constraints should not be violated by the update operations.
- Several update operations may have to be grouped together.
- Updates may **propagate** to cause other updates automatically. This may be necessary to maintain integrity constraints.



Update Operations on Relations

- In case of integrity violation, several actions can be taken:
 - Cancel the operation that causes the violation (RESTRICT or REJECT option)
 - Perform the operation but inform the user of the violation
 - Trigger additional updates so the violation is corrected (CASCADE option, SET NULL option)
 - Execute a user-specified error-correction routine.

Possible violations for each operation

- INSERT may violate any of the constraints:
 - **Domain constraint:**
 - if one of the attribute values provided for the new tuple is not of the specified attribute domain
 - **Key constraint:**
 - if the value of a key attribute in the new tuple already exists in another tuple in the relation
 - **Referential integrity:**
 - if a foreign key value in the new tuple references a primary key value that does not exist in the referenced relation
 - **Entity integrity:**
 - if the primary key value is null in the new tuple



1. Insert <'Cecilia', 'F', 'Kolonsky', null, '1960-04-05', '6357 Windy Lane, Katy, TX', F, 28000, null, 4> into EMPLOYEE.
 - This insertion violates the entity integrity constraint (null for the primary key SSN), so it is rejected.
2. Insert <'Alicia', 'J', 'Zelaya', '999887777', '1960-04-05', '6357 Windy Lane, Katy, TX', F, 28000, '987654321', 4> into EMPLOYEE.
 - This insertion violates the key constraint because another tuple with the same SSN value already exists in the EMPLOYEE relation, and so it is rejected.
3. Insert <'Cecilia', 'F', 'Kolonsky', '677678989', '1960-04-05', '6357 Windswept, Katy, TX', F, 28000, '987654321', 7> into EMPLOYEE.
 - This insertion violates the referential integrity constraint specified on DNO because no DEPARTMENT tuple exists with DNUMBER = 7.
4. Insert <'Cecilia', 'F', 'Kolonsky', '677678989', '1960-04-05', '6357 Windy Lane, Katy, TX', F, 28000, null, 4> into EMPLOYEE.
 - This insertion satisfies all constraints, so it is acceptable.



Possible violations for each operation

- DELETE may violate only referential integrity:
 - If the primary key value of the tuple being deleted is referenced from other tuples in the database
 - Can be remedied by several actions: RESTRICT, CASCADE, SET NULL (see Chapter 8 for more details)
 - RESTRICT option: reject the deletion
 - CASCADE option: propagate the new primary key value into the foreign keys of the referencing tuples
 - SET NULL option: set the foreign keys of the referencing tuples to NULL
 - One of the above options must be specified during database design for each foreign key constraint



1. Delete the WORKS_ON tuple with ESSN = '999887777' and PNO = 10.
 - This deletion is acceptable.
2. Delete the EMPLOYEE tuple with SSN = '999887777'.
 - This deletion is not acceptable, because tuples in WORKS_ON refer to this tuple. Hence, if the tuple is deleted, referential integrity violations will result.
3. Delete the EMPLOYEE tuple with SSN = '333445555'.
 - This deletion will result in even worse referential integrity violations, because the tuple involved is referenced by tuples from the EMPLOYEE, DEPARTMENT, WORKS_ON, and DEPENDENT relations.



Possible violations for each operation

- UPDATE may violate domain constraint and NOT NULL constraint on an attribute being modified
- Any of the other constraints may also be violated, depending on the attribute being updated:
 - Updating the primary key (PK):
 - Similar to a DELETE followed by an INSERT
 - Need to specify similar options to DELETE
 - Updating a foreign key (FK):
 - May violate referential integrity
 - Updating an ordinary attribute (neither PK nor FK):
 - Can only violate domain constraints

Update Operation

The **Update** operation is used to change the values of one or more attributes in a tuple (or tuples) of some relation R. It is necessary to specify a condition on the attributes of the relation to select the tuple (or tuples) to be modified. Here are some examples.

1. Update the SALARY of the EMPLOYEE tuple with SSN = '999887777' to 28000.
 - Acceptable.
2. Update the DNO of the EMPLOYEE tuple with SSN = '999887777' to 1.
 - Acceptable.
3. Update the DNO of the EMPLOYEE tuple with SSN = '999887777' to 7.
 - Unacceptable, because it violates referential integrity.
4. Update the SSN of the EMPLOYEE tuple with SSN = '999887777' to '987654321'.
 - Unacceptable, because it violates primary key and referential integrity constraints.



Transaction concepts

- A database application program running against a relational data base typically runs a series of transactions.



Relational Algebra Overview

- Relational algebra is the basic set of operations for the relational model
- These operations enable a user to specify **basic retrieval requests** (or **queries**)
- The result of an operation is a *new relation*, which may have been formed from one or more *input* relations
 - This property makes the algebra “closed” (all objects in relational algebra are relations)

Relational Algebra Overview (continued)

- The **algebra operations** thus produce new relations
 - These can be further manipulated using operations of the same algebra
- A sequence of relational algebra operations forms a **relational algebra expression**
 - The result of a relational algebra expression is also a relation that represents the result of a database query (or retrieval request)

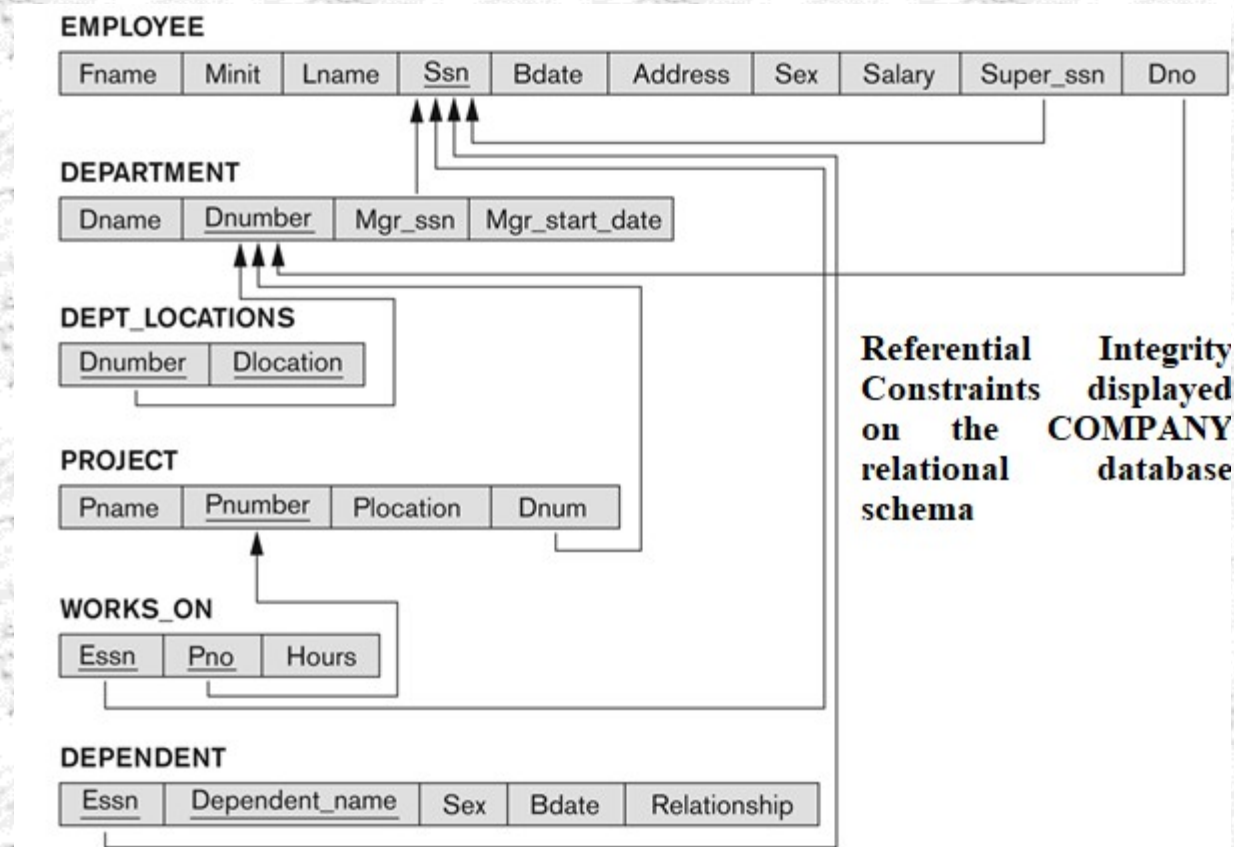


Relational Algebra Overview

- Relational Algebra consists of several groups of operations
 - **Unary Relational Operations**
 - SELECT (symbol: σ (sigma))
 - PROJECT (symbol: π (pi))
 - RENAME (symbol: ρ (rho))
 - **Relational Algebra Operations From Set Theory**
 - UNION (\cup), INTERSECTION (\cap), DIFFERENCE (or MINUS, $-$)
 - CARTESIAN PRODUCT (\times)
 - **Binary Relational Operations**
 - JOIN (several variations of JOIN exist)
 - DIVISION
 - **Additional Relational Operations**
 - OUTER JOINS, OUTER UNION
 - AGGREGATE FUNCTIONS (These compute summary of information: for example, SUM, COUNT, AVG, MIN, MAX)

Database State for COMPANY

- All examples discussed below refer to the COMPANY database shown here.





Unary Relational Operations: SELECT

- The SELECT operation (denoted by σ (sigma)) is used to select a *subset* of the tuples from a relation based on a **selection condition**.
 - The selection condition acts as a **filter**
 - Keeps only those tuples that satisfy the qualifying condition
 - Tuples satisfying the condition are *selected* whereas the other tuples are discarded (*filtered out*)
- Examples:
 - Select the EMPLOYEE tuples whose department number is 4:

$$\sigma_{DNO = 4} (EMPLOYEE)$$

- Select the employee tuples whose salary is greater than \$30,000:

$$\sigma_{SALARY > 30,000} (EMPLOYEE)$$

Select Operation – Example

- Relation r

A	B	C	D
α	α	1	7
α	β	5	7
β	β	12	3
β	β	23	10

- $\sigma_{A=B \wedge D > 5}(r)$

A	B	C	D
α	α	1	7
β	β	23	10



Unary Relational Operations: SELECT

- In general, the *select* operation is denoted by $\sigma_{\langle \text{selection condition} \rangle}(R)$ where
 - the symbol σ (sigma) is used to denote the *select* operator
 - the selection condition is a Boolean (conditional) expression specified on the attributes of relation R
 - tuples that make the condition **true** are selected
 - appear in the result of the operation
 - tuples that make the condition **false** are filtered out
 - discarded from the result of the operation

Unary Relational Operations: SELECT (contd.)

- SELECT Operation Properties

- The SELECT operation $\sigma_{\langle \text{selection condition} \rangle}(R)$ produces a relation S that has the same schema (same attributes) as R
- SELECT σ is commutative:
 - $\sigma_{\langle \text{condition1} \rangle}(\sigma_{\langle \text{condition2} \rangle}(R)) = \sigma_{\langle \text{condition2} \rangle}(\sigma_{\langle \text{condition1} \rangle}(R))$
- Because of commutativity property, a cascade (sequence) of SELECT operations may be applied in any order:
 - $\sigma_{\langle \text{cond1} \rangle}(\sigma_{\langle \text{cond2} \rangle}(\sigma_{\langle \text{cond3} \rangle}(R))) = \sigma_{\langle \text{cond2} \rangle}(\sigma_{\langle \text{cond3} \rangle}(\sigma_{\langle \text{cond1} \rangle}(R)))$
- A cascade of SELECT operations may be replaced by a single selection with a conjunction of all the conditions:
 - $\sigma_{\langle \text{cond1} \rangle}(\sigma_{\langle \text{cond2} \rangle}(\sigma_{\langle \text{cond3} \rangle}(R))) = \sigma_{\langle \text{cond1} \rangle \text{ AND } \langle \text{cond2} \rangle \text{ AND } \langle \text{cond3} \rangle}(R))$
- The number of tuples in the result of a SELECT is less than (or equal to) the number of tuples in the input relation R



The following query results refer to this database state

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

DEPT_LOCATIONS

Dnumber	Dlocation
1	Houston
4	Stafford
5	Bellaire
5	Sugarland
5	Houston

WORKS_ON

Essn	Pno	Hours
123456789	1	32.5
123456789	2	7.5
666884444	3	40.0
453453453	1	20.0
453453453	2	20.0
333445555	2	10.0
333445555	3	10.0
333445555	10	10.0
333445555	20	10.0
999887777	30	30.0
999887777	10	10.0
987987987	10	35.0
987987987	30	5.0
987654321	30	20.0
987654321	20	15.0
888665555	20	NULL

PROJECT

Pname	Pnumber	Plocation	Dnum
ProductX	1	Bellaire	5
ProductY	2	Sugarland	5
ProductZ	3	Houston	5
Computerization	10	Stafford	4
Reorganization	20	Houston	1
Newbenefits	30	Stafford	4

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1988-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse

One Possible database state for the COMPANY relational database schema



Unary Relational Operations: PROJECT

- PROJECT Operation is denoted by π (pi)
- This operation keeps certain *columns* (attributes) from a relation and discards the other columns.
 - PROJECT creates a vertical partitioning
 - The list of specified columns (attributes) is kept in each tuple
 - The other attributes in each tuple are discarded
- Example: To list each employee's first and last name and salary, the following is used:

$\pi_{\text{LNAME, FNAME, SALARY}}(\text{EMPLOYEE})$

Project Operation – Example

- Relation r :

A	B	C
α	10	1
α	20	1
β	30	1
β	40	2

- $\Pi_{A,C}(r)$

A	C
α	1
α	1
β	1
β	2

 $=$

A	C
α	1
β	1
β	2

Unary Relational Operations: PROJECT (cont.)

- The general form of the *project* operation is:

$$\pi_{\langle \text{attribute list} \rangle}(\mathbf{R})$$

- π (pi) is the symbol used to represent the *project* operation
- $\langle \text{attribute list} \rangle$ is the desired list of attributes from relation R.
- The project operation *removes any duplicate tuples*
 - This is because the result of the *project* operation must be a *set of tuples*
 - Mathematical sets *do not allow* duplicate elements.

Unary Relational Operations: PROJECT (contd.)

- PROJECT Operation Properties
 - The number of tuples in the result of projection $\pi_{\langle \text{list} \rangle}(R)$ is always less or equal to the number of tuples in R
 - If the list of attributes includes a *key* of R, then the number of tuples in the result of PROJECT is *equal* to the number of tuples in R
 - PROJECT is *not* commutative
 - $\pi_{\langle \text{list1} \rangle}(\pi_{\langle \text{list2} \rangle}(R)) = \pi_{\langle \text{list1} \rangle}(R)$ as long as $\langle \text{list2} \rangle$ contains the attributes in $\langle \text{list1} \rangle$



Examples of applying SELECT and PROJECT operations

Results of SELECT and PROJECT operations. (a) $\sigma_{(Dno=4 \text{ AND } Salary > 25000) \text{ OR } (Dno=5 \text{ AND } Salary > 30000)}(EMPLOYEE)$.
(b) $\pi_{Lname, Fname, Salary}(EMPLOYEE)$. (c) $\pi_{Sex, Salary}(EMPLOYEE)$.

(a)

Fname	Minit	Lname	<u>Ssn</u>	Bdate	Address	Sex	Salary	Super_ssn	Dno
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5

(b)

Lname	Fname	Salary
Smith	John	30000
Wong	Franklin	40000
Zelaya	Alicia	25000
Wallace	Jennifer	43000
Narayan	Ramesh	38000
English	Joyce	25000
Jabbar	Ahmad	25000
Borg	James	55000

(c)

Sex	Salary
M	30000
M	40000
F	25000
F	43000
M	38000
M	25000
M	55000



Relational Algebra Expressions

- We may want to apply several relational algebra operations one after the other
 - Either we can write the operations as a single **relational algebra expression** by nesting the operations, or
 - We can apply one operation at a time and create **intermediate result relations**.
- In the latter case, we must give names to the relations that hold the intermediate results.



Single expression versus sequence of relational operations (Example)

- To retrieve the first name, last name, and salary of all employees who work in department number 5, we must apply a select and a project operation
- We can write a *single relational algebra expression* as follows:

- $\pi_{\text{FNAME, LNAME, SALARY}}(\sigma_{\text{DNO}=5}(\text{EMPLOYEE}))$

- OR We can explicitly show the *sequence of operations*, giving a name to each intermediate relation:

- $\text{DEP5_EMPS} \leftarrow \sigma_{\text{DNO}=5}(\text{EMPLOYEE})$

- $\text{RESULT} \leftarrow \pi_{\text{FNAME, LNAME, SALARY}}(\text{DEP5_EMPS})$



Unary Relational Operations: RENAME

- The RENAME operator is denoted by ρ (rho)
- In some cases, we may want to *rename* the attributes of a relation or the relation name or both
 - Useful when a query requires multiple operations
 - Necessary in some cases (see JOIN operation later)



Unary Relational Operations: RENAME (contd.)

- The general RENAME operation ρ can be expressed by any of the following forms:
 - $\rho_S(B_1, B_2, \dots, B_n)(R)$ changes both:
 - the relation name to S , *and*
 - the column (attribute) names to B_1, B_1, \dots, B_n
 - $\rho_S(R)$ changes:
 - the *relation name* only to S
 - $\rho_{(B_1, B_2, \dots, B_n)}(R)$ changes:
 - the *column (attribute) names* only to B_1, B_1, \dots, B_n

Unary Relational Operations: RENAME (contd.)

- For convenience, we also use a *shorthand* for renaming attributes in an intermediate relation:
 - If we write:
 - $\text{RESULT} \leftarrow \pi_{\text{FNAME, LNAME, SALARY}}(\text{DEP5_EMPS})$
 - RESULT will have the *same attribute names* as DEP5_EMPS (same attributes as EMPLOYEE)
 - If we write:
 - $\text{RESULT (F, M, L, S, B, A, SX, SAL, SU, DNO)} \leftarrow \pi_{\text{FNAME, LNAME, SALARY}}(\text{DEP5_EMPS})$
 - The 10 attributes of DEP5_EMPS are *renamed* to F, M, L, S, B, A, SX, SAL, SU, DNO, respectively



Example of applying multiple operations and RENAME

(a)

Fname	Lname	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

(b)

TEMP

Fname	Minit	Lname	<u>Ssn</u>	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston,TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston,TX	M	40000	888665555	5
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble,TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

R

First_name	Last_name	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

Results of a sequence of operations.

(a) $\pi_{Fname, Lname, Salary}(\sigma_{Dno=5}(EMPLOYEE))$.

(b) Using intermediate relations and renaming of attributes.



Relational Algebra Operations from Set Theory: UNION

- **UNION Operation**

- Binary operation, denoted by \cup
- The result of $R \cup S$, is a relation that includes all tuples that are either in R or in S or in both R and S
- Duplicate tuples are eliminated
- The two operand relations R and S must be “type compatible” (or UNION compatible)
 - R and S must have same number of attributes
 - Each pair of corresponding attributes must be type compatible (have same or compatible domains)

Union Operation – Example

- Relations r, s :

A	B
-----	-----

α	1
α	2
β	1

r

A	B
-----	-----

α	2
β	3

s

$r \cup s$:

A	B
-----	-----

α	1
α	2
β	1
β	3

Relational Algebra Operations from Set Theory: UNION

- Example:

- To retrieve the social security numbers of all employees who either *work in department 5* (RESULT1 below) or *directly supervise an employee who works in department 5* (RESULT2 below)
- We can use the UNION operation as follows:

$$\text{DEP5_EMPS} \leftarrow \sigma_{\text{DNO}=5} (\text{EMPLOYEE})$$
$$\text{RESULT1} \leftarrow \pi_{\text{SSN}} (\text{DEP5_EMPS})$$
$$\text{RESULT2}(\text{SSN}) \leftarrow \pi_{\text{SUPERSSN}} (\text{DEP5_EMPS})$$
$$\text{RESULT} \leftarrow \text{RESULT1} \cup \text{RESULT2}$$

- The union operation produces the tuples that are in either RESULT1 or RESULT2 or both



Example of the result of a UNION operation

- UNION Example

Result of the
UNION operation
 $\text{RESULT} \leftarrow \text{RESULT1} \cup \text{RESULT2}$.

RESULT1

Ssn
123456789
333445555
666884444
453453453

RESULT2

Ssn
333445555
888665555

RESULT

Ssn
123456789
333445555
666884444
453453453
888665555



Relational Algebra Operations from Set Theory

- Type Compatibility of operands is required for the binary set operation UNION \cup , (also for INTERSECTION \cap , and SET DIFFERENCE $-$, see next slides)
- $R1(A1, A2, \dots, An)$ and $R2(B1, B2, \dots, Bn)$ are type compatible if:
 - they have the same number of attributes, and
 - the domains of corresponding attributes are type compatible (i.e. $\text{dom}(Ai) = \text{dom}(Bi)$ for $i=1, 2, \dots, n$).
- The resulting relation for $R1 \cup R2$ (also for $R1 \cap R2$, or $R1 - R2$, see next slides) has the same attribute names as the *first* operand relation $R1$ (by convention)



Relational Algebra Operations from Set Theory: INTERSECTION

- INTERSECTION is denoted by \cap
- The result of the operation $R \cap S$, is a relation that includes all tuples that are in both R and S
 - The attribute names in the result will be the same as the attribute names in R
- The two-operand relations R and S must be “type compatible”



Set-Intersection Operation - Example

- Relation r , s :

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- $r \cap s$

A	B
α	2



Relational Algebra Operations from Set Theory: SET DIFFERENCE (cont.)

- SET DIFFERENCE (also called MINUS or EXCEPT) is denoted by –
- The result of $R - S$, is a relation that includes all tuples that are in R but not in S
 - The attribute names in the result will be the same as the attribute names in R
- The two operand relations R and S must be “type compatible”



Set Difference Operation (MINUS)– Example

- Relations r , s :

A	B
-----	-----

α	1
α	2
β	1

r

A	B
-----	-----

α	2
β	3

s

$r - s$:

A	B
-----	-----

α	1
β	1



Example to illustrate the result of UNION, INTERSECT, and DIFFERENCE

(a) STUDENT

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

INSTRUCTOR

Fname	Lname
John	Smith
Ricardo	Browne
Susan	Yao
Francis	Johnson
Ramesh	Shah

(b)

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert
John	Smith
Ricardo	Browne
Francis	Johnson

(c)

Fn	Ln
Susan	Yao
Ramesh	Shah

(d)

Fn	Ln
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

(e)

Fname	Lname
John	Smith
Ricardo	Browne
Francis	Johnson

The set operations UNION, INTERSECTION, and MINUS. (a) Two union-compatible relations. (b) $\text{STUDENT} \cup \text{INSTRUCTOR}$. (c) $\text{STUDENT} \cap \text{INSTRUCTOR}$. (d) $\text{STUDENT} - \text{INSTRUCTOR}$. (e) $\text{INSTRUCTOR} - \text{STUDENT}$.



Some properties of UNION, INTERSECT, and DIFFERENCE

- Notice that both union and intersection are *commutative* operations; that is
 - $R \cup S = S \cup R$, and $R \cap S = S \cap R$
- Both union and intersection can be treated as n-ary operations applicable to any number of relations as both are *associative* operations; that is
 - $R \cup (S \cup T) = (R \cup S) \cup T$
 - $(R \cap S) \cap T = R \cap (S \cap T)$
- The minus operation is not commutative; that is, in general
 - $R - S \neq S - R$



Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT

- CARTESIAN (or CROSS) PRODUCT Operation
 - This operation is used to combine tuples from two relations in a combinatorial fashion.
 - Denoted by $R(A_1, A_2, \dots, A_n) \times S(B_1, B_2, \dots, B_m)$
 - Result is a relation Q with degree $n + m$ attributes:
 - $Q(A_1, A_2, \dots, A_n, B_1, B_2, \dots, B_m)$, in that order.
 - The resulting relation state has one tuple for each combination of tuples—one from R and one from S .
 - Hence, if R has n_R tuples (denoted as $|R| = n_R$), and S has n_S tuples, then $R \times S$ will have $n_R * n_S$ tuples.
 - The two operands do NOT have to be "type compatible"



Cartesian-Product Operation (cross production)-Example

Relations r, s :

A	B
-----	-----

α	1
β	2

r

C	D	E
-----	-----	-----

α	10	a
β	10	a
β	20	b
γ	10	b

s

$r \times s$:

A	B	C	D	E
α	1	α	10	a
α	1	β	10	a
α	1	β	20	b
α	1	γ	10	b
β	2	α	10	a
β	2	β	10	a
β	2	β	20	b
β	2	γ	10	b

Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT (cont.)

- Generally, CROSS PRODUCT is not a meaningful operation
 - Can become meaningful when followed by other operations
- Example (not meaningful):
 - $\text{FEMALE_EMPS} \leftarrow \sigma_{\text{SEX}='F'}(\text{EMPLOYEE})$
 - $\text{EMP_NAMES} \leftarrow \pi_{\text{FNAME, LNAME, SSN}}(\text{FEMALE_EMPS})$
 - $\text{EMP_DEPENDENTS} \leftarrow \text{EMP_NAMES} \times \text{DEPENDENT}$
- EMP_DEPENDENTS will contain every combination of EMP_NAMES and DEPENDENT
 - whether or not they are actually related



Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT (cont.)

- To keep only combinations where the DEPENDENT is related to the EMPLOYEE, we add a SELECT operation as follows
- Example (meaningful):
 - $\text{FEMALE_EMPS} \leftarrow \sigma_{\text{SEX}='F'}(\text{EMPLOYEE})$
 - $\text{EMP_NAMES} \leftarrow \pi_{\text{FNAME, LNAME, SSN}}(\text{FEMALE_EMPS})$
 - $\text{EMP_DEPENDENTS} \leftarrow \text{EMP_NAMES} \times \text{DEPENDENT}$
 - $\text{ACTUAL_DEPS} \leftarrow \sigma_{\text{SSN}=\text{ESSN}}(\text{EMP_DEPENDENTS})$
 - $\text{RESULT} \leftarrow \pi_{\text{FNAME, LNAME, DEPENDENT_NAME}}(\text{ACTUAL_DEPS})$
- RESULT will now contain the name of female employees and their dependents



Example of applying CARTESIAN PRODUCT

FEMALE_EMPS

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
Alicia	J	Zelaya	999887777	1988-07-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

EMPNames

Fname	Lname	Ssn
Alicia	Zelaya	999887777
Jennifer	Wallace	987654321
Joyce	English	453453453

The CARTESIAN PRODUCT (CROSS PRODUCT) Operations

EMP_DEPENDENTS

Fname	Lname	Ssn	Esn	Dependent_name	Sex	Bdate	...
Alicia	Zelaya	999887777	333445555	Alice	F	1988-04-05	...
Alicia	Zelaya	999887777	333445555	Theodore	M	1983-10-25	...
Alicia	Zelaya	999887777	333445555	Joy	F	1958-05-03	...
Alicia	Zelaya	999887777	987654321	Abner	M	1942-02-28	...
Alicia	Zelaya	999887777	123456789	Michael	M	1988-01-04	...
Alicia	Zelaya	999887777	123456789	Alice	F	1988-12-30	...
Alicia	Zelaya	999887777	123456789	Elizabeth	F	1967-05-05	...
Jennifer	Wallace	987654321	333445555	Alice	F	1988-04-05	...
Jennifer	Wallace	987654321	333445555	Theodore	M	1983-10-25	...
Jennifer	Wallace	987654321	333445555	Joy	F	1958-05-03	...
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	...
Jennifer	Wallace	987654321	123456789	Michael	M	1988-01-04	...
Jennifer	Wallace	987654321	123456789	Alice	F	1988-12-30	...
Jennifer	Wallace	987654321	123456789	Elizabeth	F	1967-05-05	...
Joyce	English	453453453	333445555	Alice	F	1988-04-05	...
Joyce	English	453453453	333445555	Theodore	M	1983-10-25	...
Joyce	English	453453453	333445555	Joy	F	1958-05-03	...
Joyce	English	453453453	987654321	Abner	M	1942-02-28	...
Joyce	English	453453453	123456789	Michael	M	1988-01-04	...
Joyce	English	453453453	123456789	Alice	F	1988-12-30	...
Joyce	English	453453453	123456789	Elizabeth	F	1967-05-05	...

ACTUAL_DEPENDENTS

Fname	Lname	Ssn	Esn	Dependent_name	Sex	Bdate	...
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	...



RESULT

Fname	Lname	Dependent_name
Jennifer	Wallace	Abner

Binary Relational Operations: JOIN

- JOIN Operation (denoted by \bowtie)
 - The sequence of CARTESIAN PRODECT followed by SELECT is used quite commonly to identify and select related tuples from two relations
 - A special operation, called JOIN combines this sequence into a single operation
 - This operation is very important for any relational database with more than a single relation, because it allows us *combine related tuples* from various relations
 - The general form of a join operation on two relations $R(A_1, A_2, \dots, A_n)$ and $S(B_1, B_2, \dots, B_m)$ is:
$$R \bowtie_{\langle \text{join condition} \rangle} S$$
 - where R and S can be any relations that result from general *relational algebra expressions*.

Binary Relational Operations: JOIN (cont.)

- Example: Suppose that we want to retrieve the name of the manager of each department.
 - To get the manager's name, we need to combine each DEPARTMENT tuple with the EMPLOYEE tuple whose SSN value matches the MGRSSN value in the department tuple.
 - We do this by using the join  operation.

 - $\text{DEPT_MGR} \leftarrow \text{DEPARTMENT} \underset{\text{MGRSSN=SSN}}{\text{JOIN}} \text{EMPLOYEE}$
- MGRSSN=SSN is the join condition
 - Combines each department record with the employee who manages the department
 - The join condition can also be specified as $\text{DEPARTMENT.MGRSSN} = \text{EMPLOYEE.SSN}$



Example of applying the JOIN operation

DEPT_MGR


Dname	Dnumber	Mgr_ssn	...	Fname	Minit	Lname	Ssn	...
Research	5	333445555	...	Franklin	T	Wong	333445555	...
Administration	4	987654321	...	Jennifer	S	Wallace	987654321	...
Headquarters	1	888665555	...	James	E	Borg	888665555	...

Result of the JOIN operation

Some properties of JOIN

- Consider the following JOIN operation:
 - $R(A_1, A_2, \dots, A_n) \bowtie S(B_1, B_2, \dots, B_m)$
 $R.A_i = S.B_j$
 - Result is a relation Q with degree $n + m$ attributes:
 - $Q(A_1, A_2, \dots, A_n, B_1, B_2, \dots, B_m)$, in that order.
 - The resulting relation state has one tuple for each combination of tuples— r from R and s from S , but *only if they satisfy the join condition* $r[A_i] = s[B_j]$
 - Hence, if R has n_R tuples, and S has n_S tuples, then the join result will generally have *less than* $n_R * n_S$ tuples.
 - Only related tuples (based on the join condition) will appear in the result

Some Properties of JOIN

- The general case of JOIN operation is called a Theta-join: R  S

theta

- The join condition is called *theta*
- *Theta* can be any general boolean expression on the attributes of R and S; for example:
 - $R.A_i < S.B_j$ AND $(R.A_k = S.B_l \text{ OR } R.A_p < S.B_q)$
- Most join conditions involve one or more equality conditions “AND”ed together; for example:
 - $R.A_i = S.B_j$ AND $R.A_k = S.B_l$ AND $R.A_p = S.B_q$



Binary Relational Operations: EQUIJOIN

- EQUIJOIN Operation
- The most common use of join involves join conditions with *equality comparisons* only
- Such a join, where the only comparison operator used is =, is called an EQUIJOIN.
 - In the result of an EQUIJOIN we always have one or more pairs of attributes (whose names need not be identical) that have identical values in every tuple.
 - The JOIN seen in the previous example was an EQUIJOIN.



Binary Relational Operations: NATURAL JOIN Operation

- NATURAL JOIN Operation
 - Another variation of JOIN called NATURAL JOIN — denoted by $*$ — was created to get rid of the second (superfluous) attribute in an EQUIJOIN condition.
 - because one of each pair of attributes with identical values is superfluous
 - The standard definition of natural join requires that the two join attributes, or each pair of corresponding join attributes, *have the same name* in both relations
 - If this is not the case, a renaming operation is applied first.



Natural Join Operation –

• Example

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	ϵ

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ



Binary Relational Operations NATURAL JOIN (contd.)

- Example: To apply a natural join on the DNUMBER attributes of DEPARTMENT and DEPT_LOCATIONS, it is sufficient to write:
 - $DEPT_LOCS \leftarrow DEPARTMENT * DEPT_LOCATIONS$
- Only attribute with the same name is DNUMBER
- An implicit join condition is created based on this attribute:
 $DEPARTMENT.DNUMBER = DEPT_LOCATIONS.DNUMBER$
- Another example: $Q \leftarrow R(A,B,C,D) * S(C,D,E)$
 - The implicit join condition includes *each pair* of attributes with the same name, “AND”ed together:
 - $R.C = S.C \text{ AND } R.D = S.D$
 - Result keeps only one attribute of each such pair:
 - $Q(A,B,C,D,E)$



Example of NATURAL JOIN operation

(a)

PROJ_DEPT

Pname	<u>Pnumber</u>	Plocation	Dnum	Dname	Mgr_ssn	Mgr_start_date
ProductX	1	Bellaire	5	Research	333445555	1988-05-22
ProductY	2	Sugarland	5	Research	333445555	1988-05-22
ProductZ	3	Houston	5	Research	333445555	1988-05-22
Computerization	10	Stafford	4	Administration	987654321	1995-01-01
Reorganization	20	Houston	1	Headquarters	888665555	1981-06-19
Newbenefits	30	Stafford	4	Administration	987654321	1995-01-01

(b)

DEPT_LOCS

Dname	Dnumber	Mgr_ssn	Mgr_start_date	Location
Headquarters	1	888665555	1981-06-19	Houston
Administration	4	987654321	1995-01-01	Stafford
Research	5	333445555	1988-05-22	Bellaire
Research	5	333445555	1988-05-22	Sugarland
Research	5	333445555	1988-05-22	Houston

Results of two NATURAL JOIN operations.

(a) PROJ_DEPT \leftarrow PROJECT * DEPT.

(b) DEPT_LOCS \leftarrow DEPARTMENT * DEPT_LOCATIONS.

Complete Set of Relational Operations

- The set of operations including SELECT σ , PROJECT π , UNION \cup , DIFFERENCE $-$, RENAME ρ , and CARTESIAN PRODUCT \times is called a *complete set* because any other relational algebra expression can be expressed by a combination of these five operations.
- For example:
 - $R \cap S = (R \cup S) - ((R - S) \cup (S - R))$
 - $R \bowtie_{\langle \text{join condition} \rangle} S = \sigma_{\langle \text{join condition} \rangle} (R \times S)$

Binary Relational Operations: DIVISION

- DIVISION Operation
 - The division operation is applied to two relations
 - $R(Z) \div S(X)$, where X subset Z . Let $Y = Z - X$ (and hence $Z = X \cup Y$); that is, let Y be the set of attributes of R that are not attributes of S .
 - The result of DIVISION is a relation $T(Y)$ that includes a tuple t if tuples t_R appear in R with $t_R[Y] = t$, and with \bowtie
 - $t_R[X] = t_s$ for every tuple t_s in S .
 - For a tuple t to appear in the result T of the DIVISION, the values in t must appear in R in combination with every tuple in S .



Division Operation – Example

Relations r, s :

A	B
-----	-----

α	1
α	2
α	3
β	1
γ	1
δ	1
δ	3
δ	4
\in	6
\in	1
β	2

r

B

1
2

s

$r \div s$:

A

α
β



Example of DIVISION

(a)

SSN_PNOS

Essn	Pno
123456789	1
123456789	2
666884444	3
453453453	1
453453453	2
333445555	2
333445555	3
333445555	10
333445555	20
999887777	30
999887777	10
987987987	10
987987987	30
987654321	30
987654321	20
888665555	20

SMITH_PNOS

Pno
1
2

SSNS

Ssn
123456789
453453453

(b)

R

A	B
a1	b1
a2	b1
a3	b1
a4	b1
a1	b2
a3	b2
a2	b3
a3	b3
a4	b3
a1	b4
a2	b4
a3	b4

S

A
a1
a2
a3

T

B
b1
b4

The DIVISION operation. (a) Dividing SSN_PNOS by SMITH_PNOS. (b) $T \leftarrow R \div S$.



Recap of Relational Algebra Operations

Operations of Relational Algebra

Operation	Purpose	Notation
SELECT	Selects all tuples that satisfy the selection condition from a relation R .	$\sigma_{\langle \text{selection condition} \rangle}(R)$
PROJECT	Produces a new relation with only some of the attributes of R , and removes duplicate tuples.	$\pi_{\langle \text{attribute list} \rangle}(R)$
THETA JOIN	Produces all combinations of tuples from R_1 and R_2 that satisfy the join condition.	$R_1 \bowtie_{\langle \text{join condition} \rangle} R_2$
EQUIJOIN	Produces all the combinations of tuples from R_1 and R_2 that satisfy a join condition with only equality comparisons.	$R_1 \bowtie_{\langle \text{join condition} \rangle} R_2$ OR $R_1 \bowtie_{(\langle \text{join attributes 1} \rangle), (\langle \text{join attributes 2} \rangle)} R_2$
NATURAL JOIN	Same as EQUIJOIN except that the join attributes of R_2 are not included in the resulting relation; if the join attributes have the same names, they do not have to be specified at all.	$R_1 *_{\langle \text{join condition} \rangle} R_2$ OR $R_1 *_{(\langle \text{join attributes 1} \rangle), (\langle \text{join attributes 2} \rangle)} R_2$ OR $R_1 * R_2$
UNION	Produces a relation that includes all the tuples in R_1 or R_2 or both R_1 and R_2 ; R_1 and R_2 must be union compatible.	$R_1 \cup R_2$
INTERSECTION	Produces a relation that includes all the tuples in both R_1 and R_2 ; R_1 and R_2 must be union compatible.	$R_1 \cap R_2$
DIFFERENCE	Produces a relation that includes all the tuples in R_1 that are not in R_2 ; R_1 and R_2 must be union compatible.	$R_1 - R_2$
CARTESIAN PRODUCT	Produces a relation that has the attributes of R_1 and R_2 and includes as tuples all possible combinations of tuples from R_1 and R_2 .	$R_1 \times R_2$
DIVISION	Produces a relation $R(X)$ that includes all tuples $t[X]$ in $R_1(Z)$ that appear in R_1 in combination with every tuple from $R_2(Y)$, where $Z = X \cup Y$.	$R_1(Z) \div R_2(Y)$



Additional Relational-Algebra-Operations

- Generalized Projection
- Outer Join
- Aggregate Functions

Generalized Projection

- Extends the projection operation by allowing arithmetic functions to be used in the projection list.

$$\Pi_{F_1, F_2, \dots, F_n}(E)$$

- E is any relational-algebra expression
- Each of F_1, F_2, \dots, F_n are arithmetic expressions involving constants and attributes in the schema of E .
- Given relation *credit-info(customer-name, limit, credit-balance)*, find how much more each person can spend:

$$\Pi_{\text{customer-name, limit} - \text{credit-balance}}(\text{credit-info})$$

Aggregate Functions and Operations

- **Aggregation function** takes a collection of values and returns a single value as a result.

avg: average value

min: minimum value

max: maximum value

sum: sum of values

count: number of values

- **Aggregate operation** in relational algebra

$G_1, G_2, \dots, G_n \ \sigma_{F_1(A_1), F_2(A_2), \dots, F_n(A_n)}(E)$

- E is any relational-algebra expression
- G_1, G_2, \dots, G_n is a list of attributes on which to group (can be empty)
- Each F_i is an aggregate function
- Each A_i is an attribute name



Aggregate Operation: SUM – Example

- Relation r :

A	B	C
α	α	7
α	β	7
β	β	3
β	β	10

$g_{\text{sum}(C)}(r)$

$\text{sum-}C$
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Aggregate Operation – Example

- Relation *account* grouped by *branch-name*:

<i>branch-name</i>	<i>account-number</i>	<i>balance</i>
Perryridge	A-102	400
Perryridge	A-201	900
Brighton	A-217	750
Brighton	A-215	750
Redwood	A-222	700

branch-name \mathcal{G} *sum(balance)* (*account*)

<i>branch-name</i>	<i>balance</i>
Perryridge	1300
Brighton	1500
Redwood	700



Outer Join

- An extension of the join operation that avoids loss of information.
- Computes the join and then adds tuples from one relation that do not match tuples in the other relation to the result of the join.
- Uses *null* values:
 - *null* signifies that the value is unknown or does not exist
 - All comparisons involving *null* are (roughly speaking) **false** by definition.
 - Will study precise meaning of comparisons with nulls later

Outer Join – Example

- Relation *loan*

<i>loan-number</i>	<i>branch-name</i>	<i>amount</i>
L-170	Downtown	3000
L-230	Redwood	4000
L-260	Perryridge	1700

- Relation *borrower*

<i>customer-name</i>	<i>loan-number</i>
Jones	L-170
Smith	L-230
Hayes	L-155



Outer Join – Example

- Inner Join

loan ⋈ Borrower

<i>loan-number</i>	<i>branch-name</i>	<i>amount</i>	<i>customer-name</i>
L-170	Downtown	3000	Jones
L-230	Redwood	4000	Smith

■ Left Outer Join

loan ⋈_l Borrower

<i>loan-number</i>	<i>branch-name</i>	<i>amount</i>	<i>customer-name</i>
L-170	Downtown	3000	Jones
L-230	Redwood	4000	Smith
L-260	Perryridge	1700	<i>null</i>



Outer Join – Example

- **Right Outer Join**

loan ⋈_R *borrower*

<i>loan-number</i>	<i>branch-name</i>	<i>amount</i>	<i>customer-name</i>
L-170	Downtown	3000	Jones
L-230	Redwood	4000	Smith
L-155	<i>null</i>	<i>null</i>	Hayes

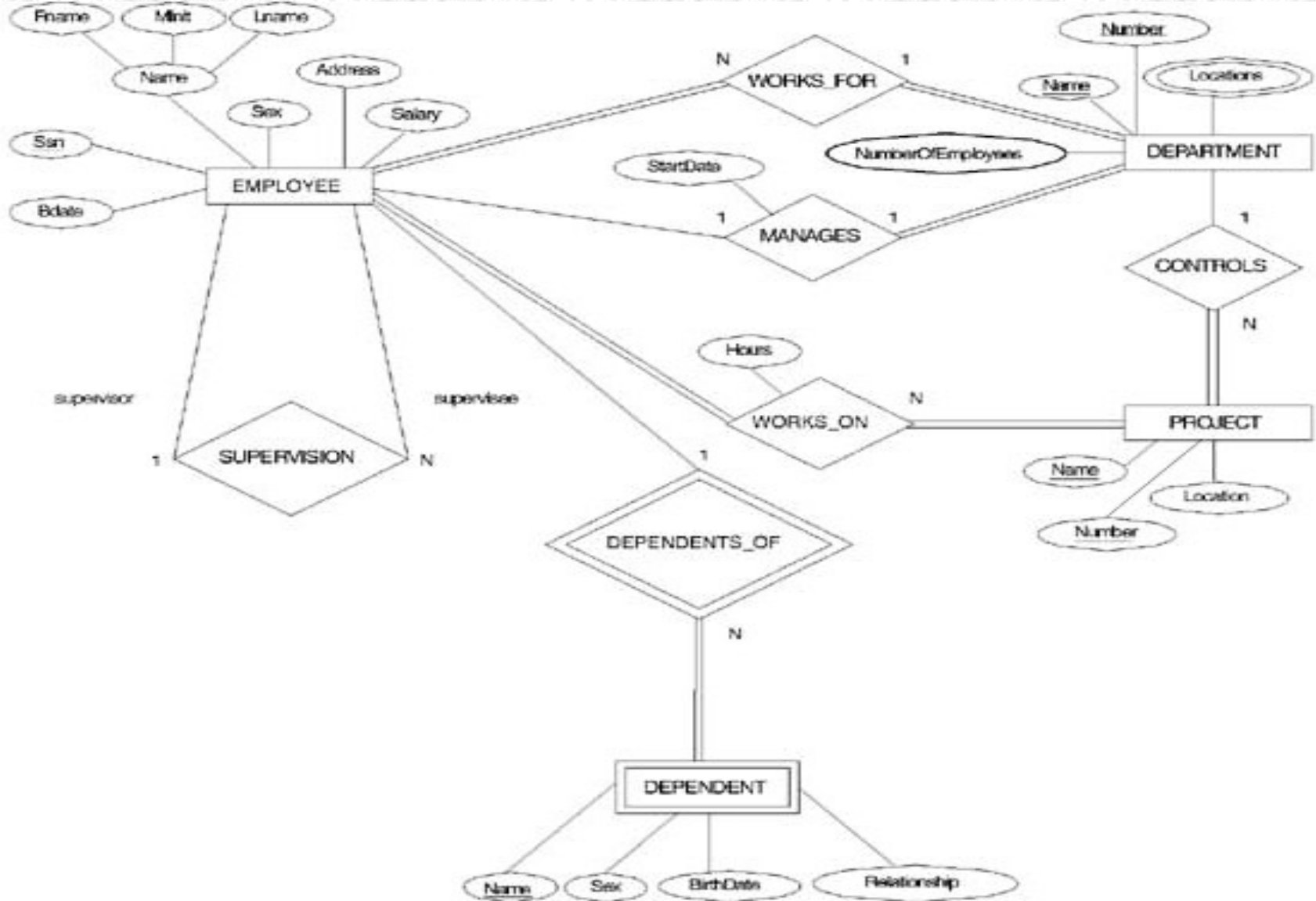
- **Full Outer Join**

loan ⋈_F *borrower*

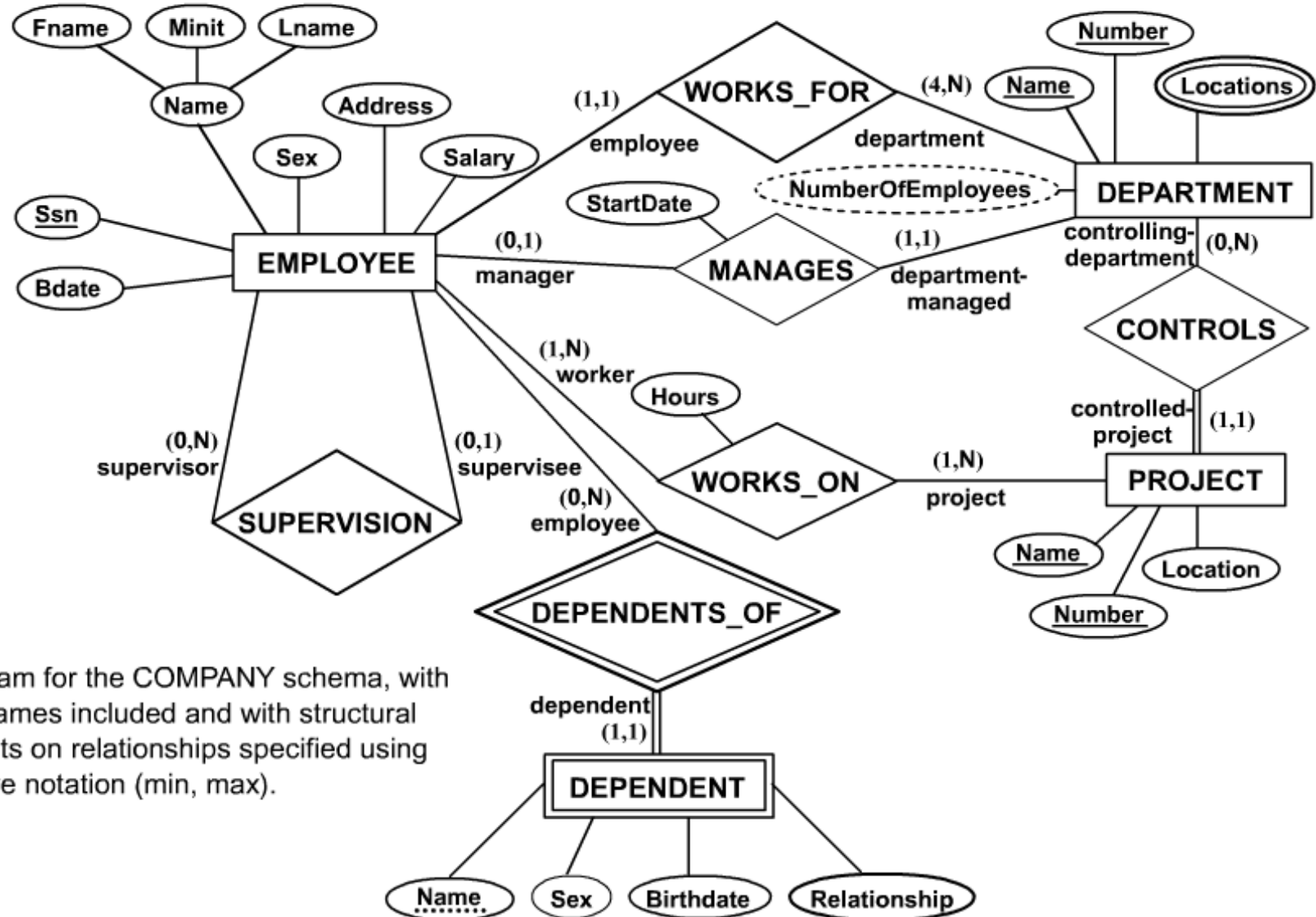
<i>loan-number</i>	<i>branch-name</i>	<i>amount</i>	<i>customer-name</i>
L-170	Downtown	3000	Jones
L-230	Redwood	4000	Smith
L-260	Perryridge	1700	<i>null</i>
L-155	<i>null</i>	<i>null</i>	Hayes



Relational DataBase Design using ER-to- Relational Mapping



Alternative ER Notations



ER diagram for the COMPANY schema, with all role names included and with structural constraints on relationships specified using alternative notation (min, max).



Relational Database Design Using ER-to-Relational Mapping

- The steps for algorithm for ER-to-Relational Mapping
- Step1: Mapping of Regular Entity types.
- Step2: Mapping of Weak Entity types.
- Step3: Mapping of Binary 1:1 Relational ship types
 - i) Foreign Key approach
 - ii) Merged relation approach
 - iii) Cross-reference or Relationship approach



- Step 4: Mapping of Binary 1:n relationship types
- Step 5: Mapping of Binary M:N relationship types
- Step 6: Mapping Multivalued Attributes
- Step 7: Mapping of N-ary Relationship types



Result of mapping the company schema into relational Database schema

Employee

Fname	Minit	Lname	<u>SSn</u>	Bdate	Addre ss	sex	salary	Super _ssn	Dno
-------	-------	-------	------------	-------	-------------	-----	--------	---------------	-----

Department

Dname	<u>Dnumber</u>	Mgr_ssn	Mgr_start_date
-------	----------------	---------	----------------

DEPT_LOCATIONS

<u>Dnumber</u>	<u>Dlocatin</u>
----------------	-----------------



Project

Pname	<u>Pnumber</u>	<u>Plocation</u>	Dnum
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Works_ON

<u>ESSn</u>	<u>Pno</u>	hours
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Dependent

<u>ESSN</u>	<u>Dependents-name</u>	Sex	Bdate	Relationship
-------------	------------------------	-----	-------	--------------