Q.2) Four cards are drawn from a pack of cards. Find probability that i) All are diamonds ii) there is one card of each suit and iii) there are two spades and two heads.

→ A pack is of 52 cards.

∴ Total number of ways drawing four cards is

$$n(S) = {}^{52}C_4$$

$$= \frac{52 \times 51 \times 50 \times 49 \times 48}{4 \times 3 \times 2 \times 1}$$

i) A → all are diamonds.

$$n(A) = {}^{13}C_4$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{{}^{13}C_4}{{}^{52}C_4} = \frac{13 \times 12 \times 11 \times 10}{52 \times 51 \times 50 \times 49}$$

$$= \frac{11}{4165}$$

2) One card of each suit :

$n(B) = {}^{13}C_1 + {}^{13}C_1 + {}^{13}C_1 + {}^{13}C_1$

$n(S) = {}^{52}C_1$

$$P(B) = \frac{13+13+13+13}{52} = \frac{52}{52} = 1$$

$P(B) = 1.$

3) Two spade and two heart :

$n(C) = {}^{13}C_2 \times {}^{13}C_2$

$n(S) = {}^{52}C_4$

$$P(C) = \frac{{}^{13}C_2 \times {}^{13}C_2}{{}^{52}C_4}$$

$$= \frac{\frac{13 \times 12 \times 13 \times 12}{2 \times 1}}{\frac{52 \times 51 \times 50 \times 49 \cancel{\times 48}}{4 \times 3 \times 2 \times 1}}$$

$$P(C) = \frac{468}{20825}$$

Q. A bag contains 40 tickets numbered 1, 2, 3, ---, 40 of which four are drawn at random & arranged - in an asending order ($t_1 < t_2 < t_3 < t_4$) Find the probability of t_3 being 25.

$$^{40}C_4 = \frac{40 \times 39 \times 38 \times 37}{4 \times 3 \times 2 \times 1}$$

$$= 91390.$$

Here,

1, - - - -, 25, - - -, 40.

1 to 24 $\Rightarrow$ $^{24}C_2$

25 to 40 $\Rightarrow$ $^{15}C_1$.

$$\therefore \frac{^{24}C_2 \times {^{15}C_1}}{^{40}C_4}$$

$$= \frac{\frac{24 \times 23}{2 \times 1} \times 15}{\frac{40 \times 39 \times 38 \times 37}{4 \times 3 \times 2 \times 1}}$$

$$= \frac{4140}{91390}$$

$$= \frac{414}{9139}$$

Q. A has one share in lottery in which there is 1 prize & 2 blanks and B has three share in a lottery in which there are 3 prizes & 6 blanks. Compare probability of A's success to that of B's success.

→ Probability of A to lose $= \frac{^2C_1}{^3C_1}$

$$= \frac{2}{3}$$

Probability of B to lose $= \frac{{}^6C_3}{{}^9C_3} = \frac{5}{21}$

$P(A \text{ to } B) = \frac{1/3}{16/21}$

$= \frac{7}{16}.$

Q. When a coin is tossed 4 times, find probabili[ty] of getting 1) Exactly one head
2) At most three heads.
3) At least two heads.

→ Total number of outcomes $= 2^4$

$= 16.$

n(S) = { {HHHH}, {HHHT}, {HHTH}, {HHTT},
{HTHH}, {HTHT}, {HTTH}, {HTTT},
{THHH}, {THHT}, {THTH}, {THTT},
{TTHH}, {TTHT}, {TTTH}, {TTTT}}

i) A = exactly one head.

$n(A) = 4$

$P(A) = \frac{n(A)}{n(S)}$

$= \frac{4}{16}$

$= \frac{1}{4}$

ii) B = at most three heads.

$n(B) = 16 - n(4 \text{ heads})$

$= 15$

$P(B) = \frac{n(B)}{n(S)} = \frac{15}{16}.$

iii) $c =$ at least two heads.

$n(c) = 11$

$$P(c) = \frac{n(c)}{n(s)}$$

$$= \frac{11}{16}.$$

Q. Ten coins are thrown simultaneously. Find the probability of getting at least 7 heads.

→ $P(\text{head}) = \frac{1}{2} = p.$

$P(\text{tail}) = 1-p = \frac{1}{2} = q.$

$n = 10$

$x =$ at least 7 heads.

$$P(X) = P(7) + P(8) + P(9) + P(10)$$

$$= {}^{10}C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{10-7} + {}^{10}C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 + {}^{10}C_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{10-9} + {}^{10}C_{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^0$$

$$= 120\left(\frac{1}{2}\right)^{10} + 45\left(\frac{1}{2}\right)^{10} + 10\left(\frac{1}{2}\right)^{10} + \left(\frac{1}{2}\right)^{10}$$

$$P(X) = \frac{176}{2^{10}}$$

$$\frac{10!}{7!\,3!}$$

Q. i) Given $P(A) = 1/2$, $P(B) = 1/3$ and $P(AB) = 1/4$. Find the value of $P(A+B)$.

→ $P(A \cap B) = \frac{1}{4}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$

$= \frac{7}{12}$

ii) A and B be two events with $P(A) = 1/2$, $P(B) = 1/3$ and $P(A \cap B) = 1/4$. Find $P(A/B)$, $P(A \cup B)$ & $P(A'/B')$.

→ $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{1/4}{1/3}$

$= \frac{3}{4}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$

$= \frac{7}{12}$

$P(A'/B') = \frac{P(A' \cap B')}{P(B')}$

$= \frac{P(A \cup B)'}{P(B')}$

$= \frac{1 - P(A \cup B)}{1 - P(B)}$

$$= \frac{1 - 7/12}{1 - 1/3}$$

$$= \frac{5}{8}.$$

Q. A can hit target 3 times in 5 shots, B 2 times in 5 chats and C 3 times in 4 shots. They have a valley. What is probability that
1) Two shots hit and 2) at least 2 shots hit the target.

→ $P(A\ hit) = 3/5$, $P(A') = 1 - 3/5 = 2/5$
$P(B\ hit) = 2/5$, $P(B') = 1 - 2/5 = 3/5$
$P(C\ hit) = 3/4$, $P(C') = 1 - 3/4 = 1/4$.

i) two shots hit
i) A & B hit and C miss
ii) A & C hit and B miss
iii) B & C hit and A miss.

i) $\frac{3}{5} \times \frac{2}{5} \times \frac{1}{4}$

ii) $\frac{3}{5} \times \frac{3}{4} \times \frac{3}{5}$

iii) $\frac{2}{5} \times \frac{3}{4} \times \frac{2}{5}$

$\therefore P(\text{two shots hit}) = \frac{3}{50} + \frac{27}{100} + \frac{3}{25}$

$$= \frac{9}{20}.$$

ii) At least 2 shots hit :

two shots hit + 3 shots hit

$$= \frac{9}{20} + \frac{3}{5} \times \frac{2}{5} \times \frac{3}{4}.$$

$$\therefore P(\text{at least 2 shots hit}) = \frac{9}{20} + \frac{9}{50}$$

$$= \frac{63}{100}.$$

Q. Suppose 5 cards are drawn at random from a pack of 52 cards. If all cards are red, what is probability that all of them are hearts?

→ $P(\text{selecting 5 red cards}) = \frac{^{26}C_5}{^{52}C_5}$

$P(\text{selecting 5 hearts}) = \frac{^{13}C_5}{^{26}C_5}$

$P(\text{selecting red \& hearts})$

$$= \frac{^{26}C_5}{^{52}C_5} \times \frac{^{13}C_5}{^{26}C_5}$$

$$= \frac{^{13}C_5}{^{52}C_5}$$

$$= \frac{33}{66640}$$

$P(\text{red \& heart}) / P(\text{red})$

$$= \frac{^{13}C_5}{^{52}C_5} \Big/ \frac{^{26}C_5}{^{52}C_5}$$

$$= \frac{{}^{13}C_5}{{}^{26}C_5}$$

$$= \frac{9}{460}.$$

Q. A pair of dice is tossed twice. Find probability of scoring 7 points (i) once (ii) at least once and (iii) twice.

→ Pair of dice is tossed twice.

{ {1,1}, {1,2}, {1,3}, {1,4}, {1,5}, {1,6},
{2,1}, {2,2}, {2,3}, {2,4}, {2,5}, {2,6},
{3,1}, {3,2}, {3,3}, {3,4}, {3,5}, {3,6}
{4,1}, {4,2}, {4,3}, {4,4}, {4,5}, {4,6}
{5,1}, {5,2}, {5,3}, {5,4}, {5,5}, {5,6}
{6,1}, {6,2}, {6,3}, {6,4}, {6,5}, {6,6} }

Total event = 36.

(1) favourable event
= {1,6}, {2,5}, {3,4}, {4,3}, {5,2}, {6,1}.

once:

$P(7 \text{ once}) = \frac{6}{36}$

$= \frac{1}{6}.$

(2) Probability of getting not 7 = $1 - \frac{1}{6} - \frac{5}{6}$

P(at least once) = $1 - P(A')$

$$= 1 - \frac{5}{6}$$

$$= \frac{1}{6}.$$

(3) Probability of getting 7 twice

$$= \frac{6}{36} \times \frac{5}{35}$$

$$= \frac{1}{42}$$