Conjuctive Normal Form.

A formula which is equivalent to a given formula and which consist of a product of elementary sums is called CNF of the given formula.

CNF = product of elementary sum.

e.g. $\neg(P \vee Q) \longleftrightarrow (P \wedge Q)$

$\Leftrightarrow \neg(P \vee Q) \rightarrow (P \wedge Q) \wedge (P \wedge Q) \rightarrow \neg(P \vee Q)$

$\Leftrightarrow [(P \vee Q) \vee (P \wedge Q)] \wedge [\neg(P \wedge Q) \vee \neg(P \vee Q)]$

$\Leftrightarrow (P \vee Q \vee P) \wedge (P \vee Q \vee Q) \wedge [(\neg P \vee \neg Q) \vee (\neg P \wedge \neg Q)]$

$\Leftrightarrow (P \vee Q) \wedge (P \vee Q) \wedge [((\neg P \vee \neg Q) \vee \neg P) \wedge (\neg P \vee \neg Q \wedge \neg Q)]$

$\Leftrightarrow (P \vee Q) \wedge (\neg P \vee \vee Q) \wedge (\neg P \vee \neg Q) \wedge (\neg P \vee \neg Q)$

$\Leftrightarrow (P \vee Q) \wedge (\neg P \vee Q P]$

Principal Disjunctive Normal Form

For a given formula, an equivalent formula consisting of Disjunction of minterms only is known as "principle disjunctive normal Form." Such a normal form is also known as Sum of "Principle disjunctive normal form." Such a normal form is also called the "Sum of products canonical form".

e.g. (1) $(\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q)$

The DNF & CNF of the statement formula is not unique. In order to obtain a unique result result of a given statement formula we introduce principle normal forms

(1) PDNF (2) PCNF.

Methods to obtain PDNF of a given formula

(1) By using truth table

(2) By without using truth table.

By using truth table.

eg. Obtain the PDNF of $P \to Q$

P	Q	$P \to Q$	minterm
T	T	T	$P \wedge Q$
T	F	F	$P \wedge \neg Q$
F	T	T	$\neg P \wedge Q$
F	F	T	$\neg P \wedge \neg Q$

* ∴ PDNF of $P \to Q$ is

$$(P \wedge Q) \vee (\neg P \wedge Q) \vee (\neg P \wedge \neg Q)$$

PDNF without using truth table.

Obtain PDNF of $P \to Q$

Sol^n: Given statement formula is,

$P \to Q$

$\Leftrightarrow \neg P \vee Q$

$\Leftrightarrow (\neg P \wedge T) \vee (Q \wedge T)$

$\Leftrightarrow \cancel{(\neg P \wedge (Q \neg Q)) \vee (Q \wedge (P \vee \neg P))}$

$\Leftrightarrow \cancel{(\neg P \wedge Q \neg Q) \vee (Q \wedge P)}$

$\Leftrightarrow [\neg P \wedge (Q \vee \neg Q)] \vee [Q \wedge (P \vee \neg P)]$

$\Leftrightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (Q \wedge P) \vee (Q \wedge \neg P)$

$\Leftrightarrow (\neg P \wedge \neg Q) \vee (Q \wedge P) \vee (Q \wedge \neg P)$

* Principle Conjunctive Normal Form (PCNF)

Def^n:- For a given formula, an equivalent formula consisting of conjunctions of the maxterms only is known as its principle conjective normal form.

⇒ PCNF is also called as a product of sum canonical form.

Method's of finding PCNF
① Using truth table
② Without using truth table

e.g. ①

P	Q	$P \leftrightarrow Q$	Maxterm
T	T	T	
T	F	F	$\neg P \vee Q$
F	T	F	$P \vee \neg Q$
F	F	T	

∴ The equivalent PCNF of $P \leftrightarrow Q$ is
$(\neg P \vee Q) \wedge (P \vee \neg Q)$

① Obtain the PCNF of $P \rightleftarrows Q$

$P \rightleftarrows Q$

$\Leftrightarrow (P \to Q) \wedge (Q \to P)$

$\Leftrightarrow (\neg P \vee Q) \wedge (\neg Q \vee P)$

The required product is,

② Obtain the PCNF of $(\neg P \to R) \wedge (Q \leftrightarrow P)$

Sol:- $(\neg P \to R) \wedge (Q \leftrightarrow P)$

$\Leftrightarrow (\neg(\neg P) \vee R) \wedge ((Q \to P) \wedge (P \to Q))$

$\Leftrightarrow (P \vee R) \wedge [(\neg Q \vee P) \wedge (\neg P \vee Q)]$

$\Leftrightarrow (P \vee R) \wedge (\neg Q \vee P) \wedge (\neg P \vee Q)$

$\Leftrightarrow ((P \vee R) \vee F) \wedge ((\neg Q \vee P) \vee F) \wedge ((\neg P \vee Q) \vee F)$

$\Leftrightarrow ((P \vee R) \vee (Q \wedge \neg Q)) \wedge ((\neg Q \vee P) \vee (R \wedge \neg R)) \wedge ((\neg P \vee Q) \vee (R \wedge \neg R))$

$\Leftrightarrow ((P \vee R \vee Q) \wedge (P \vee \neg Q \vee R)) \wedge ((P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R)) \wedge ((\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R))$

$\Leftrightarrow (P \lor Q \lor R) \land (P \lor \neg Q \lor R) \land (P \lor \neg Q \lor R) \land (P \lor \neg Q \lor \neg R)$
$\land (\neg P \lor Q \lor R) \land (\neg P \lor Q \lor \neg R).$

$\Leftrightarrow (P \lor Q \lor R) \land (P \lor \neg Q \lor R) \land (P \lor \neg Q \lor \neg R) \land$
$(\neg P \lor Q \lor R) \land (\neg P \lor Q \lor \neg R).$

. This is the required PCNF.

② Obtain the PCNF of $(P \land Q) \lor (\neg P \land Q)$

Sol^n:- $(P \land Q) \lor (\neg P \land Q)$

$\Leftrightarrow ((P \land Q) \lor \neg P) \land ((P \land Q) \lor Q)$

$\Leftrightarrow (P \lor \neg P) \land (Q \lor \neg P) \land (P \lor Q) \lor (Q \land Q)$

$\Leftrightarrow (T \land (Q \lor \neg P)) \land ((P \lor Q) \land Q)$

$\Leftrightarrow (Q \lor \neg P) \land (P \lor Q) \land Q$

$\Leftrightarrow (Q \lor \neg P) \land (P \lor Q) \land (Q \lor F)$

$\Leftrightarrow (Q \lor \neg P) \land (P \lor Q) \land (Q \lor (P \land \neg P))$

$\Leftrightarrow (Q \lor \neg P) \land (P \lor Q) \land ((Q \lor P) \land (Q \lor \neg P))$

$\Leftrightarrow (Q \lor \neg P) \land (P \lor Q) \land (Q \lor P) \land (Q \lor \neg P)$

$\Leftrightarrow (Q \lor \neg P) \land (P \lor Q)$ $\cancel{\land T}$

This is PCNF.

Example of PDNF.

Obtain the PDNF of $P \to ((P \to Q) \land \neg(\neg Q \lor \neg P))$

Sol^n: $P \to ((P \to Q) \land \neg(\neg Q \lor \neg P))$

$\Leftrightarrow P \to ((\neg P \lor Q) \land (Q \land P))$

$\Leftrightarrow$ $\cancel{P \to ((\neg P \lor Q) \land Q}$

$P \to ((\neg P \land Q \land P) \lor (Q \land Q \land P))$

$P \to ((\neg P \land P \land Q) \lor (Q \land P))$

$P \to ((F \land Q) \lor (Q \land P))$

$P \to (F \lor (Q \land P))$

$P \to (Q \land P)$

$\neg P \lor (Q \land P) \Leftrightarrow (\neg P \land T) \lor (Q \land P)$

$\cancel{(\neg P \lor Q) \land (\neg P \lor P)} \Leftrightarrow (\neg P \land (Q \lor \neg Q)) \lor (P \land Q)$

$\Leftrightarrow (\neg P \land Q) \lor (\neg P \lor \neg Q) \lor (P \land Q)$ This is PDNF.

P

PDNF form

$[\neg((P \vee Q) \wedge R)] \wedge [(P \wedge R)]$

$\Leftrightarrow [\neg(P \vee Q) \vee \neg R] \wedge [P \wedge R]$ De morgan's law

$\Leftrightarrow [(\neg P \wedge \neg Q) \vee \neg R] \wedge [P \wedge R]$

$\Leftrightarrow \neg P \wedge \neg Q \wedge P$

Inference Theory

The main aims of logic is to provide rules of inference to infer a conclusion from certain premises The Theory associated with rules of inference is known as 'Inference theory'

Valid argument or valid conclusion:

If a conclusion is derived from a set of premises by using the accepted rules of reasoning, then such a process of derivation is called a 'deduction' or "a formal proof" and the argument or conclusion is called a "valid argument" or "valid conclusion.

Note: Premises are also called as assumption (or) axioms (or) hypothesis.

Methods used to determine the conclusion:-

The following methods used to determine whether the conclusion logically follows from the given premises

1. using Truth table method
2. without using truth table method.

(I) Using Truth Table method :-

⇒ Let $P_1, P_2, \ldots, P_n$ be the variables appearing in the premises $H_1, H_2, \ldots, H_m$ and the conclusion C.

(a) Look for the rows in which all $H_1, H_2, \ldots, H_m$ have the value T, if for every such row, C also has the value T, then $H_1 \wedge H_2 \wedge \ldots \wedge H_m \Rightarrow C$ holds.

(b) Look for the rows in which C has the value 'F'. If in every such row at least one of the values of $H_1, H_2, \ldots, H_m$ is F then $H_1 \wedge H_2 \wedge \ldots \wedge H_m \Rightarrow C$.

(II) Without using truth table :

→ The truth table technique becomes tedious when the number of statement variables present in all the formulas representing the premises and the conclusion is large.

→ To overcome this disadvantage, we use the other methods without using truth table. Here we describe the process of Derivation by which one

Combinatorics

* Sum Rule (Addition principle)
If events $A_1, A_2, A_3, ---, A_n$ are mutually independent & $S = A_1 \cup A_2 \cup --- \cup A_n$ i.e. exhaus events then possible no. of ways any one of the event can happen is $n(A_1) + n(A_2) + --- + n(A_n)$

* Product Principle:
If there are 'n' events with m possible outcomes $i = 1, 2, ---, n$ then then no. of possible outcomes overall is $m_1 . m_2 . --- . m_n$.

Que. How many outcomes are possible when 3 dies are rolled if no two of them be the same?
Sol: Possible no. of way's for I^{st} die = 6
Possible no. of ways for 2^{nd} & 3^{rd} dice is 5×4
$\therefore$ Total No. of ways $= 6 \times 5 \times 4 = 120$.

Q. How many integers are there betn 1 to 1000 which are divisible by 3 or 5 or 7.
Sol:- A = no. divisible by 3
B = no. divisible by 5
C = no. ~~of~~ divisible by 7.

$$n(A) = \left[\frac{1000}{3}\right] = [333.33] = 333$$

$$n(B) = \left[\frac{1000}{5}\right] = 200$$

$$n(C) = \left[\frac{1000}{7}\right] = [142.85] = 142.$$

$$n(A \cap B) = 66 = \frac{1000}{15}$$

$n(A \cap C) = 47$, $n(B \cap C) = 28$ $n(A \cap B \cap C) = 9$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) - n(A \cap B \cap C) = 333 + 200 + 142 - 66 - 47 - 28 + 9 = 543$

Q. How many integers of 4 digits can be formed using 0, 1, 2, 3, 4, 5, 6 with repetition and without repetition. Find how many of them are divisible by 5?

→ We have to form 4 digit no.

Without repeatition $= 6 \times 6 \times 5 \times 4 = 720$

With repeatition $= 6 \times 7 \times 7 \times 7 = 2058$

Divisible by 5.

with repeatition $= 6 \times 7 \times 7 \times 2 = 588$.

without repeatition $= (6 \times 5 \times 4) + (6 \times 5 \times 4) = 220$

* Permutation of n things taken all at a time is given by '$n!$'

* Permutations of 'n' things taken r at a time is given by $^{n}P_{r} = \frac{n!}{r!}$

* Permutation with repeatition - if out of n things there are r_1 thing of one kind, r_2 thing of second kind... r_m things of m^{th} kind, Permutation of n things taken at a time is $\frac{n!}{r_1!\, r_2!\, r_3! \ldots r_m!}$

* Circular permutation :- Permutation of n things taken all at a time around circle is $(n-1)!$

Find no. of distinct permutation of letters of word MATHEMATICS. How many of them start with m, end with m. In how many permutations all vowels are together?

→ $n = 11$, M is repeated 2 times so as A & T.

Using permutation with repeatition $= \frac{11!}{2!\,2!\,2!} = \frac{11!}{8}$

Combinatorics

It deals with the arrangement of objects according to same pattern and counting the no. of ways it can be done.

e.g. There are ten true-False quetions. In how many different ways questions can be marked.

Q.1 $\rightarrow$ 2	Q6 $\rightarrow$ 2
Q.2 $\rightarrow$ 2	Q.7 $\rightarrow$ 2
Q.3 $\rightarrow$ 2	Q8 $\rightarrow$ 2
Q.4 $\rightarrow$ 2	Q.9 $\rightarrow$ 2
Q.5 $\rightarrow$ 2	Q.10 $\rightarrow$ 2

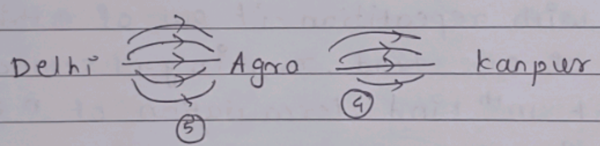
$2 \cdot 2 \cdot \ldots \cdot 2 = 2^{10}$ i.e. 1024 possibilities.

Multiplication Principle:-

Work 1 : m ways

Work 2 : n ways

both work 1 & work 2 = $(m \times n)$ ways.



Delhi $\longrightarrow$ kanpur.

$5 \times 4 = 20$

Def: If a work can be done in 'm' ways another work can be done in 'n' ways then both of the operations can be performed in $m \times n$ ways.

Addition Principle: Independent events

E_1 & E_2 (Mutually exclusive)

$E = E_1$ or E_2 occur.

$n(E) = E_1 + E_2$.

e.g. 5 veg dishes, 6 Non-veg dishes

$5+6 = 11$ ways.

Arrange - Per
select - Comb.

$A = \{-1, -2, 1, 3, 5, 6, 7, 8, 9, 10\}$

E_1 - Chosing Neg no. from A
$= \{ 2$

E_2 = Chosing an odd no. from A.
$\hookrightarrow 5$

$E = E_1$ or E_2
$= (2+5) = 7$ ways

Def^n:- If a work can done in 'n' ways and another (mutually exclusive) work can be done in 'm' ways then either of the two work can be done in $(m+n)$ ways.

Combinatorics

Combinatorics is an important part of discrete mathematics that solves counting problems without actually enumerating all possible cases.

Combinatorics deals with counting the number of ways of arranging or choosing objects from a finite set according to certain specified rules.

In other words combinatorics, is concerned with problem of permutation & combinations.

Permutation :-

An ordered ~~element~~ arrangement of r elements of a set containing n distinct elements is called an r-permutation of n-elements. and is denoted by $P(n,r)$ or ${}^nP_r$ where $r \le n$.

$$P(n,r) = \frac{n(n-1)(n-2) \cdots (n-r+1)}{}$$

$$\therefore \quad P(n,n) = \frac{n!}{(n-r)!} = n!$$

Combinations

An ordered selection of r elements of ~~a set~~ containing n distinct elements is called an ~~r~~ r-combination of n elements and is denoted by $C(n,r)$ or ~~$C(n)$~~ $({}^nC_r) = \binom{n}{r}$

$$C(n,r) = \frac{n!}{r!(n-r)!} \qquad C(n,n) = 1$$

(1) Number of permutations of n distinct objects

The number of different arrangement of n distinct objects taken all at a time is, (without repet)

$$P(n,n) = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$$

(2) Number of permutations of r objects among n distinct objects.

Suppose we are given 'n' distinct objects & wish to arrange r of the objects denoted by,

$${}^nP_r = P(n,r) = \frac{n!}{(n-r)!}$$

(3) Number of permutations of n ~~dupels~~ objects (with duplication.

It is required to find the number of permutations that can be formed from a collection of n objects of which n_1 are of the one type n_2 are of the second type, ..., n_k are of k^{th} type with $n_1 + n_2 + \dots + n_k = n$, then the number of permutations of n objects are

$$\frac{n!}{n_1! * n_2! * \cdots * n_k!}.$$

(4) Circular permutation

Permutations in a circle are called circular permutation.

The total number of ways of arranging the n persons in a circle $= (n-1)!$

Examples:

(1) How many ways are there to sit 10 boys & 10 girls around a circular table?

Sol:- Here 10 boys and 10 girls sit around a circular table.

$\therefore$ Total no. of person $= 10 + 10 = 20$

$\therefore n = 20$

Total no. of ~~circular~~ ways of circular permutations $= (n-1)!$
$= 19!$

(2) How many ways are there 3 persons sit around a round table?

Sol:- No. of persons $= 3$

$\therefore n = 3$

The total no. of ways of arranging 3 persons around a circular table $= (n-1)! = 2! = 2$

Q3 How many different arrangements of letters in the word BOUGHT?

Sol^n: The given word BOUGHT contains 6 letters that are distinct. (without duplication)

Total no. of arrangements of letters in word BOUGHT $= P(n,n) = {}^nP_n = 6!$

$= 6\times5\times4\times3\times2\times1$

$= 720.$

(4) Find the number of permutation of the letters of the word ~~a~~ success

Sol^n: No. of letters in the word.

success $= 7$ $\quad n = 7$

out of 7 letters, S are 3, C are 2, U is 1 and E is 1

$\therefore$ The number of permutations of the letters of the word SUCCESS $= \dfrac{n!}{n_1! \times n_2! \times n_3! \times n_4!}$

$= \dfrac{7!}{3! \times 2! \times 1! \times 1!}$

$= \dfrac{7\times6\times\cancel{4}\times\cancel{5}\times3!}{\cancel{3!}\times2!} = 84\times5 = 420$

(5) Find the number of permutations of the letters of the word 'STRUCTURES'.

Sol^n: Number of letters in the word ~~success~~ STRUCTURES $= 10$ $\quad\therefore n = 10$

Out of 10 letters, ~~S~~ = 2 are S, 2 are T, 2 are R, 2 are U, 1 is E, 1 is C.

$\therefore$ Total number of permutations $= \dfrac{n!}{n_1! \times n_2! \times n_3! \times n_4! \times n_5! \times n_6!}$

$$= \frac{10!}{2! * 2! * 2! * 2! * 1! * 1!}$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2!}{2! \times 2! \times 2! \times 2!}$$

$$= 226800$$

Find the number of permutations of the letters of the word MATHEMATICS?

Sol: Number of letters in the word MATHEMATICS $= 11$

$\therefore n = 11$

Out of which 2 are M, 2 are A, 2 are T, E, H, I, C, S are 1.

$$\therefore \text{Total no. of permutation} = \frac{n!}{n_1! * n_2! * n_3! * n_4! * n_5! * n_6! * n_7! * n_8!}$$

$$= \frac{11!}{2! \times 2! \times 2! \times 1! \times 1! \times 1! \times 1! \times 1!}$$

$$= \frac{11!}{8}$$

$$= 4989600$$

(7) How many different strings of length 4 can be formed using the letters of the word PROBLEM?

Sol: The given word 'PROBLEM' has 7 letters $\therefore n = 7$

The no. of different strings of length 4 can be formed by using the letters of the word problem:

$${}^{n}P_{r} = P(n, r) = P(7, 4) = \frac{7!}{(7-4)!} = \frac{7!}{3!} = 840$$

(8) How many words of three distinct letters can be formed from the letters of the word PASCAL.

Sol: The given word PASCAL contains 6 letters

$\therefore n = 6$

$\therefore$ No. of ways of 3 distinct letters can be formed by using the letters of the word PASCAL =

$$P(n,r) = {}^{n}P_{r} = \frac{6!}{(6-3)!} = \frac{6\times5\times4\times3!}{3!} = 120$$

(9) Find the number of permutations of the letters of the word 'ENGINEERING'?

Sol^n: The given word ENGINEERING has 11 letters out of which, 3 are E, 3 are N, 2 are I, 2 are G, 1 is R

$\therefore$ Total No. of permutation of the letters of the word ENGINEERING =

$$\frac{n!}{n_1! \times n_2! \times n_3! \times n_4! \times n_5!}$$

$$= \frac{11!}{3!\times3!\times2!\times2!\times1!}$$

$$= \frac{11\times10\times9\times8\times7\times6\times5\times4\times3!}{3!\times6\times4}$$

$$= 270200$$

Combinations:

Q1. How many committees of 5 with a given chair person can be selected from 12 persons?

Sol: Total number of persons = 12

Each committee consisting of 5 persons, among them, one person is chair person.

The chair person can be selected among 12 persons = 12 ways

The remaining four members or person s in the committee can be seleleted $^{11}C_4$ ways.

∴ Total no. of ways $= 12 * {}^{11}C_4$

$$= 12 \times \frac{11}{(11-4)!\,4!}$$

$$= \cancel{12} \times \frac{11 \times 10 \times 9 \times \cancel{8} \times 7!}{7!\,\cancel{4!}}$$

$$= 110 \times 36$$

$$= 3960$$

② Find the number of committes of 5 that can be selected from 7 men and 5 women. If the committee is to consist of atleast one man and atleast one women.

Sol: Total no. of persons = 7 men + 5 women

= 12

Each committee consists of 5 persons.

Number of committees of 5 that can be selected among 12 persons = $^{12}C_5$

Among these possible committes, Number of committees to select 5 men = C(7,5) & no. of committees formed to select 5 women = C(5,5)

∴ Total no. of committees formed with atleast one man & one woman = $^{12}C_5 - {}^{7}C_5 - {}^{5}C_5$

= 792 − 21 − 1 = 770.

Q. A certain question paper contains two parts A and B each containing 4 questions. How many different ways a student can answer 5 questions by selecting atleast 2 questions from each point.

Sol: Question paper contain two parts

part − A — 4 questions

part − B — 4 questions

A student can answer 5 questions atleast 2 questions from each point.

Case I : A student can select 3 Que. from part A & 2 Q from part B

No. of ways = $^4C_3 * {}^4C_2$

$= \frac{4!}{3!(4-3)!} * \frac{4!}{2!(4-2)!}$

$= \frac{4 \times 3!}{3!\ 1!} * \frac{4 \times 3 \times 2!}{2! \times 2!}$

$= 4 * 6 = 24$

Case II : A student can select 2 Que. from part A & 3 que. from part B

no. of ways = $^4C_2 * {}^4C_3$

$= \frac{4!}{2!(4-2)!} * \frac{4!}{3!(4-3)!}$

$= 6 * 4 = 24$

Que At a certain college hostel, the housing office has decided to appoint one male and one female residential advisor for each floor. How many different pairs of advisors can be selected for a seven floor building from 12 male and 15 female candidates.

Number of floors = 7

Number of male candidate = 12

Number of female ~~advisor~~ for candidate = 15

From 12 male male candidates for 7 floor can be selected in C(12,7) ways.

$C(12,7) = {}^{12}C_7 = \frac{12!}{7!(12-7!)} = \frac{12!}{7! * 5!}$

$= 792$ ways

15 female candidates, 7 females candidate for 7 floors can be selected in $^{15}C_7$ ways i.e. $C(15,7)$.

$C(15,7) = {}^{15}C_7 = \frac{15!}{7!(15-7)!} = 6435$ ways

No. of pairs of advisors can be selected for a 7 floor building $= 792 \times 6435$ ways.

Combinations with repetition

Suppose we wish to select, a combination of r objects with repetition from a set of n distinct objects. The number of such selection is given by,

$C(n+r-1, r) = C(r+n-1, n-1)$

→ The following are the other interpretations of this number.

(i) $C(n+r-1, r) = C(r+n-1, n-1)$ represents the no. of ways in which r identical objects can be distributed among n distinct container.

(ii) $C(n+r-1, r) = C(r+n-1, n-1)$ represents the number of non-negative integer solutions of the equation

Note: A non-negative integer solution of the equation $x_1 + x_2 + \dots + x_n = r$ is an n-tuple where $x_1, x_2, \dots, x_n$ are non-negative integers whose sum is r.

① How many committee of 3 people can be formed from 8 people?

$$^8C_3 = \frac{8!}{3!\,5!}$$

Example①: In how many ways can we distribute 10 identical marbles among 6 distinct containers?

Sol:

10 identical marbles $\Rightarrow r = 10$

6 distinct containers $\Rightarrow n = 6$.

Number of such selection $=$

$$C(r+n-1, r) = C(10+6-1, 10)$$
$$= C(15, 10)$$
$$= \frac{15}{10!\,(5!)} = 3003$$

Exam② Find the number of non-negative Integer solutions of the eq^n

$$x_1 + x_2 + x_3 + x_4 + x_5 = 8$$

Sol: $n = 5$ (Number of non-negative integers) $= 5$

$r = 8$

$\cancel{C(n+r-1, r)} = C(n+r-1, r) = C(5+8-1, 8)$

$\cancel{C(r+n-1)}$ $= C(12, 8)$

In how many ways can we distribute 7 apples and 6 oranges among 4 children so that each child gets atleast one apple?

→ No. of apples $= 7$

No. of oranges $= 6$

No. of children $= 4$.

each child gets atleast one apple i.e. every child may get ≥ 1 apple.

First we distribute one apple to each child. The remaining Apples $(7-4=3)$ can be distributed to 4 children

No. of ways to distribute 3 apples to 4 children

$$C(r+n-1, r) = C(3+4-1, 3) = {^6C_3} = \frac{6!}{3!\,(6-3)!} = \frac{6\times5\times4\times3!}{6\times3!}$$
$= 20$ ways

Piegon Hole Principle

If $(N+1)$ or more objects are placed into N boxes then there is at least one box containing two or more objects.

Example: If 6 colour are used to paint 37 home. Show that at least 7 home of them will be of same colour.

Sol^n:-

$$\frac{37}{6} = 6$$

$\Rightarrow$ 6 home each of 6 colour

but remainder 1 will be a colour form the 6.

$\Rightarrow$ 7 home may have a same colour.

Generalized piegonhole principle

If 'n' pigeon hole are occupied by $kn+1$ or more pigeons then atleast one piegonhole is occupied by $k+1$ or more pigeon.

Que Find the minimum number of teachers in a college to be sure that four of them are born in the same month.

Sol^n:- $n = 12$, $k+1 = 4 \Rightarrow k = 3$

$$kn+1 = 12 \times 3 + 1 = 37$$

Qu. A box contain 10 blue balls, 20 red balls 8 green balls, 15 yellow balls & 25 white balls. How many balls must be choose to ensure that we have 12 balls of the same colour.

$k+1 = 12$

$k = 11$

$n = 5$ (colour) diff^n

$\therefore kn+1 = (11\times5)+1 = 56.$

Prove that among 1,00,000 people there are two who are born on same time.

Sol: Let $P = \{P_1, P_2, P_3, \ldots, P_{100000}\}$ be the set of people.

Let $H = \{H_1, H_2, \ldots H_{24}\}$ be the set of all hours in day.

By extended pigeonhole principle there are at least $\left\lceil \frac{|P|}{|H|} \right\rceil$ persons birth during the hour.

$= \frac{100000}{24} = 4167.$

~~Now~~ $~~Q = \{Q_1, Q_2, \ldots, Q\}~~$

Then number of person born in same minute ar atleast $= \frac{4167}{60} = 70$

Greatest integer fn.

Again number of person born in same second are atleast $= \left[\frac{70}{60}\right] = 2$

If n pigeons are assigned to 'm' piegonhole and $m<n$, show that same pigeon hole contain atleast two pigeon also show that among 13 people there are atleast two people who were born in the same month.

Sol: We know that it is true.

The no. of people born in same month are $= \left[\frac{13}{12}\right] = 2.$

Show that if we allot 26 rooms to the students in a P. G. Hosted from the room numbered from 1 and 50 both inclusive at least two allotments are constitutively numbered.

Sol: $\left\lceil \frac{50}{26} \right\rceil = 2$

Q. Find min no. of students in class such that 3 of them are born in same month.

→ $n = 12$ $\quad k+1 = 3 \Rightarrow k = 2$

Let, pigeon holes be no. of months. Given at least 3 students born in same month.

∴ Required min no. of students $= kn+1 \quad (\because k=2)$

$= (2)(12)+1$

$= 25$

~~A bag contains~~

If 7 colours are used to paint 50 ~~pig~~ bycycle show that at least 8 of them will be of same colour.

Sol: Let colours denote pigeonholes & bicycles denote pigeons and $m < n$

$\frac{50}{7} = 7$ ∴ 7 bicycles each of 7 colour.

remainder 1 will be a colour from the 7

∴ 8 bicycles may have a same colour.

* Show that in a group of 50 students atleast 5 are born in same month.

Sol: There are $n = 12$ (months) pigeonhole

& $kn+1 = 50$ pigeons

$$\therefore K(12)+1 = 50$$
$$12K = 49$$
$$K = \frac{49}{12} = 4$$

remainder = 1

$\therefore$ Atleast $4+1=5$ students are born in same month.

Poisson probability distribution:

Def^n:

A discrete random variable X, denoting number of occurrences of an event during a fixed time period, interval or area, is said to follow Poisson probability distribution, if its pmf is as follows

$$p(X=x) = p(x) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad x = 1, 2, \ldots, \infty.$$

where, λ denotes average number of occurrences of an event during a fixed time period, interval or area

Properties of the poisson-probability distribution:

1. The constant λ is called the parameter of the Poisson distribution. Hence Poisson distribution can be represented by the notation $X \sim P(\lambda)$
2. The pmf of Poisson distribution satisfies the cond^n $\sum_x p(x) = 1$
3. The mean of the poisson distribution is $m = \lambda$.
4. The variance of the poisson distribution is also $m = \lambda$

If on an average, 12 accidents occur during 1 year in a textile mill, what is the probability that in the coming year there will be
(i) no accident? (ii) at most two accidents?

Sol^n:- Here X - No. of accidents during 1 year.

Let $X \sim P(\lambda)$

where,

$\lambda = 12 =$ avg no of accidents during 1 year

$$P(X=x) = p(x) = \frac{e^{-12} \cdot 12^x}{x!} \quad x = 1, 2, \ldots, \infty$$

① P{ no accidents during next year } = P{X=0}

$$= p(x) = \frac{e^{-12}\, 12^0}{0!}$$

$$= 6.144 \times 10^{-6} \cong 0$$

② P{ there are at most two accidents in 1 year }

$$= P[x \leq 2]$$

$$= P(x=0) + P(x=1) + P(x=2)$$

$$= e^{-12} + e^{-12} \times 12 + e^{-12} \times \frac{144}{2}$$

$$= 0.000006144 + 0.00007373 + 0.000\ldots$$

$$= 0.0005.$$

Q② A 100 m^2 roll of the fabric is expected to contain on an average eight weaving defects scattered uniformaly over the full fabric. If the fabric is cut into four pieces of equal size, what is the probability that

① a piece will be free from the weaving defects?

(ii) all the pieces will be free from the weaving defects?

Sol:- Here X - number of weaving defects in a piece of size 25 m^2

Let $X \sim P(\lambda)$

where,

λ average number of weaving defects in a piece of size 25 m^2 $= \frac{25 \times 8}{100} = 2$

$$P(X=x) = p(x) = \frac{e^{-2} \cdot 2^x}{x!} \quad x = 1, 2, \ldots, \infty$$

(i) P[a piece will be free from weaving defect]
$= P(x=0) = e^{-2} = 0.1353$

(ii) P[all pieces are free from weaving defects]
$= P[(1^{st}$ is free$) \cap (2^{nd}$ is free$) \cap (3^{rd}$ is free$)$
$\cap (4^{th}$ is free$)]$
$= P(1^{st}$ is free$) \cdot P(2^{nd}$ is free$) \cdot P(3^{rd}$ is free$)$
$P(4^{th}$ is free$)$
$= (0.1353)^4$
$= 0.000335$

ie. there is approximately 0% chance that all pieces are free from weaving defects.

Normal Distribution:

A continuous random variable "X" is said to follow normal probability distribution if its pdf is as follows.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad 0 < x < \infty$$

Properties of the normal probability distribution:

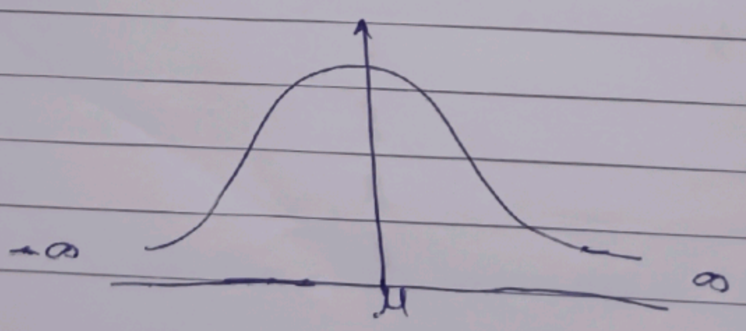
1. In case of normal distribution μ and σ are known as the parameters.
   Hence the normal distribution can be represented by the notation $X \sim N(\mu, \sigma)$ or $X \sim N(\mu, \sigma^2)$

2. The pdf of normal distribution satisfies the condition $\int f(x)\,dx = 1$.

3. The mean of normal distribution is μ.

4. The variance of the normal distribution is σ^2.

5. Graph of the normal distribution



As normal distribution is symmetric distribution
Mean = Mode = Median

Determination of probabilities in normal distribution:

$$P(X<a) = \int_{-\infty}^{a} f(x)\,dx$$

$$P(X>a) = \int_{a}^{\infty} f(x)\,dx$$

$$P(a<X<b) = \int_{a}^{b} f(x)\,dx$$

~~Probabi~~ Procedure of finding probabilities in normal probability distribution.

① Convert the normal random variable (X) into the standard normal random variable (z) using the defn

$$z = \left(\frac{X-\mu}{\sigma}\right)$$

② Use the statistical table for finding area under standard