* Probability *

mutual exclusive - no Common.

Exercise 10.1.

17

(a) does not have eith kind of defeat
(b) has only one kind of defeat

→ let event defet of 1^{st} type = A
event B = defet of 2^{nd} type

The defeats are independent

$$P(A \cap B) = P(A) \times P(B) = 0.05 \times 0.1 = 0.005$$

P [does not contain - - -] = 1 - P [Contains either

= 1 -

23 A machine made up of three component A, B, C
It works only if all three Components are working
A, B, C are Fail ate 0.01, 0.1, 0.02
what is the P(A) that machine will work.

$$P = P[A \cap B \cap C]$$
$$= P(A) \times P(B) \times P(C)$$
$$= 1 - P(A^c) \times [1 - P(B^c)] \times [1 - P(C^c)]$$

$$= 0.87$$

there is 87% chance of machine work

PAGE NO.:

10.4

1) A student can win Prize x with probability 0.4 and win at least two prize x & y with probability 0.7 Find the P(A) that he will win Prize if events independent and mutual exclusive

→ $P(A \cap B) = 0$

event A = St

2) what is the chance of 2 dice thrown where Find Sum of ~~addi~~ 5 and Sum of 11.

→ Sample Space = 36

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (

(i) Sum = 5 are → (1, 4) (4, 1) (3, 2) (2, 3)

(ii) No. of ways giving sum 11 - (5, 6) (6, 5)

n = 36 A = event contain sum5
B = -- -- sum 11

$P(\text{sum} = 5 \text{ or } 11) = P(A) + P(B) - P(A \cap B)$

$= \frac{4}{36} + \frac{2}{36} - 0$

$P(A \text{ or } B) = \frac{1}{6}$

3] from Packed card one card is dropped Find probability card is either a king an ace

$n = 52$

let a is getting a king

B is getting a ace

$\therefore P(A \cap B) = \phi$

$\therefore P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$

$= \frac{4}{52} + \frac{4}{52} - \phi$

$= \frac{8}{52}$

$= \frac{4}{26} = \frac{2}{13}$

4) For two events A and B $P(A) = 0.5$ $P(B) = 0.6$ and $P(A \cup B) = 0.8$ Find the conditional probability $P(A/B)$ $P(B/A)$

$\rightarrow$

$P(A/B) = \frac{P(A \cap B)}{P(B)}$ $\qquad P(B/A) = \frac{P(A \cap B)}{P(A)}$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.5 + 0.6 - 0.8$

$P(A \cap B) = 1.1 - 0.8 = 0.3$

$P(A/B) = \frac{0.3}{0.6}$ $\qquad P(B/A) = \frac{0.3}{0.5}$

$= \frac{1}{2} = 0.5$ $\qquad = \frac{3}{5} = 0.6$

5) A dice is the A box 'A' Contains two whites four black balls. Another box 'B' Contains 5 white 7 black balls. a black is transfered from ball 'A' to ball 'b' then ball is drawn from it. Find probability it is white.

→

let.

A white ball is transfered from box 'A' to box 'B' there where 6 white balls and 7 black balls

(1) The probability of transfering white ball from 'A' to be is $\frac{2}{6}$.

Then

getting a white ball from b is

$$\frac{6}{13}$$

P (getting a white ball from 'B' after transfering a white ball from Box B

$$= \frac{2}{\cancel{6}} \times \frac{\cancel{6}}{13} = \frac{2}{13}$$

(2) The probability of transfering black ball from 'A' to 'B' is $\frac{4}{6} \frac{4}{6} = \frac{2}{3}$

(3) ~~The probabilit~~

Then getting a ~~black~~ white ball from b is

$\frac{\cancel{8}5}{13}$

P(getting a ~~black~~ white ball from B' after transffering a ~~black~~ white ball from B)

$= \frac{2\cancel{4}}{3\cancel{8}} \times \frac{\cancel{8}5}{13} = \frac{\cancel{4}}{\cancel{13}} \frac{\cancel{16}}{\cancel{39}} \frac{10}{39}$

p(Therefore getting white ball from box b)

$= \frac{2}{13} + \frac{10}{39}$

$= \frac{16}{39}$

(6) 'A' can hit a traget 4 times in 5 ~~Sharpes~~ Shotes
B' — " — — " 3 times 4 Shater
~~t~~ C twice in 3 Shots. they Fire a valley
what is probability that at least 2 shots

→

$P(A) = \frac{4}{5}$ $P(B)1 = \frac{3}{4}$ $P(c) = \frac{2}{3}$

$P(A) \cap P(B) \cap P(C) = \phi$

∵ Three events mutually indepent

Probabity that atleast two shots hits
we may have

(i) all A, B, C will hit the target
(ii) A and B —— " ——
(iii) B and C —— " ——
(iv) A and C —— " ——

$$P(A \cup B) = P(A) + P(B)$$
$$= \frac{4}{5} + \frac{3}{4}$$

(i) P(A and B and C)

$$= P(A) \times P(B) \times P(C)$$
$$= \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{2}{5}$$

(ii) P(A & B will hit target & C miss target)

$$\cancel{PC'} = P(A) \times P(B) \times (1 - P(C))$$
$$= \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3}$$
$$= \frac{12}{60} = \frac{3}{15} = \frac{1}{5}$$

(iii) P($\cancel{B}$ & C will hit target & A miss target)

$$= P(B) \times P(C) \times (1 - P(A)]$$
$$= \frac{3}{4} \times \frac{2}{3} \times \frac{1}{5} = \frac{6}{60} = \frac{1}{10}$$

(ii) P(A & C will hit target & B miss)

$= P(A) \times P(C) \times (1 - P(B))$

$= \frac{4}{5} \times \cancel{3}\frac{2}{3} \times \frac{1}{4} = \frac{8}{60} = \frac{4}{30} = \frac{2}{15}$

P(At least 2 will heat target)

$= \frac{2}{5} + \frac{1}{5} + \frac{1}{10} + \frac{2}{15}$

$= \frac{3 \times 2 +}{5} \frac{1}{10} + \frac{2}{15}$

$= \frac{7}{10} + \frac{2}{15}$

$= \frac{105 + 20}{150}$

$= \frac{125}{150} = \frac{5}{6}$

Q) Base Theorem If $B_1, B_2, B_3 \ldots B_n$ are mutually exclusive and exhaustive of Sampe space S with $P(B_i) \neq 0$ for $i = 1, 2, \ldots n$ then for any arbitary event of S with $P(A > 0)$ we have

$$P(B_i / A) = \frac{P(B_i) \cdot P(A / B_i)}{\sum_{i=j=1}^{n} P(B_j) \cdot P(A / B_j)}$$

PAGE NO.:

Baye's Theorem :

$P(B_i/A) = P(B_i) \cdot P(A/B_i)$

Q In board factory machine A, B, C Manifacture 20% 35% 40% res of total output 5% 4% 2% of are defective bodies. a bolt is drawn at random is found to defective. what is probability that it was manfactured by machine 'B'.

→

let

E_1, E_2, E_3 denotes the event that a bolt Selected at random by machine A, B, C respectively and let H denotes event of its being defective.

$P(E_1) = \frac{25}{100} = 0.25$ $P(E_2) = \frac{35}{100} = 0.35$

$P(E_3) = \frac{40}{100} = 0.45$

Probability (of getting defaulting fault of factory A)

$P(H/E_1) = \frac{5}{100} = 0.05$ $P(H/E_2) = 0.04$

$P(H/E_3) = 0.02$

$$P(H|E_2) = \frac{\frac{35}{100} \times \frac{4}{100}}{\frac{25}{100} \times \frac{5}{100} + \frac{35}{100} \times \frac{4}{100} + \frac{40}{100} \times \frac{2}{100}}$$

$$= \frac{28}{69}$$

$$\simeq 0.405$$

Q. Countable

$$Q \cap Q = \phi$$

$$N \subset W \subset I \subset Q$$

$$\bar{X}_1 = \text{Expectation} = E(x) = \Sigma x \, (P(x)) \dots$$

$$= \int_{-\infty}^{\infty} F(x)\, dx$$

$$\text{Variance} = \frac{1}{n} \Sigma (x_i - \bar{x})^2$$

$$= \frac{1}{n} \Sigma x_i^2 - \bar{x}^2$$

$$= E(x_i - \bar{x}) = E(x^2) - (E(x))^2$$

$$= E(x_i - E(x))^2$$

Q The probabilty mass function x have follow distrubution

x	0	1	2	3	4	5	6
$P(x)$	K	$2K$	$3K$	$4K$	$5K$	$6K$	7

i) Value of K ii) $P(1<x<4)$ iii) $P(2\leq x\leq 5)$
iv) $P(x\geq 1)$ v) $P(x<5)$ vi) mean vii) Varian

→ (i)

Probability mass function satifies cond^n

$$\Sigma P(x) = 1$$

$$P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5) + P(x=6)$$

$$= K + 2K + 3K + 4K + 5K + 6K + 7K$$

$$28K = 1$$

$$K = \frac{1}{28}$$

∴ Given Probabilty distrubution becomes

x	0	1	2	3	4	5	6
$P(x)$	$\frac{1}{28}$	$\frac{2}{14}$	$\frac{3}{28}$	$\frac{4}{28}$	$\frac{5}{28}$	$\frac{6}{28}$	$\frac{7}{28}$

(ii) $P(1<x<4) = P(x=2) + P(x=3)$

$$= \frac{2}{28} + \frac{3}{28} = \frac{5}{28}$$

(iii) $P(2 \le x \le 6) = P(2) + P(3) + P(4) + P(x=5) + P(x=6)$

$= \frac{3}{28} + \frac{4}{28} + \frac{5}{28} + \frac{6}{28} + \frac{7}{28}$

$= \frac{25}{28}$

(iv) $P(x \ge 1)$

$= 1 - P(x<1)$

$= 1 - P(x=0)$

$= 1 - \frac{1}{28}$

$= \frac{27}{28}$

v) $P(x<5)$

$\cancel{P(x=1) + P(x=}$ $= 1 - P(x \ge 5)$

$= 1 - P(x=5) + P(x=6)$

$= 1 - \frac{6}{28} + \frac{7}{28}$

$= \frac{15}{28}$

vi) mean $= \Sigma P(x)x \rightarrow E(x)$

x	$P(x)$	$xP(x)$
0	1/28	0
1	2/28	2/28
2	3/28	6/28
3	4/28	12/28
4	5/28	20/28
5	6/28	30/28
6	7/28	42/28

$= \frac{2}{28} + \frac{6}{28} + \frac{12}{28} + \frac{20}{28} + \frac{30}{28} + \frac{42}{28}$

$= \frac{112}{28} = 4$

PAGE NO.

④ Variance

$= E(x^2) - (E(x))^2$

$= \frac{532}{28} - \frac{16}{} = 3$

$x^2 p(x) = 0 \quad \frac{2}{28} \quad \frac{4}{28} \quad \frac{12}{28} \quad \frac{86}{28} \quad \frac{80}{28} \quad \frac{150}{28} \quad \frac{252}{28}$

$=$

Q. A random variable x. The density function given by $F(x) = k \cdot x \cdot (x-2)$ For $0 \le x \le 2$

→ 1) k 2) mean 3) variance 4) $P(0 \le x \le 1)$

5) $P(x > 1)$

density function satisfies condition $\int_{-\infty}^{\infty} F(x)dx = 1$

$\int_0^2 K \cdot x \cdot (x-2) = 1$

$K \int_0^2 (x^2 - 2x)\,dx = 1$

$K \left[\frac{x^3}{3} - \frac{2x^2}{2} \right]_0^2 = 1$

$K \left[\frac{8}{3} - \frac{8}{2} \right] = 1$

This wrong

$K \left(\frac{16-24}{6} \right) = 1$

$\int_0^2 K(x(2-x)) = 1$

$\int_0^2 K(2x - x^2) = 1$

$K \left[\frac{8}{2} - \frac{8}{3} \right] = 1$

$K \left[\frac{24-16}{6} \right] = 1$

$\frac{8}{6} = 1$

$K = \frac{3}{4}$

PAGE NO.:

$$F(x) = \frac{3}{4} x(2-x)$$

② mean = Expection =

$$\int_{-\infty}^{\infty} xF(x)$$

$$= \int_{0}^{2} x \cdot \frac{3}{4} x(2 \cdot x)$$

$$= \frac{3}{4} \int_{0}^{2} 2x^2 - x^3$$

$$= \frac{3}{4} \left[\frac{2x^3}{3} - \frac{x^4}{4} \right]_0^2$$

$$= \frac{3}{4} \left[\frac{16}{3} - \frac{16}{4} \right]$$

$$= \frac{3}{4} \left[\frac{64-48}{12} \right]$$

$$= \frac{\cancel{3}^{1}}{\cancel{4}} \times \frac{\cancel{16}^{4}}{\cancel{12}_{4}}$$

$$= 1$$

Variance $= E(x)^2 - (F(x))^2$

$=$

PAGE NO.:

$$E(x)^2 = \int_{-\infty}^{\infty} x^2 F(x)$$

~~Variance~~ $= \int_0^2 x^2 \frac{3}{4} x(2-x)\, dx$

$$= \frac{3}{4}\int_0^2 2x^3 - x^4\, dx$$

$$= \frac{3}{4}\left[\frac{2x^4}{\cancel{4}\,2} - \frac{x^5}{5}\right]_0^2$$

$$= \frac{3}{4}\left[8 - \frac{32}{5} - 2\right]$$

$$= \frac{3}{4} \times \frac{30-32}{5}$$

$$= \frac{6}{5}$$

Variance $= \frac{6}{5} - 1 = \frac{1}{5}$

For Continous Variable $P(0 \le x \le 1)$

$P(0 < x < 1)$

$P(0 \le x < 1)$

$P(0 < x \le 1)$ Values are same because limits are same.

PAGE NO

Q is following function density function

$F(x) = e^{-x} \quad x \geq 0$

$= 0 \quad x \leq 0$

density function must satiesfes the condition

$$\int_{-\infty}^{\infty} F(x) = 1$$

$$\int_{0}^{\infty} -e^{-x} dx = -e^{x} \Big|_{0}^{\infty} =$$

$= -(e^{\infty} - e^{0})$

$= -(0-1)$

$= 1$

Q. A variance x probababability distrubation has

x	-3	6	9	Find $E(x)$ $E(x)^2$ $E(2x+1)^2$
$P(x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$	
$x\,P(x)$	$-\frac{1}{2}$	3	3	

~~E(x)~~ =

1. $E(x) = -\frac{1}{2} + 3 + 3 = 6 - \frac{1}{2} = \frac{11}{2}$

2. $E(x)^2$

$x^2(P(x)$	$\frac{3}{2}$	18	27

$E(x^2) = \frac{3}{2} + 18 + 27$

$= \frac{3}{2} + 45$

$= \frac{3+90}{2} = \frac{93}{2}$

$\frac{21}{3}$

③ $E(2x+1)^2$

$= (4x^2 + 4x + 1)$

$= 4\,[E(x)^2 + 4\,E(x) + 1$

$= 4 \times \frac{93}{2} + 4 \times \frac{11}{2} + 1$

$= \frac{372}{\cancel{2}} + \frac{44}{2} + 1$

$= 186 + 22 + 1$

$= 209$

★ Base Theorem.

There are 3 bags 1st Contaning 1 white two red 3 green balls. 2nd bags Contain 2 white 3 red and 1 green ball. 3rd bag 3 white 1 red & 2 green ball. 2 ball drawn chosen at random. These are found to be 1red and 1 white find probability that ball so drawn Come from 2nd bag.

→

$P(A/B_1)$

$= P$ (getting 1w & 1r From bag B_1)

$= \frac{{}^1C_1}{} \cdot \frac{{}^2C_1}{{}^6C_1} = \frac{1}{6} \cdot \frac{2}{6} = \frac{1}{18} \quad \frac{2}{15}$

$P(A/B_2) = 1/6$

$P(A/B_3) = 1/12$

PAGE NO.:

$$P(B_2/A) = \frac{P(B_2) \cdot P(A/B_2)}{P(B_1)\,P(A/B_1) + P(B_2) \cdot P(A/B_2) + P(B_2)\,P(A/B_3)}$$

$$P(B_2/A) = \frac{6}{11}$$

A Survey Consumers durable for the market. It was found that 3 measures Company A, B & C have a market share of 35%, 25%, 40%. out of which 2%, 1%, 3% are not up to satisfaction a consumer buys a products and dissatify with it probability that might be from E

→

A	35%	2%
B	25%	1%
C	40%	3%

$$P(A) = P(B) = P(C) = \frac{1}{3}$$

$$P(E/A) = 2\% = \frac{2}{100}$$

$$P(E/B) = 1\% = \frac{1}{100}$$

$$P(C/B) = 3\% = \frac{3}{100}$$