Suppose, we have collection of combination of these n objects is any selection of r of the objects where order of selection does not found i.e. r combination of sets of n objects is any subset of r elements for ex-

The combinations of 3 letters out of the 4 letters a, b, c, d is a, b, c
a, b, d
a, c, d
b, c, d.

∴ The formula for combination of r things out of n is given by $^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$

$= \binom{n}{r}$

$= C(n, r)$.

Q. A student is to answer 10 out of 13 questions in an exam. How many choices has he ii) How many - if he must answer first 2 que.
iii) How many - if he must answer first or 2nd but not both. iv) How many must ans exactly 3 out of 1st 5 que.

i) $^{13}C_{10} = \frac{13!}{(13-10)!\,10!}$

$= \frac{13!}{3!\,10!}$

$\boxed{286}$

6227020800

6 × 362880

2177280

ii) $^{11}C_8 = \dfrac{11!}{(11-8)!\,8!}$

$= 165$

iii) $^2C_1 \cdot {}^{11}C_9 =$

iv) $^5C_3 \cdot {}^8C_7 =$

Q. Find the distinct permutations of letters of the word 'combination'. Also, find

i) the no. of permutations which starts with N & end with N.

ii) find permutations in which all vowels comes together.

iii) No. of permutations in which no 2 vowe comes together.

O − 2

I − 2

N − 2

Distinct permutations are

$P(11,2,2,2) = \dfrac{11!}{2!\,2!\,2!}$

$= \cancel{6652300}$.

i) As 1st & last place is fixed it remains to permute only 9 letters in which I is repetated 2 times & O is repetated 2 times, distinct permutations = $\dfrac{9!}{2!\,2!}$

ii) COMBINATION

5 vowels — 1 group.

$6 + 1 = 7.$ $(\because 11 - 5 = 6).$

$= \dfrac{7!}{2!}$ $(\because N = 2)$ ~~O – 2~~ ~~1 – 2~~.

for vowels group

$= \dfrac{5!}{2!\,2!}$

O = 2
I = 2

#

$\dfrac{7!}{2!} \cdot \dfrac{5!}{2!\,2!} =$

iii) $\underbrace{\_\ \square\ \_\ \square\ \_\ \square\ \_\ \square\ \_\ \square\ \_\ \square\ \_}_{7\ \text{places.}}$

$11 - 5 = 6.$

$= {}^{6}P_{6}$

$= 6!$

vowel ⇒ $\dfrac{{}^{7}P_{5}}{2!\,2!}$

I = 2
O = 2

Ans : $6! \cdot \dfrac{{}^{7}P_{5}}{2!\,2!}$

$=$

Q. Find no. of distinct permutations of letters of word UNIVERSITY.

i> Start with R end with S.

ii> All vowels & together.

iii> which does not start with R & end with S.

10.

$I = 2$

Distinct permutations are

$$P(10,2) = \frac{10!}{2!}$$

Start with R end with S:

$$P(8,2) = \frac{8!}{2!}$$

All vowels together:

UNIVERSITY

$10 - 4 = 6$.

$6 + 1 = 7$ group.

$$= 7! \cdot \frac{4!}{2!}$$

$= 60480$

Does not start with R & end with S.

$$\frac{10!}{2!} - \cancel{60480}\ \frac{8!}{2!}$$

$=$

26-alphbates

1000 100 10 1

6 7 7 7 3

Q. find how many 4 digit no. can be form using the digits 0, 1, 2, 3, 4, 5, 6

i) odd nos.

ii) Divisible by 5

iii)

6 7 7 7.

1 st place has 6 choices. (1st digit should not be zero).

2 —— 11 —— 7 ——

3 —— 11 ——

4 —— 11 ——.

i) $6 \times 7 \times 7 \times \cancel{7} \times 3$. —— odd nos.

ii) Divisible by 5 :

$6 \times 7 \times 7 \times 2$.

If repetations is not allowed : 6

i) Total no. of 4 digits = $6 \times \cancel{4 \times 3}$. $5 \times 4 \times 3$.

$= 6 \times 6 \times 5 \times 4$.

For odd nos :

1, 3, 5 ⇒ 3 odd nos.

$\square$ $\square$ $\square$ $\square$

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$

~~5~~ ~~5~~ ~~4~~ ~~3~~

choices choices

$\cancel{5 \times 5 \times 4 \times 3} = 300$ when 0 is ~~not~~ available for 100's place

$3 \times 5 \times 5 \times 4$

Divisible by 5 :

End should be 0, 5 ⇒ Divisible

$5 \times 5 \times 4 \times 2$

1 1

$\boxed{0, 1}$

$3 \times 6 \times 5 \times 3 = 270$ when 0 is available for 100's place.

Q. In how many ways can 10 students be divided in 3 teams. One containing ~~3~~ 4 students & others 3, 3.

$^{10}C_4 \cdot {}^{6}C_3 \cdot {}^{3}C_3$

$^{n}C_n = 1.$

$^{n}P_n = n.$

Pigeon hole Principle :

If 'n' pigeon holes are occupied by n+1 pigeons then at least one pigeon hole is occupied by more than one pigeon.

Generalized pigeon hole principle or extended " " "

If n pigeon holes are occupied by kn+1 pigeons then at least one pigeon hole is occupied by kn+1 or more pigeons. The minim no. of pigeons to be selected by $\left[\frac{k-1}{n}\right]+1$.

Q. Suppose, there & 13 students then there must be at least 2 of them where born in same month.

→ Let pigeons be student &

pigeon hole be months of year.

$n=12$

∴ Using pigeon hole principle ~~E~~

if n+1 pigeons are replaced with at least 1 pigeons hole contains more than 1 pigeons.

∴ At least 2 student must born in same month.

Q. find minimum no. of students in a class to be sure that 7 of them are born on the same day of week.

→ No. of pigeon holes = 7. = days of week = n.

No. of student = piegon.

$\therefore n = 7$

$\therefore k = ?$

~~kn+1=~~

$$\left[\frac{k-1}{n}\right] + 1 = 7.$$

$$\left[\frac{k-1}{7}\right] + 1 = 7$$

$$\frac{k-1}{7} = 6$$

$$k-1 = 42$$

$$k = 43$$

$\therefore$ minimum 43 students is required.

Q. find minimum no. of bicycles to be selected to be assure that at least 9 of them is of same colour / same brand out of given 7 different colours.

→ No. of pigeon holes = 7 = different colours.

$n = 7.$

Total pigeons = 9.

$$\left[\frac{k-1}{n}\right] + 1 = 9.$$

$$\frac{k-1}{7} = 8.$$

$$k-1 = 56$$

$$k = 57.$$

and = $\cap$.

Inclusion - Exclusion Principle :

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - [n(A \cap B) + n(B \cap C) + n(A \cap C)] + n(A \cap B \cap C).$$

Q. From given 1 to 1000 integers including both how many integers & divisible by 3, or 5 or 7

→ Let A = set of integers divible by 3

B = —— " —— 5

C = —— " —— 7

$A \cap B$ denote set of integers divisible by 3 & 5 it should be divisible by 15.

$B \cap C$ = divisible by 5 & 7 i.e. 35

$A \cap C$ = —— " —— 3 & 7 i.e. 21.

$A \cap B \cap C$ = divisible by 3 & 5 & 7 i.e. 105.

$$n(A) = \left[\frac{1000}{3}\right] = [333.333]$$

$$= 333$$

$$n(B) = \left[\frac{1000}{5}\right] = 200$$

$$n(C) = \left[\frac{1000}{7}\right] = 142.8$$

$$= 142$$

$$n(A \cap B) = \left[\frac{1000}{15}\right] = 66$$

$$n(A \cap C) = \left[\frac{1000}{21}\right] = 47$$

$$n(B \cap C) = \left[\frac{1000}{35}\right] = 28.$$

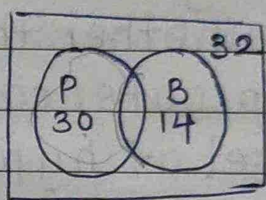
$$n(A \cap B \cap C) = \frac{1000}{105} = 9.5.$$

$n(A \cup B \cup C) = 333 + 200 + 142 - (66 + 47 + 28) + 9.$
$= 543.$

$\therefore$ 543 integers divisible by 3 or 5 or 7

Q. out of 32 people who save paper or bottles or both for recycling 30 save paper & 14 save bottles. Find the no. of people who save
1) Both
2) only paper
3) only bottles.

→ Using inclusion - exclusion principle



$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

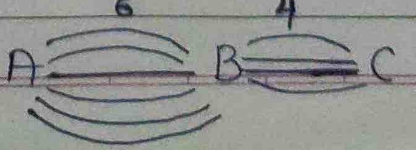
Let A = set of people saving paper
B = —— " —— bottle.

$\therefore 32 = 30 + 14 - n(A \cap B)$
$n(A \cap B) = 12.$ — Both

2) ~~P(A/B)~~
$n(A/B) = n(A) - n(A \cap B)$
$= 30 - 12$
$= 18$

3) $n(B/A) = n(B) - n(A \cap B)$
$= 14 - 12$
$= 2.$

Q. There are 6 ~~rows~~ roads bet^n A & B and 4 bet^n B & C. Find the i) no. of ways that one can drive from A to C by B ii) a round trip from A to C by the way of B iii) a ~~roup~~ round trip from A to C —— " —— without using the same road more than once.



→ i) $6 \times 4 = 24$

ii) 24×24

iii) $24 \times (5 \times 3)$

$= 24 \times 15$