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Q. Box 1 contain 5 white 6 black balls. Another box II contains 6 white & 4 black balls. A box is selected at random then a ball is drawn from it
① What is the probability that ball drawn is white ② white ball came from box I.

→ Box 1 : 5 white & 6 black = Total 11
Box 2 : 6 white & 4 black = Total 10

$P(\text{Drawing Ball From Box 1}) = \frac{1}{2} = P(A)$

$P(\text{Drawing Ball From Box 2}) = \frac{1}{2}$

$P\left(\text{Drawing white Ball From Box 1}\right) = \frac{{}^5C_1}{{}^{11}C_1} = \frac{5}{11}$

$\therefore P(\text{White ball drawn is From box I}) =$

$P(A) \times \frac{5}{11} = \frac{1}{2} \times \frac{5}{11}$

$= \frac{5}{22}$

$P(\text{White ball drawn From box I or box II})$

$= \frac{1}{2} \times \frac{5}{11} + \frac{1}{2} \times \frac{6}{10}$

$= \frac{5}{22} + \frac{6}{20}$

$= \frac{29}{55}$

Q. A purse contains 2 silver & 4 copper coins & second purse contains 4 silver & 4 copper coins. If a coin is selected at random from one of two purses what is probability that its a silver coin.

→

1st Purse : 2 silver & 4 copper coins
2nd Purse : 4 silver & 4 copper coins

P(Getting either one of the bags) = $\frac{1}{2}$

Now:

P(Getting silver coin in 1st purse) = $\frac{{}^2C_1}{{}^6C_1} = \frac{2}{6} = \frac{1}{3}$

P(Getting silver coin in 2nd Purse) = $\frac{{}^4C_1}{{}^8C_1} = \frac{4}{8} = \frac{1}{2}$

P(Getting a silver coin) $= \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{6} + \frac{1}{4}$

$= \frac{10}{24}$

$= \frac{5}{12}$

$\boxed{\therefore \text{ Probability of getting silver coin is } \frac{5}{12}}$

Q. A, B, C, in order toss a coin, First one to throw head wins. If A starts, what are their respective chances of winning?

→

Case I : A wins

i) A throws head in First attempt

$\therefore P(A \text{ win}) = \frac{1}{2}$

ii) If A throw a tail in first then for A to win B & C must throw tail for first chance

$$\therefore \begin{array}{cccc} T & T & T & H \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{array}$$

iii) If A dont win even in First two throw it will look like,

$$\begin{array}{ccccccc} T & T & T & T & T & T & H \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{array}$$

$\therefore$ Its an infinite series,

$$P(A) = \frac{1}{2} + \frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2} \dots$$

$$\therefore P(A) = \frac{1/2}{1-\frac{1}{8}}$$

$$= \frac{1/2}{7/8}$$

$$= \frac{1}{2} \times \frac{8}{7}$$

$$P(A) = \frac{4}{7}$$

Case II : B wins

simillarly, B will also have a infinite series which will look like;

$$\underset{A}{\overset{T}{\frac{1}{2}}}\,\underset{B}{\overset{H}{\frac{1}{2}}} + \underset{A}{\overset{T}{\frac{1}{2}}}\,\underset{B}{\overset{T}{\frac{1}{2}}}\,\underset{C}{\overset{T}{\frac{1}{2}}}\,\underset{A}{\overset{T}{\frac{1}{2}}}\,\underset{B}{\overset{H}{\frac{1}{2}}} + \cdots\cdots$$

$$\therefore P(B) = \frac{1/4}{1-1/8} = \frac{1/4}{7/8} = \frac{1}{4}\times\frac{8}{7} = \frac{2}{7}$$

Case III : C wins

Simillarly, C will also have a infinite series which will look like,

$$\underset{A}{\overset{T}{\frac{1}{2}}}\,\underset{B}{\overset{T}{\frac{1}{2}}}\,\underset{C}{\overset{H}{\frac{1}{2}}} + \underset{A}{\overset{T}{\frac{1}{2}}}\,\underset{B}{\overset{T}{\frac{1}{2}}}\,\underset{C}{\overset{T}{\frac{1}{2}}}\,\underset{A}{\overset{T}{\frac{1}{2}}}\,\underset{B}{\overset{T}{\frac{1}{2}}}\,\underset{C}{\overset{H}{\frac{1}{2}}} + \cdots\cdots$$

$$\therefore P(C) = \frac{1/8}{1-1/8} = \frac{1/8}{7/8} = \frac{1}{8}\times\frac{8}{7} = \frac{1}{7}$$

$$\boxed{\therefore P(A) = 4/7 \;;\; P(B) = 2/7 \;;\; P(C) = 1/7}$$

Q. A biased coin is tossed till head appears for first time. What is the probability that the number required tosses is odd.

→ From above conditions we can say that one can get head at 1^{st}, 3^{rd} & 5^{th} and so on.

∴ Lets probability of getting head P

∴ Probability of getting tail $(1-P)$

$\therefore$ It will form an infinite series of,

$$P + (1-p)^2 * p + (1-p)^4 \times p + \dots + \infty$$

$\therefore$ Its first term P, common difference $(1-p)^2$

$$\therefore \text{Sum} = \frac{p}{1-(1-p)^2} = \frac{p}{1-(1-2p+p^2)}$$

$$= \frac{p}{1-1+2p-p^2}$$

$$= \frac{1}{2-p}$$

$$\boxed{\therefore \text{Probability will be } \frac{1}{2-p}}$$

Q. The odds that book will be reviewed by three independent critic is 5 to 2, 4 to 3, 3 to 4, what is the probability that of the three review, a majority will be majority will be favourable?

→ From given,

$$P_1 = \frac{5}{7}, \quad P_2 = \frac{4}{7}, \quad P_3 = \frac{3}{7}$$

$$P_1' = \frac{2}{7}, \quad P_2' = \frac{3}{7}, \quad P_3' = \frac{4}{7}$$

For majority to be favourable at least two should be in favour,

$\therefore$ Required probability will have,

$P_1 P_2 P_3' + P_1 P_2' P_3 + P' P_2 P_3 + P_1 P_2 P_3$

$\therefore P(\text{Required}) = \frac{5}{7} \times \frac{4}{7} \times \frac{4}{7} + \frac{5}{7} \times \frac{3}{7} \times \frac{3}{7} + \frac{2}{7} \times \frac{4}{7} \times \frac{3}{7}$

$+ \frac{5}{7} \times \frac{4}{7} \times \frac{3}{7}$

$= \frac{60}{7^3} + \frac{24}{7^3} + \frac{80}{7^3} + \frac{45}{7^3}$

$$\boxed{P(\text{Req}) = \frac{209}{343}}$$

Q. The probability that a 50 year old man will be alive at 60 is 0.83, probability that a 45 year old woman will be alive at 55 is 0.87. What is the probability that a man who is 50 & his wife who is 45 will be both alive after 10 years?

→ A : Man will be alive at 60

B : Woman will be alive at 55

$A \cap B$: Both will be alive

$\therefore P(A) = 0.83, \quad P(B) = 0.97$

$\therefore P(A \cap B) = P(A) \cdot P(B)$

$= 0.83 \cdot 0.97$

$= 0.8051$

$\therefore$ Probability that both will be alive is 0.8051

UNIT : 6

• Combinatorics •

Counting Principle :-

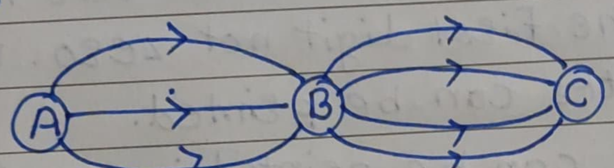
There are two basic counting principle.

1) Sum Rule

Suppose some event E can occour in m ways & a second event F can occour in 'n' ways, then E or F can occour in 'm + n' ways.

In general if $E_1, E_2, E_3 \dots, E_N$ events can occour in $n_1, n_2, n_3, \dots n_n$ ways respectively then, one of the E_i can occour in '$n_1 + n_2 + n_3 \dots + n_n$' ways

For ex :



$\therefore A \to C = 7$ ways

① Suppose there are 8 male professors & 5 female professors. a student can choose a calculus professor
$8 + 5 = 13$ ways

② Suppose E is the event of choosing a prime number less than 10 & suppos F is the event of choosing an even number less than 10 then
E can occour in 4 ways ; {2, 3, 5, 7}
F can occour in 4 ways ; {2, 4, 6, 8}

However E or F cannot occour in $4 + 4 = 8$ ways then since number 2 is common for both the events
E or F can occour in $4 + 4 - 1 = 7$ ways.

2) Product Rule :-

Suppose there is an event E which can occour in m ways. another event F independent of event E can occour in n ways. Then the combination of E & F can occour in m x n ways

In general suppose $E_1, E_2, E_3 \dots E_m$ are m independent events which can occour in $n_1, n_2, n_3 \dots, n_m$ ways respectively. Then all E_i together will happen in '$n_1 \cdot n_2 \cdot \dots \cdot n_m$' ways.

For Example:-

① Suppose a Licenceplate contain two letter followed by 3 digit with the first digit not zero. How many different plates can be printed.

Each letter can be printed in 26 different ways. the first digit is non-zero so have 9 ways, & the remaining two digit can be printed in 10 ways.

Hence the total number of ways licence plate can be printed $= 26 \times 26 \times 9 \times 10 \times 10$
$= 608400$ ways.

② In how many ways can an organisation containing 26 members elect a president, tresuror & secretory assuming no person is elected to more than one position.

For electing a president we have 26 ways.
For electing a treasuror we have $(26-1) = 25$ ways as president is already elected.
For electing a secreatory we have $(26-2) = 24$ ways as no person is elected to more than one position

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Permutation :-

Any arrangement of set of n object in a given order is called a permutation of object.

Any arrangement of any $r \le n$ of these objects in a given order is called r - permutation or permutation of n object taken r at a time.

For example:-

Permutation of the letters of the word TIME taken all at a time is -

The r permutation is denoted by ${}^nP_r$ or $P(n,r)$ & is defined as,

$$\boxed{P(n,r) = {}^nP_r = \frac{n!}{(n-r)!}}$$

There are n! permutations of n objects taken all at a time.

Ex.

Permutation with Repetation :-

Let the number of permutation of n objects of which n_1 are of one type & n_2 are of some other type... n_k are of some other type then permutation of n items taken at a time is given by,

$$\boxed{P(n,n) = \frac{n!}{n_1!.n_2!...n_k!}}$$

For Ex : Permutations of the letter of the word MOM

$$= \frac{3!}{2!} = \frac{6}{2} = 3$$

The different permutation of the five letter word DADDY can be done in $\frac{5!}{3!} = \frac{120}{6} = 20$ ways

Combination :-

Suppose we have, a collection of ~~10~~ n objects, the combination of these 'n' object taken 'r' at a time is any selection of 'r' of the objects where order of selection does not count i.e. 'r' combination of 'n' object is any subset of 'r' elements.

For example :-

The combinations of three letters of out of 4 letters of ABCD are.

a b c
a b d
a c d
b c d

∴ The formula for combination of r things out of 'n' is given by,

$$C(n,r) = {}^nC_r = \frac{n!}{(n-r)!\,r!} = \binom{n}{r}$$

Ex : 1) A student is to answer 10 out of 13 questions in an exam.

i) How many choices has he?

ii) How many if he must answer first two questions

iii) How many if he must answer first or second but not both.

iv) How many if he must answer exactly 3 out of first 5 question.

→ ① $^{13}C_{10} = \dfrac{13!}{3! \times 10!} = \dfrac{13 \times \overset{6}{\cancel{12}} \times 11}{3 \times \cancel{2} \times 1} = 13 \times 22 = 286$

② $^{11}C_8 = \dfrac{11!}{3! \times 8!} = \dfrac{11 \times \overset{5}{\cancel{10}} \times \overset{3}{\cancel{9}}}{\cancel{3} \times \cancel{2}} = 11 \times 15 = 165$

③ $^{2}C_1 \times {}^{11}C_9 = 2 \times \dfrac{11 \times 10}{2} = 110$ ways

④ $^{5}C_3 \times {}^{8}C_7 = \dfrac{5 \times \overset{2}{\cancel{4}}}{2} \times 8 = 80$ ways

9. Find the distinct permutations of the letters of the word COMBINATION. Also find,
① The number of permutation which start with N & end with N
② Find the permutation in which all vowels come together.
③ Number of permutation in which no two vowel come together.

→

COMBINATION

O - 2 , I - 2, N - 2

① ∴ Distinct Permutations $= P(11; 2, 2, 2)$
$= \dfrac{11!}{2!\,2!\,2!}$
$= 4989600$

② As first & last place is fixed, it remains to permute only 9 letters in which I is repeated 2 time & O is repeated 2 times.
∴ Distinct Permutation $= P(9; 2, 2)$
$= \dfrac{9!}{2!\,2!}$
$= 90720$

③ As we are considering vowels should be together we will make a group of vowels. So now we are left to permute 7 places in which N is repeated 2 times. But in group we can permute 5 places in which I & O is repeated 2 times each.

$$\therefore \text{Distinct Permutation} = \frac{7!}{2!} \times \frac{5!}{2! \times 2!}$$

$$= \underline{75600}$$

④ We dont want any vowel together, so we will place vowels in between consonent so we will have 6 place for consonent & 7 places for vowels. So we will permute 6 in which N is repeated 2 times & in 7 places ~~O~~, I is repeated 2 ti

$$\therefore \frac{6!}{2!} \times$$

Q. Find the number of distinct permutations of the letters of the word UNIVERSITY

① start with R end with S

② All vowels are together

③ which does not start with R end with S

I - 2,

① Now end positions are fixed so we have to permute 8 ~~10~~ places in which I is repeated 2 times

$\therefore$ Distinct Permutation $= \dfrac{8!}{2!} = 20160$

② All vowels together so we make group of vowels

$\therefore$ Places $6 + 1 = 7$

$\therefore$ Distinct Permutation $= \dfrac{7! \times 4!}{2!} = 60480$

③ ~~One place~~ For this we subtract the permutations which start with R & end with S

$\therefore \dfrac{10!}{2!} - \dfrac{8!}{2!} = 1794240$

Q. Find how many 4 digit number can be formed using the digit ~~6~~ 0, 1, 2, 3, 4, 5, 6 which are,

① odd

② Divisible by 5

$\rightarrow$

$\therefore$ Distinct Numbers $= 6 \times 7 \times 7 \times 7 = 2058$

$\therefore$ Odd Numbers $= 6 \times 7 \times 7 \times 3 = 882$

Divisible by 5 $= 6 \times 7 \times 7 \times 2 = 588$

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If repete is not allowed

1) Distinct Numbers = $6 \times 6 \times 5 \times 4 = 720$

2) Odd Numbers = ~~$5 \times 6 \times 5 \times 3$~~

i) When 0 is available for 1000's place.

$= 3 \times 6 \times 5 \times 3 = 270$

ii) When 0 is not available for 1000's place

$= 3 \times 5 \times 5 \times 4 = 300$

3) Divisible by 5

$\underline{5}\ \underline{5}\ \underline{4}\ \underline{2} = 5 \times 5 \times 4 \times 2 = 200$

Q. In how many ways can 10 students be divided into 3 teams one containing 4 students & others 3.

→ $^{10}C_4 \times {}^6C_3 \times {}^3C_3 = 4200$

Pigeon-Hole Principle :-

If n pigeons hole are occupied by '$n+1$' pigeons, then at least one pigeon hole is occoupied by more than one pigeon.

Generalized pigeon hole principle extended pigeon h.p. If 'n' pigeon holes occoupied by $kn+1$ pigeons then at least 1 pigeon hole is occoupied by $k+1$ pigeon or more pigeons.

The minimum number of pigeons to be selected are given by Formula, $\left[\frac{k-1}{n}\right]+1$

Suppose there are 13 student then there must be at least 2 student who are born in same month.

Let pigeons be the student & pigeons hole be months of year $\therefore n = 12$

$\therefore$ Using pigeon hole principle If $n+1$ pigeon are to be placed in n pigeon hole. Atleast 1 pigeon hole contain more than 1 pigeon.

$\therefore$ Two students are born in same month.

Q.1] Find the minimum number of students in a class to be sure that 7 of them are born on the same day of the week

→ No. of pigeon holes = 7

Let the number of students be pigeon & days of the week are pigeon hole. $\therefore n = 7$

$$\therefore\ 1 + \left[\frac{k-1}{7}\right] = 7$$

$$\frac{K-1}{7} = 6$$

$$K-1 = 42$$

$$\boxed{K = 43}$$

Q. Find the minimum no. of bicycles to be selected to be ~~the~~ assure that at least 9 of them are of same colour out of given 7 diff. colours.

→

pigeon hole = n = 7

$$\left[\frac{K-1}{7}\right] + 1 = 9$$

$$\frac{K-1}{7} = 8$$

$$K-1 = 56$$

$$\boxed{\therefore K = 57}$$

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - [n(A \cap B) + n(A \cap C)] + n(A \cap C) + n(A \cap B \cap C)$

Q. From given 1 to 1000 integers including both how many integers are divisible by 3 or 5 or 7.

→ Let A : Set of integer divisible by 3

B : Set of integer divisible by 5

C : Set of integer divisible by 7

$A \cap B$: Set of integer divisible by 3 & 5

$B \cap C$: Set of integer divisible by 5 & 7

$A \cap C$: set of integer divisible by 3 & 7
$A \cap B \cap C$: set of integer divisible by 3, 5 & 7.

$\therefore n(A) = \frac{1000}{3} = 333$

$n(B) = \frac{1000}{5} = 200$

$n(C) = \frac{1000}{7} = 142$

$n(A \cap B) = \frac{1000}{15} = 66$

$n(B \cap C) = \frac{1000}{35} = 28$

$n(A \cap C) = \frac{1000}{21} = 47$

$n(A \cap B \cap C) = \frac{1000}{105} = 9$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - [n(A \cap B) + n(B \cap C) + n(A \cap C) + n(A \cap B \cap C)]$

$= 333 + 200 + 144 - [66 + 28 + 47 + 9]$

$= 527$

$\therefore$ From given integer, there are 527 integer which are divisible by 3 or 5 or 7.

Camlin

Q. Out of 32 people who save paper or bottles or both for recycling ~~is~~, 30 save paper & 14 save bottles. Find number of people who save
(1) both (2) Only paper (3) Only Bottles.

→

1) Using inclusion-exclusion principle,

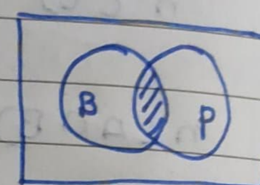
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$32 = 30 + 14 - n(A \cap B)$

$32 = 44 - n(A \cap B)$

$n(A \cap B) = 44 - 32$

$\boxed{\therefore n(A \cap B) = 12}$



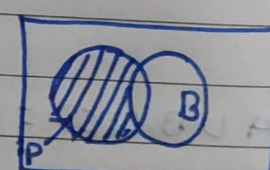
∴ People which save both are 12.

2) Now people who save only paper

$n(A/B) = n(A) - n(A \cap B)$

$= 30 - 12$

$= 18$



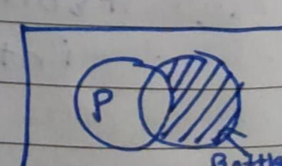
∴ 18 people save paper only

3) Now people who save only bottles

$n(B/A) = n(B) - n(A \cap B)$

$= 14 - 12$

$= 2$



∴ 2 people save bottles only.