

## *VCG Auction Experiment*

### **1. Problem Overview**

The given problem is a combinatorial auction where advertising spaces on lecture slides are being auctioned off. The professor offers two types of ad spaces: a top banner and a sidebar. There are different types of bidders, some of whom are interested only in one space, while others require both. The auction uses the Vickrey-Clarke-Groves (VCG) mechanism for allocation and pricing, which incentivizes truthful bidding. Additionally, a Second Price Auction (SPA) mechanism is implemented for comparison.

The goal is to determine:

- The allocation of the ad spaces to bidders using the VCG auction.
- The payments and total revenue for the auctioneer in VCG and SPA scenarios.
- The evaluation of VCG truthfulness and revenue properties by introducing experiments, including agents with multiple identities.

### **2. VCG Auction (Part A)**

#### **2.1 Code Explanation**

- The class `VCGAuction` is responsible for managing bids and determining the allocation of ad spaces.
- The `find_optimal_allocation` method identifies the allocation that maximizes the overall value (social welfare). It compares two cases:
  1. Allocating both spaces to a single bidder interested in both spaces.
  2. Allocating the top banner and sidebar to two separate bidders.

The allocation yielding the highest value is chosen.

- The `compute_vcg_payments` method calculates the payments for the winners. For each winning bidder:
  1. The auction is run again without this bidder to determine the social welfare without them.
  2. The difference in social welfare with and without the bidder determines their VCG payment.

#### **2.2 Input Example**

The input JSON file (`example_input.json`) contains the bids from different agents:

```
json
{
  "bids": [
    {"bidder_id": "A1", "value": 100, "space_type": "b"},
```

```
{
  "bidder_id": "B1", "value": 60, "space_type": "t"},
  {"bidder_id": "C1", "value": 50, "space_type": "s"},
  {"bidder_id": "D1", "value": 80, "space_type": "b"}
}
```

- A1 and D1 bid for both spaces (b), while B1 bids for the top banner (t) and C1 bids for the sidebar (s).

## 2.3 VCG Auction Results

When the VCG auction is executed, the following allocation is found:

- B1 is allocated the top banner.
- C1 is allocated the sidebar.

The VCG payments are calculated based on the value difference when each bidder is removed:

- Bidder B1 pays 50.
- Bidder C1 pays 40.

The total revenue from the VCG auction is 90.

```
PS C:\Users\sharm\Desktop\B21CS095_CSL7650_A2> python .\vcg_auction.py
Experiment Results:
{
  "vcg_allocation": {
    "top": "B1",
    "sidebar": "C1"
  },
  "vcg_payments": {
    "B1": 50,
    "C1": 40
  },
  "vcg_revenue": 90,
  "spa_winner": "A1",
  "spa_payment": 80
}
PS C:\Users\sharm\Desktop\B21CS095_CSL7650_A2>
```

## 2.4 Experiment Results

The following results were obtained:

```
json
{
```

```

"vcg_allocation": {
  "top": "B1",
  "sidebar": "C1"
},
"vcg_payments": {
  "B1": 50,
  "C1": 40
},
"vcg_revenue": 90
}

```

```

PS C:\Users\sharm\Desktop\B21CS095_CSL7650_A2> python .\action_test_suite.py

Key Findings:

1. Sybil Attack Analysis:
Original VCG Revenue: 50
Sybil Attack VCG Revenue: 110

2. Revenue Comparison:
Minimum SPA/VCG Revenue Ratio: 0.58
Average SPA/VCG Revenue Ratio: 1.18

3. Truthful Bidding Analysis:
True Value: 150
Truthful Bid Payment: 100
Overbid Payment: 100
Underbid Payment: 100
PS C:\Users\sharm\Desktop\B21CS095_CSL7650_A2>

```

### 3. Second Price Auction (Part C)

#### 3.1 Code Explanation

The SecondPriceAuction class simulates an auction where the highest bidder wins both ad spaces but pays the amount of the second-highest bid.

- The run\_auction method sorts the bids and allocates both spaces to the highest bidder. The payment is equal to the second-highest bid.

#### 3.2 SPA Results

In the given experiment:

- Bidder A1 wins the auction (highest bid of 100 for both spaces).
- A1 pays the second-highest bid of 80.

The output of the second price auction is:

```
json
{
  "spa_winner": "A1",
  "spa_payment": 80
}
```

## **4. Analysis (Part B and Part C Experiments)**

### **4.1 Non-Dominant Strategy Truthfulness**

By experimenting with a bidder submitting multiple identities, we can test whether VCG is dominant-strategy truthful. In scenarios where an agent creates a second identity and bids using both identities, they might achieve a higher utility than when bidding truthfully with only one identity. This demonstrates that the VCG mechanism may not be dominant-strategy truthful when such identity manipulation is possible.

### **4.2 Comparison of Revenue**

The second-price auction consistently generates at least half the revenue of the VCG auction when bidders submit truthful reports. In the current experiment:

- VCG revenue = 90
- SPA revenue = 80

## **5. Conclusion**

- The VCG auction provides a truthful mechanism (without identity manipulation) and optimizes social welfare.
- The second-price auction, though simpler, yields comparable revenue and maintains a dominant strategy of truthful bidding.

```
PS C:\Users\sharm\Desktop\B21CS095_CSL7650_A2> python .\auction_analysis.py
```

**Key Findings:**

**1. Sybil Attack Analysis:**

Original VCG Revenue: 50

Sybil Attack VCG Revenue: 110

**2. Revenue Comparison:**

Minimum SPA/VCG Revenue Ratio: 0.58

Average SPA/VCG Revenue Ratio: 1.18

**3. Truthful Bidding Analysis:**

True Value: 150

Truthful Bid Utility: 100

Overbid Utility: 100

Underbid Utility: 100

```
PS C:\Users\sharm\Desktop\B21CS095_CSL7650_A2> |
```

**Summary of the key findings from output:**

**1. Sybil Attack Analysis:**

- Original VCG Revenue: 50
- Sybil Attack VCG Revenue: 110
- This result suggests that the Sybil attack significantly increased the revenue for the auctioneer. In the original scenario, the VCG revenue was only 50, but after splitting the identity of a bidder, the revenue more than doubled. This demonstrates how vulnerable the VCG mechanism can be to such attacks.

**2. Revenue Comparison:**

- Minimum SPA/VCG Revenue Ratio: 0.58
- In one of the test cases, the second-price auction generated only 58% of the revenue compared to the VCG auction. This shows that, in certain scenarios, the second-price auction may result in much lower revenue.
- Average SPA/VCG Revenue Ratio: 1.18
- On average, the second-price auction generated about 18% more revenue than VCG across the test cases. This indicates that while VCG typically optimizes for efficiency, the second-price auction may sometimes produce better monetary outcomes.

**3. Truthful Bidding Analysis:**

- True Value: 150
- Truthful Bid Utility: 100
- Overbid Utility: 100
- Underbid Utility: 100
- In this case, it seems that bidding truthfully, overbidding, or underbidding resulted in the same utility (payment of 100). This may indicate that in this specific test scenario, the second-price auction was resilient to deviations in bidding strategy, reinforcing the idea that truthful bidding is a dominant strategy in such auctions.

## **Insightful results for the Sybil attack, revenue comparison, and truthful bidding scenarios.**

### **1. Sybil Attack Analysis:**

- Original VCG Revenue: 50
- Sybil Attack VCG Revenue: 11
- In this case, introducing a Sybil attack (where a single entity splits into two identities) increased the VCG revenue, showcasing a vulnerability where bidders can manipulate the system by splitting bids.

### **2. Revenue Comparison:**

- Minimum SPA/VCG Revenue Ratio: 0.58
- The lowest second-price auction revenue was 58% of the VCG revenue, indicating that sometimes the second-price auction generates significantly less revenue.
- Average SPA/VCG Revenue Ratio: 1.18
- On average, second-price auctions yielded slightly more revenue than VCG auctions, suggesting they can sometimes outperform VCG, depending on the bidding scenarios.

### **3. Truthful Bidding Analysis:**

- True Value: 150
- Truthful Bid Payment: 100
- Overbid Payment: 100
- Underbid Payment: 100
- In second-price auctions, whether the bidder truthfully bids their value, overbids, or underbids, the payment remains the same. This confirms the property of second-price auctions where bidders are incentivized to bid truthfully because the payment does not change based on their bid.

```
PS C:\Users\sharm\Desktop\B21CS095_CSL7650_A2> python .\experiments.py
```

```
Sybil Attack Experiment Results:
```

```
{
  "truthful": {
    "allocation": {
      "top": "B1",
      "sidebar": "C1"
    },
    "payments": {
      "B1": 30,
      "C1": 20
    }
  },
  "sybil": {
    "allocation": {
      "top": "B1",
      "sidebar": "C1"
    },
    "payments": {
      "B1": 60,
      "C1": 50
    }
  }
}
```

```
Revenue Comparison Experiment Results:
```

```
[
  {
    "vcg_revenue": 80,
    "spa_revenue": 80,
    "revenue_ratio": 1.0
  },
  {
    "vcg_revenue": 125,
    "spa_revenue": 140,
    "revenue_ratio": 1.12
  },
  {
    "vcg_revenue": 90,
    "spa_revenue": 70,
    "revenue_ratio": 0.7777777777777778
  }
]
```

```
PS C:\Users\sharm\Desktop\B21CS095_CSL7650_A2> █
```

The test suite worked well in illustrating the core concepts, including potential weaknesses in VCG auctions due to strategic behavior like Sybil attacks.