DEPARTMENT OF MATHEMATICS, IIT - GUWAHATI
Odd Semester of the Academic year 2015 - 2016
MA 322 Assignment/Problem sheet 8
Instructor: Dr. J. C. Kalita
Due before midnight of 1 November 2015

This problem concerns the diffusion equation

$$Du_{xx} = u_t + bu, \text{ for } \begin{cases} 0 < x < 1 \\ t > 0 \end{cases}$$

where $u(0,t) = u(1,t) = 0$ and $u(x,0) = g(x)$. Also D, b are positive constants. The exact solution is

$$u(x,t) = \sum_{n=1}^{\infty} A_n e^{-(b+D\lambda_n^2)t} \sin(\lambda_n x),$$

where $\lambda_n = n\pi$ and $A_n = 2\displaystyle\int_0^1 g(x)\sin(\lambda_n x)dx$.

1. Derive an explicit and implicit finite difference approximation to the problem that has truncation error $O(\Delta x^2) + O(\Delta t)$.
2. Assuming $g(x) = \sin(\pi x)$, $D = \frac{1}{10}$ and $b = 1$, use the methods derived by you to compute the solution at $t = 1$ using $\Delta t = \frac{1}{4}, \frac{1}{8}$ and $\frac{1}{16}$. On the same axes plot the exact solution at $t = 1$ and the three numerical solutions, one for the explicit and the other for the implicit method. Make sure to state how you decide on how many points to use in the x-axis.
3. Assuming $g(x) = \sin(\pi x)$, $D = \frac{1}{10}$, $b = 1$, plot the maximum error at $t = 1$ as function of M where $\Delta t = \frac{1}{M}$ and $\Delta x = \frac{1}{10}$ for $M = 4, 8, 16, 32$. On the same axes also plot the maximum error at $t = 1$ for $M = 4, 8, 16, 32$ and $\Delta x = \frac{1}{20}$. Explain the behaviour of these two curves using the stated truncation error.
4. Compare graphically your best numerical solutions with the exact solutions at time stations $t = 0.1$, 0.5, 1.0 and 5.0 by choosing suitable Δt (your choice does not necessarily have to be the same as in exercises 2 and 3). What can you say about the physics of the problem by observing the graphs?

HAPPY COMPUTING