DEPARTMENT OF MATHEMATICS, IIT - GUWAHATI
Odd Semester of the Academic year 2015 - 2016
MA 322 Assignment/Problem sheet 7
Instructor: Dr. J. C. Kalita
Due before midnight of 11 October 2015

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1. The one-dimensional radiation problem is described by

$$T' = -\alpha(T^4 - T_a^4) = f(t, T), \;\; T(0) = T_0 = 2500 \;\text{ and }\; T_a = 250.$$

(a) Find the exact solution of the above problem at $t = 1, 2, ..., 10$ by using secant method for the following equation

$$\tan^{-1}\left(\frac{T}{T_a}\right) - \tan^{-1}\left(\frac{T_0}{T_a}\right) + \frac{1}{2}\ln\left(\frac{(T_0 - T_a)(T + T_a)}{(T - T_a)(T_0 + T_a)}\right) = 2\alpha T_a^3 t$$

with $\alpha = 2.0 \times 10^{-12}$.

(b) Solve the ODE numerically by the (i) Euler explicit, (ii) Euler implicit, (iii) Modified Euler and (iv) Fourth order Runge-Kutta methods using $\Delta t = 2, 1, 0.05, 0.025$ and 0.01.

(c) Compare graphically your results obtained with the above time steps for those methods with the exact results at the points $t = 1, 2, ..., 10$. This means you must have four figures (one each for one method) with the graphs of the exact solution along with the solution obtained through those time steps.

(d) Are the rates of convergence as predicted by the order of the schemes?

2. (a) Solve the ODE of problem 1 numerically by the (i) Adams-Bashforth explicit and (ii)Adams-Bashforth implicit methods using $\Delta t = 2, 1, 0.05, 0.025$ and 0.01.

(b) Compare graphically your results obtained with the above time steps for those methods with the exact results obtained through the solution of problem 1 at the points $t = 1, 2, ..., 10$. This means you must have two figures (one each for one method) with the graphs of the exact solution along with the solution obtained through those time steps.

(c) Are the rates of convergence as predicted by the order of the schemes?

HAPPY COMPUTING