DEPARTMENT OF MATHEMATICS, IIT - GUWAHATI
Odd Semester of the Academic year 2015 - 2016
MA322 & MA311M   Assignment/Problem sheet 5
Instructor: Dr. J. C. Kalita
Due before midnight of 6 September 2015

1. Find the Newton's forward interpolating polynomial of degree 10 that interpolates the function $\tan^{-1} x$ at 11 equally spaced points in the interval $[0, 6]$. Print the coefficients of the polynomial. Compute and print the difference between the polynomial and the function at 33 equally spaced points in the interval $[0, 8]$. What conclusion can be drawn?

2. Using Thomas algorithm, solve the tridiagonal system of equations given by

$$T_{i-1} - (2 + \alpha^2 \Delta x^2)T_i + T_{i+1} = 0$$

for $1 \leq i \leq 7$ with $T_0 = 0.0$, $T_8 = 100.0$, $\alpha = 2.0$ and $\Delta x = 0.125$. Then solve the equations for $\Delta x = 0.0625$ with $T_{i_{\max}} = 100.0$. Graph the two solutions and comment on them.

(**Look for the Thomas algorithm in my extra material (THOMAS) page.**)

3. (a) Let S be the cubic spline that interpolates $f(x) = \dfrac{1}{1 + x^2}$ at 41 equally spaced knots in the interval $[-5, 5]$. Evaluate $S(x) - f(x)$ at 101 equally spaced points in the interval $[0, 5]$.

(b) Repeat first part of 3 (a) with Newton's divided difference polynomial $P(x)$ at 11 equally spaced points. Graph $P(x)$ and $f(x)$ and give your comments.

You should submit the answers to the following questions by typing your answers preferably in Latex. You may calculate manually or write your own programmes to find the answers.

4. A car traveling along a straight road is clocked at a number of points. The data from the observations are given in the following table, where the time is in seconds, the distance is in feet, and the speed in feet per second.

Time	0	3	5	8	13
Distance	0	225	383	623	993
Speed	75	77	80	74	72

(a) Use an interpolating polynomial of your choice to predict the position of the car and its speed when $t = 10s$.
(b) Does the car ever exceed a 55 mile/hour speed limit on the road? If so, what is the first time the car exceeds this speed?
(c) What is the predicted maximum speed of the car?