DEPARTMENT OF MATHEMATICS, IIT - GUWAHATI
Odd Semester of the Academic year 2015 - 2016
MA322 & MA 311M Assignment/Problem sheet 4
Instructor: Dr. J. C. Kalita
Due before midnight of 30 August 2015

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1. Consider the nonlinear system

$$f(x, y) = x^2 + y^2 - 25 = 0, \quad g(x, y) = x^2 - y - 2 = 0$$

Using software package that has 2D plotting capabilities, illustrate what is going on in solving such a system by plotting $f(x, y)$ and $g(x, y)$ and show their intersection with the xy-plane. Determine approximate roots of these equations from the graphical results.

2. Solve the above pair of simultaneous nonlinear equations by first eliminating y and then solving the resulting equation in x by Newton's method. Start with initial value $x_0 = 1$. Likewise solve the system

$$\sin(x + y) = e^{(x-y)}, \quad \cos(x + 6) = x^2y^2$$

with initial guess $(x_0, y_0) = (-0.01, -0.01)$.

3. Find the root of the equation $e^x - x - 1 = 0$ by modified Newton's method.

4. Newton's method can be defined for the equation $f(z) = g(x, y) + ih(x, y)$, where $f(z)$ is an analytic function of the complex variable $z = x + iy$ (x and y are real) and $g(x, y)$ and $h(x, y)$ are real functions for all x and y. The derivative $f'(z)$ is given by $f'(z) = g_x + ih_x = h_y - ig_y$ because the **Cauchy-Riemann Equations** $g_x = h_y$ and $h_x = -g_y$ hold. Show that Newton's method

$$z_{n+1} = z_n - \frac{f(z_n)}{f'(z_n)}$$

can be written in the form

$$x_{n+1} = x_n - \frac{gh_y - hg_y}{g_xh_y - g_yh_x}$$

$$y_{n+1} = y_n - \frac{hg_x - gh_x}{g_xh_y - g_yh_x}$$

Here all the functions are evaluated at $z_n = x_n + iy_n$.

5. Using problem 4, find a complex root of each of the following:
(a) $z^3 - z - 1 = 0$
(b) $2z^3 - 6(1 + i)z - 6(1 - i) = 0$.