DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

End-Sem Assignment   Odd Semester of 2015-2016

MA322: SCIENTIFIC COMPUTING

Instructor: Dr. Jiten C Kalita.

Instructions:

1. This question paper contains 2 questions in parts.
2. Be advised that the assignment part accounts for 25% of your overall grade.
3. The Statements you make use of to solve the problems **MUST** be clearly written.
4. You must return the assignments (**printed or written hard-copies as well as the programmes through e-mail to your TA**) before the commencement of your end-sem exam on 22nd November 2015.
5. The front-page of the assignment report must bear your group name along with the names, roll numbers and signatures of the group members.

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1. A thin rectangular homogeneous thermally conducting plate lies in the xy-plane defined by $0 \leq x \leq 4$, $0 \leq y \leq 1$. The edge $y = 0$ is maintained at temperature $200x(x-4)$, while the remaining edges are held at 0^o. The other faces are insulated and no sources and sink are present.
Set the partial differential equation governing the steady-state temperature $T(x, y)$ along with the boundary conditions. Then solve it numerically using a known finite difference scheme. Use the Gauss-Seidel and the conjugate gradient methods as the iterative solver. You must experiment
(a) with $\Delta x = 0.1,\ \ 0.05,\ \ 0.025$
(b) with three different grid aspect ratios given by $\beta = \dfrac{\Delta x}{\Delta y}$ (preferably $1,\ <1,\ >1$).
The analytical solution of this problem is given by

$$T(x,y) = \sum_{n=1}^{\infty} \operatorname{cosech}\left(-\frac{n\pi}{4}\right) \frac{12800}{n^3\pi^3}[(-1)^n - 1] \sin\left(\frac{n\pi}{4}x\right) \sinh\left[\frac{n\pi}{4}(y-1)\right]$$

Compare graphically the analytical solution along the vertical centerline with the numerical ones. (There should be three graphs, one for each β. Each figure will have the analytical solution along the vertical centerline with the numerical ones obtained for $\Delta x = 0.1,\ \ 0.05,\ \ 0.025$.)

What can you comment upon the the efficiency of the iterative solvers? Perform further experiment on the Gauss-Seidel iterative solver by using **SOR** and comment upon the optimum ω for each of the grids. (25)+(5)+(20).

2. Solve numerically the following convection-diffusion equation for $Re = 10$, 50 and 100. Use central differences for discretizing the derivatives and then solve the system of the linear algebraic equations by Thomas Algorithm.

$$-\frac{\partial^2 u}{\partial x^2} + Re\frac{\partial u}{\partial x} = 0, \quad 0 \leq x \leq 1$$

with boundary conditions $u(0) = 0$ and $u(1) = 1$. The analytical solution to this problem is

$$u(x) = \frac{e^{Rex} - 1}{e^{Re} - 1}$$

Use $\Delta x = 0.1,\ 0.05, 0.025$. Compare graphically (There should be three graphs, one for each Re. Each figure will have the analytical solution with the numerical ones obtained for $\Delta x = 0.1,\ \ 0.05,\ \ 0.025$.) the analytical and the numerical solutions obtained by you. What observation can be made based upon your numerical results with increasing Re and decreasing Δx? (30)+(20).

NOTE: Your report must include write-ups on the finite difference methods being constructed and used for solving the two partial differential equations numerically.

HAPPY COMPUTING.