EE450 Discussion #5

- Shannon’s Theorem
- Modulations
- Multiplexing

Shannon’s Theorem

- $C = B \log_2(1 + SNR)$
- Theoretical Maximum Capacity that can be obtained on a line
- Sets an Upper Bound on the capacity given the conditions
  - Used for Calculating the
    - Signal to Noise Ratio – Given the Bandwidth and capacity of the channel
    - Bandwidth - Given the SNR and Channel Capacity
    - Capacity - Given the SNR and the Bandwidth

Problem #1

- What SNR is needed to put a T-1 carrier on a 50 khz line?
  - What do we know?
    - T-1 Capacity = 1.544 Mbps
    - Bandwidth = 50 KHz
  - Move them around and Solve:
    - 1,544,000 = 50,000 log 2 (1+SNR)
    - 2^30.88 –1 = SNR

Continued

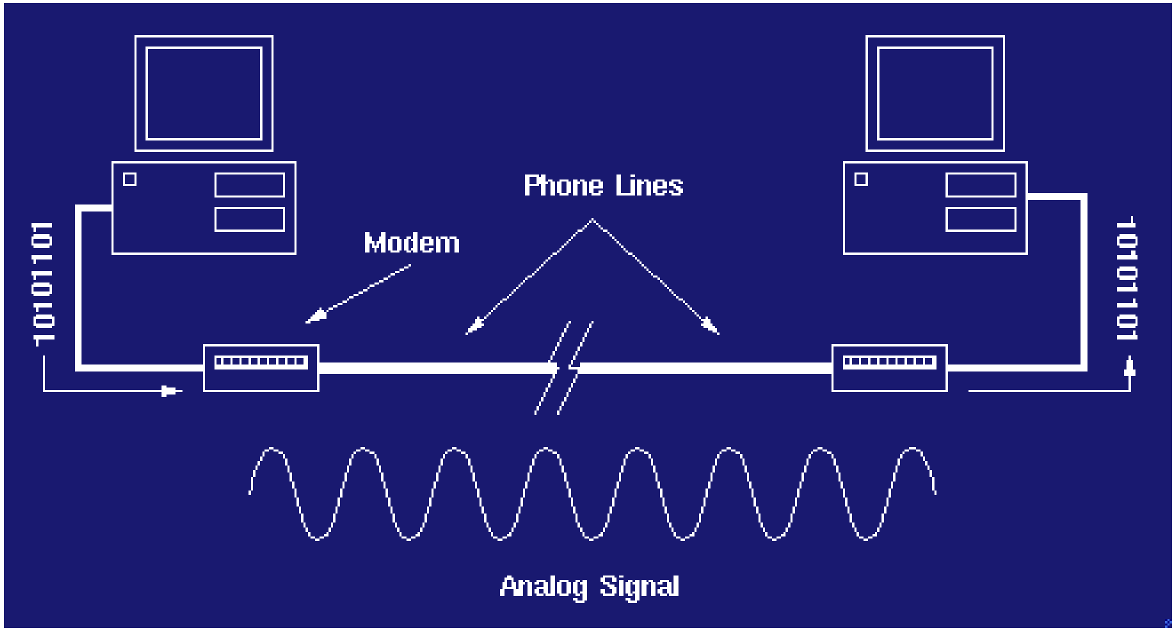
- So SNR = 1976087931
  - SNR is typically measured in DB
    - Use SNR dB = $10 \log_{10}(SNR)$
  - In this case
    - SNR dB = $10 \log_{10}(1976087931)$
    - SNR aprox. 92.9 dB
  - However you must NOT plug SNR into Shannon's theorem in dB format

Problem #2

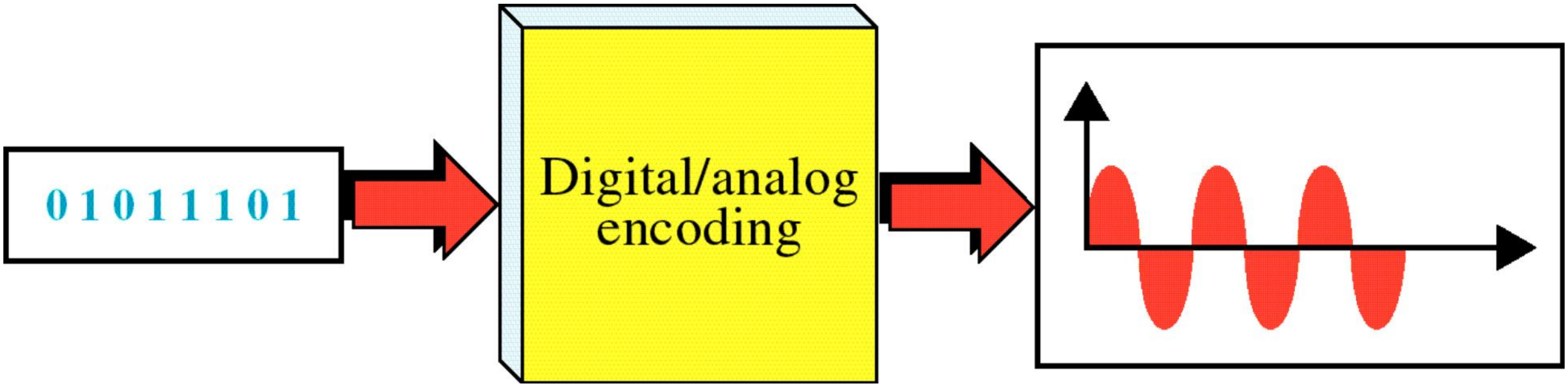
- Calculate the maximum rate supported by a telephone line with BW of 4 KHz. When the signal is 10 volts, the noise is 5 milivolts.
- SNR=Signal power/Noise Power
- Power is proportional to square of the voltage
- S/N = (10^2)/(0.005^2) = 4000000
- B = 4000 Hz
- $C = B \log_2 (1+S/N)$
  - Reminder: $\log_2 x = \ln x / \ln 2$
- $C = 4000 \log_2 (1+4000000) = 87726$ bps

Review on Modems

- Modem Stands for
  - MOdulator / DEModulator
- Uses Sine wave As the carrier Signal



Digital to Analog Encoding

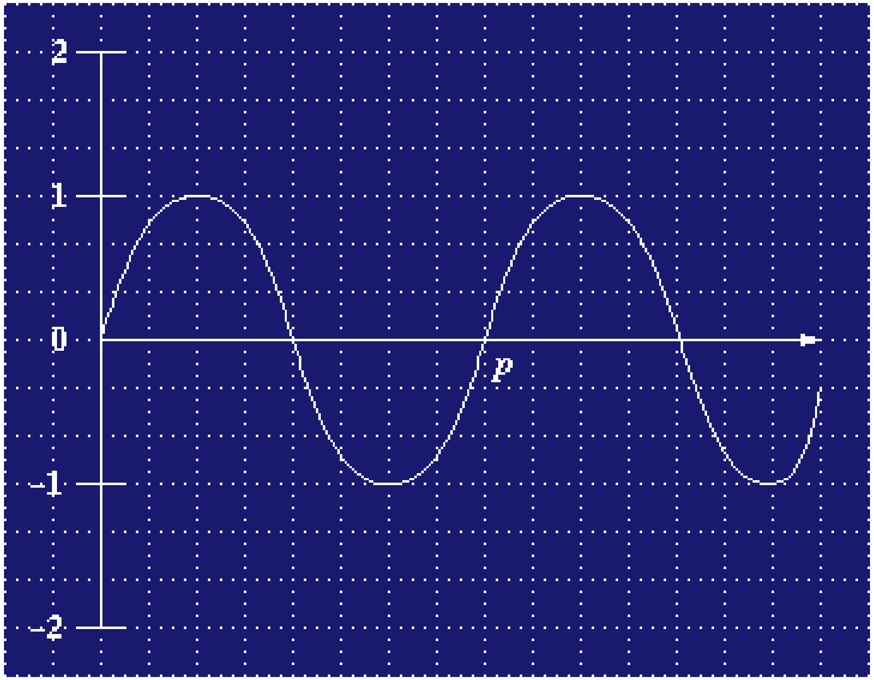


Modulation

- Need to Encode Digital Data in an Analog Signal
- In modem transmission we use different techniques for modulation
  - Amplitude Modulation
  - Frequency Modulation
  - Phase Shift

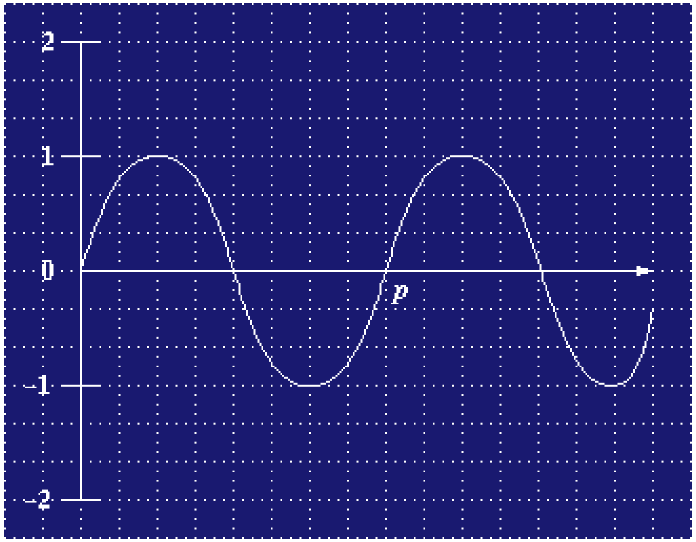
Amplitude Modulation

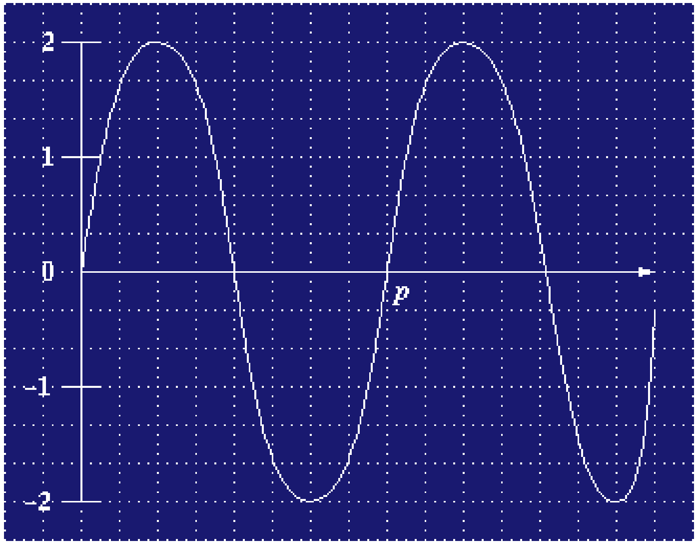
- Varies the Amplitude of the Signal



Amplitude Modulation

- Same Signal Greater Amplitude

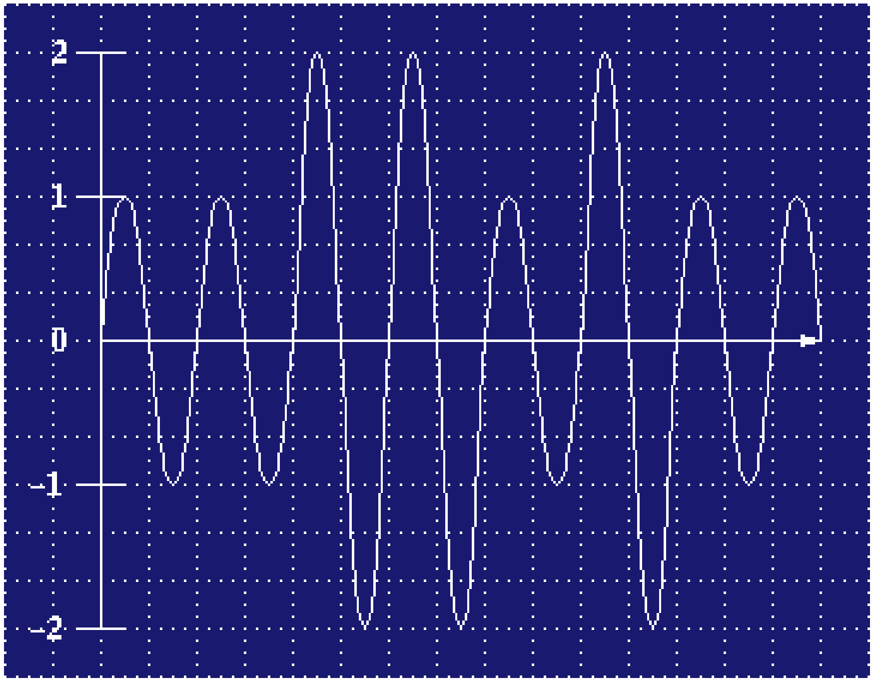




Amplitude = 1

Amplitude = 2

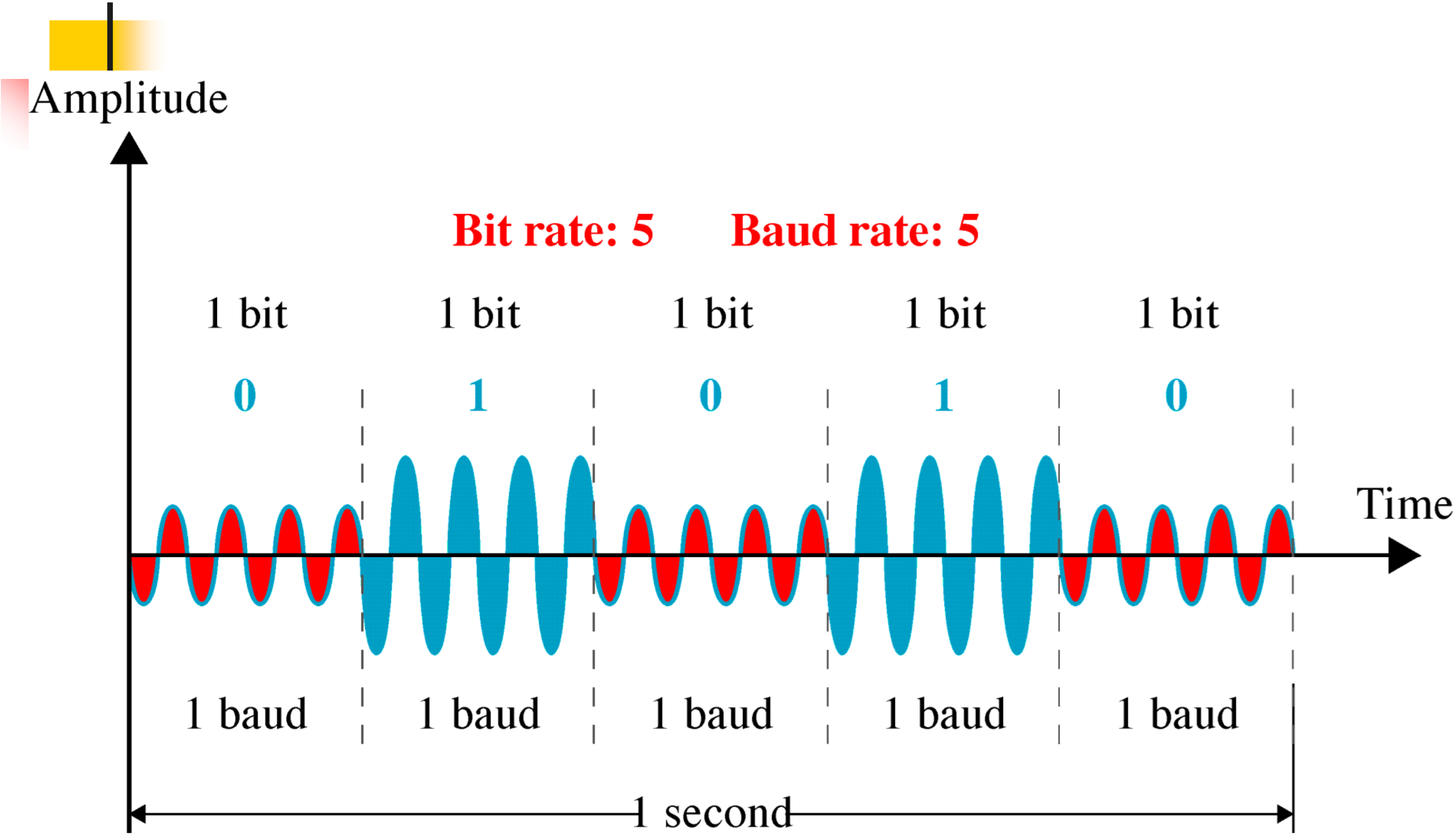
Amplitude Modulation



- Amplitude 2 = 1
- Amplitude 1 = 0

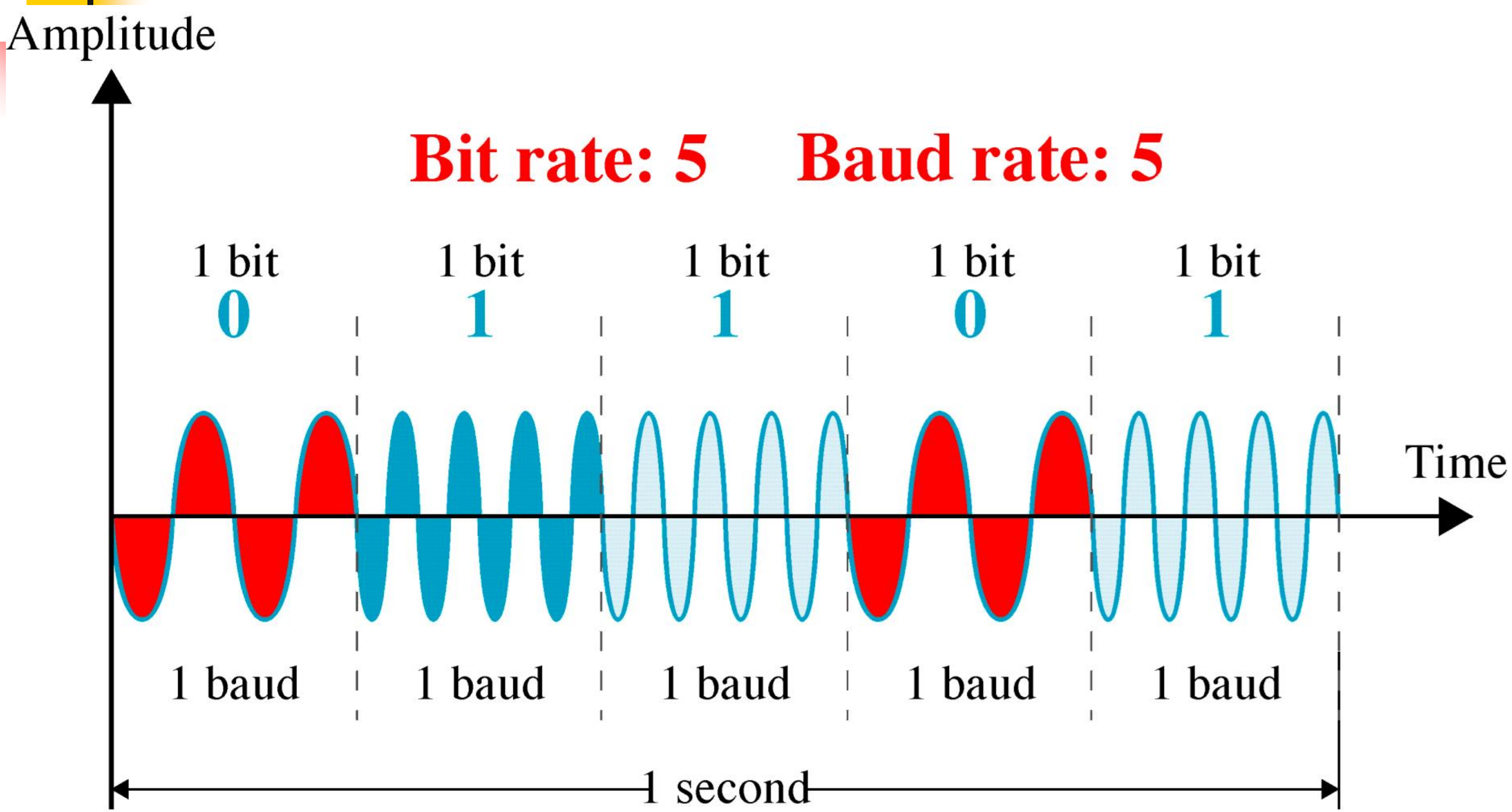
- This signal Represents:
  - 0011010

ASK



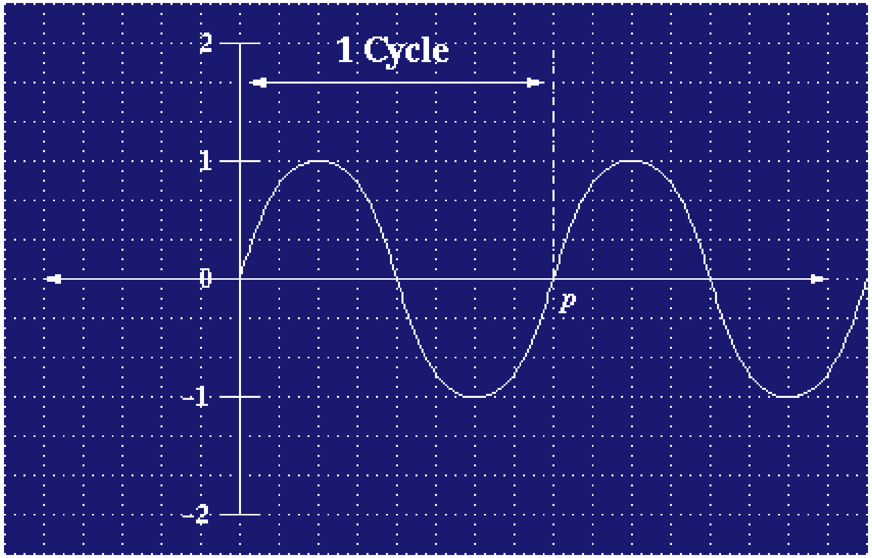
Amplitude
Bit rate: 5
Baud rate: 5
1 bit
1 bit
1 bit
1 bit
1 bit
0
1
0
1
0
Time
1 baud
1 baud
1 baud
1 baud
1 baud
1 second

FSK


Amplitude
Bit rate: 5
Baud rate: 5
1 bit
0
1 bit
1
1 bit
1
1 bit
0
1 bit
1
Time
1 baud
1 baud
1 baud
1 baud
1 baud
1 second

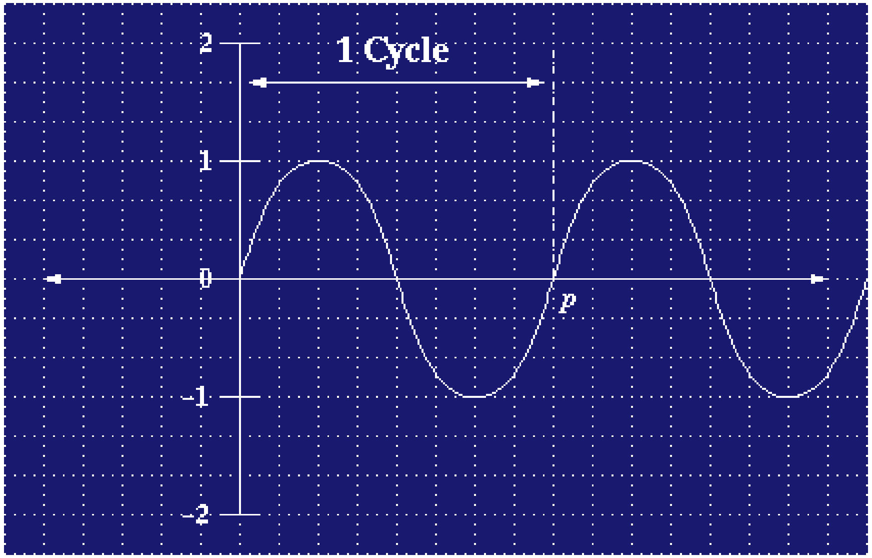
Phase-Shift Modulation

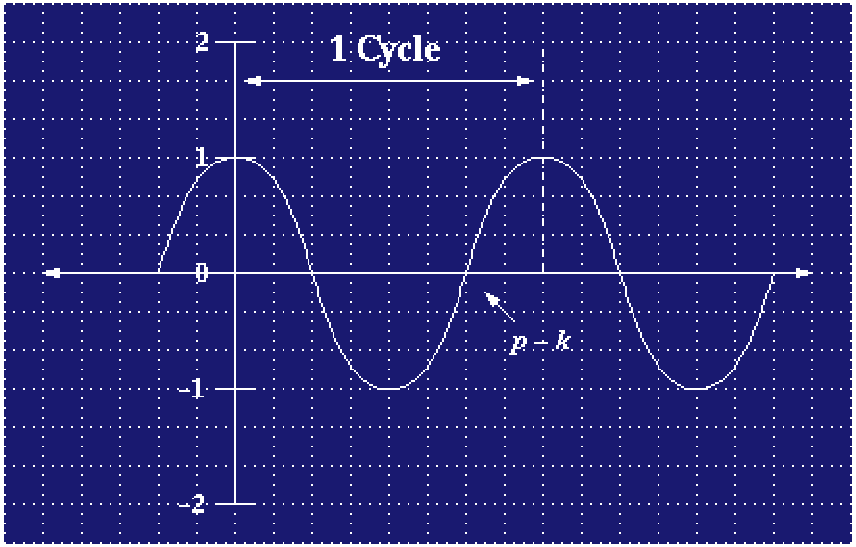
- Start with our normal sine wave
- The sine wave has a period of P
  - P may be denoted as T instead in the equations



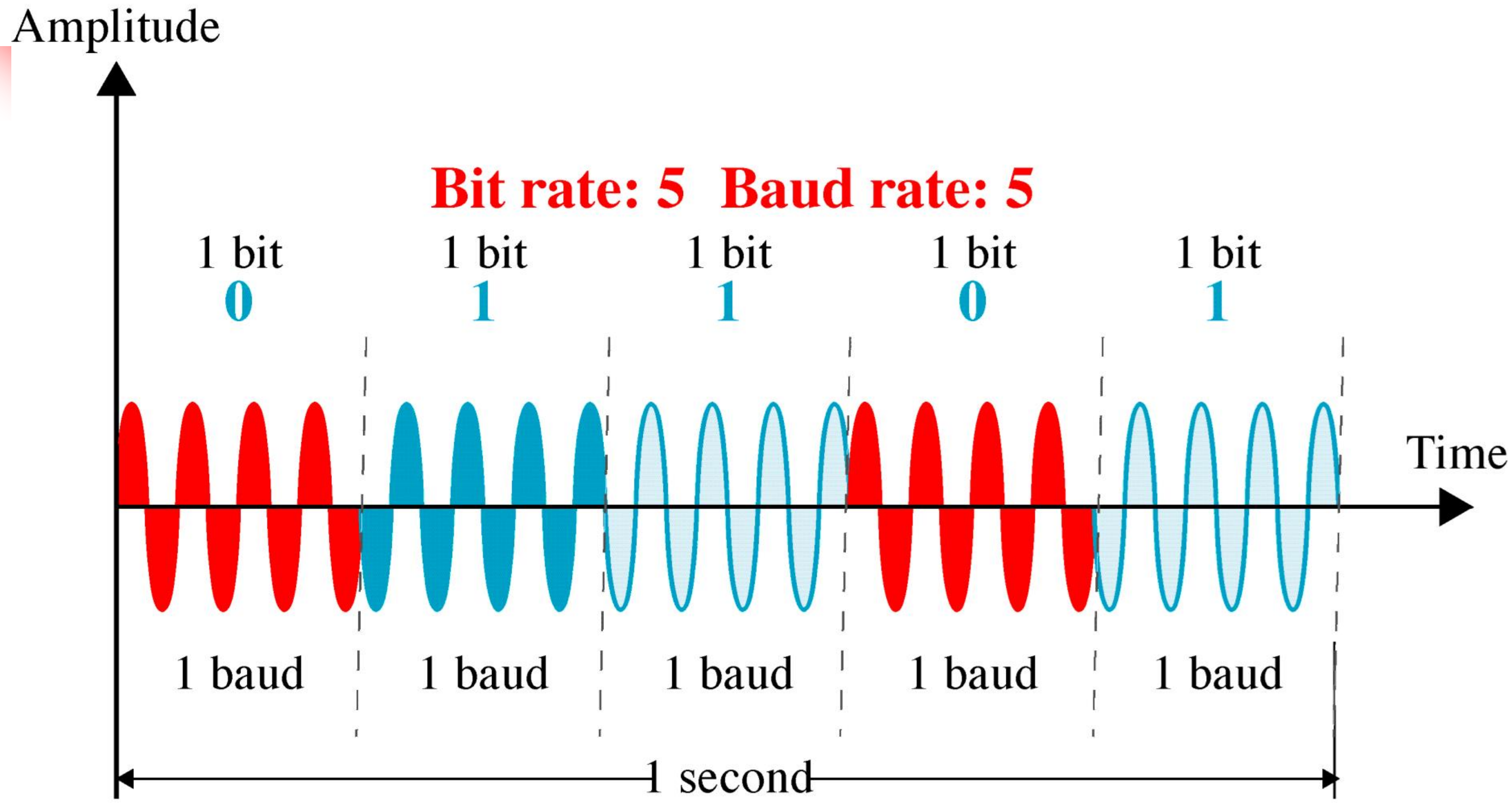
Phase-Shift Modulation

- Shift the Phase of the Sine Wave
- Shifted diagram shows that the cycle starting at 1 vs. starting at 0





PSK

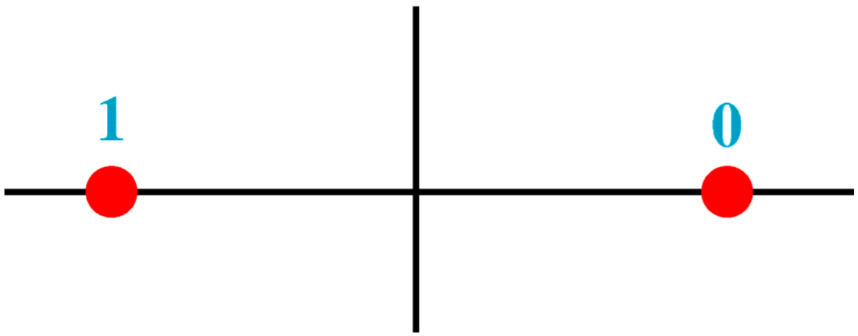


Amplitude
Bit rate: 5 Baud rate: 5
1 bit
0
1 bit
1
1 bit
1
1 bit
0
1 bit
1
Time
1 baud
1 baud
1 baud
1 baud
1 baud
1 second

PSK Constellation

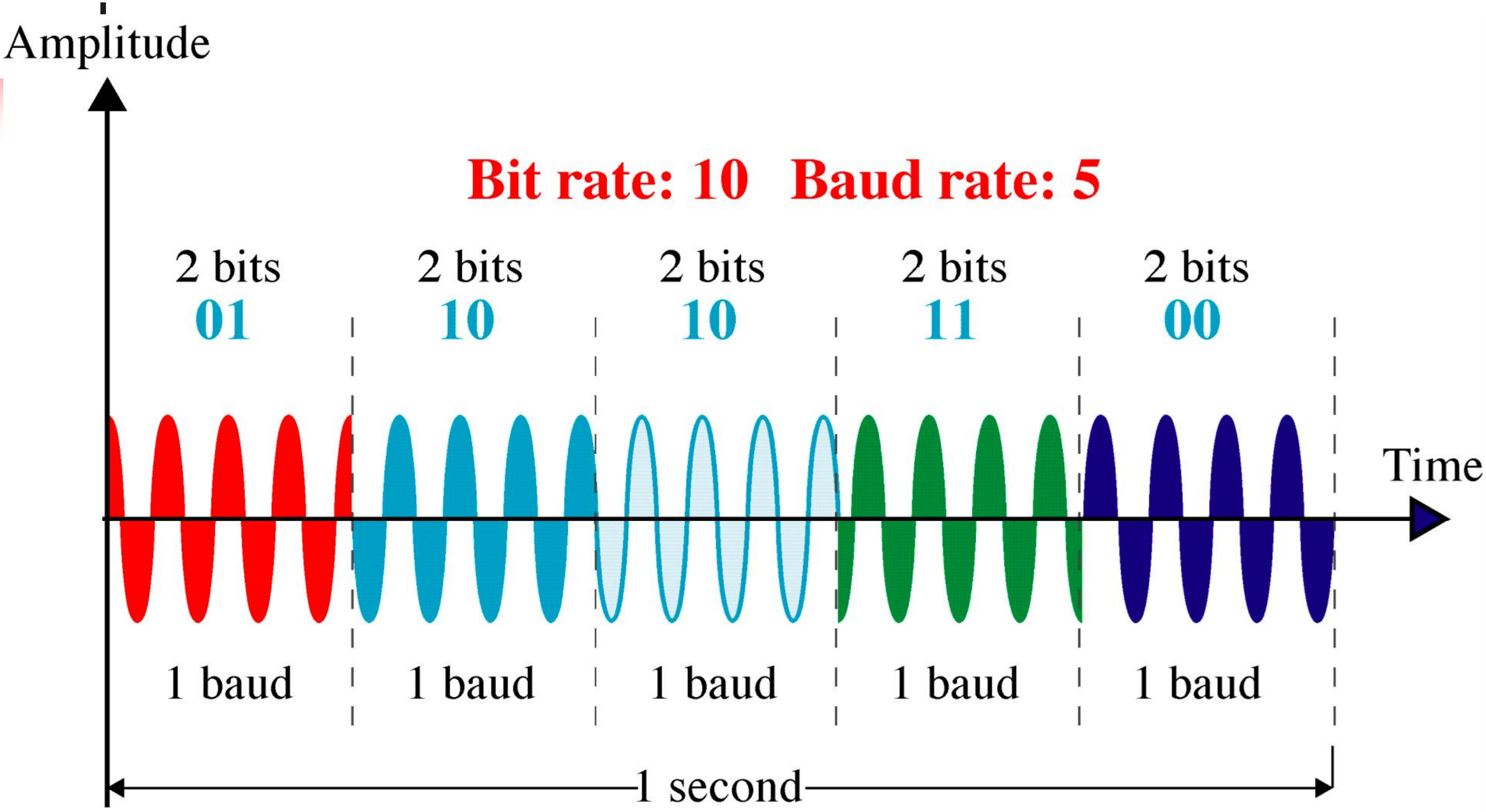
Bit	Phase
0	0
1	180

Bits



Constellation diagram

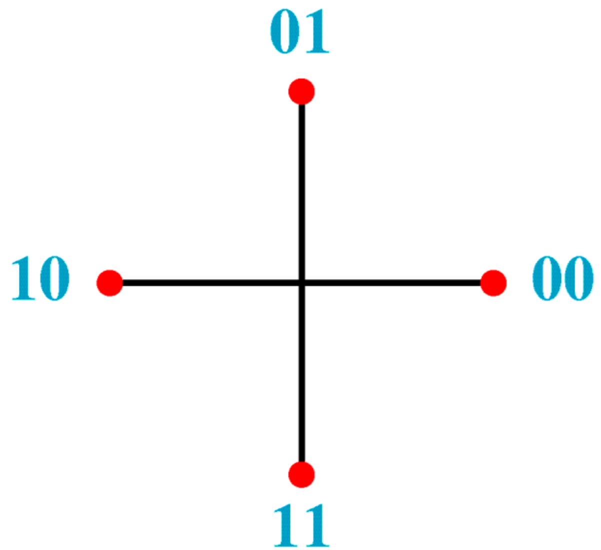
4-PSK


Amplitude
Bit rate: 10 Baud rate: 5
2 bits
01
2 bits
10
2 bits
10
2 bits
11
2 bits
00
Time
1 baud
1 baud
1 baud
1 baud
1 baud
1 second

4-PSK Constellation

Dibit	Phase
00	0
01	90
10	180
11	270

Dibit
(2 bits)

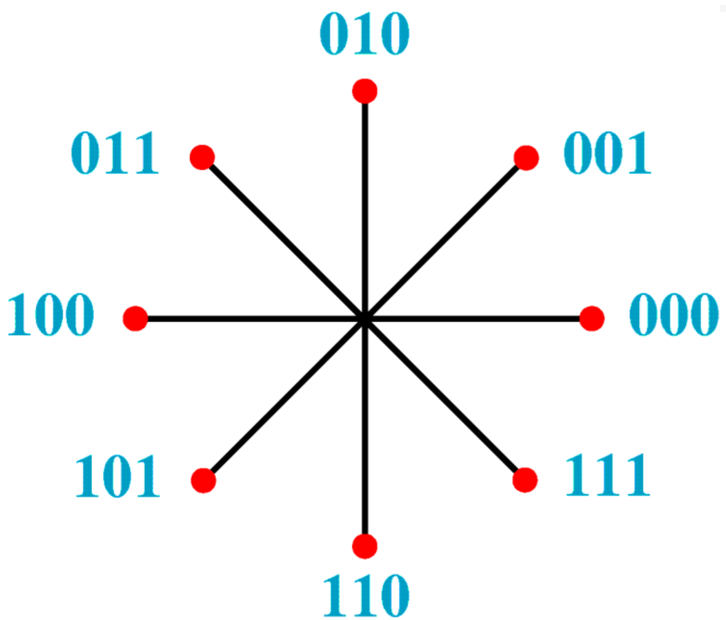


Constellation diagram

8-PSK Constellation

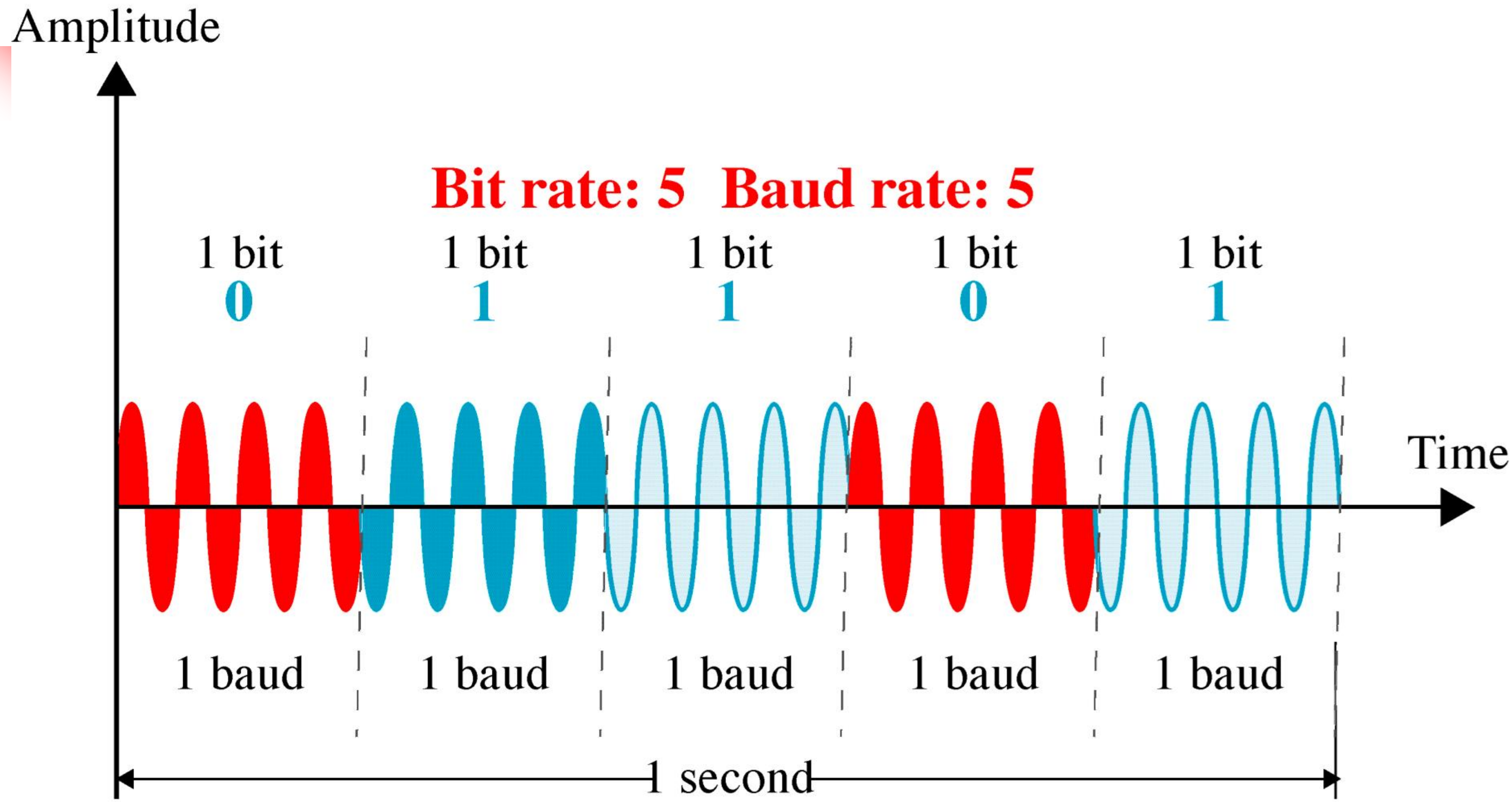
Tribit	Phase
000	0
001	45
010	90
011	135
100	180
101	225
110	270
111	315

Tribits
(3 bits)



Constellation diagram

PSK

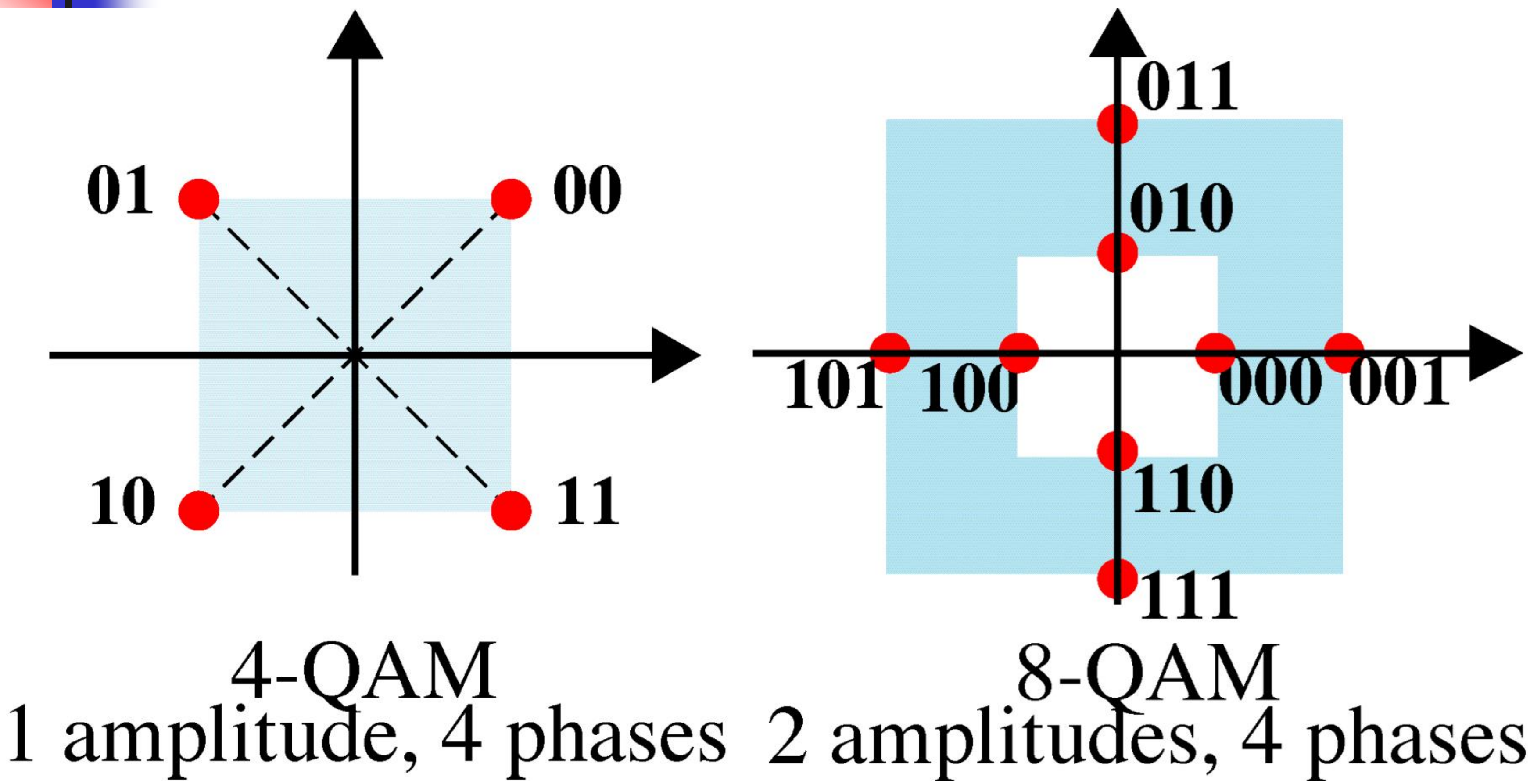


Amplitude
Bit rate: 5 Baud rate: 5
1 bit
0
1 bit
1
1 bit
1
1 bit
0
1 bit
1
Time
1 baud
1 baud
1 baud
1 baud
1 baud
1 second

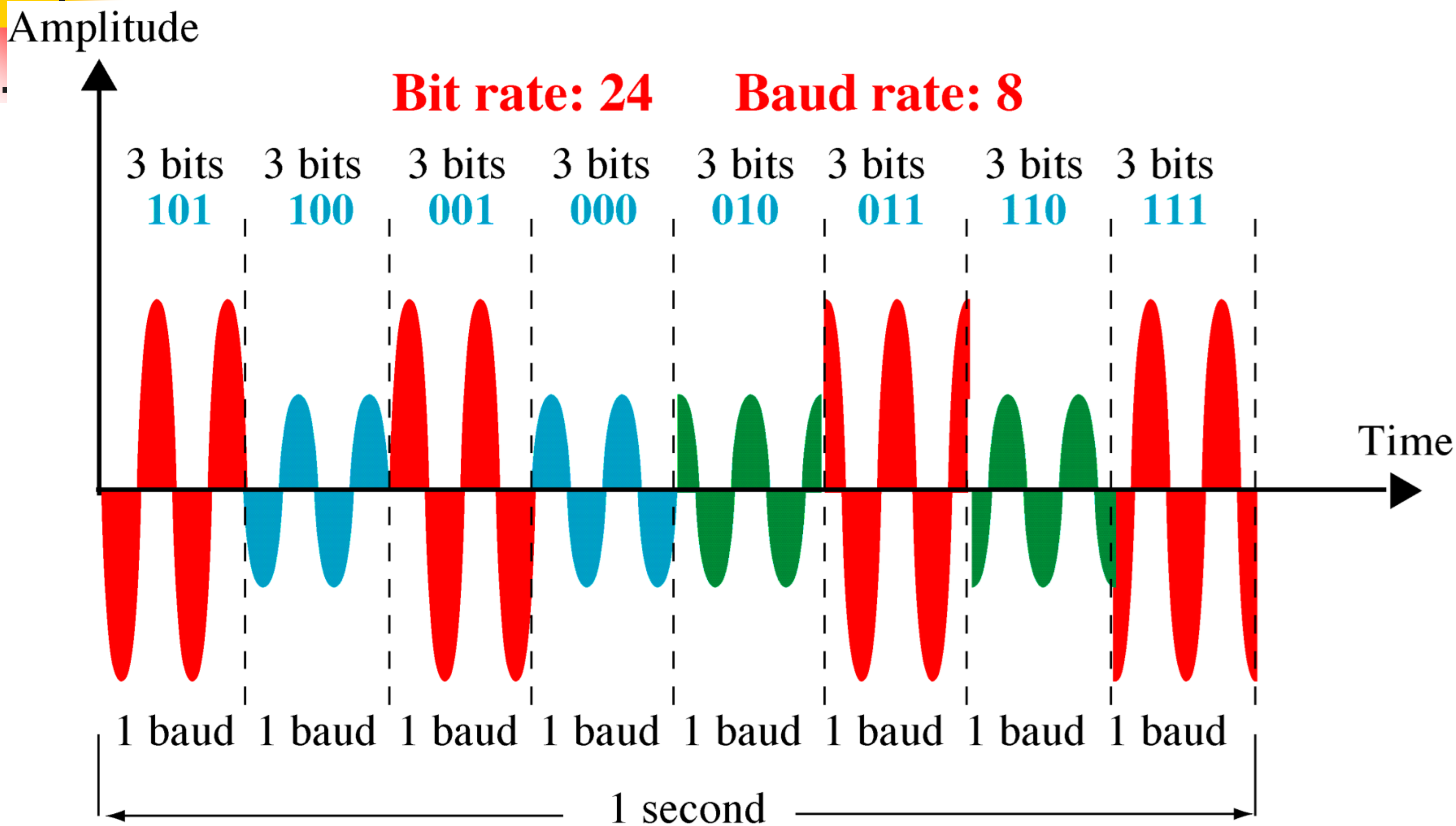
Combining Both

- Modulation used in Modern Modems
  - Uses:
    - Amplitude Modulation
    - Phase Shift Keying
- QAM
  - Quadrature Amplitude Modulation
    - Big Name – Simple Concept

4-QAM and 8-QAM Constellation

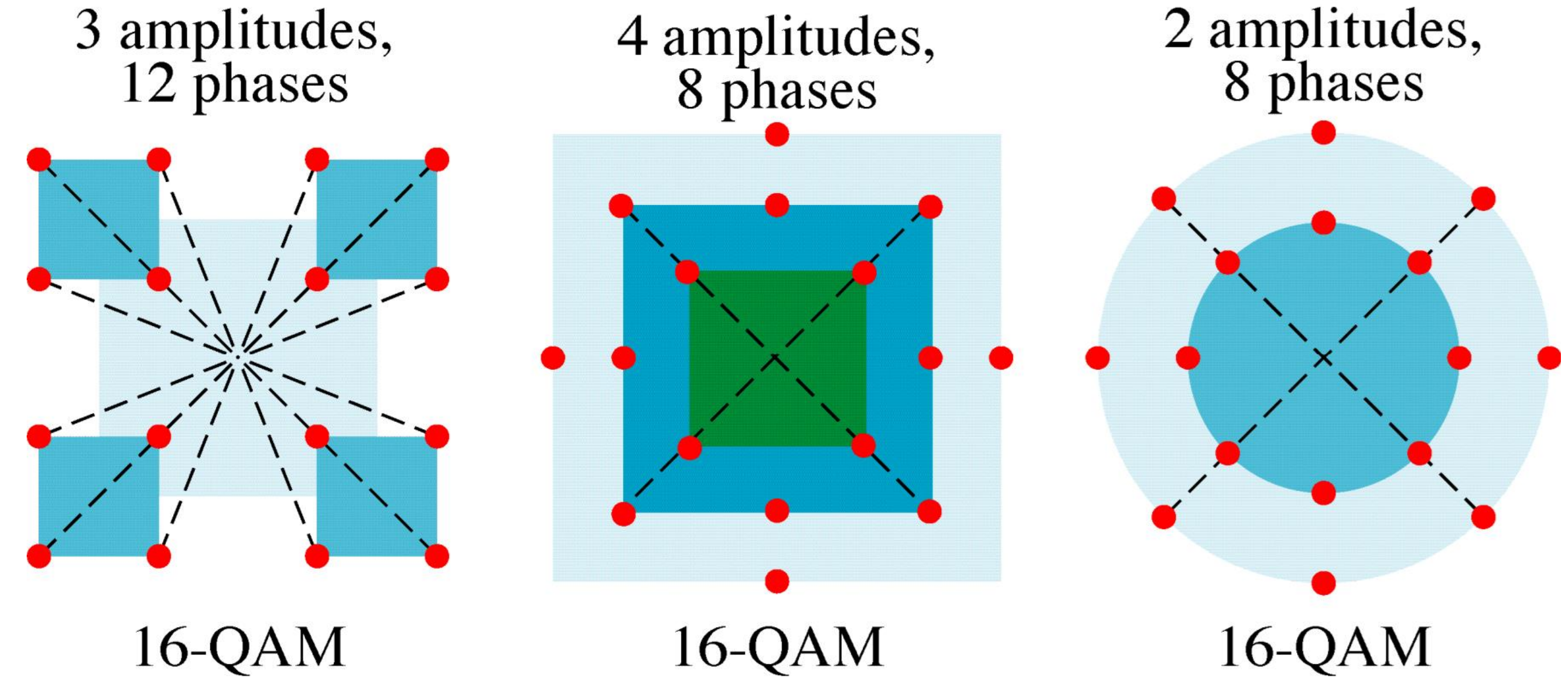


8-QAM Signal

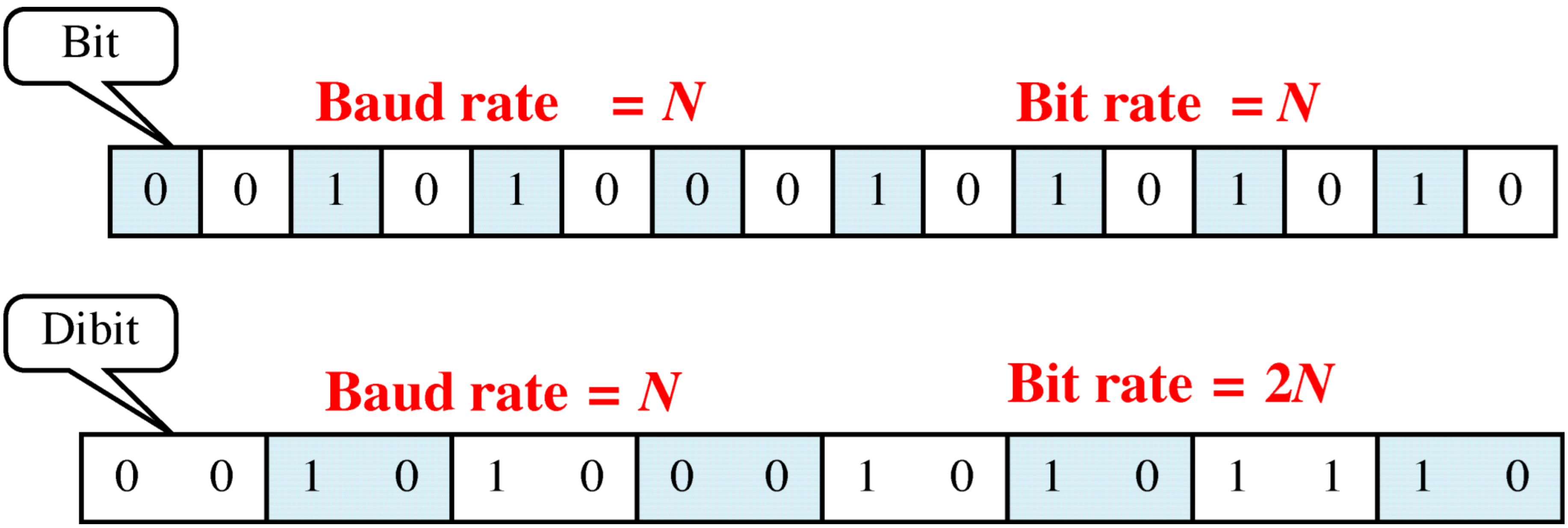


Amplitude
Bit rate: 24
Baud rate: 8
3 bits
101
3 bits
100
3 bits
001
3 bits
000
3 bits
010
3 bits
011
3 bits
110
3 bits
111
Time
1 baud
1 baud
1 baud
1 baud
1 baud
1 baud
1 baud
1 baud
1 second

16-QAM Constellation



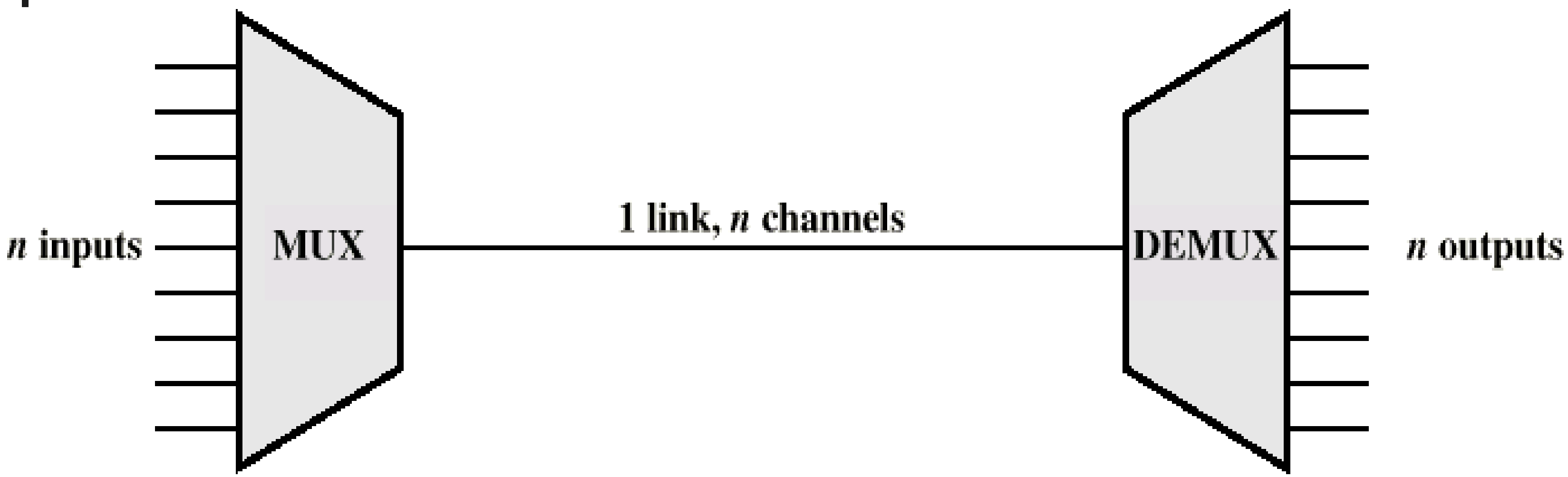
Bit Rate and Baud Rate



Problem #3

- A modem uses an 8-PSK modulation scheme supporting data rate of 4800 bps. What is the signaling rate (aka baud rate)?
- 8 PSK – (Phase Shift Keying)
  - 8 different encoding levels
  - Each encoding has $\log_2 8 = 3$ bits
  - 4800 / 3 = 1600 Baud Rate

Ways of Multiplexing/Demultiplexing



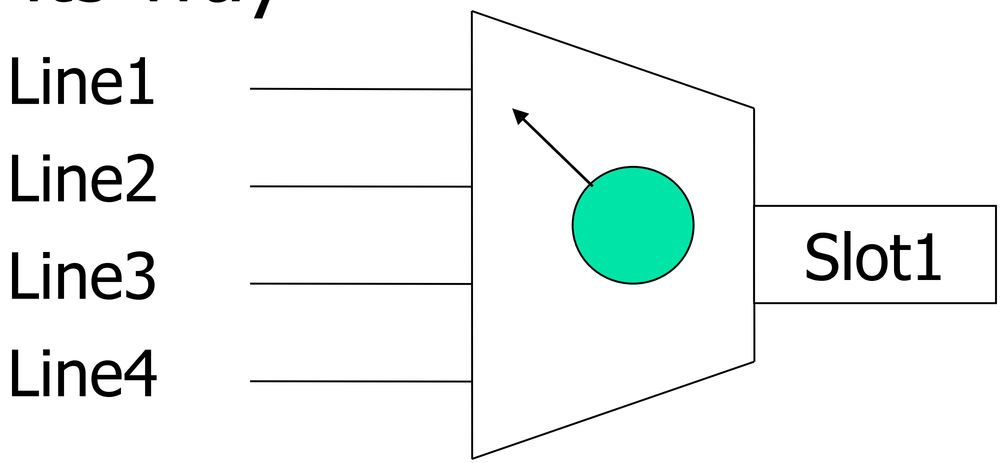
- Time Division Multiplexing (TDM)
- You have n input lines coming in
- The same # of lines going out..
- Only one line interconnecting. How?

Packing the Data In

- **Multiplexing** – A way of aggregating data onto a single line without compromising the rate at which original data is sent.
- We are not limiting anyone's channel capacity
- We are simply sending there signal through a shared line

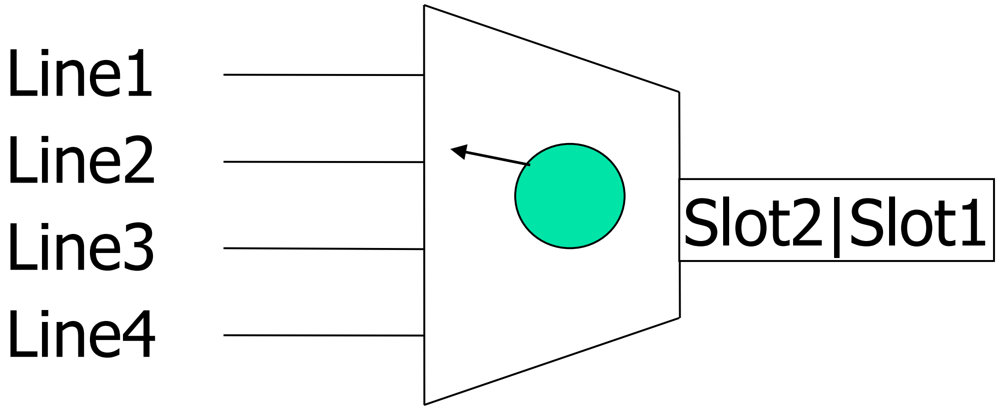
Sharing – by time slots

- Sample the Line 1 – Place its value in slot 1
- Sampling the Line – and Send its value on its way



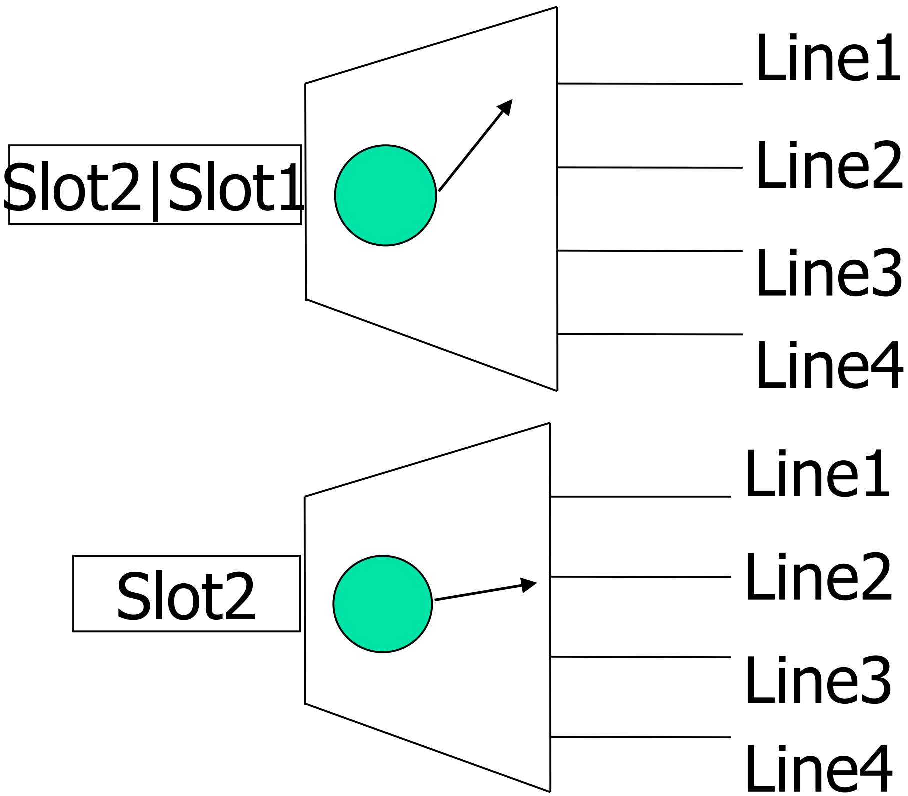
More Time Slots

- Line 2 Places its Sample on the Line
- And it goes on…

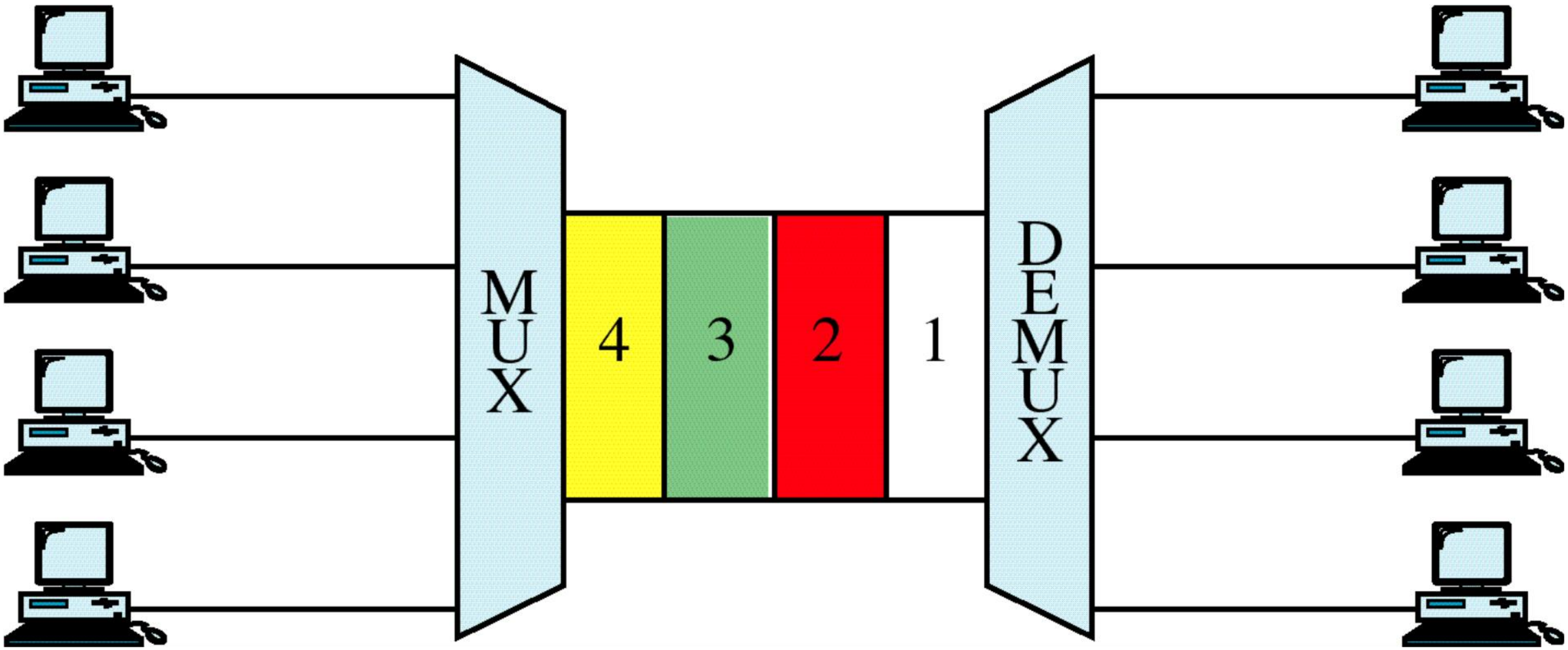


Other Side - Demultiplexing

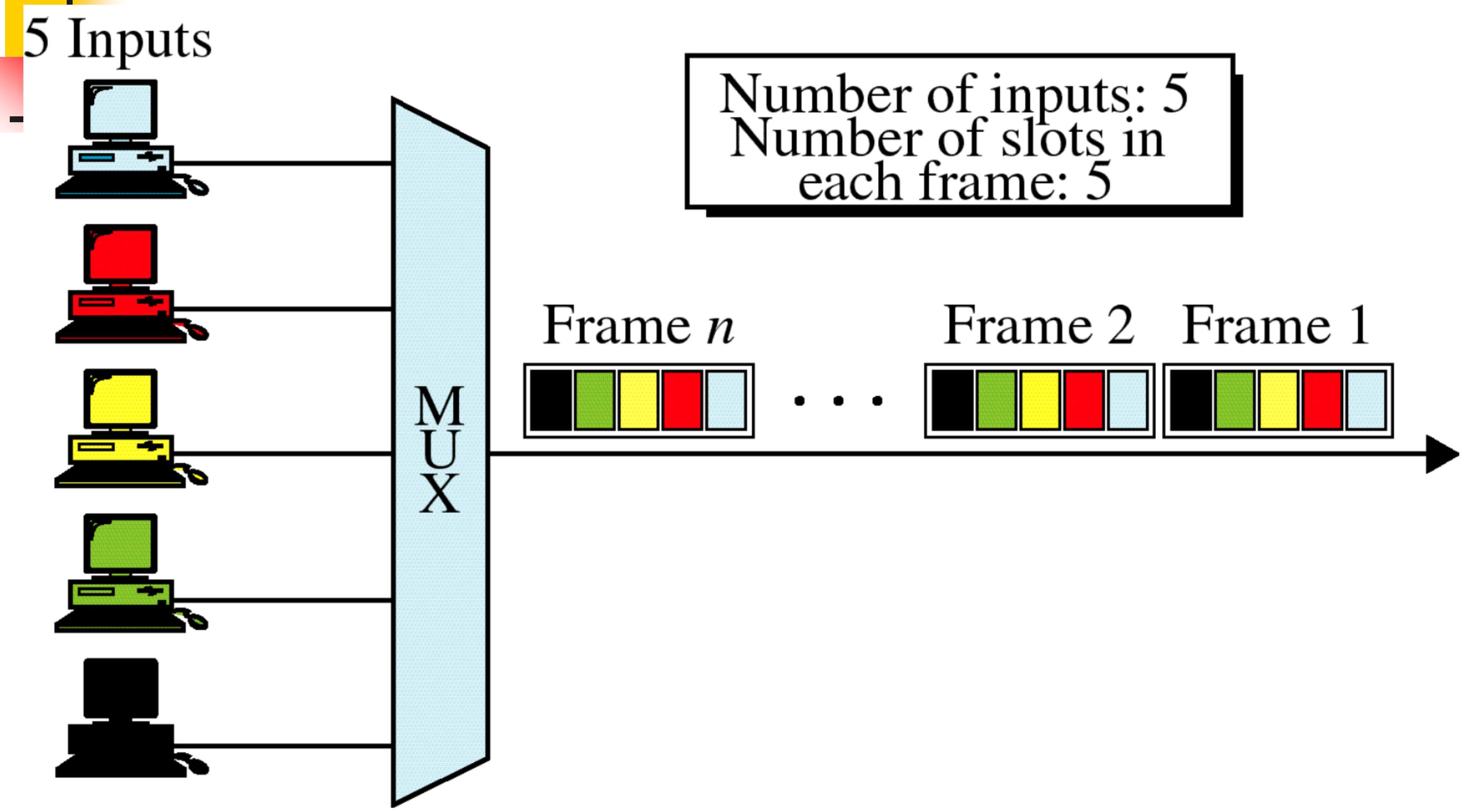
- Similar to Multiplexing just the reverse



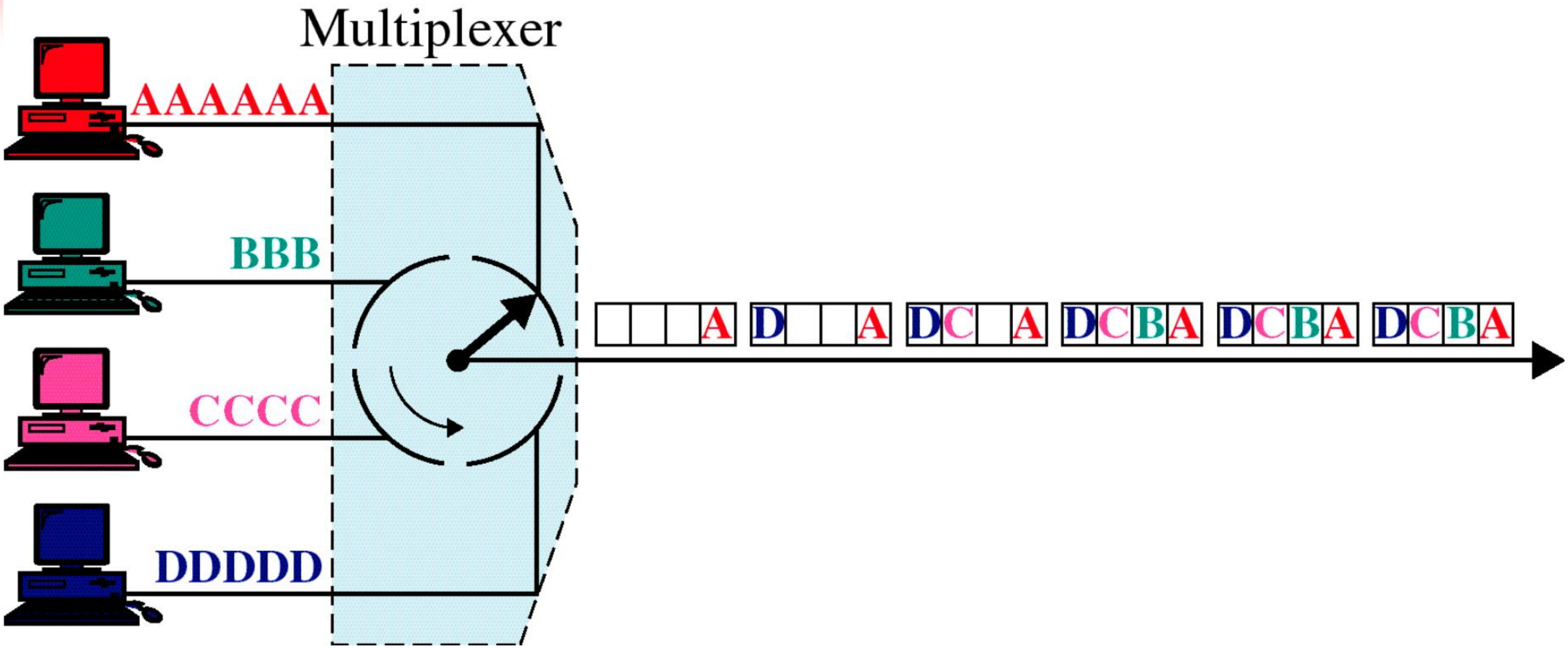
TDM



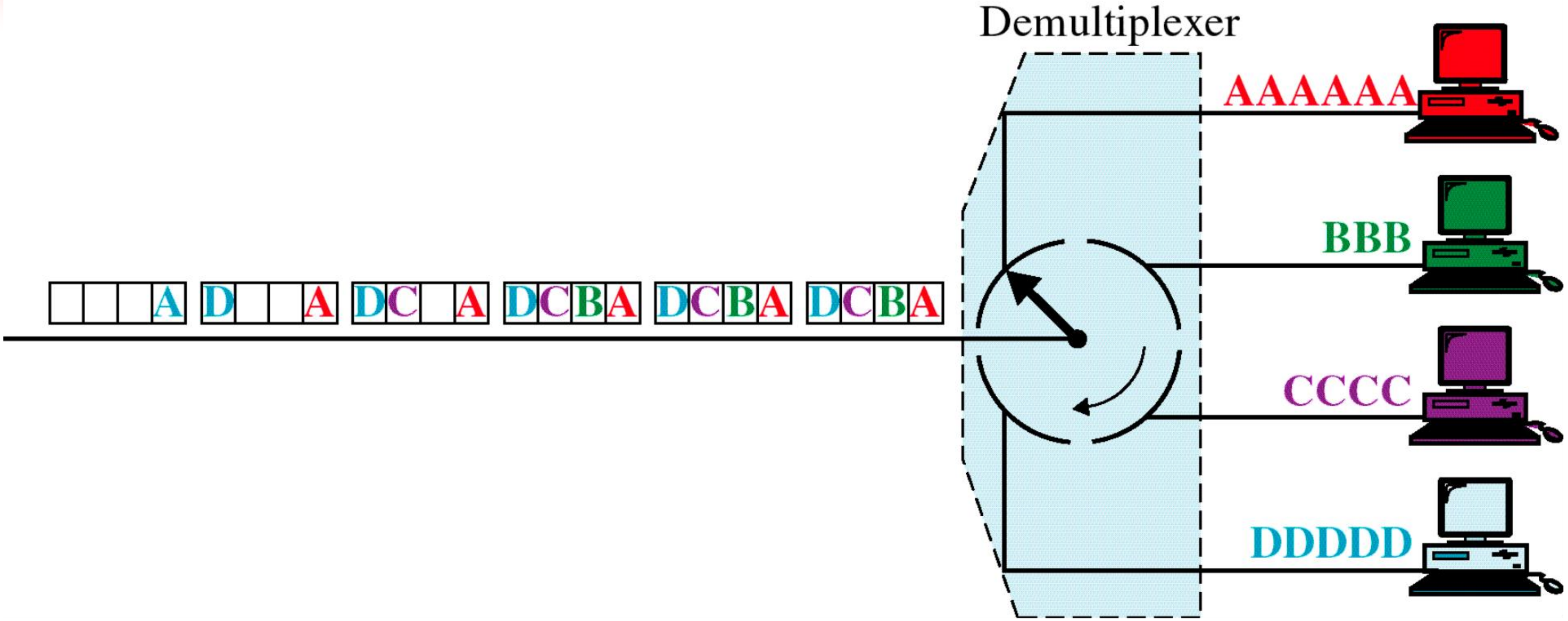
Synchronous TDM



TDM, Multiplexing

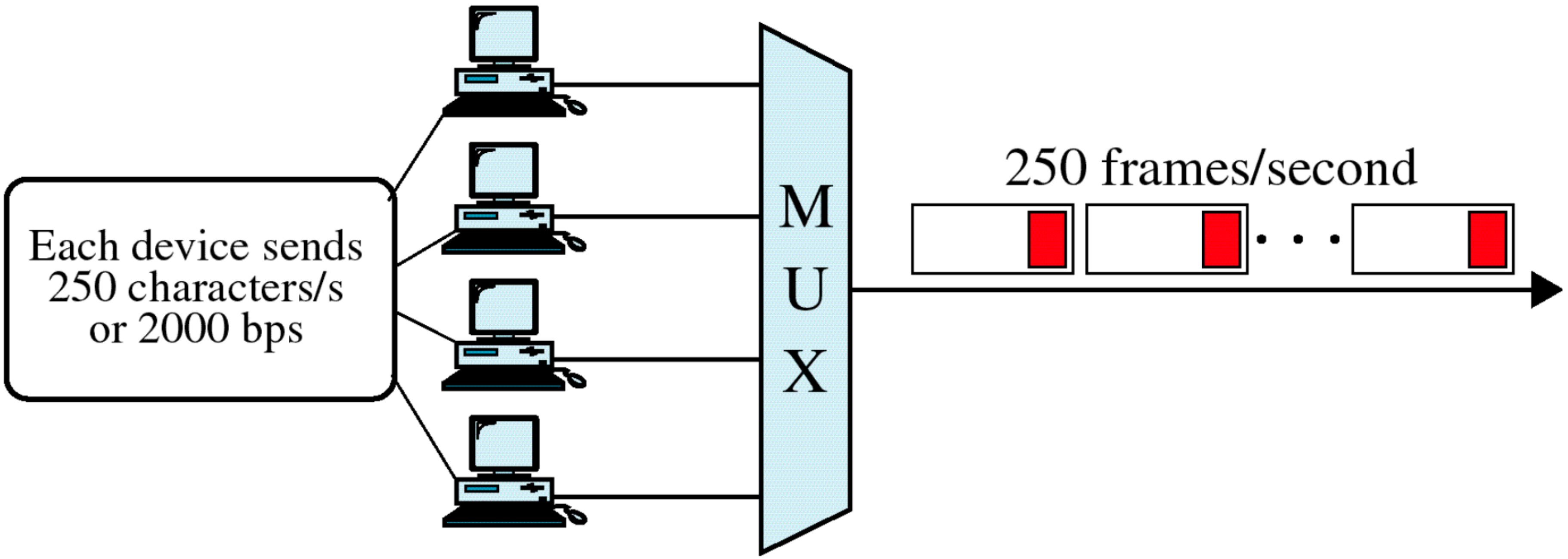


TDM, Demultiplexing



Data Rate

8250 bps = 250 frames/second x 33 bits/frame
or
8250 bps = 4 x 2000 bps + 250 synchronization bps



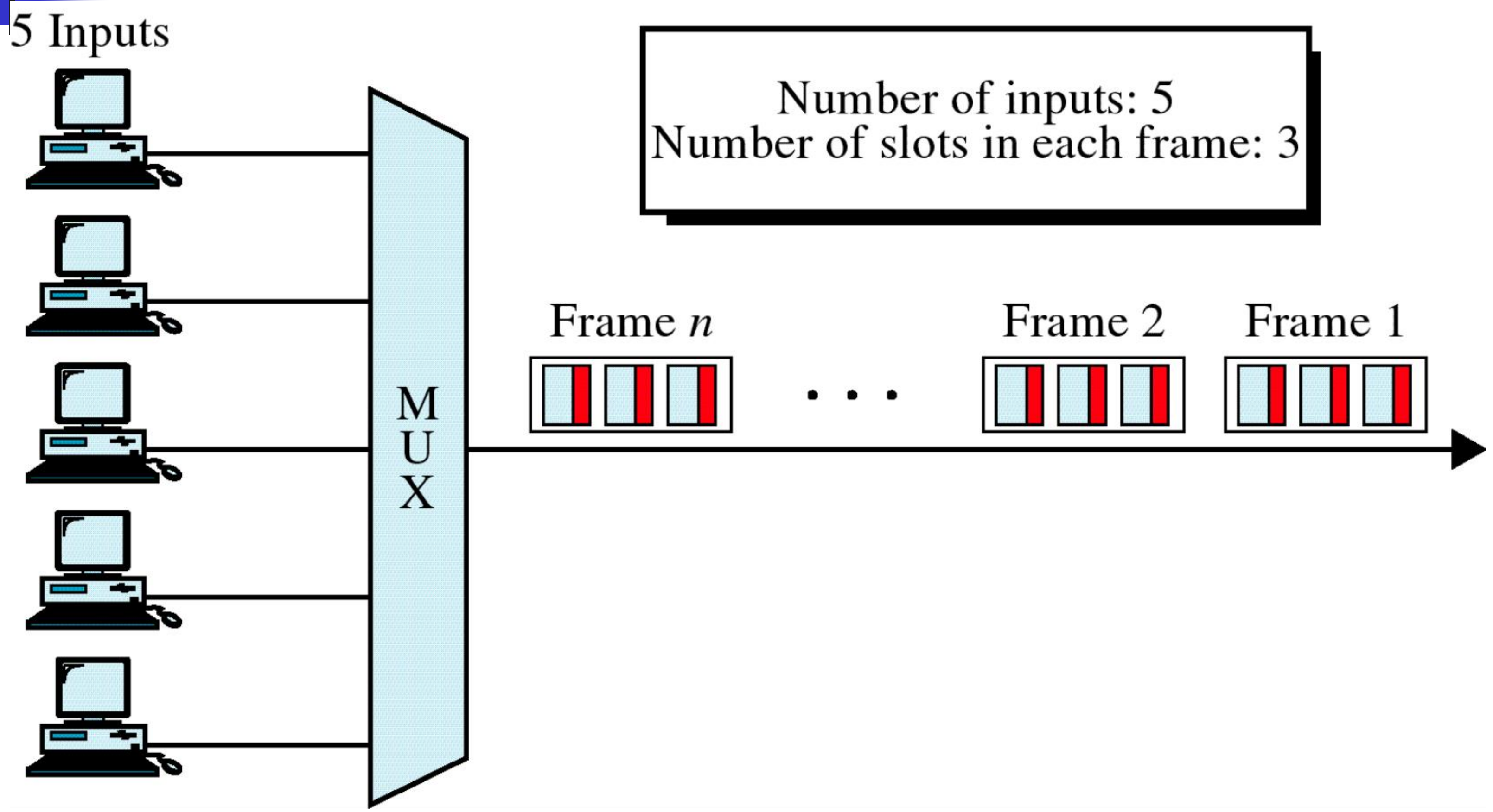
TDM - Example

- T-1 Lines
  - Carries the equivalent of 24 voice lines
  - Each analog voice line is sampled at 8000 times a second
  - Digital Sample is thrown on the Digital Carrier Line
  - On the other side Digital samples are used to reconstruct Analog Signal.

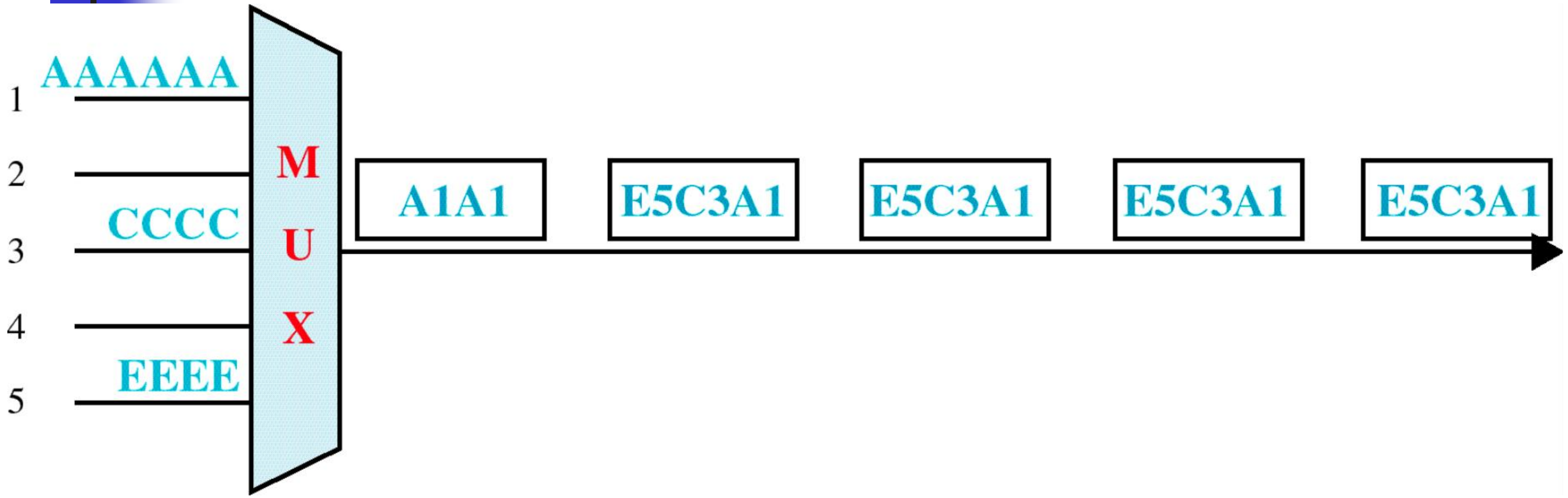
STDM (Asynchronous TDM)

- How does it Work?
  - Checks to see if there is data to transmit on input line
    - If there is transmit data
    - If not move on to next input line

Asynchronous TDM



Frames and Addresses



a. Only three lines sending data

TDM vs. STDM

- Synchronous TDM – Gray Slots are actually carrying data

Slot 1	Slot 2	Slot 3	Slot 4	Slot 1	Slot 2	Slot 3

- Statistical TDM – Uses empty time slots but does add some overhead

	Slot 1		Slot 2		Slot 1		Slot 2		Slot 3		Slot 1

Notes on TDM

- Sampling Occurs very quickly
- Applicable to fixed number of flows
- Requires Precise Timing
- Resources are guaranteed

STDM

- Statistical Time Division Multiplexing
- Similar to regular TDM but different in this:
  - Traffic is sent on demand – Only if there is data on line 1, will slot 1 be occupied by line 1
  - Resources are not guaranteed
- If we are no longer guaranteed a time slot why use it?
  - We (the carrier) can take advantage of one of the input lines not being busy
- Important distinction – STDM is used mainly for Digital Lines

TDM vs. STDM

1. Resources are guaranteed to the users
2. Sampling Occurs very quickly
3. Applicable to fixed number of flows
4. Requires Precise Timing
5. Wastes valuable carrier space

1. Traffic is sent on demand utilizing unused time slots so it benefits the Carrier
2. Resources are not guaranteed so when time slots are busy the users suffer
3. In real life there is some overhead
4. Speedup isn’t as obvious

Statistical TDM Parameters

- I = Number of Input Sources
- R = Data rate of each source (bps)
- α (Alpha) = mean fraction of time each source is transmitting
- M = Effective capacity of multiplexed line
- K = M / (I x R) = Ratio of multiplexed line capacity to total input rate
- λ (lambda) = α x I x R = Average Arrival time
- T_s = 1 / M = Service time in seconds

ρ : Line Utilization

- ρ = Fraction of total link capacity being used
- Many different forms to express line utilization
  - $\rho = \lambda T_s$
  - $\rho = (\alpha \times I \times R) / M$
  - $\rho = \alpha / K$
  - $\rho = \lambda / M$

Sample Problem #5

- Ten 9600 bps lines are multiplexed using TDM. Ignoring overhead bits what is the total capacity required for Synchronous TDM?
  - Simple: 10 X 9600 = 96 kbps (96,000)

Sample Problem #6

- Ten 9600 bps lines are multiplexed using TDM. Assuming that we limit line utilization to 0.8 and each line is busy 50 % of the time. What is the capacity required for <u>Statistical</u> TDM?
  - What do we know?
    - Line utilization – $\rho = .8$
    - Fraction of time transmitting - $\alpha = .5$
    - R Data Rate of each input source = 9600 bps
    - I number of Input Sources = 10

Continued

- The Equation:
- $\rho = \alpha \times I \times R \times /M$
  - Where M is the capacity of the multiplexed line
- Rearrange for M
  - $M = \alpha \times I \times R / \rho$
- Plug in the given parameters
  - $M = 0.5 \times 10 \times 9600 / 0.8$
  - $M = 60$ kbps

Sample Problem #7

- Calculate the capacity of a Multiplexed carrier?
  - 24 voice channels multiplexed
  - 8000 samples per second
    - Each frame lasts 1/8000 = 125 μ sec
  - Uses 8 bit encoding per sample
  - Capacity
    - 24 X 8000 samples/second X 8 bits/sample
    - 1,536,000 bits/second
  - T-1 Adds an extra bit per frame (for synchronization) which makes it 1.544 Mbps

Sample Problem #8

- What is the percent overhead on a T-1 Carrier?
  - T-1 Carrier bandwidth of 1.544 Mbps
    - Every 192 bits one more bit is added for framing.
  - What is the overhead?

Continued

- Overhead in a T-1 Line
  - Every frame consists of 193 bits
  - 24 X 8 = 192 bits
  - 193-192 = 1 overhead bit
  - 1/193 = .5%