Probabilistic Reasoning: Homework 4

Due on March 9, 2015

Prof. Nevatia

Tushar Tiwari

Problem 1

Create a corresponding clique tree

Solution

Calculations

The clique tree is constructed using the variable elimination technique when finding P(A).

$$P(A) = \sum_N \phi(N) \sum_M \phi(M,N,A) \sum_E \phi(E) \sum_B \phi(B)\phi(A,B,E) \sum_T \phi(T) \sum_J \phi(J,T,A)$$

$$\psi_1(J,T,A) = \phi(J,T,A)$$

$$\tau_1(T,A) = \sum_J \psi_1(J,T,A)$$

$$\psi_2(T,A) = \phi(T)\tau_1(TA)$$

$$\tau_2(A) = \sum_T \psi_2(T,A)$$

Since, scope(ψ_2) $\subset$ scope(ψ_1), only one clique is sufficient to represent these factors.

$$\psi_3(A,B,E) = \phi(B)\phi(A,B,E)\tau_2(A)$$

$$\tau_3(A,E) = \sum_B \psi_3(A,B,E)$$

$$\psi_4(A,E) = \phi(E)\tau_3(A,E)$$

$$\tau_4(A) = \sum_E \psi(A,E)$$

Again, scope(ψ_4) $\subset$ scope(ψ_3), only one clique is sufficient to represent these factors.

$$\psi_5(M,N,A) = \phi(M,N,A)\tau_4(A)$$

$$\tau_5(N,A) = \sum_M \psi_5(M,N,A)$$

$$\psi_6(N,A) = \phi(N)\tau_5(N,A)$$

$$\tau_6(A) = \sum_N \psi(N,A)$$

Again, scope(ψ_6) $\subset$ scope(ψ_5), only one clique is sufficient to represent these factors.
Also, the messages passed between the cliques are all $\tau_i(A)$ so the cliques will have message $\delta_{i\to j}(A)$ passed between them.

Problem 1 continued on next page...

Clique Tree

The clique tree description:

- 1:$\psi(J,T,A)$ connected to $\psi(B,E,A)$ over $\delta_{1\to2}(A)$.
- 2:$\psi(B,E,A)$ connected to $\psi(J,T,A)$ over $\delta_{2\to1}(A)$.
- 2:$\psi(B,E,A)$ connected to $\psi(M,N,A)$ over $\delta_{2\to3}(A)$.
- 3:$\psi(M,N,A)$ connected to $\psi(B,E,A)$ over $\delta_{3\to2}(A)$.

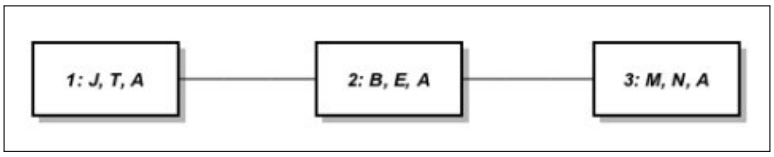

Figure 1: The clique tree

Problem 2

Show the intermediate calculations for clique tree calibration.

Solution

The initial CPDs to be multiplied are given below

$\phi(T) \times \phi(J,T,A) =$

$$\phi(T) = \begin{matrix} t^0 \\ t^1 \end{matrix} \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix} \qquad \phi(J,T,a^0) = \begin{matrix} t^0 \\ t^1 \end{matrix} \begin{matrix} \begin{matrix} j^0 & j^1 \end{matrix} \\ \begin{bmatrix} 0.98 & 0.02 \\ 0.97 & 0.03 \end{bmatrix} \end{matrix} \qquad \phi(J,T,a^1) = \begin{matrix} t^0 \\ t^1 \end{matrix} \begin{matrix} \begin{matrix} j^0 & j^1 \end{matrix} \\ \begin{bmatrix} 0.12 & 0.88 \\ 0.55 & 0.45 \end{bmatrix} \end{matrix}$$

$\phi(N) \times \phi(M,N,A) =$

$$\phi(N) = \begin{matrix} n^0 \\ n^1 \end{matrix} \begin{bmatrix} 0.75 \\ 0.25 \end{bmatrix} \qquad \phi(M,N,a^0) = \begin{matrix} n^0 \\ n^1 \end{matrix} \begin{matrix} \begin{matrix} m^0 & m^1 \end{matrix} \\ \begin{bmatrix} 0.99 & 0.01 \\ 0.999 & 0.001 \end{bmatrix} \end{matrix} \quad \phi(M,N,a^1) = \begin{matrix} n^0 \\ n^1 \end{matrix} \begin{matrix} \begin{matrix} m^0 & m^1 \end{matrix} \\ \begin{bmatrix} 0.25 & 0.75 \\ 0.75 & 0.25 \end{bmatrix} \end{matrix}$$

$\phi(E) \times \phi(B) \times \phi(B,E,A) =$

$$\phi(E) = \begin{matrix} e^0 \\ e^1 \end{matrix} \begin{bmatrix} 0.999 \\ 0.001 \end{bmatrix} \qquad \phi(B) = \begin{matrix} b^0 \\ b^1 \end{matrix} \begin{bmatrix} 0.998 \\ 0.002 \end{bmatrix} \qquad \phi(B,E,a^0) = \begin{matrix} b^0 \\ b^1 \end{matrix} \begin{matrix} \begin{matrix} e^0 & e^1 \end{matrix} \\ \begin{bmatrix} 0.988 & 0.4 \\ 0.15 & 0.07 \end{bmatrix} \end{matrix} \qquad \phi(B,E,a^1) = \begin{matrix} b^0 \\ b^1 \end{matrix} \begin{matrix} \begin{matrix} e^0 & e^1 \end{matrix} \\ \begin{bmatrix} 0.002 & 0.6 \\ 0.85 & 0.93 \end{bmatrix} \end{matrix}$$

Multiplying the factors

$\psi(J,T,A)=$

$$a^0 = \begin{matrix} \\ t^0 \\ t^1 \end{matrix} \begin{matrix} \begin{matrix} j^0 & j^1 \end{matrix} \\ \begin{bmatrix} 0.196 & 0.004 \\ 0.776 & 0.024 \end{bmatrix} \end{matrix} \quad a^1 = \begin{matrix} \\ t^0 \\ t^1 \end{matrix} \begin{matrix} \begin{matrix} j^0 & j^1 \end{matrix} \\ \begin{bmatrix} 0.024 & 0.176 \\ 0.44 & 0.36 \end{bmatrix} \end{matrix}$$

$\psi(M,N,A)=$

$$\begin{matrix} \psi(M,N,a^0) = \\ \\ n^0 \\ n^1 \end{matrix} \begin{matrix} \begin{matrix} m^0 & m^1 \end{matrix} \\ \begin{bmatrix} 0.7425 & 0.0075 \\ 0.24975 & 0.00025 \end{bmatrix} \end{matrix} \quad \begin{matrix} \psi(M,N,a^1) \\ \\ n^0 \\ n^1 \end{matrix} \begin{matrix} \begin{matrix} m^0 & m^1 \end{matrix} \\ \begin{bmatrix} 0.1875 & 0.5625 \\ 0.1875 & 0.0625 \end{bmatrix} \end{matrix} =$$

$\psi(B,E,A)=$

$$a^0 = \begin{matrix} \\ e^0 \\ e^1 \end{matrix} \begin{matrix} \begin{matrix} b^0 & b^1 \end{matrix} \\ \begin{bmatrix} 0.995008 & 0.00029967 \\ 0.0003992 & 0.00000014 \end{bmatrix} \end{matrix} \quad a^1 = \begin{matrix} \\ e^0 \\ e^1 \end{matrix} \begin{matrix} \begin{matrix} b^0 & b^1 \end{matrix} \\ \begin{bmatrix} 0.001994004 & 0.0016983 \\ 0.0005988 & 0.00000186 \end{bmatrix} \end{matrix}$$

Computing Deltas

$\delta_{1\to 2}(A) = \sum_{J,T} \psi_1(J,T,A) =$

$$\delta_{1\to 2}(A) = \begin{matrix} \\ a^0 \\ a^1 \end{matrix} \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$$

$\delta_{3\to 2}(A) = \sum_{M,N} \psi_3(M,N,A)$

$$\delta_{3\to 2}(A) = \begin{matrix} \\ a^0 \\ a^1 \end{matrix} \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$$

$\delta_{2\to 1}(A) = \sum_{B,E} \psi_2(B,E,A) \times \delta_{3\to 2}(A)$

$$\delta_{2\to 1}(A) = \begin{matrix} \\ a^0 \\ a^1 \end{matrix} \begin{bmatrix} 0.99570704 \\ 0.00429296 \end{bmatrix}$$

$\delta_{2\to 3}(A) = \sum_{B,E} \psi_2(B,E,A) \times \delta_{1\to 2}(A)$

$$\delta_{2\to 3}(A) = \begin{matrix} \\ a^0 \\ a^1 \end{matrix} \begin{bmatrix} 0.99570704 \\ 0.00429296 \end{bmatrix}$$

Problem 2 continued on next page...

Computing Beliefs

$\beta_1(J,T,A) = \psi_1(J,T,A) \times \delta_{2\to1}(A)$

$\beta_1(J,T,a_0) =$

	j^0	j^1
t^0	0.195158579056	0.003982828144
t^1	0.772668659936	0.023896968864

$\beta_1(J,T,a_1) =$

	j^0	j^1
t^0	0.000103031136	0.000755561664
t^1	0.00188890416	0.00154546704

$\beta_3(M,N,A) = \psi_3(M,N,A) \times \delta_{2\to3}(A)$

$\beta_3(M,N,a_0) =$

	m^0	m^1
n^0	0.73931247423	0.00746780277
n^1	0.248677832241	0.000248926759

$\beta_3(M,N,a_1) =$

	m^0	m^1
n^0	0.00080493075	0.00241479225
n^1	0.00080493075	0.00026831025

$\beta_2(B,E,A) = \psi_2(B,E,A) \times \delta_{1\to2}(A) \times \delta_{3\to2}(A)$

$\beta_2(B,E,a_0) =$

	b^0	b^1
e^0	0.995007996	0.0002997
e^1	0.0003992	0.00000014

$\beta_2(B,E,a_1) =$

	b^0	b^1
e^0	0.001994004	0.0016983
e^1	0.0005988	0.00000186

Problem 3

The distributions of the variables.

Solution

In Clique 1:J,T,A

$$P(A) = \begin{matrix} a^0 \\ a^1 \end{matrix} \begin{bmatrix} 0.99570704 \\ 0.00429296 \end{bmatrix} \qquad P(J) = \begin{matrix} j^0 \\ j^1 \end{matrix} \begin{bmatrix} 0.96981917 \\ 0.03018083 \end{bmatrix} \qquad P(T) = \begin{matrix} t^0 \\ t^1 \end{matrix} \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix}$$

In Clique 1:M,N,A

$$P(A) = \begin{matrix} a^0 \\ a^1 \end{matrix} \begin{bmatrix} 0.99570704 \\ 0.00429296 \end{bmatrix} \qquad P(M) = \begin{matrix} m^0 \\ m^1 \end{matrix} \begin{bmatrix} 0.98960017 \\ 0.01039983 \end{bmatrix} \qquad P(N) = \begin{matrix} n^0 \\ n^1 \end{matrix} \begin{bmatrix} 0.75 \\ 0.25 \end{bmatrix}$$

In Clique 1:B,E,A

$$P(A) = \begin{matrix} a^0 \\ a^1 \end{matrix} \begin{bmatrix} 0.99570704 \\ 0.00429296 \end{bmatrix} \qquad P(B) = \begin{matrix} b^0 \\ b^1 \end{matrix} \begin{bmatrix} 0.998 \\ 0.002 \end{bmatrix} \qquad P(E) = \begin{matrix} e^0 \\ e^1 \end{matrix} \begin{bmatrix} 0.999 \\ 0.001 \end{bmatrix}$$

Problem 4

Show the intermediate calculations for calculating updated distributions.

Solution

Updating the psi's

$\psi(j^1, T, A)=$

$$\psi(j^1, T, a^0) = \begin{matrix} & j^1 \\ t^0 \\ t^1 \end{matrix} \begin{bmatrix} 0.004 \\ 0.024 \end{bmatrix} \quad \psi(j^1, T, a^1) = \begin{matrix} & j^1 \\ t^0 \\ t^1 \end{matrix} \begin{bmatrix} 0.176 \\ 0.36 \end{bmatrix}$$

$\psi(m^0, N, A)=$

$$\psi(m^0, N, a^0) = \begin{matrix} & m^0 \\ n^0 \\ n^1 \end{matrix} \begin{bmatrix} 0.7425 \\ 0.24975 \end{bmatrix} \quad \psi(m^0, N, a^1) = \begin{matrix} & m^1 \\ n^0 \\ n^1 \end{matrix} \begin{bmatrix} 0.1875 \\ 0.1875 \end{bmatrix}$$

Compute the deltas

$\delta_{1\to 2}(A) = \sum_T \psi_1(j^1, T, A) =$

$$\delta_{1\to 2}(A) = \begin{matrix} a^0 \\ a^1 \end{matrix} \begin{bmatrix} 0.028 \\ 0.536 \end{bmatrix}$$

$\delta_{3\to 2}(A) = \sum_N \psi_3(m^0, N, A)$

$$\delta_{3\to 2}(A) = \begin{matrix} a^0 \\ a^1 \end{matrix} \begin{bmatrix} 0.99225 \\ 0.375 \end{bmatrix}$$

$\delta_{2\to 1}(A) = \sum_{B,E} \psi_2(B, E, A) \times \delta_{3\to 2}(A)$

$$\delta_{2\to 1}(A) = \begin{matrix} a^0 \\ a^1 \end{matrix} \begin{bmatrix} 0.98799031 \\ 0.00160986 \end{bmatrix}$$

$\delta_{2\to 3}(A) = \sum_{B,E} \psi_2(B, E, A) \times \delta_{1\to 2}(A)$

$$\delta_{2\to 3}(A) = \begin{matrix} a^0 \\ a^1 \end{matrix} \begin{bmatrix} 0.0278798 \\ 0.00230103 \end{bmatrix}$$

Problem 4 continued on next page...

Computing Beliefs

$\beta_1(j^1, T, A) = \psi_1(j^1, T, A) \times \delta_{2\to1}(A)$

$\beta_1(j^1, T, a^0) =$

	j^1
t^0	0.003951961225884
t^1	0.023711767355304

$\beta_1(j^1, T, a^1) =$

	j^1
t^0	0.000283335624
t^1	0.0.00057955014

$\beta_3(m^0, N, A) = \psi_3(m^0, N, A) \times \delta_{2\to3}(A)$

$\beta_3(m^0, N, a^0) =$

	m^0
n^0	0.02070074927844
n^1	0.006962979302749

$\beta_3(m^1, N, a^1) =$

	m^0
n^0	0.000431442882
n^1	0.000431442882

$\beta_2(B, E, A) = \psi_2(B, E, A) \times \delta_{1\to2}(A) \times \delta_{3\to2}(A)$

$\beta_2(B, E, a^0) =$

	b^0	b^1
e^0	0.027644307152868	0.0000083265651
e^1	0.0000110909736	0.00000000388963

$\beta_2(B, E, a^1) =$

	b^0	b^1
e^0	0.000400794804	0.0003413583
e^1	0.0001203588	0.00000037386

Calculate the probabilities

In Clique 1:J,T,A

$P(A|j^1, m^0) =$

a^0	0.96975155
a^1	0.03024845

$P(T|j^1, m^0) =$

t^0	0.14846826
t^1	0.85153174

In Clique 3:M,N,A

$P(A|j^1, m^0) =$

a^0	0.96975155
a^1	0.03024845

$P(N|j^1, m^0) =$

n^0	0.74078865
n^1	0.25921135

In Clique 2:B,E,A

$P(A|j^1, m^0) =$

a^0	0.96975155
a^1	0.03024845

$P(B|j^1, m^0) =$

b^0	0.98772856
b^1	0.01227144

$P(E|j^1, m^0) =$

e^0	0.99537879
e^1	0.00462121