(d)

~~Outbound message from clique [JTA, MNA, BEA~~

$$\cancel{\delta_{1-2}(A) = \begin{array}{cc} a^0 & a^1 \\ \end{array} \left[\begin{array}{cc} 0.024 & 0.36 \end{array}\right]}$$

$$\cancel{\delta_{3-2}(A) = \begin{array}{cc} a^0 & a^1 \\ \end{array} \left[\begin{array}{cc} 0.7425 & 0.1875 \end{array}\right]}$$

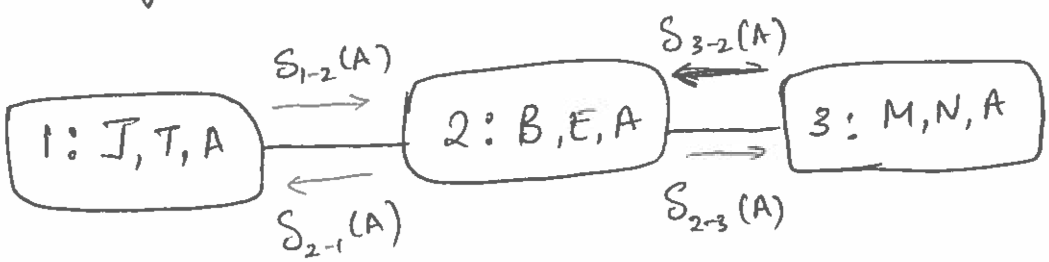
$$\cancel{\delta_{2-1}(A) = \begin{array}{cc} a^0 & a^1 \\ \end{array} \left[\begin{array}{cc} 0.73879344 & 0.0003788 \end{array}\right]}$$

$$\cancel{\delta_{2-3}(A) = \left[\begin{array}{cc} 0.0238\,8019 & 0.00071784 \end{array}\right]}$$

HW-5 TUSHAR TIWARI

Q2. (a)

Reusing tree from assignment #4



Clique tree generated from following elimination order.

$$\sum_N \phi(N) \sum_M \phi(M,N,A) \sum_E \phi(E) \sum_B \phi(B)\ \phi(A,B,E) \sum_T \phi(T) \sum_J \phi(J,T,A)$$

Q2.

b)

$\psi_1(J,T,A) =$

$$a^0 = \begin{array}{c} \\ t^0 \\ t^1 \end{array} \begin{array}{cc} j^0 & j^1 \\ \left[\begin{array}{c} 0.196 \\ 0.776 \end{array}\right. & \left.\begin{array}{c} 0.004 \\ 0.024 \end{array}\right] \end{array} \qquad a^1 = \begin{array}{c} \\ t^0 \\ t^1 \end{array} \begin{array}{cc} j^0 & j^1 \\ \left[\begin{array}{c} 0.024 \\ 0.44 \end{array}\right. & \left.\begin{array}{c} 0.176 \\ 0.36 \end{array}\right] \end{array}$$

$\psi_3(M,N,A) =$

$$a^0 = \begin{array}{c} \\ n^0 \\ n^1 \end{array} \begin{array}{cc} m^0 & m^1 \\ \left[\begin{array}{c} 0.7425 \\ 0.24975 \end{array}\right. & \left.\begin{array}{c} 0.0075 \\ 0.00025 \end{array}\right] \end{array} \qquad a^1 = \begin{array}{c} \\ n^0 \\ n^1 \end{array} \begin{array}{cc} m^0 & m^1 \\ \left[\begin{array}{c} 0.1875 \\ 0.1875 \end{array}\right. & \left.\begin{array}{c} 0.5625 \\ 0.0625 \end{array}\right] \end{array}$$

$\psi_2(B,E,A) =$

$$a^0 = \begin{array}{c} \\ e^0 \\ e^1 \end{array} \begin{array}{cc} b^0 & b^1 \\ \left[\begin{array}{c} 0.995008 \\ 0.003992 \end{array}\right. & \left.\begin{array}{c} 0.00029967 \\ 0.00000014 \end{array}\right] \end{array} \qquad a^1 = \begin{array}{c} \\ e^0 \\ e^1 \end{array} \begin{array}{cc} b^0 & b^1 \\ \left[\begin{array}{c} 0.001994004 \\ 0.0005988 \end{array}\right. & \left.\begin{array}{c} 0.0016983 \\ 0.00000186 \end{array}\right] \end{array}$$

$$\delta_{1\to 2}(A) = \max_{J,T} \psi_1(J,T,A) = \begin{array}{cc} a^0 & a^1 \\ [\,0.776 & 0.44\,] \end{array}$$

$$\delta_{3\to 2}(A) = \max_{M,N} \psi_3(M,N,A) = \begin{array}{cc} a^0 & a^1 \\ [\,0.7425 & 0.5625\,] \end{array}$$

$$\delta_{2\to 1}(A) = \max_{B,E} \psi_3(B,E,A)\,\delta_{3\to 2} = \begin{array}{cc} a^0 & a^1 \\ [\,0.73879344 & 0.00112163\,] \end{array}$$

$$\delta_{2\to 3}(A) = \max_{B,E} \psi_3(B,E,A)\,\delta_{1\to 2} = \begin{array}{cc} a^0 & a^1 \\ [\,0.7721262 & 0.00087736\,] \end{array}$$

$\beta_3(M,N,A) = \psi_3(M,N,A) \times \delta_{2\to3}(A)$

A	N	M	P
F	F	F	0.573304
F	F	T	0.005791
F	T	F	0.192839
F	T	T	0.000193
T	F	F	0.000165
T	F	T	0.000494
T	T	F	0.000165
T	T	T	0.000055

MAP value = 0.573304

$MAP(M,N,A) = (m^0, n^0, a^0)$

$\beta_2(B,E,A) = \psi_2(B,E,A) \times \delta_{1\to2}(A) \times \delta_{3\to2}(A)$

A	E	B	P
F	F	F	0.573304
F	F	T	0.000173
F	T	F	0.000230
F	T	T	0.0
T	F	F	0.000494
T	F	T	0.000420
T	T	F	0.000148
T	T	T	0.0

MAP value = 0.575304

$MAP(B,E,A) = (b^0, e^0, a^0)$

~~MAP~~

$MAP(A,T,J,B,E,\cancel{A},M,N,\cancel{B}) = (a^0, t^1, j^0, m^0, n^0, \cancel{\ }, b^0, e^0, \cancel{\ })$

(d)

Outbound messages

$$\delta_{1\to 2}(A) = \begin{array}{cc} a^0 & a^1 \\ \end{array} \left[\begin{array}{cc} 0.028 & 0.536 \end{array}\right]$$

$$\delta_{3\to 2}(A) = \begin{array}{cc} a^0 & a^1 \\ \end{array} \left[\begin{array}{cc} 0.99225 & 0.375 \end{array}\right]$$

$$\cancel{B_1}\ \beta_2(B,E,A) = \psi_2(B,E,A) \times \delta_{1-2}(A) \times \delta_{3-2}(A)$$

A	E	B	P
F	F	F	0.027644
F	F	T	0.000008
F	T	F	0.000011
F	T	T	0.0
T	F	F	0.000401
T	F	T	0.000341
T	T	F	0.000120
T	T	T	0.0

Sum over A

$\phi(B,E) =$

E	B	P
F	F	0.028045
F	T	0.000350
T	F	0.000131
F	F	0.0

$$\max_{B,E} \cancel{\Sigma}\, \phi(B,E) = 0.028045$$

MAP(B, E | J = True, M = False)

= ~~(...)~~

= (B = False, E = False)

Canonical form for X_3: $C_3(\cancel{X_3, h}.\ X_3, X_2, X_1; K_3, h_3, g_3)$

$$K_3 = \frac{1}{\sigma_3^2}\begin{bmatrix} 1 & -\beta_{3,1} & -\beta_{3,2} \\ -\beta_{3,1} & \beta_{3,1}^2 & -\beta_{3,1}\beta_{3,2} \\ -\beta_{3,2} & -\beta_{3,1}\beta_{3,2} & \beta_{3,2}^2 \end{bmatrix} \qquad h_3 = \cancel{\frac{1}{\sigma_3^2}}\begin{bmatrix} \beta_{3,0} \\ \beta_{3,1}\beta_{3,0} \\ \beta_{3,2}\beta_{3,0} \end{bmatrix}$$

$$g_3 = -\frac{1}{2}\frac{1}{\sigma_3^2}\beta_{3,0}^2 - \frac{1}{2}\ln(2\pi\sigma^2)$$

Elimination Order

$$P(x_1 \mid X_4 = x_4, X_5 = x_5) = \phi(X_1)\sum_{x_2}\phi(X_2)\sum_{x_3}\phi_3(X_3, X_2, X_1)\,\phi_4[X_4 = x_4](X_3)\,\phi_5[X_5 = x_5](X_3)$$

Eliminating X_3

$$\psi_1(X_3) = \phi_3(X_3, X_3, X_1)\,\phi_4(X_3)\,\phi_5(X_3) = C_3 + C_4 + C_5 = C_3($$

$$K_3' = \cancel{\frac{1}{\sigma_3^2}}\begin{bmatrix} \frac{1}{\sigma_4^2}\beta_{4,1}^2 + \frac{1}{\sigma_5^2}\beta_{5,1}^2 + \frac{1}{\sigma_3^2} & -\frac{1}{\sigma_3^2}\beta_{3,1} & -\frac{1}{\sigma_3^2}\beta_{3,2} \\ -\frac{1}{\sigma_3^2}\beta_{3,1} & \frac{1}{\sigma_3^2}\beta_{3,1}^2 & -\frac{1}{\sigma_3^2}\beta_{3,1}\beta_{3,2} \\ -\frac{1}{\sigma_3^2}\beta_{3,2} & -\frac{1}{\sigma_3^2}\beta_{3,1}\beta_{3,2} & \frac{1}{\sigma_3^2}\beta_{3,2}^2 \end{bmatrix}$$

$$h_3' = \cancel{\frac{1}{\sigma_3^2}}\begin{bmatrix} \frac{1}{\sigma_3^2}\beta_{3,0} + \frac{1}{\sigma_4^2}\beta_{4,1}(x_4 - \beta_{4,0}) + \frac{1}{\sigma_5^2}\beta_{5,1}(x_5 - \beta_{5,0}) \\ -\frac{1}{\sigma_3^2}\beta_{3,1}\beta_{3,0} \\ -\frac{1}{\sigma_3^2}\beta_{3,2}\beta_{3,0} \end{bmatrix}$$

Canonical form of x_2 $C_2(x_2; K_2, h_2, g_2)$

$$g_2 = -\frac{1}{2}\frac{1}{\sigma_2^2}\cancel{\beta}\mu_{x_2}^2 - \frac{1}{2}\ln(2\pi\sigma_2^2)$$

$$h_2 = \frac{1}{\sigma_2^2}\mu_{x_2}$$

$$K_2 = \frac{1}{\sigma_2^2}$$

$$\varphi_2(x_1, x_2) = \phi(x_2)\,\tau_1(x_1, x_2)$$

$$= C_3'(x_1, x_2; K_3'', h_3'', g_3'') + C_2(x_2; K_2, h_2, g_2)$$

$$= C_2'(x_1, x_2; K_2', h_2', g_2')$$

$$K_2' = \begin{bmatrix} \frac{1}{\sigma_3^4}\beta_{3,1}^2 & \frac{1}{\sigma_3^4}\beta_{3,1}\beta_{3,2} \\ \frac{1}{\sigma_3^4}\beta_{3,1}\beta_{3,2} & \frac{1}{\sigma_3^4}\beta_{3,2}^2 \end{bmatrix}\left[1 - \left(\frac{1}{\sigma_4^2}\beta_{4,1}^2 + \frac{1}{\sigma_5^2}\beta_{5,1}^2 + \frac{1}{\sigma_3^2}\right)\right]^{-1} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{\sigma_2^2} \end{bmatrix}$$

$$h_2' = h_3'' + h, \quad g_2' = g_3'' + g_2$$

Sum over x_2: $\tau_2 : C_2''(x_1; K_2'', h_2'', g_2'')$

$$K_2'' = \cancel{K'_{2x_2x_2}}$$

$$= K'_{2x_1x_1} - K'_{2x_2x_1} \times K'^{-1}_{2x_2x_2} \times K_{2x_1x_2}$$