When $O = \langle a^0, ?, ?, d^1 \rangle$

Computing $P(a^0, d^1)$

$$= \sum_{B,C} P(d^1|C)\, P(C|a^0 B)\, P(B)\, P(a^0)$$

$$= \sum_{B,C} d^1 \begin{bmatrix} c^0 & c^1 \\ 0.1 & 0.8 \end{bmatrix} \times c^c \begin{bmatrix} b^0 & b^1 \\ 0.17 & 0.91 \\ 0.83 & 0.09 \end{bmatrix} \begin{bmatrix} b^0 & b^1 \\ 0.1 & 0.9 \end{bmatrix} \times 0.7$$

$$= 0.15036$$

Computing all $P(x_i, u_i | O[m], \theta)$

$$P(b^0, a^0, c^1 | a^0, d^1) = \frac{P(d^1|c^1)\, P(c^1|a^0 b^0)\, P(a^0)\, P(b^0)}{0.15036}$$

$$= 0.8 \times 0.83 \times 0.7 \times 0.1 \,/\, 0.15036 = 0.30912$$

$$P(c^1, a^0, b^1 | a^0, d^1) = \frac{P(d^1|c^1)\, P(c^1|a^0, b^1)\, P(a^0)\, P(b^1)}{0.15036}$$

$$= \frac{0.8 \times 0.09 \times 0.7 \times 0.9}{0.15036} = 0.30168$$

$$P(c^0, a^0, b^1 | a^0, d^1) = P(d^1|c^0)\, P(c^0|a^0 b^1)\, P(a^0) P(b^1) \,/\, 0.15036$$

$$= 0.1 \times 0.91 \times 0.7 \times 0.9 \,/\, 0.15036$$

$$= 0.38128$$

$$P(c^0, a^0, b^0 \mid a^0 d^1) = P(d^1 \mid c^0) P(c^0 \mid a^0 b^0) P(a^0) P(b^0) / 0.15036$$

$$= 0.1 \times 0.17 \times 0.7 \times 0.1 / 0.15036$$

$$= 0.00792$$

$$P(d^1, c^0 \mid a^0 d^1) = P(d^1 \mid c^0) P(c^0 \mid a^0 B) P(B) P(a^0) / 0.15036$$

$$= \sum_B 0.1 \times b^0 \begin{bmatrix} 0.17 \\ 0.91 \end{bmatrix} \times \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix} \times 0.7 / 0.15036$$

$$= \sum_B \begin{matrix} b^0 \\ b^1 \end{matrix} \begin{bmatrix} 0.00119 \\ 0.05733 \end{bmatrix} / \cancel{0.3892}\ 0.15036$$

$$= 0.05852 / 0.15036 = 0.3892$$

$$P(d^1, c^1 \mid a^0, d^1) = P(d^1, c^1) P(c^1 \mid a^0 B) P(B) P(a^0) / 0.15036$$

$$= \sum_B 0.8 \times b^0 \begin{bmatrix} 0.83 \\ 0.09 \end{bmatrix} \times \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix} \times 0.7 / 0.15036$$

$$= \sum_B \begin{matrix} b^0 \\ b^1 \end{matrix} \begin{bmatrix} 0.04648 \\ 0.04536 \end{bmatrix} / 0.15036$$

$$= 0.09184 / 0.15036 = 0.6108$$

In $o' <?, b', ?, d^\circ>$

$$P(b', d^\circ) = \sum_{A,C} P(d^\circ|C)\, P(C|Ab')\, P(A)\, P(b')$$

$$= \begin{matrix} c^\circ \\ c' \end{matrix} \begin{bmatrix} 0.9 \\ 0.2 \end{bmatrix} \times \begin{matrix} \\ a^\circ \end{matrix} \begin{matrix} \\ a' \end{matrix} \begin{bmatrix} 0.91 & 0.8 \\ 0.09 & 0.2 \end{bmatrix} \times \begin{bmatrix} 0.7 & 0.3 \end{bmatrix} \times 0.9$$

$$= \sum_{A,C} \begin{matrix} c^\circ \\ c' \end{matrix} \begin{bmatrix} 0.51597 & 0.1944 \\ 0.01134 & 0.0108 \end{bmatrix} = 0.73251$$

$$P(c^\circ, a^\circ, b'|b', d^\circ)$$

$$= P(c^\circ|a^\circ b')\, P(d^\circ|c^\circ)\, P(a^\circ)\, P(b') / 0.73251$$

$$= 0.91 \times 0.9 \times 0.7 \times 0.9 / 0.73251 = 0.7044$$

$$P(c^\circ, a', b'|b', d^\circ)$$

$$= P(d^\circ|c^\circ)\, P(c^\circ|a', b')\, P(a')\, P(b') / 0.73251$$

$$= 0.9 \times 0.8 \times 0.3 \times 0.9 / 0.73251 = 0.2654$$

$$P(c', a^\circ, b'|b', d^\circ) = P(d^\circ|c')\, P(c'|a^\circ b')\, P(a^\circ)\, P(b') / 0.73251$$

$$= 0.2 \times 0.09 \times 0.7 \times 0.9 / 0.73251 = 0.01548$$

$$P(c', a', b'|b', d^\circ) = P(d^\circ, |c')\, P(c'|a'b')\, P(a')\, P(b') / 0.73251$$

$$= 0.2 \times 0.2 \times 0.3 \times 0.9 / 0.73251 = 0.01472$$

$$P(d^0 c^0 | b^1 d^0) = \sum_A P(d^0|c^0) P(c^0|b^1) P(A) P(b^1)$$

$$= \sum_A 0.9 \times \begin{matrix} a^0 \\ a^1 \end{matrix} \begin{bmatrix} 0.91 \\ 0.8 \end{bmatrix} \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix} \times 0.9 \Big/ 0.73251$$

$$= \begin{bmatrix} 0.51597 \\ 0.1944 \end{bmatrix} = 0.71037 / 0.73251 = 0.9698$$

$$P(d^0 c^1 | b^1 d^0) = \sum_A P(d^0|c^1) P(c^1|Ab^1) P(A) P(b^1) / 0.7325)$$

$$= \sum_A 0.2 \times \begin{bmatrix} 0.09 \\ 0.2 \end{bmatrix} \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix} \times 0.9 / 0.73251$$

$$= 0.0302.$$

$$\bar{M}_{\theta^t}[c^1, a^0, b^0] = 0.30912$$

$$\bar{M}_{\theta^t}[c^1, a^0, b^1] = 0.01548 + 0.30168 = 0.31716$$

$$\bar{M}_{\theta^t}[c^1, a^1, b^1] = 0.01472$$

$$\bar{M}_{\theta^t}[d^1, c^0] = 0.3892 + 0.00792 + 0.38128 = 0.7784$$

$$\bar{M}_{\theta^t}[d^1, c^1] = 0.6108 + 0.30168 + 0.30912 = 1.0216$$

$$\theta^{t+1}_{d^1|c^0} \leftarrow \frac{\bar{M}_t[d^1, c^0]}{\bar{M}_t[c^0]} = \frac{0.7784}{2.718} = 0.2864$$

$$M_t[c^0] = 0.28\ 0.7784 + 0.7044 + 0.2654 + 0.9698$$

$$= 2.718$$

$$\theta^{t+1}_{d'|c'} = \frac{\bar{M}_t[d', c']}{M_t[c']} = \frac{1.2216}{1.282} = 0.9529$$

$$M_t[c] = 1.2216 + 0.01548 + 0.01472 + 0.0302 = 1.282$$

$$\theta^{t+1}_{c'|a^\circ b^\circ} = \frac{\bar{M}_t[c', a^\circ, b^\circ]}{\bar{M}_t[a^\circ, b^\circ]} = \frac{0.30912}{0.31704} = 0.975$$

$$M_t[a^\circ, b^\circ] = 0.30912 + 0.00792 = 0.31704$$

$$\theta^{t+1}_{c'|a'b^\circ} = 0$$

$$\theta^{t+1}_{c'|a^\circ b'} = \frac{\bar{M}_t[c', a^\circ, b']}{\bar{M}_t[a^\circ, b']} = \frac{0.31716}{1.40284} = 0.2261$$

$$\bar{M}_t[a^\circ, b'] = 0.31716 + 0.38128 + 0.7044 = 1.40284$$

$$\theta^{t+1}_{c'|a'b'} = \frac{\bar{M}_t[c', a', b']}{\bar{M}_t[a', b']} = \frac{0.01472}{0.28012} = 0.0525$$

$$\bar{M}_t[a', b'] = 0.01472 + 0.2654 = 0.28012$$