

Lecture 1: January 12, 2015  
cs 573: Probabilistic Reasoning  
Professor Nevatia  
Spring 2015

## Introduction

- Course name: Probabilistic Reasoning
  - csci 573
- Instructor: Prof. Ram Nevatia
  - My background, interests
- Attendance sheet, in class students only
- Today's objective
  - Describe course content
  - Conduct of the class
  - Required work, grading
  - Pre-requisites

## General Information

- Course web page <https://courses.usc.edu/net>
  - Must be registered for class to have access
  - Usually, a preliminary version of slides will be posted prior to class, complete version after the class.
  - Lecture videos, course notes....
- Office hours:
  - Instructor: MW 1:30-3:00 P.M., PHE 204, 213-740-6427, [nevatia AT usc.edu](mailto:nevatia@usc.edu)
  - This week only: Today, 1- 2 P.M., none on Wednesday
  - TA: Tom Collins, [collinst AT usc.edu](mailto:collinst@usc.edu) , office hours and place TBD
- Book:
  - Required: *Probabilistic Graphical Models* by Koller and Friedman, MIT Press 2010. Note: errata for early printings may be found at <http://pgm.stanford.edu/errata/>
- Koller video lectures available at:  
<https://class.coursera.org/pgm/lecture/preview>

## About Enrollment

- On-campus section is fully subscribed
- Capacity limited by physical space and ability to provide individual attention to students
  - Not possible to add another section due to non-availability of a qualified instructor
- Heavy demand is a surprise to us
  - Previous years have seen much lower demand and enrollments
- Common for students to drop in first 1-2 weeks of class
  - Will fill in if and as students drop

## Prerequisites

- Undergraduate level course in probability theory
- Good skills with basic mathematics such as calculus and linear algebra
- Programming skills: ability to convert algorithms into programs
- May have some overlap with cs561 (basic AI course) and cs567 (Machine Learning).
  - Neither is a pre-requisite

## Requirements and Grading

- Assignments: 7-8 assignments, mostly written (mathematical) but 1-2 programming assignments may be included. *Large* projects are not planned.
- Grading:
  - Assignments 30%,
  - Exams 1 and 2: 30% each
  - Class attendance and participation 10% (does not apply to DEN students)
- Exams:
  - Exam 1, during 7<sup>th</sup> or 8<sup>th</sup> week of classes; exact date will be announced at least 1 week in advance
  - Exam 2, April 29, 2015; last day of scheduled classes
- Programming
  - No programming projects are planned but some programming may be necessary to solve numerical problems
  - Students may choose their own language but some packages may be specific to some languages

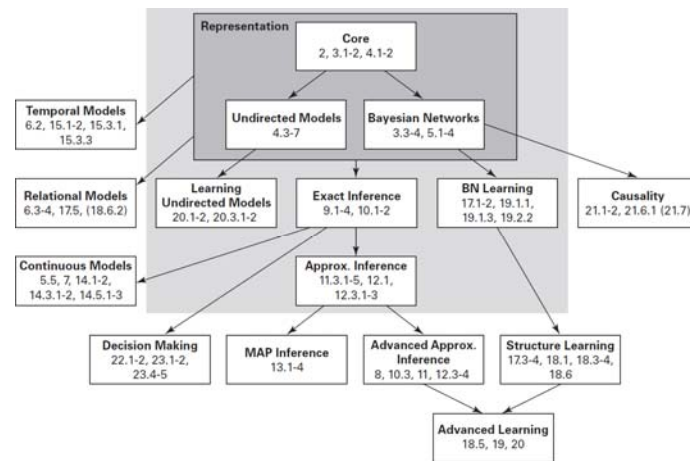
## Course Objectives

- In-depth coverage of issues related to probabilistic reasoning. These include:
  - Probabilistic Representations (~ 4 Weeks)
    - Bayesian networks, undirected graphs, dynamic networks
  - Probabilistic inference (~ 4 Weeks)
    - Exact and approximate, including for temporal graphs
  - Learning of parameters and structure of probabilistic graphical models (~ 3 Weeks)
  - Decision making under uncertainty (~ 2 Weeks)
- First two topics will get the most attention. There are other courses that cover learning and planning in more detail.
- Focus on concepts and algorithms, not applications or commercial systems

## Syllabus in terms of Book Sections

- Note: This is only a plan, actual coverage may vary some. Not all material in each section may be covered and some external material may be included.
- Representations: Chapters 2-5 all except sections marked with “\*” in the book, Chapter 6: 6.1, 6.2 only
- Inference: Chapter 9 (except 9.5 and 9.6), 10, 11.1 to 11.3, 12.1 to 12.3, 13.1 to 13.3, 15.1, 15.2; additional notes
- Parameter Estimation: 17.1, 17.2 (exclude 17.2.4, 17.2.5), 19.2.2 (exclude 19.2.2.5, 19.2.2.6), Additional Notes
- Decision Theory: Ch. 22
- Selected parts of Ch. 7, 14, 11.4, 23

## Book Plan



## Course Style

- Interactive to the extent possible
- Will follow book closely; however, we will skip some details such as proofs of most theorems.
  - Assignments may require more detailed knowledge than covered in class
  - Preliminary version of lecture slides will be posted in advance of the lectures
- We will discuss algorithms at a relatively high level, almost never at the code or data structure levels
  - In English, in pseudo-code, by examples
  - Ability to convert high level descriptions to code is assumed

## Reasoning under Uncertainty

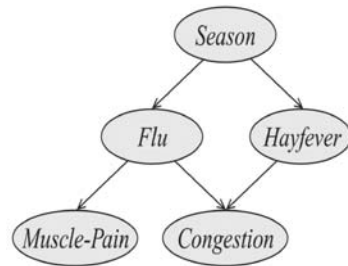
- Uncertainty is almost always present in solving real problems
  - State of the world is not known precisely or not even knowable in principle
  - Some aspects of the state can't be measured directly (*e.g.* cause of some types of sickness)
  - Effects of actions are uncertain
    - *e.g.* What route to take to go to the airport? When to start? What courses to take to succeed in a cs career?
- Rational decision (under uncertainty)
  - Consider the relative importance of various goals and the *likelihood* that they will be achieved.
  - Rationality does not *guarantee* success

## Possible Application Areas

- Originally developed for “expert systems”
  - Medical diagnosis (Pathfinder), Prospector...
  - Attempt to systematize reasoning about uncertain knowledge
- Now, virtually all aspects of computer science
  - AI, robotics, vision, speech
  - e-commerce, search...
  - Networks, OS, software engineering...
- Outside CS
  - Economics, finance, weather prediction, political science....
- Likely to become as important in CS study as discrete algorithms taught in courses such as cs570/670

## An Example Graph

- Note: only the *structure* of relations is shown
- Methodology attempts to combine rule-based representations with probabilistic reasoning



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## Denoising an Image



Observed, noisy image



Model



Estimated Image

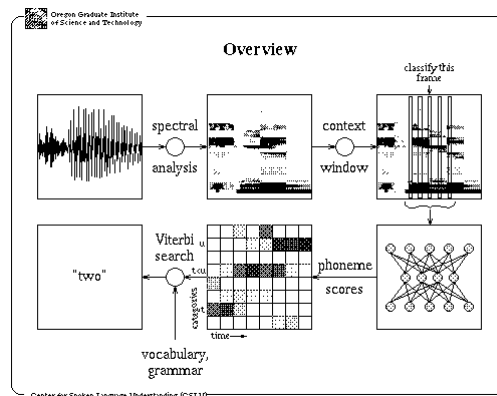
From: [http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL\\_COPIES/AV0809/ORCHARD/](http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL_COPIES/AV0809/ORCHARD/)

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## Speech Recognition



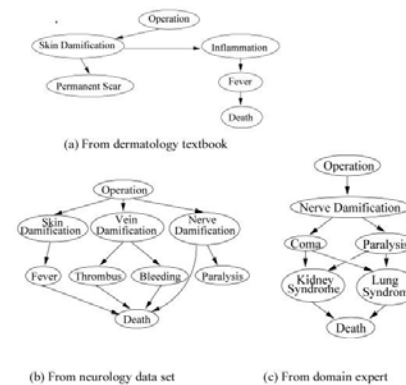
[http://www.cslu.ogi.edu/tutordemos/nnet\\_recog/overview.gif](http://www.cslu.ogi.edu/tutordemos/nnet_recog/overview.gif)

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## Medical Diagnosis



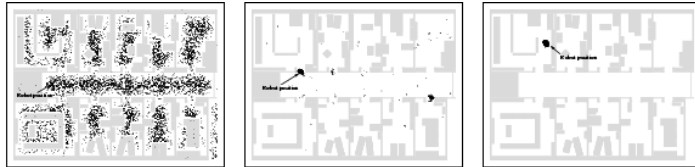
From C. Jiang et al: AMIA Annual Proceedings 2005, 370-374

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## Robot Localization



[http://www.cc.gatech.edu/~dellaert/assets/images/autogen/a\\_sonar.gif](http://www.cc.gatech.edu/~dellaert/assets/images/autogen/a_sonar.gif)

## Nature of Study

- Develop a representation for a problem domain
  - We will not study specific domains, just the methodologies
  - Will normally consist of choice of *random variables* and relations between them, represented in a graph
- Given values of some variables, compute probability distributions of others. Some types of *queries*:
  - *Forensic/diagnostic*: causes given evidence
  - *Predictions*: observables given cause
  - *Explanations*: of observed phenomenon
  - *Actions*: optimal actions given a model and observations
- Learning of the parameters (sometimes also the structure) of the graph from examples

## Basic Probability: Informal concepts

- *Unconditional* or *prior* probability that a proposition A is true
  - Let  $P(\text{the event of rain falling on Jan 12 in L.A. is True}) = 0.2$
  - $P(\text{fair\_coin\_toss:head}) = 0.5$
- *Conditional* probability  $P(\text{rain\_today}|\text{cloudy\_today})$
- *Joint* probability  $P(\text{rain\_Jan12, cloudy\_Jan12})$
- *Independence* of variables: coin toss and weather

## Probability Theory: Some notations

- Event *space*, space of all possible outcomes,  $\Omega$ 
  - For outcome of roll of dice:  $\Omega = \{1,2,3,4,5,6\}$
  - Each event is a subset of  $\Omega$ , e.g.  $\{3\}$ ,  $\{2,4,6\}$  (even outcome)
- Set of measurable events:  $S$ 
  - We assign probabilities to elements of  $S$
  - Contains *empty event*  $\emptyset$  and *trivial event*  $\Omega$
  - Closed under union (if  $\alpha$  and  $\beta \in S$ , then so is  $\alpha \cup \beta$ )
  - If  $\alpha \in S$ , so is  $\Omega - \alpha$

## Probability Distributions

- Probability distribution  $P$  defined over  $(\Omega, S)$ :
  - Mapping from events in  $S$  to real values (probabilities)
    - $P(\alpha) \geq 0$  for all  $\alpha \in S$
    - $P(\Omega) = 1$
    - If  $\alpha, \beta \in S$ , and  $\alpha \cap \beta = \emptyset$ , then  $P(\alpha \cup \beta) = P(\alpha) + P(\beta)$
- It follows that  $P(\emptyset) = 0$  and
  - $P(\alpha \cup \beta) = P(\alpha) + P(\beta) - P(\alpha \cap \beta)$
- Common interpretation of probability is as frequency of events
  - Examples: coin toss, dice roll, weather (how many outcomes?)

## Conditional Probability, Bayes' Rule

- Conditional probability:  $P(\beta | \alpha)$ 
    - Probability that  $\beta$  is true, given that  $\alpha$  is true
    - e.g.  $P(\text{rain} | \text{cloudy})$ ,  $P(\text{grade A} | \text{high\_intell})$
    - $P(\beta | \alpha) = P(\alpha \cap \beta) / P(\alpha)$
  - Chain rule:  $P(\alpha \cap \beta) = P(\alpha) P(\beta | \alpha)$
- $$P(\alpha_1 \cap \dots \cap \alpha_k) = P(\alpha_1) P(\alpha_2 | \alpha_1) \dots P(\alpha_k | \alpha_1 \cap \dots \cap \alpha_{k-1})$$
- Bayes' rule  $P(\alpha | \beta) = P(\beta | \alpha) P(\alpha) / P(\beta)$ 
    - It may be easier to gather  $P(\beta | \alpha)$  than  $P(\alpha | \beta)$ , for some situations (e.g. probability of symptoms given disease rather than the disease given symptoms)

$$P(\alpha | \beta \cap \gamma) = \frac{P(\beta | \alpha \cap \gamma) P(\alpha | \gamma)}{P(\beta | \gamma)}$$

## Random Variables

- Random variables take on different values with probabilities given by a distribution function
  - e.g. coin toss, roll of dice, grades in a course etc
  - Formally, defined by function that associates a real value to each outcome in  $\Omega$ .
  - Random variables normally denoted by upper case letters  $X, Y, Z, \dots$
- Discrete random variable: takes one of a finite set of values:  $Val(X)$  denotes the set of values of  $X$
- $P(X = x^i)$  denotes the probability that  $X$  takes value  $x^i$
- Often abbreviated to  $P(x^i)$ 
  - $\sum_x$  denotes summation over all possible values of  $X$
  - Sum over all possible values must equal 1 for a discrete variable, i.e.  $\sum_x P(x) = 1$
  - Distribution is called multinomial
    - Binomial or Bernoulli for a binary variable
- Bold letters used to denote a *set* of variables, e.g.  $\mathbf{X}, \mathbf{Y}, \mathbf{Z}, \dots$ 
  - $x, y, z$  denote values of variable in these sets

## Marginal and Joint Distributions

- Joint distribution*: probabilities for each combination of values of the random variables.
  - $P(x, y)$  is used to denote joint probability of  $X = x$  and  $Y = y$
- Figure 2.1, example of  $P(\text{Intelligence}, \text{grade})$

		Intelligence		
		low	high	
Grade	A	0.07	0.18	0.25
	B	0.28	0.09	0.37
	C	0.35	0.03	0.38
		0.7	0.3	1

- Marginal distribution*: Distribution of one variable, regardless of the values of others;
  - obtained by summing over all other variables from the joint distribution, e.g.  $P(\text{Intelligence} = \text{high})$  or  $P(\text{grade} = A)$ ;

## Conditional Distribution

- $P(X | Y)$ :
  - for each value of  $Y$ , assign a distribution over values of  $X$ .

- Chain rule:  $P(X, Y) = P(X) P(Y | X)$

$$P(X_1, \dots, X_k) = P(X_1)P(X_2 | X_1) \cdots P(X_k | X_1, \dots, X_{k-1})$$

- Bayes' rule  $P(X | Y) = P(X) P(Y | X) / P(Y)$

## Independence

- Event  $\alpha$  is independent of event  $\beta$  in  $P$ , denoted as  $P \models \alpha \perp \beta$ , if  $P(\alpha | \beta) = P(\alpha)$ , or if  $P(\beta) = 0$
- Follows that  $P(\alpha \cap \beta) = P(\alpha) P(\beta)$
- Examples: toss two coins; coin toss and weather...
- Full independence is rare, *conditional independence* where two events are independent, given a third event
- Conditional independence
  - $P(\text{USC} | \text{UCLA}, \text{GradeA}) = P(\text{USC} | \text{GradeA})$   
(USC means admitted to USC, similar for UCLA)
  - $P(\text{Congestion} | \text{Flu}, \text{Hayfever}, \text{Season}) = P(\text{Congestion} | \text{Flu}, \text{Hayfever})$
- Event  $\alpha$  is independent of event  $\beta$  in  $P$ , given event  $\gamma$ , denoted as  $P \models (\alpha \perp \beta | \gamma)$  if  $P(\alpha | \beta \cap \gamma) = P(\alpha | \gamma)$ , or if  $P(\beta | \gamma) = 0$
- Follows that  $P(\alpha \cap \beta | \gamma) = P(\alpha | \gamma) P(\beta | \gamma)$

## Next Class

- Read sections 2.1, 2.2 and 3.1 of the KF book