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Q1. Derive a generalized version of Bayes' rule stated as

$$P(x|Y,z) = \frac{P(Y|x,z) * P(x|z)}{P(Y|z)}$$

A generalized version would be $P(x|Y,z,\lambda)$

$$P(x|Y,z,\lambda) = \frac{P(Y,x,z,\lambda)}{P(Y,z,\lambda)}$$

Applying chain rule to numerator and denominator.

$$= \cancel{\frac{P(Y|x,z,\lambda)}{P(Y|z,\lambda)}}$$

$$= \frac{P(Y|x,z,\lambda)\,P(x|z,\lambda)\,\cancel{P(z|\lambda)}\,\cancel{P(\lambda)}}{P(Y|z,\lambda)\,\cancel{P(z|\lambda)}\,\cancel{P(\lambda)}}$$

$$\boxed{= \frac{P(Y|x,z,\lambda)\,P(x|z,\lambda)}{P(Y|z,\lambda)}}$$

Q2. <u>Symmetry</u>

$$(X \perp Y \mid Z) \Rightarrow (Y \perp X \mid Z)$$

Assume $(X \perp Y \mid Z)$ holds

then

$$P(x, y \mid z) = P(x \mid z)\, P(y \mid z)$$

$\because$ Multiplication is commutative

$$P(x, y \mid z) = P(y \mid z)\, P(x \mid z) = P(y, x \mid z)$$

Thus $(Y \perp X \mid Z)$ also holds.

<u>Decomposition</u>

Assume $(X \perp Y, W \mid Z)$ holds.

Then $P(X, Y, W \mid Z) = P(X \mid Z)\, P(Y, W \mid Z)$

$\because P(\alpha \cap \beta \mid \gamma) = P(\alpha \cap \gamma)\, P(\beta \cap \gamma)$

No we show,

$$P(X, Y \mid Z) = \cancel{P(X \mid Z)\, P(Y, W \mid Z)}$$

~~Summing over W~~

$\cancel{P(X \mid Z) \sum_w P(Y, w \mid Z)}$

$\cancel{\sum_w P(X \mid Z)\, P(Y, w \mid Z)}$

$$= \sum_w P(X, Y, w \mid Z)$$

$$= \sum_w P(X \mid Z)\, P(Y, w \mid Z)$$

$$= P(X \mid Z) \sum_w P(Y, w \mid Z)$$

$$= P(X \mid Z)\, P(Y \mid Z)$$

$\therefore$ ~~By applyin~~

Q3.

Variance is written as

$$Var_p[X] = E_p[(X - E_p(X))^2]$$

Expanding square

$$= E_p[\,X^2 + E_p^2[X] - 2X E_p[X]]$$

$\because$ linearity

$$= E_p[X^2] + E_p^2[X] - 2\,E_p[X E_p[X]]$$

$$\because E_p[E_p[X]] = E_p[X] \quad \cancel{\text{and}}$$

$$= E_p[X^2] + E_p^2[X] - 2\,E_p^2[X]$$

$$= E_p[X^2] - E_p^2[X]$$

$$= \boxed{\therefore E[X^2] - (E[X])^2}$$

Q4.

$$E[X] = \mu$$

$$E[Y] = E[aX+b]$$

$$= aE[X] + E[b] \quad \text{Applying linearity}$$

$$= aE[X] + b$$

$$= a\mu + b$$

$$Var[X] = \sigma^2 = E[X^2] - (E[X])^2$$

$$Var[Y] = E[(aX+b)^2] - (E[(aX+b)])^2$$

$$= E[a^2X^2 + b^2 + 2abX] - (aE[X]+b)^2$$

$$= a^2E[X^2] + \cancel{b^2} + \cancel{2abE[X]}$$
$$- (a^2(E[X])^2 + \cancel{b^2} + \cancel{2abE[X]})$$

$$= a^2E[X^2] - a^2(E[X])^2$$

$$= a^2(E[X^2] - (E[X])^2)$$

$$= a^2\sigma^2$$