

# Probabilistic Reasoning: Homework 4

Due on March 9, 2015

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## Problem 1

Create a corresponding clique tree

## Solution

### Calculations

The clique tree is constructed using the variable elimination technique when finding  $P(A)$ .

$$P(A) = \sum_N \phi(N) \sum_M \phi(M, N, A) \sum_E \phi(E) \sum_B \phi(B) \phi(A, B, E) \sum_T \phi(T) \sum_J \phi(J, T, A)$$

$$\psi_1(J, T, A) = \phi(J, T, A)$$

$$\tau_1(T, A) = \sum_J \psi_1(J, T, A)$$

$$\psi_2(T, A) = \phi(T) \tau_1(T, A)$$

$$\tau_2(A) = \sum_T \psi_2(T, A)$$

Since,  $\text{scope}(\psi_2) \subset \text{scope}(\psi_1)$ , only one clique is sufficient to represent these factors.

$$\psi_3(A, B, E) = \phi(B) \phi(A, B, E) \tau_2(A)$$

$$\tau_3(A, E) = \sum_B \psi_3(A, B, E)$$

$$\psi_4(A, E) = \phi(E) \tau_3(A, E)$$

$$\tau_4(A) = \sum_E \psi_4(A, E)$$

Again,  $\text{scope}(\psi_4) \subset \text{scope}(\psi_3)$ , only one clique is sufficient to represent these factors.

$$\psi_5(M, N, A) = \phi(M, N, A) \tau_4(A)$$

$$\tau_5(N, A) = \sum_M \psi_5(M, N, A)$$

$$\psi_6(N, A) = \phi(N) \tau_5(N, A)$$

$$\tau_6(A) = \sum_N \psi_6(N, A)$$

Again,  $\text{scope}(\psi_6) \subset \text{scope}(\psi_5)$ , only one clique is sufficient to represent these factors.

Also, the messages passed between the cliques are all  $\tau_i(A)$  so the cliques will have message  $\delta_{i \rightarrow j}(A)$  passed between them.

### Clique Tree

The clique tree description:

- 1:  $\psi(J, T, A)$  connected to  $\psi(B, E, A)$  over  $\delta_{1 \rightarrow 2}(A)$ .
- 2:  $\psi(B, E, A)$  connected to  $\psi(J, T, A)$  over  $\delta_{2 \rightarrow 1}(A)$ .
- 2:  $\psi(B, E, A)$  connected to  $\psi(M, N, A)$  over  $\delta_{2 \rightarrow 3}(A)$ .
- 3:  $\psi(M, N, A)$  connected to  $\psi(B, E, A)$  over  $\delta_{3 \rightarrow 2}(A)$ .

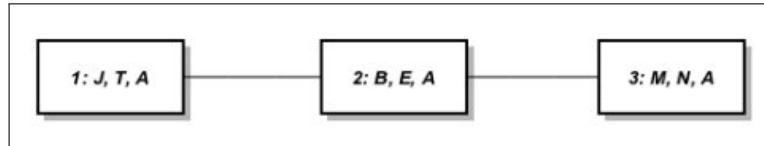


Figure 1: The clique tree