Probabilistic Reasoning: Homework 6

Due on April 13, 2015

Prof. Nevatia

Tushar Tiwari

Problem 1

Description of Homework

Solution

Description of code

- Reduced all initial factors based on evidence j^1, m^0
- Maintained a hashmap that maps variable name to current assignment.
- Maintained hashmaps that maps variable name to distribution in current and older iteration.
- Initial assignment given to variables $e^1,\ b^1,\ t^1,\ n^1,\ a^0$
- Computed distribution of the variable under consideration, by reducing the factors that contain the variable based on evidence specified in the hashmap. Multiply the reduced factors to obtain distribution. Saved the value of the distribution in the current iteration hashmap against the variable name.
- Used random.uniform() in python to obtain a random number in [0, 1]. Select an assignment based on this random number and save this in the hashmap against variable name.
- After doing the above computation for all variables, check if the time step is a multiple of 1000000 and $|\ old\ distribution\ -\ new\ distribution\ | <\ 0.00000000000001$ for all variables. If true, break. Convergence is achieved. Else, copy all distributions to the old_iteration hashmap.

Final Values

T :$Time\_Step$	
$Variable$:$Assignment$	$[False\_Value, True\_Value]$
T :3000000	
A :0	[0.992466248795, 0.0075337512054]
B :0	[0.999698887087, 0.000301112913328]
E :0	[0.999598958093, 0.000401041906874]
T :1	[0.142857142857, 0.857142857143]
N :0	[0.748299319728, 0.251700680272]

Problem 1 continued on next page. . .

Intermediate Values

The intermediate values are taken when time step T is a factor of 500000.

T :$Time\_Step$	
$Variable$:$Assignment$	$[False\_Value, True\_Value]$
T :0	
A :1	[0.00531385805244, 0.994686141948]
B :1	[0.540043290043, 0.459956709957]
E :0	[0.998905985319, 0.00109401468097]
T :0	[0.328358208955, 0.671641791045]
N :1	[0.5, 0.5]
T :500000	
A :0	[0.992466248795, 0.0075337512054]
B :0	[0.999698887087, 0.000301112913328]
E :0	[0.999598958093, 0.000401041906874]
T :1	[0.142857142857, 0.857142857143]
N :0	[0.748299319728, 0.251700680272]
T :1000000	
A :0	[0.978218253104, 0.0217817468961]
B :0	[0.999698887087, 0.000301112913328]
E :0	[0.999598958093, 0.000401041906874]
T :0	[0.142857142857, 0.857142857143]
N :0	[0.748299319728, 0.251700680272]
T :1500000	
A :0	[0.992466248795, 0.0075337512054]
B :0	[0.999698887087, 0.000301112913328]
E :0	[0.999598958093, 0.000401041906874]
T :1	[0.142857142857, 0.857142857143]
N :0	[0.748299319728, 0.251700680272]
T :2000000	
A :0	[0.992466248795, 0.0075337512054]
B :0	[0.999698887087, 0.000301112913328]
E :0	[0.999598958093, 0.000401041906874]
T :1	[0.142857142857, 0.857142857143]
N :0	[0.748299319728, 0.251700680272]

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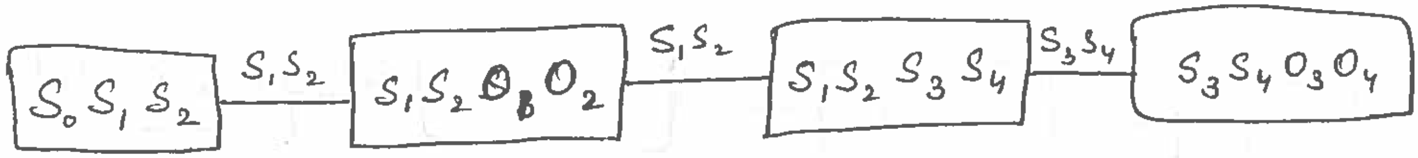
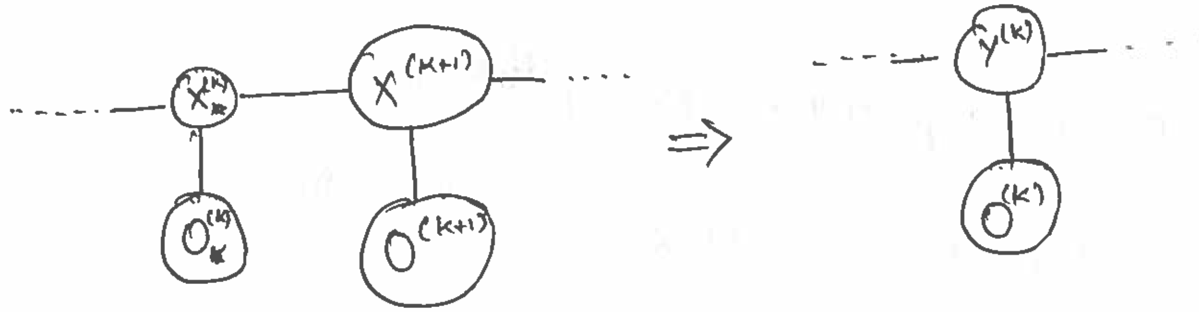
T :2500000	
A :0	[0.978218253104, 0.0217817468961]
B :0	[0.999698887087, 0.000301112913328]
E :0	[0.999598958093, 0.000401041906874]
T :0	[0.142857142857, 0.857142857143]
N :0	[0.748299319728, 0.251700680272]

a)



(b) To derive the recursive function we modify the HMM to combine $X^{(1)} X^{(2)}$ to $Y^{(1)}$ and $X^{(3)} X^{(4)}$ to $Y^{(2)}$ and so on.



The CPD's for the observations can be multiplied by $P(O^{(k)} | X^{(k)})$, $P(O^{(k+1)} | X^{(k+1)})$ and CPD's for the stated can be combined by $P(X^{(k)} | X^{(k-1)}, X^{(k-2)})$, $P(X^{(k+1)} | X^{(k)}, X^{(k-1)})$

~~we get~~ $\cancel{P(X^{(k+1)}, X^{(k)} | X^{(k)} X}$ to get $P(X^{(k+1)}, X^{(k)} | X^{(k-1)}, X^{(k-2)})$

which is rewritten as $P(Y^{i+1} | Y^{i})$ and

$P(O^{(k)}, O^{(k+1)} | X^{(k+1)} X^{(k)})$ which is rewritten as $P(O^{i+1} | Y^{i+1})$

and $\sigma^{(t+1)}(Y^{i+1}) = P(Y^{i+1} | O^{(1:i+1)})$

Compute $\sigma^{(t+1)}(y^{(t+1)})$. This is already known.

$$\sigma^{(t+1)}(y^{(t+1)}) = \frac{P(\theta^{(t+1)} \mid y^{(t+1)})\,\sigma^{(\cdot t+1)}(y^{(t+1)})}{P(\theta^{(t+1)} \mid \theta^{(1:t)})}$$

Substituting for all the Y's and θ's and the appropriate t!

$$\boxed{\sigma^{(t+1)}(x^{(t+1)}, x^{(t)}) = \frac{P(o^{(t+1)}, o^{(t)} \mid x^{(t+1)}, x^{(t)})\,\sigma^{(\cdot t+1)}(x^{(t+1)}, x^{(t)})}{P(o^{(t+1)}, o^{(t)} \mid o^{(1:t-1)})}}$$

Compute $\sigma^{(\cdot t+1)}(x^{(t+1)}, x^{(t)})$ by substituting in $\sigma^{(\cdot t+1)}(y^{(t+1)})$

$$\sigma^{(\cdot t+1)}(y^{(t+1)}) = P(y^{(t+1)} \mid \theta^{(1:t)})$$

$$= \sum_{y^{(t)}} P(y^{(t+1)} \mid y^{(t)})\,\sigma^{(t)}(y^{(t)})$$

$$\sigma^{(\cdot t+1)}(x^{(t+1)}, x^{(t)}) = \sum_{x^{(t-1)}, x^{(t-2)}} P(x^{(t+1)}, x^{(t)} \mid x^{(t-1)}, x^{(t-2)})\,\sigma^{(t-1)}(x^{(t-1)}, x^{(t-2)})$$

$$= \sum_{x^{(t-1)}, x^{(t-2)}} P(x^{(t+1)} \mid x^{(t)}, x^{(t-1)}) . P(x^{(t)} \mid x^{(t-1)}, x^{(t-2)})\,\sigma^{(t-1)}(x^{(t-1)}, x^{(t-2)})$$

$$\boxed{\sigma^{(t+1)}(x^{(t+1)}) = \sum_{x^{(t)}} \sigma^{(t+1)}(x^{t+1}, x^{(t)})}$$