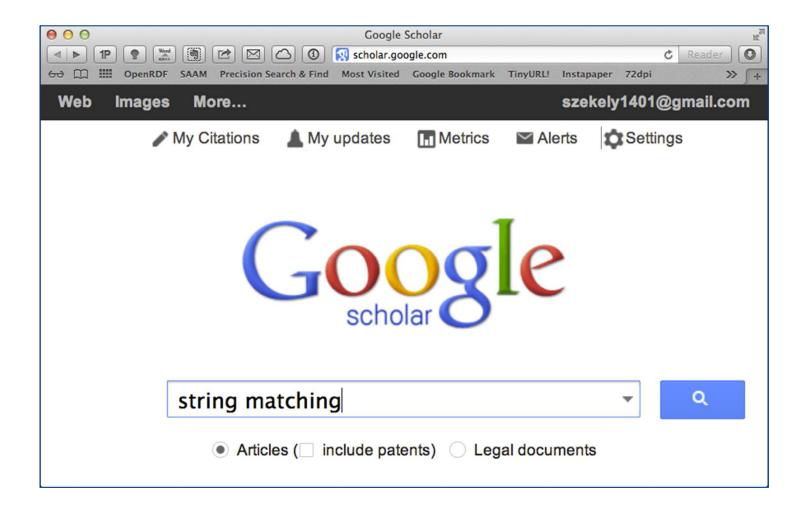
String Matching

Pedro Szekely & Craig Knoblock University of Southern California

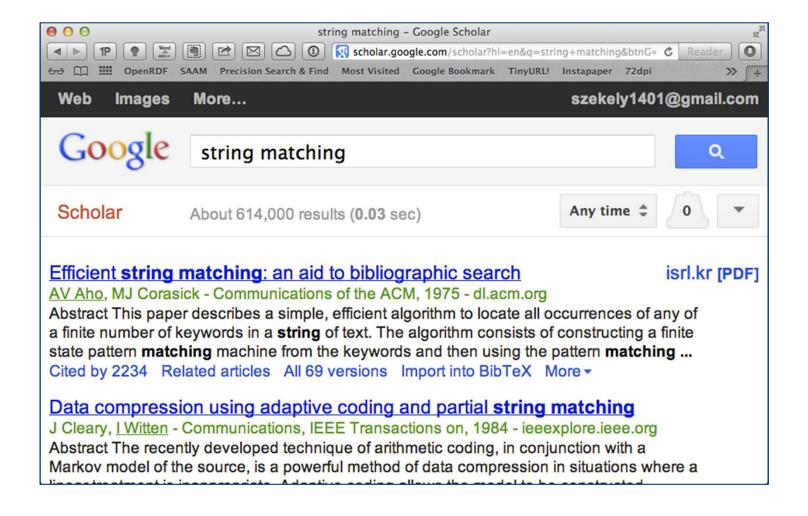
Isn't the Problem Solved?

String.equalsIgnoreCase(String x)

How Many Publications?



Why So Many?



Multiple John Singer Sargents?

```
dallas:John_Singer_Sargent
  a foaf:Person;
  :dateOfBirth "1856";
  :dateOfDeath "1925";
  :name "John Singer Sargent".
```

```
ima:John_Singer_Sargent
  a foaf:Person;
  :dateOfBirth "1856";
  :dateOfDeath "1925";
  :name "John S. Sargent".
```

Multiple John Singer Sargents?

```
dallas:John_Singer_Sargent
  a foaf:Person;
  :dateOfBirth "1856";
  :dateOfDeath "1925";
  :name "John Singer Sargent".
```

) = 55.5

string_match(

```
a foaf:Person;
:dateOfBirth "1856";
:dateOfDeath "1925";
:name "John S. Sargent".
```



String Matching Problem

myMatchFunction(x, y)

yourMatchFunction(x, y)

What does it mean that one is better than the other?

Problem Definition

Given X and Y sets of strings

Find pairs (x, y)
such that both x and y
refer to the same real world entity

"John S. Sargent"

"John Singer Sargent"



Problem Definition

Given X and Y sets of strings

```
Find pairs (x, y)
such that both x and y
refer to the same real world entity
```

We can use precision and recall to evaluate algorithms



Problem Definition

Given X and Y sets of strings

```
Find pairs (x, y)
such that both x and y
refer to the same real world entity
```

fraction of pairs found that are correct

We can use precision and recall to evaluate algorithms





Why Strings Don't Match Perfectly?

```
typos "Joh" vs "John"
```

```
OCR errors "J0hn" vs "John"
```

```
formatting conventions
                         "03/17" vs "March 17"
```

```
abbreviations
             "J. S. Sargent" vs "John Singer Sargent"
```

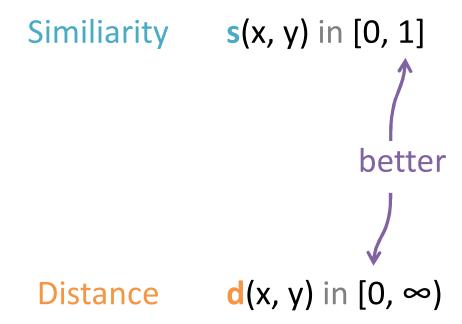
```
nick names "John" vs "Jock"
```

word order





Similiarity Measure



Types of Similarity Metrics

- Sequence based
- Set based
- Hybrid
- Phonetic

Sequence Based Metrics

Edit Distance

"J0n Singer Sargent"

insert character

delete character

substitute character

transpose character

• •

"John S. Sargent"

Edit Distance

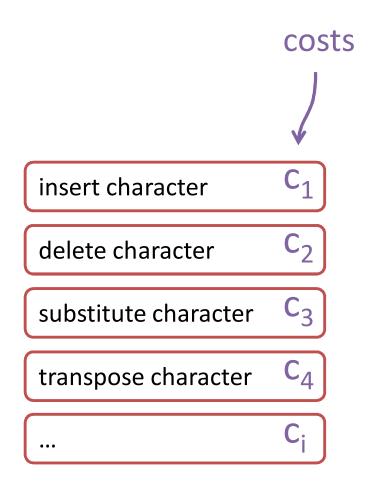
"J0n Singer Sargent"

costs insert character C_2 delete character **C**₃ substitute character transpose character

"John S. Sargent"

Edit Distance

```
"J0n Singer Sargent"
                    \sum_{C_i}
```



"John S. Sargent"

Levenshtein Distance

Edit distance: insert character 1

delete character 1

substitute character 1

lev(x, y) is the minimum cost to transform x to y

Online calculator: http://planetcalc.com/1721/



lev(x, y) is the minimum cost to transform x to y

Definitions

```
x = x_1x_2 ... x_n
y = y_1y_2 ... y_m
d(i,j) = lev(x_1x_2 ... x_i, y_1y_2 ... y_j)
d(0,0) = lev((3),(3)) = 0
```

We want d(n, m)



$$d(0,0) = 0$$

$$d(0,0) = 0$$

$$x_1 x_2 \dots x_{i-1} x_i$$
 is a suffix o

$$x_1x_2 \dots x_{i-1}x_i$$
 is a suffix of x $y_1y_2 \dots y_{j-1}y_j$ is a suffix of y

$$d(0,0) = 0$$

$$x_1x_2$$
 ... $x_{i-1}x_i$ is a prefix of x

$$x_1x_2 \dots x_{i-1}x_i$$
 is a prefix of x $y_1y_2 \dots y_{j-1}y_j$ is a prefix of y

Case	Distance	Operation
$x_i = y_j$	d(i-1,j-1)	keep x _i

$$d(0,0) = 0$$

$$x_1x_2$$
 ... $x_{i-1}x_i$ is a prefix of x

$$x_1x_2 \dots x_{i-1}x_i$$
 is a prefix of x $y_1y_2 \dots y_{j-1}y_j$ is a prefix of y

Case	Distance	Operation		
$x_i = y_j$	d(i-1,j-1)	keep x _i		
$x_i != y_j$		delete X _i		
		insert y _j after x _i		
		replace x_i with y_j		

$$d(0,0) = 0$$

$$x_1x_2$$
 ... $x_{i-1}x_i$ is a prefix of x

$$x_1x_2 \dots x_{i-1}x_i$$
 is a prefix of x $y_1y_2 \dots y_{j-1}y_j$ is a prefix of y

Case	Distance	Operation		
$x_i = y_j$	d(i-1,j-1)	keep X _i		
$x_i != y_j$	d(i-1,j) + 1	delete X _i		
		insert y _j after x _i		
		replace x_i with y_j		

$$d(0,0) = 0$$

$$x_1x_2$$
 ... $x_{i-1}x_i$ is a prefix of x

$$x_1x_2 \dots x_{i-1}x_i$$
 is a prefix of x $y_1y_2 \dots y_{j-1}y_j$ is a prefix of y

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		replace x_i with y_j	

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	d(i-1,j-1) + 1	replace x_i with y_j		

$$d(0,0) = 0$$

$$x_1x_2$$
 ... $x_{i-1}x_i$ is a prefix of x

$$x_1x_2 \dots x_{i-1}x_i$$
 is a prefix of x $y_1y_2 \dots y_{j-1}y_j$ is a prefix of y

Case	Distance	Operation		
$x_i = y_j$	d(i-1,j-1)	keep x _i		
$x_i != y_j$	d(i-1,j) + 1	delete x _i		
	d(i,j-1) + 1	insert y _j after x _i		
	d(i-1,j-1) + 1	replace x_i with y_j		

$$d(i,j) = minimum \int$$



Computing Levenshtein Distance Using Dynamic Programming

x = dva, y = dave

		y0	y1	y2	y3	y4
			d	a	V	e
x0		0	1	2	3	4
x 1	d	1	0 🛨	- 1		
x2	V	2				
x3	a	3				

		y0	y1	y2	у3	y4
			d	a	V	e
x0		0	1	2	3	4
x 1	d	1	0	- 1 -	- 2 ←	- 3
x2	V	2	1	1	1 ←	- 2
x3	a	3	2	1 🛨	$\frac{1}{2}$	2

$$x = d - v a$$

| | | | |
 $y = d a v e$

substitute a with e insert a (after d)

Cost of dynamic programming is O(|x||y|)

Levenshtein Distance Complexity

Dynamic programming algorithm

Time Complexity = O(N * M), Space Complexity = O(N * M)

Polylogarithmic Approximation for Edit Distance and the Asymmetric Query Complexity

Alexandr Andoni, Robert Krauthgamer, Krzysztof Onak

We present a near-linear time algorithm that approximates the edit distance between two strings within a polylogarithmic factor; specifically, for strings of length n and every fixed epsilon>0, it can compute a $(\log n)^{O(1/epsilon)} \text{ approximation in } n^{(1+epsilon)} \text{ time.}$

http://arxiv.org/abs/1005.4033



```
lev(John Singer Sargent,
        John S. Sargent)
```

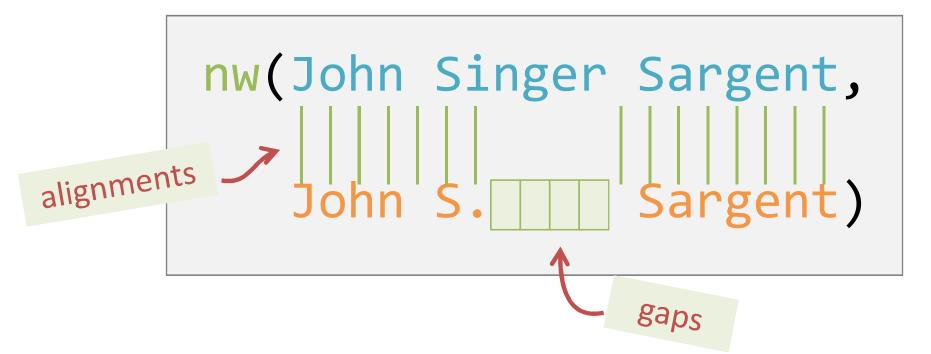
```
Too high a cost for deleting a sequence of characters

lev(John Singer Sargent, Sargent) = 5
```

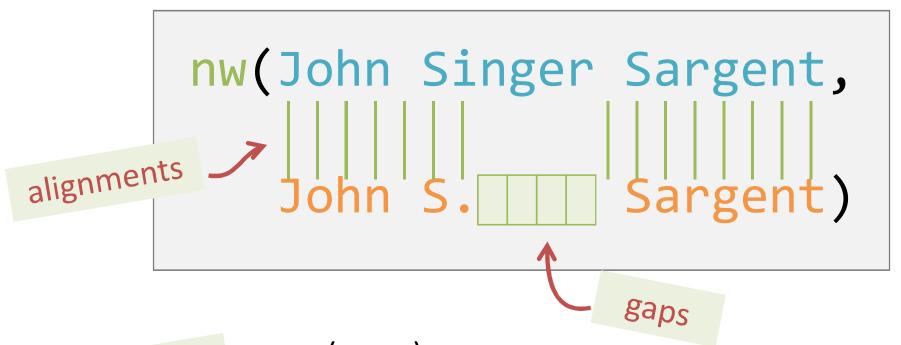
```
lev(John Singer Sargent,
    Jane Klinger Sargent) = 5
```

Needleman-Wunch Measure

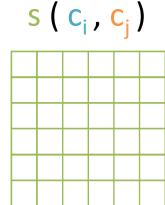
Generalization of levenstein(x, y)



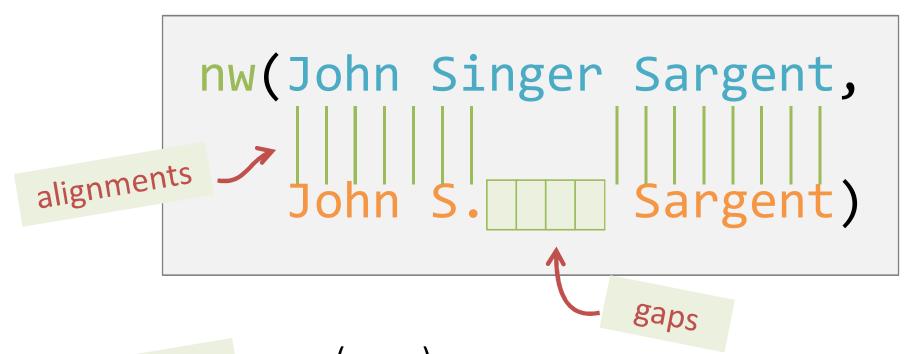
Generalization of levenstein(x, y)



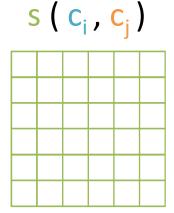
alignment score matrix



Generalization of levenstein(x, y)



Alignment score matrix



gap score

Generalization of levenstein(x, y)

$$s(c_{i}, c_{j}) = \begin{cases} 2 & \text{if } c_{i} = c_{j} \\ -1 & \text{if } c_{i} ! = c_{j} \end{cases}$$

$$gap\text{-score} = -0.5$$



Generalization of levenstein(x, y)



$$s(c_{i},c_{j}) = \begin{cases} 2 & \text{if } c_{i} = c_{j} \\ -1 & \text{if } c_{i} ! = c_{j} \end{cases}$$

gap-score =
$$-0.5$$



Comparison

	Levenshtein	Needleman-Wunch
Costs	1	matrix
Operations	insert/delete	gaps
Result	distance	similarity

OCR errors "John" vs "John"

$$score(0,0) = -0.2$$

$$score(m, 0) = -1.0$$

lower penalty





Needleman-Wunch Example

```
nw(John Singer Sargent,
    John S. Sargent)
```

$$2 * 14 + (-1) * 1 + (-0.5) * 4 = 25$$

$$2 * 14 + (-1) * 3 + (-0.5) * 1 = 24.5$$

nw(John S inger Sargent,
 Jane Klinger Sargent)



Needleman-Wunch Example

```
nw(John Singer Sargent,
    John S. Sargent)
```

$$2 * 14 + (-1) * 1 + (-0.5) * 4 = 25$$

$$2 * 14 + (-1) * 1 + (-0.5) * 8 = 23$$

nw(John Stanislaus Sargent,
 John S. Sargent)

Needleman-Wunch Example

```
nw(John Singer Sargent,
    John S. Sargent)
```

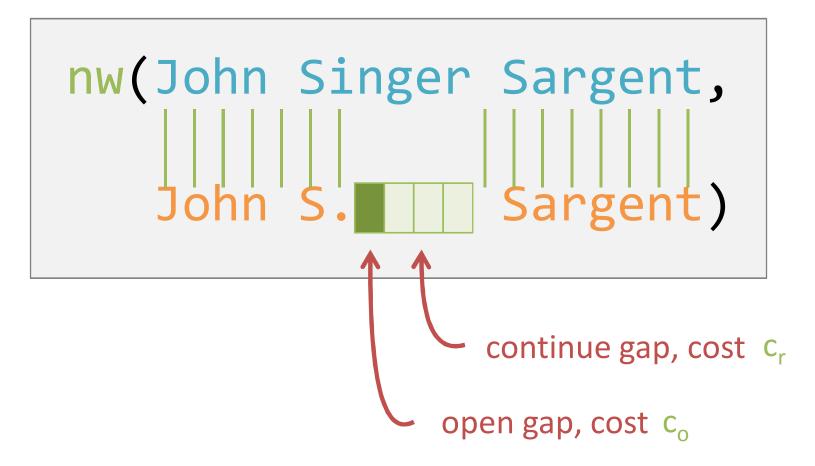
Longer gaps are penalized more

$$+ (-1) * 1 + (-0.5) * 4 = 25$$

nw(John Stanislaus Sargent,
 John S. Sargent)

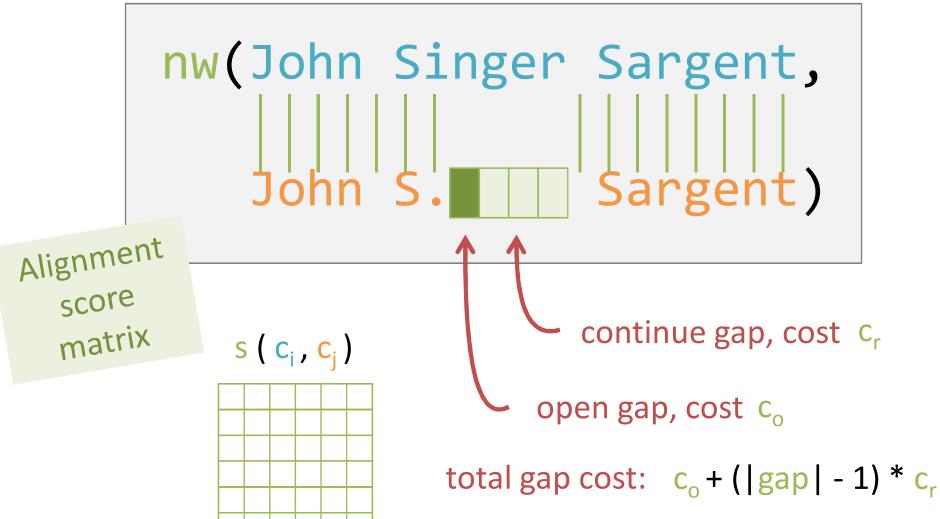
Affine Gap Measure

Generalization of needleman-wunch(x, y)



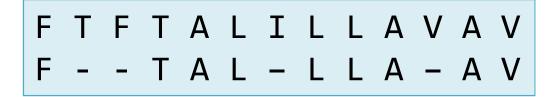
Affine Gap Measure

Generalization of needleman-wunch(x, y)



Smith-Waterman

Global alignment fully align sequences, opening gaps as needed



Needleman-Wunch

Local alignment find best subsequences to align

F T F T A L I L A V A V - F T A L - L A V - -

Smith-Waterman: local alignment version of Needleman-Wunch



Smith-Waterman Example

```
match(John Sargent, american painter,
American artist John S. Sargent)
```

Needleman-Wunch: significant gap penalty



Smith-Waterman Example

Needleman-Wunch: significant gap penalty

Smith-Waterman Example

match(John Sargent, american painter,
American artist John S. Sargent)

Needleman-Wunch: significant gap penalty

Smith-Waterman: identifies similar subsequences



Jaro Similarity Measure

- Get points for having characters in common
 - but only if they are "close by"

- Get points for common characters in the same order
 - lose points for transpositions

Jaro Similarity Measure jaro(x, y)

$$max-distance = \frac{max(|x|,|y|)}{2} - 1$$

m = number of matching characters

t = number of transpositions
 (of matching characters)



Jaro Similarity Measure

$$max-distance = \frac{max(|x|,|y|)}{2} - 1$$

m = number of matching characters

t = number of transpositions

$$jaro(x, y) = \begin{cases} 0 & \text{if } m = 0 \\ \frac{1}{3} \left(\frac{m}{|x|} + \frac{m}{|y|} + \frac{m - t}{m} \right) \end{cases}$$

```
lev(DIXON, DICKSONX) = 4 (4/8 = 0.5)
jaro(DIXON, DICKSONX) = ???
```

	D	I	X	0	N
D	1	0	0	0	0
I	0	1	0	0	0
С	0	0	0	0	0
K	0	0	0	0	0
S	0	0	0	0	0
0	0	0	0	1	0
N	0	0	0	0	1
X	0	0	1	0	0

	D	I	X	0	N
D	1	0	0	0	0
I	0	1	0	0	0
С	0	0	0	0	0
K	0	0	0	0	0
S	0	0	0	0	0
0	0	0	0	1	0
N	0	0	0	0	1
X	0	0	0	0	0

	D	I	X	0	N
D	1	0	0	0	0
I	0	1	0	0	0
С	0	0	0	0	0
K	0	0	0	0	0
S	0	0	0	0	0
0	0	0	0	1	0
N	0	0	0	0	1
X	0	0	0	0	0

$$|x| = 5$$
 $|y| = 8$
 $m = 4$
 $t = 0$

$$= 0.767$$



```
lev(DIXON, DICKSONX) = 4 (4/8 = 0.5)
jaro(DIXON, DICKSONX) = 0.767
```

Jaro-Winkler Measure

Give a bonus if there is a common prefix

```
jwProximity(x,y,boostThreshold,prefixSize)
     = jaro(x,y) <= boostThreshold</pre>
     ? jaro(x,y)
     : jaro(x,y)
       + 0.1 * prefixMatch(x,y,prefixSize)
             * (1.0 - jaro(x, y))
prefixMatch(x,y,prefixSize) =
  min(prefixSize, common-prefix(x,y))
boostThreshold = 0.7
    prefixSize = 4
```



Jaro-Winkler Measure

```
jwProximity(x,y,boostThreshold,prefixSize)
                                              boostThreshold = 0.7
    = jaro(x,y) <= boostThreshold</pre>
                                                  prefixSize = 4
    ? jaro(x,y)
    : jaro(x,y)
      + 0.1 * prefixMatch(x,y,prefixSize)
            * (1.0 - jaro(x,y))
jaro(DIXON, DICKSONX) = 0.767
jwProximity(DIXON, DICKSONX) = 0.767 + 0.1*2*(1 - 0.767)
                                  = 0.767 + 0.2*0.233
                                  = 0.767 + 0.0466
                                  = 0.8136
```

Set-Based Metrics

Set-Based Metrics

Generate set of tokens from the strings

Measure similarity between the sets of tokens

Tokenizing a String

Words

Tokenizing a String

Words

q-grams: substrings of length q

"david smith" 3-grams {##d, #da, dav, avi, ..., h##}

Jaccard Measure

```
B_{x} = tokens(x)
B_{y} = tokens(y)
jaccard(x,y) = \frac{|B_{x} \cap B_{y}|}{|B_{x} \cup B_{y}|}
```

```
jaccard(dave, dav)
B<sub>x</sub> = {#d, da, av, ve, e#}
B<sub>y</sub> = {#d, da, av, v#}
jaccard(x,y) = 3/6
```

TF = term frequency

IDF = inverse document frequency

```
x = Apple Corporation, CA
y = IBM Corporation, CA
z = Apple Corp
...
blah blah Corporation
```

TF = term frequency

IDF = inverse document frequency

```
x = Apple Corporation, CA
y = IBM Corporation, CA
z = Apple Corp
...
blah blah Corporation
```

... but intuitively (x, z) is a better match



TF = term frequency

IDF = inverse document frequency

x = Apple Corporation, CA
y = IBM Corporation, CA
z = Apple Corp
...
blah blah Corporation

frequent term should not carry as much weight

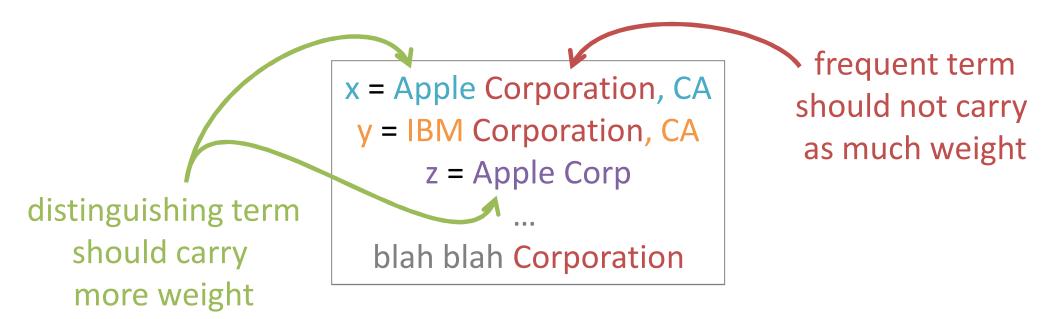
lev(x, y) > lev(x, z)

... but intuitively (x, z) is a better match



TF = term frequency

IDF = inverse document frequency



lev(x, y) > lev(x, z)

... but intuitively (x, z) is a better match



Term Frequencies and Inverse Document Frequencies

- Assume x and y are taken from a collection of strings
- Each string is converted into a bag of terms called a document
- term frequency tf(t,d) =
 - number of times term t appears in document d
- inverse document frequency idf(t) =
 - N / N_d , number of documents in collection divided by number of documents that contain t
 - note: in practice, idf(t) is often defined as $log(N / N_d)$

Example

$$x = aab$$
 \Rightarrow $B_x = \{a, a, b\}$
 $y = ac$ \Rightarrow $B_y = \{a, c\}$
 $z = a$ \Rightarrow $B_z = \{a\}$

tf(a, x) = 2 idf(a) =
$$3/3 = 1$$

tf(b, x) = 1 idf(b) = $3/1 = 3$
... idf(c) = $3/1 = 3$
tf(c, z) = 0

Feature Vectors

- Each document d is converted into a feature vector V_d
- \mathbf{v}_{d} has a feature $\mathbf{v}_{d}(t)$ for each term t
 - value of v_d(t) is a function of TF and IDF scores
 - here we assume $v_d(t) = tf(t,d) * idf(t)$

$$x = aab$$
 \implies $B_x = \{a, a, b\}$
 $y = ac$ \implies $B_y = \{a, c\}$
 $z = a$ \implies $B_z = \{a\}$

tf(a, x) = 2 idf(a) =
$$3/3 = 1$$

tf(b, x) = 1 idf(b) = $3/1 = 3$
... idf(c) = $3/1 = 3$
tf(c, z) = 0

	a	b	c
v _x	2	3	0
v _y	3	0	3
v _z	3	0	0

TF/IDF Similarity Score

- Let p and q be two strings, and T be the set of all terms in the collection
- Feature vectors v_p and v_q are vectors in the |T|-dimensional space wher each dimension corresponds to a term
- TF/IDF score of p and q is the cosine of the angle between \mathbf{v}_p and \mathbf{v}_q
 - $s(p,q) = \sum_{t \in T} v_p(t) * v_q(t) / [\sqrt{\sum_{t \in T} v_p(t)^2} * \sqrt{\sum_{t \in T} v_q(t)^2}]$

TF/IDF Similarity Score

- Score is high if strings share many frequent terms
 - terms with high TF scores
- Unless these terms are common in other strings
 - i.e., they have low IDF scores
- Dampening TF and IDF as commonly done in practice
 - use v_d(t) = log(tf(t,d) + 1) * log(idf(t)) instead of v_d(t) = tf(t,d) * idf(t)
- Normalizing feature vectors

•
$$v_d(t) = v_d(t) / \sqrt{\sum_{\{t \in T\}} v_d(t)^2}$$

Hybrid Similarity Measures

Hybrid Measures

Do the set-based thing but use a similiarity metric for each element of the set

x = Apple Corporation, CA
y = IBM Corporation, CA
z = Aple Corp
...
Aple mispelt
blah blah Corporation

Generalized Jaccard Measure

Jaccard measure

- considers overlapping tokens in both x and y
- a token from x and a token from y must be identical to be included in the set of overlapping tokens
- this can be too restrictive in certain cases

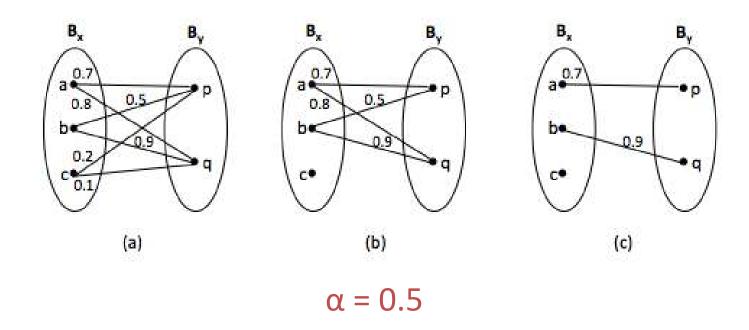
• Example:

- matching taxonomic nodes that describe companies
- "Energy & Transportation" vs. "Transportation, Energy, & Gas"
- in theory Jaccard is well suited here, in practice Jaccard may not work well if tokens are commonly misspelled
 - e.g., energy vs. energg
- generalized Jaccard measure can help such cases

Generalized Jaccard Measure

- Let $B_x = \{x_1, ..., x_n\}, B_y = \{y_1, ..., y_m\}$
- Step 1: find token pairs that will be in the "softened" overlap set
 - apply a similarity measure s to compute sim score for each pair (x_i, y_i)
 - keep only those score > a given threshold α, this forms a bipartite graph G
 - find the maximum-weight matching M in G
- Step 2: return normalized weight of M as generalized Jaccard score
 - GJ(x,y) = $\sum_{(x_i,y_i) \text{ in M}} s(x_i,y_i) / (|B_x| + |B_y| |M|)$

Generalized Jaccard Example



• Generalized Jaccard score: (0.7 + 0.9)/(3 + 2 - 2) = 0.53

The Soft TF/IDF Measure

- Similar to generalized Jaccard measure, except that it uses TF/IDF measure as the "higher-level" sim measure
 - e.g., "Apple Corporation, CA", "IBM Corporation, CA", and "Aple Corp", with Apple being misspelled
- Step 1: compute close(x,y,k): set of all terms t $\in B_x$ that have at least one close term u $\in B_y$, i.e., s'(t,u) >= k
 - s' is a basic sim measure (e.g., Jaro-Winkler), k prespecified
- Step 2: compute s(x,y) as in traditional TF/IDF score, but weighing each TF/IDF component using s'
 - $s(x,y) = \sum_{t \in close(x,y,k)} v_x(t) * v_y(u^*) * s'(t,u^*)$
 - $u^* \in B_y$ maximizes s'(t,u) $\forall u \in B_y$

Soft TF/IDF Example

Monge-Elkan Measure

- Break strings x and y into multiple substrings
 - $x = A_1 ... A_n, y = B_1 ... B_m$
- Compute
 - $s(x,y) = 1/n * \sum_{i=1}^{n} max_{j=1}^{m} s'(A_i,B_j)$
 - s' is a secondary sim measure, such as Jaro-Winkler
 - Intuitively, we ignore the order of the matching of substrings and only consider the best match for substrings of x in y

Monge-Elkan Measure

$$s(x,y) = 1/n * \sum_{i=1}^{n} max_{j=1}^{m} s'(A_{i},B_{j})$$

$$x = A_{1}A_{2} \qquad y = B_{1}B_{2}B_{3}$$

$$max(s'(A_{1},B_{1}), s'(A_{1},B_{2}), s'(A_{1},B_{3}))$$

$$+ max(s'(A_{2},B_{1}), s'(A_{2},B_{2}), s'(A_{2},B_{3}))$$

$$2$$

s' could be any metric, e.g., levenshtein



Monge-Elkan Measure

$$s(x,y) = 1/n * \sum_{i=1}^{n} max_{j=1}^{m} s'(A_i,B_j)$$

x = Comput. Sci. and Eng. Dept., University of California, San Diego

y = Department of Computer Science, Univ. of Calif., San Diego

what s' should we use?

levenshtein needleman-wunch affine-gap smith-waterman jaro jaro-winkler



Phonetic Similarity Measures

Phonetic Similarity Measures

- Match strings based on their sound, instead of appearances
- Very effective in matching names, which often appear in different ways that sound the same
 - e.g., Meyer, Meier, and Mire; Smith, Smithe, and Smythe
- Soundex is most commonly used

The Soundex Measure

- Used primarily to match surnames
 - maps a surname x into a 4-letter code
 - two surnames are judged similar if share the same code
- Algorithm to map x into a code:
 - Step 1: keep the first letter of x, subsequent steps are performed on the rest of x
 - Step 2: remove all occurences of W and H. Replace the remaining letters with digits as follows:
 - *replace B, F, P, V with 1, C, G, J, K, Q, S, X, Z with 2, D, T with 3, L with 4, M, N with 5, R with 6
 - Step 3: replace sequence of identical digits by the digit itself
 - Step 4: Drop all non-digit letters, return the first four letters as the soundex code

The Soundex Measure

- Example: x = Ashcraft
 - after Step 2: A226a13, after Step 3: A26a13, Step 4 converts this into A2613, then returns A261
 - Soundex code is padded with 0 if there is not enough digits
- Example: Robert and Rupert map into R163
- Soundex fails to map Gough and Goff, and Jawornicki and Yavornitzky
 - designed primarily for Caucasian names, but found to work well for names of many different origins
 - does not work well for names of East Asian origins
 - *which uses vowels to discriminate, Soundex ignores vowels

Other Readings

- http://en.wikipedia.org/wiki/String_metric
- http://en.wikipedia.org/wiki/Approximate_string_matching
- http://en.wikipedia.org/wiki/Edit_distance
- http://en.wikipedia.org/wiki/Levenshtein_distance
- http://en.wikipedia.org/wiki/Jaro—Winkler_distance
- http://en.wikipedia.org/wiki/Smith–Waterman_algorithm
- http://en.wikipedia.org/wiki/Jaccard_index
- http://alias-i.com/lingpipe/demos/tutorial/stringCompare/read-me.html
- http://www.gettingcirrius.com/2011/01/calculating-similarity-part-2-jaccard.html
- http://en.wikipedia.org/wiki/Sequence_alignment#Pairwise_alignment

Software

- http://code.google.com/p/javasimilarities/source/browse/trunk/simmetrics/src/main/java/uk/ac/shef/wit/simmetrics/
- http://sourceforge.net/projects/secondstring/
- http://planetcalc.com/1721/ (Levenshtein calculator)

Directories	Filename
▼simmetrics arbitrators basiccontainers math metrichandlers ▼similaritymetrics costfunctions task tokenisers utils wordhandlers	AbstractStringMetric.java
	BlockDistance.java
	ChapmanLengthDeviation.java
	ChapmanMatchingSoundex.java
	ChapmanMeanLength.java
	ChapmanOrderedNameCompoundSimilarity.java
	CosineSimilarity.java
	DiceSimilarity.java
	EuclideanDistance.java
	InterfaceStringMetric.java
	JaccardSimilarity.java
	Jaro.java
	JaroWinkler.java
	Levenshtein.java
	MatchingCoefficient.java
	MongeElkan.java
	NeedlemanWunch.java
	OverlapCoefficient.java
	QGramsDistance.java

