CS670 Midterm - Prof. Ming-Deh Huang

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Be concise and accurate. Give the best solution you have even if you cannot solve a problem completely.

1. Describe an algorithm that given two arrays $A[1, ..., n]$ and $B[1, ..., n]$ of natural numbers, decides if there are i and j such that $A[i] = B[j]$ in $O(n \log n)$ time. Justify your algorithm and its running time. (15%)

2. Let $A[1, ..., n]$ be an array of distinct positive integers. For $i = 1, ..., n$, let b_i be the i-th smallest element in the array, and let $b_{n+1} = 0$. Denote by d_A the smallest i such that $b_1 + ... + b_i > b_{i+1}$. Describe an algorithm that given an array $A[1, ..., n]$ of positive integers outputs $b_1, ..., b_{d_A}$ in time $O(n + d_A \log n)$. Justify your algorithm and its running time. (20%)

3. Given a sequence $A = a_1, ..., a_n$ of natural numbers, the weight of the sequence is defined as $\sum_{i=1}^{n} a_i$. Given two sequences A and B of natural numbers we would like to find a common subsequence of A and B with maximum possible weight. (1) Show that if A and B end with the same number q, then every common subsequence of A and B with maximum weight ends with q. (10%) (2) Describe an algorithm that finds a common subsequence of maximum weight given a sequence A of n natural numbers and a sequence B of m numbers in $O(nm)$ time. Justify your algorithm and its running time. (15%)

4. Consider the problem of making change for n cents using the fewest number of coins. Suppose the available coins are in the denominations of 1, 10 and 100. (1) Show that an optimal solution must have exactly q coins of denomination 100 where $n = 100q + r$ with $0 \leq r < 100$. (10%) (2) Show that there is a unique optimal solution and describe the solution. (15%)

5. There were initially n_0 baseball clubs. It is possible for a club to split into two and it is also possible for two clubs can merge into one. We assume that cost of splitting a club into two is A and the cost of merging two into one is B. We forgot to keep track of the number of merges but we do remember the number of splits is N. Show that the overall cost of all the merges and splits is bounded by $NA + NB + n_0B$. (15%)