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Topic: Maths Assignment   Date

Q1 - find the rank of the matrix A by reducing in Row reduced echelon form

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_2 \longrightarrow R_2 - 2R_1$
$R_3 \longrightarrow R_3 - 3R_1$
$R_4 \longrightarrow R_4 - 6R_1$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_2 \longrightarrow R_3$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_2 \longrightarrow R_2/4$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$$R_1 \longrightarrow R_1 - 2R_2$$

$$A = \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$$R_4 \longrightarrow R_4 + 4R_2$$

$$A = \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$$R_3 \longrightarrow -R_3/3$$

$$A = \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$$R_1 \longrightarrow R_1 + R_3$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

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$$R_2 \to R_2 - 2R_3$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 0 & 7/12 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$$R_4 \Rightarrow R_4 + 3R_3$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 0 & 7/12 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of Matrix is 3 Ans

Q2 – Let W be the vector space of all symmetric 2×2 matrices and let $T: W \to P_2$ be, the Linear Transformation defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$

find ~~x~~ rank & nullity of T.

Maximum degree of polynomial $T \to 2$

So $\dim(P_2) \to 3$

Kernel : So a subset of Kernel T is $T(A) = 0$

$$(a-b) + (b-c)x + (c-a)x^2 \to 0$$

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$$[a = b = c = t \text{ (Let)}]$$

New Matrix $\begin{bmatrix} t & t \\ t & d \end{bmatrix}$

dimension of, kernel is 1, because there's only one independent parameter as 't'

According to rank nullity Theorem →

$$\text{rank}(T) + \text{nullity}(T) \rightarrow \dim(W)$$

$$\text{rank}(T) + 1 = 4$$

So, rank of T is 3, & nullity is 1.

Q3 – Let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ find the eigen values and eigen vectors of A^{-1} and $A + 4I$.

Soln –

$$A - \lambda I \rightarrow 0$$

$$\det \begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)^2 - 1 = 0$$

$$2 - \lambda = \pm 1$$

$$\lambda = 1, 3$$

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OR $\boxed{\lambda=1}$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x - y = 0$$
$$x \Rightarrow y$$

Let $x = t$
$y = t$

Eigen vector V_1, $t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ Ans

$\boxed{\text{for } \lambda = 3}$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-x - y = 0$$
$$\boxed{x = -y}$$

Let $x = t$
$y = -t$

So, Eigen value $V_2 = t \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Now, find for A^{-1}

→ Eigen values of A^{-1} will be $\frac{1}{\lambda_1}$ &

$\frac{1}{\lambda_2} \Rightarrow 1, \frac{1}{3}$

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$\Rightarrow$ and eigen vectors are same as of A

$$V_1 = t\begin{bmatrix}1\\1\end{bmatrix} \qquad V_2 = t\begin{bmatrix}-1\\1\end{bmatrix}$$

Now, for $A + 4I$

$\rightarrow$ eigen values for $A + 4I$ will be

$$\lambda_1 + 4, \ \lambda_2 + 4 \Rightarrow 5, 7$$

$\rightarrow$ and eigen vectors are same as of A

$$V_1 = t\begin{bmatrix}1\\1\end{bmatrix} \qquad V_2 = t\begin{bmatrix}-1\\1\end{bmatrix}$$

Q4 – Solve by Gauss - Seidal Method (Take three iteration)

$$3x - 0.1y - 0.2z = 7.85$$
$$0.1x + 7y - 0.3z = -19.3$$
$$0.3x + 0.2y + 10z = 71.4$$

with initial values $x(0) = 0, \ y(0) = 0, \ z(0) = 0$

equation $\rightarrow$ $$x^{k+1} = \frac{7.85 + 0.1y^{k} + 0.2z^{k}}{3}$$

$$y^{k+1} = \frac{-19.3 - 0.1x^{k+1} - 0.3z^{k}}{-7}$$

$$z^{k+1} = \frac{71.4 - 0.3x^{k+1} + 0.2y^{k+1}}{10}$$

we know $x(0) = 0, \ y(0) = 0, \ z(0) = 0$

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Iteratⁿ-1 :-

$$x(1) = \frac{7.85 + 0.1(0) + 0.2(0)}{3} \Rightarrow 2.6167$$

$$y(1) = \frac{-19.3 - 0.1(2.6167) - 0.3(0)}{-7} \Rightarrow 2.7956$$

$$z(1) = \frac{71.4 - 0.3(2.6167) - 0.2(3.9756)}{10} = 7.1373$$

Iteratⁿ-2 :-

$$x(2) \Rightarrow \frac{7.85 + 0.1(2.7956) + 0.2(7.1373)}{3} = 3$$

$$y(2) = \frac{-19.3 - 0.1(3) - 0.3(7.1373)}{-7} = 3$$

$$z(2) = \frac{71.4 - 0.3(3) - 0.2(3)}{10} = 3$$

Iteratⁿ-3 :-

$$x(3) = (7.85 + 0.1(3) + 0.2(3))/3 \rightarrow 3$$

$$y(3) = (-19.3 - 0.1(3) - 0.3(3))/-7 \Rightarrow 3$$

$$z(3) \rightarrow (71.4 - 0.3(3) + 0.2(3))/10 = 3$$

After 3 iteration $x, y, z \simeq 3$

so value of $x=3$, $y=3$, $z=3$

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(5) Define consistent or inconsistent system of equation.

Consistent
(atleast one solution)

Dependent
(infinite sol^n)

Independent
↓
unique sol^n

Inconsistent
(No solution)

$$A = \left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$, $R_4 \to R_4 + 2R_2$

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$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\rho(A) \Rightarrow 2$$
$$\rho(A:B) \Rightarrow 2$$
$$n \Rightarrow 3$$

$$\rho(A) = \rho(A:B) \neq n$$

Consistent, but infinite solⁿ.

Q6- Determine whether the function $T: P_2 \to P_2$ is linear transformation or not.

$$T(a + bx + cx^2) = (a+1) + (b+1)x + (c+1)x^2$$

Additive

$$T(u+v) = T(u) + T(v)$$
$$u = a_1 + b_1 x + c_1 x^2$$
$$v = a_2 + b_2 x + c_2 x^2$$

$$T(u+v) = T\left((a_1+a_2) + (b_1+b_2)x + (c_1+c_2)x^2\right)$$

$$\Rightarrow (a_1+a_2+1) + (b_1+b_2+1)x + (c_1+c_2+1)x^2$$

$$\Rightarrow (a_1+1) + (b_1+1)x + (c_1+1)x^2 + (a_2+1) + (b_2+1)x + (c_2+1)x^2$$

$$\rightarrow T(u) + T(v)$$ Hence Proved

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2) Homogenity

$$T(Ku) = KT(u)$$

$$T(K(a + bx + cx^2))$$

$$T(Ka + Kbx + Kcx^2)$$

$$\Rightarrow (Ka + Kb + Kc + 1) + (Ka + Kb + Kc + 1)x + (Ka + Kb + Kc + 1)x^2$$

$$= K(a+1) + K(b+1)x + K(c+1)x^2$$

$$= KT(u) \quad \underline{\text{Hence proved}}$$

$\Rightarrow$ It is a linear Transformation.

Q7- Determine whether the set $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ is a basis of $V_3(R)$. In case S is not a basis determine the dimension and the basis of the sub space spanned by S.

Sol^n

$$a(1,2,3) + b(3,1,0) + c(-2,1,3) = (0,0,0)$$

$$a + 3b - 2c = 0$$
$$2a + b + c = 0$$
$$3a + 3c = 0$$

$$c = -a, \quad b = a$$

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Only one soln is possible is $a=b=c=0$.

So linearly Independent.

Since, dimn of $V_3(R)$ is 3 and S also Contains 3 vector.
$\therefore$ S $\rightarrow$ then, it spans $V_3(R)$.
making it a basis for $V_3(R)$.

Q8 - Using Jacobi's method (perform 3 iteratns)

$$3x - 6y + 2z = 23$$
$$-4x + y - z = -15$$
$$x - 3y + 7z = 16$$

$x_0 = 1, \quad y_0 = 1, \quad z_0 = 1$

$\rightarrow$ first equation $x = \frac{1}{3}(23 + 6y - 2z)$

second equation. $y = (-15 + 4x + z)$

third equation $z = \frac{1}{7}(16 - x + 3y)$

$x(0) = 1, \quad y(0) = 1, \quad z(0) = 1$

$\boxed{\text{Iteratn-1}}$:-

$$x(1) = (23 + 6 - 2)/3 = 9$$
$$y(1) = (-15 + 4 + 1) = -10$$
$$z(1) = (16 - 1 + 3)/7 = 18/7$$

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Q9 - Explain an application of matrix operations in image processing with example.

Affine Transformation Rotation

Suppose we have a 2-D image represented as grid or pixels. We can use A T matrix to rotate around centre.

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ rotation of image at θ to rotate it around centre.

1. Translation to origin :-

Translate the image, so that it centre alligns with origin.

2.) Rotation :-

Apply rotation Matrix

3. Translation Back :-

Translate it back with its original position by adding coordinates of centre.

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Q10 - Give a brief description of Linear transformations for computer Vision for rotating 2D image.

→ Linear Transformation for rotation 2D images involves applying a rotation matrix to each pixels coordinate. This matrix rotates points counter clockwise by an angle θ around the origin. Rotation is essential in tasks like image alignment and object detection in computer vision.

x ______ x