EE208:CONTROL ENGINEERING LAB 01

Analyzing Dynamic Response Of A Linear Analog System Using MATLAB

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TABLE OF CONTENTS

1. OBJECTIVE

The project's objective is to analyze and discuss the dynamic response of a given linear analog system in terms of different design and analytical performance measures (peak overshoot, peak time, delay time, settling time).

2. GIVEN

The second order analog OLTF (which has no zeros) of an oven temperature system is provided, for which a given PD controller is to be considered in

- Cascade
- Feedback.

$G_{OL}(s)$ = $\frac{K}{s^2 + 3s + 10}$

PD Control = $80\,(s\, +\, 5)$

3. THEORY

3.1 Cascaded Block Diagram & Closed Loop Transfer Function

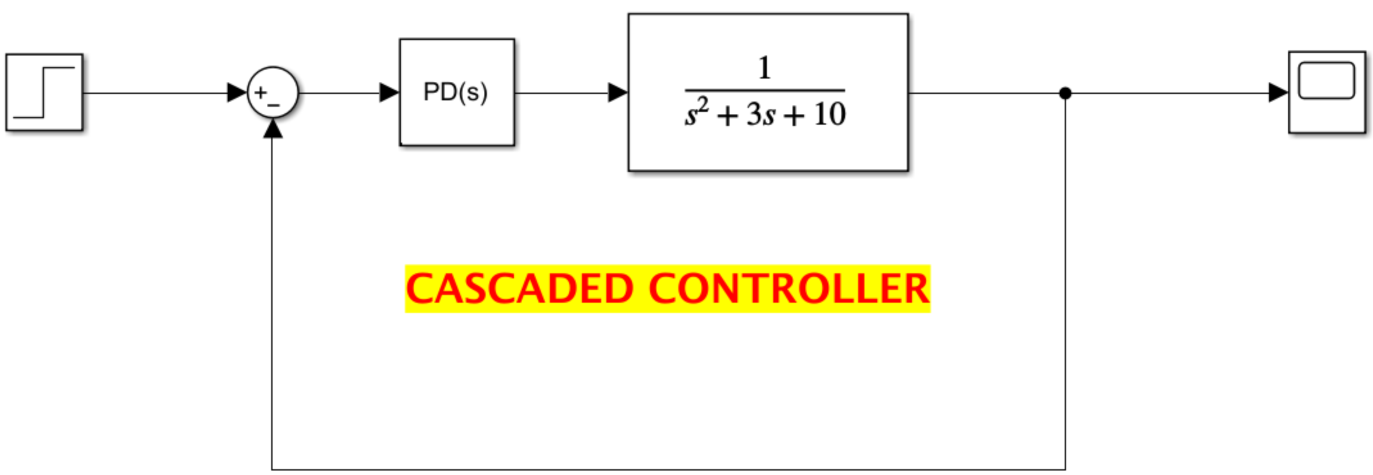


Figure 3.1 PD control cascaded to given OLTF

$$G_{CL}(s) = \frac{80K(s+5)}{s^2 + (3+80K)s + (10+400K)}$$

3.2 Feedback Block Diagram & Closed Loop Transfer Function

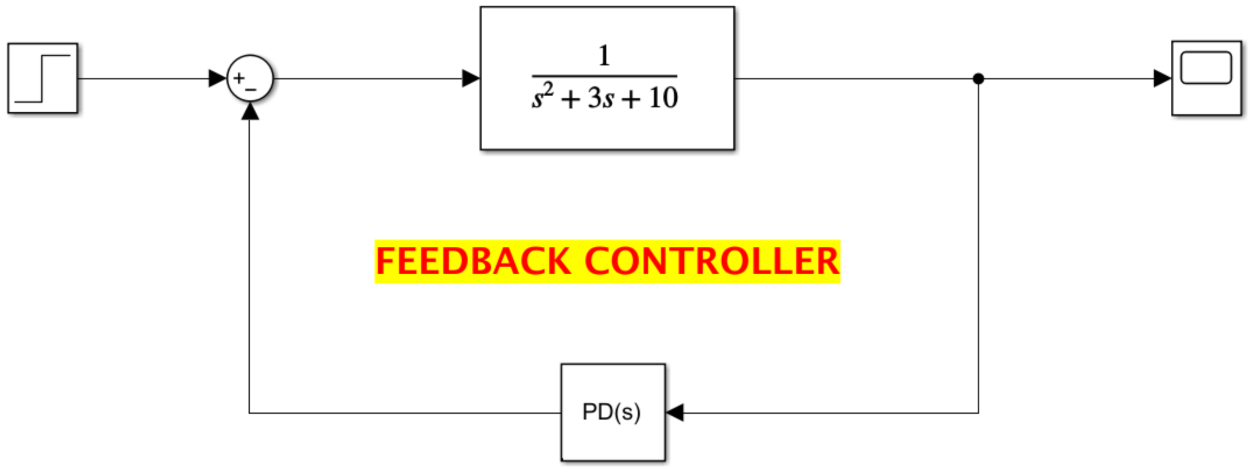


Figure 3.2 PD Control used as feedback to given OLTF

$$G_{CL}(s) = \frac{K}{s^2 + (3+80K)s + (10 + 400K)}$$

$$G_{CL}(s) = \frac{K}{s^2 + (3+80K)s + (10 + 400K)}$$

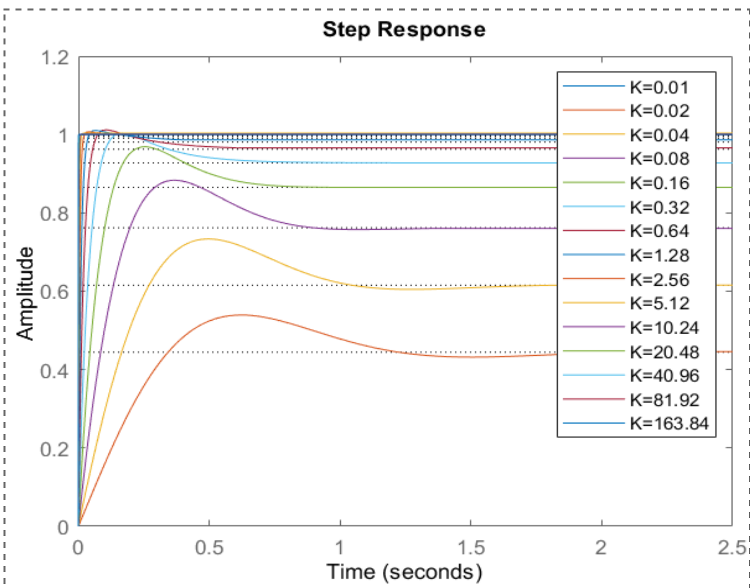
- **PEAK TIME** of a response is defined as the time required for the response to reach its first peak overshoot value and is represented generally by t_p.
- **PEAK OVERSHOOT** is defined as the maximum amount by which the response value exceeds the steady state value, which is obvious to occur at the first peak only. It is in general measured and written as a percentage of the steady state value.It is represented by M_p.
- **SETTLING TIME** is defined as the time taken for the oscillations to die away. It is measured for a given percentage of the amplitude of the response. For example, 2% settling time implies that

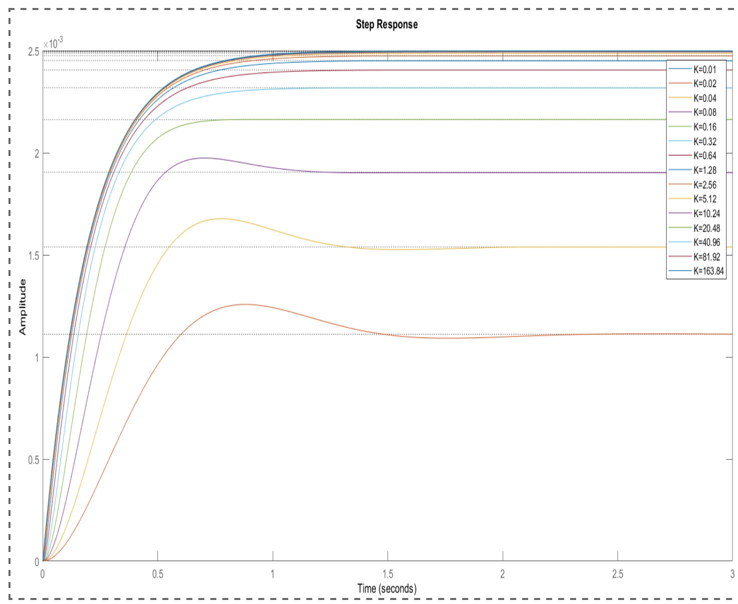
after this span of time, the value of the response remains restricted to a deviation of not more than 2% of its desired steady state value. It is generally represented as t_s .

- **DELAY TIME** of a response is defined as the time required by the response to reach half of its final value, and is denoted by t_d .
- **STEADY STATE ERROR** of a response is defined as the deviation of the output when it reaches the steady state, from the expected value of the output as desired from the control system, it is represented as e_{ss} .

4. OBSERVATIONS

Step responses of the Cascaded and Feedback Systems





🔺 **STEP RESPONSE OF CASCADED**

🔺 **STEP RESPONSE OF FEEDBACK**

From the step response graphs of the Cascaded and Feedback systems we infer that:

- The overshoot is higher for the cascaded system than the feedback system for a given value of ‘K’.
- The overshoot value diminishes in case of the feedback system as we increase the value of ‘K’.

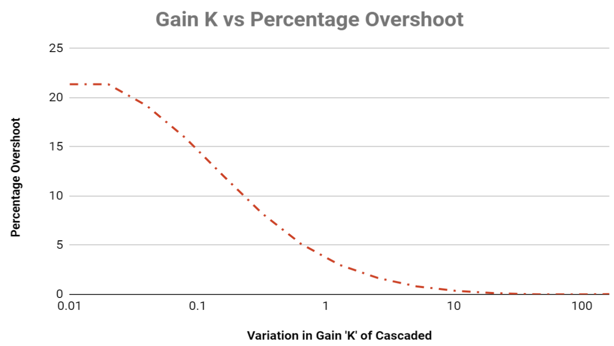
- The value of peak time reduces on increasing the value of K for cascaded systems, whereas the opposite trend can be noticed in case of feedback systems.

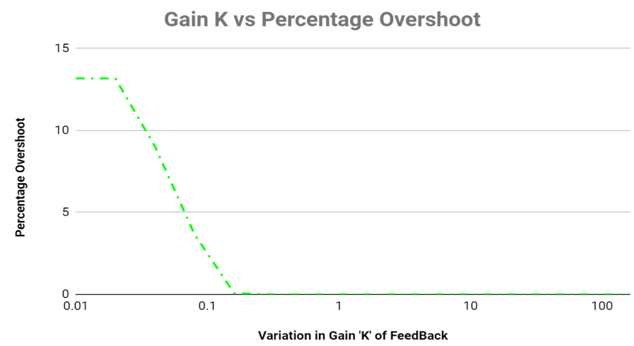
We obtained various Closed Loop Transfer Functions for different values of K. For this experiment, the value of K was taken in the range of **K ∈ (0.01, 163.84)**. A sample of fifteen values were taken at a factor of 2. The following observations were made on the peak overshoot, peak time, settling time, steady state error.

4.1. PEAK OVERSHOOT

PEAK OVERSHOOT FOR VARIATIONS IN K		
K	**CASCADED**	**FEEDBACK**
0.01	21.3706	13.176
0.02	21.3706	13.176
0.04	19.1884	9.0197
0.08	15.925	3.6547
0.16	12.0083	0.0682
0.32	8.2109	0
0.64	5.1454	0
1.28	3.0035	0
2.56	1.6638	0
5.12	0.8184	0
10.24	0.3698	0
20.48	0.1359	0
40.96	0.0164	0
81.92	0	0
163.84	0.0278	0

CASCADED	FEEDBACK

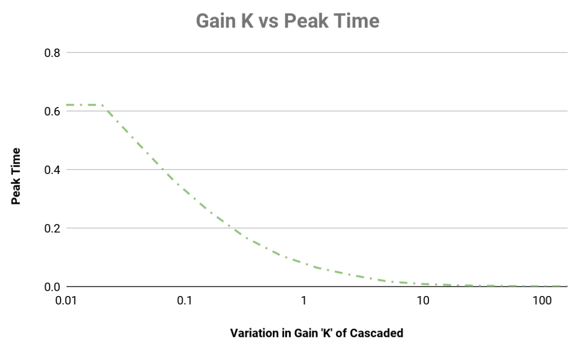
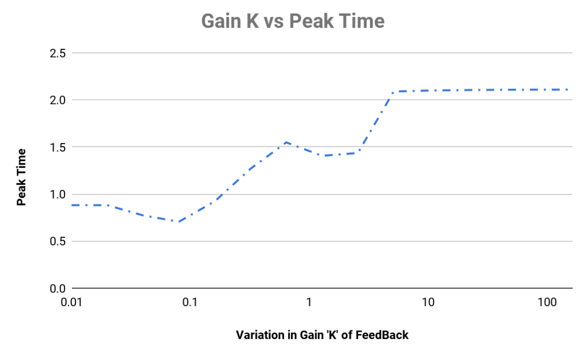




- The graph showcases a smooth decrease in values of % overshoot with the increase in value of gain K
- This graph's infers that the change is steep after a certain value of gain K (ie. between K=0.08 to K=0.16).
- The value of % overshoot becomes zero at a higher value of gain K (ie K =81.92)
- The % overshoot becomes zero very early (ie K =0.32)

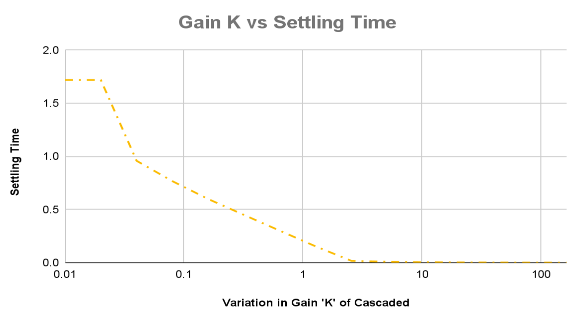
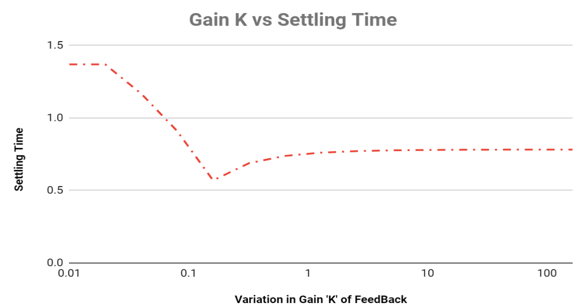
4.2. PEAK TIME

PEAK TIME FOR VARIATIONS IN K		
K	**CASCADED**	**FEEDBACK**
0.01	0.6207	0.881
0.02	0.6207	0.881
0.04	0.4902	0.7725
0.08	0.3625	0.7055
0.16	0.2565	0.921
0.32	0.1682	1.2675
0.64	0.1058	1.5476
1.28	0.0644	1.4054
2.56	0.0384	1.4354
5.12	0.0168	2.0884
10.24	0.0084	2.0988
20.48	0.0042	2.104
40.96	0.0021	2.1066
81.92	0.001	2.1079
163.84	0.0008	2.1085

4.2.1 CASCADED	4.2.2 FEEDBACK
	
• According to the graph the peak time of the cascaded system is decreasing as the value of gain K is increasing	• We infer a nonuniform rise of Peak Time as we increase the gain K

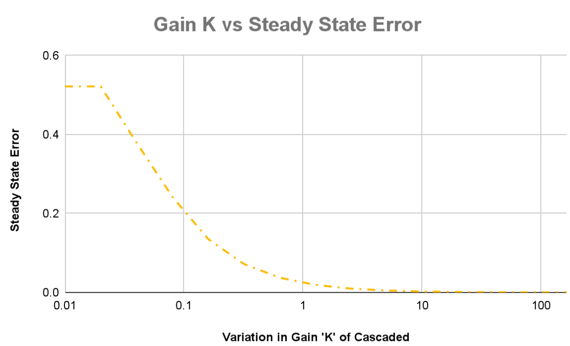
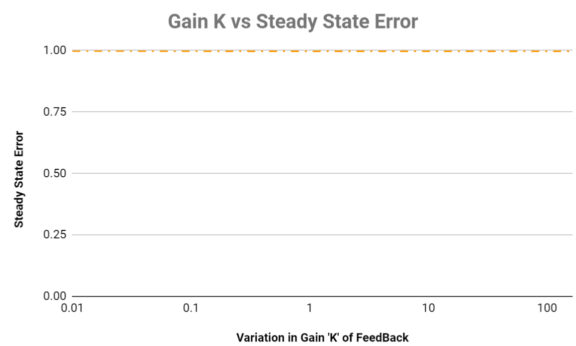
4.3. SETTLING TIME

SETTLING TIME FOR VARIATIONS IN K		
K	CASCADED	FEEDBACK
0.01	1.7195	1.3696
0.02	1.7195	1.3696
0.04	0.9585	1.1684
0.08	0.7665	0.9105
0.16	0.5984	0.5699
0.32	0.4484	0.6886
0.64	0.3003	0.7386
1.28	0.1527	0.7611
2.56	0.0161	0.7719
5.12	0.0087	0.7772
10.24	0.0045	0.7798
20.48	0.0023	0.7811
40.96	0.0012	0.7818
81.92	0.0006	0.7821
163.84	0.0003	0.7823

4.3.1 CASCADED	4.3.2 FEEDBACK
	
• Remains constant for smaller values of K	• Remains constant for very small value of K
• Starts decreasing after K=0.02	• Decreases uniformly upto K=0.16
• Tends to 0 after K=1.28	• After reaching minima it increases a little and then remains almost constant

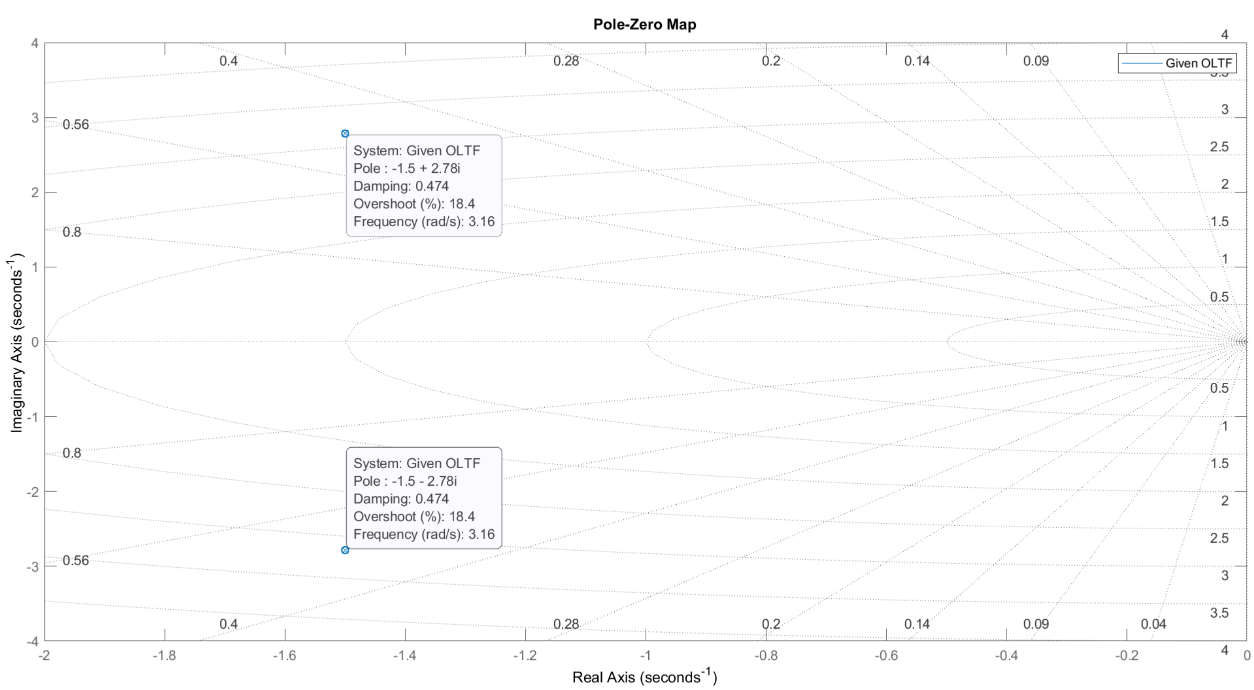
4.4. STEADY STATE ERROR

STEADY STATE ERROR FOR VARIATIONS IN K		
K	**CASCADED**	**FEEDBACK**
0.01	0.5216	0.9988
0.02	0.5216	0.9988
0.04	0.3789	0.9984
0.08	0.2419	0.9981
0.16	0.135	0.9978
0.32	0.0718	0.9977
0.64	0.0372	0.9976
1.28	0.0189	0.9976
2.56	0.0095	0.9975
5.12	0.0048	0.9975
10.24	0.0024	0.9975
20.48	0.0012	0.9975
40.96	0.0006	0.9975
81.92	0.0003	0.9975
163.84	0.0002	0.9975

<table>
<tr><th>4.4.1 CASCADED</th><th>4.4.2 FEEDBACK</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>The steady state error for cascaded system Decreases with the increase of gain K</td><td>The steady state error does not change much for variation in gain K</td></tr>
</table>

5. ANALYSIS

5.1 Reason for difference in responses of Cascaded System and Feedback System ?



$$G_{OL} = \frac{1}{s^2 + 3s + 10}$$

The overall closed loop transfer functions for both the cases whether the PD control is connected in cascade or in feedback, has the same denominator polynomial and hence the same set of poles, which depends on the value of gain K. The factor that differentiates the closed loop transfer functions of cascaded system and feedback system are their numerator polynomials. The feedback system's closed loop transfer function has got no zeroes, but the cascaded system has a zero present at s = -5, which influences various specifications like the peak overshoot, the peak time, the settling time and the steady state error show different trends in these two systems.

5.2 Effect of variation of OL gain of the oven temperature OLTF on the four quantities

From the tables and graphs we have concluded that the effect of variation of OL gain on the four quantities are -

- The value of peak overshoot generally decreases with an increase in the value of gain K for both the cascaded and the feedback system.
- With increment in the value of gain K, the peak time reduces for the cascaded system, whereas it increases in case of the feedback system.
- The value of settling time also decreases for both the cascaded and the feedback system with increasing value of K, but the decrement effect is more pronounced in the case of the cascaded system as compared to the feedback system.
- For sufficiently low values of K, the steady state error for the cascaded system remains around 50% but it decreases significantly with increasing values of K, almost to around 0.02% for K as high as 163.84 , but in case of the feedback system the variation of this large range of values of K does not affect it significantly, rather the steady state error mostly remains well above 99%, which is highly unexpected for a good control system.

5.3 Which controller should be used for controlling Oven Temperature?

- Heating processes require power and temperature controllers and sensors. These sensors operate in sync to complete the thermal cycling process. In most cases, this cycle is controlled by a single-loop controller. The **environment reacts slower than the control system of the device** (Oven). This delay can cause poor application performance.

- Cascade control involves the use of two discrete control loops. The first control loop provides the set point for the PD control loop. This system is designed to provide **improved response to gains and reduce disturbances** in heating systems. They make the heating process more efficient with reduced lags and fewer disturbances.

- Feedback control systems action is entirely empirical, as long as adjustments are made,i.e more heat means increasing temperature and vice-versa, the control system would remove the effect of external disturbance. But, the **main disadvantage is that disturbances** are corrected after output has moved. However certain devices allow fluctuations to some extent, but in case of disturbance of large magnitude the overall effectiveness of feedback control is quite unsatisfactory. The other disadvantage is **time delay**.

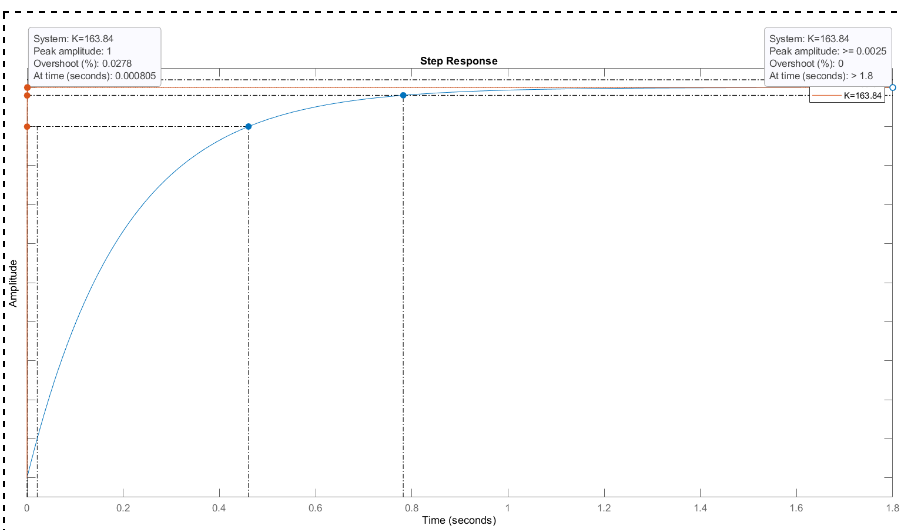


FIGURE 5.3.1 COMPARISON BETWEEN CASCADED CLTF & FEEDBACK CLTF (K=163.84)

- So, a **Cascade Control system with high Gain 'K'** value must be used to control Oven Temperature. It provides less settling time, less overshoot value and the steady state error is minimum.

5.4 How is the step response when the CLTF is critically damped ($\zeta = 1$) and undamped ($\zeta = 0$) ? What will be the value of ω_n for this?

The system response depends directly on the values of the damping ratio ζ and the undamped natural frequency ω_n. Two separate cases are described below:

Underdamped System ($0 \leq \zeta < 1$): The two roots of the characteristic equation are complex conjugates with negative real parts. The response may be determined by substituting the values of the roots. The initial condition response for an underdamped system is a damped cosine function, oscillating at the damped natural frequency ω_d with a phase shift ψ, and with the rate of decay determined by the exponential term $e^{-\zeta\omega nt}$. As the damping ratio increases from zero, the frequency of oscillation ωd decreases, until at a damping ratio of unity, the value of $\omega_d = 0$ and the response consists of a sum of real decaying exponentials.

Critically Damped System ($\zeta = 1$): The roots of the characteristic equation are real and identical. This response form is known as a critically damped response because it marks the transition between the non-oscillatory overdamped response and the oscillatory response .

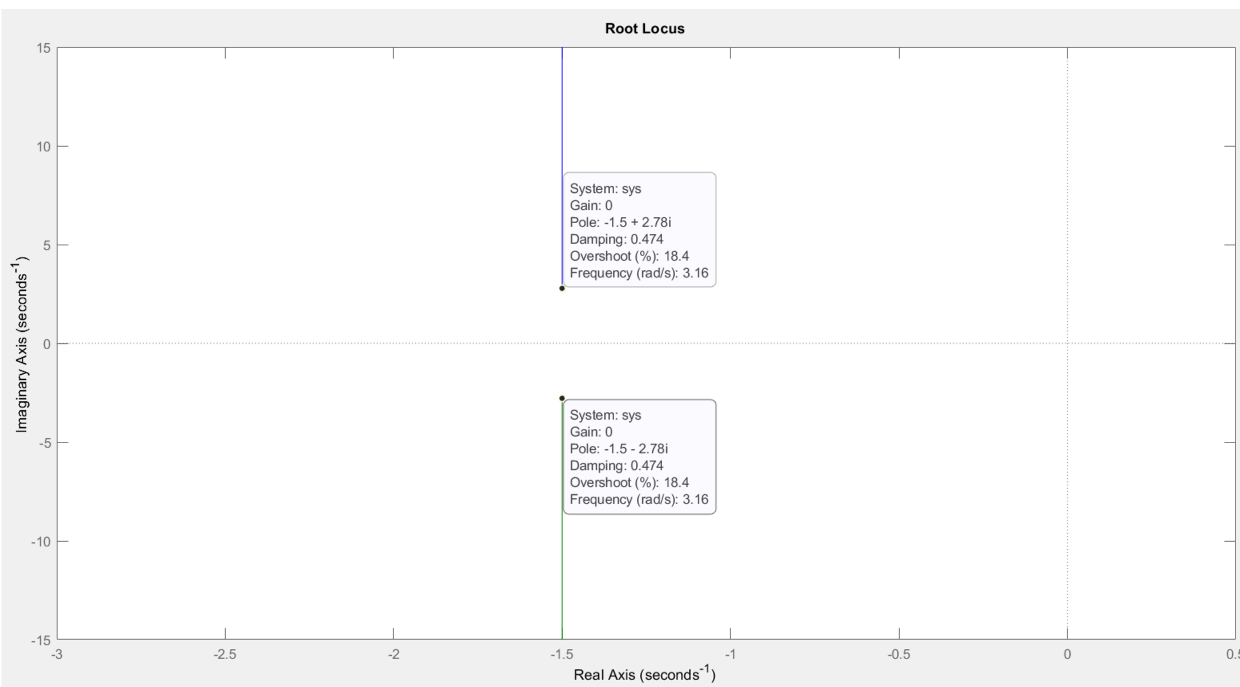


FIGURE 5.4.1 : ROOT LOCUS FOR THE GIVEN OPEN LOOP TRANSFER FUNCTION

6. CONCLUSION

- The given analog systems' step responses were thoroughly analysed with respect to their performance measures using MATLAB
- Changes in peak overshoot, peak time, settling time, and steady state error due to variation in the value of K were observed.
- Further, graphs were plotted for each of the specifications mentioned using the data obtained and filled in the respective tables for both the cascaded and feedback systems.
- Finally, after observing the trends of variation of various specification parameters with increment of value of the gain K, for both the systems, it was concluded that the Cascaded system with a sufficiently high gain would serve the purpose of a very good controller for the desired application in case of a unit step input, and is far better than the feedback system which is completely undesirable.

7. MATLAB SCRIPTS

exp1.m exp1.m

```
import mlreportgen.dom.*

[C_Cltf,F_Cltf,valueOfK] = loopK(0.0001,24,1.55);

% VARIABLES TO STORE VALUES OF OVERSHOOT, PEAK TIME, SETTLING TIME,
% STEADYSTATE ERROR FOR EACH CLTF

overShootCLTF_C = [stepinfo(C_Cltf(1)).Overshoot];
peakTimeCLTF_C = [stepinfo(C_Cltf(1)).PeakTime];
settlingTimeCLTF_C = [stepinfo(C_Cltf(1)).SettlingTime];
tmp = step(C_Cltf(1),1);
ssErrorCLTF_C = [abs(tmp(end)-1)];

overShootCLTF_F = [stepinfo(F_Cltf(1)).Overshoot];
peakTimeCLTF_F = [stepinfo(F_Cltf(1)).PeakTime];
settlingTimeCLTF_F = [stepinfo(F_Cltf(1)).SettlingTime];
tmp = step(F_Cltf(1),1);
ssErrorCLTF_F = [abs(tmp(end)-1)];

for i = 2:length(C_Cltf)

    overShootCLTF_C(end+1) = stepinfo(C_Cltf(i)).Overshoot;
    peakTimeCLTF_C(end+1) = stepinfo(C_Cltf(i)).PeakTime;
    settlingTimeCLTF_C(end+1) = stepinfo(C_Cltf(i)).SettlingTime;
    tmp = step(C_Cltf(i),1);
    ssErrorCLTF_C(end+1) = [abs(tmp(end)-1)];

    overShootCLTF_F(end+1) = stepinfo(F_Cltf(i)).Overshoot;
    peakTimeCLTF_F(end+1) = stepinfo(F_Cltf(i)).PeakTime;
    settlingTimeCLTF_F(end+1) = stepinfo(F_Cltf(i)).SettlingTime;
    tmp = step(F_Cltf(i),1);
    ssErrorCLTF_F(end+1) = [abs(tmp(end)-1)];
end

displayValues(overShootCLTF_C,peakTimeCLTF_C,settlingTimeCLTF_C,ssErrorCLTF_C,overShootCLTF_F,peakTimeC
plotGraph(C_Cltf);
plotGraph(F_Cltf);
```

```
exp1.m exp1.m

%    UNCOMMENT THE GRAPH YOU WANT TO SEE

    step(C(1),C(2),C(3),C(4),C(5),C(6),C(7),C(8),C(9),C(10),C(11),C(12),C(13),C(14),C(15))
    pzmap(C(1),C(2),C(3),C(4),C(5),C(6),C(7),C(8),C(9),C(10),C(11),C(12),C(13),C(14),C(15))
    rlocus(C(1),C(2),C(3),C(4),C(5),C(6),C(7),C(8),C(9),C(10),C(11),C(12),C(13),C(14),C(15))
    legend('K=0.01','K=0.02','K=0.04','K=0.08','K=0.16','K=0.32','K=0.64','K=1.28','K=2.56','K=5.12','K=
end


% DISPLAY THE VALUES OF STEP RESPONSE FOR EACH CLTF
function displayValues(overShootCLTF_C,peakTimeCLTF_C,settlingTimeCLTF_C,ssErrorCLTF_C,overShootCLTF_F,p
    disp("   Overshoot Cascaded");
    disp(overShootCLTF_C);
    disp("   Peak Time Cascaded");
    disp(peakTimeCLTF_C);
    disp("   Settling Time Cascaded");
    disp(settlingTimeCLTF_C);
    disp("   Steady State Cascaded");
    disp(ssErrorCLTF_C);

    disp("   Overshoot Feedback");
    disp(overShootCLTF_F);
    disp("   Peak Time Feedback");
    disp(peakTimeCLTF_F);
    disp("   Settling Time Feedback");
    disp(settlingTimeCLTF_F);
    disp("   Steady State Feedback");
    disp(ssErrorCLTF_F);
    disp("   Value of K");
    disp(valueOfK);
end


%FUNCTION TO FIND THE CLTF FOR EACH VALUE OF 'K'
function [cascadedCLTFs,feedbackCLTFs,valueOfK] = loopK(K_begin,vals,interval)
    [tmpCLTF_Cascade,tmpCLTF_FeedBack] = Calc_CLTF(interval*K_begin);
    cascadedCLTFs = [tmpCLTF_Cascade];
    feedbackCLTFs = [tmpCLTF_FeedBack];
    valueOfK = [K_begin];
    tmp = interval;
    for i = 1:vals
        [tmpCLTF_Cascade,tmpCLTF_FeedBack] = Calc_CLTF(K_begin*tmp);
        cascadedCLTFs(end+1) = tmpCLTF_Cascade;
```

C exp1.m exp1.m

```
    legend('K=0.01','K=0.02','K=0.04','K=0.08','K=0.16','K=0.32','K=0.64','K=1.28','K=2.56','K=5
end


% DISPLAY THE VALUES OF STEP RESPONSE FOR EACH CLTF
function displayValues(overShootCLTF_C,peakTimeCLTF_C,settlingTimeCLTF_C,ssErrorCLTF_C,overShoot
    disp("    Overshoot Cascaded");
    disp(overShootCLTF_C);
    disp("    Peak Time Cascaded");
    disp(peakTimeCLTF_C);
    disp("    Settling Time Cascaded");
    disp(settlingTimeCLTF_C);
    disp("    Steady State Cascaded");
    disp(ssErrorCLTF_C);

    disp("    Overshoot Feedback");
    disp(overShootCLTF_F);
    disp("    Peak Time Feedback");
    disp(peakTimeCLTF_F);
    disp("    Settling Time Feedback");
    disp(settlingTimeCLTF_F);
    disp("    Steady State Feedback");
    disp(ssErrorCLTF_F);
    disp("    Value of K");
    disp(valueOfK);
end


%FUNCTION TO FIND THE CLTF FOR EACH VALUE OF 'K'
function [cascadedCLTFs,feedbackCLTFs,valueOfK] = loopK(K_begin,vals,interval)
    [tmpCLTF_Cascade,tmpCLTF_FeedBack] = Calc_CLTF(interval*K_begin);
    cascadedCLTFs = [tmpCLTF_Cascade];
    feedbackCLTFs = [tmpCLTF_FeedBack];
    valueOfK = [K_begin];
    tmp = interval;
    for i = 1:vals
        [tmpCLTF_Cascade,tmpCLTF_FeedBack] = Calc_CLTF(K_begin*tmp);
        cascadedCLTFs(end+1) = tmpCLTF_Cascade;
        feedbackCLTFs(end+1) = tmpCLTF_FeedBack;
        valueOfK(end+1) = K_begin*tmp;
        tmp = interval*tmp;
    end
end
```