Experiment 11

Nonlinear system dynamics on Simulink for different Lyapunov control designs.

Background:

We have studied two aspects of the following system in Expt #8 and #10 respectively. The details are presented once again just to complete this problem.

$$[m_L + m_C]\cdot\ddot{x}_1(t) + m_L l\cdot\left[\ddot{x}_3(t)\cdot\cos x_3(t) - \dot{x}_3^2(t)\cdot\sin x_3(t)\right] = u(t)$$
$$m_L\cdot[\ddot{x}_1(t)\cdot\cos x_3(t) + l\cdot\ddot{x}_3(t)] = -m_L g\cdot\sin x_3(t)$$

where:

m_C: Mass of trolley; 10 kg.
m_L: Mass of hook and load; the hook is again 10 kg, but the load can be zero to several hundred kg's, but constant for a particular crane operation..
l: Rope length; 1m or higher, but constant for a particular crane operation.
g: Acceleration due to gravity, 9.8 ms^{-2}.

Variables for the problem include:

Input:
u: Force in Newtons, applied to the trolley.

Output:
y: Position of load in metres, $y(t) = x_1(t) + l\cdot\sin x_3(t)$.

States:
x_1: Position of trolley in metres.
x_2: Speed of trolley in m/s.
x_3: Rope angle in rads.
x_4: Angular speed of rope in rad/s

Objective:

Dynamic studies of the crane trolley system introduced in Expt. #8 and #10, with which different Lyapunov control designs are to be incorporated.

The systems are to be individually simulated on Simulink; each using a detailed nonlinear state space system simulation in four state variables..

Tutorial:

Brush up your Lyapunov design concepts very thoroughly for this one ! I suppose they must be a bit rusty by now !!

Project:

Consider four different energy-based function components, additive combinations of which can be used as different Lyapunov functions:

A. Proportionate to *square of linear potential energy* $:= K^{l}_{PE} \cdot (x_{1,ref} - x_1)^2$
B. Proportionate to *linear kinetic energy* $:= K^{l}_{KE} \cdot x_2^2$
C. Proportionate to *square of rotary potential energy* $:= K^{r}_{PE} \cdot x_3^2$
D. Proportionate to rotary kinetic energy $:= K^{r}_{KE} \cdot x_4^2$

where the design problem amounts to determination of *K*-constants by **Lyapunov's Second Method** (which is the formal name for what we have been referring to as "Lyapunov control design").

- For different control designs, Lyapunov functions can be generated by linear combinations in ones, twos, threes, or all out of "A, B, C, D". Accordingly, Lyapunov control gains will evolve.
- Be careful, because not all control logics may be equally effective !
- Note that as in Physics concepts, *potential energy* is proportionate to *differences of distance*, or *angle*. Function components "A" and "C" have been defined as square of these, simply for ease of differentiability. *Kinetic energy* terms are proportionate to *square of speeds*, so these have been retained without squaring.
- Interestingly, "A" resembles our standard state feedback error, and can be correlated to conventional state feedback principles.

Create combinations of components for appropriate Lyapunov functions, and design control by each.

Note: You may find that while some components are *unavoidable or indispensable* (without including them you will not be able to control effectively), some others are redundant, that is their presence makes no difference to control.

For observations and discussions:

Using step, impulse, and ramp inputs for $x_{1,\,ref}$ (the reference position of the carriage), study dynamics of the crane from one steady-state to another, using traces of all four state variables as well as the single output variable.

The studies should be done for each Lyapunov control design you work out. Accordingly, conclude the best Lyapunov control for the nonlinear system to work with (you may provide a ranking between the various controls that you find to be effective). You are expected to carefully reason out your preferences.

Repeat the studies for different values of load mass that the crane is lifting. Discuss thoroughly if the load mass makes any difference to system control by Lyapunov method.

Make sure your submission includes the ***detailed patched block diagram*** *for the overall system from the Simulink main screen. A brief write-up on the simulation, involving notes and comments, could add credit to your work.*