Experiment 8

Stability and instability of nonlinear systems.

Background:

As part of the theory course, we have worked on concepts of linearisation of nonlinear systems about different equilibria, and we know that stability features can change depending on *where* a system is to be linearised.

The problem can become a more involved one in systems for which *some of the parameters may assume **arbitrary**, but **constant** values.* Our objective in this experiment is to study details of one such problem, and to check is out for ***global*** and ***local stability/instability***.

Objective:

To examine stability features of a given nonlinear system by observing *movement of eigenvalues* across the *s*-plane..

Tutorial:

In the MATLAB platform, go through the procedural steps as described for:

- *Create state-space model:* https://in.mathworks.com/help/control/ref/ss.html
- *Working with state space quadruple models.*
- *Eigenvalues and Eigenvectors:* https://in.mathworks.com/help/symbolic/eigenvalues-and-eigenvectors.html?searchHighlight=eigenvalues&s_tid=doc_srchtitle

Project:

The following differential equations represent a simplified model of an overhead crane:

$$[m_L + m_C]\cdot\ddot{x}_1(t) + m_L l\cdot\left[\ddot{x}_3(t)\cdot\cos x_3(t) - \dot{x}_3^2(t)\cdot\sin x_3(t)\right] = u(t)$$

$$m_L\cdot[\ddot{x}_1(t)\cdot\cos x_3(t) + l\cdot\ddot{x}_3(t)] = -m_L g\cdot\sin x_3(t)$$

It is of course, easy to see that the model will involve certain parameters that are *constant*, some that are *constant but adjustable*, and some that are *constant and arbitrary*; but here is a complete list:

m_C: Mass of trolley; 10 kg.

m_L: Mass of hook and load; the hook is again 10 kg, but the load can be zero to several hundred kg's, but constant for a particular crane operation..

l: Rope length; 1m or higher, but constant for a particular crane operation.

g: Acceleration due to gravity, 9.8 ms^{-2}.

Variables for the problem include:

Input:

u: Force in Newtons, applied to the trolley.

Output:

y: Position of load in metres, $y(t) = x_1(t) + l \cdot \sin x_3(t)$.

States:

x_1: Position of trolley in metres.

x_2: Speed of trolley in m/s.

x_3: Rope angle in rads.

x_4: Angular speed of rope in rad/s

We are required to locate the *different equilibria* for the problem, and examine *stability/instability* in terms of movement of eigenvalues across the *s*-plane.

For observations and discussions:

- In particular, check .the **local** and **global nature** of stability and instability points by tracking the *movement of eigenvalues* as parameters and variables assume different values and ranges.

- The stability features as above should be examined for different values assigned to the parameters m_L and l, since these can be "set" or "made to assume" different values.

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