1. **(c) :** Let f be focal length of the convex mirror.
According to new cartesian sign convention
Object distance, $u = -f$, focal length $= +f$
According to mirror formula

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

$$\frac{1}{-f}+\frac{1}{v}=\frac{1}{f} \text{ or } \frac{1}{v}=\frac{1}{f}+\frac{1}{f}=\frac{2}{f} \text{ or } v=\frac{f}{2}$$

The image is formed at a distance $\frac{f}{2}$ behind the mirror.
It is a virtual image.

Magnification, $m=-\frac{v}{u}=-\frac{(f/2)}{(-f)}=\frac{1}{2}$

Also, $m=\frac{\text{Height of image}(h_I)}{\text{Height of object}(h_O)}$

$\therefore \quad h_I = mh_O = \frac{1}{2}(1\,\text{m}) = 0.5\,\text{m}$

2. **(c) :** For spherical mirror, $f=\frac{R}{2}$
Here, $R = 35.0$ cm

$\therefore \quad f=\frac{35}{2}=17.5$ cm

Now, $\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$

Also, magnification $m=\frac{-v}{u}$ or $v=-mu$

$\therefore \quad \frac{1}{f}=\frac{1}{u}-\frac{1}{mu}$

or $\quad u = f\left(1-\frac{1}{m}\right)$

$$=17.5\left(1-\frac{1}{2.5}\right)=17.5\times(0.6)=10.5 \text{ cm}$$

3. **(b):** According to mirror formula,

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

Differentiating with respect to t, we get

$\therefore \quad -\frac{1}{u^2}\frac{du}{dt}-\frac{1}{v^2}\frac{dv}{dt}=0 \quad (\because f \text{ is constant})$

or $\quad \frac{dv}{dt}=-\left(\frac{v}{u}\right)^2\frac{du}{dt}$

$$v_i=-\left(\frac{v}{u}\right)^2 v_o \quad \left(\because \frac{dv}{dt}=v_i \text{ and } \frac{du}{dt}=v_o\right)$$

Substituting the given values, we get

$$v_i=-\left(\frac{10}{30}\right)^2\times 9=-1\text{ m s}^{-1}$$

$|v_i| = 1$ m s^{-1}

4. **(a) :** Here, $h_1 = 2$ cm, $u = -16$ cm
$h_2 = -3$ cm (since image is real and inverted)

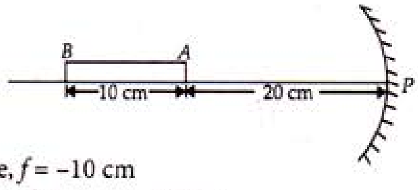
$\therefore \quad m=\frac{-h_2}{h_1}=\frac{v}{u}$

$\therefore \quad v=\frac{-h_2}{h_1}u=\frac{3}{2}\times(-16)=-24$ cm

Now, $\frac{1}{f}=\frac{1}{v}+\frac{1}{u}=-\frac{1}{24}-\frac{1}{16}=\frac{-2-3}{48}=\frac{-5}{48}$

$f=\frac{-48}{5}=-9.6$ cm

5.

(d) : 

Here, $f = -10$ cm
For end A, $u_A = -20$ cm
Image position of end A,

$$\frac{1}{v_A}+\frac{1}{u_A}=\frac{1}{f}$$

$$\frac{1}{v_A}+\frac{1}{(-20)}=\frac{1}{(-10)} \quad \text{or} \quad \frac{1}{v_A}=\frac{1}{-10}+\frac{1}{20}=-\frac{1}{20}$$

$v_A = -20$ cm
For end B, $u_B = -30$ cm
Image position of end B,

$$\frac{1}{v_B}+\frac{1}{u_B}=\frac{1}{f}$$

$$\frac{1}{v_B}+\frac{1}{(-30)}=\frac{1}{(-10)} \quad \text{or} \quad \frac{1}{v_B}=\frac{1}{-10}+\frac{1}{30}=-\frac{2}{30}$$

$v_B = -15$ cm
Length of the image

$$=|v_A|-|v_B|=20\text{ cm}-15\text{ cm}=5\text{ cm}$$

6. **(a) :** Apparent depth = $\frac{\text{Real depth}}{a_{\mu_l}}$

Here, Real depth = 12.5 cm and $a_{\mu_l} = 1.33$

$\therefore$ Apparent depth = $\frac{12.5}{1.63}=7.67$ cm

Now the microscope will have to shift from its initial position to focus 9.4 cm depth object to focus 7.67 cm depth object.
Shift distance = 9.4 – 7.67 = 1.73 cm

7. **(b):** ${}^a\mu_g=\frac{\sin 60^\circ}{\sin 35^\circ}$ ... (i)

${}^a\mu_w=\frac{\sin 60^\circ}{\sin 41^\circ}$ ... (ii)

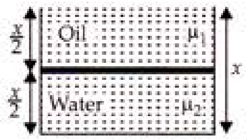
${}^w\mu_g=\frac{\sin 41^\circ}{\sin\theta}$ ... (iii)

${}^a\mu_w \times {}^w\mu_g = {}^a\mu_g$

$\frac{\sin 60^\circ}{\sin 41^\circ}\times\frac{\sin 41^\circ}{\sin\theta}=\frac{\sin 60^\circ}{\sin 35^\circ}$ (Using (i), (ii) and (iii))

$\sin\theta = \sin 35^\circ$
$\theta = 35^\circ$

8. **(a)**:



As refractive index, $\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$

$\therefore$ Apparent depth of the vessel when viewed from above is

$$d_{\text{apparent}} = \frac{x}{2\mu_1} + \frac{x}{2\mu_2} = \frac{x}{2}\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)$$
$$= \frac{x}{2}\left(\frac{\mu_2 + \mu_1}{\mu_1\mu_2}\right) = \frac{x(\mu_1 + \mu_2)}{2\mu_1\mu_2}$$

9. **(d)**: Let x_1 be apparent depth of the dot when seen from air.

$\therefore \mu_1 = \frac{h/3}{x_1}$

(Here, $h/3$ is real depth of the dot under liquid of density d_1)

$\Rightarrow x_1 = \frac{h}{3\mu_1}$

Similarly, apparent depths of the dot when seen from air through two other liquids are

$x_2 = \frac{h}{3\mu_2}$ and $x_3 = \frac{\mu}{3\mu_3}$

$\therefore$ Apparent depth of the dot $= x_1 + x_2 + x_3$

$$= \frac{h}{3\mu_1} + \frac{h}{3\mu_2} + \frac{h}{3\mu_3} = \frac{h}{3}\left[\frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3}\right]$$

10. **(c)**: Here, $i = 45°$

Applying Snell's law at air-glass surface, we get

$\mu_a \sin i = \mu_g \sin r'$

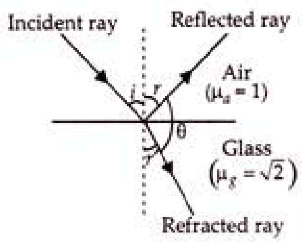
$1 \sin i = \sqrt{2} \sin r'$

$\sin r' = \frac{1}{\sqrt{2}} \sin i$

$= \frac{1}{\sqrt{2}} \sin 45°$

$\Rightarrow \sin r' = \frac{1}{2}$

$r' = \sin^{-1}\left(\frac{1}{2}\right) = 30°$



From figure,

$r + \theta + r' = 180°$

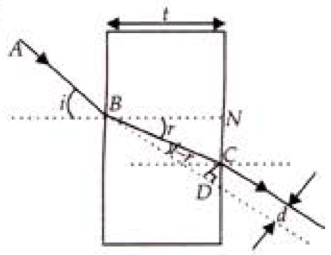
$i + \theta + 30° = 180°$ $(\because i = r)$

$45° + \theta + 30° = 180°$

$\theta = 180° - 75° = 105°$

Hence, the angle between reflected and refracted rays is 105°.

11. **(c)**:



From figure, in right angled ΔCDB

$\angle CBD = (i - r)$

$\therefore \sin(i - r) = \frac{CD}{BC} = \frac{d}{BC}$

or $d = BC \sin(i - r)$ ... (i)

Also, in right angled ΔCNB

$\cos r = \frac{BN}{BC} = \frac{t}{BC}$

or $BC = \frac{t}{\cos r}$ ... (ii)

Substitute equation (ii) in equation (i), we get

$d = \frac{t}{\cos r} \sin(i - r)$

For small angles $\sin(i - r) \approx i - r$; $\cos r \approx 1$

$d = t(i - r)$, $d = it\left[1 - \frac{r}{i}\right]$

12. **(c)**: Actual depth of the needle in water

$h_1 = 12.5$ cm

Apparent depth of needle in water

$h_2 = 9.4$ cm

$\therefore \mu_{\text{water}} = \frac{h_1}{h_2} = \frac{12.5}{9.4} = 1.33$

Hence, $\mu_{\text{water}} = 1.33$, when water replaced by a liquid of refractive index $\mu' = 1.63$.

The actual depth remains the same, but its apparent depth changes.

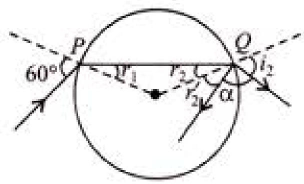
Let H be the new apparent depth of the needle

$\therefore \mu' = \frac{h_1}{H}$ or $H = \frac{h_1}{\mu'} = \frac{12.5}{1.63} = 7.67$ cm

Here, H is less than h_2. Thus to focus the needle again, the microscope should be moved up. Distance by which the microscope should be moved up

$= 9.4 - 7.67 = 1.73$ cm

13. **(c)**: Refraction at P,



$\frac{\sin 60°}{\sin r_1} = \sqrt{3}$

$\sin r_1 = \frac{1}{2}$

or $r_1 = 30°$

Since, $r_2 = r_1$

$\therefore r_2 = 30°$

Refraction at Q, $\frac{\sin r_2}{\sin i_2} = \frac{1}{\sqrt{3}}$ or $\frac{\sin 30°}{\sin i_2} = \frac{1}{\sqrt{3}}$ or $i_2 = 60°$

At point Q, $r_2' = r_2 = 30°$

$\therefore \alpha = 180° - (r_2' + i_2) = 180° - (30° + 60°) = 90°$

14. **(c)**: For total internal reflection,

$\sin i > \sin C$

where,

i = angle of incidence, C = critical angle

But, $\sin C = \frac{1}{\mu}$ $\therefore \sin i > \frac{1}{\mu}$ or $\mu > \frac{1}{\sin i}$

$$\mu > \frac{1}{\sin 45^\circ} \qquad (i = 45^\circ \text{ (Given)})$$

$$\mu > \sqrt{2}$$

Hence, option (c) is correct.

15. (C)

16. **(d):** Here, $v_A = 1.8 \times 10^8$ m s^{-1}.
$v_B = 2.4 \times 10^8$ m s^{-1}

Light travels slower in denser medium. Hence medium A is a denser medium and medium B is a rarer medium. Here, light travels from medium A to medium B. Let C be the critical angle between them.

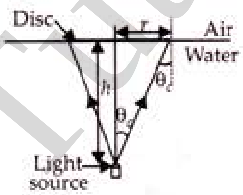
$$\therefore \ \sin C = {}^{A}\mu_B = \frac{1}{{}^{B}\mu_A}$$

Refractive index of medium B w.r.t. to medium A is

$${}^{A}\mu_B = \frac{\text{Velocity of light in medium } A}{\text{Velocity of light in medium } B} = \frac{v_A}{v_B}$$

$$\therefore \ \sin C = \frac{v_A}{v_B} = \frac{1.8 \times 10^8}{2.4 \times 10^8} = \frac{3}{4} \quad \text{or} \quad C = \sin^{-1}\left(\frac{3}{4}\right)$$

17. **(a):** The figure shows incidence from water at critical angle θ_c for the limiting case.



Now, $\sin\theta_c = \dfrac{1}{\mu}$

so that $\tan\theta_c = \dfrac{1}{(\mu^2 - 1)^{1/2}}$

From figure, $\tan\theta_c = \dfrac{r}{h}$

where r is the radius of the disc.
Therefore, diameter of the disc is

$$2r = 2h\tan\theta_c = \frac{2h}{(\mu^2 - 1)^{1/2}}$$

18. **(b):** $\sin\theta = \dfrac{\mu_2}{\mu_1} = \dfrac{v}{v'}$

where v and v' are the speeds of light in medium (i) and medium (ii) respectively.

$$v' = \frac{v}{\sin\theta}$$

19. **(b):** Since refraction occurs from denser to rarer medium,

Therefore, $-\dfrac{\mu_2}{u} + \dfrac{\mu_1}{v} = \dfrac{\mu_1 - \mu_2}{R}$

$$-\frac{3}{2(-3)} + \frac{1}{v} = \frac{1 - \frac{3}{2}}{-5} = \frac{1}{10} \quad \text{or} \quad \frac{1}{v} = \frac{1}{10} - \frac{1}{2} = -\frac{4}{10}$$

$\Rightarrow v = -2.5$ cm

20. **(d):** Here, $P_1 = 10$ D and $P_2 = -5$ D
Therefore, power of the combined lens is
$P = P_1 + P_2 = +10 + (-5) = +5$ D

Now, magnification, $m = \dfrac{f}{u + f}$

Here, $m = 2$ and $f = \dfrac{1}{P} = \dfrac{1}{5} = 0.2$ m $= 20$ cm

$$\therefore \ 2 = \frac{20}{u + 20} \Rightarrow u + 20 = 10$$

$\Rightarrow u = -10$ cm

21. **(a):** $\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

Here, $f = \dfrac{2}{3}R$, $R_1 = +R$, $R_2 = -R$

$$\therefore \ \frac{1}{(2/3)R} = (\mu - 1)\left(\frac{1}{R} + \frac{1}{R}\right) = \frac{(\mu - 1) \times 2}{R}$$

$$\mu - 1 = \frac{3}{4} = 0.75$$

$\Rightarrow \mu = 1.75$

22. **(d):** The far point of 6.0 m tell us that the focal length of the lens is $f = -6.0$ m, $u = -18$ m and $h = 2$ m

Using, $\dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u} \ \Rightarrow \ \dfrac{1}{v} = \dfrac{1}{f} + \dfrac{1}{u} = \dfrac{1}{-6.0} - \dfrac{1}{18.0}$

$\Rightarrow v = -4.5$ m

$\therefore$ The image size, $h' = h\left(\dfrac{-v}{u}\right) = 2 \times \left(-\dfrac{-4.5}{18.0}\right) = 0.50$ m

23. **(b):** Power of the lens combination is

$$P = P_1 + P_2 = \frac{1}{f_1(\text{in m})} + \frac{1}{f_2(\text{in m})} = \frac{1}{+0.40\,\text{m}} + \frac{1}{-0.25\,\text{m}}$$
$$= -1.5 \text{ m}^{-1} = -1.5 \text{ D}.$$

24. **(a):** Area of a square card $= 1$ mm $\times$ 1 mm $= 1$ mm^2
Focal length of magnifying lens (converging lens),
$f = +10$ cm
Object distance, $u = -9$ cm
According to thin lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{+10\text{ cm}} + \frac{1}{-9\text{ cm}} = \frac{1}{10\text{ cm}} - \frac{1}{9\text{ cm}}$$

or $v = -90$ cm

Magnification, $m = \dfrac{v}{u} = \dfrac{-90\text{ cm}}{-9\text{ cm}} = 10$

$\therefore$ Apparent area of the card through the lens
$= 10 \times 10 \times 1$ mm^2
$= 100$ mm^2 $= 1$ cm^2

25. **(b):** Lens maker formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, $f = 20$ cm, $\mu = 1.55$, $R_1 = R$ and $R_2 = -R$

$$\therefore \ \frac{1}{20} = (1.55 - 1)\left(\frac{1}{R} - \frac{1}{(-R)}\right) = 0.55 \times \left(\frac{1}{R} + \frac{1}{R}\right)$$

$$\frac{1}{20} = 0.55 \times \frac{2}{R}$$

$R = 1.1 \times 20 = 22$ cm

26. **(d):** $\frac{1}{f}=\frac{2}{f_L}+\frac{1}{f_M}$

For a plane mirror, $f_M = \infty$

$\therefore\ \frac{1}{f}=\frac{2}{f_L}$

or $f=\frac{f_L}{2}=\frac{20\text{ cm}}{2}=10\text{ cm}$

27. **(d):** According to lens maker's formula

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Here, $f = 30$ cm, $R_1 = \infty$ (For plane surface)

$R_2 = -10$ cm

$$\therefore\ \frac{1}{30}=(\mu-1)\left(\frac{1}{\infty}-\frac{1}{-10}\right) \Rightarrow \frac{1}{30}=\frac{(\mu-1)}{10}$$

$3(\mu-1)=1$

$3\mu = 4$

$\mu=\frac{4}{3}$

28. **(c):** $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

Here, $R_1 = 20$ cm, $R_2 = -40$ cm, $f = 20$ cm

$$\therefore\ \frac{1}{20}=(\mu-1)\left(\frac{1}{20}+\frac{1}{40}\right)$$

$$\frac{1}{20}=(\mu-1)\frac{3}{40} \Rightarrow (\mu-1)=\frac{2}{3} \text{ or } \mu=\frac{2}{3}+1$$

$\mu=\frac{5}{3}$

29. **(b):** Focal length for lens

$$\frac{1}{f_L}=(\mu_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Here, $\mu_g = 1.5$, $R_1 = 20$ cm, $R_2 = -30$ cm

$$=(1.5-1)\left(\frac{1}{20}-\frac{1}{-30}\right)=0.5\times\left(\frac{5}{60}\right)$$

or $f_L=\frac{120}{5}=24$ cm

Focal length for mirror

$$f_m=\frac{R}{2}=\frac{-20}{2}=-10\text{ cm}$$

$\therefore$ Equivalent focal length

$$f_{eq}=\frac{-2}{f_L}+\frac{1}{f_m}=\frac{-2}{24}+\frac{1}{-10}=\frac{-11}{60}$$

$\Rightarrow f_{eq}=\frac{-60}{11}$ cm

Hence, this system behaves like a concave mirror of focal length $\frac{-60}{11}$ cm.

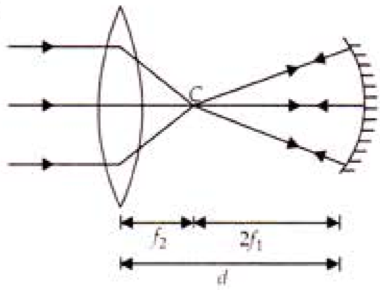
30. **(c):** Here, $m=\frac{v}{u}=-4$ or $u=\frac{-v}{4}$

Also, $|u|+|v|=1.5$

$\frac{v}{4}+v=1.5$ or $v=1.2$ m and $u=\frac{-1.2}{4}=-0.3$ m

$$\therefore\ f=\frac{uv}{u-v}=\frac{-0.3\times1.2}{-0.3-1.2}=0.24\text{ m}$$

31. **(c):**



$\therefore\ d=2f_1+f_2$

32. **(b):** Using, $A+\delta=i+e$

Here, $A = 60°$, $\mu = 1.5$,

$$i=e=\frac{3}{4}A=\frac{3}{4}\times60°=45°$$

$\therefore\ 60°+\delta=45°+45° \Rightarrow \delta=90°-60°=30°$

33. **(c):** As refracted ray emerges normally from opposite surface, $r_2 = 0$

As $A=r_1+r_2\quad\therefore\quad r_1=A$

Now, $\mu=\frac{\sin i_1}{\sin r_1}\approx\frac{i_1}{r_1}=\frac{i}{A}$ or $i=\mu A$

34. **(c):** Magnification, of compound microscope

$$m=m_o\times m_e=\left(\frac{L}{f_o}\right)\times\left(\frac{D}{f_e}\right)$$

Here, $L = 20$ cm, $D = 25$ cm (near point), $f_o = 1$ cm and $f_e = 2$ cm

$$\therefore\ m=\frac{20}{1}\times\frac{25}{2}=250$$

35. **(d):** As $u >> f_0$, $v = f_0 = 19$ m.

Now $u = -3.8\times10^8$ m. Therefore, magnification produced by the objective is

$$m_0=\frac{v}{u}=-\frac{19}{3.8\times10^8}=-0.5\times10^{-7}$$

$\therefore$ Diameter of the image of moon is

$3.5\times10^6\times0.5\times10^{-7}=0.175\text{ m}=17.5\text{ cm}$

36. **(d):** Here, $f_o = 1.5$ cm, $f_e = 6.25$ cm, $u_o = -2$ cm, $v_e = -25$ cm

For objective,

$$\frac{1}{v_o}-\frac{1}{u_o}=\frac{1}{f_o}\ \therefore\ \frac{1}{v_o}-\frac{1}{-2}=\frac{1}{1.5}$$

$$\frac{1}{v_o}=\frac{1}{1.5}-\frac{1}{2} \text{ or } v_o=6\text{ cm}$$

For eye piece

$$\frac{1}{v_e}-\frac{1}{u_e}=\frac{1}{f_e}$$

$$\frac{1}{-25}-\frac{1}{u_e}=\frac{1}{6.25}$$

$$-\frac{1}{u_e}=\frac{1}{6.25}+\frac{1}{25} \text{ or } u_e=-5 \text{ cm}$$

Distance between two lenses = $|v_o|+|u_e|$

$= 6 \text{ cm} + 5 \text{ cm} = 11 \text{ cm}$

37. **(d)**: In normal adjustment,
Length of telescope tube, $L = f_o + f_e$
Here, f_o = 20 m and f_e = 2 cm = 0.02 m
$\therefore$ $L = 20 + 0.02 = 20.02$ m

and magnification, $m=\frac{f_o}{f_e}=\frac{20}{0.02}=1000$

The image formed is inverted with respect to the object.

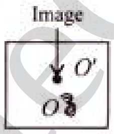
38. **(b)**: Here, f_o = 15 m = 15×10^2 cm, f_e = 1.0 cm

$\therefore$ magnification, $m=\frac{f_o}{f_e}=\frac{15\times10^2}{1}$

$m = 1500$

39. **(d)**: In the later case microscope will be focussed for O'.
So, it is required to be lifted by distance OO'.
OO' = real depth of O – apparent depth of O

$$=3-\frac{3}{1.5} \qquad \left[\mu=\frac{\text{real depth}}{\text{apparent depth}}\right]$$



$$=3\left[\frac{1.5-1}{1.5}\right]=\frac{3\times0.5}{1.5}=1 \text{ cm (upward)}$$

40. **(a)** : Here, d = 25 cm, f_o = 8.0 mm, f_e = 2.5 cm,
u_o = –9.0 mm = –0.9 cm

Now, $\frac{1}{v_e}-\frac{1}{u_e}=\frac{1}{f_e}$ $\quad\therefore\quad$ $\frac{1}{u_e}=\frac{1}{v_e}-\frac{1}{f_e}=\frac{1}{-25}-\frac{1}{2.5}$

$=\frac{-1-10}{25}=\frac{-11}{25}$ $\qquad (\because v_e=-d=-25 \text{ cm})$

$u_e=\frac{-25}{11}=2.27$ cm

Again, $\frac{1}{v_o}-\frac{1}{u_o}=\frac{1}{f_o}$

$\frac{1}{v_o}=\frac{1}{f_o}+\frac{1}{u_o}=\frac{1}{0.8}+\frac{1}{-0.9}=\frac{0.9-0.8}{0.72}=\frac{0.1}{0.72}$

$v_o=\frac{0.72}{0.1}=7.2$ cm

Therefore, separation between two lenses
$= u_e + v_o = 2.27 + 7.2 = 9.47$ cm

Magnifying power, $m=\frac{v_o}{u_o}\left(1+\frac{d}{f_e}\right)=\frac{7.2}{0.9}\left(1+\frac{25}{2.5}\right)=88$