Ray Optics and Optical Instruments

Optics deals origin, motion and detection of light. It is classified in to three.

1) **Ray optics (Geometrical optics):** Deals light as ray- stream of corpuscles (Tiny, weightless elastic particles emitted by luminous body) moving along a straight line.

2) **Wave Optics (Physical Optics):** Deals light as wave.

3) **Photon optics:** Deals light as waves with discreate Photon.

1. What is the nature light?
Maxwell and Hertz realised that light is EM wave

2. Name the important phenomena of light.

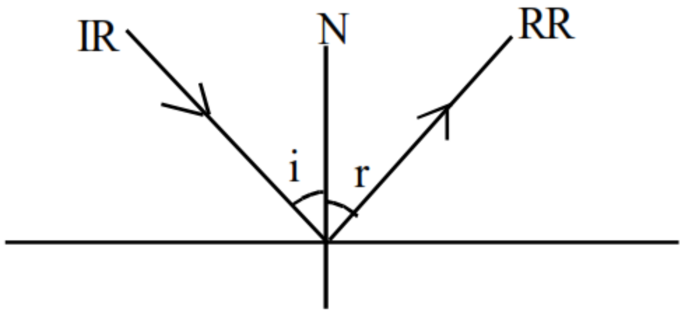
- Rectilinear propagation of light
- Reflection
- Refraction
- Diffraction
- Scattering
- Polarization
- Interference
- Colour vision
- Photoelectric effect

Reflection

3. Define Reflection.
The process of rebouncing beam of light from a polished surface.
(Note: Beam of light - Combination of light rays).

4. Write laws of reflection

- IR and RR and Normal (N) to the point of incidence all lie in the same plane.
- Angle of incidence (i) is equal to Angle of reflection (r)



IR - Incident Ray

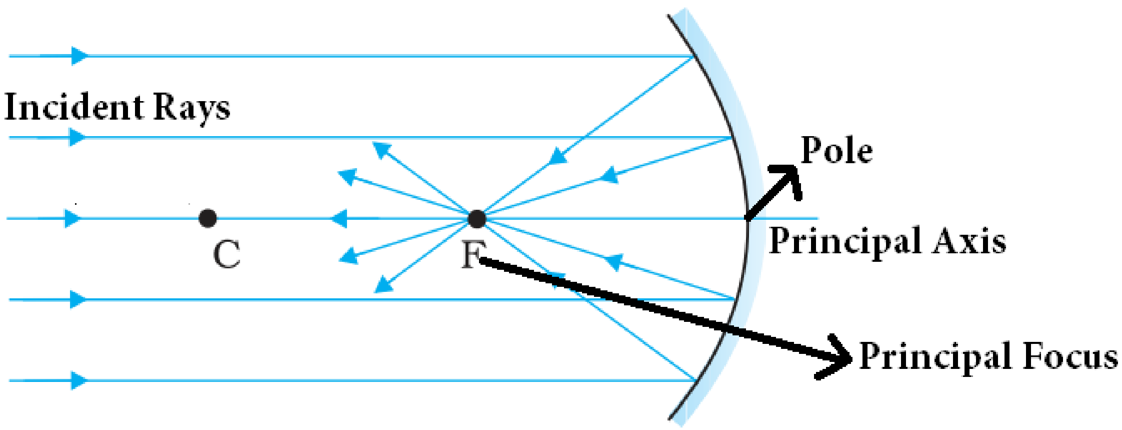
RR - Reflected Ray

5. Spherical Mirrors:

(i) **Concave:** Reflecting side bend inward

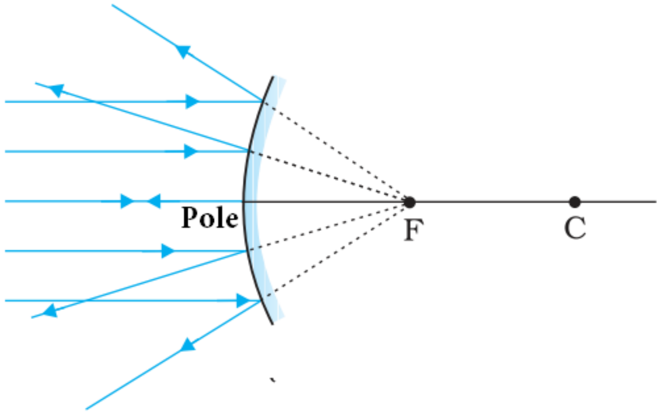
(ii) **Convex:** Reflecting side bend outward.

6. Define principal focus of a concave mirror.



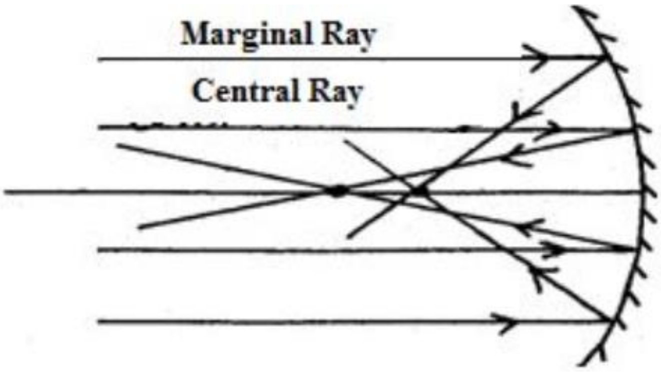
The light rays which are coming parallel to the principal axis after reflection converge to a point on the principal axis. This point is called the principal focus of a concave mirror.

7. Define principal focus of a convex mirror.



The light rays which are coming parallel to the principal axis after reflection from the convex mirror appears to diverge from a point on the principal axis, on the other side of the mirror. This point is called the principal focus of the convex mirror.

8. What is spherical aberration?



The marginal rays in mirrors of very large apertures are focused relatively closer to the vertex (pole) as compared to the central rays, which converge at F. As a result of this the image formed is not sharp but fuzzy. The fuzzy formation of image which arises due to large aperture of mirror is called spherical aberration.

In other words, the inability of a spherical mirror of large aperture to focus the marginal rays and central rays at a single point is called spherical aberration.

9. How can you minimise spherical aberration?

Spherical aberration can be minimised

- by using stop for marginal rays so that only central rays are allowed to pass through

or

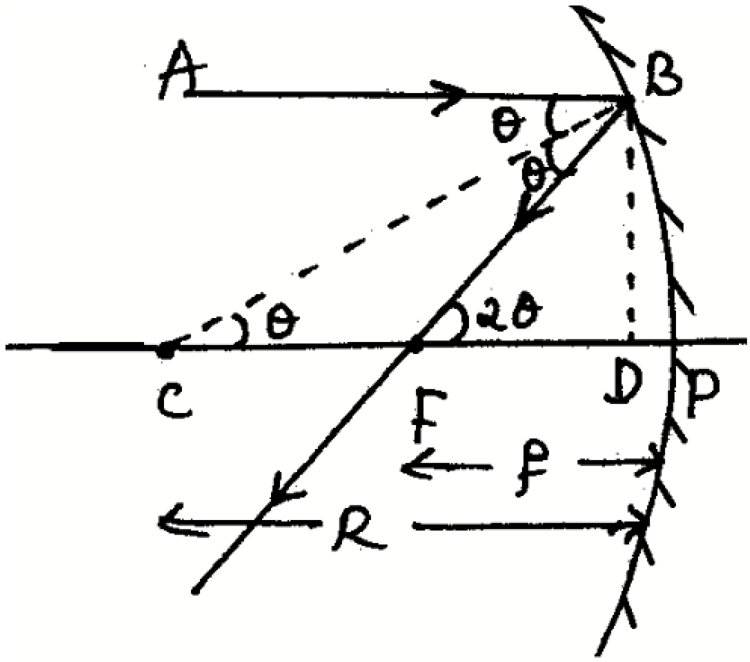
- by using paraboloidal mirrors.

10. Derive the relation between R and f for a spherical mirror.

Or

Show that Principal focus is the mid - point of radius of curvature

Ans:



Let us consider a ray **AB** parallel to the principal axis is incident on a concave mirror at B which is very close to the pole 'P'. After reflection the ray passes through 'F'.

From ΔBCD,

$$\tan\theta = \frac{BD}{CD} \cdots\cdots\cdots (1)$$

From ΔBFD,

$$\tan 2\theta = \frac{BD}{FD} \cdots\cdots\cdots (2)$$

If θ is very small,

$\tan\theta \approx \theta$ and $\tan 2\theta \approx 2\theta$

$\therefore$ eqns (1) and (2) $\rightarrow$

$$\theta = \frac{BD}{CD} \text{ and}$$

$$2\theta = \frac{BD}{FD}$$

Now we have

$$\frac{BD}{FD} = 2\theta \Rightarrow \frac{\cancel{B}D}{FD} = 2\times\frac{\cancel{B}D}{CD}$$

$$\Rightarrow \frac{1}{FD} = \frac{2}{CD}$$

$$\Rightarrow CD = 2FD \cdots\cdots\cdots (3)$$

But $FD \approx f$ and $CD \approx R$

$\therefore$ equ $(3) \rightarrow R = 2f$

Also, $f = R/2$

11. Explain new Cartesian sign convention.

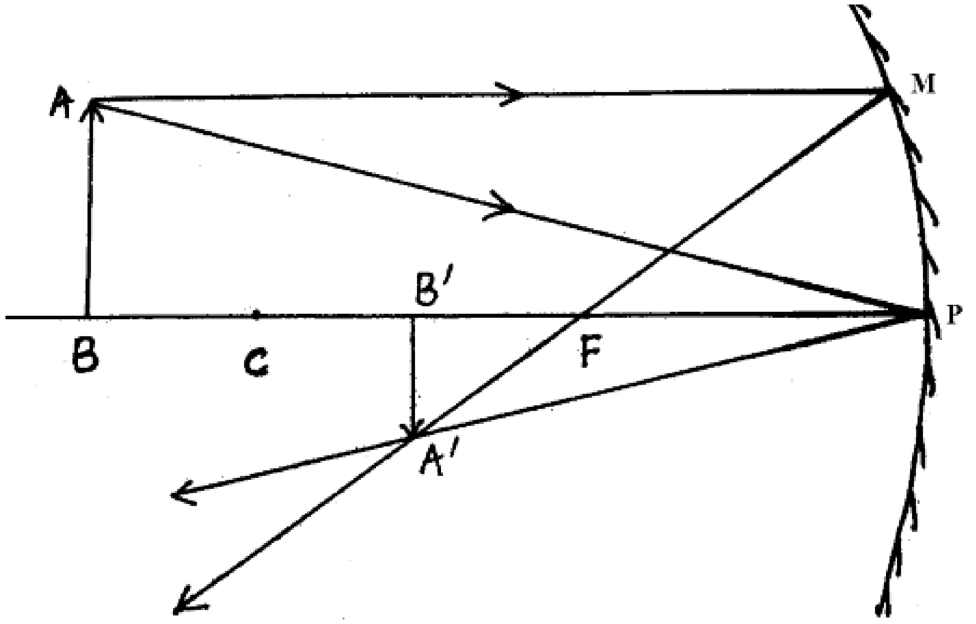
The main conventions in the new Cartesian sign convention are:

- All distances are measured from the pole of the mirror.
- All distances measured in the direction of incident ray are taken as positive.
- All distances measured against the direction of incident ray are taken as negative.
- Heights measured perpendicular to the principal axis and upwards are taken as positive.

- Heights measured perpendicular to the principal axis and downwards are taken as negative.

12. Derive the **mirror formula.**

Ans:



Let AB is a linear object placed on the principal axis of a concave mirror. A' B' is the image of AB.

The two right triangle ΔA' B' F and ΔMPF are similar.

$$\therefore \frac{A'B'}{MP} = \frac{B'F}{PF}$$

But MP = AB

$$\therefore \frac{A'B'}{AB} = \frac{B'F}{PF} \ldots\ldots\ldots\ldots(1)$$

Also triangles A ' B' P and ABP are similar.

$$\therefore \frac{A'B'}{AB} = \frac{B'P}{BP} \ldots\ldots\ldots\ldots(2)$$

From equation (1) and (2)

$$\frac{B'F}{PF} = \frac{B'P}{BP}$$

$$\Rightarrow \frac{B'P - PF}{PF} = \frac{B'P}{BP} \ldots\ldots(3)$$

By applying new Cartesian sign Convention

$$B'P = -v,\ PF = -f,\ BP = -u$$

$$\therefore \text{equ}(3) \rightarrow$$

$$\frac{-v+f}{-f} = \frac{-v}{-u}$$

$$\Rightarrow \frac{-v+f}{-f} = \frac{v}{u}$$

$$\Rightarrow \frac{v}{f} - 1 = \frac{v}{u}$$

Dividing by 'v', we get

$$\frac{1}{f} - \frac{1}{v} = \frac{1}{u}$$

$$\Rightarrow \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \qquad \boxed{\frac{1}{u} + \frac{1}{v} = \frac{1}{f}}$$

This is called mirror formula.

Note: Put $f = \frac{R}{2}$, $\quad {}^{1}\!/_{u} + {}^{1}\!/_{v} = \frac{2}{R}$

13. Define linear magnification produced by a spherical mirror.

It is the ratio of height of the image to the height of the object.

$$m = \frac{\text{Height of Image}}{\text{Height of Object}} = \frac{-h_e}{h_o} \quad \text{or} \quad m = \frac{\text{Image Distance}}{\text{Object Distace}} = \frac{-v}{-u}$$

$$m = \frac{h_e}{h_o} = \frac{-v}{u}$$

14. Derive the relation between

a) m, u and f

b) m, v and f

Relation between m, u and f

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

Multiplying by u

$$1+\frac{u}{v}=\frac{u}{f}$$

$$\frac{-u}{v}=1-\frac{u}{f}$$

$$\frac{1}{m}=\frac{f-u}{f}$$

$$m=\frac{f}{f-u}$$

Relation between m, v and f

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

Multiplying by v

$$\frac{v}{u}+1=\frac{v}{f}$$

$$\frac{v}{u}=\frac{v}{f}-1$$

$$-m=\frac{v-f}{f}$$

$$m=\frac{f-v}{f}$$

15. Significance of magnification ‘m’

- When ‘m’ is positive, the image is erect (virtual)
- When ‘m’ is negative, the image is inverted (real)
- For enlarged image, $m > 1$
- For diminished image, $m < 1$.

16. What are the uses of spherical mirrors?

Concave mirrors

- Used as reflectors of table lamps to direct light in a given area.
- Concave mirrors of large aperture are used in reflecting type astronomical telescopes.
- Shaving mirrors are made slightly concave to get erect enlarged image of the face.

Convex mirrors

They are used in automobiles as rear view mirrors because of the two reasons:

- A convex mirror always produces an erect image.
- The image is diminished in size, so that it gives a wide field of view.

17. Write the difference between real and virtual images?

- Real image - Formed on a screen, inverted
- Virtual image - Cannot formed on a screen, erect.

18. Virtual images cannot formed on a screen but can be seen - Justify

Virtual image formed acts as virtual object to the eye lens which produces real image on the retina.

19. Nature of image formed by a plane mirror is

virtual, erect, same size, laterally inverted and u=v

20. Focus of a plane mirror in at infinity- Justify

Reflected rays do not converge

21. Why convex mirror is used as rear view mirror in automobiles?

Wider field of view.

22. 'OBJECTS IN THE MIRROR ARE CLOSER THAN THEY APPEAR' - This is written on rear view mirror - Justify

It cannot give exact distance of approaching vehicle
(For a convex mirror if object is placed at anywhere in front of the mirror the image is formed in between the mirror and Principle focus).

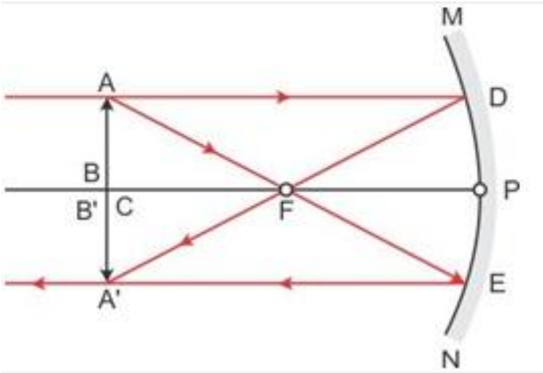
23. What do you meant by conjugate focus of concave mirror?
The positions of the object and real image formed by a concave mirror are interchangeable. Pairs of such points on the principal axis are conjugate foci.

24. Will focal length of a mirror changes if it is placed in water?
No Focal length of a mirror is independent of the medium.

25. What is the minimum distance between the object and it is real image formed by a concave mirror - Explain.

The minimum distance between an object and its real image in the case of a concave mirror is Zero.

When the object is at the 2F distance i.e. the center of curvature, then the image is real and formed inverted at the same distance.



26. Show that the image formed by a convex mirror is virtual erect and diminished

$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$, for convex mirror u is -ve, f is +ve.

$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}$

Hence v is the +ve - image is formed behind the mirror (virtual)

$m=\frac{f}{f-u}$

Put sign convention, $m=\frac{f}{f+u}<1$, diminished. (u is -ve)

m is +ve errect image.

27. Show that for a plane mirror, u = v

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

By sign convention, u is -ve, v is +ve

For a plane mirror, $f=\alpha$, $\frac{1}{f}=0$

$$0=\frac{1}{-u}+\frac{1}{v}, \quad \therefore u=v.$$

28. If a and b are the distances of the object and the real image from the focus of a concave mirror of focal length f. Show that $f=\sqrt{ab}$

$u = -(f+a)$

$v = -(f+b)$

For concave mirror,

$$-\frac{1}{f}=\frac{1}{-(f+a)}+\frac{1}{-(f+b)}$$

$$-\frac{1}{f}=\frac{-f-b-f-a}{(f+a)(f+b)}$$

$-f^2 - fb - af - ab = -f^2 - fb - f^2 - fa$

$f^2 = ab$

$f=\sqrt{ab}$, Newtons formula

Refraction

29. What is refraction? What is the reason?

Bending of light at the surface of separation of two media. It is because of difference in optical density between the media.

30. Difference between optical density and mass density

- Optical density - Opposition offered by a medium to the propagation of light
- Mass density - Mass per unit volume, called density.

31. Define angle of deviation (d)

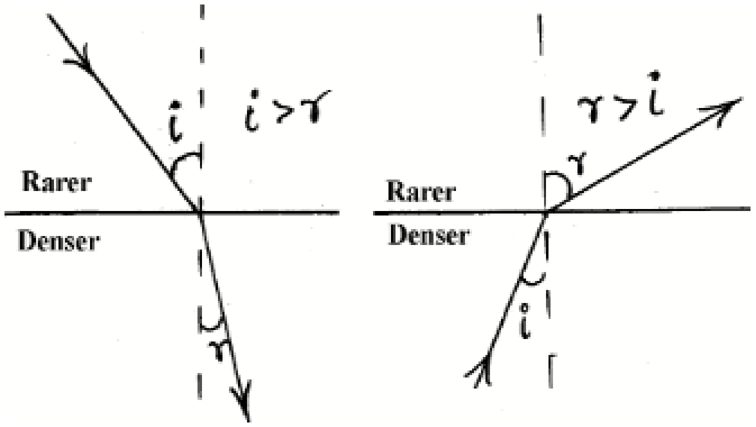
Angle between the incident ray and refracted ray.

32. State the **laws of refraction.**

- Incident ray, refracted ray, and the normal to the point of incidence lie in the same plane.
- Snell's law: - The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant.

$$\text{i.e., } \frac{\sin i}{\sin r} = \text{Constant}$$

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = n_{21}$$



i → angle of incident, r → angle of refraction.

33. Define Refractive Index (n) of a medium

Refractive index of a medium is defined as the ratio of velocity of light in vacuum to the velocity of light in the medium.

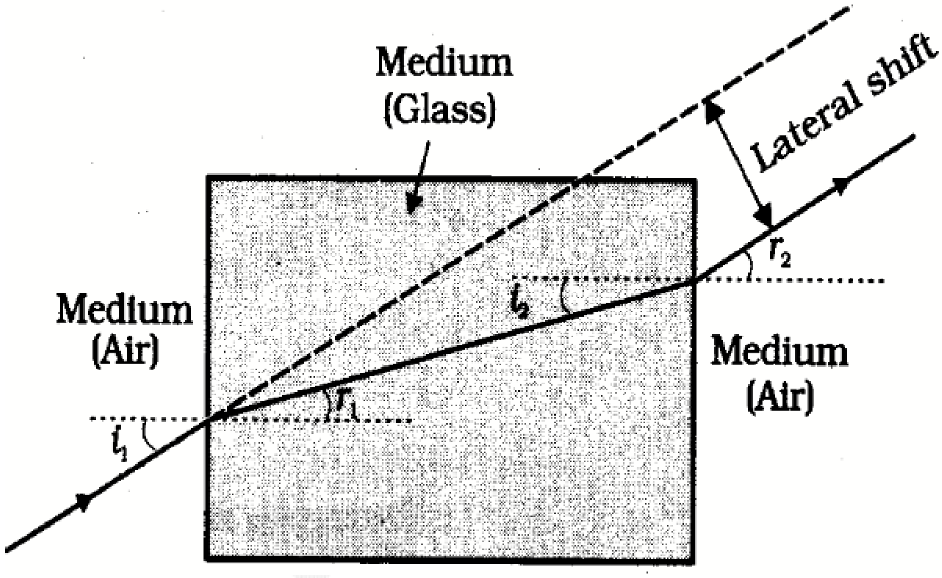
$$n = \frac{c}{v}$$

$n_{air} = 1$	$n_{glass} = 1.5$
$n_{water} = 1.33$	$n_{diamond} = 2.42$

$v \propto \frac{1}{n}$ greater the refractive index smaller is the velocity of light.

34. Draw the refraction through a glass slab and show that the incident ray and emergent ray are parallel.

Let a glass slab of refractive index n_2 be placed in air of refractive index n_1.



At the air- glass interface, we can write the Snell's law:

$$\frac{n_2}{n_1} = \frac{\sin i_1}{\sin r_1} \text{..........(1)}$$

At the glass-air interface, we can write the Snell's law:

$$\frac{n_1}{n_2} = \frac{\sin i_2}{\sin r_2}$$

Taking reciprocal, we get

$$\frac{n_2}{n_1} = \frac{\sin r_2}{\sin i_2} \quad ....(2)$$

From equations (1) and (2), we get

$$\frac{\sin i_1}{\sin r_1} = \frac{\sin r_2}{\sin i_2} \quad ....(3)$$

But from the figure, it is clear that $r_1 = i_2$

$\therefore$ eqn(3) $\Rightarrow \sin i_1 = \sin r_2$

$\Rightarrow i_1 = r_2$

i.e., angle of incidence = angle of emergence

Or the incident ray and the emergent ray are parallel.

35. Refractive indices of water and glass are 4/3 and 3/2 respectively. A light ray travelling in water is incident on water glass interface at 30°. What is 1) n_{gw} 2) Angle of refraction at water - glass interface.

$$n_{gw} = \frac{n_{ga}}{n_{wa}} = \frac{3/2}{4/3} = \frac{9}{8}$$

$$\frac{\sin r_1}{\sin r_2} = n_{gw} = \frac{9}{8}$$

$$\sin r_2 = \frac{8}{9} \text{ Sin } 30$$

$$\sin r_2 = \frac{8}{9} \times 0.5$$

$$r_2 = \sin^{-1}(0.44)$$

36. Show that, $n_{21} = \frac{n_2}{n_1}$

$$n_{21} = \frac{v_1}{v_2}$$

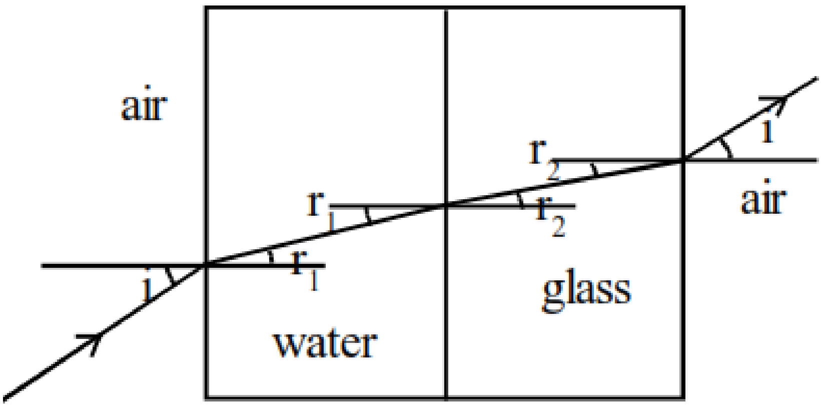
$$n_{21} = \frac{v_1}{c} \cdot \frac{c}{v_2} \quad = \frac{1}{n_1} . n_2$$

$$n_{21} = \frac{n_2}{n_1}$$

37. State whether given statement is true or false – 'Refractive index decreases with temperature'

True

38. Show that $n_{gw} = \frac{n_{ga}}{n_{wa}}$



$$n_{wa} = \frac{\sin i}{\sin r_1}, \quad n_{gw} = \frac{\sin r_1}{\sin r_2}, \quad n_{ag} = \frac{\sin r_2}{\sin i}$$

since as the interfaces are parallel

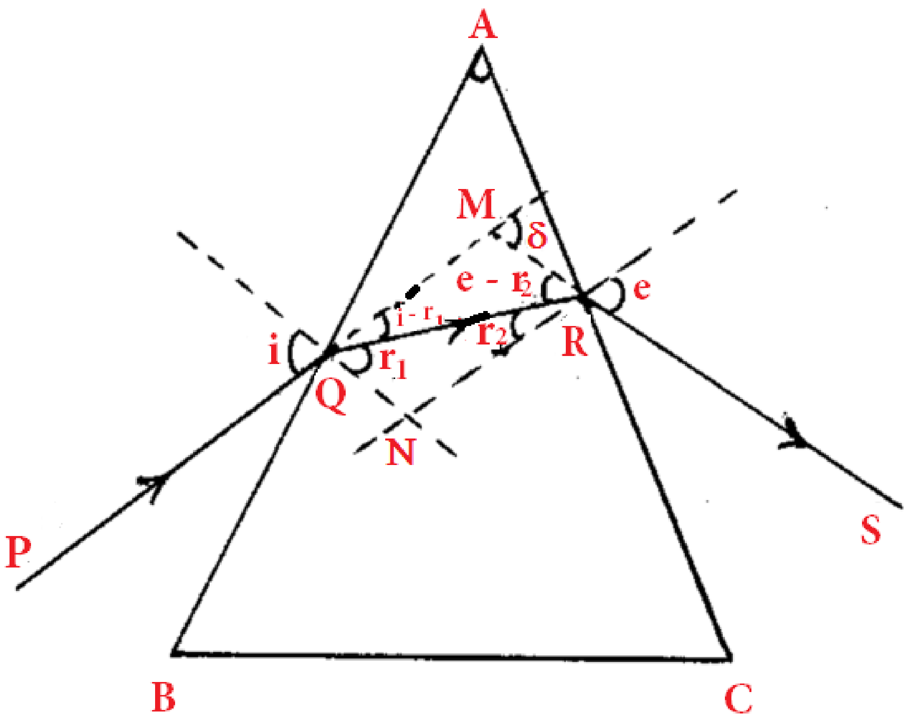
angle of incidence = angle of emergence (i)

$$n_{wa} \cdot n_{gw} \cdot n_{ag} = \frac{\sin i}{\sin r_1} \cdot \frac{\sin r_1}{\sin r_2} \cdot \frac{\sin r_2}{\sin i}$$

$$n_{wa} \times n_{gw} = \frac{1}{n_{ag}} = \frac{\frac{1}{n_a}}{n_g} = \frac{n_g}{n_a} = n_{ga}$$

$$n_{gw} = \frac{n_{ga}}{n_{wa}}$$

39. Explain the refraction through a prism. Derive an expression for the refractive index of the material of the prism (prism formula)



- ABC - Glass Prism
- AB, AC - Refracting sides,
- BC- Base of the Prism
- $\angle A$ - Angle of the Prism –

  Angle between the refracting sides.
- PQ - Incident Ray
- QR - Refracted ray
- RS - Emergent ray

In the quadrilateral AQNR

$$\angle Q + \angle R = 180^0$$

$$\therefore A + \angle N = 180^0 \ldots\ldots(1)$$

from Δ QNR,

$$r_1 + r_2 + \angle N = 180^0 \ldots(2)$$

from (1) and (2), we get

$$r_1 + r_2 = A \ldots\ldots\ldots\ldots\ldots(3)$$

From ΔQRM,

Exterior angle = Sum of opposite interior angles

$$i - r_1 + e - r_2 = \delta$$

$$i + e - r_1 - r_2 = \delta$$

$$i + e - (r_1 + r_2) = \delta$$

$$\left[\text{from (3) } r_1 + r_2 = A\right]$$

$$\therefore i + e - A = \delta$$

or

$$i + e = A + \delta \ldots\ldots\ldots(4)$$

If we increase the angle of incidence, the angle of deviation decreases, reaches a minimum value and then increases.

At the minimum deviation condition

$i = e$, $r_1 = r_2 = r$ and $\delta = D_m$

$D_m \rightarrow$ angle of minimum **deviation**

At the minimum Deviation position

$(3) \rightarrow r + r = A$

$2r = A$

$r = \frac{A}{2}$

$(4) \rightarrow i + i = A + D_m$

$2i = A + D_m$

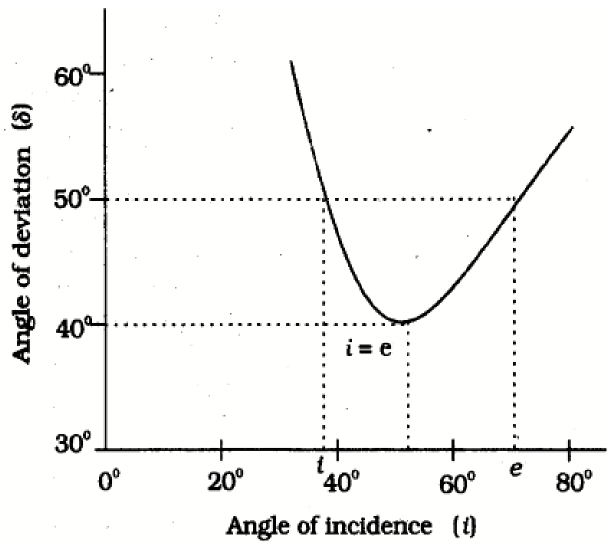
$i = \frac{A + D_m}{2}$

By Snell's law, refractive index of the material of the prism,

$$n_{21} = \frac{\sin i}{\sin r}$$

$$\boxed{n_{21} = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}}$$

40. Draw the i –d Curve. Define angle of minimum deviation.



- It is the graph between angle of incidence (i) and angle of deviation (d).
- *As the angle of incidence increases the angle of deviation decreases at first, reaches a minimum value, and then increases. The minimum value of deviation is called angle of minimum deviation.*

41. Write the prism formula for a small angled prism.

We know,

$$n_{21} = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

For a small angled prism D_m is also very small, and then we have

$$n_{21} = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \approx \frac{\frac{A+D_m}{2}}{\frac{A}{2}}$$

$$\Rightarrow n_{21} = \frac{A+D_m}{A} = 1 + \frac{D_m}{A}$$

$$\Rightarrow \frac{D_m}{A} = n_{21} - 1$$

$$\Rightarrow \boxed{D_m = (n_{21} - 1)A}$$

42. Derive the curved surface formula.

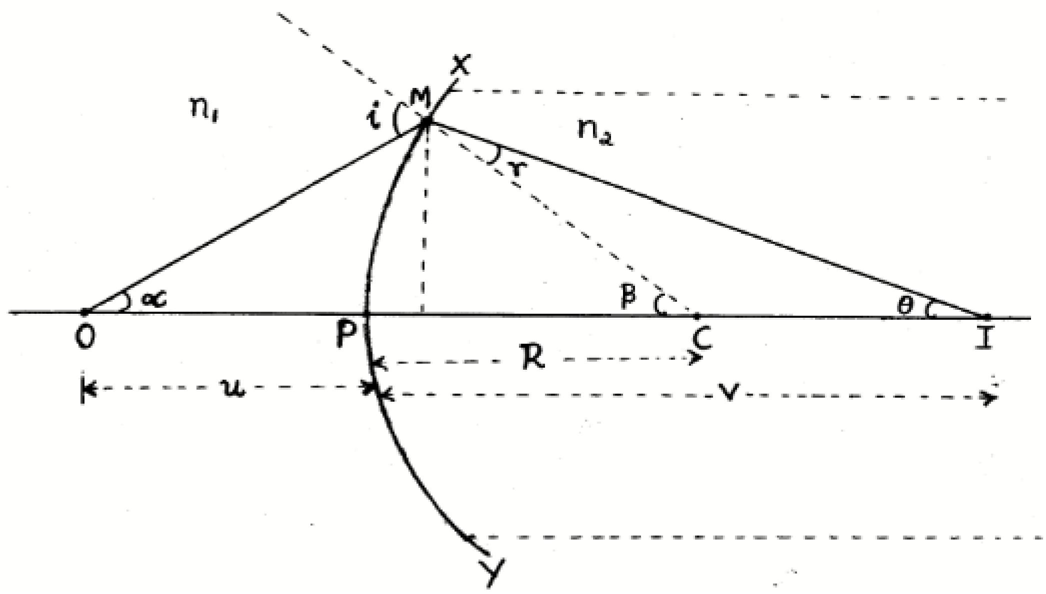
Or

Derive the expression for refraction at a convex surface.

Or

Explain refraction at a spherical surface.

Ans:



- XPY - spherical refracting surface made of glass of refractive index of n_2.
- i - Angle of incidence
- r - angle of refraction
- O - object
- I - real image
- C - Centre of curvature
- OP = -u , Object distance (-ve symbol according to sign convention)
- PI = v, image distance.

We know that for small angles, $\tan\theta \approx \theta$

From Δ**OMP** $\tan\alpha \approx \alpha = \dfrac{PM}{PO}$

From ΔPMI, $\tan\theta \approx \theta = \dfrac{PM}{PI}$

From ΔPCM, $\tan\beta \approx \beta = \frac{PM}{PC}$

From, ΔOMC,

Exterior angle = sum of opposite interior angles

i.e., $i = \alpha + \beta$

$$= \frac{PM}{PO} + \frac{PM}{PC} \quad \text{.............(1)}$$

From ΔIMC,

$$\beta = r + \theta$$

$$\Rightarrow r = \beta - \theta$$

$$= \frac{PM}{PC} - \frac{PM}{PI} \quad \text{.........(2)}$$

By Snell's law,

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$

If i and r are small,

$\sin i \approx i$ and $\sin r \approx r$

$$\frac{i}{r} = \frac{n_2}{n_1}$$

$$n_1 i = n_2 r$$

Substituting for i and r, we get

$$n_1\left(\frac{PM}{PO} + \frac{PM}{PC}\right) = n_2\left(\frac{PM}{PC} - \frac{PM}{PI}\right)$$

$$n_1\frac{PM}{PO} + n_1\frac{PM}{PC} = n_2\frac{PM}{PC} - n_2\frac{PM}{PI}$$

$$\frac{n_1}{PO} + \frac{n_1}{PC} = \frac{n_2}{PC} - \frac{n_2}{PI}$$

$$\frac{n_1}{PO} + \frac{n_2}{PI} = \frac{n_2 - n_1}{PC} \quad \text{.............(3)}$$

But, PO = -u, PI = v, PC = R

$$(3) \rightarrow \frac{n_1}{-u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

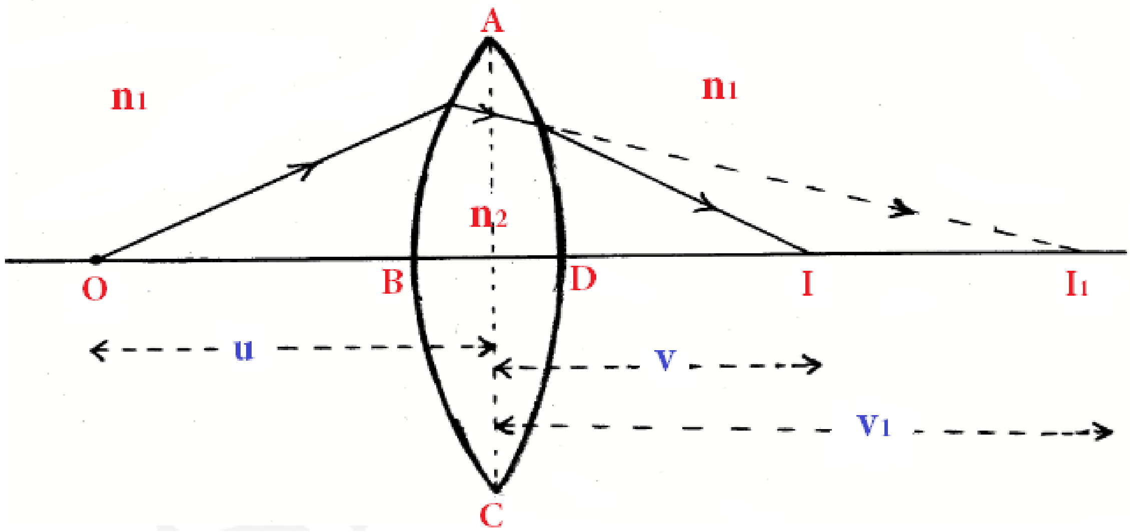
i.e., $$\boxed{\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}}$$

This is the equation for refraction at a convex surface.

Note: If the object is in glass and the ray is refracted to air, n_1 and n_2 are interchanged.

$$\boxed{\frac{n_1}{v} - \frac{n_2}{u} = \frac{n_1 - n_2}{R}}$$

43. Derive Lens Maker's formula. Also obtain the thin lens formula.



Here, image formation has two steps:

(i) The first refracting surface forms the image I_1 of the object O.

(ii) The image formed by the first refracting surface acts as the virtual object for the second refracting surface and the final image is formed at I.

We have the curved surface formula,

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

<u>For refraction at the surface ABC</u>: Light ray travels from n_1 to n_2.

O is the object and $\mathbf{I_1}$ is the image. Let the radius of curvature of ABC be $\mathbf{R_1}$.

$$u \rightarrow u \ , v \rightarrow v_1, R \rightarrow R_1$$

$$\therefore \frac{n_2}{v_1} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} \ldots\ldots\ldots\ldots(1)$$

<u>For refraction at the surface ADC</u> : Light ray travels from n_2 to n_1.
$\mathbf{I_1}$ is the object and **I** is the image. Let the radius of curvature of ADC be $\mathbf{R_2}$.

$$n_1 \leftrightarrow n_2, u \rightarrow v_1, v \rightarrow v, R_1 \rightarrow R_2$$

$$\therefore \frac{n_1}{v} - \frac{n_2}{v_1} = \frac{n_1 - n_2}{R_2}$$

$$\frac{n_1}{v} - \frac{n_2}{v_1} = \frac{-(n_2 - n_1)}{R_2} \ldots\ldots\ldots (2)$$

$$(1) + (2) \rightarrow$$

$$\frac{\cancel{n_2}}{\cancel{v_1}} - \frac{n_1}{u} + \frac{n_1}{v} - \frac{\cancel{n_2}}{\cancel{v_1}}$$

$$= \frac{n_2 - n_1}{R_1} - \frac{n_2 - n_1}{R_2}$$

$$\frac{n_1}{v} - \frac{n_1}{u} = (n_2 - n_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Dividing by n_1,

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2 - n_1}{n_1}\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2}{n_1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{v} - \frac{1}{u} = (n_{21} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \ldots\ldots.(3)$$

If the object is at infinity, image formed at the principal focus.

i.e., If $u = \infty \quad v = f$

$$\therefore \ (4) \rightarrow \frac{1}{f} - \frac{1}{\infty} = (n_{21} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\therefore \frac{1}{f} = (n_{21} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \ldots\ldots\ldots\ldots (4)$$

This is lens makers formula

If the first medium is air,

$n_1 = 1$ and let $n_2 = n$, then

$$n_{21} = \frac{n_2}{n_1} = n$$

$$\frac{1}{f} = (n - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

from (4) and (3)

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

This is called thin lens formula.

44. Define linear magnification of a lens.

Linear magnification,

$$m = \frac{\text{Height of the image}}{\text{Height of the object}}$$

$$m = \frac{h_i}{h_o}$$

m is positive for virtual images and negative for real images.

We can prove that,

$$m = \frac{v}{u}$$

45. Write the expression for refractive index of the material (glass)lens
(For both convex and concave lens)

$$\frac{1}{f} = (n-1)(\frac{1}{R_1} + \frac{1}{R_2})$$

$$n-1 = \frac{R_1 R_2}{f(R_1 + R_2)}$$

$$n = 1 + \frac{R_1 R_2}{f(R_1 + R_2)}$$

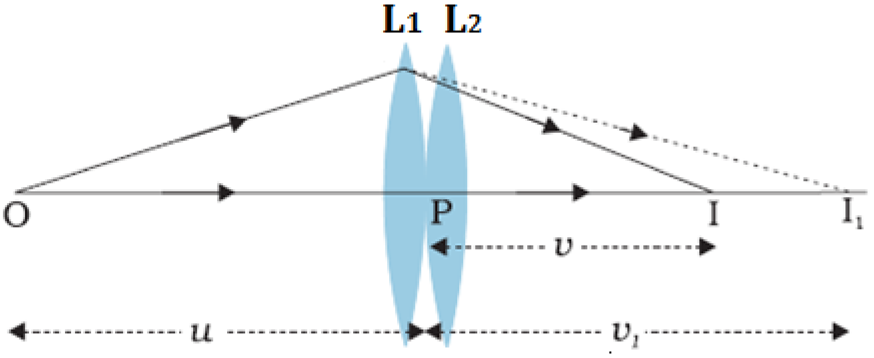
46. Define power of a lens (P)

Power of a lens is the reciprocal of focal length expressed in metre.

$$P = 1/f$$

- Ability to converge or diverge beams of light.
- SI unit of power of lens is diopetre (D)
- For convex lens, P is +ve
- For concave lens, P is -ve

47. Derive an expression for the effective (i) focal length (ii) power for the combination of two thin lenses in contact. Also write equation for effective magnification.



For the 1st lens, object is at ‘O’ and image is at ‘I_1’.

$$u \to u,\ v \to v_1,\ f \to f_1$$

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \ldots\ldots\ldots\ldots (1)$$

For the second lens object is I_1 and image is I

$$u \to v_1,\ v \to v,\ f \to f_2$$

$$\therefore \frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$(1) + (2) \Rightarrow$$

$$\frac{1}{v_1} - \frac{1}{u} + \frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \ldots\ldots\ldots\ldots (3)$$

If the combination of the two lenses is replaced by a single lens of focal length **f** such that the image of the same object is formed at the same position.

Then we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \ldots\ldots\ldots\ldots (4)$$

from (3) and (4)

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}}$$

Power, $P = P_1 + P_2$

Magnification $m = m_1 \times m_2$

In general,

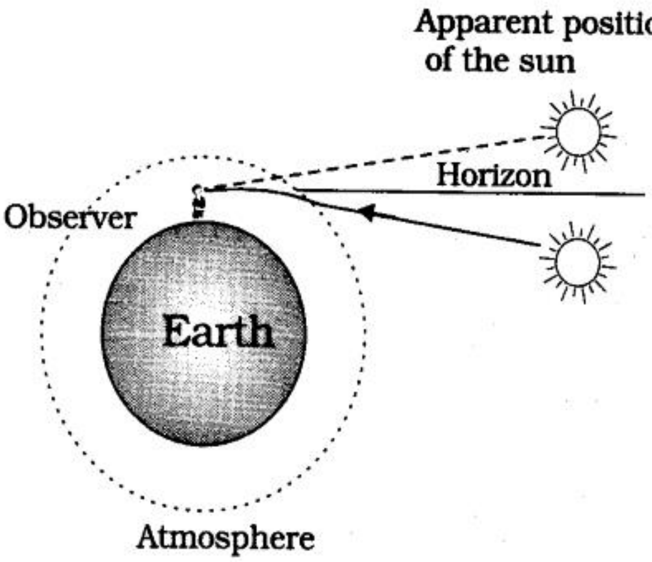
$$\boxed{\begin{aligned} &\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ldots\ldots\ldots \\ &P = P_1 + P_2 + P_3 + \ldots\ldots\ldots \\ &m = m_1 \times m_2 \times m_3 \times \ldots\ldots\ldots \end{aligned}}$$

48. What are the applications of refraction?

i) Twinkling of stars

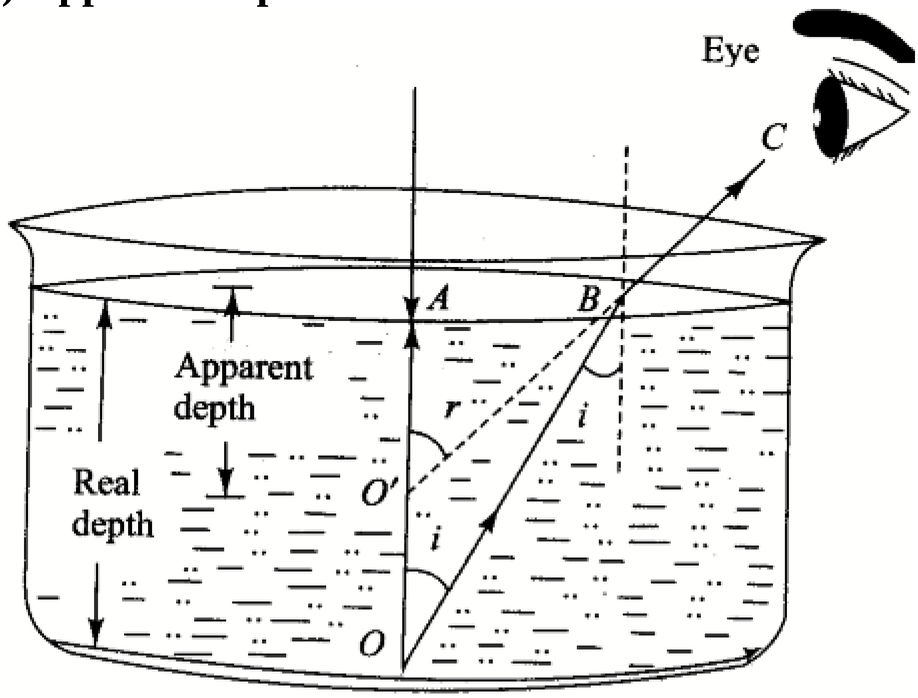
As we go up, the density of air in the atmosphere continuously decreases. Therefore, the light coming from the star is travelling from a rarer part of air to denser part. Therefore, it bends towards the normal. Thus we see the star at an apparent position. But the density of air in the atmosphere continuously changes. Therefore the apparent position also continuously changes. Thus the star appears to be twinkling.

ii) Early sunrise and delayed sunset



As we go up, the density of air in the atmosphere continuously decreases. Therefore the light coming from the sun is travelling from a rarer medium to denser medium. Therefore it bends towards the normal. Thus we see the sun at an apparent position raised above the horizon. This is the reason for early sunrise and delayed sunset.

iii) Apparent depth



If an object in a denser medium is viewed from a rarer medium the image appears to be raised towards the surface. Refractive index,

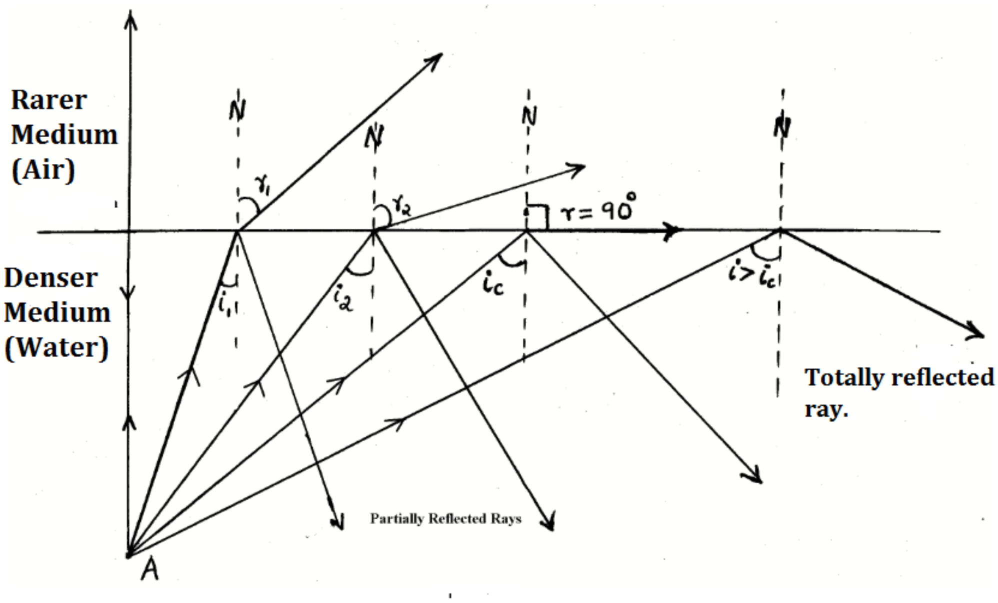
$$n_{21} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

$$\text{Apparent depth} = \frac{\text{Real depth}}{n_{21}}$$

49. Define total internal reflection.

When light travels from a denser medium to a rarer medium, if the angle of incidence is greater than the critical angle it gets totally reflected in to the same medium. This phenomenon is called total internal reflection.

50. Explain total internal reflection?



When a ray of light is incident on the surface separating two media, a part of light is reflected and the other part is transmitted (refracted). When light travel from a denser medium to a rarer medium, the refracted ray bends away from the normal. As the angle of incidence increases, the angle of refraction also increases. For a particular angle of incidence the angle of refraction becomes 90^0. If the angle of incidence is further increased the ray gets totally reflected into the same medium. This phenomenon is called total internal reflection.

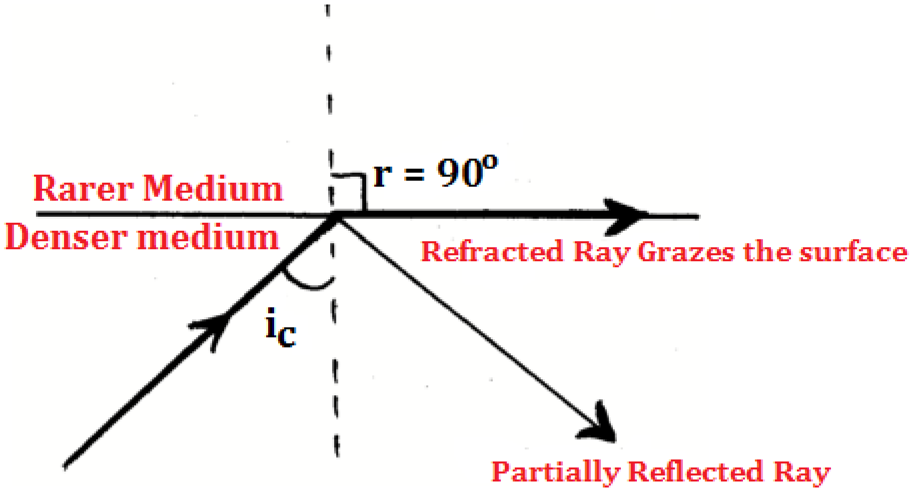
51. What are the necessary conditions for total internal reflection to occur?

- The light ray should travel from denser medium to rarer medium.
- The angle of incidence should be greater than the critical angle.

52. Define critical angle.

It is the angle of incidence in the denser medium for which the angle of refraction becomes 90^0.

53. Derive the relation between critical angle and refractive index of the denser medium?



By Snell's law, $\dfrac{\sin i}{\sin r} = \dfrac{n_1}{n_2}$,

[Here the ray goes from n_2 to n_1]

When $i = i_c$, $r = 90^0$

$$\frac{\sin i_c}{\sin 90^0} = \frac{n_1}{n_2}$$

$$\frac{\sin i_c}{1} = \frac{n_1}{n_2}$$

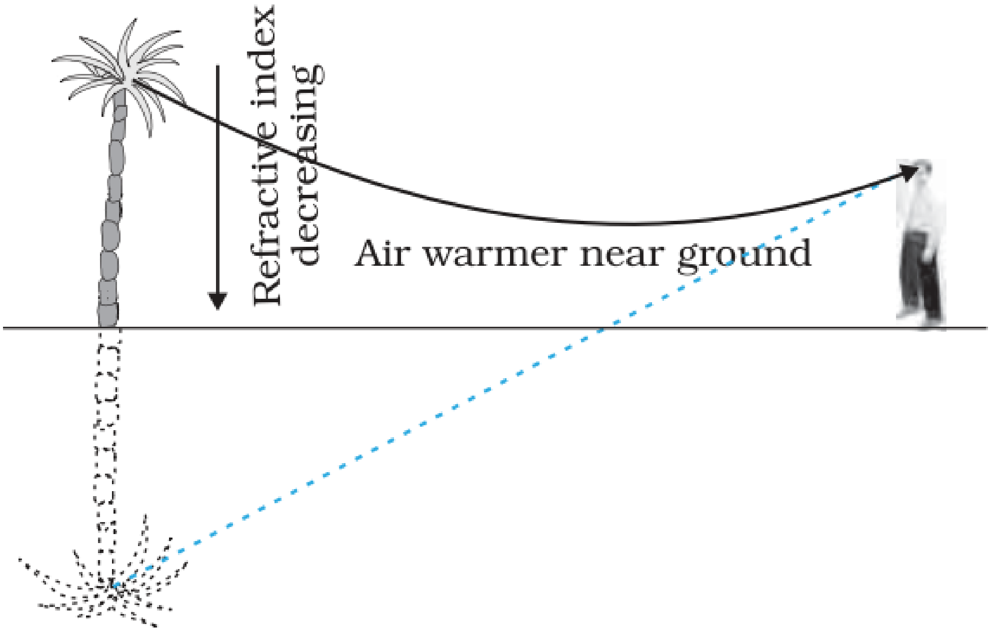
$$\frac{n_2}{n_1} = \frac{1}{\sin i_c}$$

If the rarer medium is air, then $n_1 = 1$ and let $n_2 = n$.

$$\mathbf{n = \frac{1}{\sin i_c}}$$

54. What are the applications of total internal reflection?

- **Mirage**



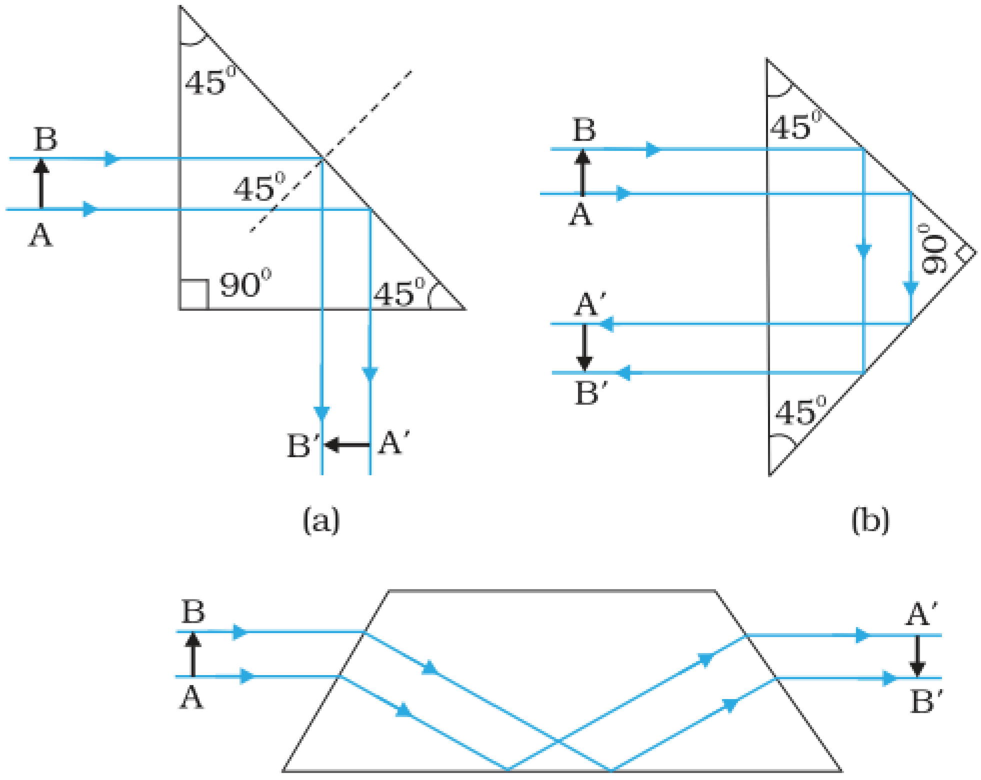
In hot sunny days the layer of air in contact with sand in a desert (or tar road), becomes hot and rarer. The upper layers are comparatively cooler and denser. Therefore the ray of light coming down from a distant object like a tree is travelling from a denser medium to a rarer medium and it suffers total internal reflection. Thus for an observer the image of a distant object is seen inverted. This makes the illusion that the tree is standing near a pool of water. This phenomenon is called mirage.

- **Brilliance of diamond**

The refractive index of diamond is very high [n = 2.42] and the critical angle is very low [c = 24.4^0]. Moreover the faces of diamond are cut in such a way that, a ray entering the diamond, undergoes multiple total internal reflections inside it and finally comes out only through few faces. These faces appear glittering.

- **Total reflecting prisms**

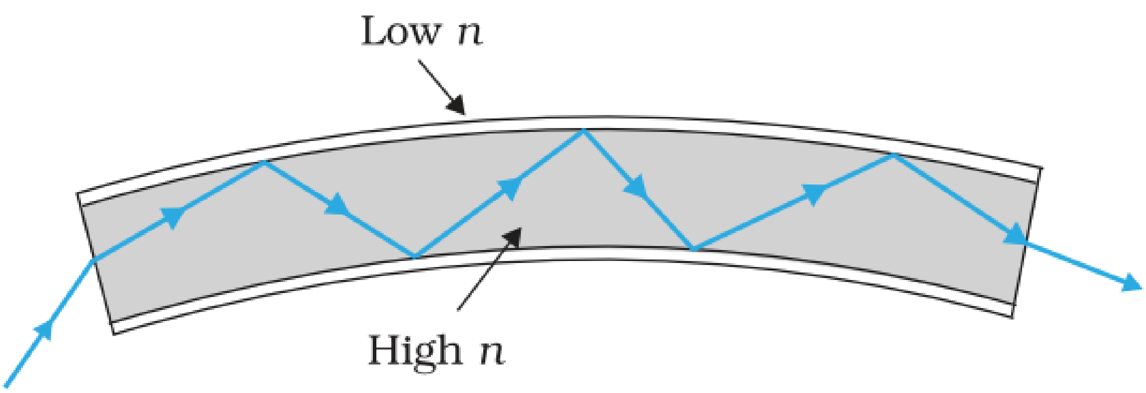
Total reflecting prisms are designed to bent light or to invert images without changing their size, based on total internal reflection. (Used in periscopes)



(a) (b)

(c)

- **Optical fibres**



Core of optical fibre is made up of glass or quartz [n = 1.7]. There is a thin layer of outer coating called **cladding**. Cladding is made up of a material of lower refractive index [n = 1.5]. When a ray of light enters the fibre at one end, it undergoes multiple total internal reflections inside the fibre and finally comes out at the other end.

55.What are the uses of Optical fibres?

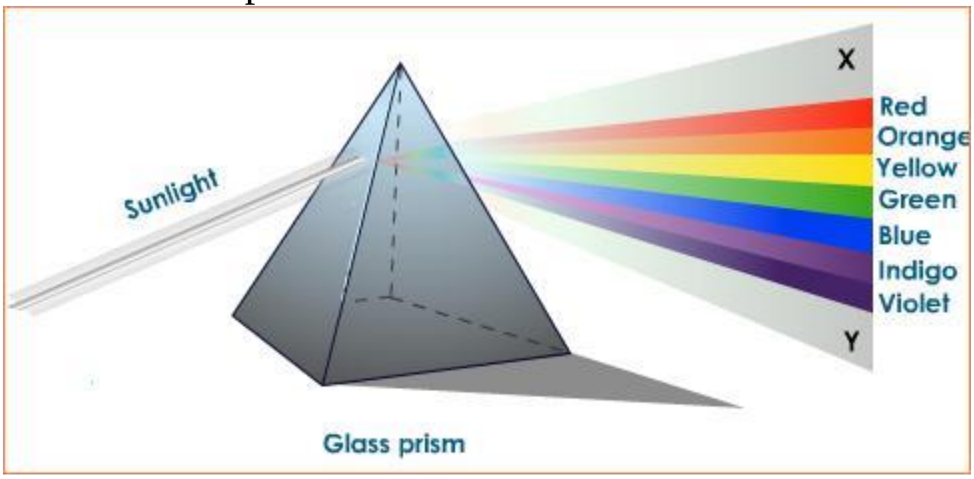
i) Optical fibres are used as a light pipe for visual examination of internal organs.

ii) Optical fibres are used to carry electrical signals which are converted to light.

Dispersion of light

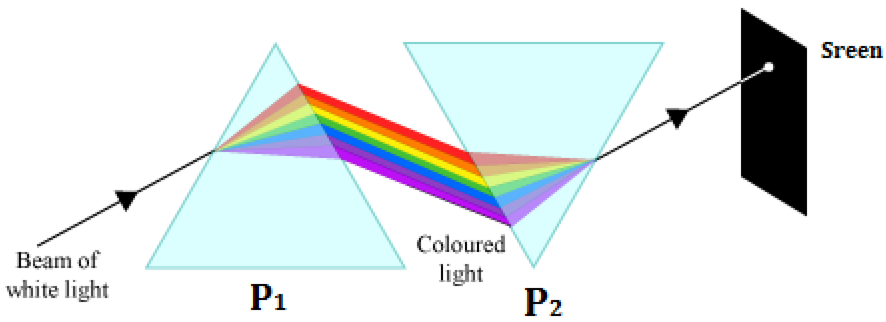
56. Explain dispersion of light

The phenomenon of splitting of composite light into its component colours is called dispersion.



When white light is passed through a prism, it splits into its seven component colours (**VIBGYOR).**

Note:



If we place a second prism in an inverted position, close to the first prism, the second prism recombines the colours and we get white light.

57. What is the cause of dispersion?

Colour of light is associated with wavelength of light. Dispersion takes place because the refractive index of medium is different for different wave lengths (colours). Refractive index of the medium for violet light is greater than that for red light. So red light travels faster than violet in a prism.

58. Distinguish between dispersive medium and non-dispersive medium.

The medium in which the different colours of light travel with different velocities is called a dispersive medium.

Eg: Glass

The medium in which the different colours of light travel with the same velocity is called non-dispersive medium.

Eg: Vacuum

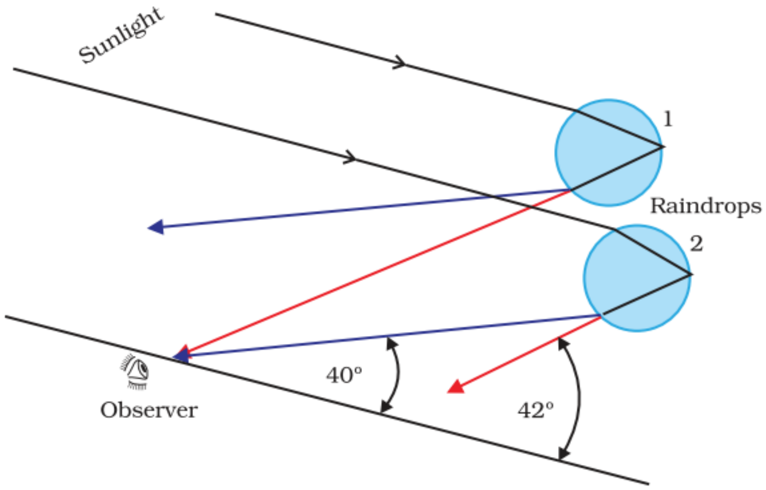
59. How rainbow is formed?

Rainbow is formed due to the combined effect of dispersion, refraction and total internal reflection of sunlight by the rain drops.

60. What is the condition for a person to see rainbow?

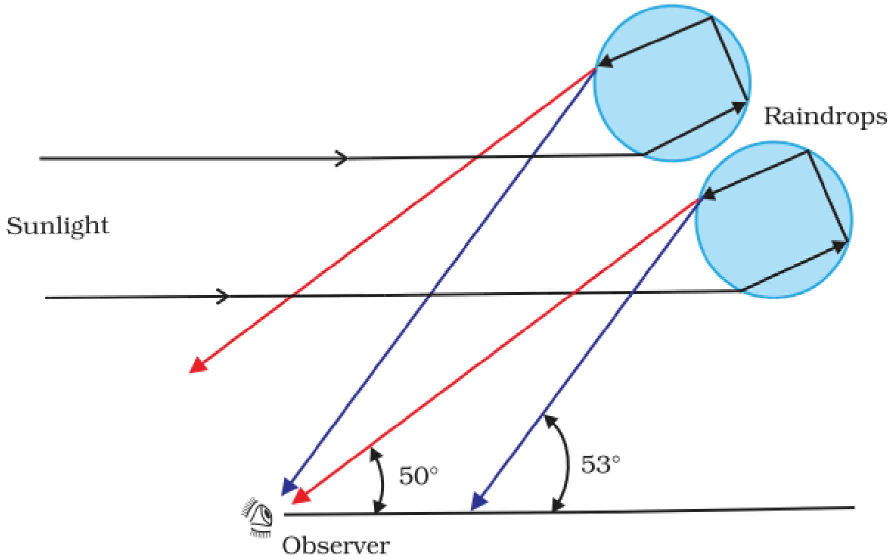
The condition for observing a rainbow is that the sun should be shining in one part of the sky while it is raining in the opposite part of the sky. An observer can therefore see a rainbow only when his back is towards the sun.

61. What are the differences in the formation and appearance of primary and secondary rainbows?



- A primary rainbow is a result of three-step process: refraction, total internal reflection and again refraction.

- In a primary rainbow the violet light emerges from raindrops at an angle of 40^{o} relative to the incoming sunlight and red light emerges at an angle of 42^{0}. Thus an observer sees a primary rainbow with red colour on the top and violet on the bottom.



- A secondary rainbow is a result of four- step process: refraction, total internal reflection, again total internal reflection and refraction.
- In a secondary rainbow the violet light emerges from the raindrops at an angle of 53^{0} relative to the incoming sunlight and red light emerges at an angle of 50^{0}. Thus an observer sees a secondary rainbow with violet colour on the top and red on the bottom.
- Secondary rainbow is fainter than primary rainbow.

Scattering

62. What is scattering?

The irregular and partial reflection of light at the dust particles and air molecules in the atmosphere is called scattering.

63. State Rayleigh's scattering law.

According to Rayleigh's scattering law the intensity of the scattered light is inversely proportional to forth power of wave length.

$$I_s = \frac{1}{\lambda^4}$$

64. Why sky appears blue?

When sun light comes through the atmosphere it undergoes scattering at the dust particles and air molecules. Thus the low wavelength region (bluish region) is more scattered. Since our eyes are more sensitive to blue than violet, sky appears blue.

65. Why sun appears red during sunrise and sunset?

During sunrise and sunset light has to travel more distance through the atmosphere. Thus most part of low wavelength region is scattered away and the least scattered longer wavelength region (reddish region) reaches our eye. Therefore the sun appears red.

66. Why sea appears blue?

It is due to the scattering of sunlight at the water molecules and dust particles. By Rayleigh's scattering law, low wavelength region (blue region) is more scattered. Therefore, sea appears blue.

67. Why cloud appears white?

The particles of cloud are comparatively bigger in size. Therefore, all colours of sunlight are almost equally scattered. Thus clouds appear white.

Optical Instruments

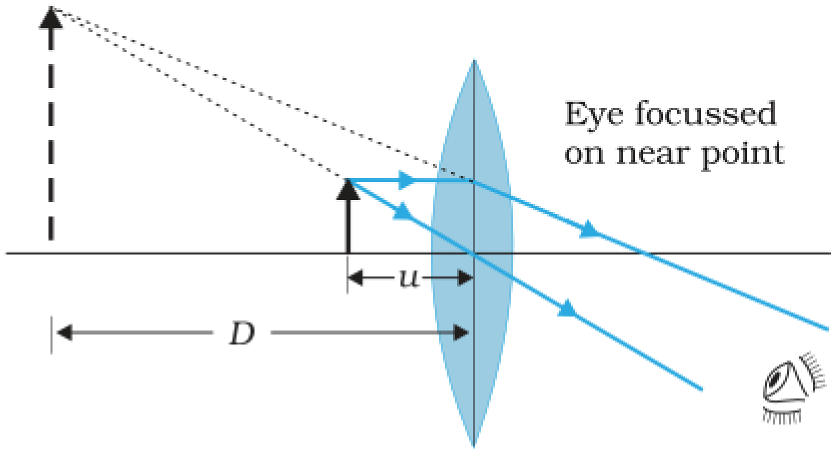
Microscopes

68. What is the use of a microscope?

Microscope is used to get magnified images of near objects.

69.

- By drawing a neat ray diagram, explain the image formation in a **simple microscope**.
- Derive the equation for magnification.
- Write the nature of the image formed?
- What is the limitation of a simple microscope?



Converging lens (convex lens) of small focal length is used as a simple microscope. If the object is at the focus, the image is at infinity. If the object is brought closer, then the image is formed at a distance closer than infinity. The position of the object can be adjusted so that the image is formed at the least distance of distinct vision.

Magnification

$$m = \frac{v}{u} = v.\frac{1}{u}$$

We know,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Then $$\frac{1}{v} - \frac{1}{f} = \frac{1}{u}$$

Therefore,

$$\mathbf{m} = v.(\frac{1}{v} - \frac{1}{f})$$

$$m = 1 - \frac{v}{f}$$

If the image is formed at the near point, v = – D

$$\therefore \boxed{m = 1 + \frac{D}{f}}$$

If the image is formed at infinity,

$$\boxed{m = \frac{D}{f}}$$

70. Nature of the image

The image is erect, magnified and virtual.

71. Limitation of simple microscope

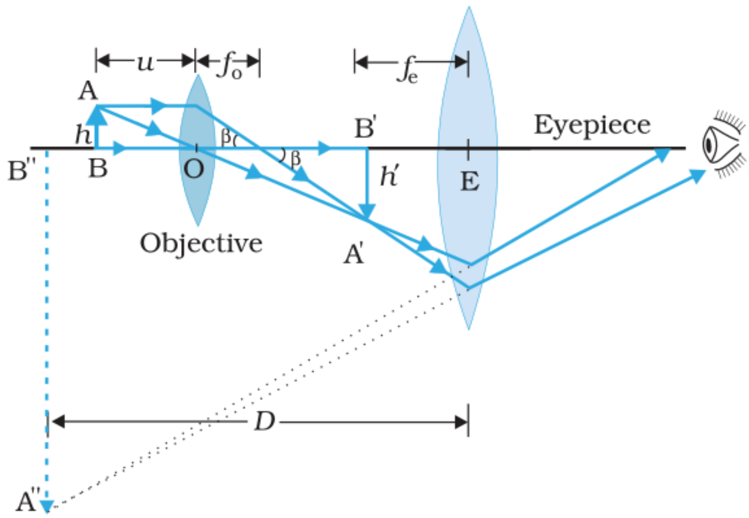
The magnification of the simple microscope, m = ≤ 9. To get further magnification we use compound microscope.

72.

1) By drawing a neat ray diagram explain the image formation in a compound microscope.

2) What is the nature of image formed?

3) Derive an expression for the magnification produced by compound microscope.



- Compound microscope consists of two convex lenses objective and eye piece.
- The focal length and aperture of objective is less than those of eye piece.
- When an object is placed beyond the focal length of the objective, a magnified, real and inverted image is formed beyond the '2f' of the objective on the other side. The distance between the lenses is adjusted so that this image falls within the

focal length of the eye piece. Now the eyepiece acts as a simple microscope and the final image is formed at the least distance of distinct vision.

2) Nature of final image

The final image is enlarged, inverted and virtual w. r. t. the object.

3) Magnification

If m_o - Magnification of object and m_e - Magnification of image
Then, the magnification of the compound microscope is given by
$m = m_o \times m_e$

$$= \frac{v_o}{u_o} \times \left[1 + \frac{D}{f_e}\right]$$

Since the object is placed very near to the focus of the objective,
$u_o \approx f_o$ and $v_o \approx L$, length of microscope tube

$$\boxed{m = \left(\frac{L}{f_o}\right)\left(1 + \frac{D}{f_e}\right)}$$

If the final image is formed at infinity

$$\boxed{m = \left(\frac{L}{f_o}\right)\left(\frac{D}{f_e}\right)}$$

From the above equation, it is very clear that, to achieve a large magnification of a small object, the objective and eyepiece should have small focal lengths.

73. Define the resolving power of a microscope.

Resolving power of a microscope is defined as the reciprocal of minimum separation between two point objects which can be distinctly seen by it.

$$\textbf{Resolving Power} = \frac{1}{d_{min}} = \frac{2n\sin\beta}{1.22\lambda}$$

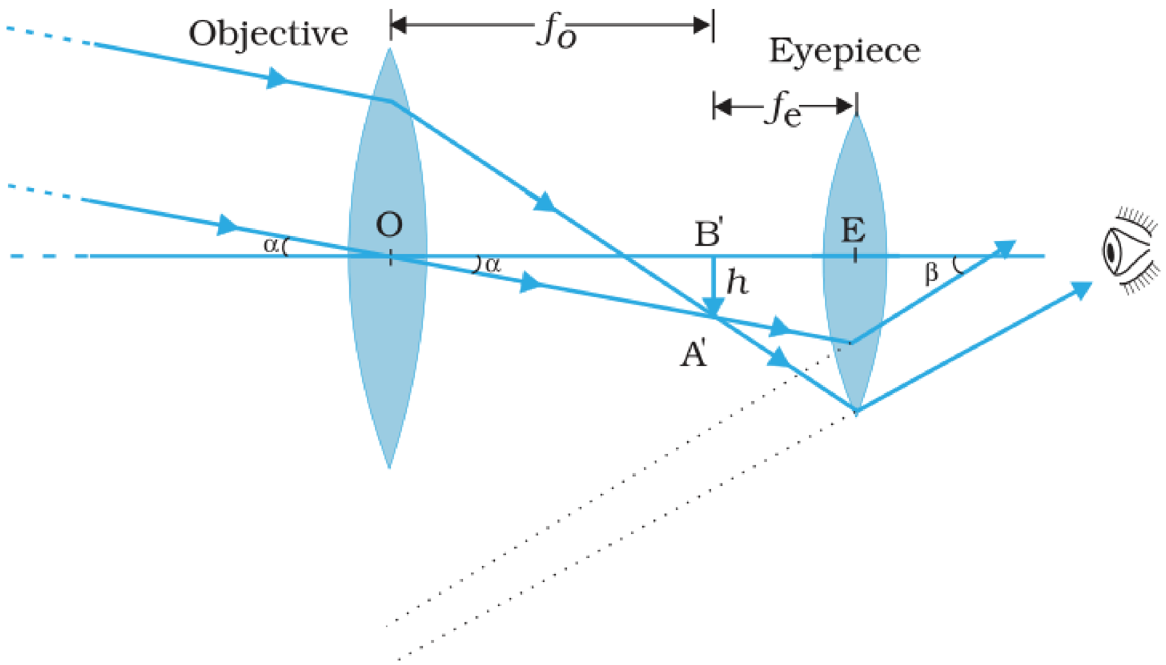
The quantity $n\sin\beta$ is called the numerical aperture.

Telescopes

Telescope is used to provide angular magnification of distant objects.

74.
1) By drawing a neat ray diagram explain the image formation in a refracting type telescope.
2) Derive an expression for the magnification produce by it.
3) What is the nature of the image?

Ans:
1)



- In a telescope there are two convex lenses- the objective and eyepiece.
- The objective has a large focal length and much larger aperture than the eyepiece.
- Light from a distant object enters the objective and a real and inverted image is formed at its focus (F_o). The eyepiece magnifies this image producing a final inverted image with respect to the object.

2) Magnification (m)
The magnifying power 'm' is the ratio of the angle 'β' subtended by the final image at eye to the angle 'α' subtended by the object at the lens or eye.

$$m = \frac{\beta}{\alpha} \quad \text{But we have,}$$

$$\beta \approx \tan\beta = \frac{h}{f_e} \text{ and } \alpha \approx \tan\alpha = \frac{h}{f_0}$$

$$\therefore m = \frac{h/f_e}{h/f_0} = \frac{f_0}{f_e} \qquad \boxed{m = \frac{f_0}{f_e}}$$

The above equation shows that to have greater magnification for the telescope, the focal length of the objective should be large and that of the eye piece should be small.

3) Nature of final image

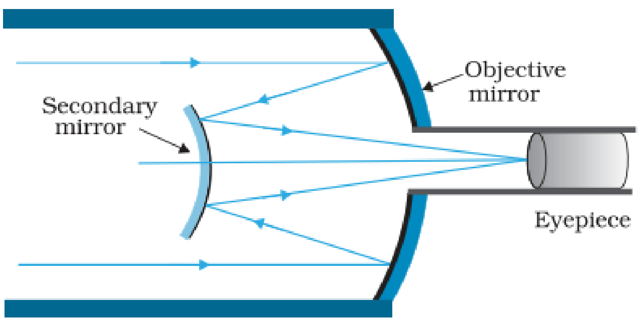
The final image is enlarged, inverted and virtual w. r. t. the object.

75. Give the expression for the length of an astronomical telescope.

The length of a telescope is the separation between the objective lens and eye piece.

$$L = f_o + f_e$$

76. By drawing a neat ray diagram explain the image formation by a reflecting type telescope.



- In reflecting type telescopes, a concave mirror is used as objective instead of convex lens.
- The light from the object is reflected by the concave mirror to the secondary mirror, which again reflects the light into the eyepiece. This type of reflecting telescope is known as cassegrain telescope.

77. What are the disadvantages of refracting telescope?

- In refracting telescopes, to get better resolving power objective lens of large aperture is needed. Big lenses are very heavy and therefore, difficult to support by their edges.
- It is difficult and expensive to make such large sized lenses.
- Chromatic aberration is a main defect in a lens.

78. What are the advantages of a reflecting type telescope?

- There is no chromatic aberration in a mirror.
- If a parabolic mirror is chosen as the objective, spherical aberration can be removed.
- Mechanical support is much less of a problem since a mirror weighs much less than a lens of equivalent optical quality, and can be supported over its entire back surface, not just over its rim.

79. How can you remove spherical aberration in a reflecting type telescope?

Spherical aberration can be removed by using parabolic concave mirror.

80. Define the resolving power of a telescope. Give its equation

The resolving power of a telescope is defined as the reciprocal of smallest angular separation between two distant objects whose images are distinctly separated by the telescope.

Resolving Power $= \frac{1}{\Delta\theta} = \frac{d}{1.22\lambda}$, where d is the diameter of telescope objective and λ is the wavelength of light used. $\Delta\theta$ is called the limit of resolution of the telescope.

Important Solved Problems

1. An object is placed at (i) 10 cm, (ii) 5cm in front of a concave mirror of radius of curvature 15 cm. find the position, nature and magnification of the image in each case.

(i) u = -10 cm, R = -15 cm

f = R/2 = -7.5 cm

$$\mathbf{\frac{1}{v}+\frac{1}{u}=\frac{1}{f}}$$

$\mathbf{\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{-7.5}-\frac{1}{-10}=\frac{-7.5+10}{-75}=\frac{2.5}{-75}}$

$\mathbf{v=\frac{-75}{2.5}=-30cm}$

Since v is negative, image formed is 30 cm from the mirror on the same side of the object.

Image is real.

$m = -v/u = \frac{-(-30)}{-10} = -3$

Since, m is negative, image is inverted, real, magnified.

(ii) $u = -5$ cm

$R = -15$ cm; $f = -7.5$ cm

$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{-7.5}-\frac{1}{-5}=\frac{2.5}{37.5}$

$v = \frac{37.5}{2.5} = 15$ cm

Image formed behind the mirror

$$m=\frac{-v}{u}=\frac{-15}{-5}=3$$

Image is virtual, magnified 3 times.

2. Light from a point source in air falls on a spherical glass surface (n= 1.5 and radius of curvature = 20 cm). The distance of the light source from the glass surface is 100 cm. At what position the image is formed?

$n_1 = 1$, $n_2 = 1.5$, $R = 20$ cm, $f = R/2 = 10$ cm, $u = -100$ cm

$$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$$

$$\frac{1.5}{v}-\frac{1}{-100}=\frac{1.5-1}{20}$$

$$\frac{1.5}{v}+\frac{1}{100}=\frac{.5}{20}$$

$$v = 100 \text{ cm}$$

Image is virtual, inverted. Image is formed behind the spherical surface.

3. A magician during a show makes a glass lens with n = 1.47 disappears in a trough of liquid. What is the refractive index of the liquid? Could the liquid be water?

The refractive index of the liquid must be equal to 1.47 in order to make the lens disappear. This means $n_1 = n_2$. This gives $1/f = 0$ or $f \to \infty$. The lens in the liquid will act like a plane sheet of glass. No, the liquid is not water. It could be glycerin.

4. (i) If f = 0.5 m for a glass lens, what is the power of the lens? (ii) The radii of curvature of the faces of a double convex lens are 10 cm and 15 cm. The focal length is 12 cm. What is the refractive index of glass?

i) $f = 0.5$ m

$P = 1/f = 1/0.5 = 2$ D

ii) $R_1 = 10$ cm ; $R_2 = -15$ cm; $f = 12$ cm

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{12} = (n-1)\left[\frac{1}{10} - \frac{1}{-15}\right]; \quad n = 1.5$$

5. An equilateral glass prism is placed on a horizontal surface. A ray PQ is incident on it. For minimum deviation

a. PQ is horizontal

b. QR is horizontal

c. RS is horizontal

d. None of these

Answer

QR is horizontal.

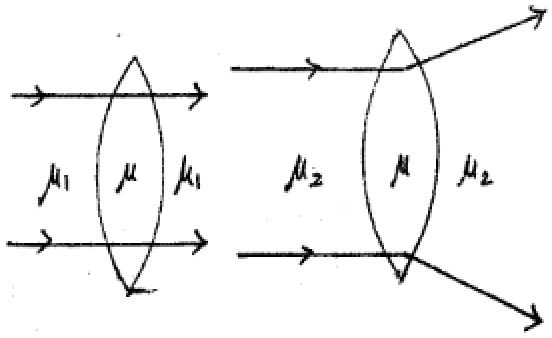
6. A thick lens gives coloured images due to ……

Answer: Chromatic aberration.

7. The critical angle for diamond is 30°. What is its refractive index?

$$n = \frac{1}{\sin C} = \frac{1}{\sin 30^{o}} = \frac{1}{\frac{1}{2}} = 2$$

8. The path of light rays through a convex lens when it is placed in two different media is shown in the figure.



9. What is the relation between the refraction indices μ, μ1 and μ2

Answer: $\mu = \mu_1$ and $\mu_2 > \mu$

10. A thin convex lens of focal length 5 cm is used as a simple microscope by a person with a normal near point (25 cm). What is the magnifying power of the microscope?

$$f = 5 \text{ cm}; D = 25 \text{ cm}$$

$$m = 1 + \frac{D}{f} = 1 + \frac{25}{5} = 6$$

11. Using the data given below, state as which of the given lenses will you prefer to use as

(i) An eye-piece and

(ii) An objective to construct an astronomical telescope? Give reason for your answer.

(iii) What will be the magnifying power and the normal length of the telescope tube so constructed?

Lens	Power	Aperture
L_1	1D	0.1m
L_2	10D	0.05m
L_3	10D	0.02m
L_4	20D	0.02m

Answer

(i) L_4 (Convex lens of maximum power, minimum aperture)
(ii) L_1 (Minimum power, maximum aperture)

12. Assume that light of wavelength 6000A° is coming from a star. What is the limit of resolution of a telescope whose objective has a diameter of 100 inch (254 cm)?

$\lambda = 6000 \times 10^{-10}$ m $= 6 \times 10^{-7}$ m
d = 2.54 m

Resolving power of telescope = $\frac{1}{\Delta\theta} = \frac{d}{1.22\lambda}$

Limit of resolution, $\Delta\theta = \frac{1.22\,(6\times10^{-7})}{2.54} = 2.88 \times 10^{-7}\text{rad}$

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