Homework Assignment 8

Due: 16:00pm Tuesday, March. 28, 2023

Problem 1. The value of a message signal varies within $[-5, 5]$.

(a) Design a 5-level *uniform* quantizer for the message by specifying the quantization regions and quantization levels.

(b) The following sequence of message samples is input to the quantizer: $\{-2.3, 1.7, 4.2, -0.6, 1.4\}$. Find the output of the quantizer you designed in Part (a).

Problem 2. A message sample is distributed according to a triangular PDF as follows

$$f_M(m) = \begin{cases} \frac{m}{4} + \frac{1}{2} & m \in [-2, 0] \\ -\frac{m}{4} + \frac{1}{2} & m \in (0, 2] \end{cases}.$$

It is quantized using the 2-level (binary) uniform quantizer

$$Q(m) = \begin{cases} 1 & 0 < m \leq 2 \\ -1 & -2 \leq m < 0 \end{cases}.$$

(a) Calculate the MSE of the quantization.

(b) Calculate the SQNR.

(c) Let V be the quantization result, which is a random variable. Determine the probability mass function (PMF) of V.

Problem 3. This problem addresses the digitalization and transmission of a signal using pulse-coded modulation. The signal bandwidth is 4 MHz. Specifications of the modulator include the following:

Sampling:	at sample rate f_s with a guard band of 12.5% of the Nyquist rate
Quantization:	uniform quantization with 1024 levels
Encoding:	Binary natural coding

(a) Determine the sampling rate f_s.

(b) Determine the maximum permissible bit duration and the corresponding bit rate.

Problem 4. The message

$$m(t) = 2\cos(2\pi t) + \cos(4\pi t)$$

is transmitted using a 3-bit PCM system (thus $N = 2^3 = 8$.).

(a) Design a uniform quantizer for this system.

(b) Assume a sampling rate of 3 samples per second with samples taken at

$$t = \cdots, -5/6, -1/2, -1/6, 1/6, 1/2, 5/6, \cdots.$$

What is the quantizer output?

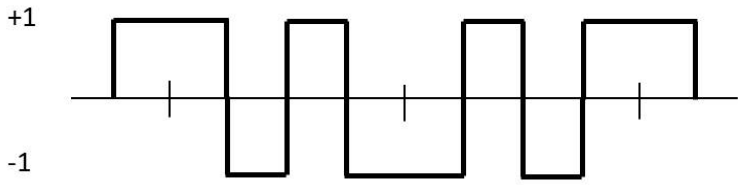

Figure 1: PCM message in Problem 5.

Problem 5. Figure 1 shows a PCM signal in which the amplitude levels of $+1$ and -1 volt are used to represent binary symbols 1 and 0, respectively. Natural coding is used where each codeword consists of 2 bits. The message takes values in $[-1, 1]$ and uniform quantizer is used. What are the quantization outputs that produce this PCM signal?

Problem 6. Consider the following sequence of 1s and 0s:

(a) 10101010,

(b) 11110000,

(c) 11110111.

Sketch the waveform of each of the sequences using the following methods of representing symbols 1 and 0:

(1) non-return-to-zero signaling,

(2) differential encoding.