Homework Assignment 6

Due: 16:00pm Tuesday, Match 14, 2021

Problem 1. A baseband signal $m(t)$ is the periodic sawtooth signal shown in Fig. 1, where $T_0 = 1, A = 1$.

(a) Sketch the FM wave for this signal $m(t)$ if $f_c = 4$ and $k_f = 10$.

(b) Estimate the bandwidth of the FM wave. Assume the bandwidth of $m(t)$ is defined by the fifth harmonic frequency of $m(t)$.

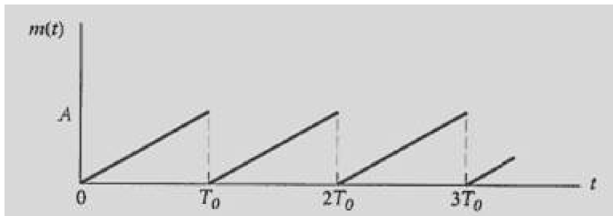

Figure 1: Message signal in Problem 1.

Problem 2. Given $m(t) = \sin[2\pi(1000)t]$, $k_f = 100,000$ and $k_p = 10$.

(a) Estimate the bandwidth of FM and PM waves using Carson's rule.

(b) Repeat part (a) if the message signal amplitude is doubled.

(c) Repeat part (a) if the message signal frequency is doubled.

(d) Comment on the sensitivity of FM and PM bandwidths to the spectrum of $m(t)$.

Problem 3. (Haykin and Moher Problem 4.11) The sinusoidal wave

$$m(t) = A_m \cos(2\pi f_m t)$$

is applied to a phase modulator with phase sensitivity k_p. The unmodulated carrier wave has frequency f_c and amplitude A_c. Find the spectrum of the resulting phase-modulated (PM) wave, assuming that the maximum phase deviation $\beta = k_p A_m$ is sufficiently small.

Note: Use the approximations $\sin x \approx x$ and $\cos x \approx 1$ for $|x| \ll 1$.

Problem 4. (Haykin and Moher Problem 4.24 modified) An FM wave is given as

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\right),$$

where the message bandwidth is W and the maximum frequency deviation is $\Delta f_{\max}$. Consider a memoryless channel characterized by the following non-linear input-output relationship:

$$v_0(t) = a_1 s(t) + a_2 s^2(t) + a_3 s^3(t),$$

where $v_0(t)$ is the system output and $s(t)$ is the input.

(a) By using the generalized Carson's rule, show that if

$$f_c > 3\Delta f_{\max} + 2W,$$

the effect of the non-linear distortion can be removed by band-pass filtering. In other words, by applying $v_0(t)$ to a band-pass filter, the FM wave $s(t)$ can be recovered.

(b) How to design the pass-band of the filter in Part (a)?

Note: $\cos^2 x = \frac{1}{2}[1 + \cos(2x)]$. $\cos^3 x = \frac{1}{4}[3\cos(x) + \cos(3x)]$.