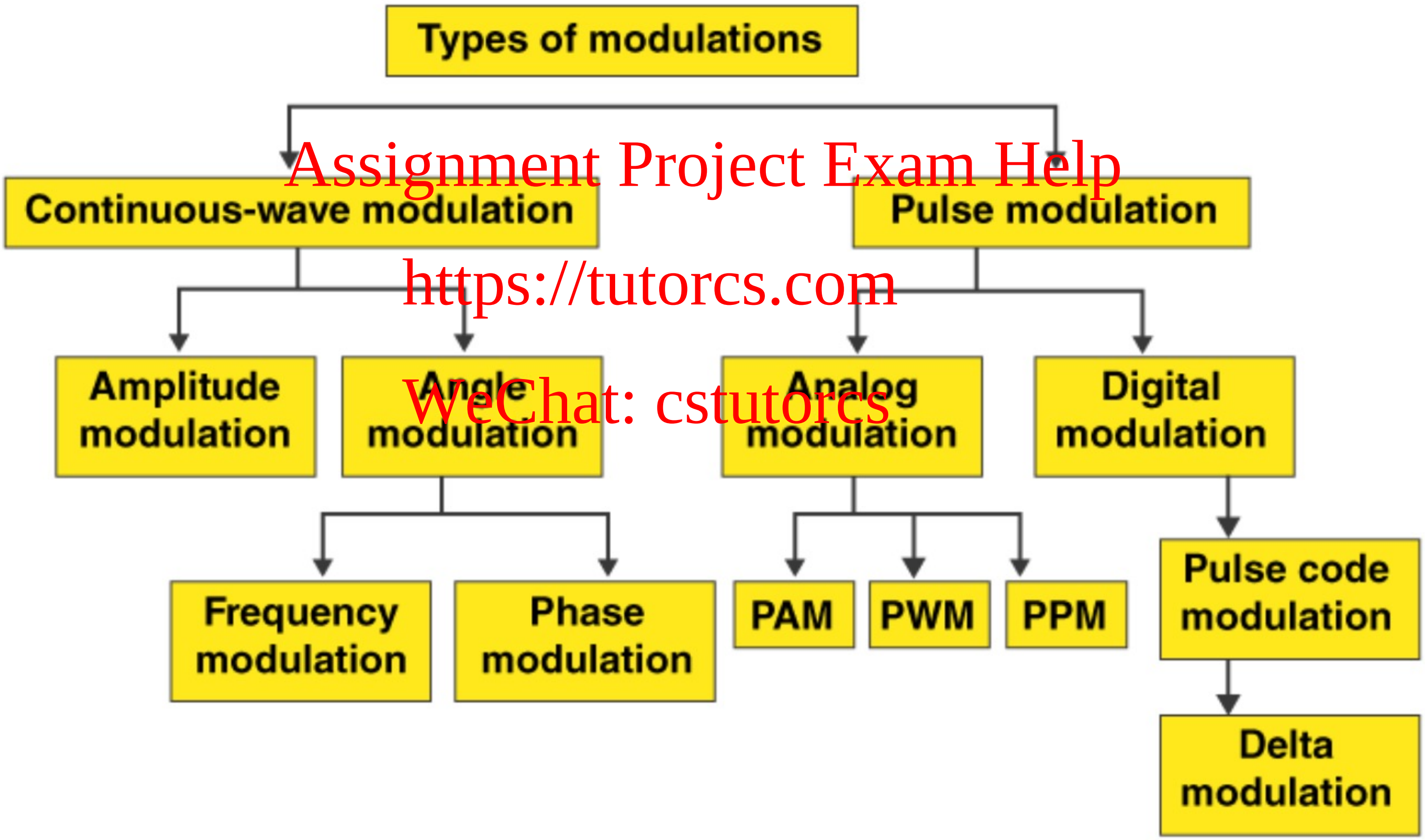
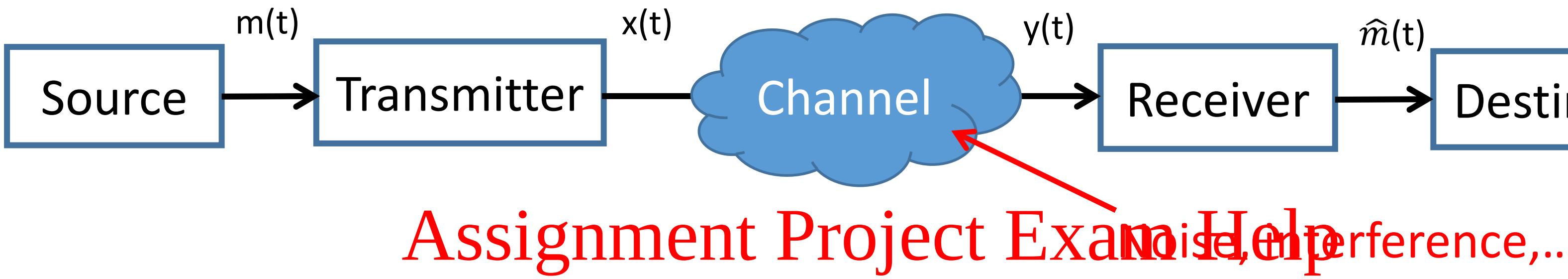
Chapter 5. Pulse Modulation

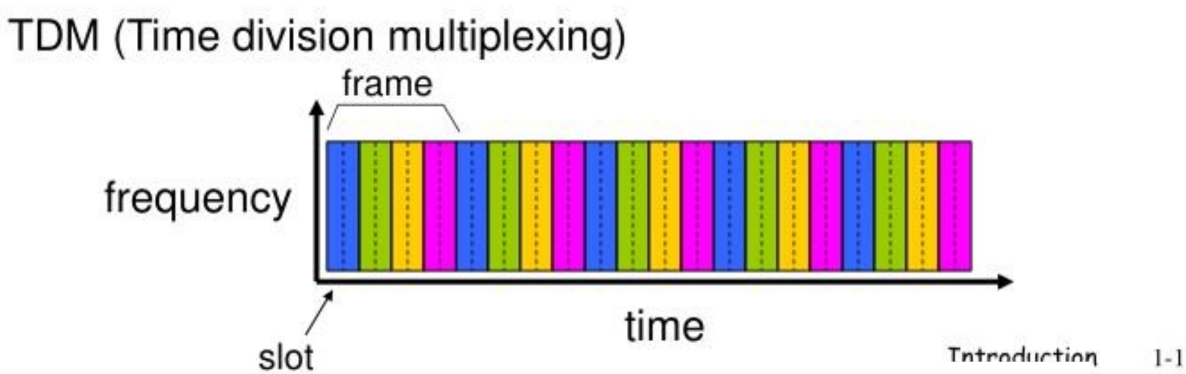


Why we need Pulse modulation?



Time-division multiplexing (TDM)

- Use pulse modulation to send messages in different time slices



Analog Pulse modulation: modulate discrete-time message (e.g., samples of signals) on a pulse train.

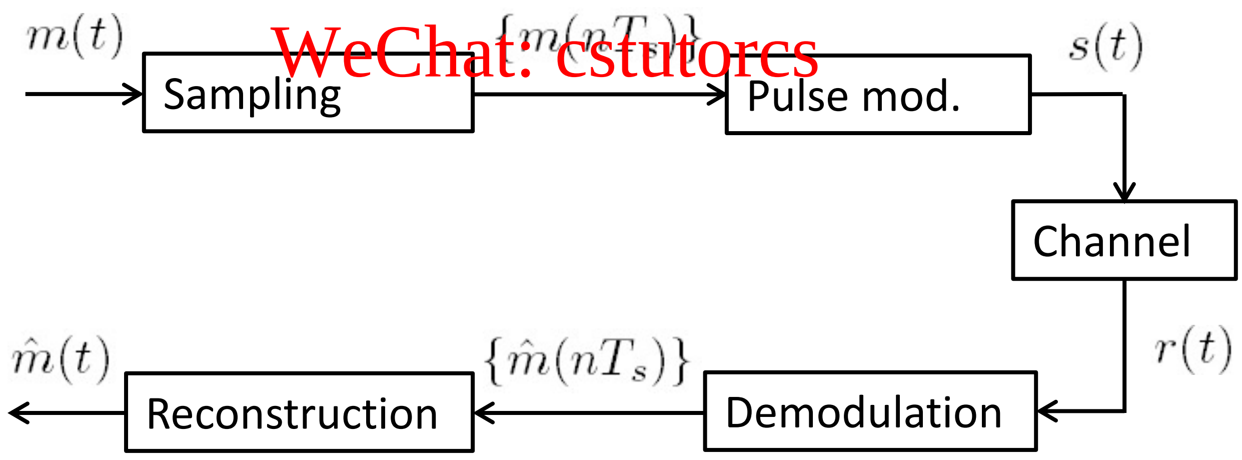
- Message signal is discrete-time and analog.
    - Sampling processing **(partial Haykin & Moher 5.1)**

- Some feature of each pulse (e.g., amplitude, duration) is varied in a continuous manner in accordance with the sample value of the message.
    - Pulse-Amplitude Modulation **(Haykin & Moher 5.2)**
    - Pulse-Position Modulation **(Haykin & Moher 5.4 and 5.3)**

- A bit more about TDM.

With sampling and reconstruction, the communications of a continuous-time signal is converted to the communications a discrete-time sequence of sample values.

Pulse modulation and demodulation: For communications of a discrete-time sequence of analog values via pulse train.



5.1 Sampling Process

- **Sampling process:** To covert a continuous-time signal into a discrete-time signal

  - to obtain a sequence of values (called samples) that are usually spaced uniformly in time.

- **Signal reconstruction:** To reconstruct the original continuous-time signal from its samples.

- Questions to be answered:
  - How to sample and reconstruct?
  - When is perfect reconstruction possible?
  - If not possible, how to control and analyze the error?

Instantaneous (ideal) sampling:

- Given continuous-time energy signal $g(t)$, sample the signal instantaneously at a uniform rate at every T_s seconds.

$$\boxed{g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s)}$$

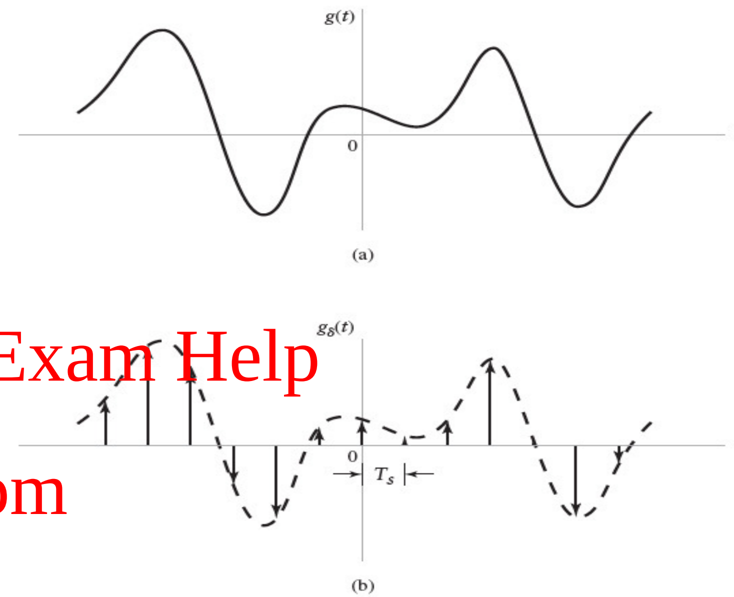


FIGURE 5.1 Illustration of the sampling process. (a) Analog waveform $g(t)$. (b) Instantaneously sampled representation of $g(t)$.

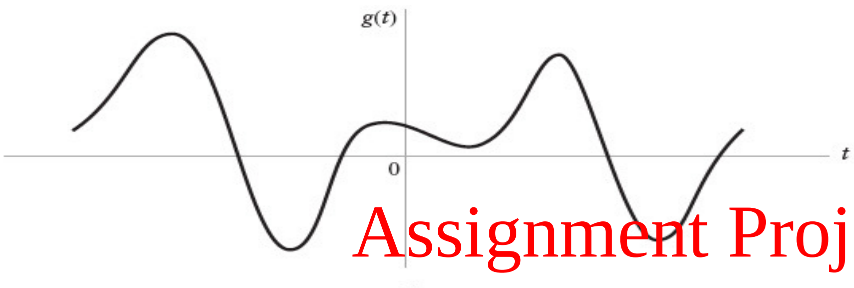
- Sampled sequence: $\{g(nT_s),\ n \in \mathbb{Z}\}$

$$\cdots, g(-2T_s), g(-T_s), g(0), g(T_s), g(2T_s), \cdots$$

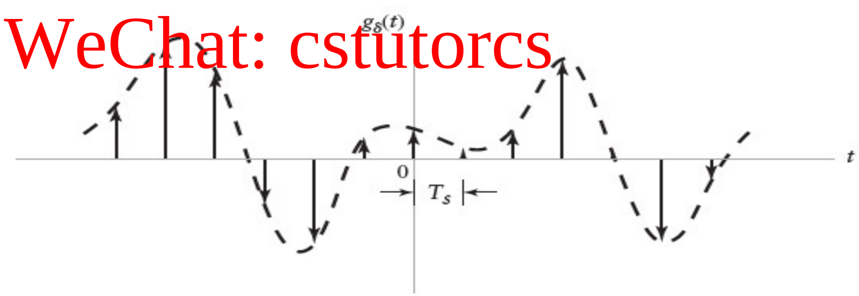
- **Sampling period/ sampling interval:** T_s
- **Sampling rate (**# of samples per second): $f_s = 1/T_s$

Instantaneous (ideal) sampling – Time domain

$$\delta_{T_s}(t) = \sum_{n=-\propto}^{\propto} \delta(t - nT_s)$$



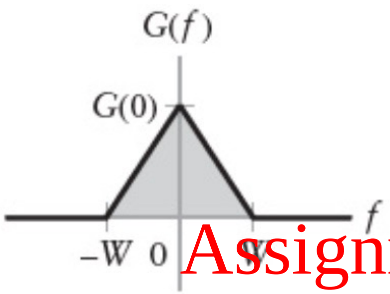
Multiplication in time domain



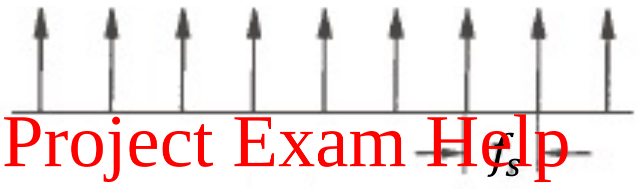
$$g_\delta(t) = g(t) \times \delta_{T_s}(t) = \mathrm{g(t)} \times \textstyle\sum_{n=-\propto}^{\propto} \delta(t - nT_s)$$

$$= \sum_{n=-\propto}^{\propto} g(t)\delta(t - nT_s) = \sum_{n=-\propto}^{\propto} g(nT_s)\delta(t - nT_s)$$

Instantaneous (ideal) sampling – Frequency domain



$\delta_{T_s}(f) = f_s \sum_{n=-\propto}^{\propto} \delta(f - nf_s), \;\; f_s = 1/T_s$



Convolution in frequency domain

$$G_\delta(f) = G(f) * \delta_{T_s}(f) = \mathrm{G(f)}* f_s \sum_{n=-\propto}^{\propto} \delta(f - nf_s)$$

$$= f_s \sum_{n=-\propto}^{\propto} G(f) * \delta(f - nf_s) = f_s \sum_{n=-\propto}^{\propto} G(f - nf_s)$$

Aliasing phenomenon

Aliasing will be produced by the sampling process, where a high-frequency component of the signal seemingly takes on the identity of a lower frequency in the spectrum of its sampled version.

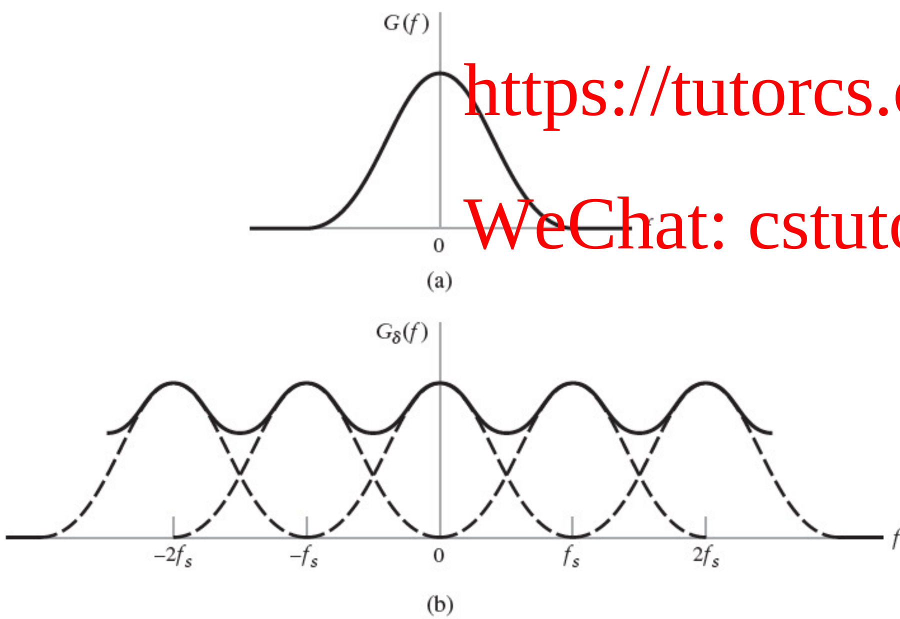


Fig. 5.3. (a) Spectrum of a signal. (b) Spectrum of an under-sampled version of the signal, exhibiting the aliasing phenomenon.

Solution:

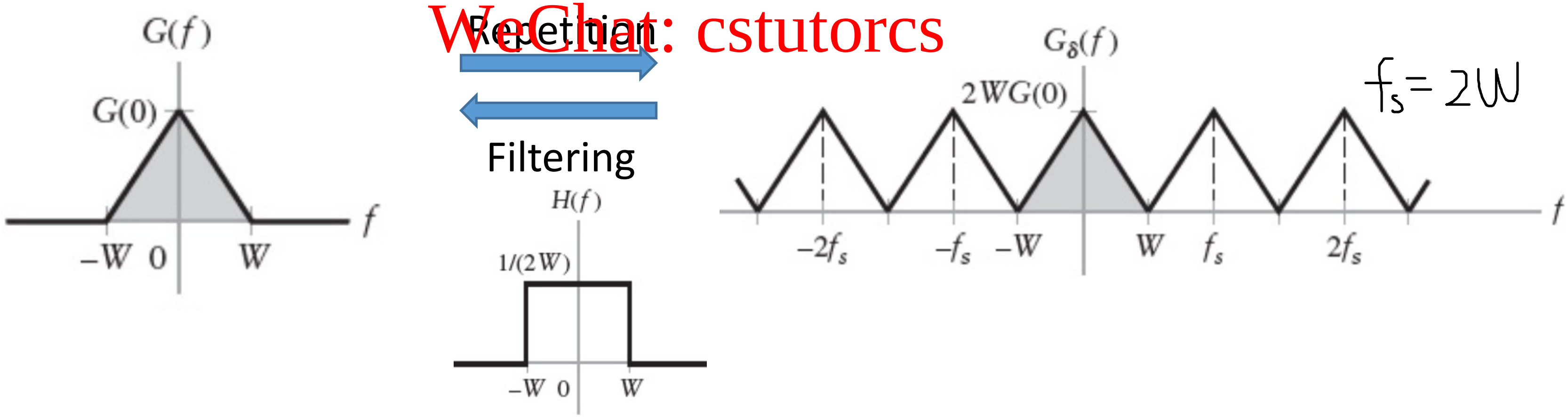
- Use anti-alias filter with cut-off frequency f_c before sampling, s.t. $f_c < f_s/2$
- Increase sample rate T_s, s.t. $T_s < 1/(2W)$

Reconstruct scheme: recover the signal $g(t)$ from the sampled sequence $\{g(nT_s),\ n \in \mathbb{Z}\}$.

<u>In frequency domain – low-pass filtering</u>:

$$G_\delta(f) = f_s \sum_{m=-\infty}^{\infty} G(f - mf_s) = f_s G(f) + f_s \sum_{m=-\infty, m\neq 0}^{\infty} G(f - mf_s).$$

- When $T_s \le 1/(2W)$, $G_\delta(f) = f_s G(f) \quad \text{for } -W \le f \le W$.



$$G(f) = G_\delta(f) \times \frac{1}{f_s} rect(\frac{f}{2W})$$

Reconstruct scheme: recover the signal $g(t)$ from the sampled sequence $\{g(nT_s),\ n \in \mathbb{Z}\}$

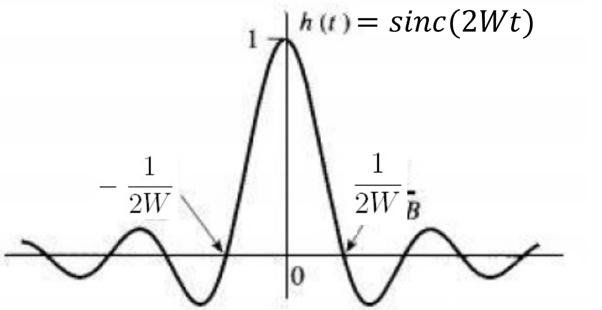
<u>In time domain - interpolation</u>

$$g(t) = g_\delta(t) * \frac{1}{f_s} 2W sinc(2Wt)$$

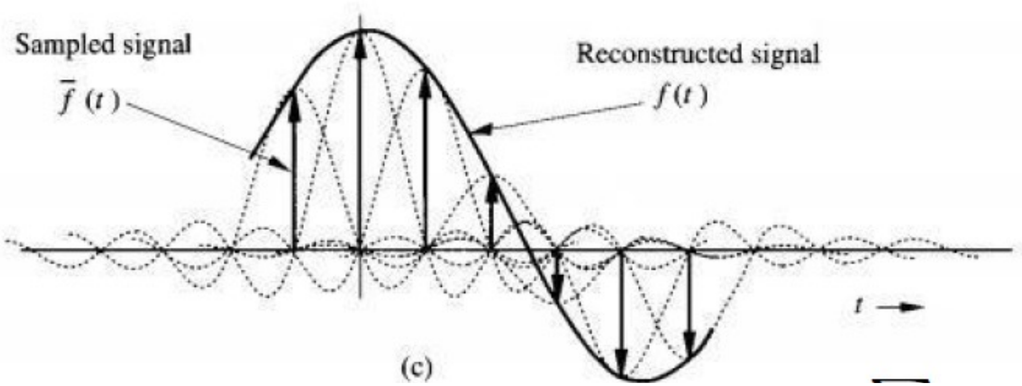
$$= \sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s) * \frac{1}{f_s} 2W sinc(2Wt)$$

$$= \sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s) * \frac{1}{f_s} 2W sinc(2Wt)]$$

$$= \frac{2W}{f_s} \sum_{n=-\infty}^{\infty} g(nT_s) sinc[2W(t - nT_s)]$$



- The sinc-function is the **interpolation function**.



Nyquist's Sampling Theorem:

Consider a baseband signal $g(t)$ bandlimited to W, i.e. $G(f) = 0, |f| > W$. $g(t)$ can be uniquely recovered from its samples $f(nT_s), n = -\infty, \ldots, \infty$, if $T_s < \frac{1}{2W}$ or $f_s > 2W$.

- For a signal with bandwidth W, the sampling rate of $2W$ to allow perfect reconstruction is called **the Nyquist rate**.

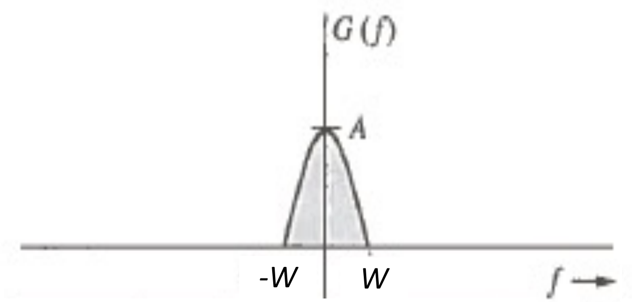
Practical Sampling and Reconstruction:

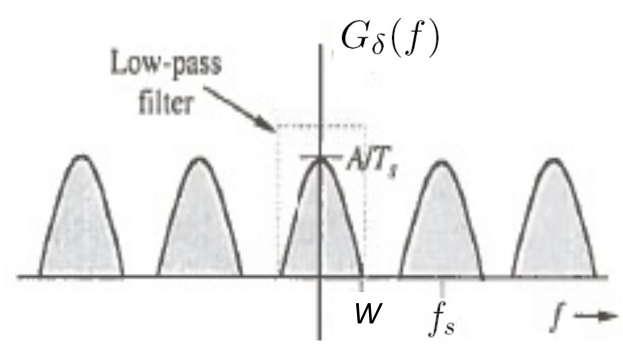
- Anti-aliasing filter $g(t) \rightarrow$ LPF ($f_s/2$) $\rightarrow g_{f_s}(t)$

- Narrow pulses instead of impulses

- Sampling faster than the minimum rate (due to non-existence of ideal LPF)

Guard band: the gap between adjacent pulses in $G_\delta(f)$. It equals $\frac{f_s}{2} - W$.





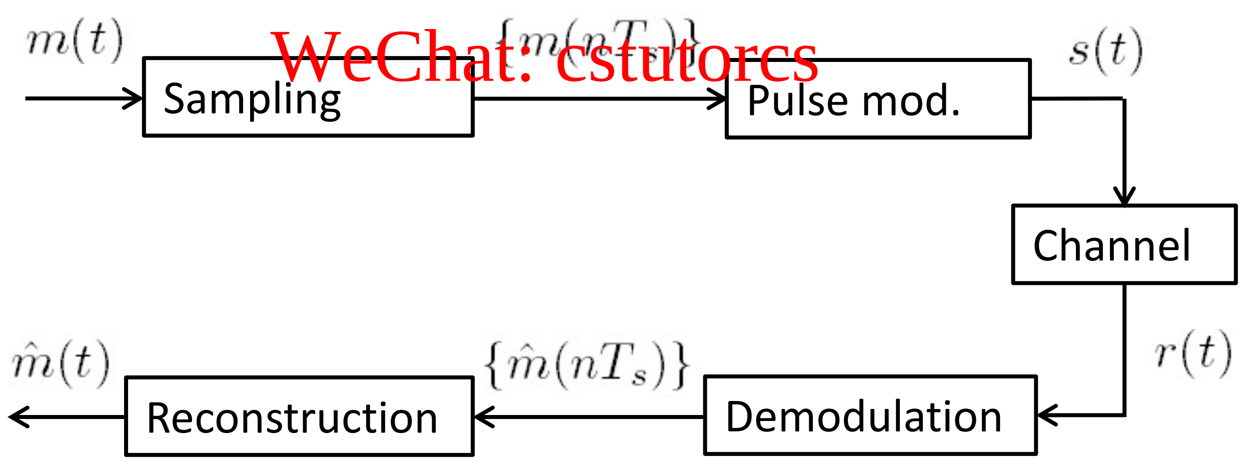
Example: A bandlimited signal has a bandwidth 3400Hz.

(a) What is the Nyquist rate for this signals?

(b) If a guard band of 600Hz is desired, what should the sampling rate be?

With sampling and reconstruction, the communications of a continuous-time signal is converted to the communications a discrete-time sequence of sample values.

Pulse modulation and demodulation: For communications of a discrete-time sequence of analog (or digital) values via pulse train.



5.2 Pulse-Amplitude Modulation

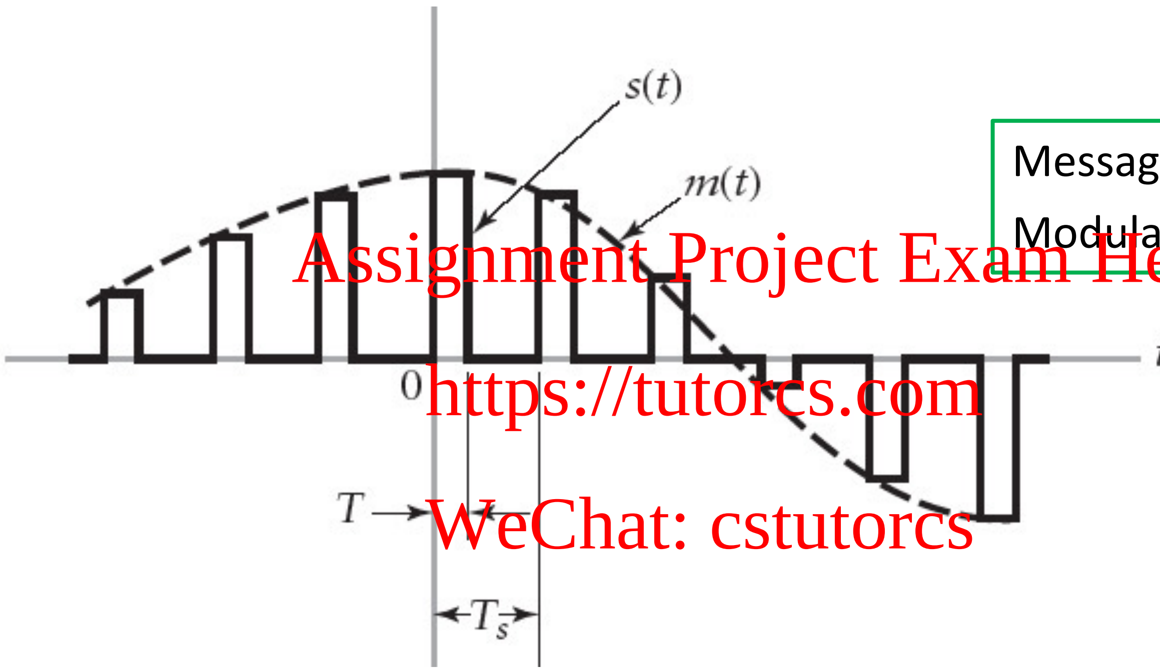
For communications of discrete-time sequence of analog (or digital) values via pulse train.

Pulse-amplitude modulation (PAM): The amplitudes of pulses are varied in proportional to the discrete-time sample values of a continuous-time message.

- Information is in the amplitudes of pulses.
- The pulses can be rectangular ones or others.

Demodulation: Obtain the pulse amplitudes, rescale to get the sample values, reconstruct the message.

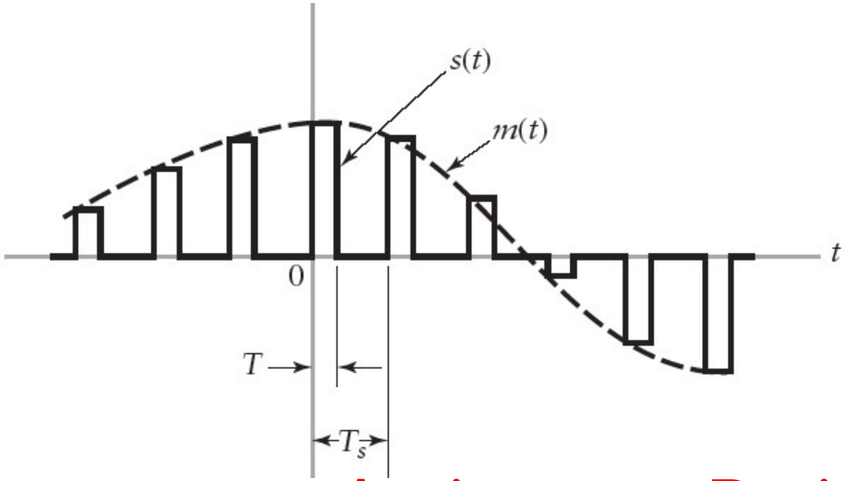
- Combine PAM with rectangular pulses and the sampling process. The process can be seen as **sample-and-hold**.

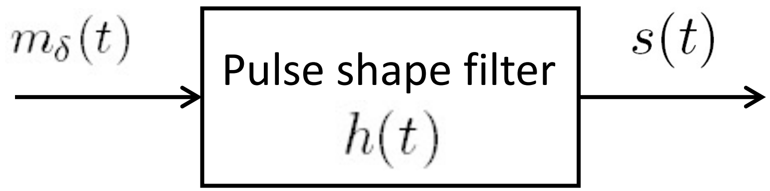


Message: $m(t) \rightleftharpoons M(f)$

Modulated signal: $s(t) \rightleftharpoons S(f)$

1. Sample the message signal $m(t)$ every T_s seconds.
2. Lengthening the duration of each sample (hold the value) for T seconds to generate $s(t)$.





$$h(t) = \mathrm{rect}\left(\frac{t - T/2}{T}\right)$$

$$H(f) = T\,\mathrm{sinc}(fT)\exp(-j\pi fT).$$

The PAM signal $s(t)$ is mathematically equivalent to the convolution of $m_\delta(t)$, the instantaneously sampled version of the message $m(t)$, and the pulse shape function $h(t)$

The sampling and modulation process can be represented as passing the instantaneously sampled signal through a filter whose frequency response represents the pulse shape.

$$s(t) = m_\delta(t) \star h(t). \rightleftharpoons \quad S(f) = M_\delta(f)H(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)H(f).$$

5.3 Pulse-Position Modulation

- **Pulse-duration modulation (PDM):** The duration (length) of pulses are varied in proportional to the discrete-time sample values of a continuous-time message.
- **Pulse-position modulation (PPM):** The position (time of occurrence) of pulses are varied in proportional to the discrete-time sample values of a continuous-time message.

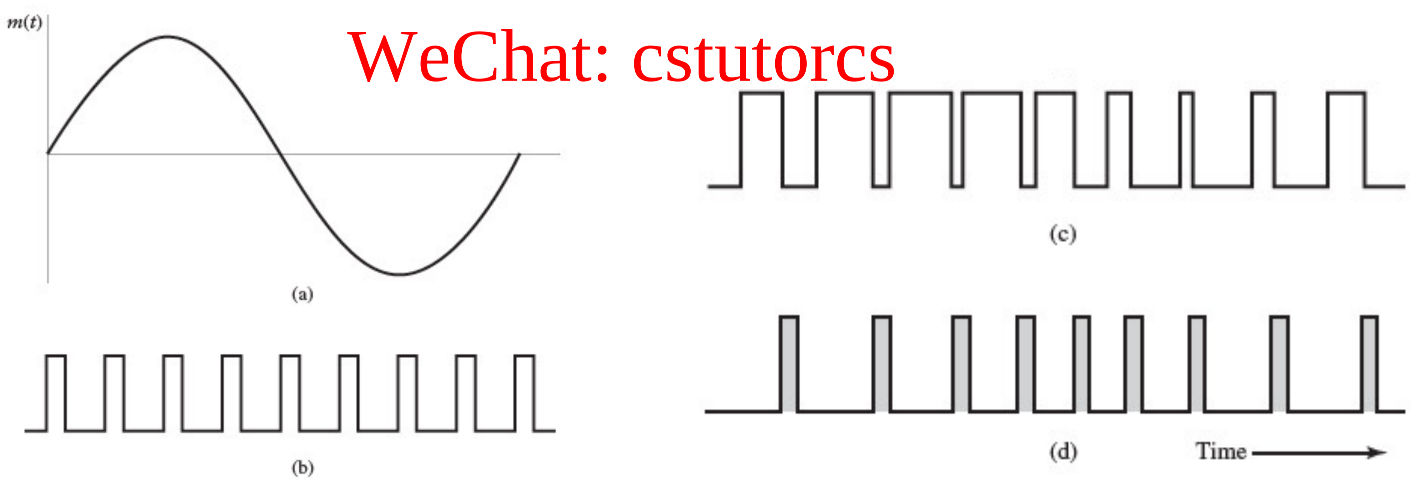


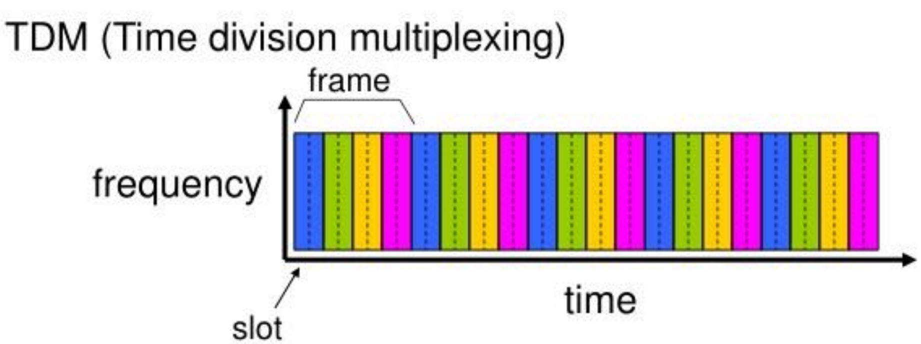
FIGURE 5.8 Illustration of two different forms of pulse-time modulation for the case of a sinusoidal modulating wave. (*a*) Modulating wave. (*b*) Pulse carrier. (*c*) PDM wave. (*d*) PPM wave.

5.4 Time-Division Multiplexing (TDM)

Sampling brings conservation of time:

- Transmission of continuous-time signal becomes transmission of samples at discrete-time instances, which engages the channel for only a fraction of the sampling interval.
- Some of the time interval between adjacent samples is cleared for use by other independent messages.

TDM: Time-shared to enable multiple messages to use a common channel without interference.



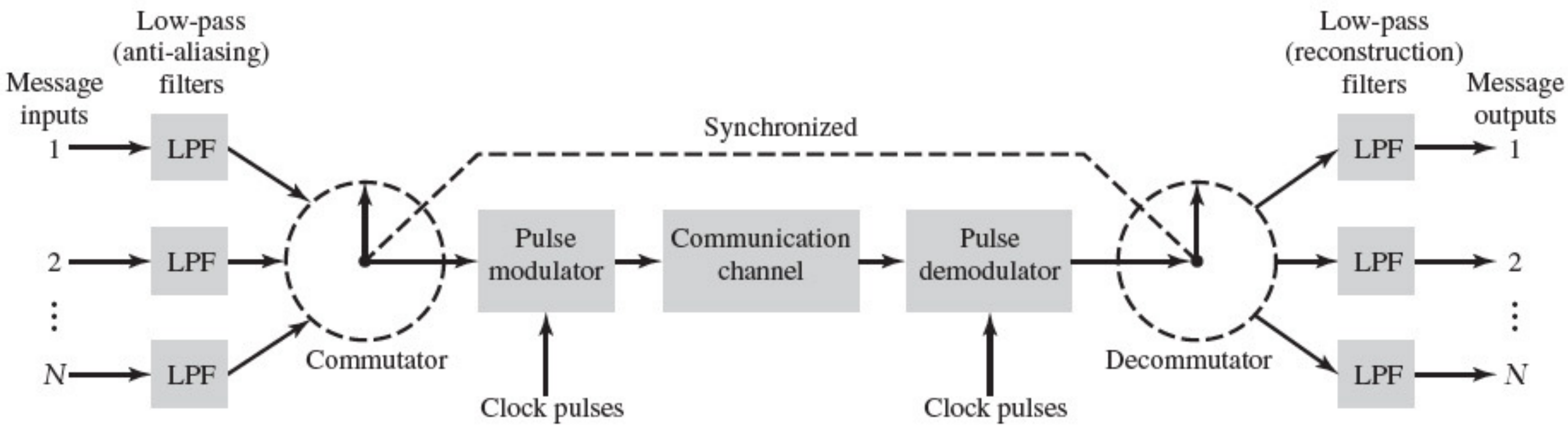


FIGURE 5.21 Block diagram of TDM system.

LPFs on the left: Restrict the bandwidths of messages.

Commutator: Electronic switch 1) to take samples of the messages and 2) to sequentially interleave the N samples from N different messages within a sampling period.

Pulse modulator: To transform the multiplexed signals into a form suitable for transmission.

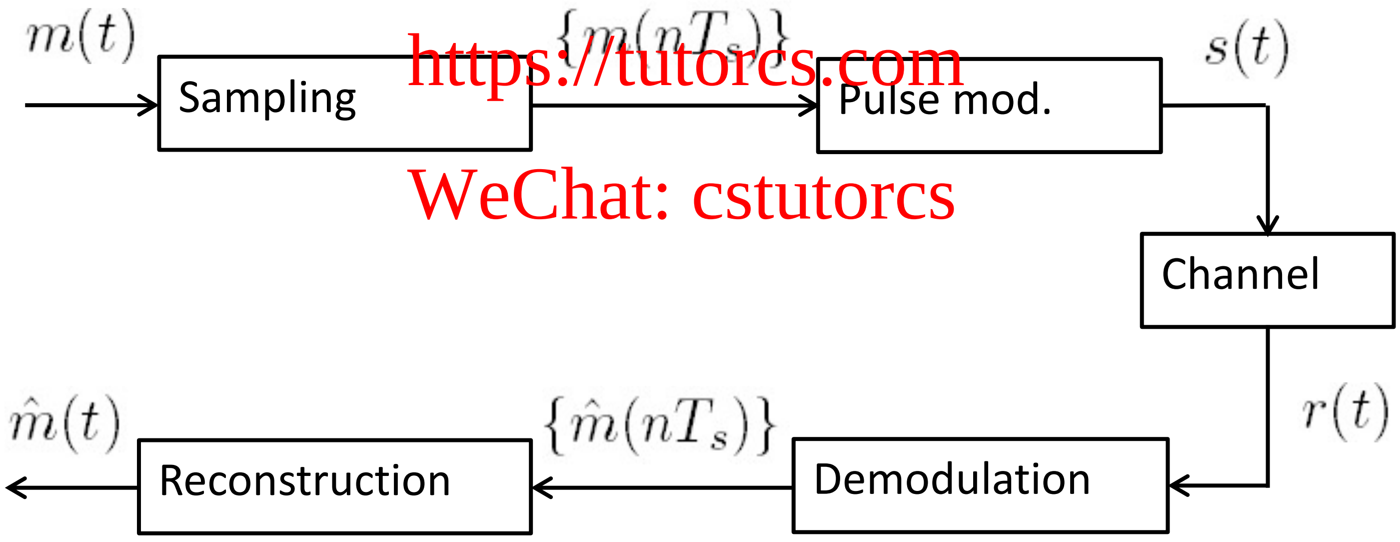
Pulse demodulator: inverse of pulse modulator.

Decommutator: distribute samples/signals to theirs right destinations (synchronized with commutator).

LPFs on the right: reconstruct messages from their samples.

5.5 Quantization: Transition from Analog to Digital

Pulse modulation and demodulation: For communications of a discrete-time sequence of analog (or digital) values via pulse train.



Quantization Process

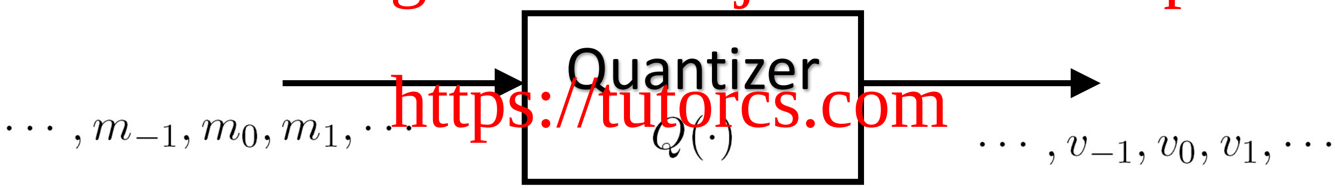
From analog values to digital values.

Message	Communication schemes	
Continuous-time analog	AM	FM/PM
Sampling process (Nyquist's sampling theorem)		
Discrete-time analog (sequence of analog values)	Analog PAM	Analog PPM/PDM
Quantization process		
Discrete-time digital (sequence of digital values)	Digital pulse modulation	

Quantization: convert analog values to digital values with finite possibilities.

- An approximation/rounding process.

Quantizer: The system that performs the quantization process.



$m_i's$: Sampled analog values

$v_i's$: Quantized digital values

Memoryless quantization: The quantization of each sample is not affected by other samples. $\boxed{v = Q(m)}$

Criterion of quantization performance: *distortion/error* between the quantized values and the original analog values.

Mean squared error (MSE) as distortion measure

- If m is the sampled analog value and $v = Q(m)$ is the quantized value, the squared error is

$$e = [m - Q(m)]^2 = (m - v)^2.$$

- The sampled value is not fixed and is usually modeled as a random variable M, following some distribution $f_M(m)$.

- Consider the expected value of the squared error:

$$\boxed{\mathrm{MSE} = \mathrm{E}\{[M - Q(M)]^2\} = \int_{-\infty}^{\infty} [m - Q(m)]^2 f_M(m)dm,}$$

where $f_M(m)$ is the probability density function (PDF) of M.

Signal-to-quantization-noise-ratio (SQNR): signal power over the quantization noise/error power (MSE):

$$\mathrm{SQNR} = \frac{\mathrm{E}[M^2]}{\mathrm{E}\{[M - Q(M)]^2\}}$$

Where $E[M^2] = \int_{\infty}^{\infty} m^2 f_m(m)dm$

$$E\{[M - Q(M)]^2\} = \int_{\infty}^{\infty} [m - Q(m)]^2 f_m(m)dm$$

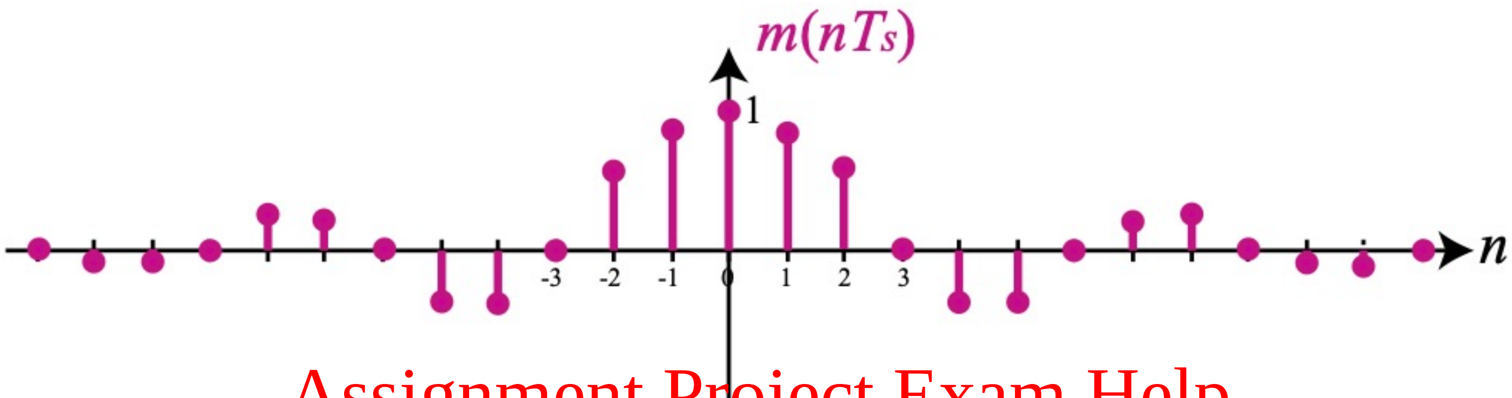
Memoryless quantization (N-level).

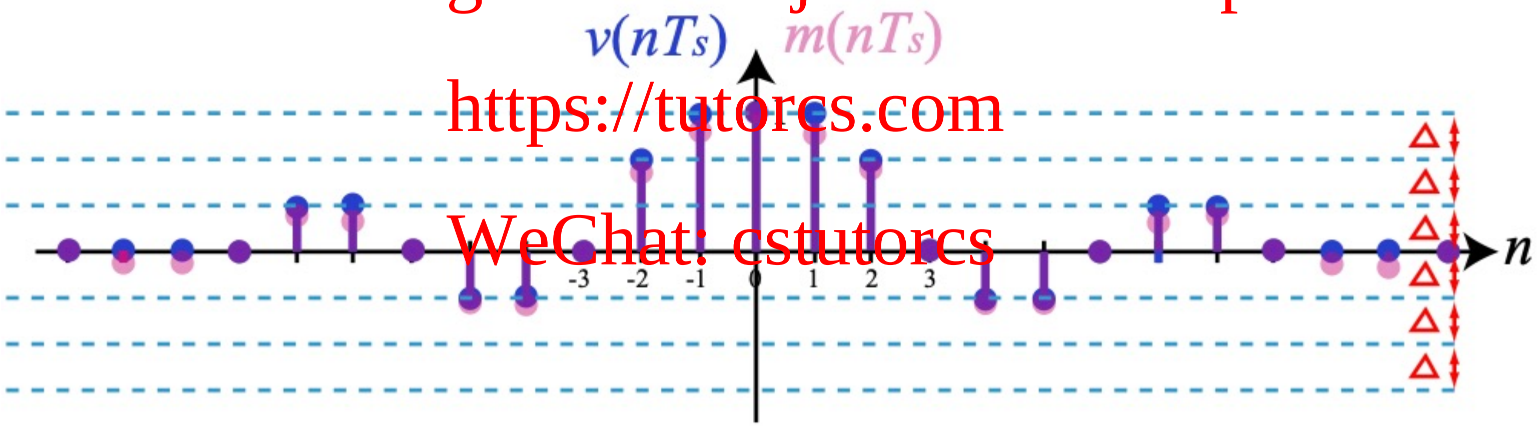
- The set of real numbers $\mathbb{R}$ is partitioned into N disjoint subsets, denoted by $\mathcal{R}_1, \cdots, \mathcal{R}_N$. Each subset is called a **quantization region** defined by its boundaries a_i and a_{i+1}.
- Corresponding to each quantization region $\mathcal{R}_k$, a representation point v_k called **quantization level/value** is chosen.

- If the sampled signal m, belongs to $\mathcal{R}_k$, then it is represented by v_k, i.e., $Q(m) = v_k$.

Thus, v_k is the quantized version of m.

The quantizer design problem is the design of both the quantization regions and the quantization levels to achieve the lowest distortion level.





. . . result of rounding to the nearest quantization level.

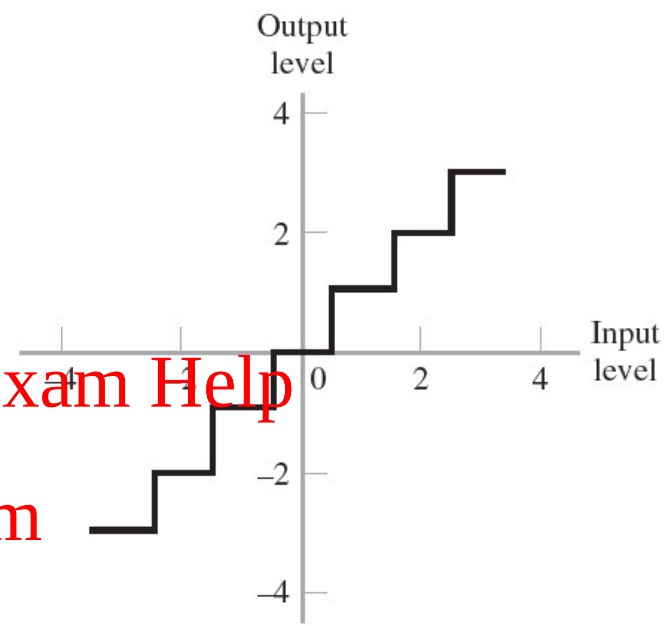
Uniform quantization:

- Except for the two end ones, the quantization regions have equal length.

$$\Delta = a_{i+1} - a_i, \text{ for } i = 1, \cdots, N-1$$

- Except for the two end ones, the quantization level for each region is the mid-point of each region.

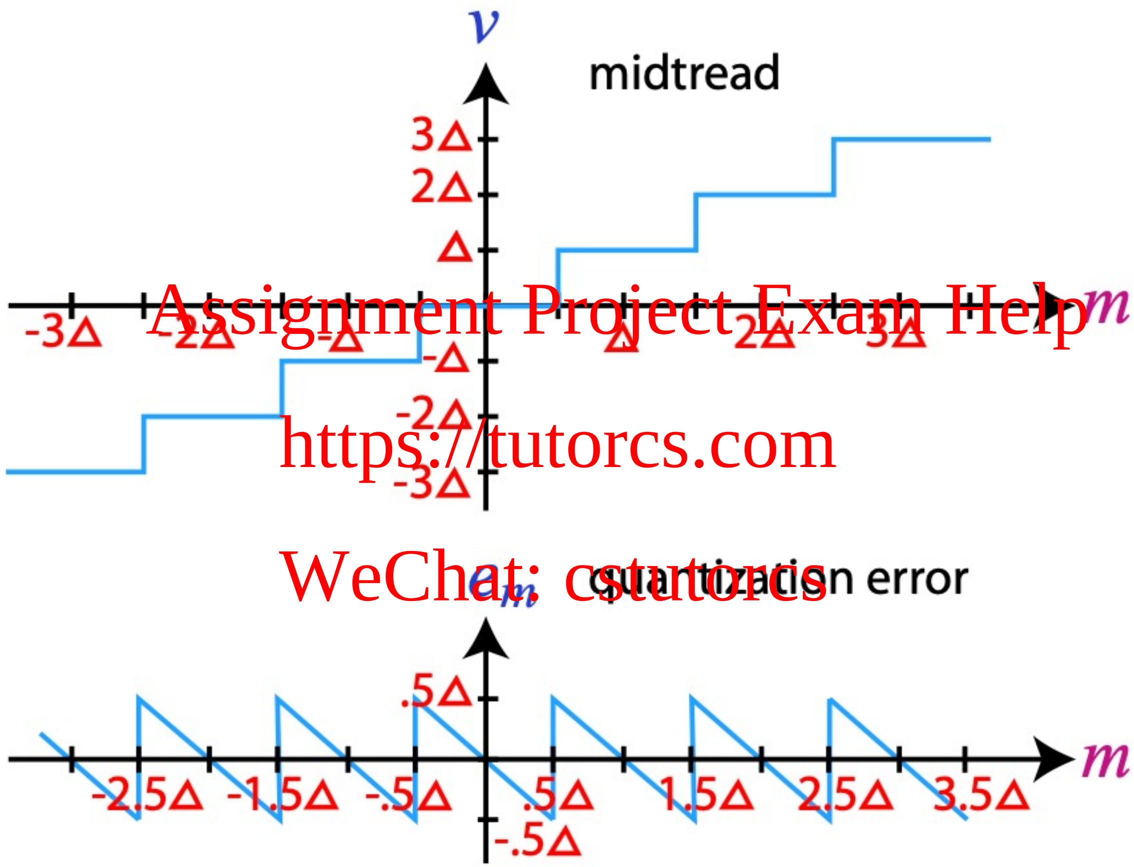
$$v_i = \frac{a_{i-1} + a_i}{2} = a_{i-1} + \frac{\Delta}{2} = a_i - \frac{\Delta}{2}, \text{ for } i = 2, \cdots, N-1.$$



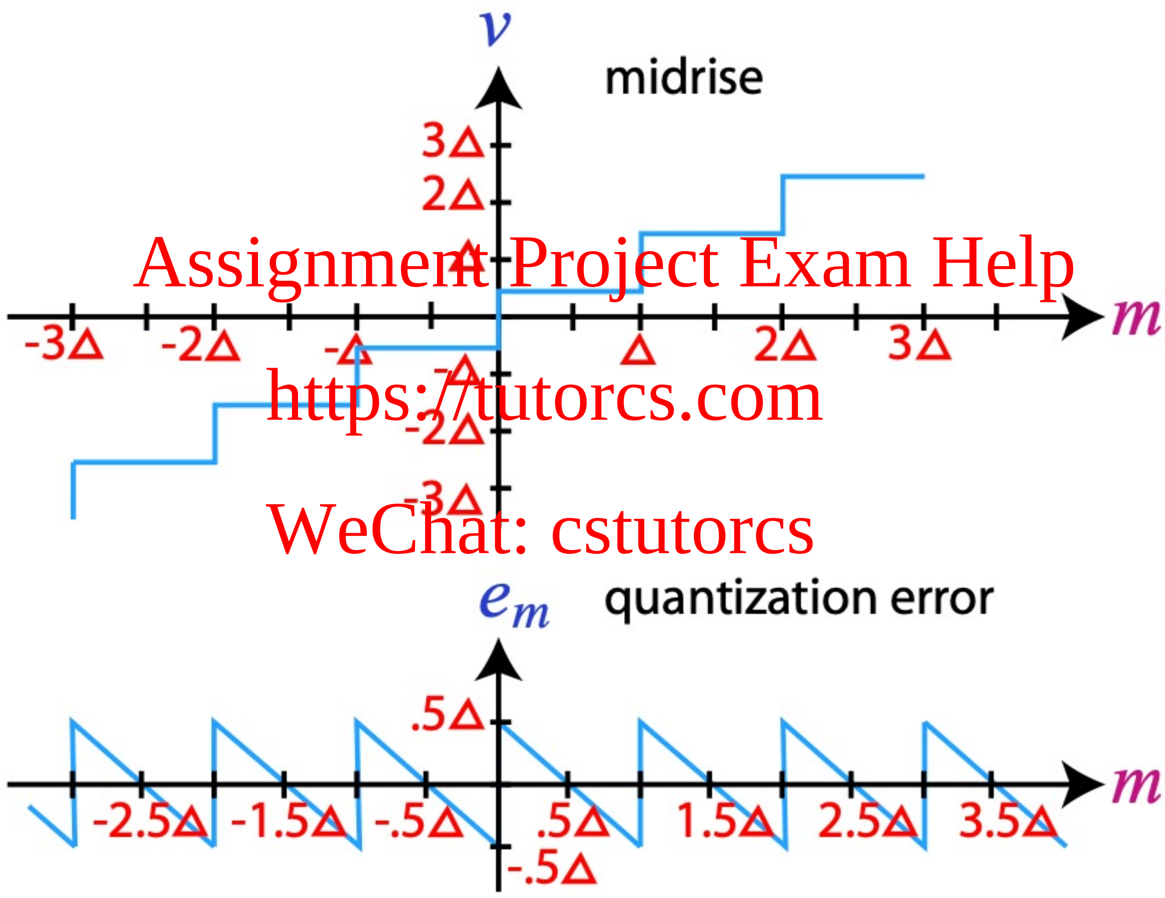
Midtread uniform quantization

> Uniform quantizers are the simplest, but generally speaking, is not the best.

Uniform quantization:



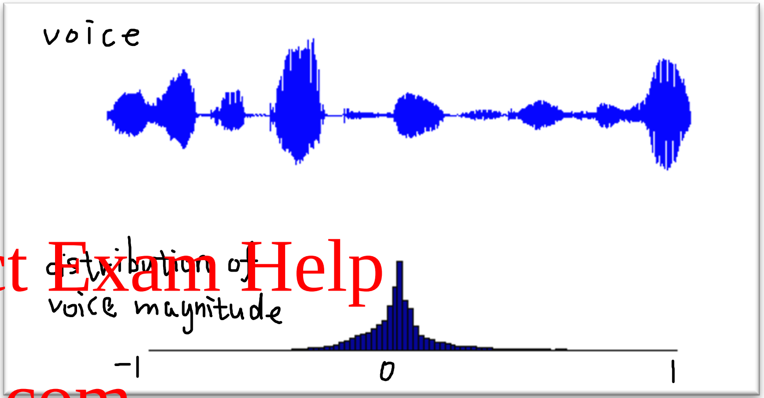
Uniform quantization:



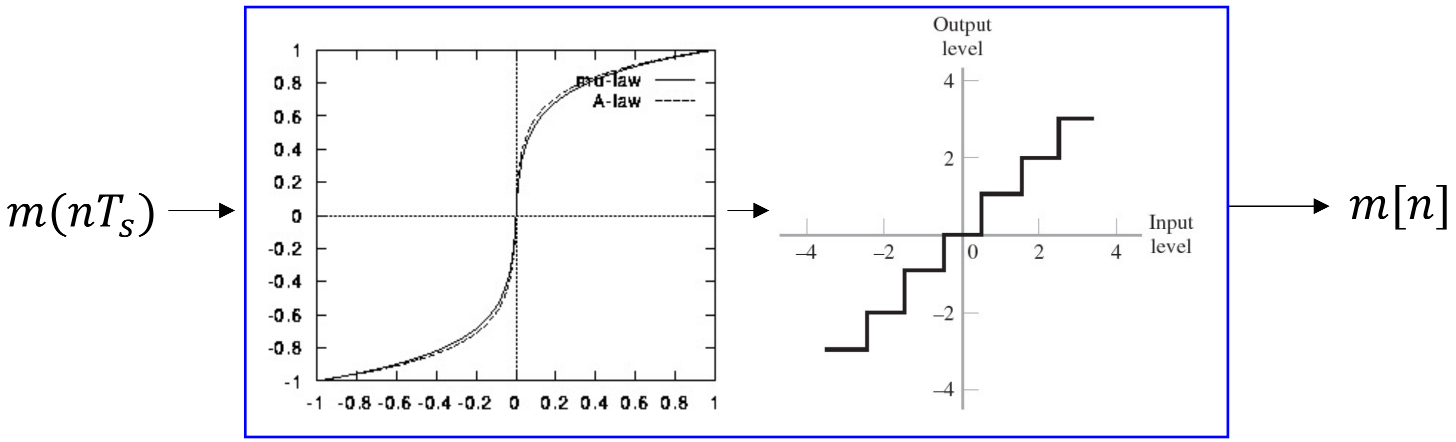
Companding law: Nonuniform quantization of voice signal

Variable quantization step for high-fidelity voice signals.

- Most voice signals have smaller magnitudes
- Minimizing quantization error by small quantization steps for smaller magnitude.



The figure is from http://www.seas.ucla.edu/dsplab/sqc/over.html.



Pulse-Code Modulation (PCM): a method used to digitally represent sampled analog signals.

- Standard form of digital audio in computers, digital telephony, and other digital audio communication applications.

Coverage:

5.6 Pulse-Code Modulation: **(Haykin & Moher 5.6)**

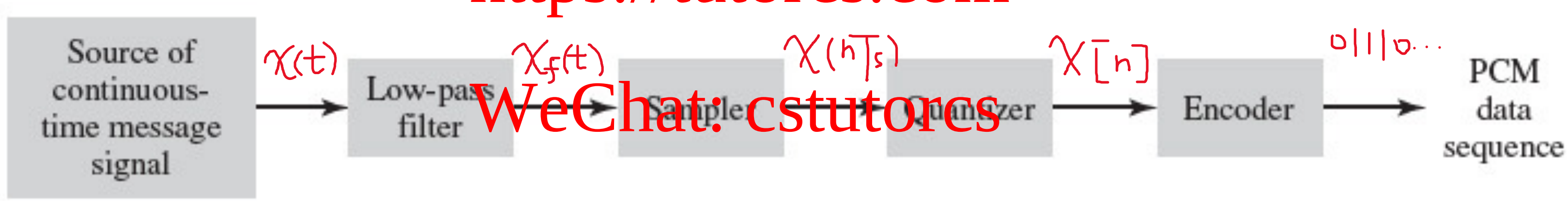
5.8 Differential Pulse-Code Modulation **(Haykin & Moher 5.8)**

5.7 Delta Modulation **(Haykin & Moher 5.7)**

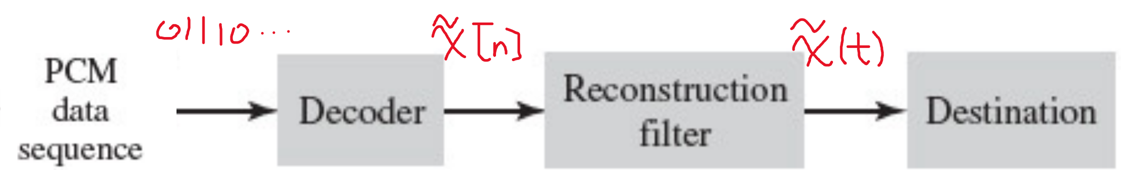
5.9 Linear Codes **(Haykin & Moher 5.9)**

Pulse-Code Modulation: To represent a message signal in discrete form in both time and amplitude, by a sequence of coded pulses.

Encoding: After quantization, the sample values are digital, but not in the form best suited for transmission. Thus, encoding is used to translate the digital sample values to a more appropriate form, usually binary sequences.



Decoding: the opposite of encoding.



Encoding: to translate the digital sample values to binary sequences.

- R bits can represent 2^R levels of amplitudes (possible values).
- To represent a quantized value with L levels, need $log_2 L$ bits.
- Bit_rate = sample_rate*R

Many ways to encode digital values into bit sequence.

- ***Natural coding:*** to express the ordinal number of the representation level as a binary number.

Ordinal number of representation level	0	1	2	3
Binary number	00	01	10	11

- **Gray code**: two adjacent values differ in only one bit.

Ordinal number of representation level	0	1	2	3
Binary number	00	01	11	10

- **Huffman coding**: an adaptive coding method based on source statistics.

Example: Consider a CD that uses PCM to record audio signals with bandwidth $W = 15KHz$. The PCM system uses the Nyquist sampling rate and 512-level uniform quantization for signal representation. Please find

(a) The Nyquist rate;

(b) The minimum bit rate and maximum permitted bit duration.

5.7 Differential Pulse-Code Modulation (DPCM)

Motivation: Since $m(nT_s)$ and $m((n+1)T_s)$ are usually close to each other, it is more efficient to transmit the difference.

$$e(nT_s) = m(nT_s) - \hat{m}(nT_s).$$

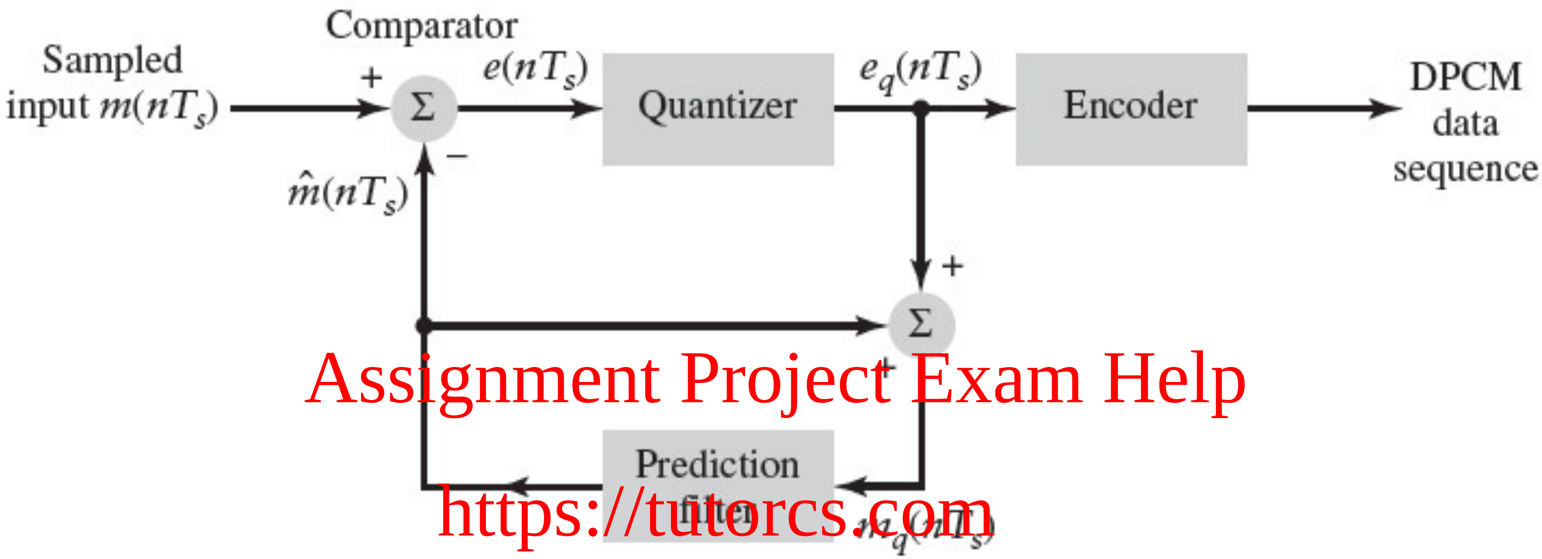
Strategy:

- Prediction/inference about the current or future value of a signal based on its past values
- Quantize and encode the difference $e(nT_s)$

DPCM output: Bit sequence representing $e(nT_s)$.

Advantages: Less redundancy, higher efficiency.

Differential PCM transmitter:



Quantization error: $q(nT_s) = e_q(nT_s) - e(nT_s).$

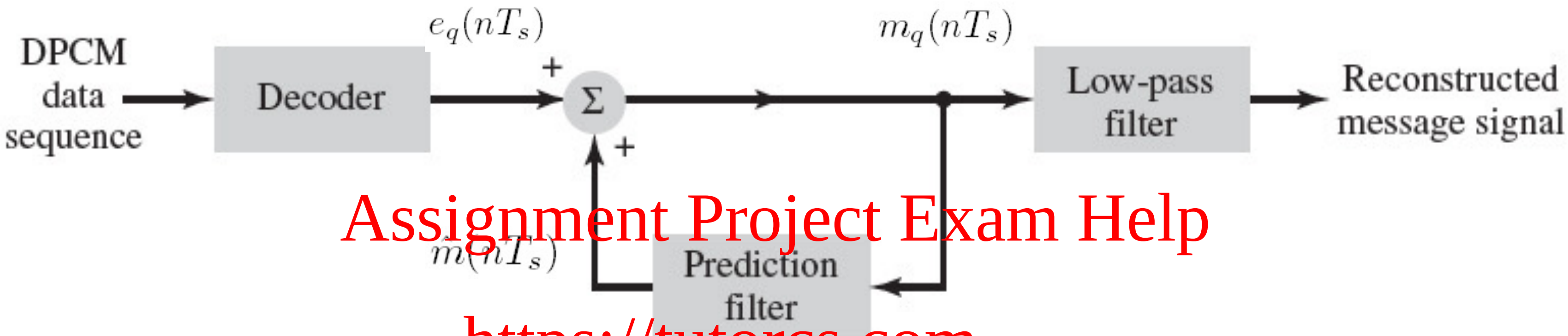
Prediction filter input:

$$m_q(nT_s) = \hat{m}(nT_s) + e_q(nT_s) = \hat{m}(nT_s) + e(nT_s) + q(nT_s).$$

Thus, $m_q(nT_s) = m(nT_s) + q(nT_s).$

- Irrespective of the prediction filter design, the quantized signal $m_q(nT_s)$ differs from the message samples by the quantization error.

Differential PCM receiver:



$$m_q(nT_s) = \hat{m}(nT_s) + e_q(nT_s)$$

- Message reconstructed from quantized samples via the low-pass reconstruction filter.
- If no channel noise and proper sampling and reconstruction design, the only distortion comes from the quantization error.

5.8 Delta Modulation (DM)

DM is a special case of DPCM with two specific designs:

- Use a zero-order prediction (single delay) as the prediction filter:

$$\hat{m}(nT_s) = m_q((n-1)T_s).$$

$$e(nT_s) = m(nT_s) - m_q((n-1)T_s)$$

- Use 1-bit (2-level) quantizer $e(nT_s)$ in DM.

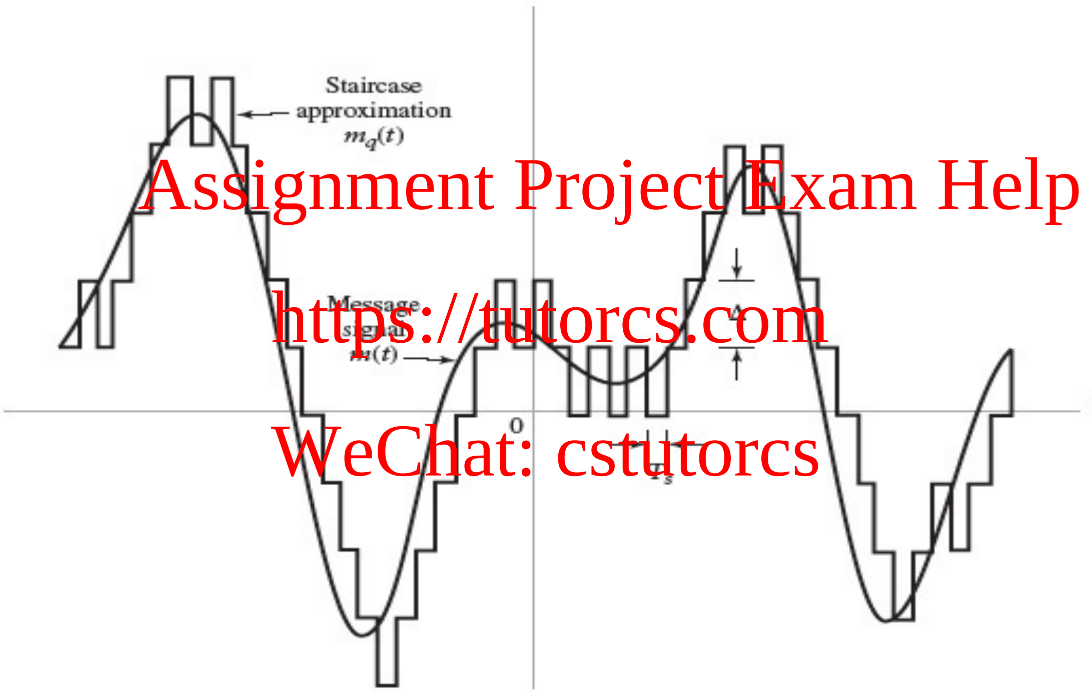
$$Q[e(nT_s)] = \begin{cases} 1 & if\ e(nT_s) > 0 \\ 0 & if\ s(nT_s) \leq 0 \end{cases}$$

Summary of DM: Quantize the difference between the sample values and their approximations into two levels, corresponding to positive and negative differences.

- DM is a simplified system, less costly in implementation compared to PCM.

DM provides a stair case approximation of the message.

$$m_q(nT_s) = m_q\big((n-1)T_s\big) + Q[e(nT_s)]\times\Delta$$



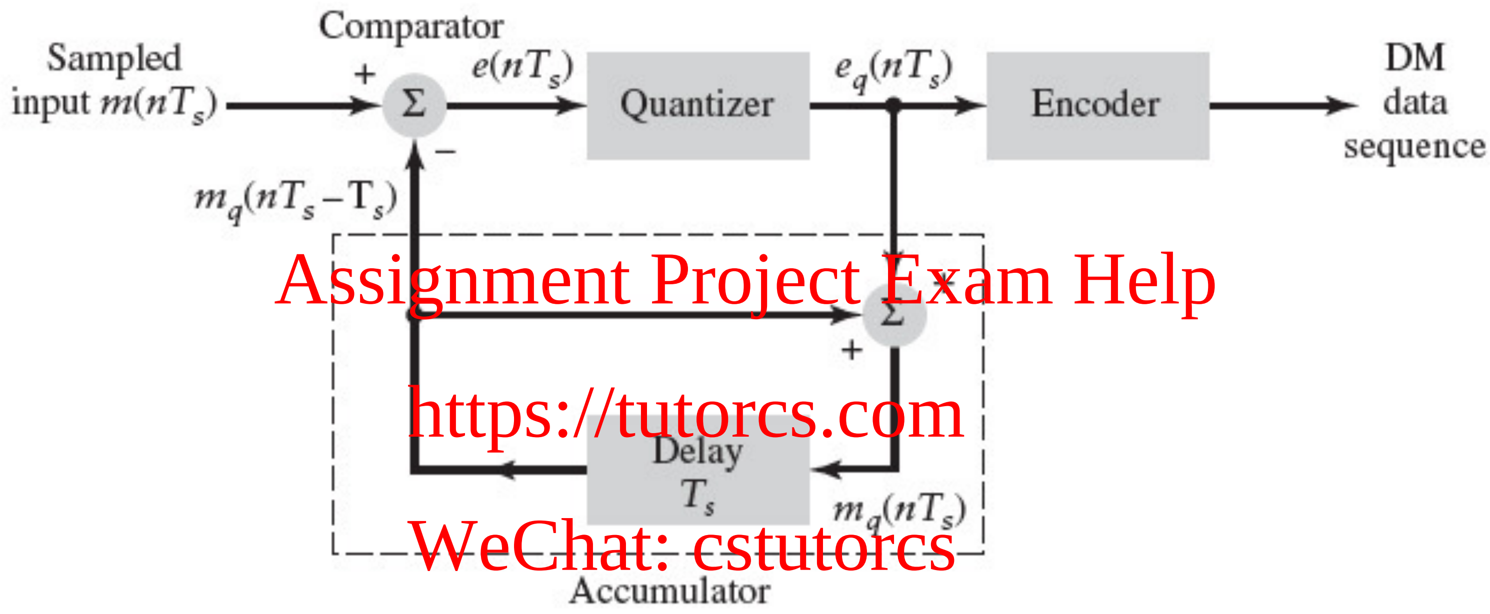
(a)

Binary sequence at modulator output 101111010000000001111110100101011110100000000110111

(b)

FIGURE 5.14 Illustration of delta modulation. (*a*) Analog waveform $m(t)$ and its staircase approximation $m_q(t)$. (*b*) Binary sequence at the modulator output.

Transmitter of DM system:



$$m_q(nT_s) = m_q((n-1)T_s) + e_q(nT_s)$$

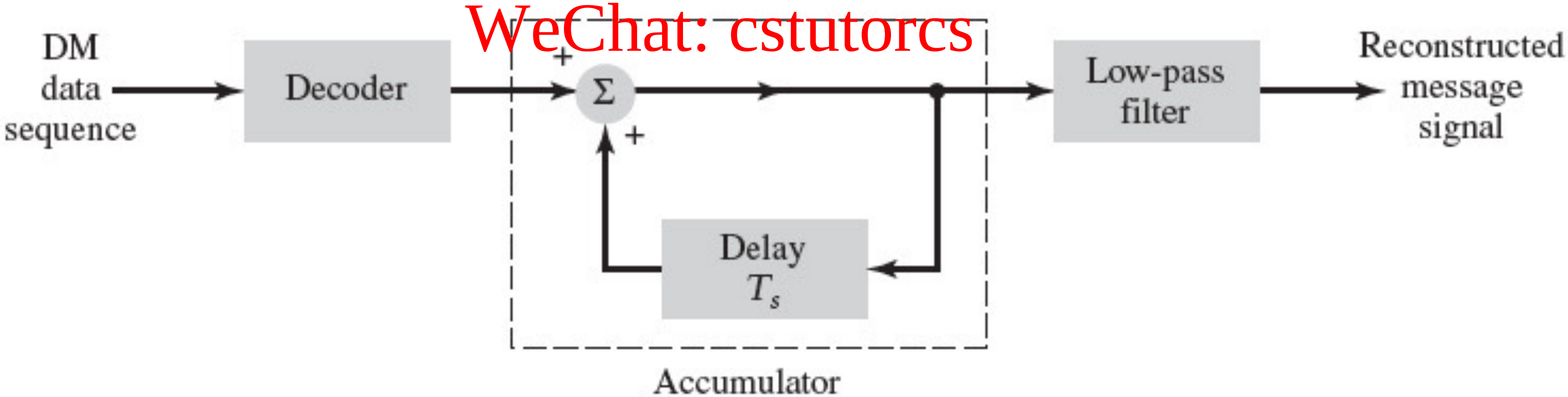
$$m_q(nT_s) = \sum_{i=1}^{n} e_q(iT_s).$$

Reconstruction:

The quantized sample values $m_q(nT_s)$ can be calculated from the binary sequence.

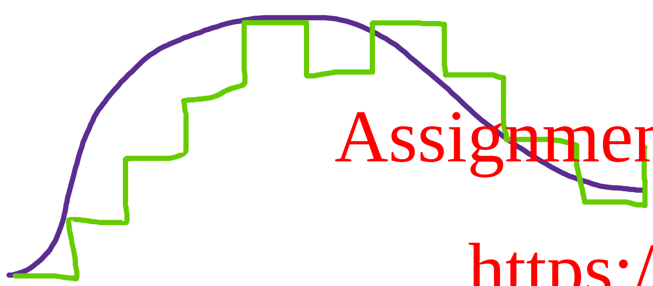
Use the quantized sample values to reconstruct the message.

Receiver of DM system:



Discussions - sampling rate in DM

Dense sample v.s. loose sample



Loose sampling with large T_s

Dense sampling with small T_s

Summary:

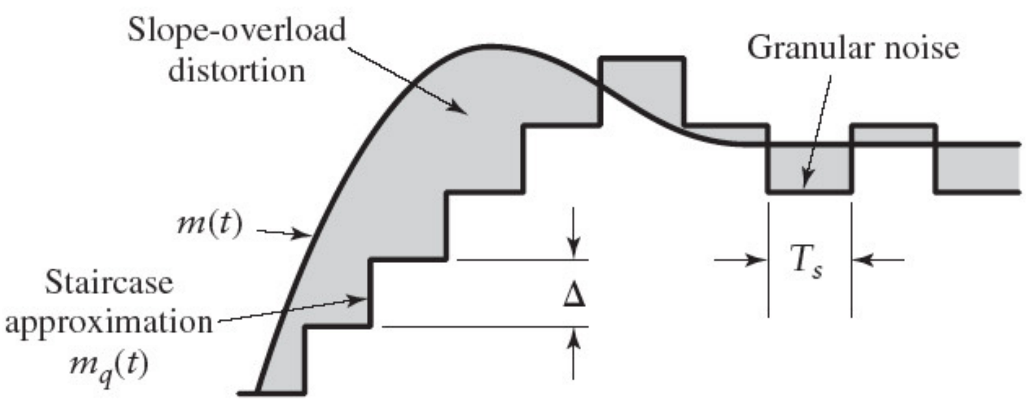
- In DM, incoming signal is usually oversampled to increase the correlation between adjacent samples.
- DM provides a staircase approximation of the message.

Discussions – distortions in DM:

Slope overload: when the message variation (slope) is very large. To avoid it, make sure $\frac{\Delta}{T_s} \geq \max|\frac{dm(t)}{dt}|$ by

- increasing step size Δ
- decreasing sampling interval T_s such that

Granular noise: if Δ is too large, the staircase approximation may hunt around a flat segment of $m(t)$, causing distortion.



5.9 Linear Codes

Electrical representation of binary sequence.

- Generate continuous-time signals for transmission.

	1	0
On-off signaling	A pulse of constant amplitude for the bit duration	No pulse
Nonreturn-to-zero signaling	A pulse of constant positive amplitude for the bit duration	A pulse of equal but negative amplitude for the bit duration
Return-to-zero signaling	A pulse of constant positive amplitude for 1/2 bit duration	No pulse

	1	0
Bipolar return-to-zero signaling	Positive and negative pulses used alternatively for the bit duration	No pulse
Split-phase (Manchester code)	Positive pulse with ½ bit duration then negative pulse for the 2nd half	Negative pulse with ½ bit duration then positive pulse for the 2nd half
Differential encoding	No transition	Transition (e.g., 0 to A or A to 0)

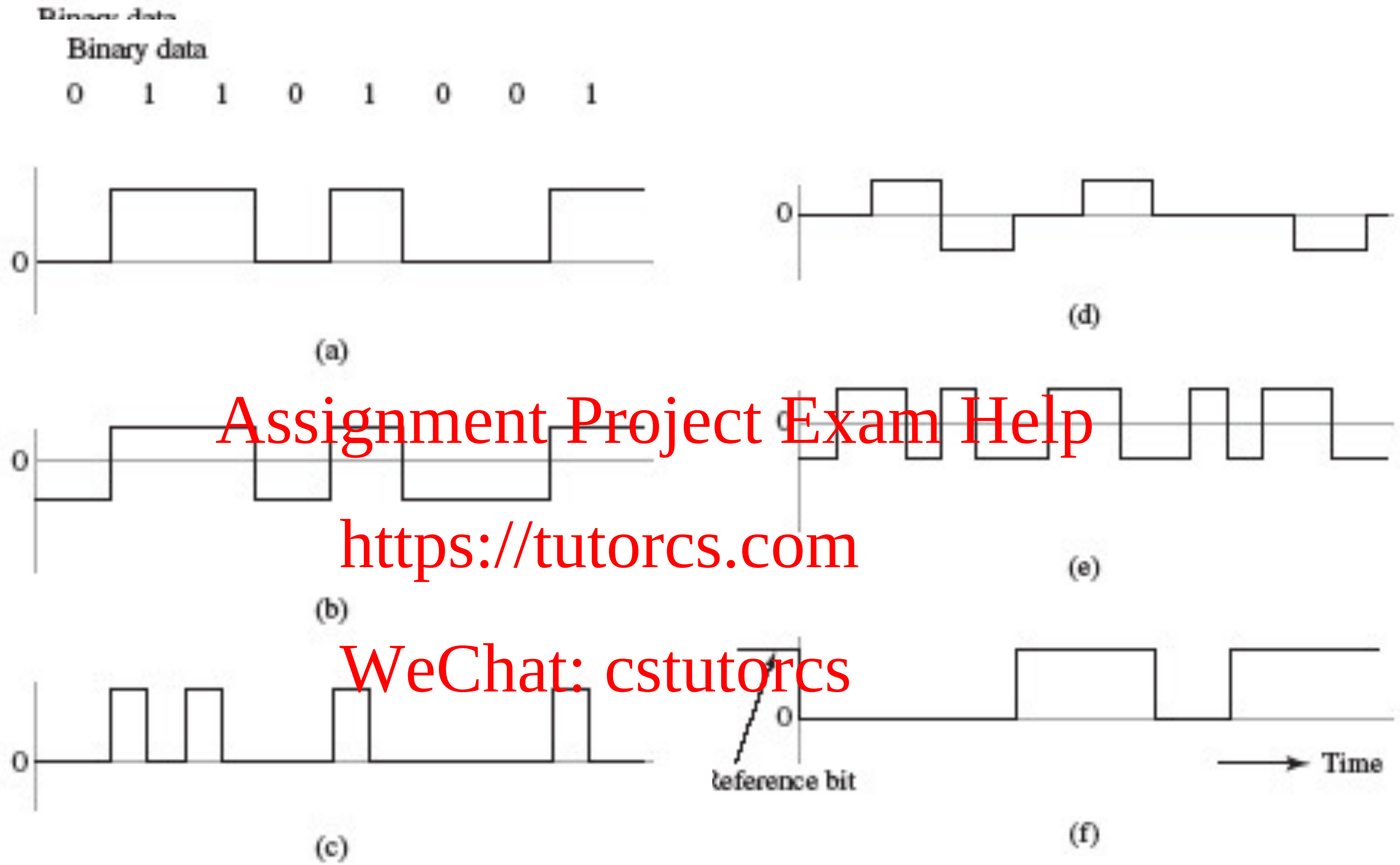


FIGURE 5.20 Line codes. (*a*) On–off signaling. (*b*) Nonreturn-to-zero signaling. (*c*) Return-to-zero signaling. (*d*) Bipolar return-to-zero signaling. (*e*) Split-phase or Manchester encoding. (*f*) Differential encoding.