Solution to Homework Assignment 10

Solution to Problem 1: Following Nyquist Channel Theorem, the Nyquist bit rate for error-free communication is $bitrate = 2B\log_2 M$. From the problem, $B = 1400Hz$ and $bitrate \geq 9600$. That is,

$$2 \times 1400 \times \log_2 M \geq 9600$$

Hence, we have $\log_2 M \geq 24/7 \approx 3.43$ bits/symbol. Let's use 4 bits to represent 1 symbol, then the symbol rate is $9600/4 = 2400$ symbol/second, with a constellation size of $2^M = 2^4 = 16$.

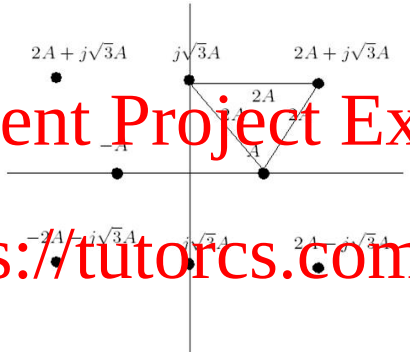
Solution to Problem 2:

(a) There are 4 points with energy $4A^2$ and 4 points with energy $4A^2 + 4A^2 = 8A^2$. Thus

$$E_{\mathcal{A}} = \frac{1}{8}\left(4 \times 4A^2 + 4 \times 8A^2\right) = 6A^2.$$

The minimum distance is $d_{\min} = 2A$.

(b) There are multiple possibilities, one of which is shown in the following figure:



The minimum distance of this new constellation is also $2A$. There are 2 points with energy A^2, 2 points with energy $3A^2$, and 4 points with energy $(2A)^2 + (\sqrt{3}A)^2 = 7A^2$. The average transmit energy of the new constellation $\mathcal{A}'$ is thus

$$E_{\mathcal{A}'} = \frac{1}{8}\left(2 \times A^2 + 2 \times 3A^2 + 4 \times 7A^2\right) = \frac{9}{2}A^2 < E_{\mathcal{A}}.$$

The new design has smaller energy with the same minimum distance, thus better.

Solution to Problem 3:

(a) The minimum decision rule is

$$\hat{a} = \arg \min_{a \in \{-1,1\}} |y - a|.$$

This is to check if y is closer to -1 or 1. Thus, the decision rule is:

$$\hat{a} = 1 \text{ if } y \geq 0 \text{ and } \hat{a} = -1 \text{ if } y < 0.$$

(b) When $a = 1$, error happens when $\hat{a} = -1$, which happens when $y < 0$. Thus

$$P_e|(a = 1) = \mathrm{P}[y < 0|a = 1] = \mathrm{P}[1 + n < 0] = \mathrm{P}[n < -1] = Q(1/\sigma).$$