Solution to Homework Assignment 7

Solution to Problem 1:

$G(f) = \frac{1}{400}\Delta\left(\frac{f}{800}\right)$. The message bandwidth is $W = 400$ Hz. Thus the Nyquist rate is $f_s = 2W = 800$ Hz and the Nyquist interval is $T_s = 1/f_s = 1.25$ms (milliseconds).

Solution to Problem 2: The instantaneous sampled signal is

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s)$$

and its Fourier transform is

$$G_\delta(f) = f_s \sum_{m=-\infty}^{\infty} G(f - mf_s).$$

Notice

$$G(f) = \frac{1}{2j}[\delta(f - 1/2) - \delta(f + 1/2)]$$

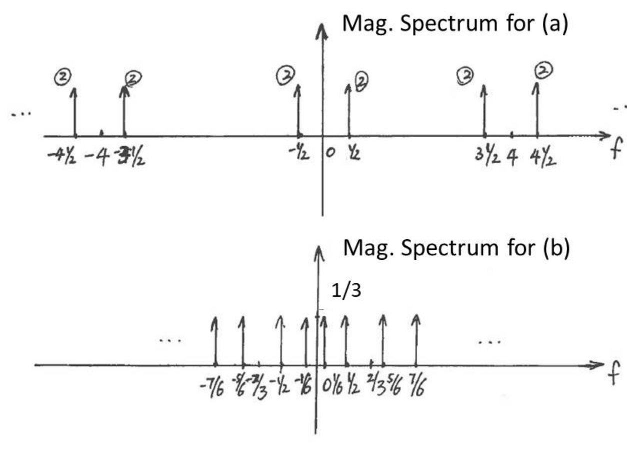
(a) $T_s = 0.25$ and $f_s = 4$. Thus

$$G_\delta(f) = \frac{2}{j} \sum_{m=-\infty}^{\infty} [\delta(f - 1/2 - 4m) - \delta(f + 1/2 - 4m)]\,.$$

(b) $T_s = 1.5$ and $f_s = 2/3$. Thus

$$G_\delta(f) = \frac{1}{3j} \sum_{m=-\infty}^{\infty} [\delta(f - 1/2 - 2m/3) - \delta(f + 1/2 - 2m/3)]\,.$$

The magnitude spectra are shown as following:



Solution to Problem 3: Signal bandwidth $W = 50$. Sampling at Nyquist rate, thus, $f_s = 2W = 100$ and $T_s = 1/f_s = 0.01$. From the problem,

$$g(-T_s) = g(-2T_s) = -1, \quad g(T_s) = g(2T_s) = 1,$$

and all other samples are 0.

(a) From the reconstruction formula

$$
\begin{aligned}
g(t) &= \sum_{n=-\infty}^{\infty} g(nT_s)\mathrm{sinc}(2Wt-n) \\
&= -\mathrm{sinc}(100t+2) - \mathrm{sinc}(100t+1) + \mathrm{sinc}(100t-1) + \mathrm{sinc}(100t-2).
\end{aligned}
$$

Thus

$$
\begin{aligned}
g(0.015) &= -\mathrm{sinc}(3.5) - \mathrm{sinc}(2.5) + \mathrm{sinc}(0.5) + \mathrm{sinc}(-0.5) \\
&= \frac{2}{7\pi} + \frac{2}{5\pi} + 2\frac{2}{\pi} \approx \frac{4.69}{\pi} \approx 1.49.
\end{aligned}
$$

(b) Since sinc-function is an energy function, $g(t)$ is also an energy function.

Solution to Problem 4:

$$m(t) = 2\sin(0.4\pi t).$$

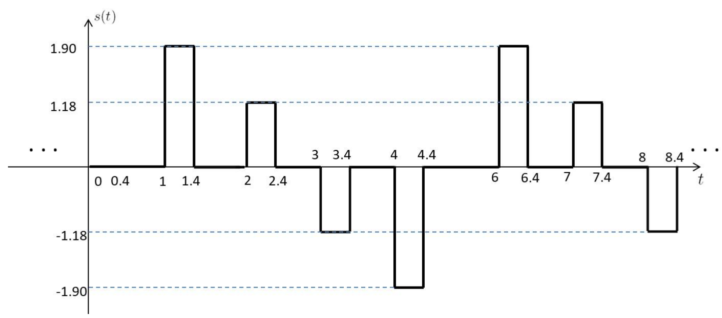
The sample values for $t = \cdots, 0, T_s, 2T_s, \cdots$ are

$$\{\cdots, 0, 1.90, 1.18, -1.18, -1.90, 0, 1.90, 1.18, -1.18, \cdots\}$$

(a) The PAM wave is

$$
\begin{aligned}
s(t) = \cdots &+ 1.9\,\mathrm{rect}\left(\frac{t-1.2}{0.4}\right) + 0.18\,\mathrm{rect}\left(\frac{t-2.2}{0.4}\right) \\
&-0.18\,\mathrm{rect}\left(\frac{t-3.2}{0.4}\right) - 1.9\,\mathrm{rect}\left(\frac{t-4.2}{0.4}\right) \\
&+0.9\,\mathrm{rect}\left(\frac{t-6.2}{0.4}\right) + 1.18\,\mathrm{rect}\left(\frac{t-7.2}{0.4}\right) - 1.18\,\mathrm{rect}\left(\frac{t-8.2}{0.4}\right) + \cdots.
\end{aligned}
$$

The waveform is shown in the following figure.



(b) $M(f) = -j\left[\delta(f-0.2) + \delta(f+0.2)\right]$, $H(f) = 0.4\,\mathrm{sinc}(0.4f)\,e^{-j0.4\pi f}$ and $f_s = 1$.

$$
\begin{aligned}
S(f) &= f_s \sum_{k=-\infty}^{\infty} M(f-kf_s)H(f) = \sum_{k=-\infty}^{\infty} M(f-k)H(f) \\
&= -0.4j \sum_{k=-\infty}^{\infty} \Big[\mathrm{sinc}(0.4k+0.08)\,e^{-j\pi(0.4k+0.08)}\delta(f-k-0.2) \\
&\qquad -\mathrm{sinc}(0.4k-0.08)\,e^{-j\pi(0.4k-0.08)}\delta(f-k+0.2)\Big].
\end{aligned}
$$