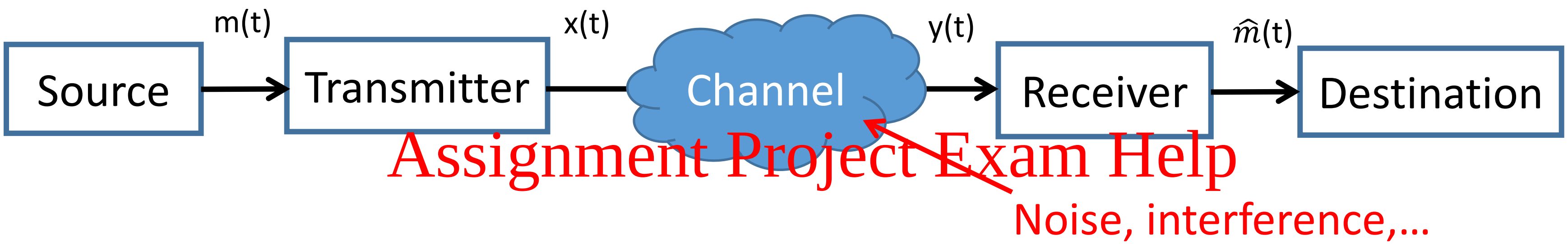
1.1 Communication System Structure

Information/data exchange between two or more parties.



- **Source:** Produces message signal m(t) in some format
- **Transmitter:** Maps the message signal m(t) to a waveform x(t) appropriate for transmission over the channel.
- **Channel:** Fades, distorts, and adds noise to the transmitted signal.
- **Receiver:** Tries to undo some of the effects of the channel. It maps back the received signal y(t) to an estimate of the message signal $\hat{m}$(t).
- **Destination:** Accept message

Chapter 3. Amplitude Modulation

Modulation: the process by which *some characteristics* of a carrier wave is varied in accordance with an information-bearing signal.

- **Carrier:** used to facilitate the transmission of messages, e.g., sinusoid waves.

$$c(t) = A_c \cos(2\pi f_c t + \theta)$$

Q: Why we need modulation in communication systems?

A: To move baseband signals to desired higher frequency band:

- Reduce antenna size
- Avoid mixing of signals

Chapter 3. Amplitude Modulation

Continuous-wave modulation: information-bearing signal $m(t)$ is continuous-time and analog.

For carrier $c(t) = A_c \cos(2\pi f_c t + \theta)$:

- Amplitude modulation (AM): the amplitude A_C is varied in accordance with $m(t)$.

- Frequency modulation (FM): the frequency f_c is varied in accordance with $m(t)$.

- Phase modulation (PM): the phase θ is varied in accordance with $m(t)$.

Summary: Different linear modulation strategies in the AM family, frequency analysis, demodulation designs.

3.1 Fundamentals of AM and Conventional AM **(Haykin & Moher 3.1, 3.2)**

3.2 Double Sideband-Suppressed Carrier Modulation **(Haykin & Moher 3.3, 3.4)**

3.3 Quadrature-Carrier Multiplexing **(Haykin & Moher 3.5)**

3.4 Frequency-Division Multiplexing **(Haykin & Moher 3.9 partial)**

3.5 Single Sideband Modulation **(Haykin & Moher 3.6)**

3.6 Vestige Sideband Modulation **(Haykin & Moher 3.7)**

3.1 Fundamentals of AM and Conventional AM

Modulation, demodulation, time-domain and frequency-domain analysis, virtues and limits.

- **A sinusoidal carrier wave:**

$$c(t) = A_c \cos(2\pi f_c t),$$

where f_c is the carrier frequency, A_c is the carrier amplitude.

- **Message signal/information-bearing signal:** $m(t)$

Amplitude modulation is defined as a process in which the *amplitude* of the carrier wave $c(t)$ is varied about a mean value, linearly with the message signal $m(t)$.

An AM wave (signal) can be described as the following time function:

$$s(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t),$$

where k_a is a constant called the **amplitude sensitivity**.

For AM, information of the message signal $m(t)$ resides in the *envelope* (amplitude) of the modulated wave $s(t)$, which is

$$|A_c[1 + k_a m(t)]|$$

If the following conditions are satisfied, the envelope has the same shape as the message signal.

- Condition 1: $f_c \gg W$, where W is the message bandwidth.
  Otherwise, an envelope cannot be visualized and detected satisfactorily.

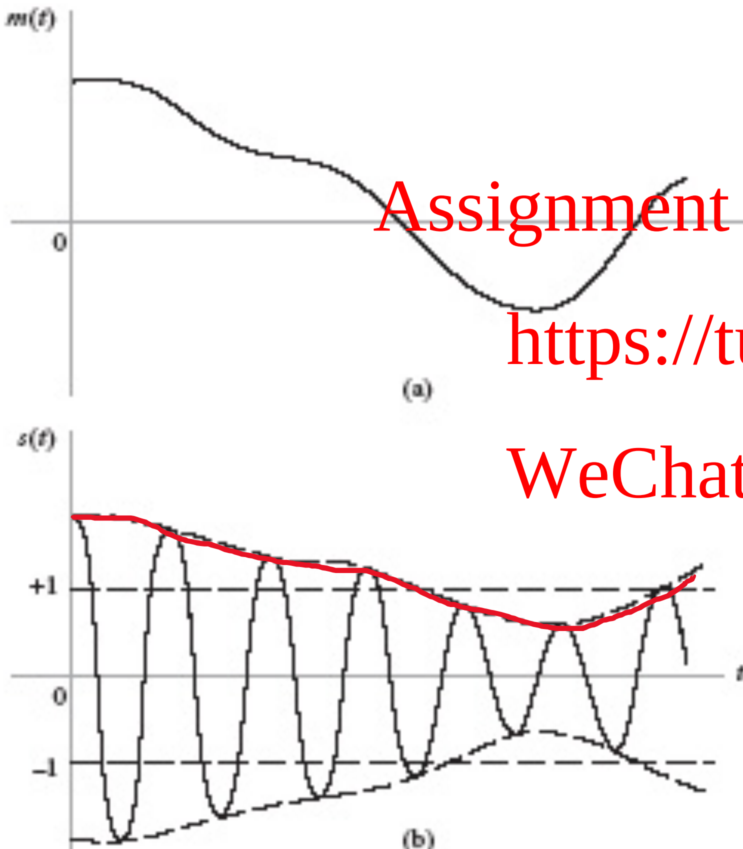
- Condition 2: $|k_a m(t)| \leq 1, \text{ for all } t.$

  To ensures $1 + k_a m(t)$ to be non-negative. Thus the envelope is simply $A_c[1 + k_a m(t)]$; Otherwise, the carrier wave may be over modulated, resulting in phase reversal and *envelop distortion*.

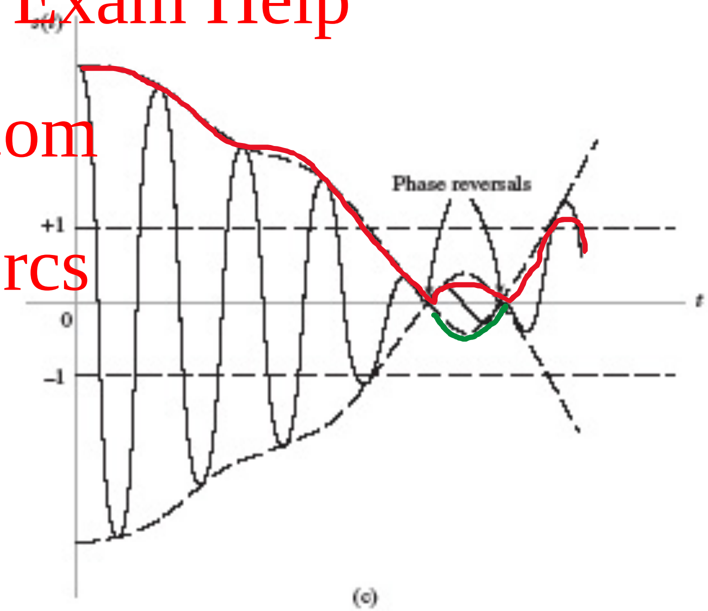
  Percentage modulation: $\max\{k_a m(t)\}$

Figure 3.1 Illustration of the amplitude modulation process. (*a*) Message signal $m(t)$. (*b*) AM wave for $k_a m(t) < 1$ for all t. (*c*) AM wave for $|k_a m(t)| > 1$ for some t.

$$s(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t),$$



Under-modulated signal



Over-modulated signal

Frequency-domain description of AM

Let $m(t) \rightleftharpoons M(f)$

$M(f)$: *message spectrum with message bandwidth W.*

- AM wave:

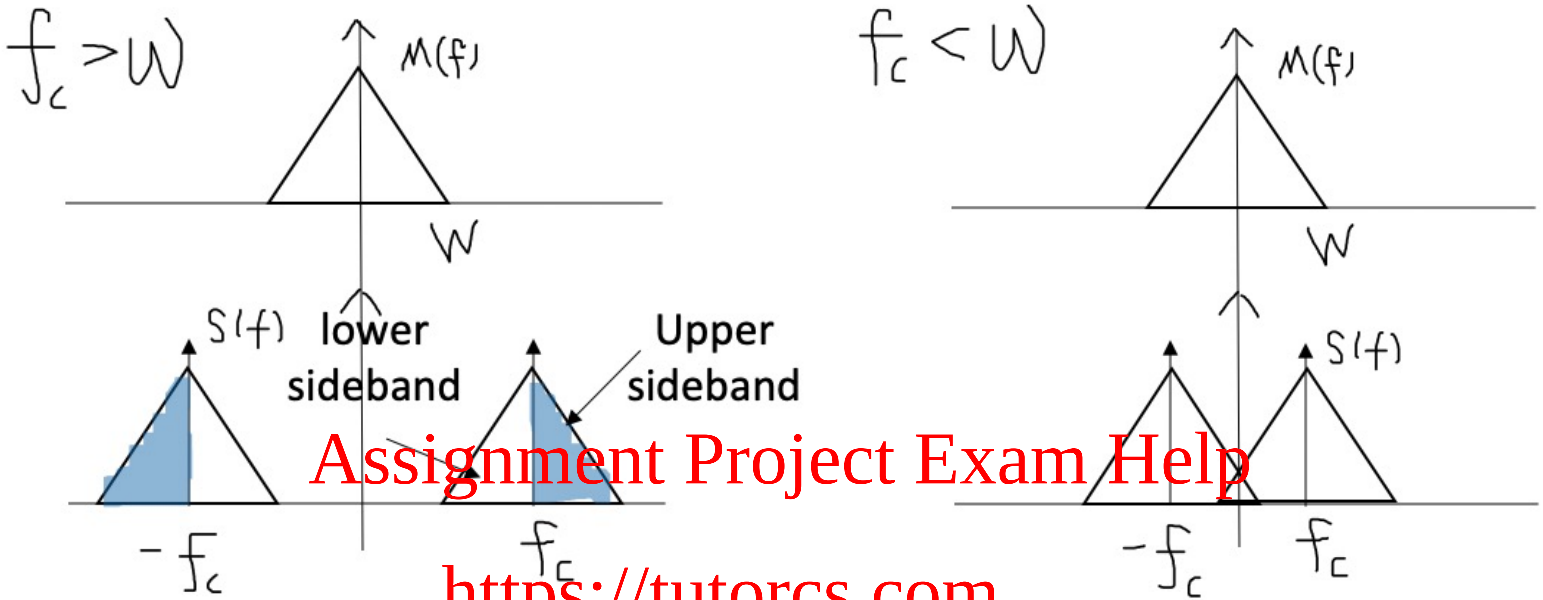
$$s(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t)$$
$$= A_c \cos(2\pi f_c t) + A_c[k_a m(t)] \cos(2\pi f_c t)$$

- Spectrum of AM wave:

$$S(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{k_a A_c}{2}\left[M(f - f_c) + M(f + f_c)\right]$$

The spectrum of the AM wave consists of

- 2 delta functions occurring at $\pm f_c$ and weighted by $A_c/2$;
- 2 versions of the message spectrum shifted to the frequency bands centered at $\pm f_c$ and scaled by $k_a A_c/2$.



Observations:

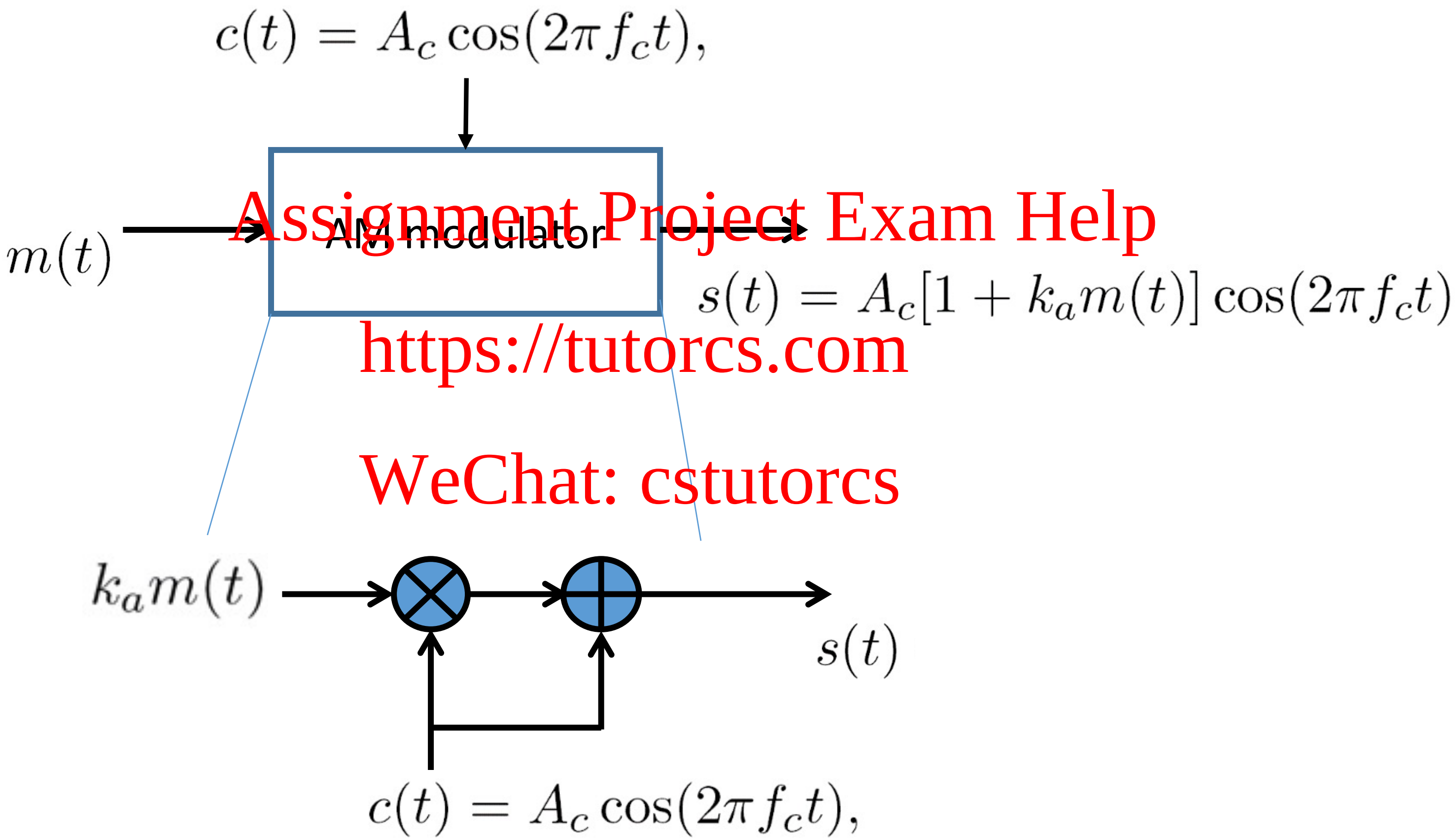
(1) For positive frequencies, the portion of the AM wave spectrum above carrier frequency f_c is referred to as the *upper sideband* and the potion below f_c is referred to as the *lower sideband*.

(2) The condition $f_c \gg W$ ensures non-overlapping sidebands.

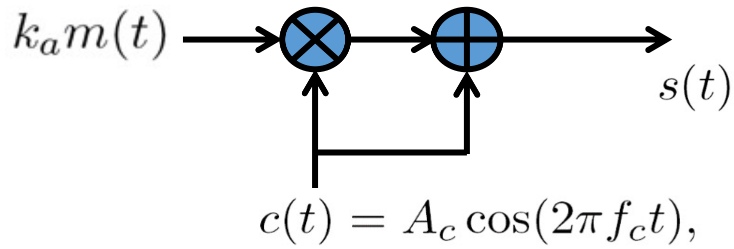
- the spectrum of the message for negative frequencies (from $-W$ to 0) becomes visible for positive frequencies.
- The bandwidth of the AM wave B_T is *twice* the message bandwidth: $\boxed{B_T = 2W}$.

AM and demodulation diagrams:

Modulation:



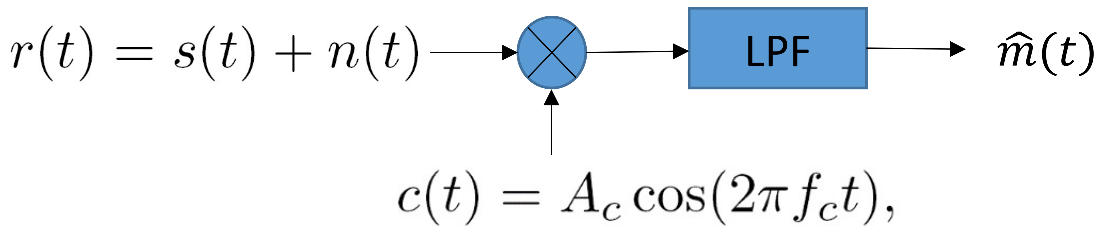
Modulation:



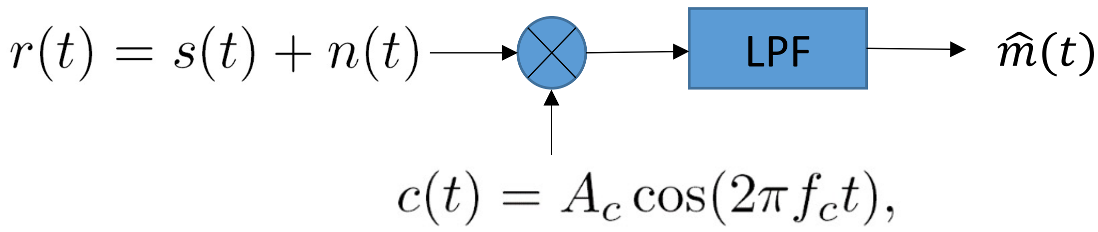
Alternative: **square-law modulator with bandpass filter**:

- step 1: $v_1(t) = A_c \cos(2\pi f_c t) + m(t)$
- step 2: $v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$
- step 3: bandpass filter centered at f_c

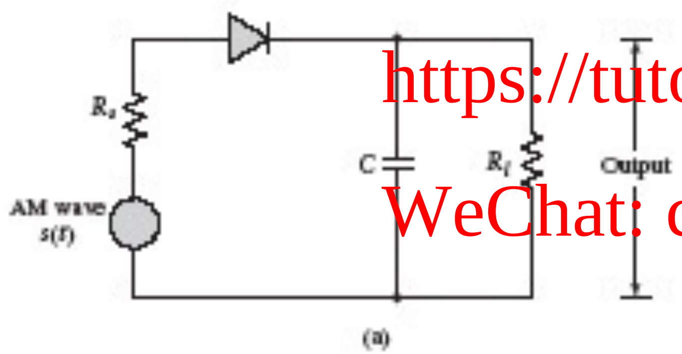
Demodulation:



Demodulation:



Alternative: **Envelope detector**



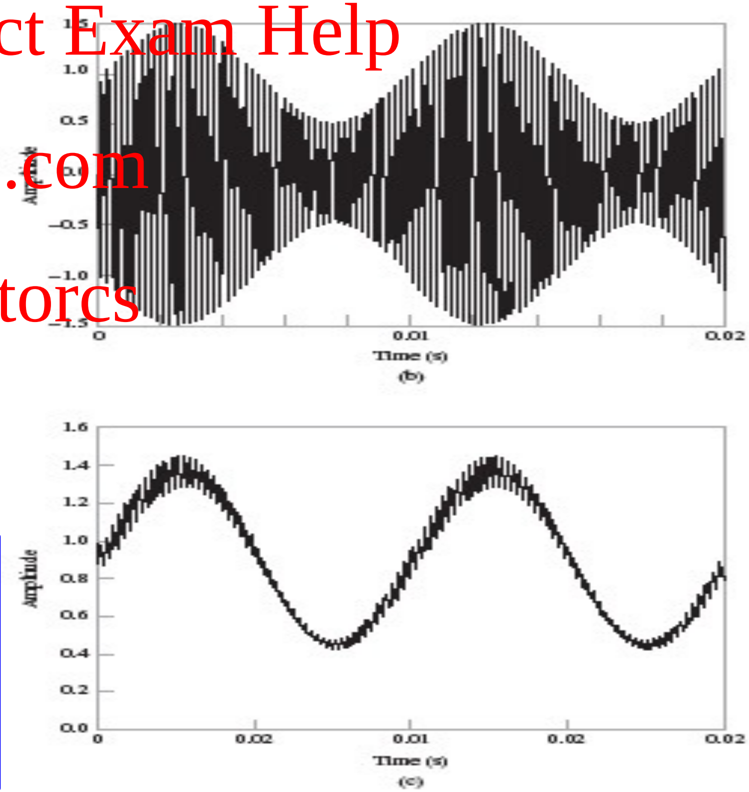


FIGURE 3.9 Envelope detector. (*a*) Circuit diagram. (*b*) AM wave input. (*c*) Envelope detector output

Easy and simple.
Need no information about carrier.
There are other de-mod schemes.

Example: Single-tone modulation.

Message signal:

$$m(t) = A_m \cos(2\pi f_m t)$$
$$M(f) = \frac{A_m}{2}\left[\delta(f - f_m) + \delta(f + f_m)\right]$$

Let $\mu = k_a A_m$

AM wave: $s(t) = A_c[1 + \mu \cos(2\pi f_m t)]\cos(2\pi f_c t)$

$$S(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]$$
$$+\frac{A_c}{4}\mu\left[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)\right]$$
$$+\frac{A_c}{4}\mu\left[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)\right]$$

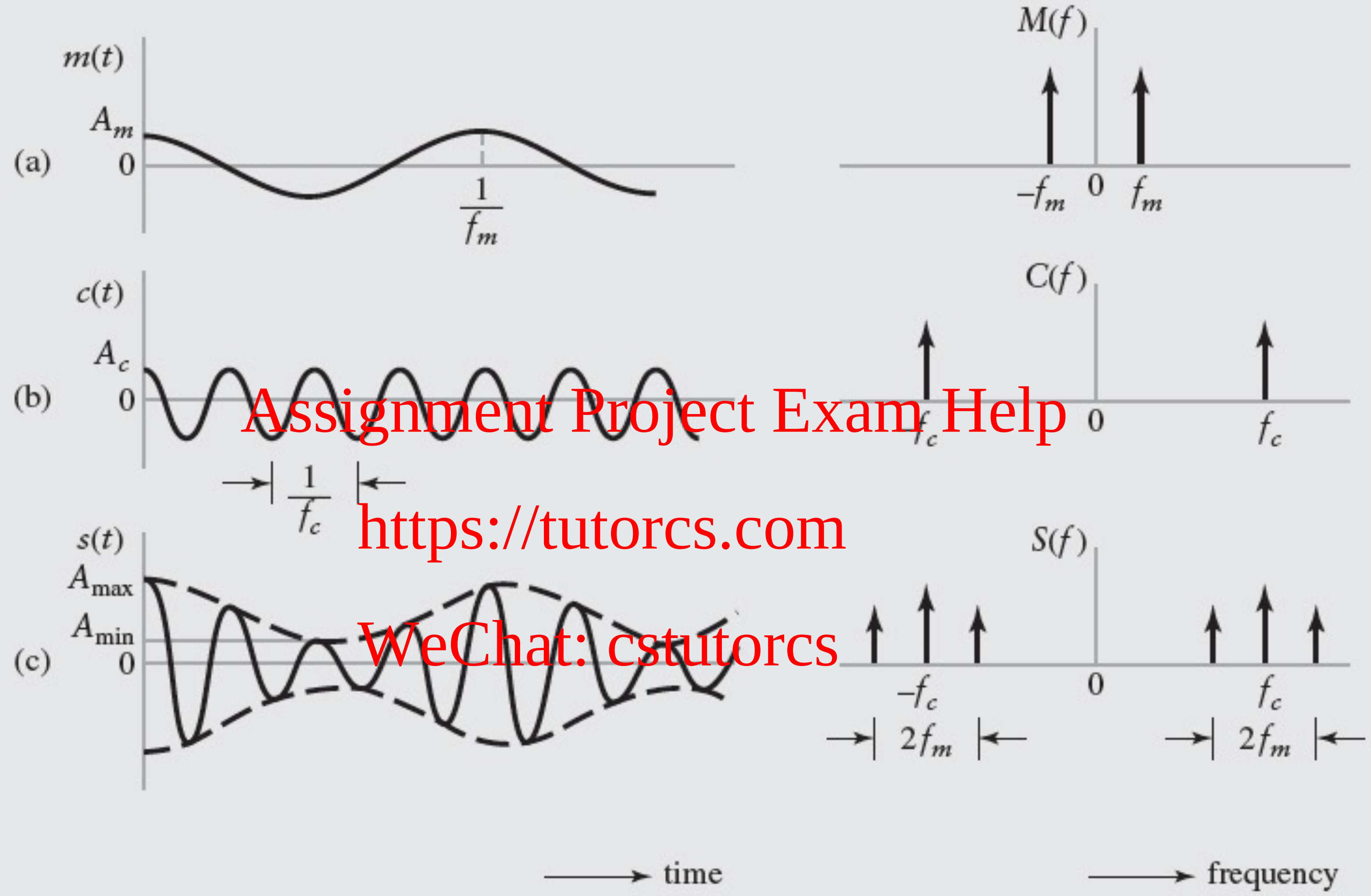


FIGURE 3.3 Illustration of the time-domain (on the left) and frequency-domain (on the right) characteristics of amplitude modulation produced by a single tone. (*a*) Modulating wave. (*b*) Carrier wave. (*c*) AM wave.

Power efficiency of single-tone AM wave:

$$\text{Carrier power} = \frac{1}{2}A_c^2$$

$$\text{Upper side-frequency power} = \frac{1}{8}\mu^2 A_c^2$$

$$\text{Lower side-frequency power} = \frac{1}{8}\mu^2 A_c^2$$

$$\frac{\text{message power (total sideband power)}}{\text{total power in the modulated wave}} = \frac{\mu^2}{2+\mu^2}$$

From $|k_a m(t)| = |k_a A_m \cos(2\pi f_m t)| \le 1 \Rightarrow \mu \le 1$,

$$\boxed{\text{efficiency} \le \frac{1}{3}}$$

Virtues, limitations, and modifications of AM

Virtue: Easy implementation and inexpensive.

Limitations:

1. Low power efficiency (wasteful of transmitted power): A large potion of the power is used on the transmission of the carrier, not the signal.

2. Wasteful of channel bandwidth: The transmission bandwidth is twice the signal bandwidth. The upper and lower sidebands contain the same information. Only one sideband is necessary.

Modifications: Trade-off complexity for better efficiency in the use of communication resources.

- Double sideband-suppressed carrier (DSB-SC) modulation: Save transmission power via the suppression of carrier wave.
- Single sideband (SSB) modulation: Use only one of the sidebands to save both power and bandwidth.
- Vestigial sideband (VSB) modulation: Use one sideband and a vestige of the other sideband. A balance between resources (power and bandwidth) and complexity.

3.2 Double Sideband-Suppressed Carrier Modulation

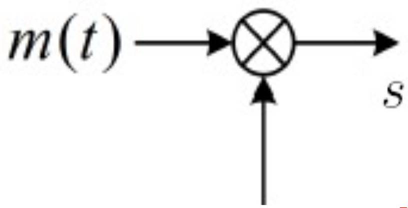
Modulation and demodulation (detection), time-domain and frequency-domain, comparison with conventional AM.

- **Message signal:** $m(t)$
- **Double sideband-suppressed carrier (DSB-SC) wave:**

$$s(t) = A_c m(t) \cos(2\pi f_c t).$$

- **Frequency-domain:**

$$S(f) = \frac{A_c}{2}\left[M(f - f_c) + M(f + f_c)\right]$$



$$s(t) = A_c m(t)\cos(2\pi f_c t).$$

$$c(t) = A_c \cos(2\pi f_c t).$$

- Product of message signal and carrier.
- The device to generate the DSB-SC wave is called <u>product modulator</u>.

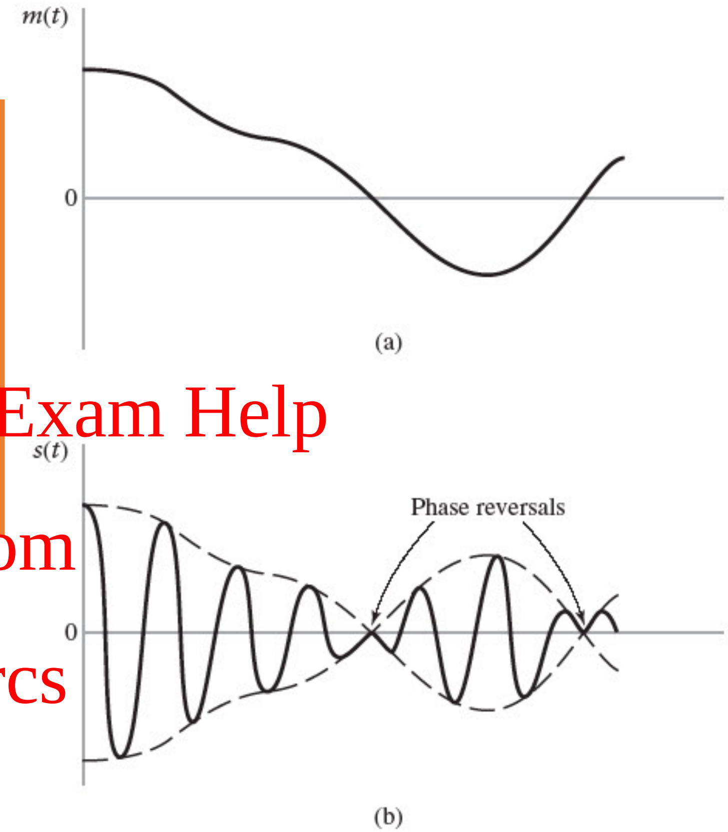


FIGURE 3.10 (*a*) Message signal $m(t)$. (*b*) DSB-SC modulated wave $s(t)$.

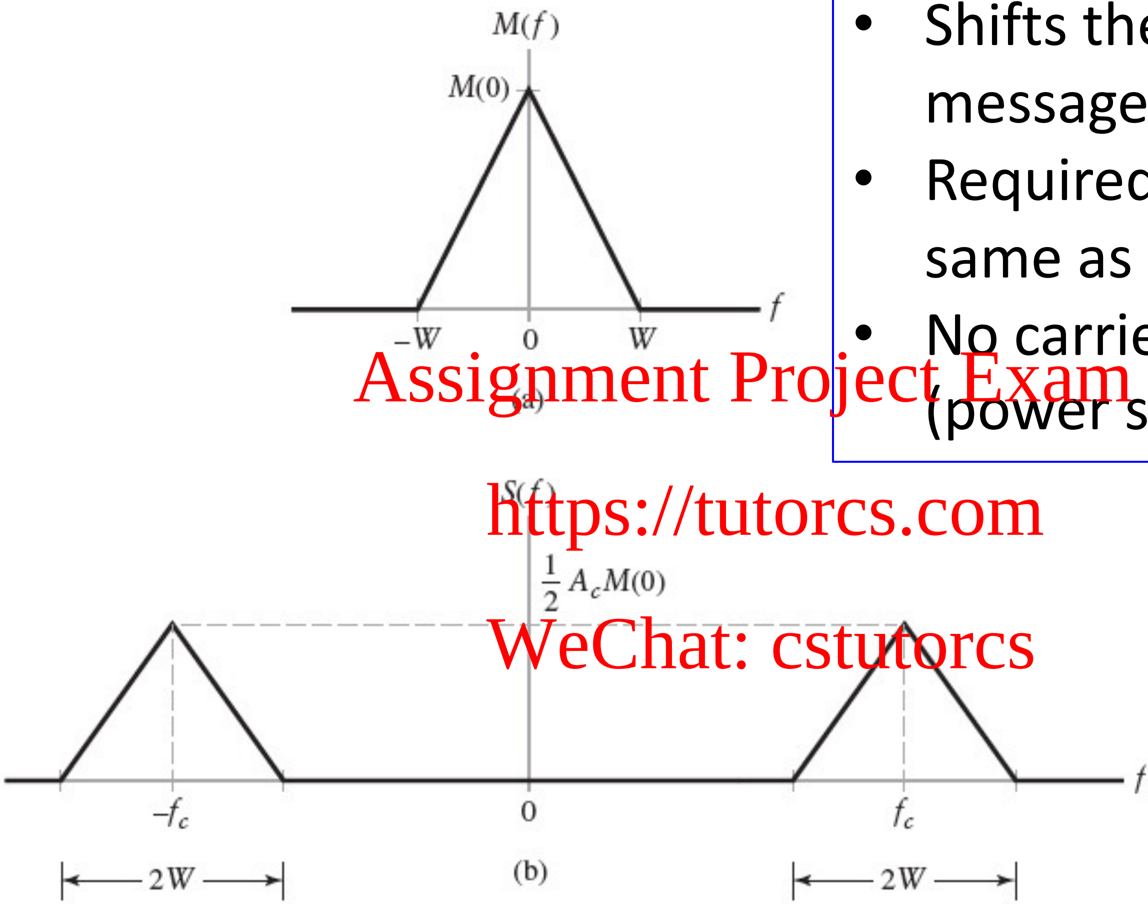


FIGURE 3.11 (*a*) Spectrum of message signal $m(t)$. (*b*) Spectrum of DSB-SC modulated wave $s(t)$.

- Shifts the spectrum of the message signal by f_c and $-f_c$.
- Required bandwidth is $2W$, same as AM.
- No carrier in modulated wave (power saving).

Example: Single-tone modulation.

Message signal: $m(t) = A_m \cos(2\pi f_m t)$

$$M(f) = \frac{A_m}{2}\left[\delta(f - f_m) + \delta(f + f_m)\right]$$

DSB-SC AM wave:

$$s(t) = A_c A_m \cos(2\pi f_m t)\cos(2\pi f_c t)$$

$$S(f) = \frac{A_c A_m}{4}\left[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)\right]$$

$$+\frac{A_c A_m}{4}\left[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)\right]$$

$$\text{Upper side-frequency power} = \frac{1}{8}A_c^2 A_m^2$$

$$\text{Lower side-frequency power} = \frac{1}{8}A_c^2 A_m^2$$

$$\text{Power of the DSB-SC wave} = \frac{1}{4}A_c^2 A_m^2$$

Coherent detection

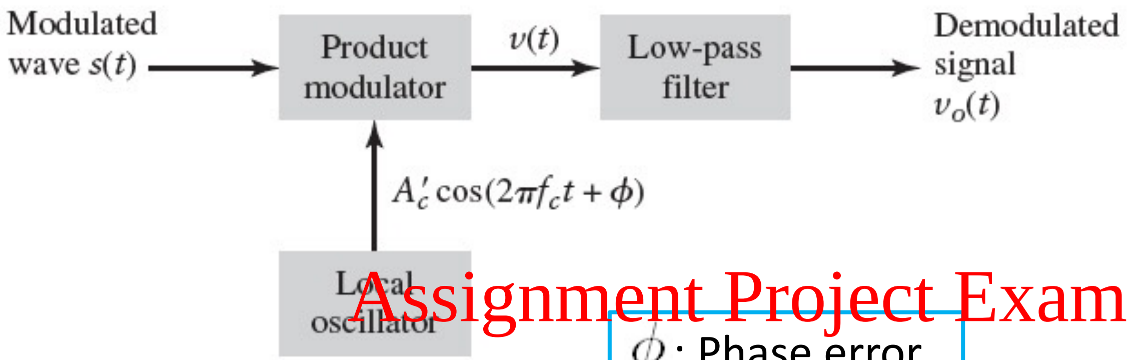


FIGURE 3.12 Block diagram of coherent detector, assuming that the local oscillator is out of phase by ϕ with respect to the sinusoidal carrier oscillator in the transmitter.

ϕ : Phase error.

$$
\begin{aligned}
v(t) &= A'_c \cos(2\pi f_c t + \phi) s(t) \\
&= A_c A'_c \cos(2\pi f_c t) \cos(2\pi f_c t + \phi) m(t) \\
&= \frac{1}{2} A_c A'_c \cos(4\pi f_c t + \phi) m(t) + \frac{1}{2} A_c A'_c \cos(\phi) m(t)
\end{aligned}
$$

Output of LPF: $v_o(t) = \frac{1}{2} A_c A'_c \cos(\phi) m(t).$

Detect the message when $\phi \neq \pm\frac{\pi}{2}.$

Coherent detection continued

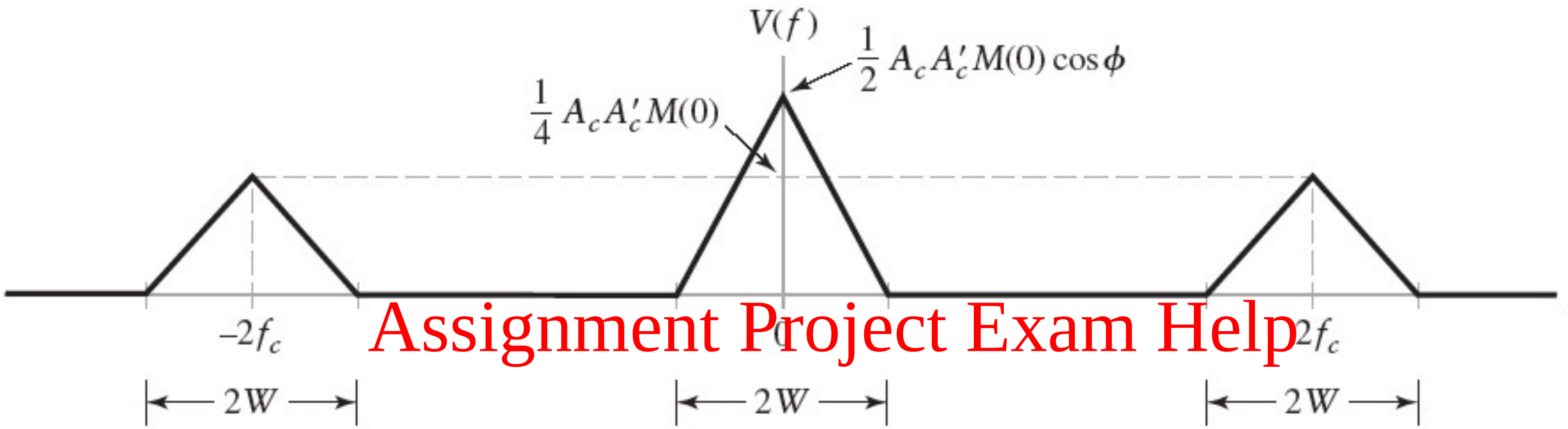


FIGURE 3.13 Illustration of the spectrum of product modulator output $v(t)$ in the coherent detector of Fig. 3.12, which is produced in response to a DSB-SC modulated wave as the detector input.

Quadrature null effect: Cannot demodulate when $\phi = \pm\frac{\pi}{2}$.

Need synchronization on both carrier frequency and phase.

- Using costas receiver to lock phase.

3.3 Multiplexing and QAM

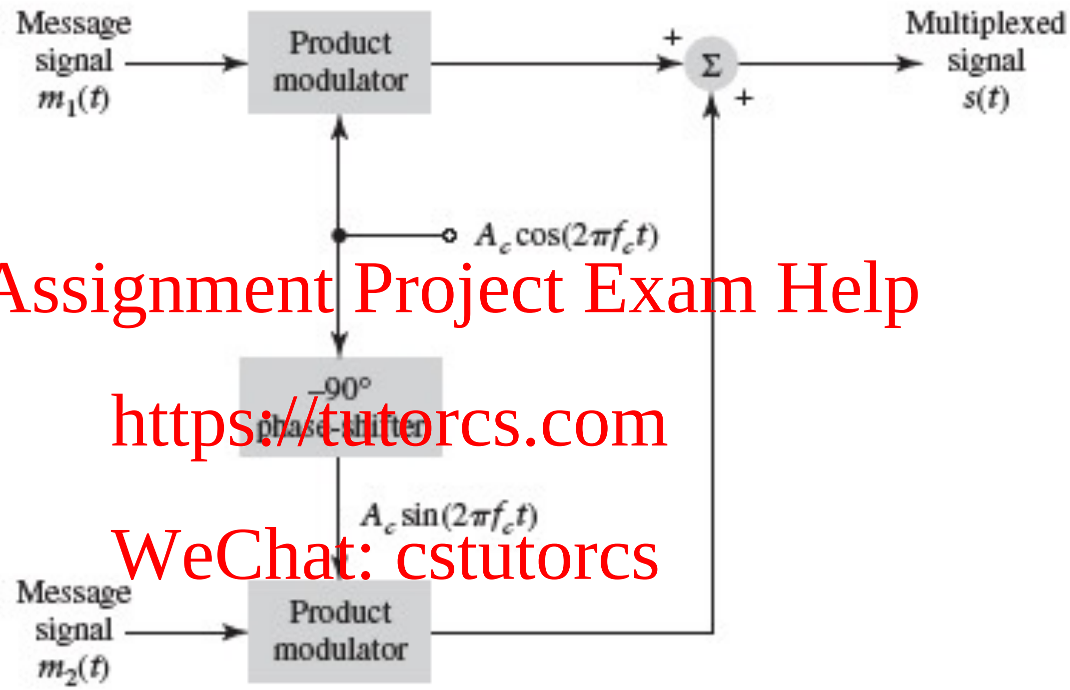
- Multiplexing: to send multiple message simultaneously

- Quadrature Amplitude Multiplexing (QAM): (quadrature-carrier multiplexing)

  - amplitude modulation scheme that enables two DSB-SC waves with independent message signals to occupy the same channel bandwidth (i.e., same frequency channel) yet still be separated at the receiver.

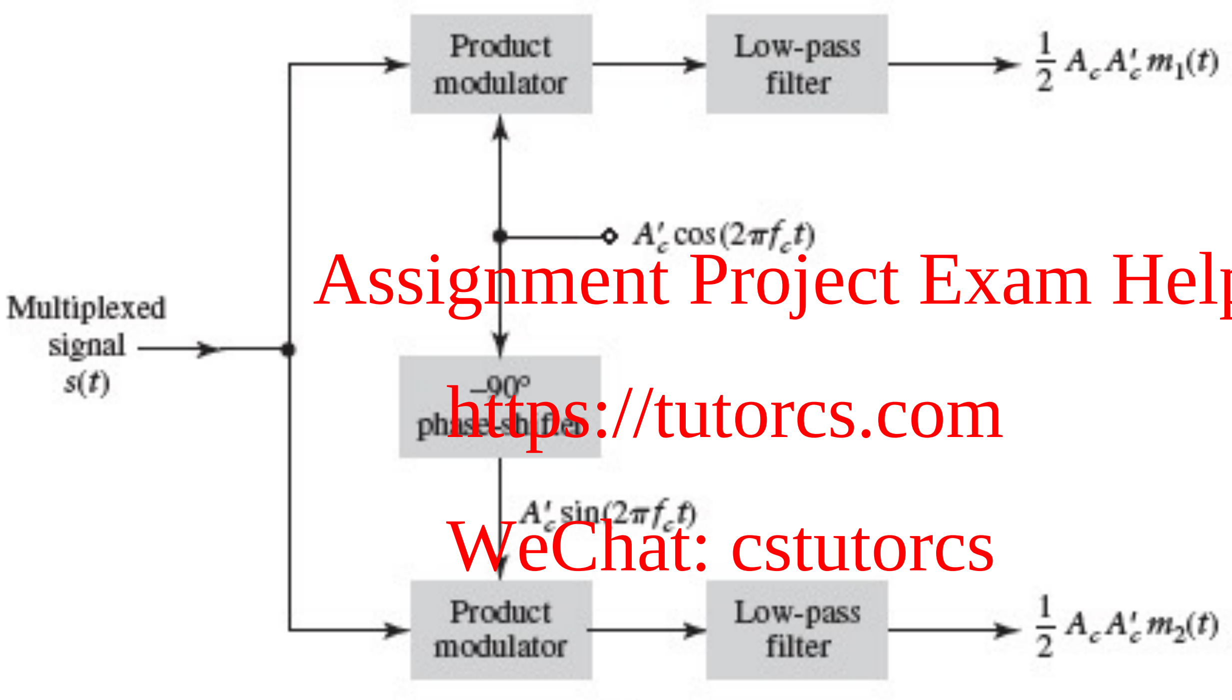
  - quadrature null effect.

Modulation of quadrature-carrier multiplexing:



$$s(t) = \underbrace{A_c m_1(t)}_{\text{In-phase component}} \cos(2\pi f_c t) + \underbrace{A_c m_2(t)}_{\text{Quadrature component}} \sin(2\pi f_c t).$$

Demodulation of quadrature-carrier multiplexing:



- Require very high synchronization level for both phase and frequency.

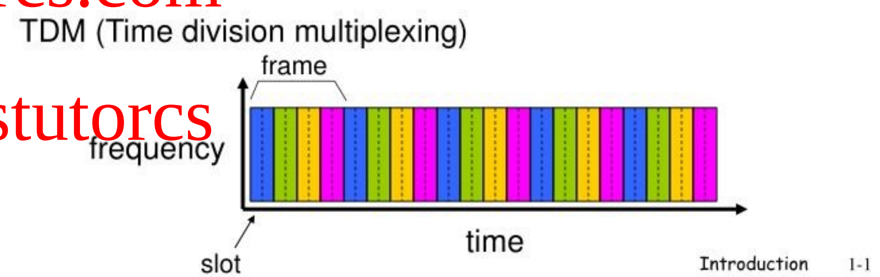
3.4 Other Multiplexing Schemes

Multiplexing: to send multiple message simultaneously *over a common channel*.

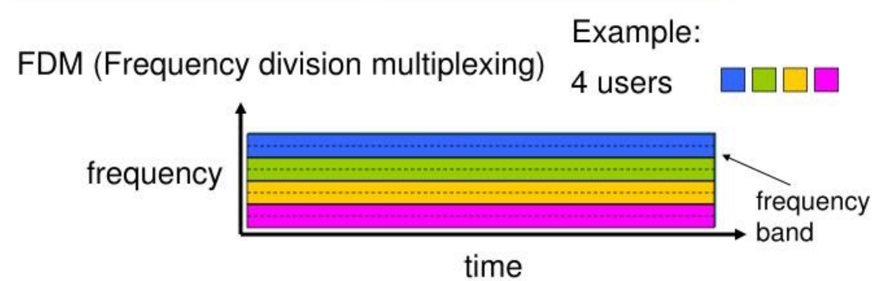
- For successful detection, signals must be kept apart by some means to avoid interference.

QAM: Signals are separated by quadrature null effect.

Time-Division Multiplexing (TDM): Signals are separated in time.



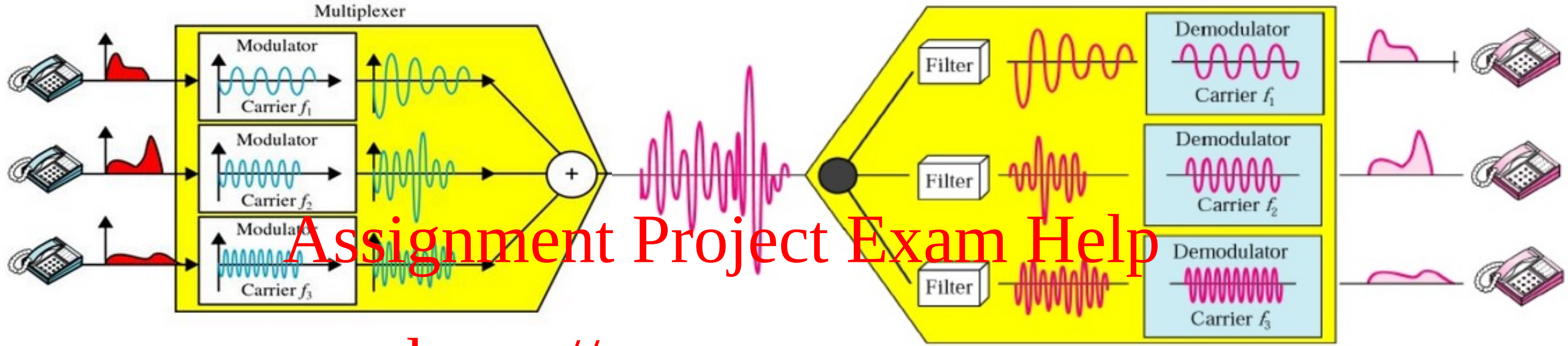
Frequency-Division Multiplexing (FDM): Signals are separated in frequency.



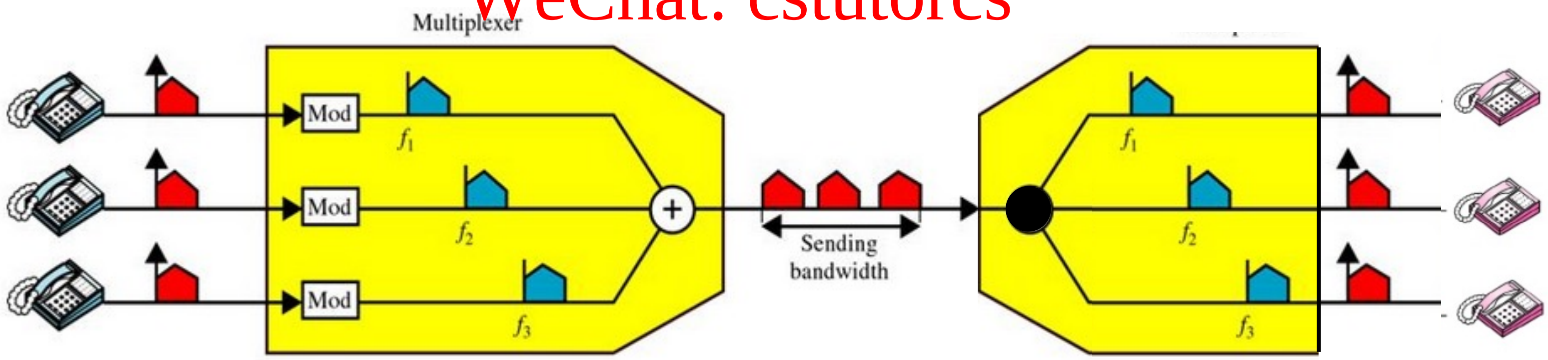
Images copied from https://www.slideserve.com/lulu/circuit-switching-fdm-and-tdm

Frequency-Division Multiplexing (FDM):

In time domain:



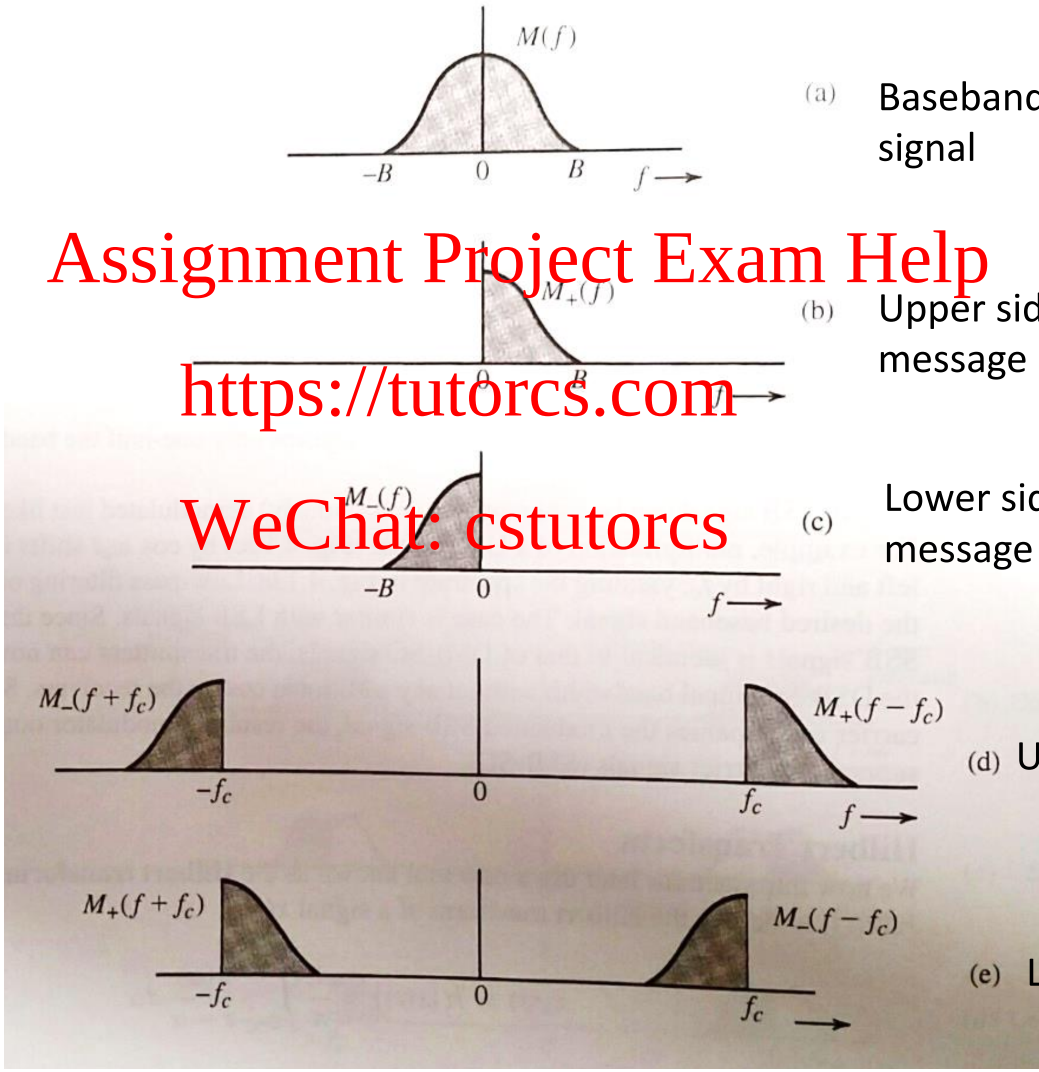
In frequency domain:



Figures are copied from http://www.slideshare.net/soumya604159/ch-06

3.5 Single-sideband (SSB) modulation

Use one of the sidebands (upper sideband or lower sideband) to save bandwidth.



Baseband message signal

Upper sideband of message signal

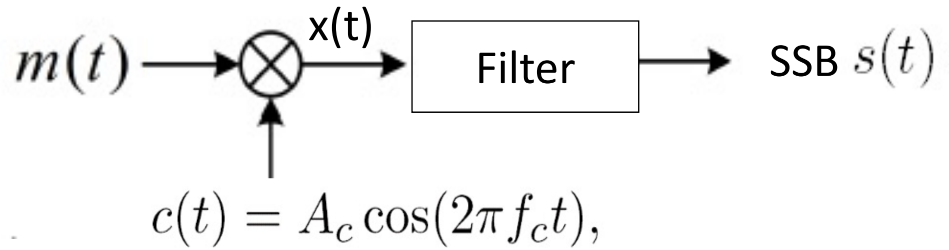
Lower sideband of message signal

Upper SSB wave

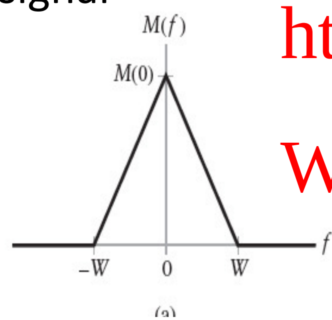
Lower SSB wave

SSB modulation methods:

Frequency Discrimination Method (Filtering method)

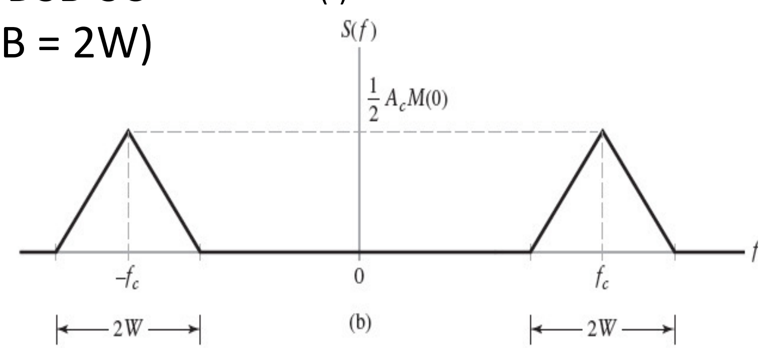


(1) Message signal (B = W)

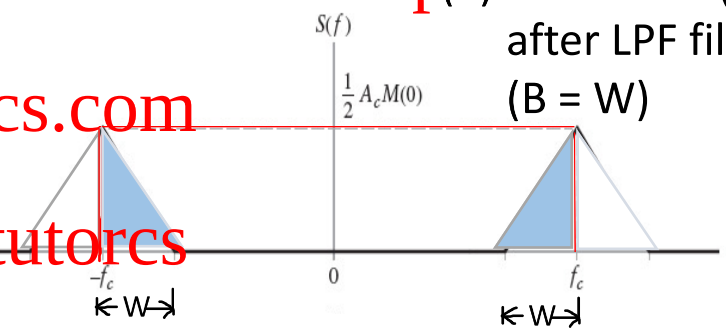


(2) DSB-SC (B = 2W)

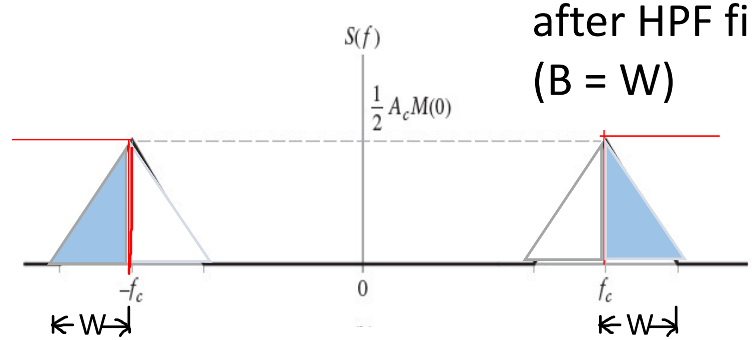
X(f)



(3) Lower SSB (LSSB) after LPF filtering (B = W)

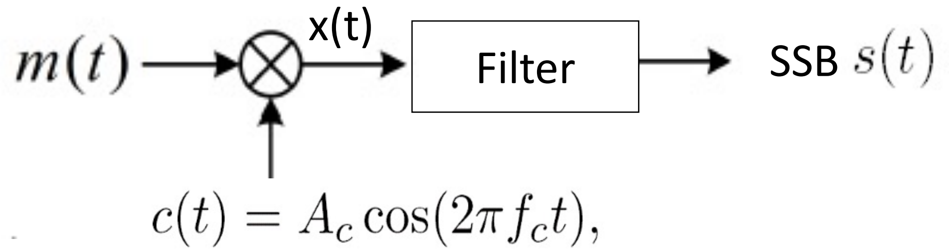


(4) Upper SSB (USSB) after HPF filtering (B = W)

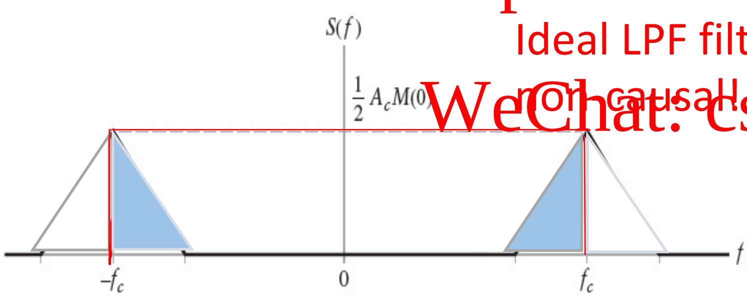


SSB modulation methods:

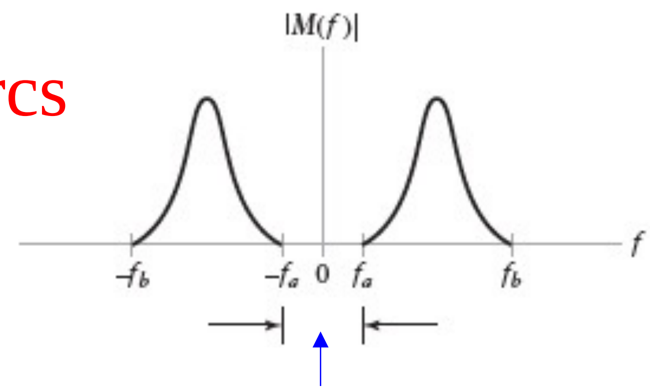
Frequency Discrimination Method (Filtering method)



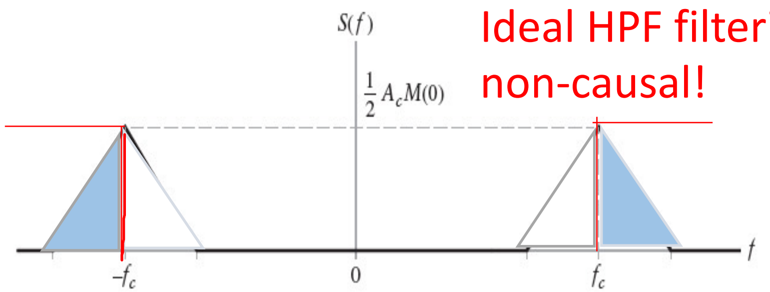
Note: filtering method needs an energy gap: a certain separation between the two sidebands for non-ideal filter design.



Ideal LPF filtering is non-causal!



Energy gap for non-ideal filter design



Ideal HPF filtering is non-causal!

SSB modulation methods:

Phase Discrimination Method by Hilbert Transfer

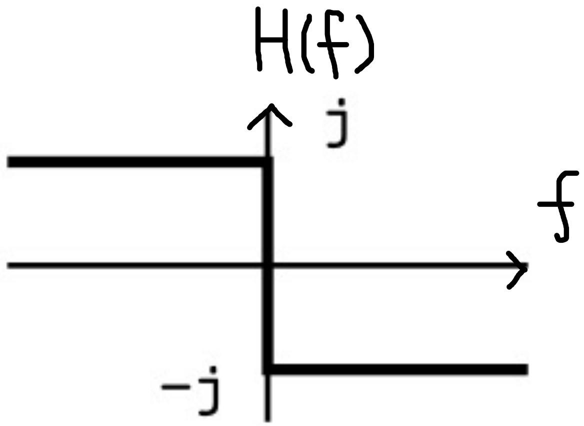
Hilbert Transfer filter: $h(t) = \frac{1}{\pi t} \rightleftharpoons H(f) = -jsgn(f)$

- Hilbert filtering system is a non-causal LTI system.

- It is an ideal phase shifter:

  - $\pi/2$ phase shift for f<0

  - $-\pi/2$ phase shift for f>0



Hilbert transform of a signal $m(t)$

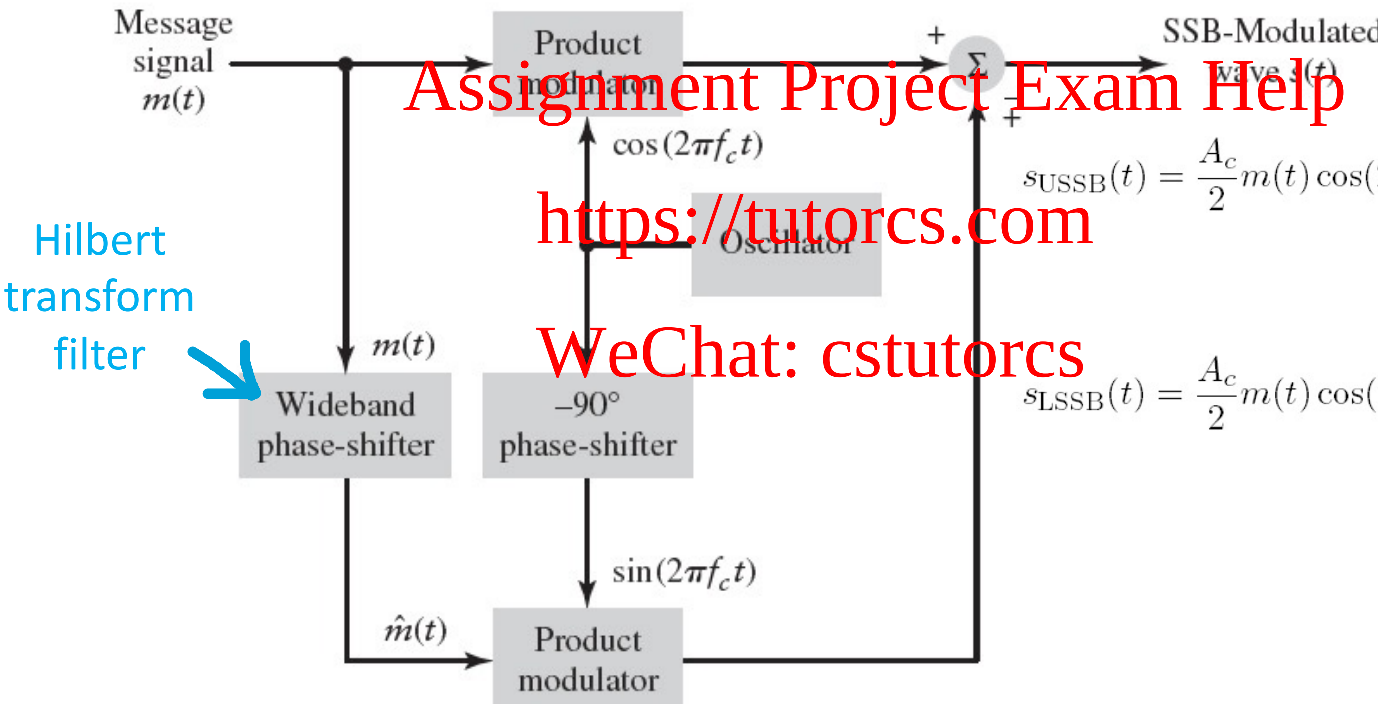
$$\hat{m}(t) = m(t) * h(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(\tau)}{t-\tau} d\tau$$

$$\hat{M}(f) = -jM(f)\mathrm{sgn}(\mathrm{f})$$

SSB modulation methods:

Phase Discrimination Method by Hilbert Transfer

$$s_{\mathrm{SSB}}(t) = \frac{A_c}{2} m(t) \cos(2\pi f_c t) \mp \frac{A_c}{2} \hat{m}(t) \sin(2\pi f_c t).$$

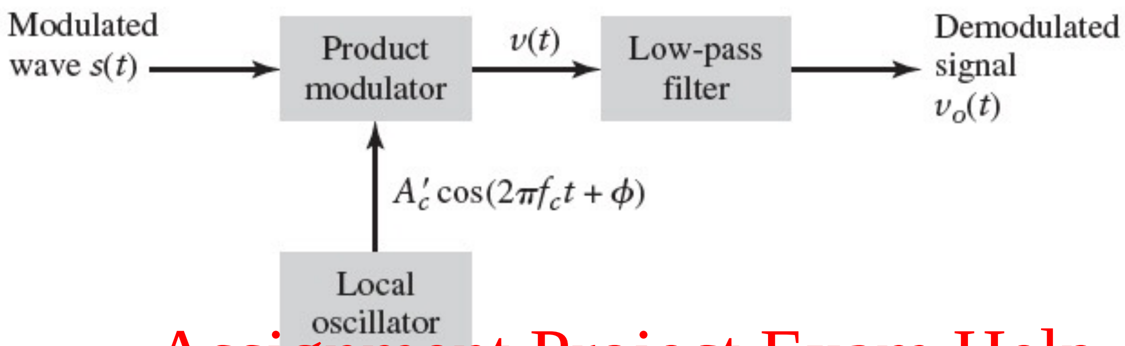


$$s_{\mathrm{USSB}}(t) = \frac{A_c}{2} m(t) \cos(2\pi f_c t) - \frac{A_c}{2} \hat{m}(t) \sin(2\pi f_c t).$$

$$s_{\mathrm{LSSB}}(t) = \frac{A_c}{2} m(t) \cos(2\pi f_c t) + \frac{A_c}{2} \hat{m}(t) \sin(2\pi f_c t).$$

FIGURE 3.20 Phase discrimination method for generating a SSB-modulated wave. Note: The plus sign at the summing junction pertains to transmission of the lower sideband and the minus sign pertains to transmission of the upper sideband.

Coherent detection of SSB: same as that for DSB-SC AM.



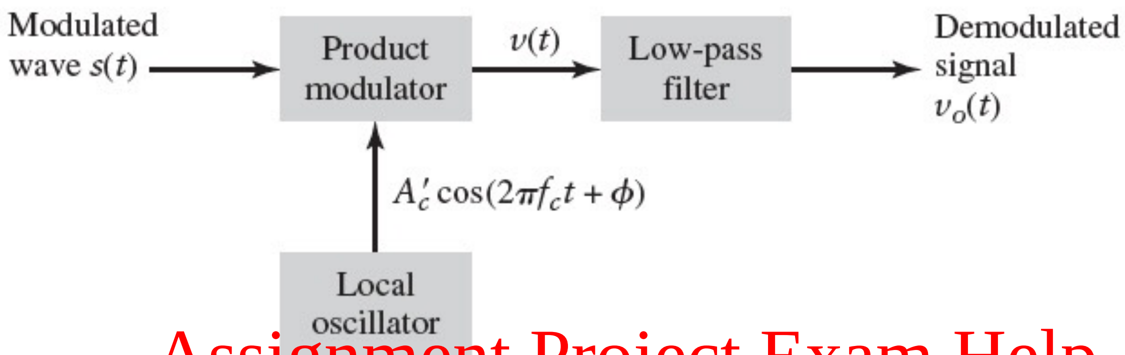
$$
\begin{aligned}
v(t) &= A_c' \cos(2\pi f_c t + \phi) s_{\mathrm{LSSB}}(t) \\
&= \frac{A_c A_c'}{2} [m(t) \cos(2\pi f_c t) + \hat{m}(t) \sin(2\pi f_c t)] \cos(2\pi f_c t + \phi) \\
&= \frac{A_c A_c'}{4} m(t) \cos(\phi) + \frac{A_c A_c'}{4} \hat{m}(t) \sin(\phi) + \text{higher frequency terms}
\end{aligned}
$$

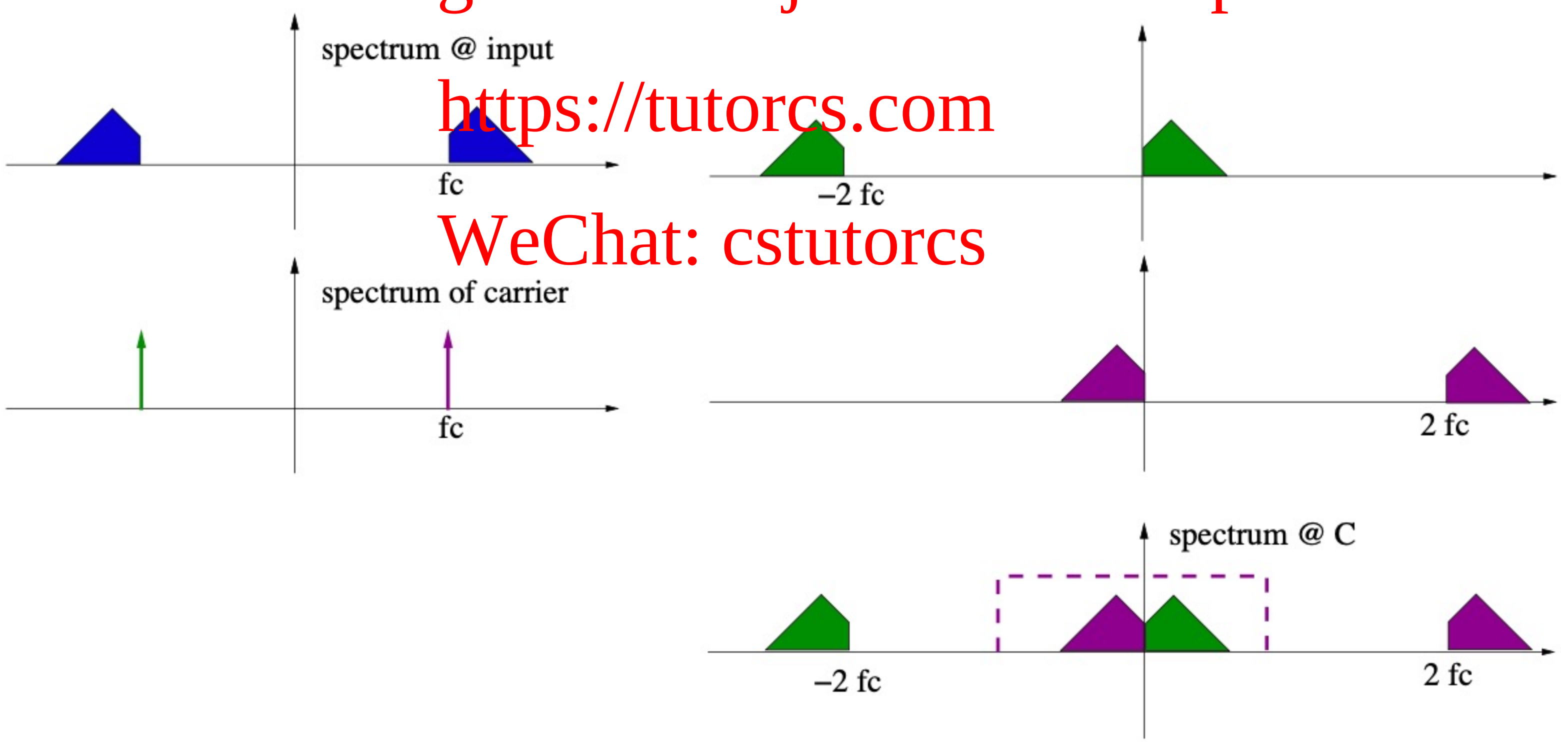
Output of LPF: $v_o(t) = \frac{A_c A_c'}{4} m(t) \cos(\phi) + \frac{A_c A_c'}{4} \hat{m}(t) \sin(\phi).$

Detect the message when $\phi = 0.$

Need accurate synchronization.

Coherent detection of SSB: same as that for DSB-SC AM.



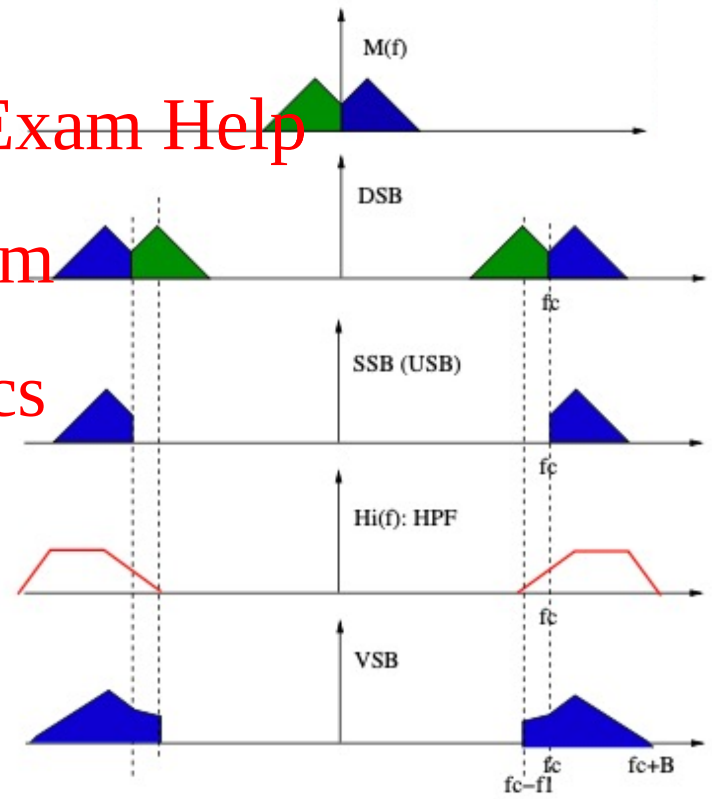


Figures are copied fromhttps://www.ee.ryerson.ca/~lzhao/ELE635/Chap3_AM-notes%20Part%202.pdf

3.6 Vestigial Sideband (VSB) Modulation

SSB modulation does not work well for signals containing significant low frequency components. Need a compromise method between DSB-SC and SSB.

- Instead of completely removing a sideband, a *vestige* of that sideband is transmitted (called 'vestigial sideband').
- *Almost the whole* of the other sideband is also transmitted.
- Bandwidth: $B_T = f_v + W$, where f_v is the vestige bandwidth (typically 25% of W), W is the message bandwidth.



Figures are copied fromhttps://www.ee.ryerson.ca/~lzhao/ELE635/Chap3_AM-notes%20Part%202.pdf

Modulation of VSB with frequency discrimination

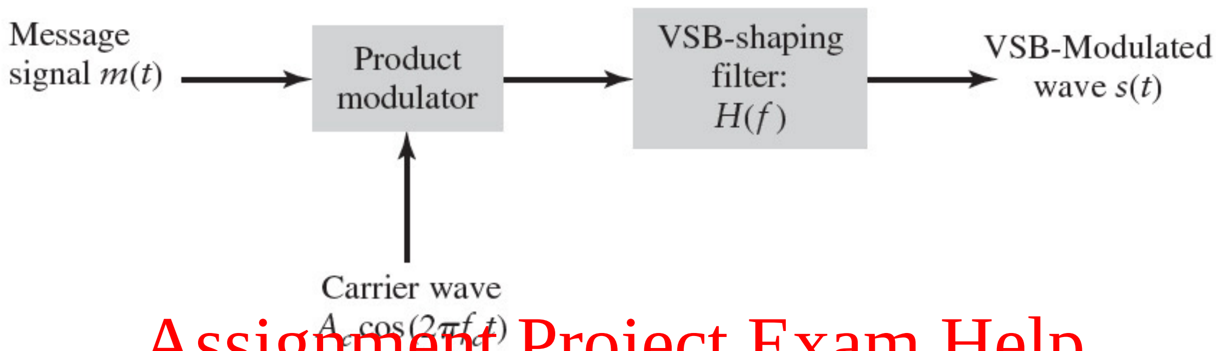


FIGURE 3.23 VSB modulator using frequency discrimination.

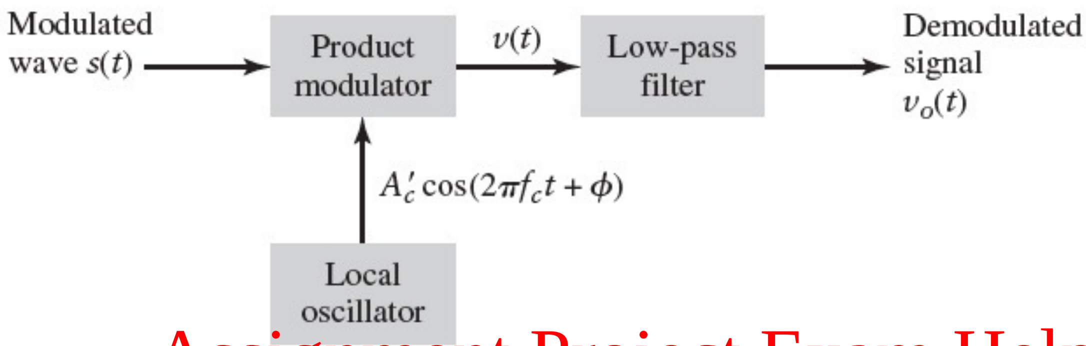
$$\boxed{S(f) = \frac{A_c}{2}\left[M(f - f_c) + M(f + f_c)\right] H(f).}$$

<u>Sideband shaping filter:</u> The transmitted vestige must compensate for the portion missing from the other sideband.

$$\boxed{H(f + f_c) + H(f - f_c) = 1, \ \text{for} \ -W \le f \le W.}$$

No constraint for $|f| > W$.

Coherent detection of VSB: same as that for DSB-SC AM.



Assume perfect synchronization. $\phi = 0$.

$$V(f) = \frac{A_c'}{2}\left[S(f - f_c) + S(f + f_c)\right]$$

$$= \frac{A_c A_c'}{4} M(f)\left[H(f - f_c) + H(f + f_c)\right]$$

$$+ \frac{A_c A_c'}{4} M(f - 2f_c) H(f - f_c) + \frac{A_c A_c'}{4} M(f + 2f_c) H(f + f_c)$$

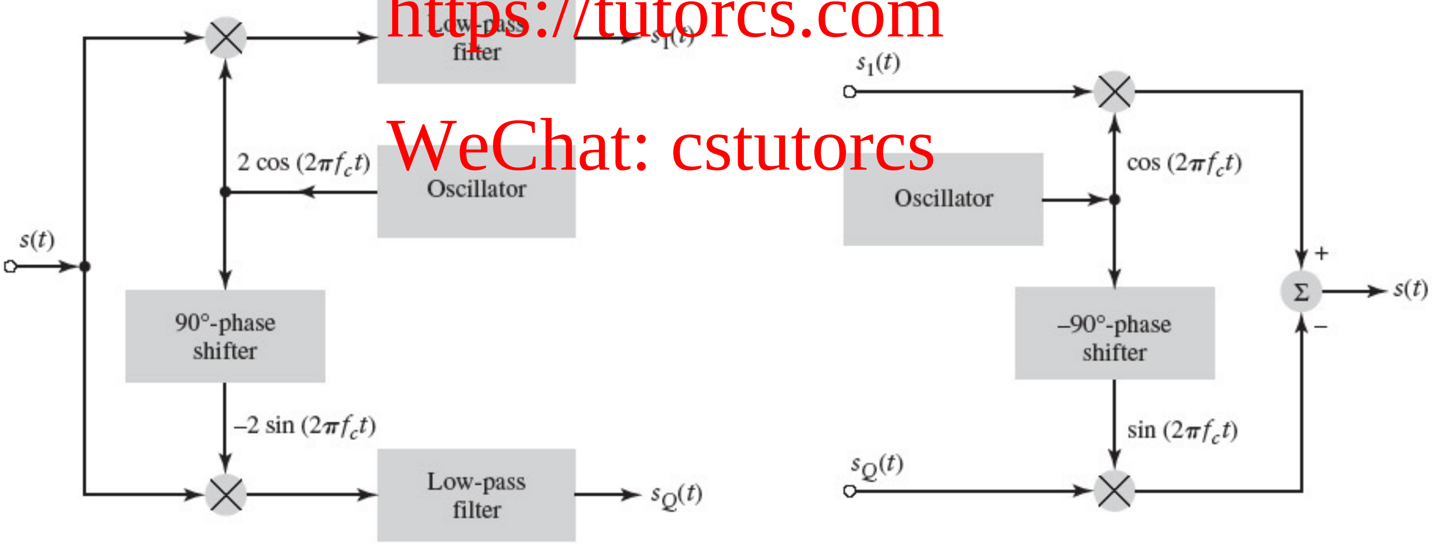
$$= \frac{A_c A_c'}{4} M(f) + \text{higher frequency terms}.$$

Output of LPF: $\frac{A_c A_c'}{4} M(f)$

A generic linearly modulated wave (assume unit carrier amplitude)

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t).$$

- $\cos(2\pi f_c t)$ is the carrier with carrier frequency f_c
- $\sin(2\pi f_c t)$ is the quadrature-phase version of the carrier.
- $s_I(t)$ is the in-phase component.
- $s_Q(t)$ is the quadrature component.



(a) (b)

FIGURE 3.25 (*a*) Scheme for deriving the in-phase and quadrature components of a linearly modulated (i.e., band-pass) signal. (*b*) Scheme for reconstructing the modulated signal from its in-phase and quadrature components.

Type of modulation	*In-phase component* $s_I(t)$	*Quadrature component* $s_Q(t)$	*Comments*
AM	$1 + k_a m(t)$	0	k_a = amplitude sensitivity $m(t)$ = message signal
DSB-SC	$m(t)$	0	
SSB:			
(a) Upper sideband transmitted	$\frac{1}{2}m(t)$	$\frac{1}{2}\hat{m}(t)$	$\hat{m}(t)$ = Hilbert transform of $m(t)$ (see part (i) of footnote 4)
(b) Lower sideband transmitted	$\frac{1}{2}m(t)$	$-\frac{1}{2}\hat{m}(t)$	
VSB:			
(a) Vestige of lower sideband transmitted	$\frac{1}{2}m(t)$	$\frac{1}{2}m'(t)$	$m'(t)$ = response of filter with transfer function $H_Q(f)$ due to message signal $m(t)$. The $H_Q(f)$ is defined by the formula (see part (ii) of footnote 4)
(b) Vestige of upper sideband transmitted	$\frac{1}{2}m(t)$	$-\frac{1}{2}m'(t)$	$H_Q(f) = -j[H(f + f_c) - H(f - f_c)]$ where $H(f)$ is the transfer function of the VSB sideband shaping filter.