Solution to Homework Assignment 8

Solution to Problem 1: For uniform quantization, since the quantization regions are the same size and there are 5 regions, the size of each quantization region is $\Delta = 10/5 = 2$. The quantization regions are:

$$[-5,-3] \quad (-3,-1] \quad (-1,1] \quad (1,3] \quad [3,5].$$

The quantization levels are the mid-points of the regions. They are

$$-4, \quad -2, \quad 0, \quad 2, \quad 4.$$

(b) The output is the following sequence: $\{-2, 2, 4, 0, 2\}$.

Solution to Problem 2:

(a) From the quantizer

$$\begin{aligned}\text{MSE} &= \mathrm{E}[(M-V)^2] \\ &= \int_{-2}^{0}(m+1)^2 f_M(m)dm + \int_{0}^{2}(m-1)^2 f_M(m)dm \\ &= 2\int_{0}^{2}(m-1)^2\left(-\frac{m}{4}+\frac{1}{2}\right)dm \qquad \text{(by symmetry)} \\ &= \frac{1}{3}.\end{aligned}$$

(b)

$$\mathrm{E}[M^2] = \int_{-2}^{2} m^2 f_M(m)dm = 2\int_{0}^{2} m^2\left(-\frac{m}{4}+\frac{1}{2}\right)dm = \frac{2}{3}.$$

The SQNR is

$$\text{SQNR} = \frac{\mathrm{E}[M^2]}{\text{MSE}} = 2 \approx 3 \text{ dB}.$$

(c) V can take 2 values: -1 and 1.

$$\mathrm{P}[V=-1] = \mathrm{P}[M \in (0,2]] = \int_{0}^{2} f_M(m)dm = \int_{0}^{2}\left(-\frac{m}{4}+\frac{1}{2}\right)dm = \frac{1}{2}.$$

$$\mathrm{P}[V=1] = \mathrm{P}[M \in [-2,0]] = \int_{-2}^{0} f_M(m)dm = \int_{-2}^{0}\left(\frac{m}{4}+\frac{1}{2}\right)dm = \frac{1}{2}.$$

Solution to Problem 3:

(a) The Nyquist rate is $2W = 8$M Hz. So the guard band has bandwitdth $8M * 0.125 = 1M$Hz. Since guard band $= f_s/2 - W$, The actual sampling rate is

$$f_s = (1M + 4M) \times 2 = 10 \text{ MHz}.$$

(b) The sampling period is

$$T_s = 1/f_s = \frac{1}{10}\mu s = 0.1\mu s.$$

With 1024 levels, each sample is encoded into $\log_2 1024 = 10$ bits. To send 10 bits for every $0.1\mu s$, the maximum permissible bit duration is

$$T_b = \frac{T_s}{10} = 0.01\mu s$$

The bit rate is $f_b = 1/T_b = 10/T_s = 10f_s = 100$ million bits per second.

Solution to Problem 4:

(a) For any sample of $m(t)$, we have $m(t) \in [-1.5, 3]$. To have 8 regions of the same size, the size of each region is $\Delta = 4.5/8 = 0.5625$. The quantization regions for the 8-level uniform quantizer are

$$[-1.5, -0.9375],\ (-0.9375, -0.375],\ (-0.375, 0.1875],\ (0.1875, 0.75],$$
$$(0.75, 1.3125],\ (1.3125, 1.875],\ (1.875, 2.4375],\ (2.4375, 3].$$

The quantization levels for the 8 regions are, respectively

$$-1.21875, -0.65625, -0.09375, 0.46875, 1.03125, 1.59375, 2.15625, 2.71875.$$

(b) The sampled analog sequence is

$$\cdots,\ 0.5,\ -1,\ 0.5,\ 0.5,\ -1,\ 0.5,\ \cdots.$$

The output digital sequence of the quantizer is

$$\cdots,\ 0.46875,\ -1.21875,\ 0.46875,\ 0.46875,\ -1.21875,\ 0.46875,\ \cdots$$

Solution to Problem 5: The codewords are 11 01 00 10 11. $N = 2^2 = 4$. For the range $[-1, 1]$, the representation levels for the uniform quantizer are

$$-3/4, -1/4, 1/4, 3/4.$$

The codewords corresponding to these levels are

$$00, 01, 10, 11.$$

Thus the quantized values are:

$$3/4,\ -1/4,\ -3/4,\ 1/4,\ 3/4.$$

Solution to Problem 6:

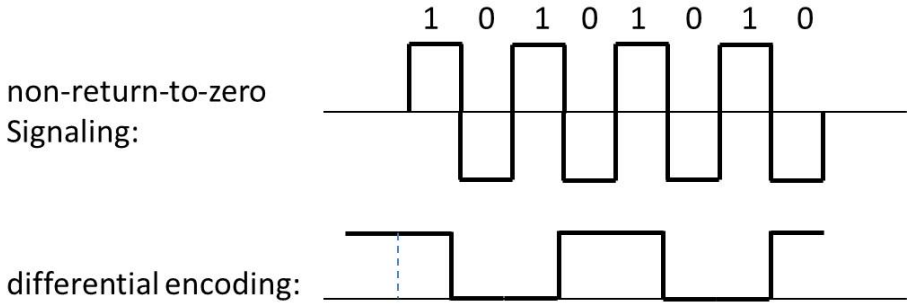

Figure 1: Line codes for Sequence P6(a).

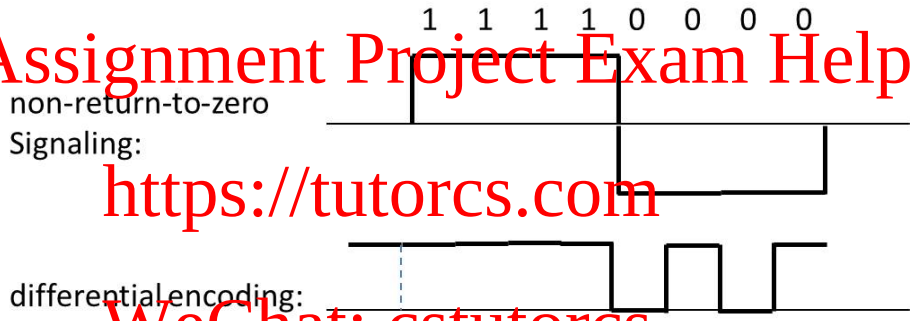

Figure 2: Line codes for Sequence P6(b).

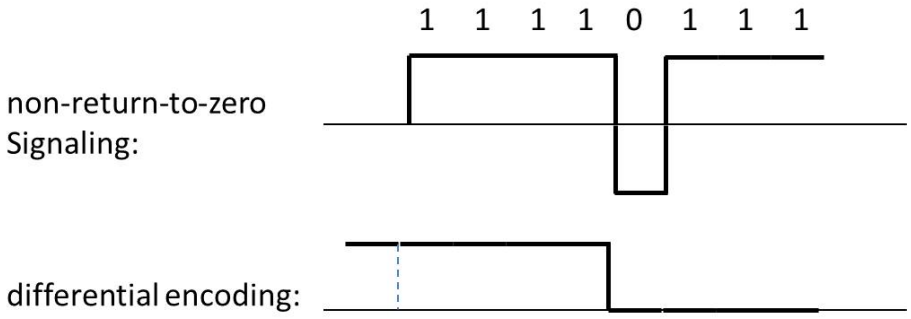

Figure 3: Line codes for Sequence P6(c).