Solution to Homework Assignment 5

Solution to Problem 1: The bandwidth of the message is 5k Hz.

a) For LSSB modulation, the modulated signal has the same bandwidth as the message: $B_T = 5$k Hz.

b) For DSB-SC AM, the bandwidth of the modulated signal is twice the bandwidth of the message: $B_T = 10$k Hz.

b) For conventional AM, the bandwidth of the modulated signal is twice the bandwidth of the message: $B_T = 10$k Hz.

Solution to Problem 2:

a)

$$\text{sinc}(2Bt) \rightleftharpoons \frac{1}{2B}\text{rect}\left(\frac{f}{2B}\right).$$

$$\text{sinc}^2(2Bt) \rightleftharpoons \frac{1}{4B^2}\text{rect}\left(\frac{f}{2B}\right) * \text{rect}\left(\frac{f}{2B}\right) = \frac{1}{2B}\Delta\left(\frac{f}{4B}\right).$$

$$m(t) = 2B\text{sinc}^2(2Bt) \rightleftharpoons M(f) = \Delta\left(\frac{f}{4B}\right).$$

Thus,

$$\hat{M}(f) = -j\text{sgn}(f)M(f) = -j\text{sgn}(f)\Delta\left(\frac{f}{4B}\right).$$

The frequency spectra are as shown in Figure 1 (where $B = 2$k).

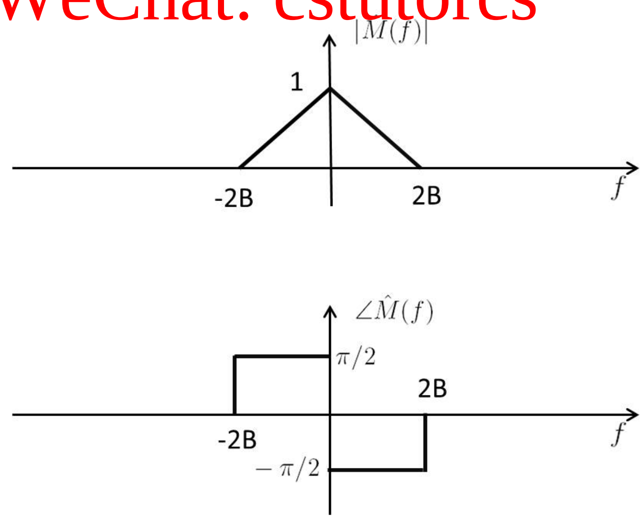

Figure 1: Frequency spectrum for the message in Problem 3.

b) $A_c = 100$, $f_c = 10$k, and $B = 2$k.

$$s_{\text{USSB}}(t) = 50m(t)\cos(20,000\pi t) - 50\hat{m}(t)\sin(20,000\pi t),$$

By following similar frequency analysis for LSSB AM in lecture notes, it can be shown that (details are omitted)

$$S_{\text{USSB}}(f) = 50\left[M_{+}(f-10,000) + M_{-}(f+10,000)\right]$$
$$= \begin{cases} 50\left[-\frac{f-10,000}{4,000}+1\right] & 10,000 < f < 14,000 \\ 50\left[\frac{f+10,000}{4,000}+1\right] & -14,000 < f < -10,000 \\ 0 & \text{otherwise} \end{cases}.$$

The magnitude spectrum is shown in Figure 2.

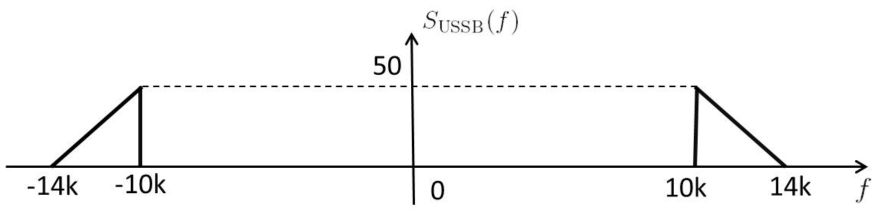

Figure 2: The magnitude spectrum of the USSB wave.

Solution to Problem 3: In the QAM system,

(a)

$$\varphi_{QAM}(t) = m_1(t)\cos(2\pi f_c t) + m_2(t)\sin(2\pi f_c t)$$
$$= sinc(t)\cos(2\pi f_c t) + sinc^2(t)\sin(2\pi f_c t)$$

To calculate the bandwidth of $\varphi_{QAM}(t)$, we should find the Fourier transform of $m_1(t)$ and $m_2(t)$, i.e.,

$$m_1(t) = sinc(t) \Leftrightarrow M_1(f) = rect(f) \quad \Rightarrow B_1 = \frac{1}{2}\text{ Hz}$$
$$m_2(t) = sinc^2(t) \Leftrightarrow M_2(f) = tri(f/2) \quad \Rightarrow B_2 = 1\text{ Hz}$$
$$\Rightarrow B_{\varphi_{QAM}} = 2\,max\{B_1, B2\} = 2\text{ Hz}.$$

(b) The required bandwidth of the low-pass filter is half of $B_{\varphi_{QAM}}$, i.e., $B_{LPF} = 1$ Hz.

(c)

$$m_1(t) = sinc(t) \Leftrightarrow M_1(f) = rect(f) \quad \Rightarrow B_1 = \frac{1}{2}\text{ Hz}$$
$$m_2(t) = \frac{1}{2}sinc^2(2t) \Leftrightarrow M_2(f) = \frac{1}{4}tri(f/4) \quad \Rightarrow B_2 = 2\text{ Hz}$$
$$\Rightarrow B_{\varphi_{QAM}} = 2\,max\{B_1, B2\} = 4\text{ Hz}$$
$$\Rightarrow B_{LPF} = \frac{B_{\varphi_{QAM}}}{2} = 2\text{ Hz}$$

(d) We get the LSB version of the SSB signal.