Solution to Homework Assignment 9

Solution to Problem 1:

(a)

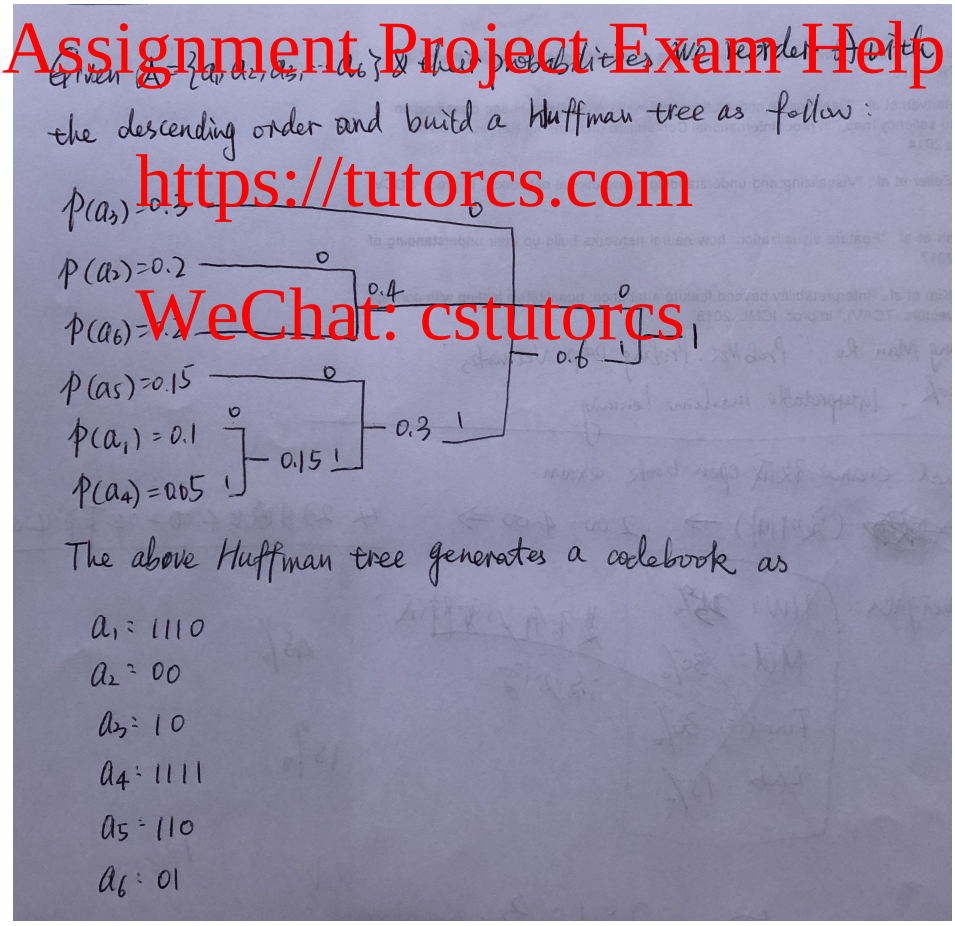
$$H(X) = -\sum_{i=1}^{6} p_i \log_2 p_i = -(0.1 \log_2 0.1 + 0.2 \log_2 0.2 + 0.3 \log_2 0.3 + 0.05 \log_2 0.05 + 0.15 \log_2 0.15 + 0.2 \log_2 0.2)$$
$$= 2.4087 \quad \text{bits/symbol}$$

(b) If the source symbols are equiprobable, then $p_i = \frac{1}{6}$ and

$$H_u(X) = -\sum_{i=1}^{6} p_i \log_2 p_i = -\log_2 \frac{1}{6} = \log_2 6 = 2.5850 \quad \text{bits/symbol}$$

As it is observed the entropy of the source is less than that of a uniformly distributed source.

Solution to Problem 2:



Solution to Problem 3:

(a) Not a codeword since the last parity check equation does not hold.

(b) The decoded information bits are: 0011.

(c) The coded sequence for 0001 is 0001111. To receive 0011101, errors occur on the 3rd and 6th bits and do not occur on the other 5 bits. The probability is

$$0.02^2 \times 0.98^5 \approx 0.036\%.$$