Homework Assignment 4

Due: 16:00pm Tuesday, March. 7, 2023

Problem 1. In conventional AM, the carrier is $c(t) = A_c \cos(2\pi f_c t)$ and the message signal is

$$m(t) = \text{sinc}(t) + \text{sinc}^2(t).$$

Let the modulation index be $k_a = 1$ and $f_c \gg 1$.

a) Find the frequency-domain representation and draw the frequency spectrum of the modulated signal.

b) What is the bandwidth of the modulated signal.

Problem 2. (Haykin and Moher Problem 3.17) In lectures, we focused on

$$c(t) = A_c \cos(2\pi f_c t)$$

as the sinusoid carrier wave. Suppose we choose

$$c(t) = A_c \sin(2\pi f_c t)$$

as the sinusoid carrier wave to modulate the following single-tone signal

$$m(t) = A_m \sin(2\pi f_m t).$$

a) Evaluate the spectrum of the new AM wave:

$$s(t) = A_c[1 + k_a m(t)] \sin(2\pi f_c t).$$

b) Compare the result derived in Part a) with those shown in lectures and discuss.

Problem 3. Suppose the signal $g(t) = m(t) + \cos(2\pi f_c t)$ is applied to a nonlinear system whose output is

$$y(t) = g(t) + \frac{1}{2} g^2(t).$$

Determine and sketch the spectrum of $y(t)$ when $M(f)$ is as shown in Figure 1, where $W \ll f_c$.

Problem 4. Consider the AM system shown in Fig. 2. The message signal is $m(t) = 4\,\text{sinc}(4t)$.

i). If $f_c = 2$, sketch the frequency spectra of the signals at points (a), (b), (c), (d), and (e).

ii). Find the minimum value of f_c for which the signal at point (e) is equal to the signal at point (b).

Problem 5. Show that the Hilbert transform of $e^{j2\pi f_0 t}$ is $-j\text{sgn}(f_0)e^{j2\pi f_0 t}$.

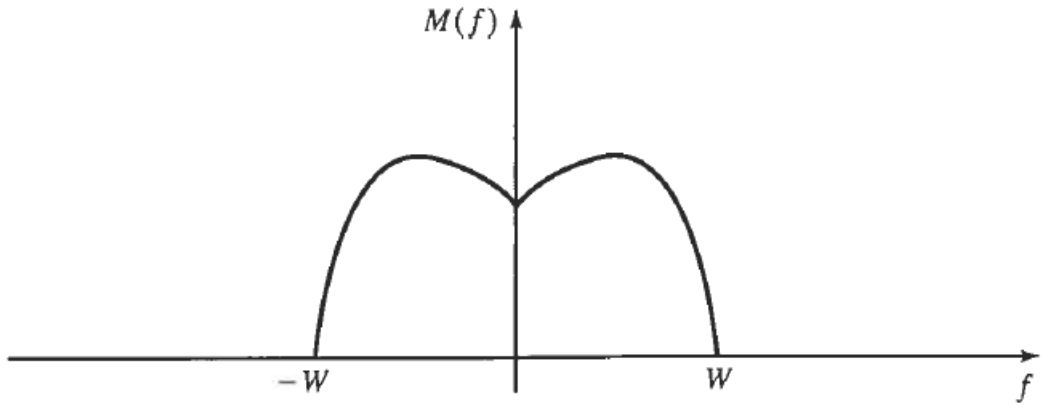

Figure 1: Frequency spectrum of $m(t)$ for Problem 3.

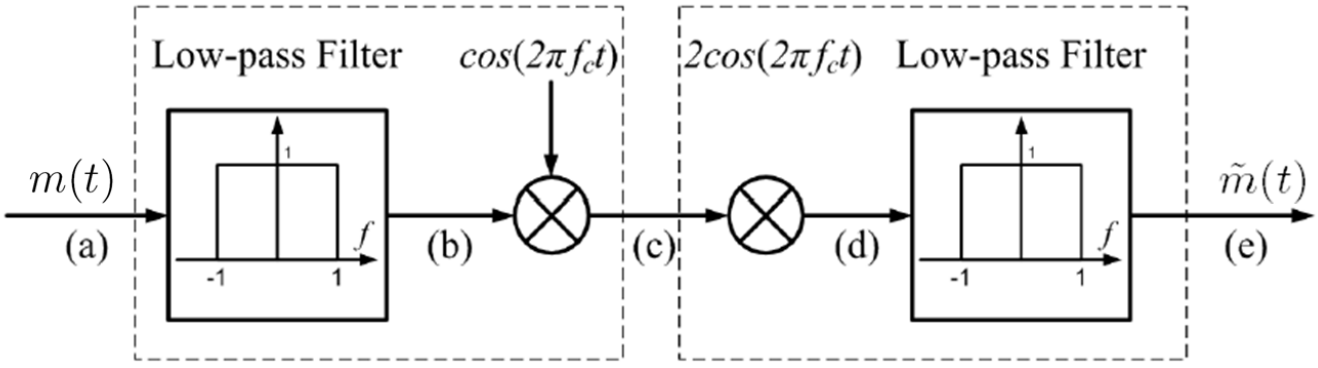

Figure 2: Fig. 2: Frequency spectrum for $m(t)$ in Problem 4.