Homework Assignment 2

Due: 16:00pm Tuesday, Feb. 14, 2023

Problem 1. Given $g_1(t) \rightleftharpoons G_1(f), g_2(t) \rightleftharpoons G_2(f)$, please use the definitions of FT and inverse FT to proof the the following FT properties.

a) The differentiation property: $\frac{d}{dt}g_1(t) \rightleftharpoons j2\pi f G_1(f)$.
b) The convolutional property: $g_1(t) * g_2(T) \rightleftharpoons G_1(f)G_2(f)$.
c) Parseval's theorem: $E_g = \int_{-\infty}^{\infty} |g_1(t)|^2 dt = \int_{-\infty}^{\infty} |G_1(f)|^2 df$.

Problem 2. a) Find the energy spectral density of the signal $g(t) = e^{-|t|}$.
b) Show that the signal $g_1(t) = e^{-|t-2|}$ has the same energy spectral density as $g(t)$.

Problem 3. Let $g_{T_0}(t)$ be a periodic signal with period π. Over the period $0 \leq t < \pi$, it is defined by $g_{T_0}(t) = \cos t$. Find the Fourier transform of $g_{T_0}(t)$ and draw the frequency spectrum.
Note: $\cos x \cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$,
$\sin x \cos y = \frac{1}{2}[\sin(x-y) + \sin(x+y)]$,
$\int e^{ax} \cos(bx)dx = \frac{e^{ax}}{a^2+b^2}[a\cos(bx) + b\sin(bx)]$.