

Problem Set 2 (Business Cycle Models)

ECON 6002

Due date: Monday, 8 May, 6pm

**NOTE:** To receive full marks, it is crucial to show all of your workings and not just provide a final algebraic or numerical answer. Solve to a higher number of decimals, but report final numerical answers to two decimal and place a box around the answer. It is also important that you provide answers in your own words. Any quotations from the textbook or other sources must be in quotation marks and attributed to the original source.

1. Abstracting from long-run growth by setting  $n = g = 0$  and from persistent shocks by setting  $\rho_A = \rho_G = 0$ , with  $\tilde{A}_t \equiv \ln A_t - \ln \bar{A}$  and  $\tilde{G}_t \equiv \ln G_t - \ln \bar{G}$ , and normalizing the population to  $N = 1$ , the following nine equations describe the “baseline” RBC model in Chapter 5:

$$Y_t = C_t + I_t + G_t \quad (1)$$

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha} \quad (2)$$

$$K_{t+1} = K_t + I_t - \delta K_t \quad (3)$$

$$\tilde{A}_t = \epsilon_{A,t} \quad (4)$$

$$\tilde{G}_t = \epsilon_{G,t} \quad (5)$$

$$r_t = \alpha (A_t L_t / K_t)^{1-\alpha} - \delta \quad (6)$$

$$w_t = (1 - \alpha) (K_t / A_t L_t)^\alpha A_t \quad (7)$$

$$\frac{1}{\bar{C}_t} = e^{-\rho} E_t \left[ \frac{1}{\bar{C}_{t+1}} (1 + r_{t+1}) \right] \quad (8)$$

$$\frac{C_t}{1 - E_t} = \frac{w_t}{1 - E_t} \quad (9)$$

- (a) Find the steady state for this economy under the following calibration:  $\alpha = \frac{1}{3}$ ,  $\delta = 0.05$ ,  $\bar{r} = 0.03$ ,  $\bar{A} = 1$ , and  $\bar{L} = 0.5$  and choose  $\bar{G}$  such that  $\bar{G}/\bar{Y} = 0.2$ . In particular, you should find the remaining parameters values  $b$  and  $\rho$  that are consistent with steady state and determine steady-state values for the endogenous variables,  $\bar{Y}$ ,  $\bar{C}$ ,  $\bar{I}$ ,  $\bar{G}$ ,  $\bar{K}$ , and  $\bar{w}$ . (Hint: first solve for  $\rho$  using (8), then solve for  $\bar{K}$  using (6), then  $\bar{I}$  using (3), then  $\bar{w}$  using (7), then  $\bar{Y}$  using (2), then  $\bar{C}$  using (1), then  $b$  using (9).)
- (b) Now consider the special case of the model where  $\delta = 1$  instead of  $\delta = 0.05$  and  $G_t = 0$  for all  $t$  (note:  $\rho$  will remain the same and  $b$  will be different, but you do not need to solve for it). Solve for  $Y_t$ ,  $C_t$ ,  $I_t$ ,  $K_{t+1}$ ,  $r_t$ , and  $w_t$  as analytical expressions of exogenous and predetermined variables  $A_t$  and  $K_t$  and constants. (Hint: with 100% depreciation, there is a constant saving rate  $s = \alpha e^{-\rho}$  and constant labour supply  $L_t = \bar{L}$ . Given this solution to the household optimization problem, first solve for  $Y_t$ ,  $r_t$ , and  $w_t$  from equations (2), (6), and (7) and then the solutions for  $C_t$ ,  $I_t$ , and  $K_{t+1}$  are straightforward.)
- (c) Again, for the special case of the model, what is the percentage change in output and percentage point change in the interest rate if the economy is at steady state at time  $t - 1$ , but there is a shock  $\epsilon_{A,t} = 0.25$  (i.e., 25%) at time  $t$ ? Explain the economic

intuition behind the responses of output and the interest rate in terms of the marginal products of labour and capital. (Hint: note that the  $\epsilon_{A,t} = 0.25$  shock is to  $\ln A_t$ , but the model solution is for the level of  $A_t$ . First solve for the steady-state level of output and then solve for output and the real interest rate given the shock.)

2. Consider Calvo price-setting firms with partial indexation. That is, if a firm is not visited by the Calvo tooth fairy in period  $t$ , its price in  $t$  is the previous period's price plus  $\gamma\pi_{t-1}$ ,  $0 \leq \gamma \leq 1$ . The average price in period  $t$  is  $p_t = \alpha x_t + (1 - \alpha)(p_{t-1} + \gamma\pi_{t-1})$ , where  $\alpha$  is the fraction of firms visited by the Calvo tooth fairy in any given period and  $x_t$  is the price they set. The resulting Phillips curve is a hybrid one:  $\pi_t = \frac{\gamma}{1+\beta\gamma}\pi_{t-1} + \frac{\beta}{1+\beta\gamma}E_t\pi_{t+1} + \frac{1}{1+\beta\gamma}\kappa\tilde{y}_t$ , where  $\beta > 0$  is the discount factor,  $\kappa = \frac{\alpha}{1-\alpha}[1 - \beta(1 - \alpha)]\phi > 0$  determines the slope of the Phillips curve,  $\tilde{y}_t$  is the output gap, and  $E_t\pi_{t+1}$  is the expectation (taken at time  $t$ ) of inflation at  $t + 1$ .

- (a) Show that  $x_t - p_t = \frac{1-\alpha}{\alpha}(\pi_t - \gamma\pi_{t-1})$ .
- (b) Use the result in (a) and the representative firm's optimal price under Calvo pricing with partial indexation being  $x_t = p_t + (1 - \beta(1 - \alpha))\phi\tilde{y}_t + \beta(1 - \alpha)(E_t(x_{t+1} - p_{t+1}) + E_t\pi_{t+1} - \gamma\pi_t)$  to derive the hybrid Phillips curve.
- (c) What value of  $\gamma$  would lead to the highest degree of inflation persistence? Why?
- (d) Now assume that  $\beta = 0.9$ ,  $\gamma = 0.5$ , and  $\kappa = 0.1$ . Assume that the central bank has some control of the evolution of  $\tilde{y}$ . Suppose that the central bank announces a *permanent* and *fully credible* reduction in its target or steady-state inflation rate from 7% to 2% at  $t = 1$  (prior to this, the economy was at the steady state with 7% inflation and zero output gap). Determine the cost of this disinflation episode. How much is the output gap reduced?

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