Digital Modulation.

Baseband transmission requires wire links. For long distance transmission we need radio frequency (RF) signals which can be frequency multiplexed. This requires modulation to generate a Band Pass (BP) signal as shown in Figure 1.

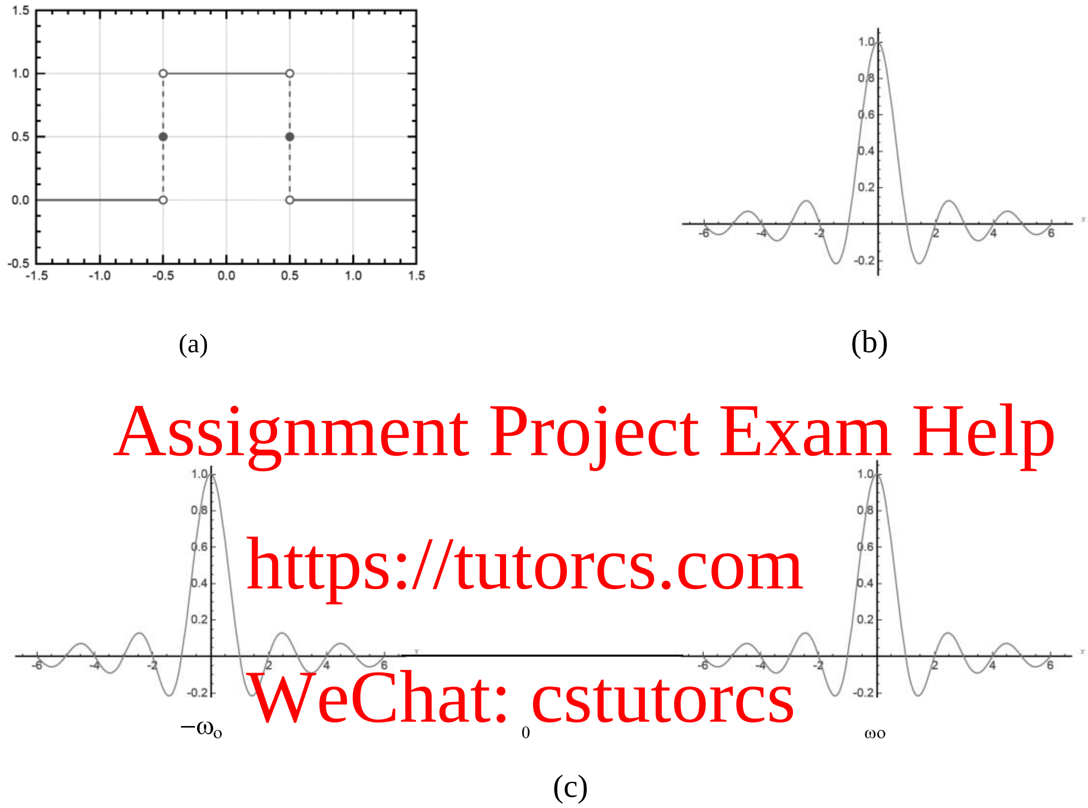


Figure 1. (a) Baseband signal, $f(t)$ (b) corresponding baseband spectrum, $F(\omega)$ (c) bandpass spectrum resulting from modulation, $f(t)\cos\omega_o t$.

In L3 you would have studied the three basic CW modulation techniques: Amplitude Modulation, AM, Frequency Modulation, FM and Phase Modulation, PM.

In digital modulation the modulating signal comprises of discrete levels (2 to 3 levels) therefore we rename the modulation schemes for digital data as: Amplitude Shift Keying, ASK, Frequency Shift Keying, FSK, and Phase Shift Keying, PSK as illustrated in figure 2.

In general, signals are detected and decoded in their Band Pass (BP) form rather than returning them to baseband.

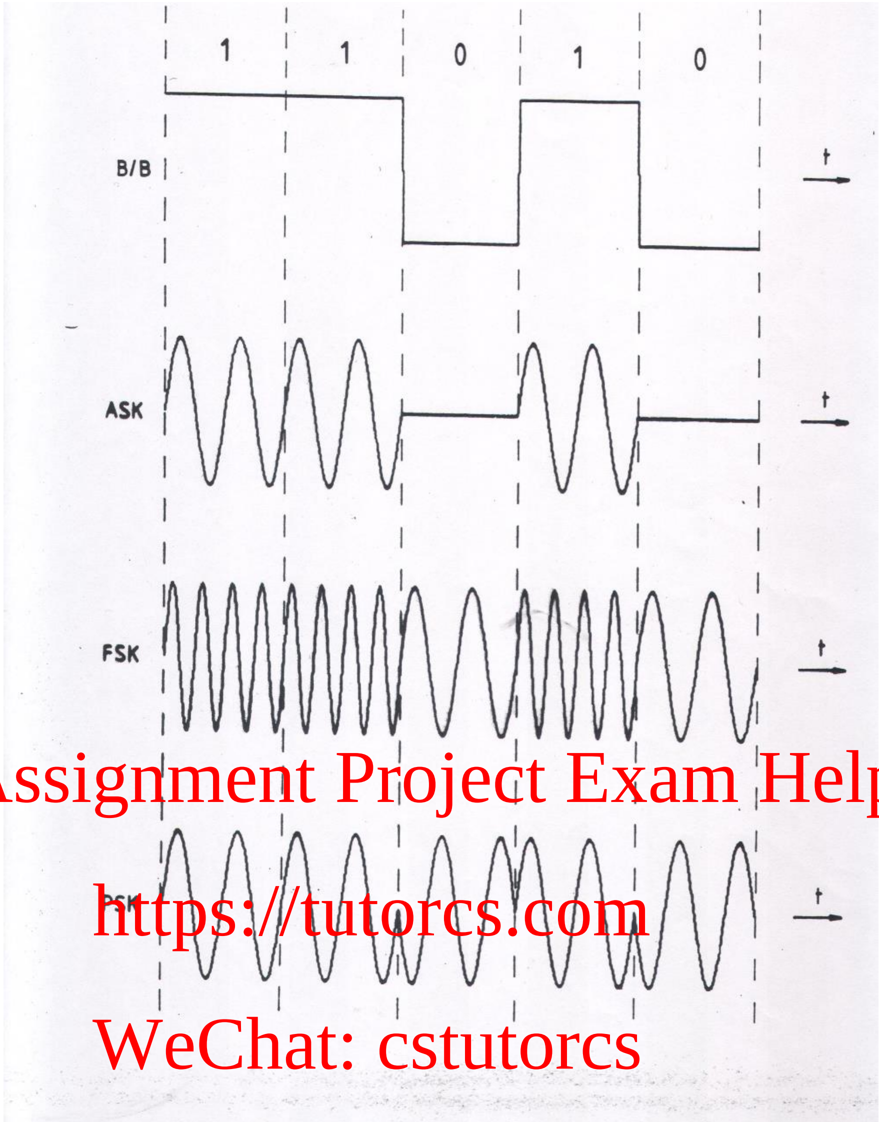


Figure 2. Digital Modulation Techniques

Regardless whether the coded signal is transmitted at baseband or at RF, an important aspect is detection. In the following we will study detection of both baseband and bandpass signals in the presence of additive noise. Thus in the first part we will need to quantify noise and its properties, then we will study two optimum receivers namely the matched filter and the coherent or correlation detector and finally we will study the error caused by the presence of noise. We will also give expressions for sub-optimum receivers and compare these with the performance of the optimum receivers.

Detection of digital signals in the presence of noise

Detection implies identification or recovery of the transmitted signal. The problem of detection arises when the signal is contaminated by noise during transmission and at the receiver.

Noise refers to anything that is undesirable or unwanted; that is unwanted waves that tend to disturb the transmission and processing of signals in physical systems and over which we have incomplete control.

Sources of noise are two types:

1. **External to the system**: These include sky noise, man-made noise such as ignition, and power lines which radiate Electromagnetic (EM) waves.

Sky noise is attributed to a number of sources which generate electromagnetic noise which is picked up by an antenna intended to receive a signal. Below 30 MHz the principal source is lightning discharges (also known as atmospheric noise) propagated from all around the world. Above 30 MHz cosmic noise, generated by the radiation from outer space like stars and pulsars predominate.

2. **Internal to the system**: These arise from spontaneous fluctuations of current or voltage in electrical circuits and are usually classified as shot noise and thermal noise.

Shot noise: arises from the random movement of discrete charged particles across a potential gradient. For example the electrons emitted from the cathode of a tube produce a specific current. However fluctuations in the current occur due to the variations of the number of particles that are emitted.

Another example is a diode where carriers crossing the junction vary in number due to the random recombination and generation of hole- electron pairs.

Thermal noise (Johnson): arises due to the random motion of thermally excited free electrons in a conducting medium such as a resistor. The path of each electron in motion is randomly oriented due to collisions. The net effect is a random current with zero mean value as illustrated in Figure 3. This generates a noise voltage across the terminals of a resistor. Thus a noisy resistor as in Figure 4 can be modelled as a noiseless resistor in series with a noise voltage. Due to the randomness of noise, the voltage is usually represented by its *rms* value as shown in Figure 4.b.

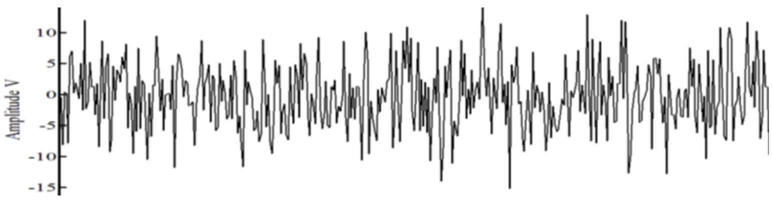


Figure 3 Example of thermal noise

Figure 4 (a) Noisy resistor, (b) model of a noisy resistor as a noiseless resistor in series with *rms* voltage.

In Figure 4.b the mean square voltage $\overline{v_n^2}$ is the mean square open circuit voltage generated by a resistor R in B Hz and is given by:

$$\overline{v_n^2} = 4kTRB \text{ volts}^2 \tag{1}$$

k= Boltzmann's constant=1.38X10^{-23}Joules/K^{o}., T= Temp. in Kelvin.
B= Bandwidth of the measuring device (Hz), R= Resistance in Ω.

<u>Note</u>:

Thermal noise arises from a large number of electrons whose motion is independent and hence it can be represented by a Gaussian distribution with zero mean given by

$$p(v) = \frac{1}{\sqrt{2\pi \overline{v_n^2}}} \exp\left(-\frac{v^2}{2\overline{v_n^2}}\right) \tag{2}$$

where $p(v)$ gives the probability that the noise voltage has the value v and the probability that the noise voltage is between v_1 and v_2 is given by

$$p(v_1 < v < v_2) = \int_{v_1}^{v_2} p(v)dv \tag{3}$$

The previous sources of noise can all be modelled by <u>white noise</u>: white because white colour contains all frequencies of visible light. Similarly white noise has a spectrum which covers all frequencies. White noise has a double sided power spectral density equal to $N_o/2$ Watts/Hz as shown in Figure 5.

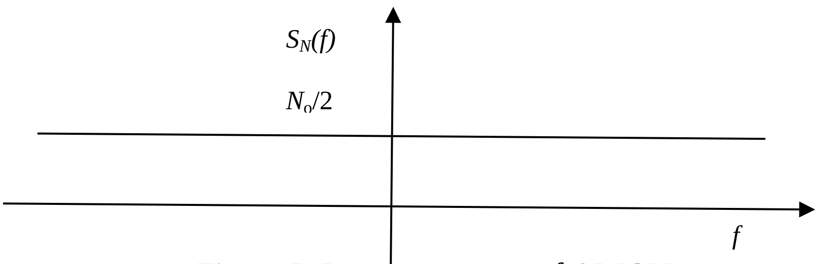


Figure 5: Power spectrum of AWGN

The 1/2 is included to cover the double sided spectrum.

<u>Note</u>: $N_o/2$ gives power/Hz. Thus to get the power we would need to integrate with frequency i.e.

$$\text{Power} = \int_{-\infty}^{\infty} \frac{N_o}{2} df \rightarrow \infty$$

Hence, it is not physically realisable. However for practical systems, only a finite bandwidth (BW) needs to be considered, and for all practical purposes, noise can be considered to be white. For a Low Pass Filter (LPF) with bandwidth equal to B as in Figure 6, the power at the output of the filter is equal to N_o B Watts

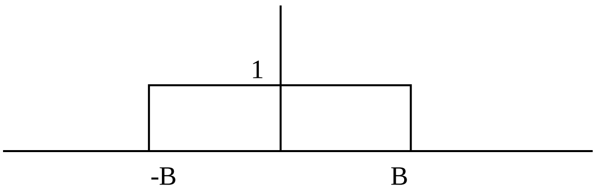


Figure 6. LPF used to pass noise within its bandwidth B

Taking the case of thermal noise for maximum transfer of noise power we need the load resistor R_L in Figure 7 to be equal to the noisy resistor R. In this case the power transferred, *P* is equal to:

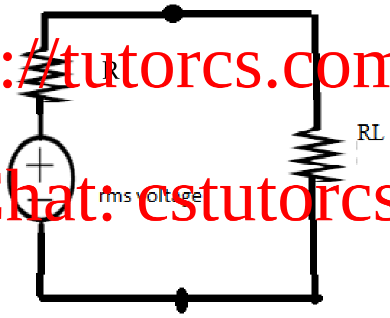


Figure 7 Configuration for maximum power transfer

$$P = \left[\frac{\sqrt{\overline{v_n^2}}}{2}\right]^2 \frac{1}{R} = \frac{\overline{v_n^2}}{4R}\ Watts$$

Replacing $\overline{v_n^2} = 4kTRB$, we get

$$P = kTB \tag{4}$$

Thus in B Hz, the power spectral density is equal to kT Watts/Hz. This means that the noise power density $N_o = kT$.

The LPF in Figure 6 is an ideal filter and cannot be realised. In the following we will learn how to find the output noise power for a practical system with transfer function H(*f*) as shown in Figure 8.

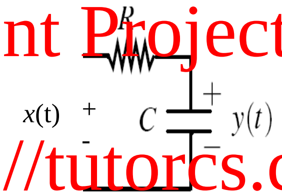
Figure 8 Block diagram of a general system for the evaluation of noise power

For a network with a current or voltage transfer function H(f), the power transfer function is $|H(f)|^2$

Thus assuming white noise at the input, the output noise power spectral density is given by

$$PSD\,|_{output} = \frac{N_o}{2}|H(f)|^2 Watts/Hz$$

Example: For the RC network with input x(t) and output y(t), the transfer function of the system H(f) is given by



$$H(f) = \frac{1}{1 + j2\pi fRC}$$

The output spectral density of noise is then given by

$$PSD\,|_{output} = \frac{N_o}{2}|H(f)|^2 = \frac{N_o}{2}\frac{1}{1+(2\pi fRC)^2} Watts/Hz$$

Integrating over frequency gives the total power as

$$P\,|_{output} = \frac{N_o}{2}\int_{-\infty}^{\infty}\frac{1}{1+(2\pi fRC)^2}df = \frac{N_o}{2}\alpha\left(\tan^{-1}(2\pi f/\alpha)\right)|_{-\infty}^{\infty}\ Watts$$

$$= \frac{N_o\alpha}{4} \text{ where } \alpha = \frac{1}{RC}$$