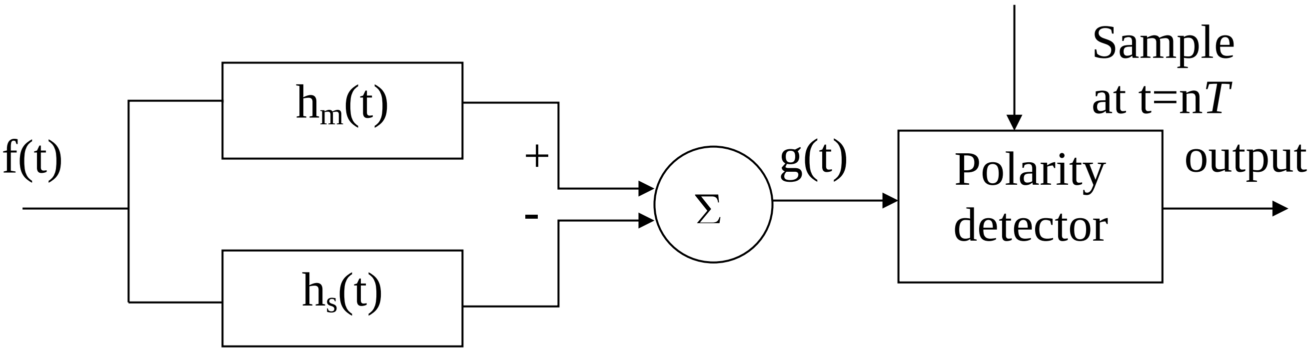
The general matched filter detector for binary signals

The previous example of the baseband signal is a special case.

In general we need two matched filters, one matched to the mark signal (binary 1) and one matched to the space signal (binary 0)

Assuming the mark and space signals to be of the same duration T and energy E, the detector is of the following form:



$f_m(t)$ ---- signal representing mark (1)
$f_s(t)$ ---- signal representing space (0)

$$\boxed{\begin{aligned} h_m(t) &= f_m(T-t) \\ h_s(t) &= f_s(T-t) \end{aligned}} \tag{17}$$

Assume that the input is a mark signal, the output g(t) is then given by

$$g(t) = f_m(t) * h_m(t) - f_m(t) * h_s(t) \tag{18}$$

$$g(t) = \int_0^t f_m(\tau) f_m(T-t+\tau) d\tau - \int_0^t f_m(\tau) f_s(T-t+\tau) d\tau$$

$$g(T) = \int_0^T f_m(\tau) f_m(\tau) d\tau - \int_0^T f_m(\tau) f_s(\tau) d\tau$$

Define $$\rho = \frac{\int_0^T f_m(\tau) f_s(\tau) d\tau}{\sqrt{\int_0^T f_m^2(\tau) d\tau \int_0^T f_s^2(\tau) d\tau}} = \frac{\int_0^T f_m(\tau) f_s(\tau) d\tau}{E} \tag{19}$$

$g(T) = E\,(1-\rho)$ for a mark signal (20)

ρ: is known as the correlation coefficient which gives an idea about the similarity between the two functions and $-1 \leq \rho \leq 1$

Similarly, g(t) can be found when a space signal is received to be :

$$\boxed{g(T) = -E\,(1-\rho\,)} \qquad (21)$$

From equations 16 and 20-21 we note that the detection depends on:

1) Correlation coefficient, ρ
2) Energy in the signal, E
3) Noise power as shown for the matched filter in equation 16, $(SNR_o)\big|_{t=T} = \dfrac{E}{\frac{N_o}{2}}$

Let us take special cases of ρ:

1) ρ = -1, then the signals are anti-phase, i.e. $f_m(t) = -f_s(t) \Rightarrow$ maximum difference between the two signals. From equations 20 and 21:

g(T)=2E for mark and = -2E for space

2) ρ =0 $\Rightarrow$ the signals are orthogonal since:

$$\int_0^T f_m(t).f_s(t).dt = 0 \qquad (22)$$

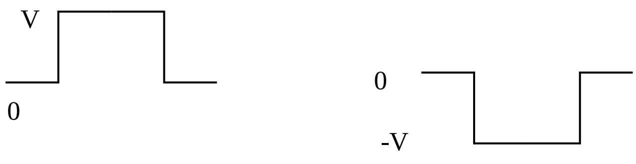
g(T)=E for mark and = -E for space

3) ρ =+1 $\Rightarrow$ $f_m(t) = f_s(t)$ i.e. we cannot distinguish between the two signals.

The above implies that a careful choice of signals to represent the *mark* and *space* signals is important.

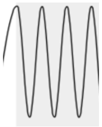
Examples:

1. Polar NRZ

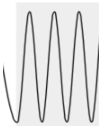


Binary 1, +V $\rho = -1$ Binary 0, -V,

2. PSK



$\rho = -1$



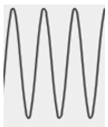
Binary 1 Binary 0

For the antipodal signals with $\rho = -1$ one matched filter is adequate since the polarity of the output at t=nT indicates the data.

Examples of orthogonal signals are the binary FSK signal and the split Manchester code shown in the figure below.

1. FSK

$\rho = 0$

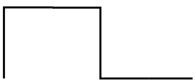


Binary 1 Binary 0

2. Split phase Manchester code $\rho = 0$

V

0



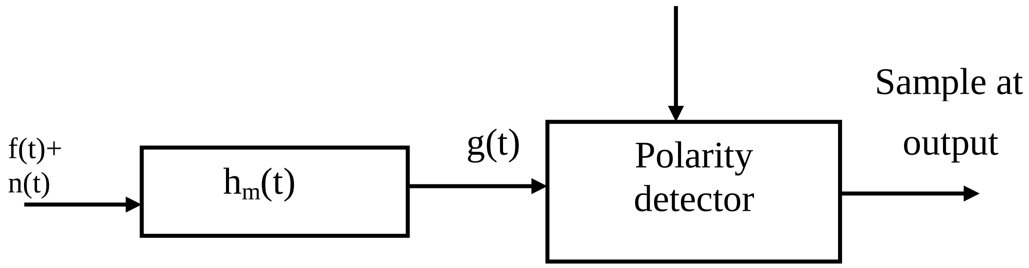
$\rho = 0$

V

0

Binary 1 Binary 0

Coherent detection of PSK (ρ = -1)



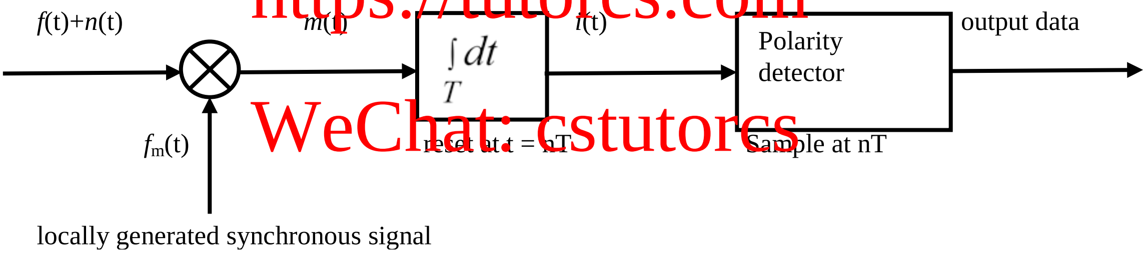
Assume that the filter's impulse response is matched to the mark signal. The output of the matched filter detector for a mark signal is expressed as

$$= \int_0^t \left(f_m(\tau) + n(\tau) \right) h(t-\tau) d\tau = \int_0^t \left(f_m(\tau) + n(\tau) \right) f_m(T - t + \tau) d\tau$$

At t=T

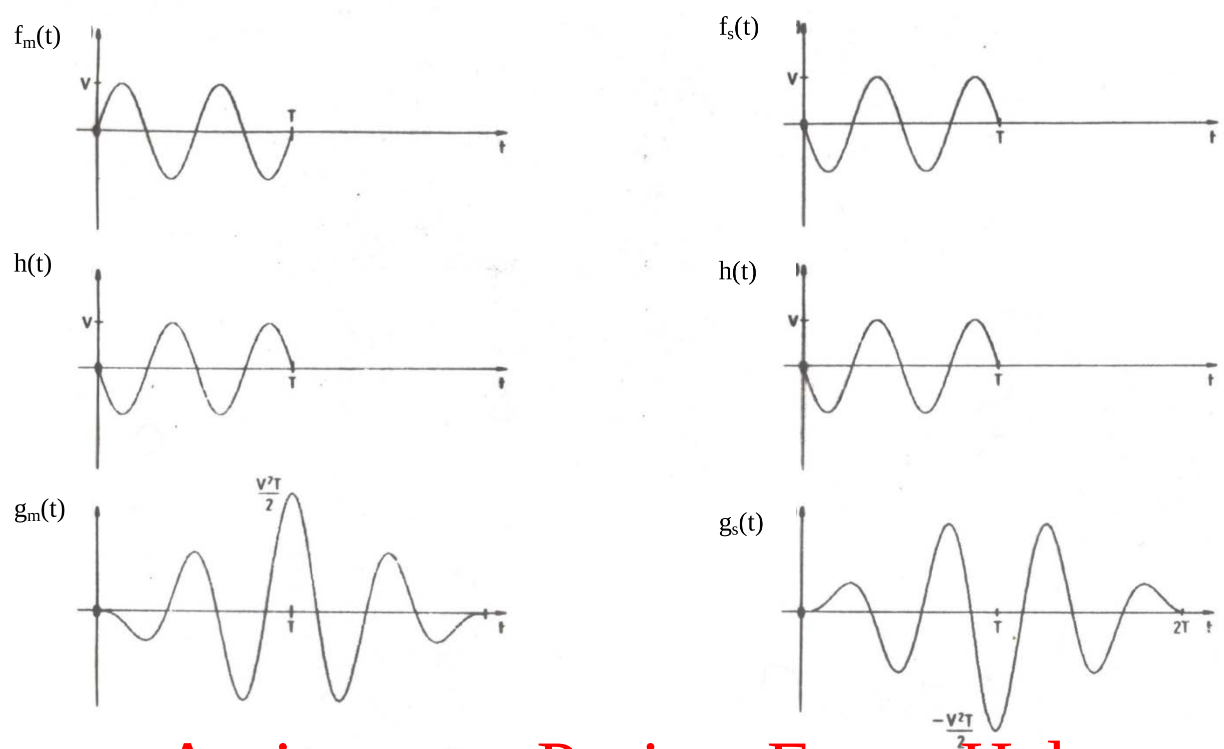
$$= \int_0^T f_m^2(\tau) d\tau + \int_0^T n(\tau) f_m(\tau) d\tau = E + Noise$$

This equation can be implemented with a correlation detector as follows:



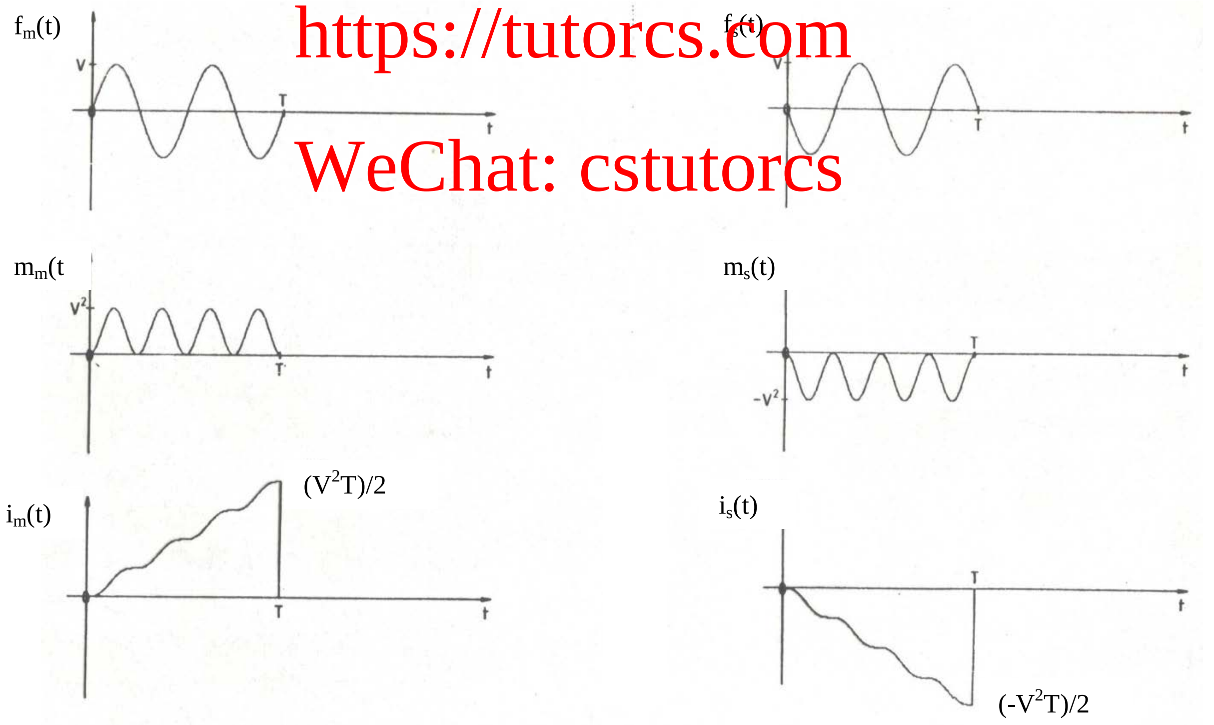
$$= \int_0^T f_m^2(\tau) d\tau + \int_0^T n(\tau) f_m(\tau) d\tau = E + Noise$$

That is at t=*T*, both block diagrams would give the same output but not necessarily look the same at other instants in time. The figure below illustrates this by showing the outputs of the matched filter and the output of the correlation detector due to the mark and space signals which have been phase modulated i.e. PSK signals. We can see from the figure that at the sampling instant T, both detectors give the same value which is equal to the energy in the signal but at other instants in time the outputs look completely different. Hence to get the best response we need to ensure that we sample the output of the detector at the correct instant in time.



(a) (b)

Output of matched filter (a) mark input, (b) space input



(a) (b)

Output of correlation detector (a) due to mark signal, (b) due to space signal.