- **<u>Small-Scale Fading and Multipath Propagation in a Mobile Radio Channel</u>**

In built-up areas the mobile antenna is well below the surrounding buildings, so there is no-line-of–sight path to the transmitter. Propagation is therefore mainly via reflection and scattering from the surfaces of buildings or diffraction over and/or around them. In practice energy arrives via several paths simultaneously and a multipath situation is said to exist in which the various incoming waves arrive from different directions and with different time delays as illustrated in Fig. 12. They combine as vectors at the receiver's antenna to give a resultant signal, which can be large or small depending on the distribution of phases amongst the component waves. As the mobile moves the relative phases between the multipath components vary leading to variations in the received signal level on the order of tens of dB as shown by the solid line in Fig. 13. These variations are known as fast or rapid fading and are usually differentiated from slow fading. Slow fading also called shadow fading arises from variations in the signal strength due to the movement of the vehicle over large distances. It can be estimated from fast fading by taking a moving average of the received signal strength as shown in the dotted line in Fig. 13.

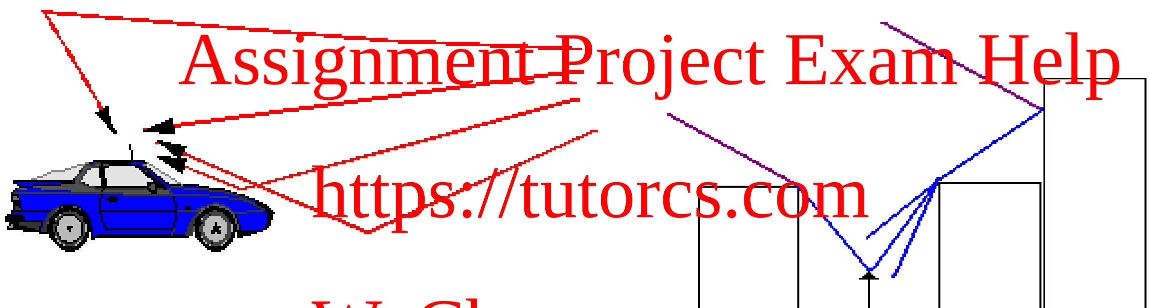


Figure 12 Multipath reception.

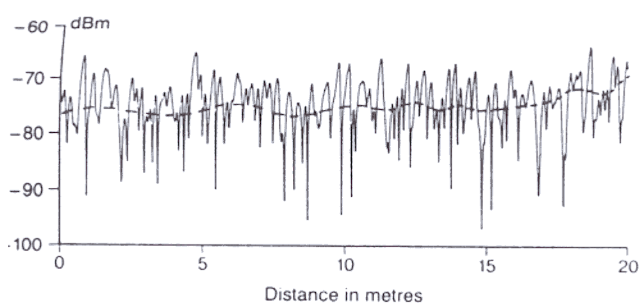


Figure 13 Experimental record of received signal envelope in an urban area (after Parsons)

- **The nature of multipath propagation**

Multipath refers to the situation where energy travels between the transmitter and receiver via several paths. The effects of multipath depend on whether the transmitted signal is narrowband or wideband. In the narrowband case it is assumed that the transmitted signal is an unmodulated carrier. Two situations with regard to the environment might occur:

- "*Static Multipath*" situation: In this case several versions of the transmitted CW signal arrive sequentially at the receiver. The differential time delay between the different paths introduces relative phase shifts between the component waves and superposition of these leads to either constructive or destructive addition (at one instance of time). Fig. 14 shows the simple case of two paths where in one case they add constructively and in the other they add destructively.

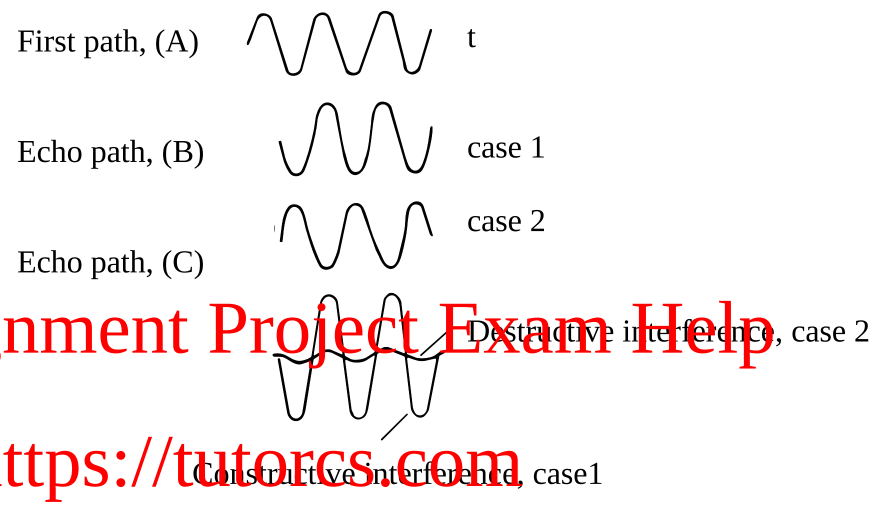


Figure 14. Constructive and destructive addition of two transmission paths (after Parsons).

- "*Dynamic multipath*" situation: in this case the movement of either the transmitter or receiver or the motion of vehicles in the surrounding environment causes a continuous change in the electrical length of every propagation path which introduces a change in the relative phase shifts as a function of spatial location. At some positions there is constructive addition, whilst at others there is almost complete cancellation as illustrated in Figure 15.

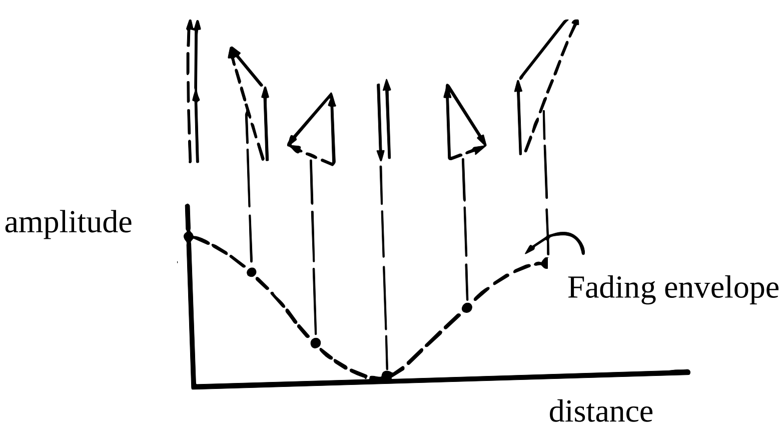


Figure 15. Envelope fading as two incoming signals combine with different phases (after Parsons).

The time variations, or dynamic change of phase, due to motion, are known as a *Doppler frequency shift* in each propagation path. The Doppler shift can be explained by viewing figure 16 in which a mobile is moving with velocity ν along the path *AA′* and it is receiving a wave from scatterer *S*. The incremental distance $d = v\,\Delta t$ gives an incremental change in path length of the wave of $\Delta l = d\cos\alpha$. The corresponding phase change is then given by

$$\Delta\Phi = \frac{2\pi}{\lambda}\Delta l = \frac{2\pi \upsilon \Delta t}{\lambda}\cos\alpha \qquad (39)$$

This in turn gives an apparent change in the carrier frequency known as the *Doppler Shift* as given by equation 40

$$f = \frac{1}{2\pi}\frac{\Delta\Phi}{\Delta t} = \frac{v}{\lambda}\cos\alpha \qquad (40)$$

Waves arriving from ahead of the mobile have a positive Doppler Shift, or an increase in frequency, whilst the reverse is the case for waves arriving from behind the mobile. These cases give the maximum rate of change of phase, i.e. $f_m = v/\lambda$.

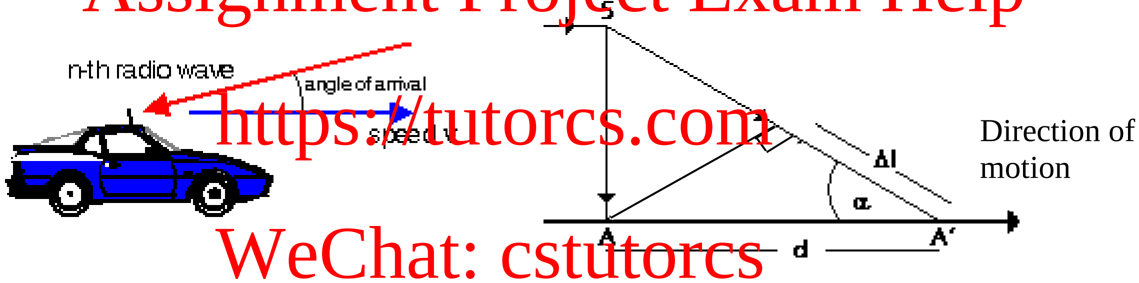


Figure 16. Illustration of Doppler shift

Thus, changes in the phase with time for each component give a corresponding Doppler shift for that component and if several components are present, the overall envelope of the received signal experiences maxima and minima. So the Doppler shift determines the rate at which the amplitude of the resulting composite signal changes.

- **Short term (fast) fading - The scattering model**

In a practical situation, the relative phases of the received components will vary continuously and randomly with time. Hence, the resultant envelope of the received signal and its phase will also be varying randomly. A number of statistical models have been suggested to explain the observed statistical behaviour of the received signal. These include the two dimensional Clarke's model, and the generalised three dimensional Clarke's model developed by Aulin.

In the three dimensional model the vertical plane is taken into consideration as in Fig. 17.

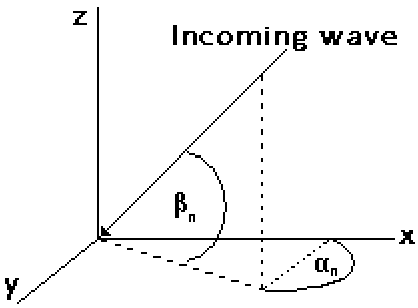


Figure 17. The incoming wave

The scattering model assumes that at every receiving point the signal is the resultant of N plane waves. The n^{th} incoming wave has an amplitude C_n, a phase of Φ_n with respect to an arbitrary reference, and spatial angles of arrival α_n and β_n, fig.(17). All parameters are random and statistically independent.

In the azimuth (x-y) plane, the waves are assumed to arrive from all the angles with equal probability. This gives a PDF for α as in equation 41:

$$P_a(\alpha) = \frac{1}{2\pi} \tag{41}$$

The PDF for the elevation angle depends on the used model.

1. Clarke's model

This is only a two-dimensional model and hence β is equal to zero. The model gives a Doppler spectrum which is strictly band-limited within the max Doppler shift ($f_m = v/\lambda$), but becomes infinite at $f_c \pm f_m$ as in Fig. 18.a

(2) <u>Aulin's model</u>

Aulin assumes the following PDF for the elevation angle:

$$P(\beta) = \begin{cases} \dfrac{\cos\beta}{2\sin\beta_n} & |\beta| \le |\beta_m| \le \dfrac{\pi}{2} \\ 0 & \text{Elsewhere} \end{cases} \tag{42}$$

The resulting Doppler spectrum is shown in Fig. 18.b

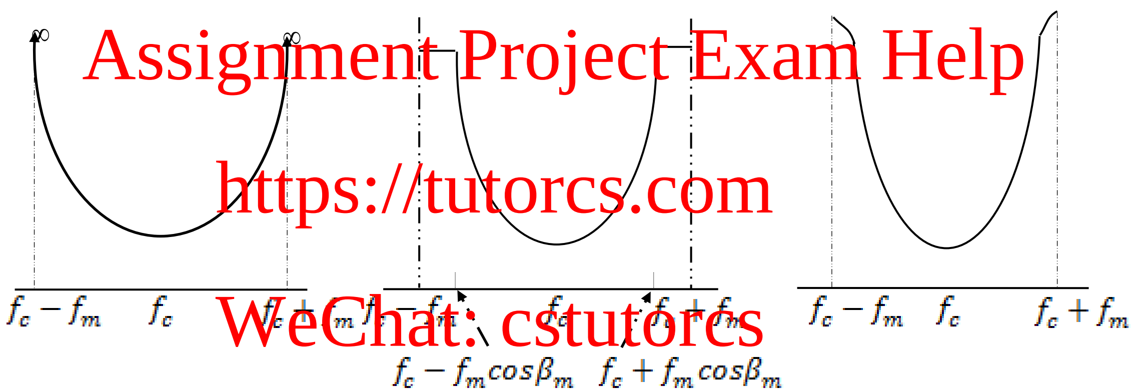
3. Parsons model

Pasrson's model assumes that the majority of incoming waves travel in a nearly horizontal direction with a PDF for β as given in equation 43 which has a mean value of 0° and is heavily biased towards small angles. It does not extend to infinity and has no discontinuities. Using numerical techniques, the baseband power spectrum can be evaluated as in Fig. 18.c.

$$P_B(\beta) = \begin{cases} \dfrac{\pi}{4|\beta_m|}\cos\left(\dfrac{\pi}{2}\dfrac{\beta}{\beta_m}\right) & |\beta| \le |\beta_m| \le \dfrac{\pi}{2} \\ 0 & \text{Elsewhere} \end{cases} \tag{43}$$

All the above spectra are strictly bandlimited to $|f| \le f_m$. Cases (2) and (3) are always finite, case (2) is constant for $f_m cos\beta_m \le |f| \le f_m$ which is unrealistic.

In conclusion, the RF signal spectrum is strictly bandlimited to a range $\pm f_m$ around the carrier frequency. However within those limits the power spectral density depends on the PDF associated with the spatial angles of arrival α and β. The limits of the Doppler spectrum can be quite high, for example v=30m/sec and f_c = 900 MHz $\Rightarrow$ f_m = 90 Hz. Frequency shifts of this magnitude can cause interference with the message information.



(a) (b) (c)

Figure 18 Doppler spectra of received signal using different parameters for the scattering model, (a) Clarke's model, (b) Aulin's model, and (c) Parsons' model.

The Received Signal Envelope

The random variations of the received signal envelope *r(t)* are normally represented by different PDF's such as Rayleigh, Rice, and Nakagami.

The Rayleigh density function is given by

$$P(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{\sigma^2}\right) \tag{44}$$

where σ^2 is the mean power, $r^2/2$ is the short – term signal power.

The probability that the envelope doesn't exceed a specified value *R* is given by:

$$\mathrm{Prob}[r \le R] = \int_0^R \Pr(r)dr = 1 - \exp\left(-\frac{R^2}{2\sigma^2}\right) \quad (45)$$

The *mean value* (or expectation) of the envelope *E{r}* is:

$$r_{mean} = E\{r\} = \int_0^\infty r\mathrm{Prob}(r)dr = \sigma\sqrt{\frac{\pi}{2}} = 1.2533\sigma$$

The *mean square value* is:

$$E\{r^2\} = \int_0^R r^2 \mathrm{Prob}(r)dr = 2\sigma^2$$

The variance is:

$$\sigma_r^2 = \mathrm{E}\{r^2\} - \mathrm{E}\{r\}^2 = 2\sigma^2 - \frac{\sigma^2\pi}{2} = 0.4292\sigma^2 \quad (46)$$

The median value r_m for which *Prob*(r_m)=0.5 is :

$$0.5 = 1 - Exp\left(-\frac{r_m^2}{2\sigma^2}\right) \Rightarrow r_m = 2\sigma^2 \ln 2 = 1.1774\sigma \quad (47)$$

- **Level Crossing Rate (LCR) and Average Fade Duration (AFD)**

As illustrated in Fig. 19, as the mobile moves the signal envelope suffers from fading i.e. signal level drops in level. *The quantitative description of the rate at which fades at a particular depth occur and the average duration of a fade below any given depth* are usually used for the design of communication systems. *These are given in terms of Level Crossing Rate (LCR) and Average Fade Duration (AFD) below a certain level R (see figure 19).*

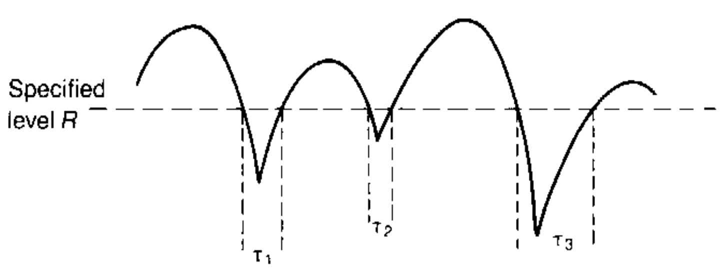


Figure 19. LCR and AFD; LCR = average number of positive going crossings per second, AFD = average of τ_1, τ_2, τ_3, ...,τ_n

<u>Level Crossing Rate</u> (LCR) at any specified level is defined as the expected rate at which the envelope crosses that level in a positive-going (or negative) direction per second, fig.(19).

For a Rayleigh distribution the average Crossing Rate (average number of crossings per second) at a level R is given by:

$$N_R = \sqrt{\frac{\pi}{\sigma^2}} R f_m \exp\left(-\frac{R^2}{2\sigma^2}\right) \qquad (48)$$

where $2\sigma^2$ is the mean square value and hence, $\sqrt{2}\sigma$ is the RMS value. Eq. 48 can therefore be expressed as:

$$N_R = \sqrt{2\pi} f_m \rho \exp(-\rho^2) \qquad (49)$$

where,

$$\rho = \frac{\mathrm{R}}{\sqrt{2}\sigma} = \frac{\mathrm{R}}{\mathrm{R}_{\mathrm{RMS}}} \qquad (50)$$

Eqs.(48-49) show that the average number of crossings per second, is a function of the mobile speed and this is apparent from the appearance of f_m in the equation.

Average Fade Duration, For a Rayleigh distribution, the average period of a fade below a specified level R is given by

$$E\{z_R\} = \frac{\mathrm{Prob}(R)}{\mathrm{N_R}} \Rightarrow \ldots \Rightarrow E\{z_R\} = \sqrt{\frac{\sigma^2}{\pi}} \frac{\exp\left(\frac{R^2}{2\sigma^2}\right) - 1}{R f_m} \qquad (51)$$

Multiplying by f_m enables us to express the average duration in wavelengths:

$$L_R = \sqrt{\frac{\sigma^2}{\pi}} \frac{\exp\left(\frac{R^2}{2\sigma^2}\right) - 1}{R} \qquad (52)$$

$$L_R = \frac{1}{\sqrt{2\pi \ln 2}} \frac{2^{\left(R/r_m\right)^2} - 1}{R/r_m} \quad \text{(in terms of the median value)} \qquad (53)$$

$$\mathrm{L_R} = \frac{\exp(\rho^2) - 1}{\rho f_m \sqrt{2\pi}} \qquad \text{(in terms of RMS)} \qquad (54)$$

AFD and LCR indicate how often a Rayleigh – fading signal needs to be sampled in order to ensure that an “average duration” fade below the specified level will be detected. For example, in order to detect about 50% of fades 30 dB below the median level, the signal must be sampled every *(AFD)*λ*=0.01*λ. At 900 MHz this is 0.33 cm.

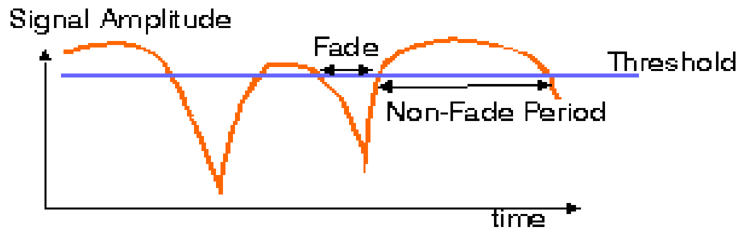


Figure 20. Fade and Non-Fade duration for a sample of a fading signal.

In a Rayleigh fading channel with fade margin M the Average Non-Fade Duration (*ANFD*), fig.(20), is:

$$ANFD = \frac{\sqrt{M}}{\sqrt{2\pi}\, f_D} \tag{55}$$

where f_D is the Doppler spread, M is the ratio of the local-mean signal power and the minimum (threshold) power needed for reliable communication.

The ANFD is proportional to the speed of the mobile user. Channel fading occurs mainly because the user moves. If the user is stationary almost no time variations of the channel occur (except if reflecting elements in the environment move). The ANFD increases in proportion to the square root of the fade margin. The non-fade duration is not so sensitive to whether the signal experiences fades below a constant noise-floor or a fading interfering signal.