Examination 2021

Question 1.a

(a) A frequency shift keying (FSK) communication system transmits $s_o(t)$ to represent binary 1 (mark) and $s_1(t)$ to represent binary 0 (space), where

$$s_o(t) = Acos2\pi f_o t \qquad 0 < t < T$$

$$s_1(t) = Acos2\pi f_1 t \qquad 0 < t < T$$

Assuming that $T \gg 1/f_o$ and $T \gg 1/f_1$

(i) Find the energy per bit.

(ii) Find an expression for the correlation coefficient, ρ between the mark and space signals.

(iii) Deduce the relationship that gives zero correlation coefficient.

Solution

Energy per bit is found from either the mark or space signal as

$$E = \int_0^T A^2 cos^2 2\pi f_o t dt = \int_0^T \left(\frac{A^2}{2} + \frac{A^2}{2} cos 4\pi f_o t\right) dt$$

The integral becomes equal to

$$E = \left(\frac{A^2 T}{2} + \frac{A^2}{8\pi f_o} sin 4\pi f_o T\right)$$

For $T \gg \frac{1}{f_o}$

The second term is much smaller than the first term which gives the energy per bit as

$$E = \frac{A^2 T}{2}$$

Solution Cont.

The correlation coefficient is

$$\rho = \frac{\int_0^T cos2\pi f_o t . cos2\pi f_1 t . dt}{E = A^2T/2} \text{ X A}^2$$

$$\rho = \frac{T}{2E}\left(\frac{sin2\pi(f_o + f_1)T}{2\pi(f_o + f_1)T} + \frac{sin2\pi(f_1 - f_0)T}{2\pi(f_1 - f_o)T}\right) \text{ X A}^2$$

Solution Cont.

$$\rho = \frac{T}{2E}\left(\frac{sin2\pi(f_o + f_1)T}{2\pi(f_o + f_1)T} + \frac{sin2\pi(f_1 - f_0)T}{2\pi(f_1 - f_o)T}\right) \text{ X } A^2$$

For $T \gg \frac{1}{f_o}$

The first term is much smaller than the second term

$$\rho = \frac{T}{2E}\frac{sin2\pi(f_1 - f_0)T}{2\pi(f_1 - f_o)T} \quad \text{X } A^2$$

For zero correlation

$sin2\pi(f_1 - f_0)T = sin\pi n$ or equivalently

$f_1 - f_0 = n/2T$

Q. 1.b

- Assume binary coded information is transmitted at 10 kb/s using FSK signal. The received amplitude of each tone is $2\text{x}10^{-2}$ V. The additive single sided noise power density spectrum is $10\text{x}10^{-9}$ W/Hz. Find the bit error rate of a coherent detector using the table of the complementary error function for correlation coefficients of (i) 0 and (ii) 0.3 and comment on the result.

$$P_e = \frac{1}{2} erfc\sqrt{\frac{E(1-\rho)}{2N_o}} \quad \text{and} \quad \rho = \frac{\int_0^T s_{mark}(t)s_{space}(t)dt}{\sqrt{\int_0^T s(t)_{mark}^2 dt \int_0^T s(t)_{space}^2 dt}}$$

$$\cos(2\pi f_1 t).\cos(2\pi f_2 t) = \frac{1}{2}\{\cos(2\pi f_1 + 2\pi f_2)\,t + \cos(2\pi f_1 - 2\pi f_2)\,t\}$$

Solution 1.b

$$P_e = \frac{1}{2} erfc \sqrt{\frac{E(1-\rho)}{2N_o}}$$

For zero correlation

$$P_e = \frac{1}{2} erfc \sqrt{\frac{E}{2N_o}}$$

$$E = \frac{A^2 T}{2}$$

The bit duration is 1/10,000

Substituting for the amplitude and T the Energy=2x10^{-8}

$$P_e = \frac{1}{2} erfc \ (1) = (0.1572992)/2 = 0.0786$$

Solution 1.b cont

$$For\ 0.3\ correlation$$

$$P_e = \frac{1}{2} erfc \sqrt{\frac{0.7\ E}{2N_o}}$$

$$= \frac{1}{2} erfc\ \ (0.8367)$$

From the error function tables
For 0.8 the value is 0.2578990 and for 0.85 the value is 0.2293319
which gives =0.1184

The error rate has increased due the correlation being greater than zero.

Question 1.c

A time division multiplexing pulse analogue modulated system transmits eight audio telephony signals with baseband bandwidth equal to 3.4 kHz and two music signals with baseband bandwidth equal to 15 kHz. For an 8-bit analogue to digital converter determine the required bandwidth of transmission for

(i) Unipolar non return to zero (NRZ).

(ii) Unipolar return to zero (RZ).

(iii) Manchester code.

Solution Q.1.c

For equal sampling the number of samples is equal to

15x10x2=300 k samples per second.

Each sample is represented by 8 bits so the total number of bits per second is equal to 300x8=2400 kbps

Solution cont.

Unipolar NRZ is represented by one +V for mark signal and 0 for space.

For binary NRZ the highest data rate is when we transmit alternate ones and zeros.

In this case the spectrum is that of a square wave and has sinx/x function with the first null at 1/Tb which in this case is equal to

2400 kHz.

Solution cont.

Unipolar return to zero would represent the mark by +V for half the bit and 0 for the second half while the space is represented by 0 for the duration of the bit.

For bandwidth requirements the Unipolar RZ would require twice the bandwidth assuming a long sequence of mark bits.

For RZ the bandwidth would be twice that of NRZ at 4800 kHz.

Solution Cont.

The Manchester code has +V for half the bit duration followed by -V for the second half.

So for a mark it could be +V followed by -V and the complement for a space i.e. -V followed by +V.

So the bandwidth requirements are similar to the Unipolar RZ.

Question 2.a

A mobile user is travelling at a speed of 50 km/hr as shown in Figure Q.2.1.a-b. Assume that the mobile phone requires a signal to noise ratio of 18 dB and that the noise floor of the receiver is -100 dBm.

Assume that both base stations transmit 20 dBm and use antennas with 3 dB gain, while the mobile uses an antenna with 0 dB gain and that the foliage attenuation is 10 dB. The base stations have dual frequency bands at 900 MHz and 1800 MHz.

i. Find the antenna gains in linear scale.

ii. Find the transmit power in mW for the base station.

Solution

(i)

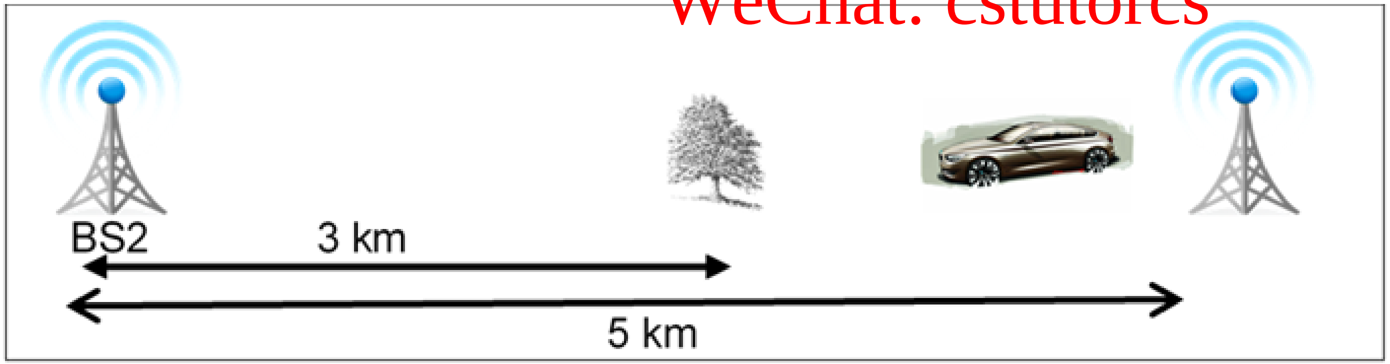
Gain in linear scale is 2 for the base station and 1 for the mobile computed on the basis of 10log(G)=3, 0

(ii)

Power of the transmitter at base station is 10log(p)=20 dBm

gives 100 mW

Question 2.a



Question 2.a

Given the relationship in equation 2.1 for free space path loss, determine the time at which the mobile phone would need to be handed over from base station 1 BS1 to base station 2 BS2 for 900 MHz and 1800 MHz operating frequencies for Figures Q2.1.a and Q.2.1.b for a 2 dB margin for hand off.

$$\frac{P_R}{P_T} = G_T G_R \left[\frac{c}{4\pi f d} \right]^2$$

where PT and PR are the transmit and receive powers respectively, G_T and G_R are the gains of the transmit and receive antennas respectively,

d is the distance from the transmitter and

f is the transmission frequency.

Solution

For hand off the minimum received signal strength should be

18+2=20 dB above noise floor

So the received signal power is -100+20=-80 dBm

$$\frac{P_R}{P_T} = G_T G_R \left[\frac{c}{4\pi f d} \right]^2$$

$$10logP_R = 10logP_T + 10logG_R + 10logG_T + 20log\frac{c}{4\pi} - 20logf - 20logd$$

Solution

For 900 MHz we get

-80=20+3+0+147.55-179-20logd

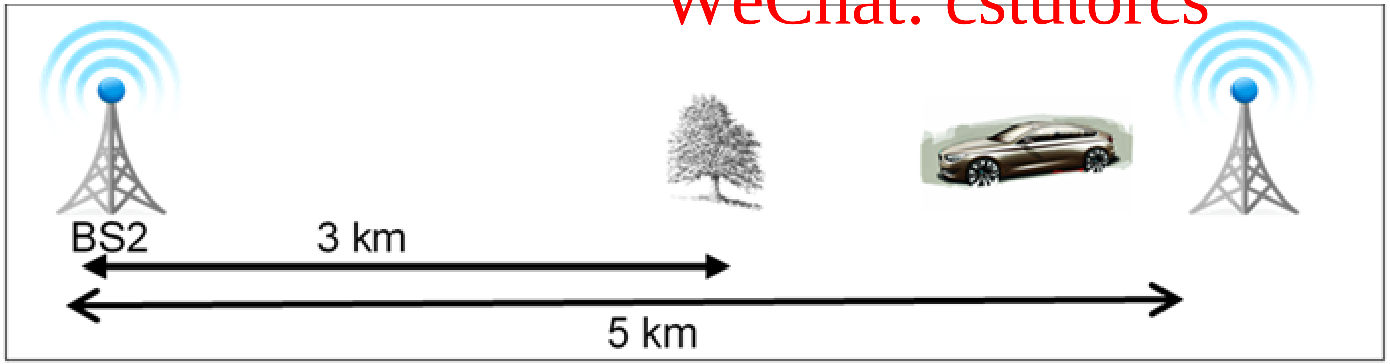
-71.55=-20logd

This gives a range of 3.78 km from BS1
Travelling at 50 km/hr this would mean a hand off after 4.536 minutes

For 1800 MHz we get
-80=20+3+0+147.55-185-20logd

This gives 1.89 km from BS1
Travelling at 50 km/hr this would mean a hand off after 2.268 minutes

Question 2.a



(iv) Comment on the success of the handover strategy for both scenarios at the two frequencies.

Solution cont.

- Scenario 1

For the 900 MHz, the handoff would be successful.

For the 1800 MHz the distance to BS2 is larger than 1.89 km, then the mobile will lose communication unless the power level is increased to permit longer distance prior to hand off is initiated.

Solution cont.

In the second scenario, the tree is within the distance of the hand off of the 900 MHz band. This adds another 10 dB loss.

To estimate the point of the hand off we first find the received signal strength at 2 km i.e. at the point of the tree.

Path loss at 2 km is equal to 94.566 dB at 900 MHz and 100.566 dB at 1800 MHz

The received signal strength at the tree is then equal to

-74.566 dBm at 900 MHz and -80.566 dBm at 1800 MHz

Solution cont.

- Scenario 2

At 900 MHz, immediately after traveling past the tree, the received signal strength from BS1 will drop to -84.566 which is below the acceptable level for handoff. The received signal strength from BS2 at the tree -78 dBm which is acceptable. However, due to the insufficient hand off margin, the mobile will lose communication resulting in unsuccessful handoff.

For the 1800 MHz, the handoff will still occur before the tree but will be unsuccessful due to higher losses.

Question 2.b

A cellular system has 15 channels to be multiplexed. Each user has a data rate of 10 kbps. Determine the overall bandwidth required for the system using frequency division multiple access.

Solution

For FDMA there are 15 users and each would require 10 kHz bandwidth.

So the overall bandwidth is 150 kHz for the uplink and 150 kHz for the downlink. So overall 300 kHz for the system.

Question 2.c

Discuss the different mechanisms of propagation that connect the transmitter and a receiver.

1. Free space which is LoS,
2. diffraction when an obstacle is between the transmitter and receiver.
3. Scattering when the wave is incident on a surface which has undulations which are proportional to the wavelength and in this case the waves scatter in all directions leading to high losses.
4. Reflection is when the wave is incident on a surface where the dimensions are of undulations are much larger than the wavelength and this leads to a relatively stronger received signal than the scattered signal.
5. The other mechanism is refraction like light incident on a prism.