Examination paper 2020

Question 1

(a) A baseband transmission system transmits the Manchester code where binary 1 is represented by +V for the first half of the bit duration and -V for the second half.

(i) Give the representation for binary 0.

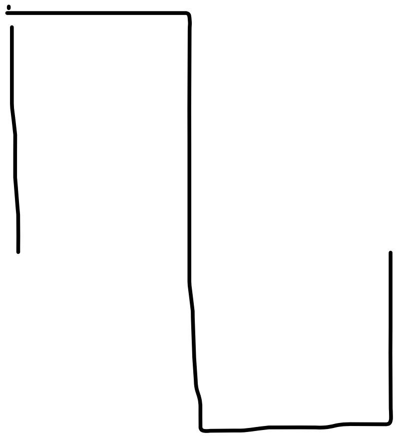
(ii) Determine the correlation coefficient between the two baseband signals representing the one and the zero.

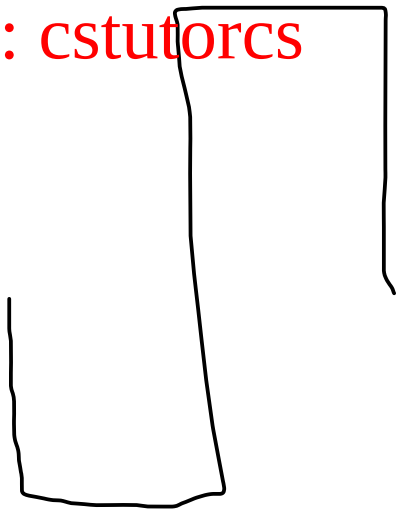
(iii) Design a suitable matched filter detector and sketch its output due to an input sequence 1101.

Answer

(i)

The representation for the zero is –V for half of the bit followed by +V for the second half.





Answer (a) (ii)

The correlation coefficient between two signals is given by

$$\rho = \frac{\int_0^T s_{mark}(t) s_{space}(t) dt}{\sqrt{\int_0^T s(t)_{mark}^2 \, dt \int_0^T s(t)_{space}^2 \, dt}}$$

which can be rewritten as

$$\rho = \frac{\int_0^T f_m(\tau) f_s(\tau) d\tau}{E}$$

where E is the energy per bit and fm, and fs are the mark and space signals, respectively.

Answer (a) (ii)

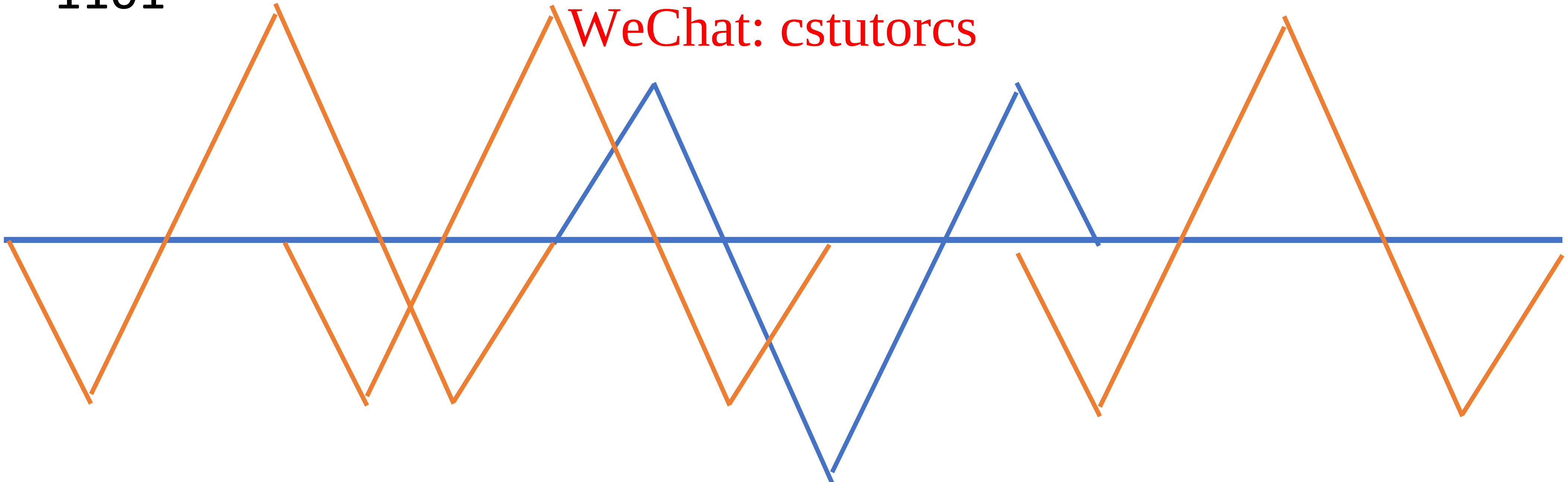
- for the Manchester code the representation for the space and mark is the same except that one signal is the negative of the other so the above expression reduces to

$$\rho = -\frac{\int_0^T f_m^2(\tau)d\tau}{E} = \frac{-V^2 T}{\int_0^T f_m^2(\tau)d\tau} = \frac{-V^2 T}{V^2 T} = -1$$

Answer (a) (iii)

- The signals for the mark and space are antipodal, so a matched filter can be used.

- The matched filter's impulse response can be given as fm (T-t) . The output for each individual bit is sketched below for the sequence 1101



Question 1 b

A binary frequency shift keying communication system transmits

s_o (t)=1.414cos(1000t) to represent binary 1 (mark) and

s_1 (t)=1.414cos(1010t) to represent binay 0 (space). Find the probability of error assuming equal probability of transmission of mark and space signals, a single sided noise power spectral density equal to 0.08 W/Hz and bit duration, T=1 second.

Solution

- Energy per bit is found from either the mark or space signal. For a sinusoid with peak amplitude A the average power is $A^2/2$. Since the average power is energy per unit time, the energy per bit is

$$E = \frac{A^2 T}{2}$$

$$E = \frac{A^2 T}{2} = \frac{(1.414)^2}{2} = 1\,Joules$$

The correlation coefficient is

$$\rho = \frac{\int_0^T s_{mark}(t)s_{space}(t)dt}{\sqrt{\int_0^T s(t)_{mark}^2 dt \ \int_0^T s(t)_{space}^2 dt}}$$

Since the denominator is the $\sqrt{E^2}$, the correlation coefficient can be rewritten as

$$\rho = \frac{\int_0^T cos2\pi f_o t . cos2\pi f_1 t . dt}{E = A^2 T/2}$$

$$\rho = \frac{sin2\pi(f_o + f_1)T}{2\pi(f_o + f_1)T} + \frac{sin2\pi(f_1 - f_0)T}{2\pi(f_1 - f_o)T}$$

$$\rho = \frac{sin(2010)}{2010} + \frac{sin10}{10} = -0.054$$

The probability of error is then

$$P_e = \frac{1}{2} erfc \sqrt{\frac{E(1-\rho)}{2N_o}}$$

$$P_e = \frac{1}{2} erfc \sqrt{\frac{1.054}{2(0.08)}}$$

$$= (1/2)erfc(2.5666)$$

=(1/2)x3.2150e-004= 1.6075e-004

Question 1 (c)

What is the sampling instant signal to noise ratio in dB at the output of a filter matched to a rectangular pulse of height 10 mV and width 1 ms if the noise at the input to the filter is white with a power spectral density of 1×10^{-9} W/Hz?

Solution

The signal to noise is equal to

2E/No

The energy in the signal is equal to

$y(t) = E = \int_0^T s^2(t)dt = A^2T$=(10x$10^{-3}$)^2 x(1x10^{-3})=1x10^{-7} Joules

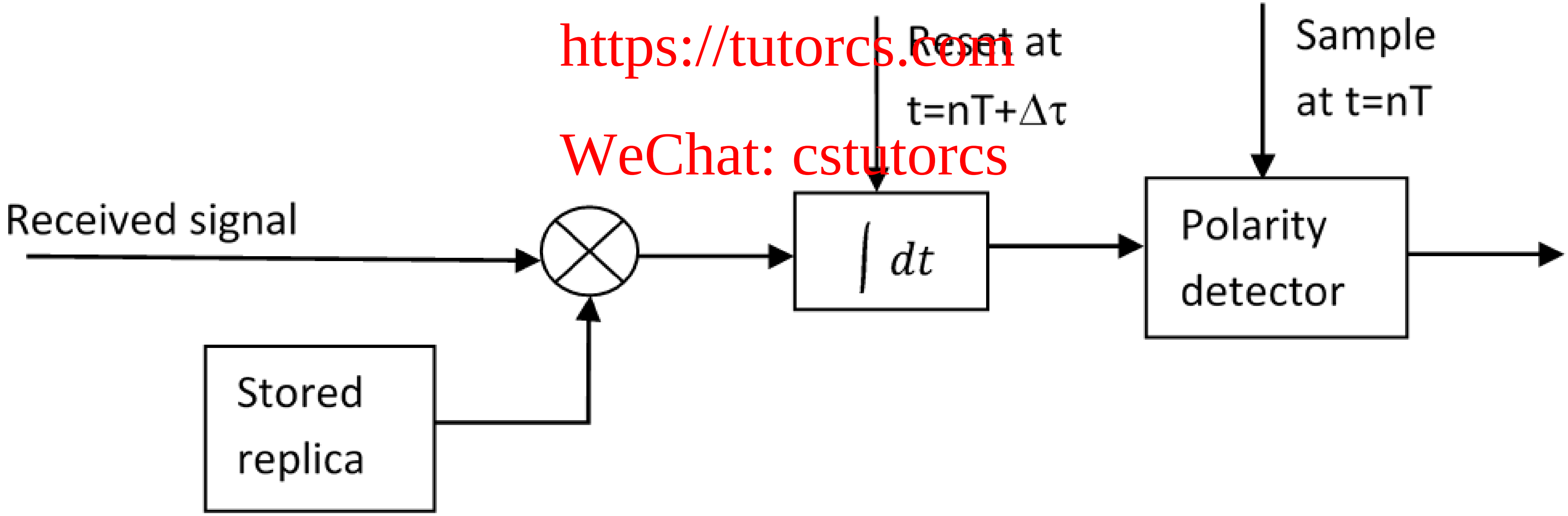
The energy to signal ratio is equal to

2E/No =(2x10^{-7})/(1x10^{-9})=200

=23.03 dB

Question 1 (d)

Figure Q.1.a shows the correlation detector of a phase shift keying (PSK) signal. Explain its functionality and discuss its synchronisation requirements for optimum performance.



Solution Q.1 (d)

The correlation detector multiplies the incoming signal with a stored replica which has to be phase coherent and identical in frequency to the incoming signal. The output of the multiplier is then integrated over one bit duration to give the energy per bit at t=T. When samples at t=T it gives the best possible detection value with respect to Gaussian additive noise. The output is E when a mark signal is received and –E when a space signal is received. This also implies that the receiver has time synchronisation in order to sample the output at multiples of T.

Question 2 (a)

A mobile receiver is located 5 km away from a base station and uses a vertical $\lambda/4$ monopole antenna with a gain of 2.55 dB to receive cellular radio signals. The free space E-field at 1 km from the transmitter is equal to 10^{-3} V/m. The carrier frequency used for this system is 900 MHz.

(i) Find the length and the gain of the receiving antenna in the linear scale.

Solution

At 900 MHz the wavelength is equal to c/*f* where c is the speed of light

$\lambda = 3x10^8/900x10^6 = 0.333m$

therefore the length is $\lambda/4 = 8.33$ cm

Gain of antenna =2.55 dB

On a linear scale this is equal $10^{(2.55/10)} = 1.8$

Q. 2 (a) (ii)

Find the received power at the mobile using the 2-ray ground reflection model assuming the height of the transmitting antenna is 50 m and the receiving antenna is 15 m above ground.

For the ground reflection model, the received electric field is given by

$$E = 2E_o \frac{2\pi}{\lambda} \frac{h_T h_R}{d}$$

where E_o is the free space electric field, h_T, h_R, are the height of the transmitter and the receiver above ground respectively; and d is the distance between the transmitter and the receiver.

Use can be made of $$\frac{E^2}{\eta} = \frac{P_T G_T}{4\pi d^2}$$

where d is the distance between the transmitter and receiver,
P_T is the transmitted power, G_T is the gain of the transmit antenna,
$\eta = 377\ \Omega$ is the free space impedance.

Solution

The free space electric field at 1 km is given but it is required to find the electric field at 5 km. So we need first to find the free space electric field at 5 km in relation to the 1 km. This can be found using the following relationship

$$\frac{E^2}{\eta} = \frac{P_T G_T}{4\pi d^2}$$

which gives the electric field at d.

Solution

- Taking the ratio of electric fields at two points in space

$$\frac{E_{d1}{}^{2}}{E_{d2}^{2}} = \frac{d_2^2}{d_1{}^{2}} \Rightarrow \frac{E_{d1}}{E_{d2}} = \frac{d_2}{d_1}$$

- Substituting in the 2 ray model

$$E_{d2} = 2E_{od2}\frac{2\pi}{\lambda}\frac{h_T h_R}{d_2}$$

$$E_{d2} = 2E_{od1}\frac{d_1}{d_2}\frac{2\pi}{\lambda}\frac{h_T h_R}{d_2}$$

$$E_{d2} = 2\times1\times10^{-3}\,\frac{1\times10^{3}}{5\times10^{3}}\,\frac{2\pi}{0.333}\,\frac{50\times15}{5\times10^{3}} = 1.12\ \mathrm{mV/m}$$

The received power is

$$P_R = \frac{\left(1.132\times10^{-3}\right)^2}{377}\,\frac{\lambda^2}{4\pi}G_R = \frac{1.28\times10^{-6}}{377}\,\frac{0.333^2}{4\pi}\times1.8 = 5.3998\times10^{-11}\,\mathrm{W} = -102.67\mathrm{dBW}$$

Question 2 (b)

The first generation analogue mobile radio system in North America AMPS, was designed for voice communication. It uses the band between 824 to 849 MHz for reverse link and the band between 869 to 894 MHz for the forward link. Using frequency division multiple access FDMA with 30 kHz separation between channels, and two service providers determine the following:

(i) Total number of available channels for each service provider.

(ii) Assume that each service provider allocates 21 channels for control. Determine the number of channels per cell for a cluster size of 7.

(iii) Explain how the number of users can be increased in such a system

Solution

Bandwidth available is 25 MHz for the uplink and 25 MHz for the downlink

Since system is FDD then

Total number of channels is 25 MHz/30 kHz=833 channels with each service provider having 416.

For cluster of 7 cells, we divide the channels between the 7 cells.

Each cell will have 3 control channels and the remaining 395 voice channels can be divided as follows:

4 cells with 56 channels and 3 cells with 57 channels.

Solution

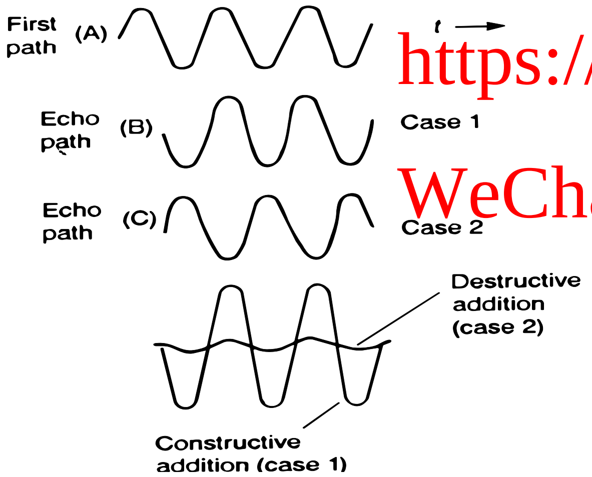
Number of users is increased by reusing the frequency channels by repeating the cluster as shown below. This was used in first and second generation mobile radio systems.

Question 2 (c)

Explain the difference between fast and slow fading and how they are modelled.

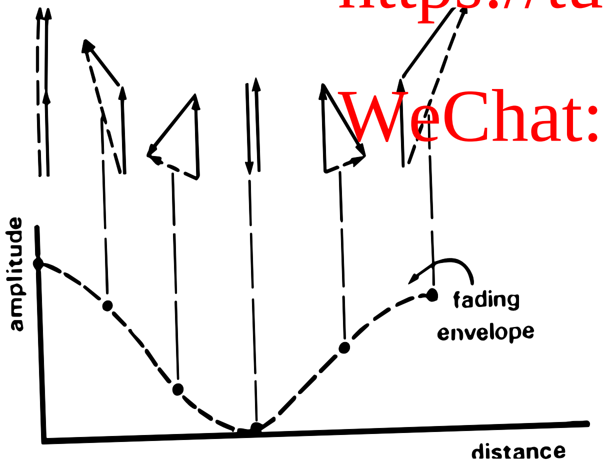
Solution

Fast and slow fading are caused by multipath which refers to the situation where energy travels between the transmitter and receiver via several paths. The effects of multipath depend on whether the transmitted signal is narrowband or wideband. If the user and the environment are static then the resultant can either have constructive or destructive interference as illustrated in the figure for a CW signal with 2 components.



constructive and destructive addition of two transmission paths

Fading occurs to due to the movements of the user or changes in the environment which is referred to as "*Dynamic multipath*" situation. In this case the movement of either the transmitter or receiver or the motion of vehicles in the surrounding environment causes a continuous change in the electrical length of every propagation path which introduces a change in the relative phase shifts as a function of spatial location. At some positions there is constructive addition, whilst at others there is almost complete cancellation.



Envelope fading as two incoming signals combine with different phases.

The fading envelope over a short distance is characterised as fast fading as the signal envelope changes over short distances and it is usually modelled by PDF such as Rice or Rayleigh depending on whether a line of sight component is present or not, respectively. Slow fading is estimated when the mobile moves over a large distance by taking the moving average of the fast fading envelope and is usually modelled by path loss models with distance coefficient