University of Exeter
Centre for Smart Grid

Optimal Power Flow

Power Flow

Power flow is well known as "load flow." This is the name given to a network solution that shows currents, voltages, and real and reactive power flows at every bus in the system.

1. The line flows
2. The bus voltages and system voltage profile
3. The effect of changes in circuit configuration, and incorporating new circuits on system loading
4. The effect of temporary loss of transmission capacity and (or) generation on system loading and accompanied effects
5. The effect of in-phase and quadrature boost voltages on system loading.
6. Economic system operation
7. system transmission loss minimization
8. Transformer tap settings for economic operation and
9. Possible improvements to an existing system by change of conductor sizes and system voltages.

Power Flow

In the network at each bus or node there are four variables viz.

(i) Voltage magnitude

(ii) Voltage phase angle

(iii) Real power and

(iv) Reactive power.

Bus	Specified variables	Computed variables
Slack – bus	Voltage magnitude and its phase angle	Real and reactive powers
Generator bus (PV – bus or voltage controlled bus)	Magnitudes of bus voltages and real powers (limit on reactive powers)	Voltage phase angle and reactive power.
Load bus	Real and reactive powers	Magnitude and phase angle of bus voltages

Power Flow

$$S_i = P_i + jQ_i = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^* = \sum_{k=1}^{n} |V_i||V_k| e^{j\theta_{ik}} (G_{ik} - jB_{ik})$$

$$= \sum_{k=1}^{n} |V_i||V_k| (\cos\theta_{ik} + j\sin\theta_{ik})(G_{ik} - jB_{ik})$$

Resolving into the real and imaginary parts:

$$P_i = P_{Gi} - P_{Di} = \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik})$$

$$Q_i = Q_{Gi} - Q_{Di} = \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik})$$

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Optimal Power Flow Definition

Optimal Power Flow Definition

Optimal load flow studies are concerned with economic operation of the system in all aspects.

The goal of OPF is to find the optimal settings of a given power system network that optimizes the system objective functions such as total generation cost, system loss, bus voltage deviation, emission of generating units, number of control actions, and load shedding while satisfying its power flow equations, system security, and equipment operating limits.

Optimal Power Flow Overview

- Optimization problem
- Classical objective function
  - Minimize the cost of generation

- Equality constraints
  - Power balance at each node - power flow equations
- Inequality constraints
  - Network operating limits (line flows, voltages)
  - Limits on control variables

Mathematical Formulation of the OPF

Mathematical Formulation of the OPF

- Optimization problem

$$\min_{\boldsymbol{u}} f(\boldsymbol{u})$$

$$\text{Subject to: } \boldsymbol{G}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{y}) = 0$$

$$\boldsymbol{H}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{y}) \geq 0$$

Mathematical Formulation of the OPF

- Optimization problem
- Classical objective function
  - Classical objective function:
    - Minimize total generating cost: $\min_{u} \sum_{i=1}^{N_G} C_i(P_i)$
  - Many other objective functions are possible:
    - Minimize changes in controls: $\min_{u} \sum_{i=1}^{N_U} c_i \mid u_i - u_i^0 \mid$
    - Minimize system losses
    - …

Mathematical Formulation of the OPF

- Decision variables (control variables)
  - Active power output of the generating units
  - Voltage at the generating units
  - Position of the transformer taps
  - Position of the phase shifter (quad booster) taps
  - Status of the switched capacitors and reactors
  - Control of power electronics (HVDC, FACTS)
  - Amount of load disconnected
- Vector of control variables: u

Mathematical Formulation of the OPF

- Equality constraints:
  - Power balance at each node - power flow equations

$$P_k^G - P_k^L = \sum_{i=1}^{N} V_k V_i [G_{ki} \cos(\theta_k - \theta_i) + B_{ki} \sin(\theta_k - \theta_i)]$$

$$Q_k^G - Q_k^L = \sum_{i=1}^{N} V_k V_i [G_{ki} \sin(\theta_k - \theta_i) - B_{ki} \cos(\theta_k - \theta_i)]$$

$$k = 1, \ldots N$$

- Compact expression:

$$G(x, u, y) = 0$$

Mathematical Formulation of the OPF

- Inequality constraints:
  - Limits on the control variables: $\underline{\boldsymbol{u}} \leq \boldsymbol{u} \leq \overline{\boldsymbol{u}}$
  - Operating limits on flows: $\left|\boldsymbol{F}_{ij}\right| \leq \overline{\boldsymbol{F}_{ij}}$
  - Operating limits on voltages $\underline{\boldsymbol{V}_j} \leq \boldsymbol{V}_j \leq \overline{\boldsymbol{V}_j}$
- Compact expression: $\boldsymbol{H}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{y}) \geq 0$

Mathematical Formulation of the OPF

According to the selected objective functions, and constraints, there are different mathematical formulations for the OPF problem. They can be broadly classified as follows

1. Linear problem in which objectives and constraints are given in linear forms with continuous control variables.
2. Nonlinear problem where either objectives or constraints or both combined are nonlinear with continuous control variables.
3. Mixed-integer linear problems with both discrete and continuous control variables.

Mathematical Formulation of the OPF

$$\min_{u} f(u)$$

Subject to: $G(x, u, y) = 0$

$$H(x, u, y) \geq 0$$

Solving the OPF

Solving the OPF

- Optimization problem
- Classical objective function
- Equality constraints
- Inequality constraints

Various techniques were developed to solve the OPF problem. The algorithms may be classified into three groups: (1) conventional optimization methods, (2) intelligence search methods, and (3) nonquantitative approach to address uncertainties in objectives and constraints.

OPF Challenges

OPF Challenges

- Size of the problem
  - 1000’s of lines, hundreds of controls
  - Which inequality constraints are binding?
- Problem is non-linear
- Problem is non-convex
- Some of the variables are discrete
  - Position of transformer and phase shifter taps
  - Status of switched capacitors or reactors

Optimal Power Flow

Questions and answers