Math 191 Sample Midterm 2A
Spring 2023, Wilkening

Your Name:

1a. (5 points) Suppose A is an $n \times n$ matrix with complex entries and $A^* = A$. Suppose V is a subspace of $\mathbb{C}^n$ such that $AV \subset V$, i.e., $x \in V \;\Rightarrow\; Ax \in V$. Show that $A(V^\perp) \subset V^\perp$.

1b. (5 points) Suppose A is a real, symmetric, $n \times n$ matrix. Show that e^A is positive definite.

2a. (2 points) Write down the overdetermined linear system $Ax = b$ whose least squares solution $x = \begin{pmatrix} C \\ D \end{pmatrix}$ gives the best-fit line $y(t) = C + Dt$ to the following points (t_i, b_i)

t_i	$-4/3$	1	4	5
b_i	$-13/4$	$-1/4$	$9/4$	$13/4$

in the sense that $\|r\|_2$ is minimized, where $r_i = b_i - y(t_i)$.

2b. (5 points) Find the Householder transformation $H_1 = I - \tau_1 v_1 v_1^T$ that reflects the first column of the matrix from part (a) to lie along the b_1 axis. (Find τ_1 and v_1, following the convention that $(v_1)_1 = 1$.)

2c. (3 points) After applying H_1 and computing and applying a second householder transformation, the above system becomes

$$\begin{pmatrix} -2 & -13/3 \\ 0 & -5 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} -1 \\ -5 \\ -0.3 \\ -0.4 \end{pmatrix}.$$

Compute C, D and the norm $\|r\|_2$ of the minimum residual.

3. (10 points) Let $A = \begin{pmatrix} 6 & 8 \\ 4 & -3 \end{pmatrix}$. Find all rank-1 matrices B such that $\|A - B\|_2$ is minimized.
Hint: if you can't figure out the SVD by inspection, AA^T is simpler than A^TA as a starting point to compute the SVD systematically.

4. (10 points) Compute the pseudo-inverse of

$$A = \begin{pmatrix} 1 & 2 \\ -2 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & 1 \end{pmatrix}.$$

You can leave your answer as a product of 3 matrices if you wish, but compute each entry of each of those matrices.