CMPSC464 **Name:** _____
Fall 2021
Midterm Exam 2
11/04/2021
Time Limit: 75 Minutes _____

This exam contains 10 pages (including this cover page, double-sided) and 6 questions.
Total of points is 100.
This will contribute to 25 % of your total grade

Grade Table (for grader use only)

Question	Points	Score
1	25	
2	15	
3	20	
4	10	
5	15	
6	15	
Total:	100	

1. (25 points) True or False Questions (Leaving an empty box will result in 0 pt)

   (a) (1 point) 3SAT is in NP.

   (b) (1 point) 2SAT is in NP.

   (c) (1 point) If L is NP-hard and is in NP, then L is NP-complete.

   (d) (1 point) If there exists a language L that is in NP, but not in P, then P is equal to NP.

   (e) (1 point) Graph Isomorphism is known to be in P.

   (f) (1 point) If a NP-complete language is in P, then P **is equal** to NP.

   (g) (1 point) If L is in NTIME($T(n)$), then it is known to be in DTIME($T(n)$).

   (h) (1 point) CLIQUE is NP-complete.

   (i) (1 point) 3SAT is NP-hard.

   (j) (1 point) If 3SAT is in P then P **is not equal** to NP.

(k) (1 point) Cook-Levin Theorem states that 3SAT is NP-complete.

(l) (1 point) k-SAT with $k \geq 2$ are all NP-complete.

(m) (1 point) If L is NP-complete, then 3SAT$\leq_p L$

(n) (1 point) Suppose $A \leq_p B$, then $B \leq_p A$.

(o) (1 point) If $A \leq_p B$ and $B \leq_p C$ then $A \leq_p C$

(p) (2 points) All NP languages are known to be NP-complete.

(q) (2 points) Any L in NP can be decided by a deterministic Turing machine in 2^{n^c}-time for some $c > 1$.

(r) (2 points) Any L in P can be solved by a non-deterministic Turing machine in $O(\log n)$-time.

(s) (2 points) Suppose there exists a language L that is in NP, but not in P. Then all NP-complete languages are not in P.

(t) (2 points) FACTORING is known to be NP-complete.

2. (15 points) Show that the respective languages are in P. Leaving Blank will result in 1 point each.

   (a) (5 points) CONNECTED – the set of all connected graphs.

   (b) (5 points) TRIANGLEFREE – the set of all graphs that do not contain a triangle (i.e. a 3-clique).

   (c) (5 points) Suppose A is in P. Then show that A^c (A complement) is also in P.

3. (20 points) Show that the respective languages are in NP. Leaving Blank will result in 1 point each.

   (a) (5 points) k-CLIQUE – the set of all graphs containing k-sized clique.

   (b) (5 points) k-INDSET – the set of all graphs containing independent set of size k.

(c) (10 points) Let A be some language in NP. Show that A^* is in NP as well. Leaving Blank will result in 2 points.

4. (10 points) Recall that Independent Set composes of (G, k) pairs where G contains an independent set of size k. Show that Independent Set is NP-hard. You may assume that Vertex Cover and CLIQUE are NP-complete. Leaving Blank will result in 2 points.

5. (15 points) Suppose SAT is in P. Then define SAT2 as the set of Boolean SAT formula with **at least two** satisfying assignments. Then show that SAT2 is in P as well. Leaving Blank will result in 3 points.

6. (15 points) Define integer programs as a list of m linear inequalities with rational coefficients over n variables $x_1, \ldots, x_n$. We say that the integer program is satisfiable if there exists $\{0, 1\}$ assignments to $x_1, \ldots, x_n$ that satisfies all linear inequalities.

   For example the following integer program is satisfiable by assigning $x_1, x_2 = 1$ and $x_3 = 0$

$$x_1 + x_2 \geq 2$$
$$x_1 + x_3 \leq 1$$
$$x_2 + x_3 \leq 1$$

   Let 0/1-IPROG be the set of satisfiable 0/1 integer programs. The goal of this problem is to show that 0/1 IPROG is NP-complete.

   (a) (5 points) Show that 0/1-IPROG is in NP. Leaving Blank will result in 1 point.

(b) (10 points) Now show that 0/1-IPROG is NP-hard. (Hint: You can use Cook-Levin Theorem as given) Leaving Blank will result in 2 points.