CMPSC 464: Introduction to the Theory of Computation **(Date: 10/05/2022)**

Recitation #6 Solution

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Problem 1

Theorem 4.4 of the book.

Problem 2

Proof by reduction from A_{TM}. Suppose that L_{PSU} were decidable and let R be a Turing machine deciding it. We use R to construct a Turing machine deciding A_{TM}. S is constructed as follows:

$S =$ "On input is $\langle M, w\rangle$, where M is the code for a Turing Machine and w is a string:

1. Construct a new Turing machine M_w as follows:
   $M_w =$ " On input string x:
   (a) Erase the input x and replace it with the constant string w.
   (b) Simulate M on w."
2. Feed $\langle M_w\rangle$ to R. If R accepts, accept. If R rejects, reject."

If M accepts w, the language of M_w contains all strings and, thus, the string "PSU". If M doesn't accept w, the language of M_w is the empty set and, thus, doesn't contain the string "PSU". So $R(\langle M_w\rangle)$ accepts exactly when M accepts w. Thus, S decides A_{TM}.

But we know that A_{TM} is undecidable. So S can't exist. Therefore we have a contradiction. So L_{PSU} must have been undecidable.