

Assignment Project Exam Help

Lecture 21

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Last time : TQBF

$Q_1 x_1 Q_2 x_2 Q_3 x_3 \dots Q_n x_n \underbrace{\varphi(x_1 \dots x_n)}$.

$Q_1 \dots Q_n \Rightarrow \exists \quad \leftrightarrow \text{SAT.}$

$Q_1 \dots Q_n \Rightarrow \forall \quad \leftrightarrow \text{UNSAT.}$

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TQBF is in PSPACE

$$S(n) \leq S(n-1) + O(m)$$

$\exists x_1$

$\rightarrow x_1 = 1 \quad Q_2 x_2 \dots Q_n x_n \underbrace{\psi(x_2 \dots x_n)}_{m} |_{x_1=1}$

$\rightarrow x_1 = 0 \quad Q_2 x_2 \dots Q_n x_n \psi(x_2 \dots x_n) |_{x_1=0}$

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$$S(n) \leq S(n-1) + O(m) + \underline{O(1)}$$

WeChat: cstutorcs $\Rightarrow S(n) \leq O(m \cdot n)$
polynomial in n

$\forall x_1$

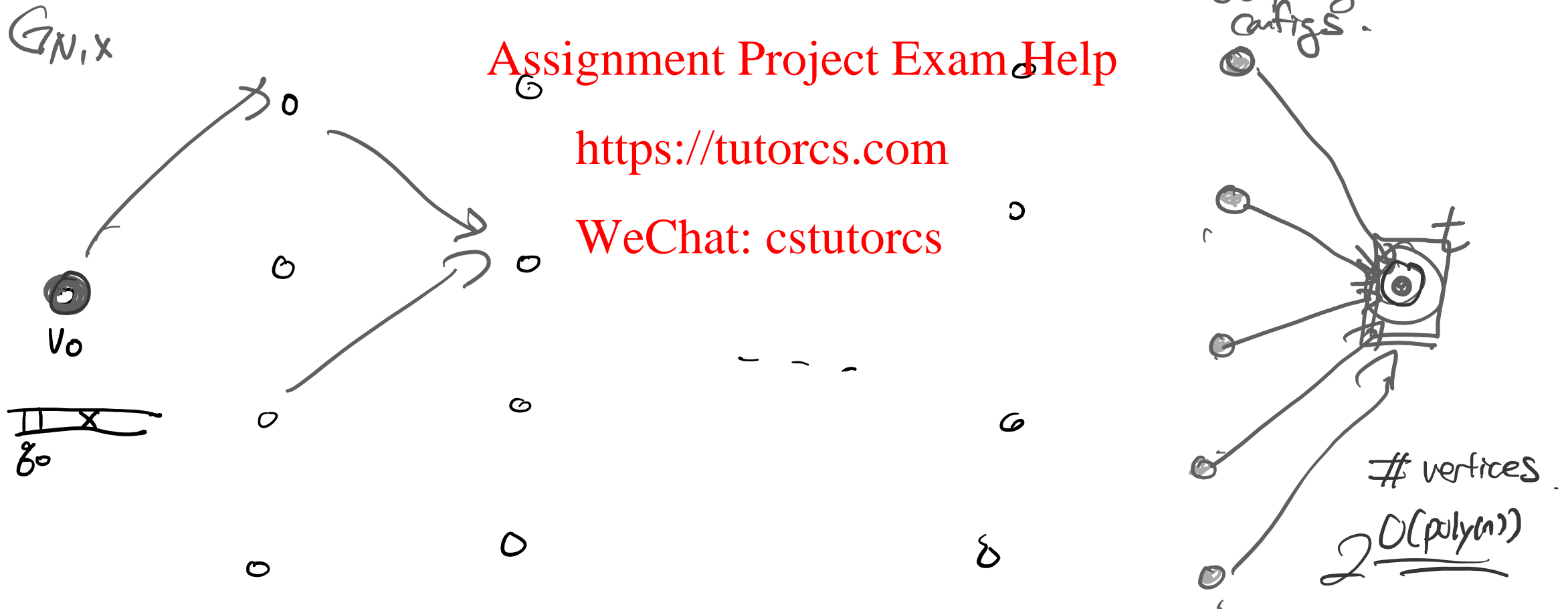
TQBF is PSPACE hard – reachability

$L \in \text{PSPACE}$. $\rightarrow \exists \text{ N.T.M. } N$ which decides L in poly space.
 on input \underline{x} $x \in L$ iff v_0 is connected to t accepting configs. in $G_{N,x}$

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Continued

$\exists \varphi_N(V_1, V_2)$ such that $\boxed{\varphi_N(V_1, V_2) = 1}$ iff V_2 is yielded by V_1
 $i = \underline{\text{poly}(n)}$

Cook-Levin.

$\varphi_i(V_1, V_2)$ true iff V_2 is reachable from V_1 in 2^i steps.

$$\varphi_i(V_1, V_2) = (\exists m) (\varphi(V_1, m) \wedge \varphi_{i-1}(m, V_2))$$

$$\varphi_0 = \varphi_N \stackrel{SC(1)}{=} 2^{SC(1)} = 2^{2 \cdot SC(1-2) + O(1)} \Rightarrow \exists m' \varphi(V_1, m') \wedge \varphi_{i-2} \rightarrow 2^{\text{poly}(n)}$$

$$\varphi_i(V_1, V_2) = \exists m \exists D_1, D_2 ((D_1 = V_1 \wedge D_2 = m) \vee (D_1 = m \wedge D_2 = V_2)) \wedge \varphi_{i-1}(D_1, D_2)$$

$2^i (SC(0) + 2) \Rightarrow \text{poly-time}$

$SC(i) = SC(i-1) + O(\text{poly}(n))$ (poly(n) m poly-time)

Motivations for studying PSPACE

- Chess, Go, etc all board games. Finding the best strategy ?

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L and NL

Logarithmic Space?

- Well input is of length n , even storing space takes linear space ... ?

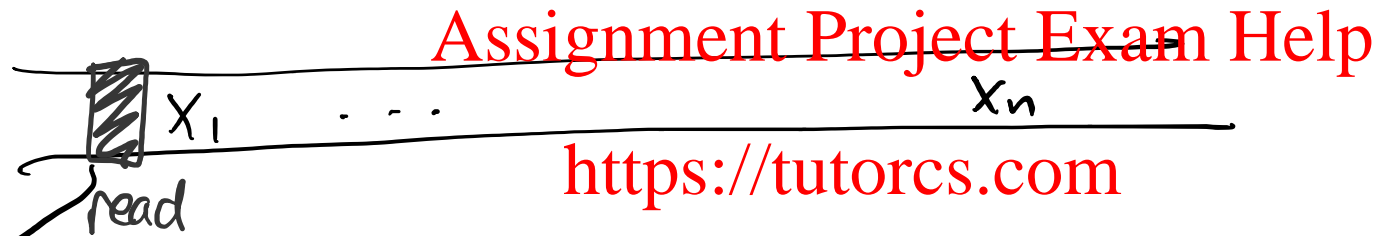
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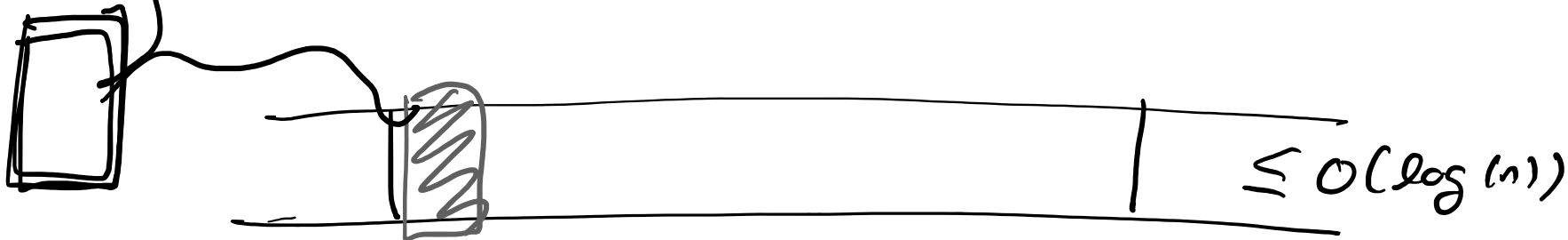
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Multitape Turing Machine

- Input is written on the read only tape



- Working tape is of length logarithmic



L and NL definition

$L \in L$ if L can be decided by a det machine
using log space

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$L \in NL$

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non-det

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Alternative Definition of NL

- Certificate Tape, Input tape, Work tape

proof

$L \in NL$

Assignment Project Exam Help that N makes

\exists det machine M

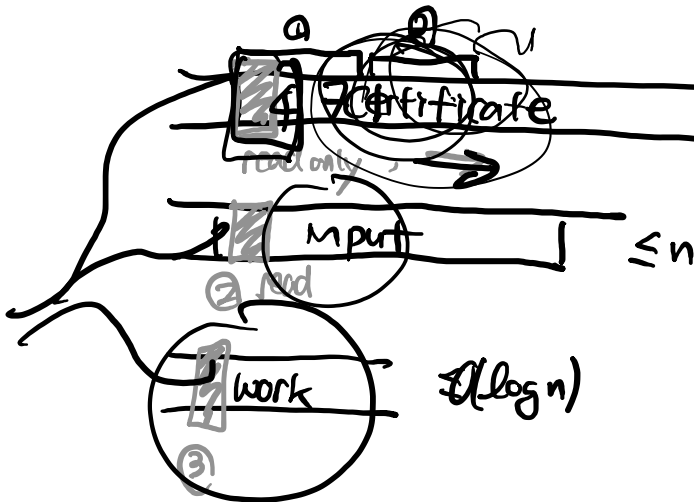
such that

decides

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$L \leq \text{poly}(n)$



$\left\{ \begin{array}{l} \text{if } x \in L, \exists c(x) \\ \text{such that } M \text{ accepts} \\ \text{if } x \notin L \quad \forall c(x) \\ M \text{ rejects.} \end{array} \right.$

(L) and NL are contained in P $\Rightarrow NL \subseteq P$
 $L \subseteq NL \subseteq P$

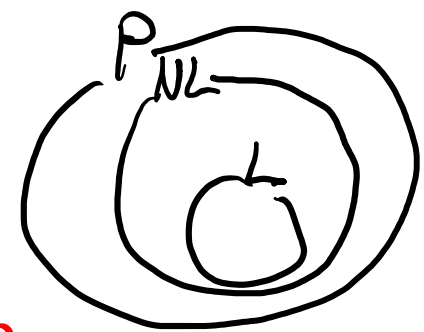


$O(n)$

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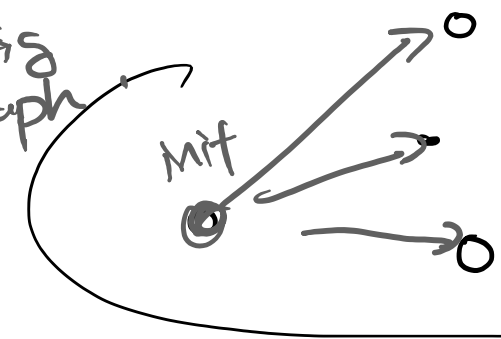


$$O(\log n) \leq c \cdot \log n = (2^{\log n})^c = n^c = \text{poly}(n)$$

Total # of config $\text{poly}(n)$

$\text{poly}(n) \cdot O(n) \rightarrow n$
 n^2, n^3, n^4, \dots

config graph



accept
 $\text{poly}(n)$

reachability
 on this graph
 $\text{poly}(n)$

Relationship between L and NL

- Savitch's Theorem?

$$L \subseteq NL \quad \cancel{L}$$

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$$NL \subseteq DSPACE(\underbrace{\log^2 n}_{\text{workspace}})$$

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$$NSPACE(S(n)) \subseteq DSPACE(S(n)^2)$$

even when

$$S(n) \ll n$$

L/NL completeness

- If we just use Karp reduction, what goes wrong?



$$\tilde{A} \leq_p B \rightarrow \log \text{space}$$

Logspace Reductions

- Reduction function is logspace computable

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Transitivity holds

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If A reduces to B and B is in L , then A is in L

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NL-completeness?

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PATH is NL-complete

- $\text{PATH} = \{ (G,s,t) \mid t \text{ is reachable from } s \text{ in } G \}$

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PATH is in NL

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PATH is NL-hard

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