**CMPSC 464: Introduction to the Theory of Computation** **(Date: 09/22/2022)**

# Recitation #4

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Notice: Try to solve these problems before the recitation session by yourself. Sometimes we put more problems than one is able to discuss in 50 minutes. We want to make sure that we do not run out of the problems during each session.

## Problem 1

Consider the following claim:

**Claim**: If $L$ is a decidable language and $L' \subseteq L$, then $L'$ is a decidable language too.

Is this claim true? If yes, prove it. If not, give a counter-example.

## Problem 2

Using the diagonalization method show that the language:

$$HALT_{TM} \stackrel{\text{def}}{=} \{ \langle M, w \rangle \mid M \text{ is a TM and } M \text{ on } w \text{ halts either in accept or reject state}\}$$

is undecidable. Hint: Modify slightly the proof of undecidability $A_{TM}$.



## Problem 3

Let EVEN= $\{\langle M, x\rangle \mid M$ is a Turing machine that runs on input $x$ for an even number of steps$\}$. Of course, a Turing machine running for infinitely many steps neither runs for an odd nor an even number of steps. We want to show that EVEN is undecidable by a proof from scratch (i.e., by diagonalization not by a reduction). First we assume $EVEN$ is decidable, i.e., there is a TM $H$ that accepts $\langle M, x\rangle$ if $M$ is a Turing machine that runs on input $x$ for an even number of steps, and rejects otherwise. We want to do the diagonalization in two stages.



(a) First, describe a TM $\tilde{\text{D}}$ (based on the supposedly existing TM $H$) accepting $\langle M\rangle$ if and only if $M$ is a Turing machine that runs on input $\langle M\rangle$ for an even number of steps. Otherwise $\tilde{\text{D}}$ rejects.
Note that you may assume the correctness of the Church-Turing thesis, i.e., instead of describing a Turing machine in detail, you can just describe an algorithm in pseudo–code.

(b) Now define a TM $D$ from $\tilde{\text{D}}$ that runs on $\langle M\rangle$ for an odd number of steps if $\tilde{\text{D}}$ accepts $\langle M\rangle$. Otherwise $D$ runs for an even number of steps.

(c) How does the existence of this produce a contradiction?

(d) What does the contradiction prove?