Lecture 4

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Recap : Regular closed under ...

- $L, \overline{L}$
- $L_1, L_2 \rightarrow L_1 \cup L_2, L_1 \cap L_2, L_1 \bullet L_2$
- $L \longrightarrow L^*$

NFA and DFA are equivalent

$P(Q)$ $\tilde{q}$ is a subset of Q.

$\delta(\tilde{q}, x)$

$:= \bigcup_{q \in \tilde{q}} \delta(q, x) \in P(Q)$

$\delta(\tilde{q}, x) := N_\epsilon^*(\quad)$

$N \rightsquigarrow M$

$Q \longrightarrow P(Q) \Rightarrow$ finite # of states.

$q_0 \longrightarrow \{q_0\}$

$\delta \longrightarrow$

$S \longrightarrow \{\tilde{q} \mid \tilde{q} \cap S \neq \emptyset\}$

$\Sigma \longrightarrow$ remain the same.

$\bigcup_{j=1}^{k} \delta(q_t^j, x)$

$N_\epsilon(\cdots N_\epsilon(N_\epsilon(N_\epsilon(\tilde{q}))))$

$:= \bigcup_{q \in \tilde{q}} \delta(q, \epsilon) \cup \tilde{q}$

$N_\epsilon^*(\tilde{q})$

q_t^2

$q'_{current} = \{q_t^1, \ldots, q_t^k\}$

$\delta(q_t^1, x)$ $\delta(q_t^2, x)$ . . . . $\delta(q_t^k, x)$

Proof Ctd

w is accepted by N,

then w is accepted by M.

DFA

M $\quad w_1 \quad w_2 \quad \ldots \quad w_n$

$\{q_0\} \quad \{\ \} \quad \{\ \} \quad \ldots$

set of states reachable in N with input w_1

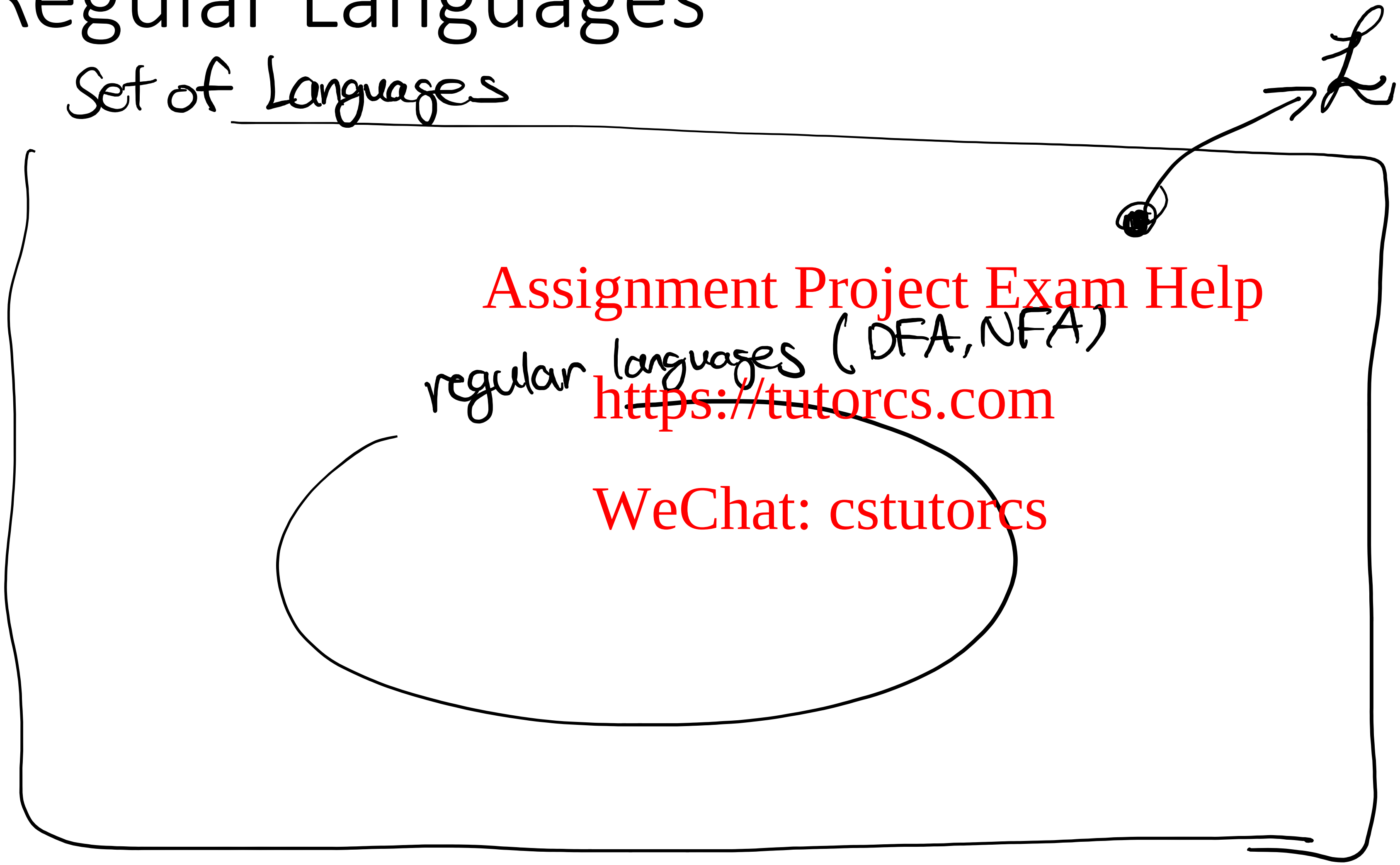
" w_2

set of states reachable in N $w_1 \ldots w_n$

w is not accepted by N.

" not accepted by M.

Regular Languages



But are all languages regular ? (Diagram)

Runtime for NFA ?

$$\leq \underbrace{|Q|}_{\text{finite}} \cdot n \simeq O(n)$$

How to show that a language is not regular ?

- EX) same number of 0’s and 1’s.

$\{ w \in \{0,1\}^* \mid \#\text{ of } 0\text{'s} = \#\text{ of } 1\text{'s} \}$

Pumping Lemma

- If L is regular, then there is a number p where if s is any string in A of length at least p, then s may be divided into three pieces s = xyz ;
  - For each i, xy^iz is also in L
  - y is non empty
  - xy is less than p

s=0110

0 11 0

x , y , z

011110

01111110

⋮

L. → then ∃ a number p.

any string s whose length is >p.

s

xy → ≤p

x y z

not empty

xyyz

xyyyz ∈ L

⋮

What is the intuition?

Pigeonhole Principle (PHP)

n pigeon holes, $n+1$ pigeons,

all pigeons go into pigeonholes.

$\Rightarrow \exists$ a pigeon hole with ≥ 2 pigeons.

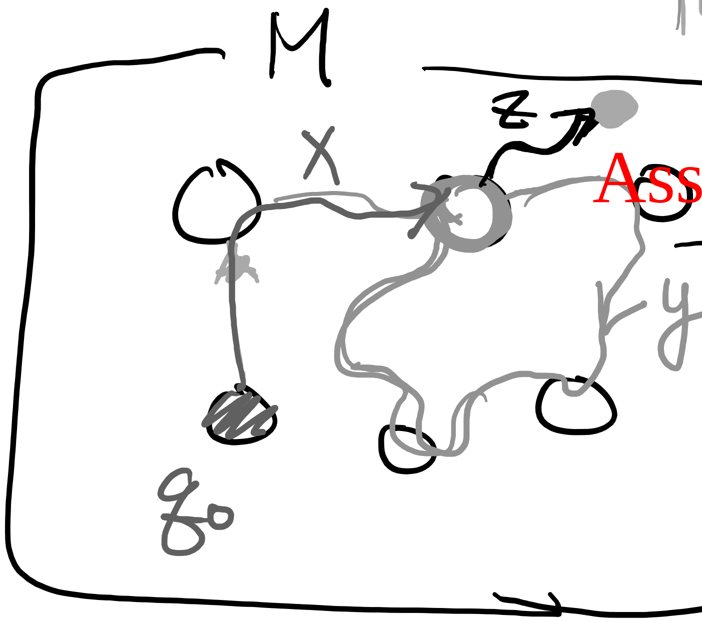
Proof Idea

Given L (regular)

p many states.

$|xy| \le p$.

$|w| > p$



$x \; y \; z = w$

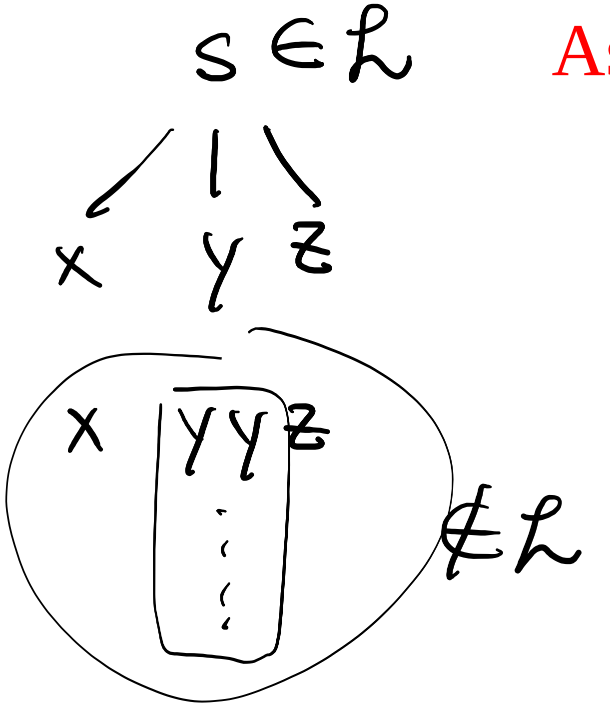
$x \; yy \; z$

circle must exist due to PHP.

for all $i \ge 1$ $\quad xy^i z \in L$.

What do we therefore want?

- Contrapositive of the statement



example (proof by contradiction)

$L = \{0^n 1^n \mid n \geq 1\}$.

Suppose L is regular,

then $\exists$ DFA M which has $L(M) = L$.

# of states for M is p.

Consider string

$s = \underbrace{0^p}_{p} 1^p$

No matter what xyz you take, xy must all be 0

$|xy| \leq p$

$xyyz$

$xyyyz \in L$.

$xyyyyz$

$\Rightarrow\Leftarrow$

Exercise : Same # of zeroes and ones

$L_= = \{ w \in \{0,1\}^* \mid$ equal # of zeroes and ones$\}$,

Assume $L_=$ is regular,

$\exists\, p > 0$ such that all $w \in L_=, |w| \geq p$.

$w = \underbrace{0 \ldots 0}_{p} \underbrace{1 \ldots 1}_{p} \quad w \in L$.

$\boxed{x\ y}\ z$

$x\,y\,y\,z \in L_=$ → has more zeroes than 1's.

contradiction. $\Rightarrow\!\Leftarrow$

Exercise : ww

$L_{ww} = \{ ww \mid w \in \{0,1\}^* \}$

0101

1111

0000

$0110 \notin L_{ww}$

L_{ww} is regular. $\exists p > 0$. such that

$$S = \underbrace{0 \ldots 0}_{p}\underbrace{1 \ldots 1}_{p}\underbrace{0 \ldots 0}_{p}\underbrace{1 \ldots 1}_{p}.$$

x y z

0^i 1^j where i > j

Picture now

Why is regular language not that powerful?