CMPSC 464: Introduction to the Theory of Computation Date: 10/19/2022

Recitation #7

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Notice: Try to solve these problems before the recitation session by yourself. Sometimes we put up more problems than one is able to discuss in 50 minutes. We want to make sure that we do not run out of problems during each session.

Problem 1

For any two NP languages L_1 and L_2, let M_1 and M_2 be the NTMs that decide them in polynomial time. We construct a NTM M' that decides L_1L_2 in polynomial time:

M' = "On input $\langle w \rangle$:

1. For each way to cut w into two substrings $w = w_1w_2$:
2. Run M_1 on w_1.
3. Run M_2 on w_2. If both accept, accept; otherwise continue with the next choice of w_1 and w_2.
4. If w is not accepted after trying all the possible cuts, reject".

In both stages 2 and 3, M' uses its nondeterminism when the machines being run make nondeterministic steps. M' accepts w iff w can be expressed as w_1w_2 such that M_1 accepts w_1 and M_2 accepts w_2. Therefore M' decides the concatenation of L_1 and L_2. Stage 2 run in polynomial time and is repeated for at most $O(n)$ time, so the algorithm runs in polynomial time.

Problem 2

Solution: Let $G = (V, E)$ be a graph with a set V of vertices and a set E of edges.We enumerate all triples (u, v, w) with vertices $u, v, w \in V$ and $u < v < w$, and then check whether or not all three edges $(u, v), (v, w)$ and (u, w) exist in E. Enumeration of all triples requires $\mathcal{O}(|V|^3)$ time. Checking whether or not all three edges belong to E takes $\mathcal{O}(|E|)$ time. Thus, the overall time is $\mathcal{O}(|V|^3|E|)$, which is polynomial in the length of the input $\langle G \rangle$. Therefore,$TRIANGLE \in P$.