# Lecture 21

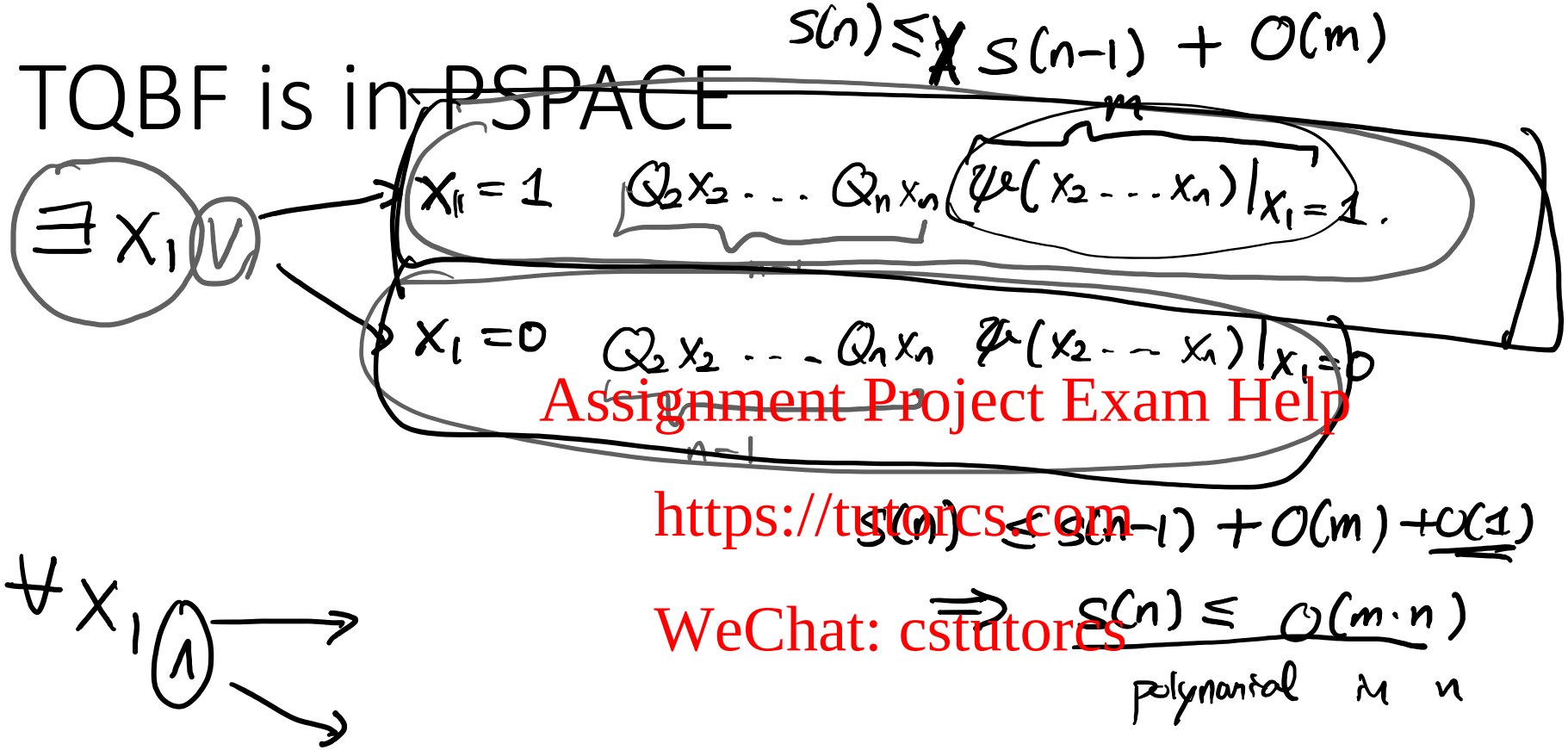
# Last time : TQBF

$Q_1 x_1 Q_2 x_2 Q_3 x_3 \ldots Q_n x_n \underbrace{\varphi(x_1 \ldots x_n)}$ .

$Q_1 \ldots Q_n \Rightarrow \exists \quad \Leftrightarrow$ SAT.

$Q_1 \ldots Q_n \Rightarrow \forall \quad \Leftrightarrow$ $\overline{\text{SAT}}$.

# TQBF is in PSPACE






$S(n) \le S(n-1) + O(m) + O(1)$

$\Rightarrow S(n) \le O(m \cdot n)$

polynomial in $n$

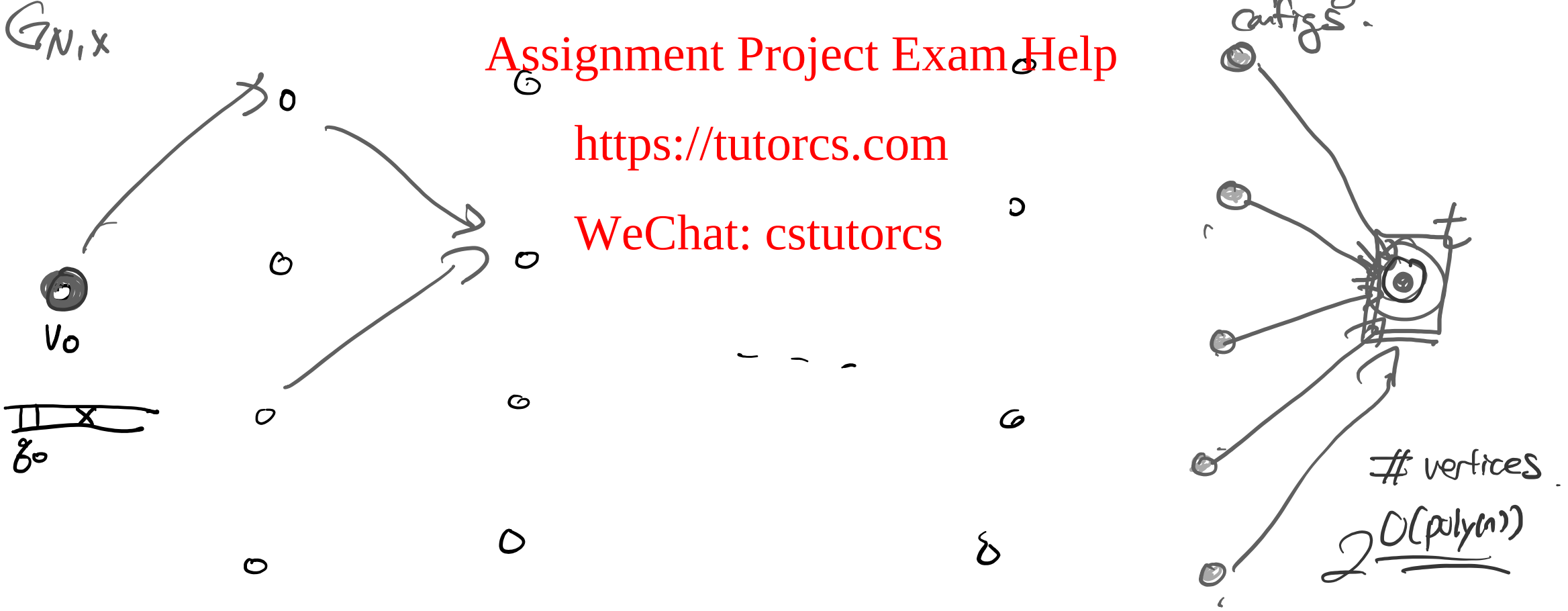
# TQBF is PSPACE hard – reachability

$\mathcal{L} \in PSPACE \longrightarrow \exists$ N.T.M. $N$ which decides $\mathcal{L}$ in poly space.

on input $x$

$x \in \mathcal{L}$ iff $v_0$ is connected to $t$.





# Continued

$\exists \psi_N(V_1, V_2)$ such that $\psi_N(V_1, V_2) = 1$ iff $V_2$ is yielded by $V_1$ in $2^i$ steps.

→ Cook-Leum.

$i = \text{poly}(n)$

$\psi_i(V_1, V_2)$ true iff $V_2$ is reachable from $V_1$



$\psi_i(V_1, V_2) = (\exists m)(\psi_{i-1}(V_1, m) \wedge \psi_{i-1}(m, V_2))$

$s(i) = 2s(i-1) + O(\text{poly}(n))$

$= 2(2 \cdot s(i-2)) + O(1)$

$\psi_0 = \psi_N$

$\exists m' \; \psi_{i-1}(V_1, m') \wedge \psi_{i-2}$

$\hookrightarrow 2^{\text{poly}(n)}$

$2^i s(0) + 2^i$

$\psi_i(V_1, V_2) = (\exists m)(\forall D_1, D_2)((D_1 = V_1 \wedge D_2 = m) \vee (D' = m \wedge D_2 = V_2)) \Rightarrow \psi_{i-1}(D_1, D_2)$

$s(i) = s(i-1) + O(\text{poly}(n))$

$\Rightarrow$ poly-time

$\psi_{\text{poly}(n)}$ is poly-time.

true

# Motivations for studying PSPACE

- Chess, Go, etc all board games. Finding the best strategy ?

# L and NL

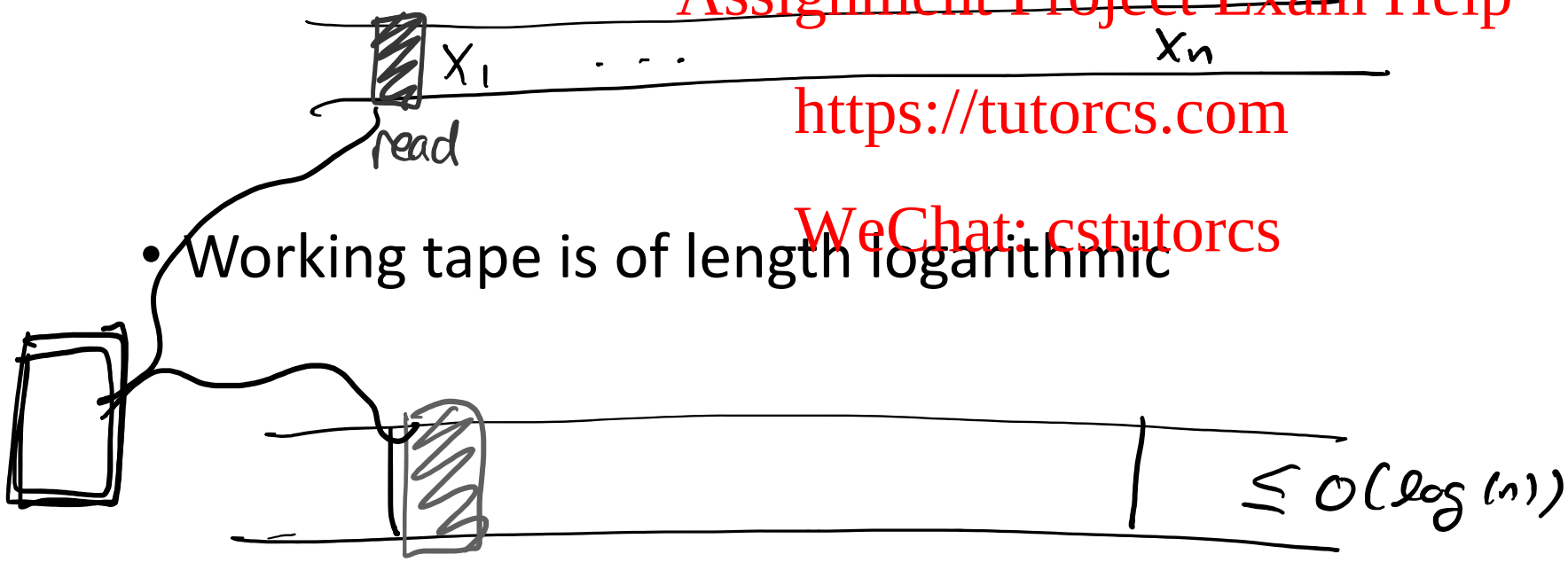
# Logarithmic Space?

- Well input is of length n, even storing space takes linear space … ?

# Multitape Turing Machine

- Input is written on the read only tape






- Working tape is of length logarithmic

# L and NL definition

$\mathcal{L} \in L$ if $\mathcal{L}$ can be decided by a det machine using log space



$\mathcal{L} \in NL$

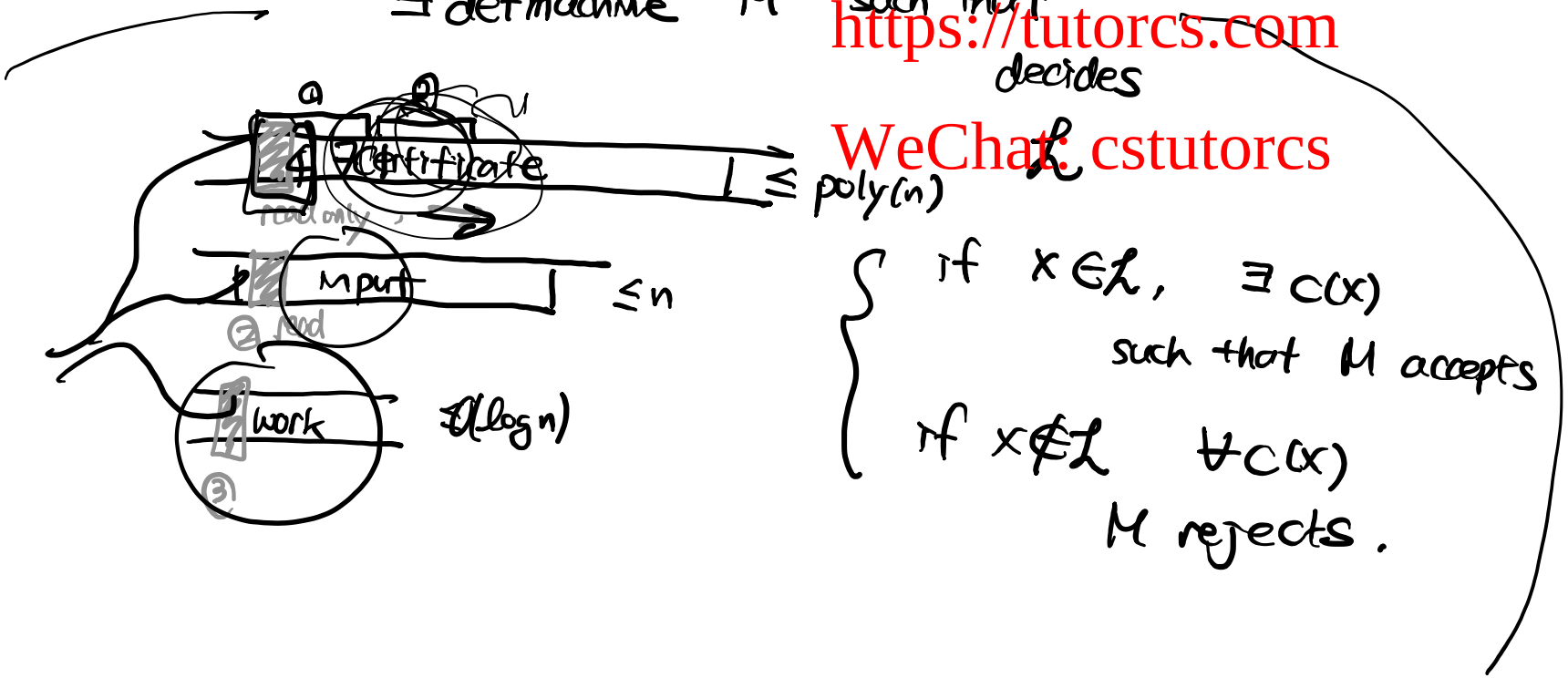


non-det

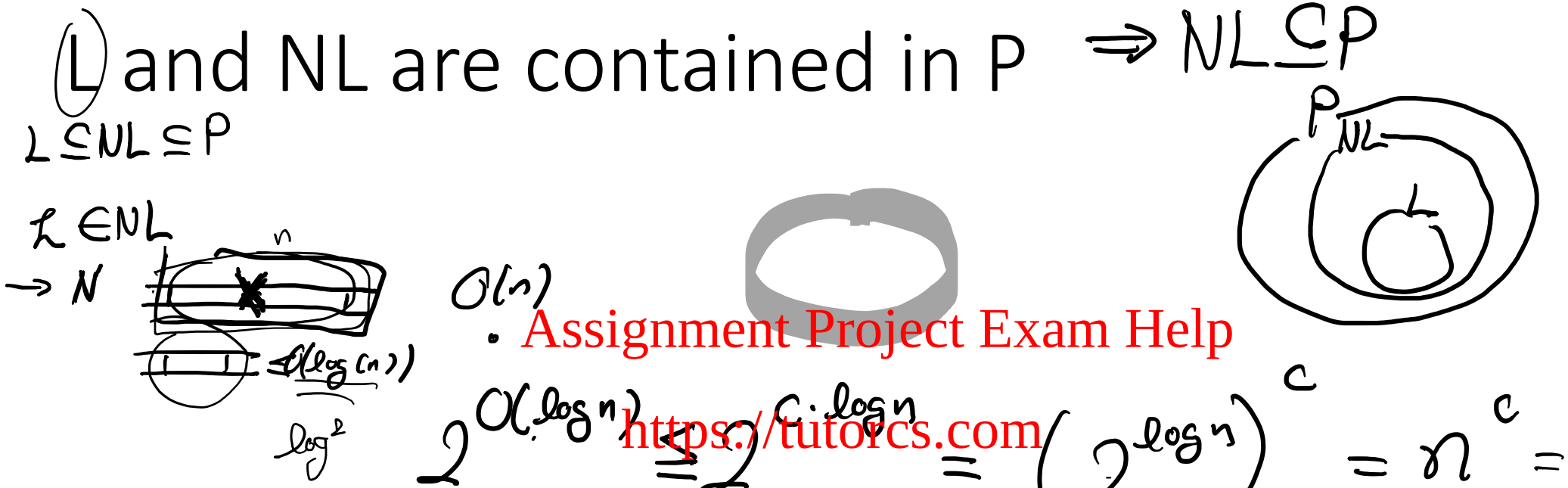
# Alternative Definition of NL

- Certificate Tape, Input tape, Work tape





# L and NL are contained in P



# Relationship between L and NL

- Savitch's Theorem?

$L \subseteq NL \subseteq L$



$NL \subseteq DSPACE(\log^2 n)$

workspace

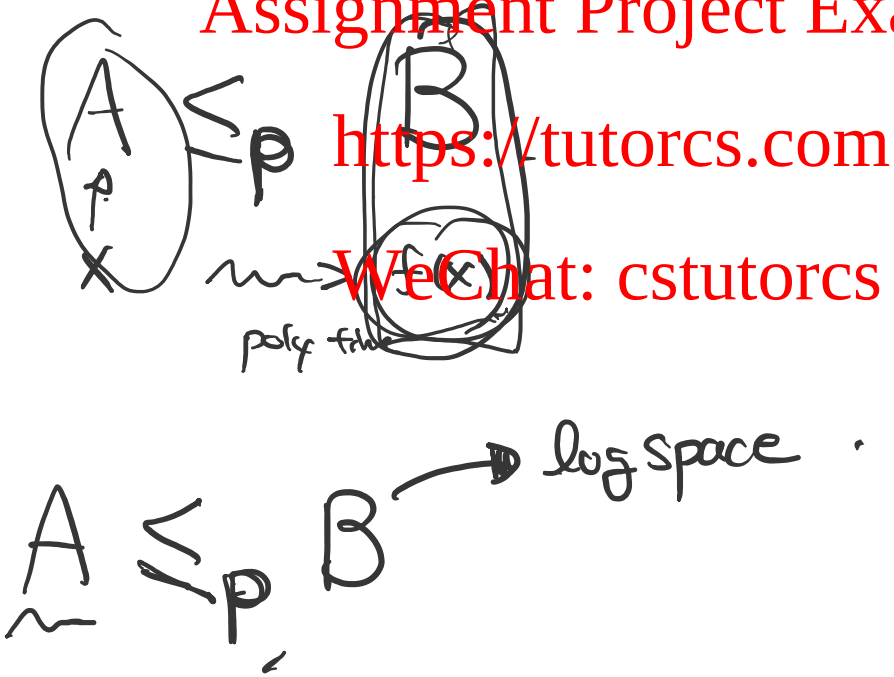
even when $S(n) \ll n$

$NSPACE(S(n)) \subseteq DSPACE(S(n)^2)$

# L/NL completeness

- If we just use Karp reduction, what goes wrong?





# Logspace Reductions

- Reduction function is logspace computable

# Transitivity holds

# If A reduces to B and B is in L, then A is in L

# NL-completeness?

# PATH is NL-complete

- PATH = { (G,s,t) | t is reachable from s in G }

# PATH is in NL

# PATH is NL-hard