Lecture 3

Recap : regular language

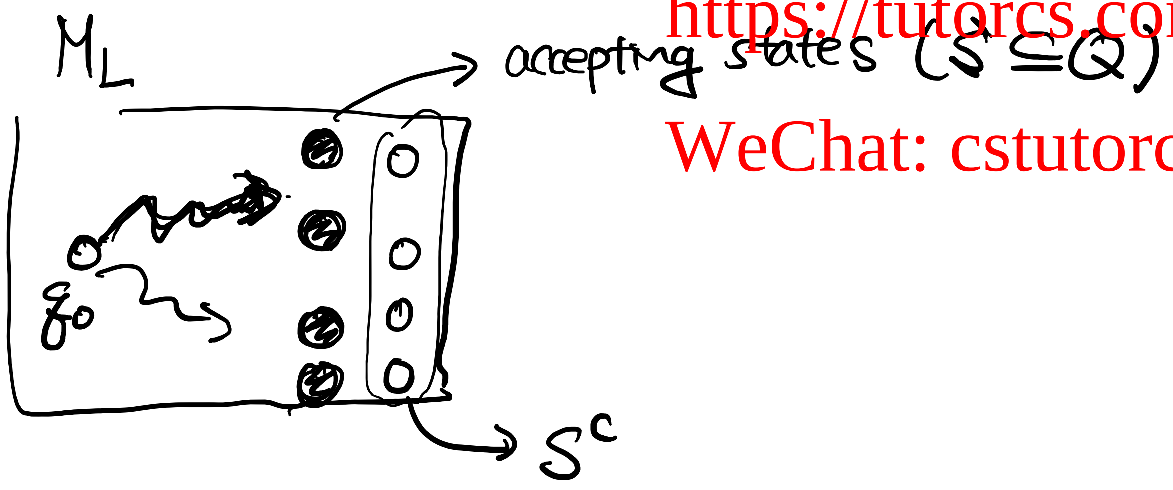
L is regular if $L = L(M)$ for some finite automata M.

Closed under Complement

If $\underline{L \text{ is regular}}$, then $\bar{L}$ is regular.

M_L whose language is L $\Longrightarrow$ $M_{\bar{L}}$ $\longrightarrow$ just change the accepting states of M_L from S to $S^c = Q - S$

w



Closed under Union

If L_1 is regular and L_2 is regular, then

$L_1 \cup L_2$ is also regular

$L_1 \cap L_2$ "

$M_{L_1} = (Q_1, q_0^1, \delta_1, S_1, \Sigma)$

$M_{L_2} = (Q_2, q_0^2, \delta_2, S_2, \Sigma)$

$M = \left(Q_1 \times Q_2, (q_0^1, q_0^2), \delta, \boxed{\begin{matrix} S_1 \times Q_2 \\ \cup (Q_1 \times S_2) \end{matrix}}, \Sigma\right)$

$S_1 \times S_2$

$\delta : (Q_1 \times Q_2) \times \Sigma \to Q_1 \times Q_2$

$\delta((q_1, q_2), x) := (\delta_1(q_1, x), \delta_2(q_2, x))$

Closed under Concat / Star?

$L_1 \circ L_2 = \{ w_1 \cdot w_2 \mid w_1 \in L_1, w_2 \in L_2 \}$

$L^* = \bigcup_{k=0} L^k \qquad L^k = \underbrace{L \circ \cdots \circ L}_{k \text{ times}}$

$L_1 = \{ 00, 001 \}$

$L_2 = \{ 11 \}$.

$\underbrace{00}_{w_1} | \underline{111} \in L_1 \circ L_2$

$001 | 11$

Nondeterminism

For deterministic M.

Deterministic → Given the input ω

$\underbrace{q_0, q_1, q_2 \cdots}_{\text{fixed}}$.

Nondeterministic →

not fixed

of each step.

you are allowed to make choice!!

Nondeterministic Finite Automata (NFA)

D Finite Automata

Q

q_0

$\delta : Q \times \Sigma \to Q$

S

Σ

$\delta(q_{current}, x) = q_{next}$

Non deterministic FA.

Q

q_0

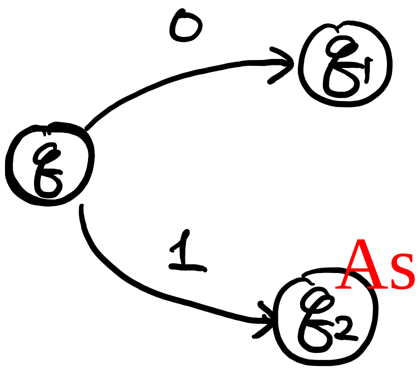
$\delta : Q \times \Sigma \to P(Q)$

S

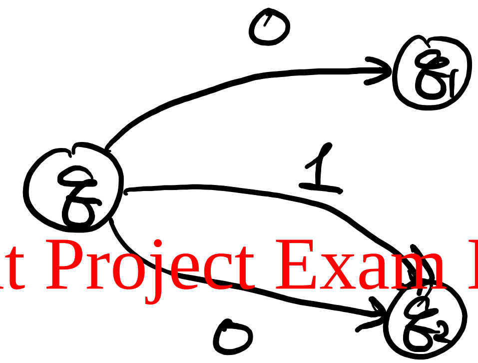
Σ

$\delta(q_{current}, x) = \{ \underbrace{q^1_{next}, q^2_{next} \cdots q^k_{next}}_{\text{set of possible next state.}} \}.$

Transition Function



DFA

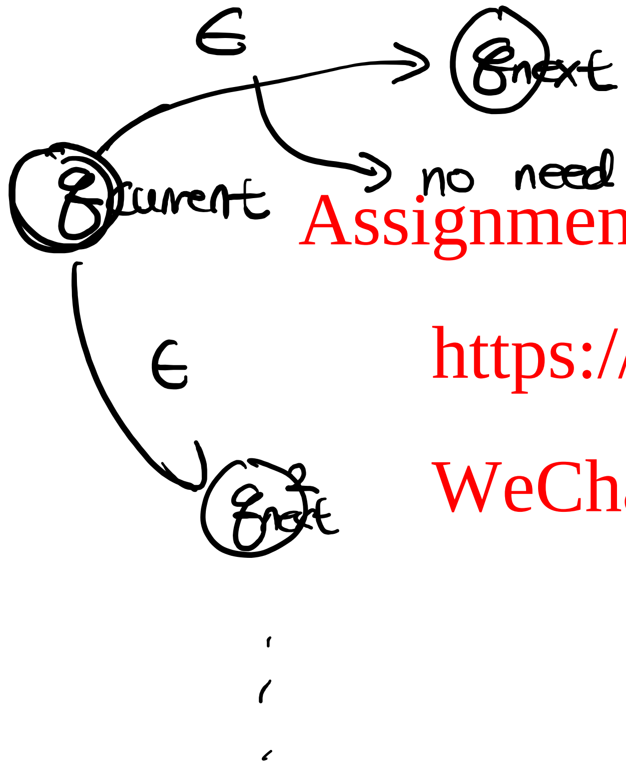


NFA

$\Sigma \cup \{\epsilon\}$.

Ability to "Jump" via eps (ϵ)

$\delta(q_{current}, \epsilon) = \{q^1_{next}, \ldots\}$



no need to read input string

Example

0 , 1

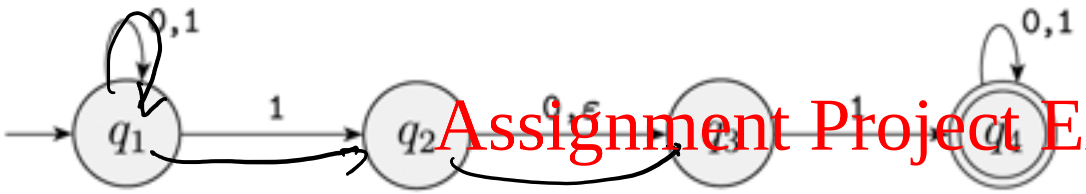


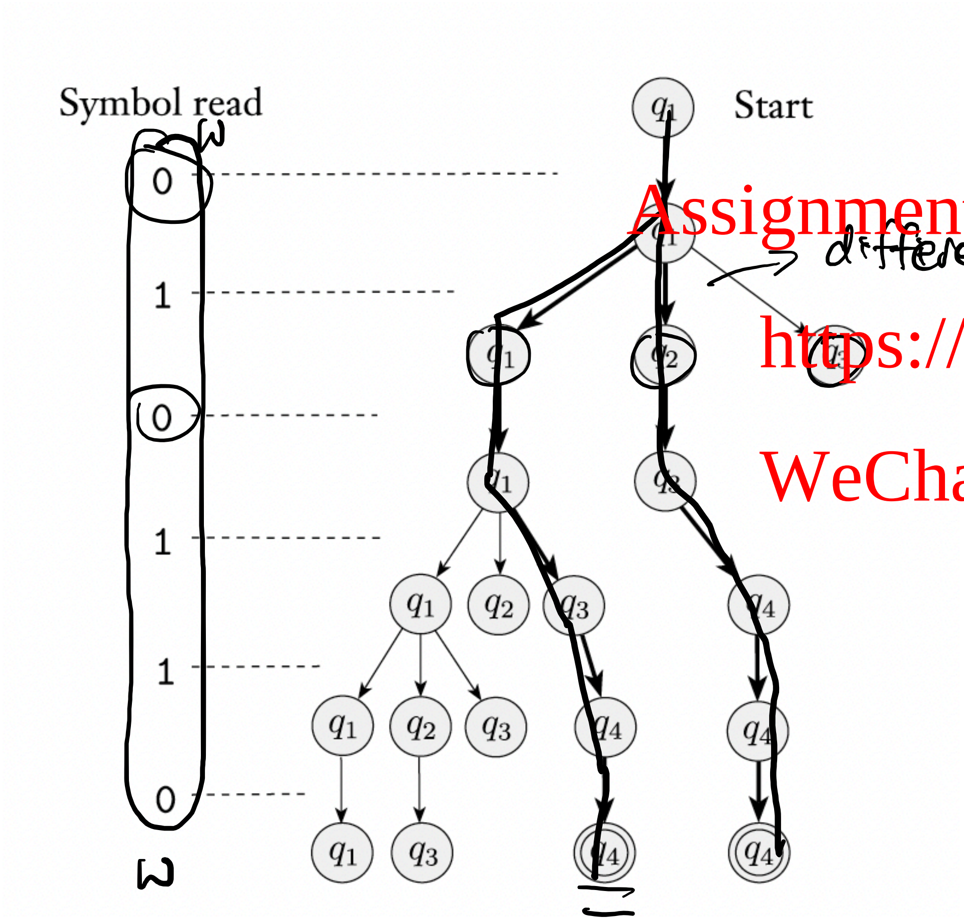
FIGURE 1.27
The nondeterministic finite automaton N_1

N FA

NFA accepts w if …

Computation Tree

DFA accepts ω
if there exists
"accepting path"



→ different possibility of choice

NFA is stronger than DFA (why?)

DFA is NFA without ϵ operation

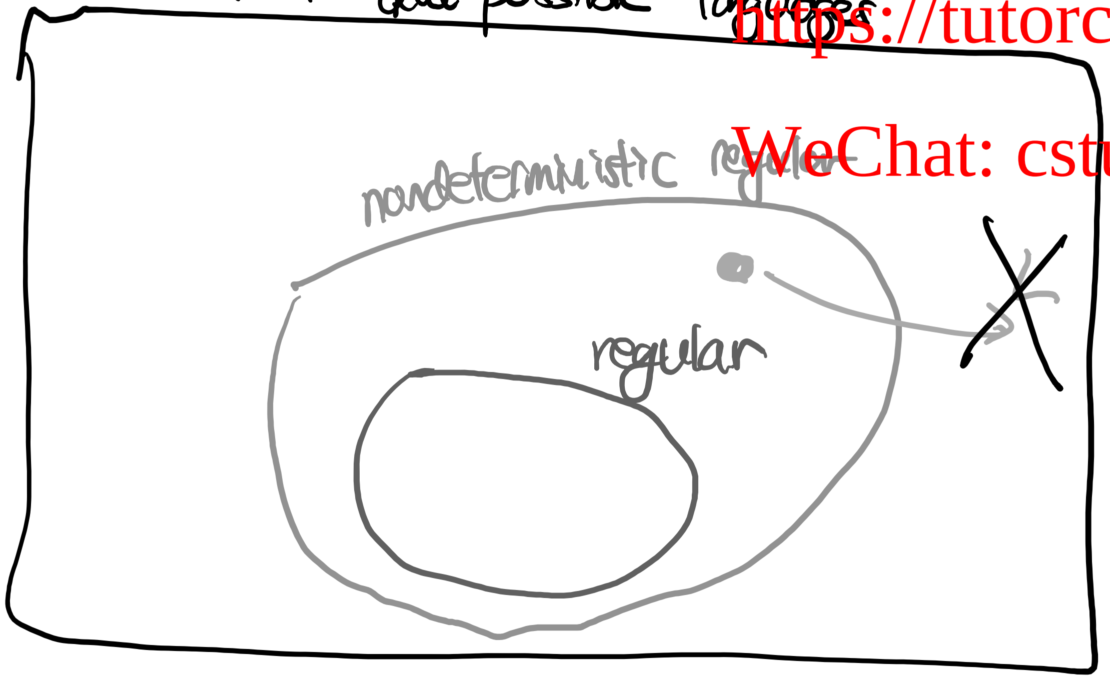
$$|\delta(q, x)| = 1$$

In terms of regular language ?

nondeterministic regular.

L s.t. $\exists$ NFA N such that $L(N) = L$

set of all possible languages



Every NFA has equivalent DFA !

(NFA DFA Equivalence)
If L is accepted by some NFA N
then $\exists$ DFA M_N s.t. $L(M_N) = L(N) = L$

regular = non-deterministic

regular

Proof Continued

So regular defined using NFA and DFA ?

nondeterministic regular

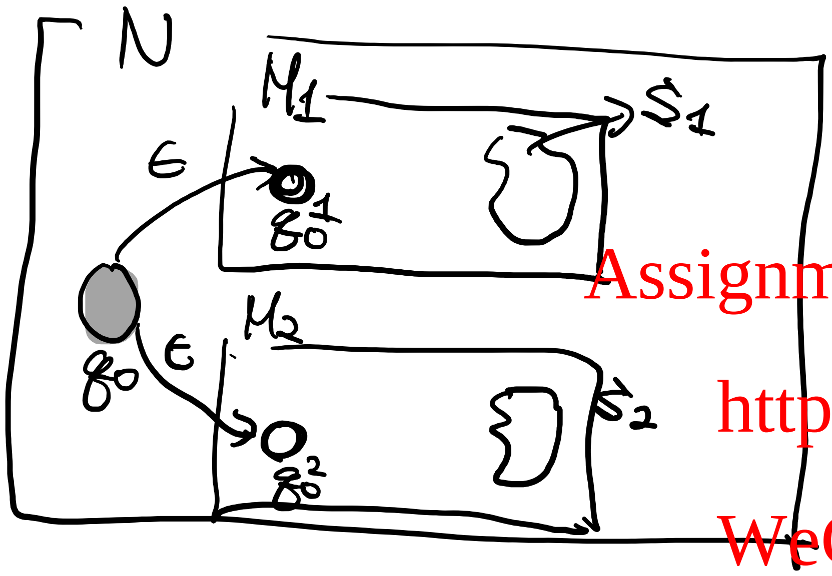
= deterministic regular (regular)

If I show L has corresponding NFA N

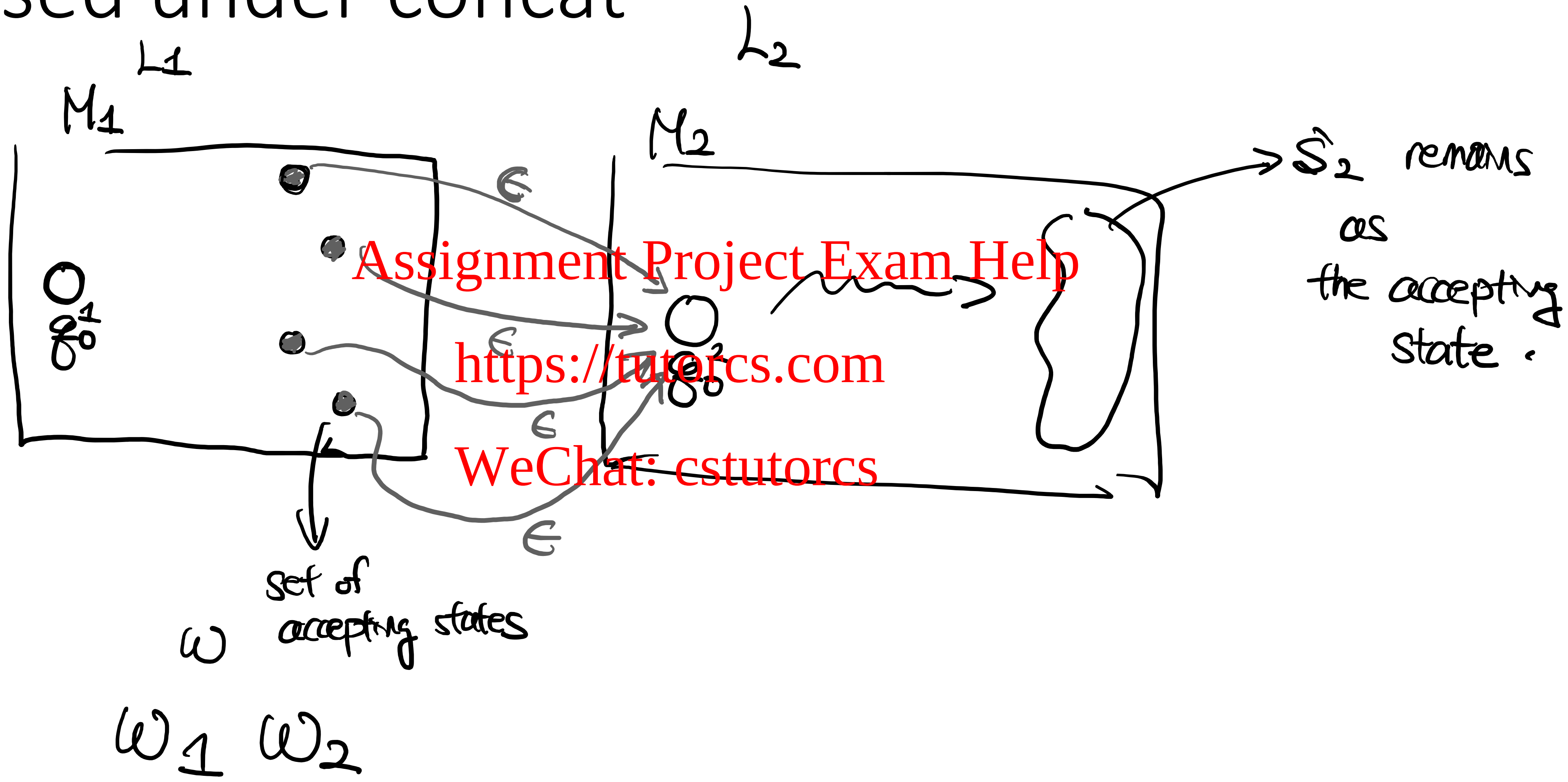
DFA M

⇒ L is regular.

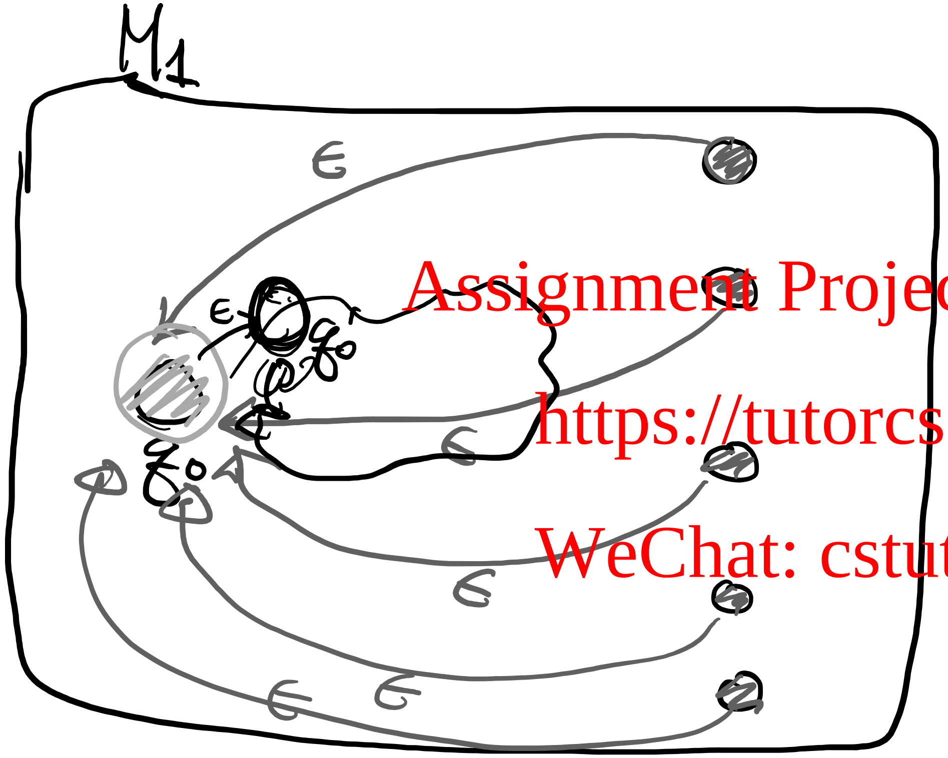
Now : Closed under Union



Closed under concat



Closed under star



L^1

L^2

$w_1 \; w_2$

L^3

$\vdots$

L^k

Closed under intersection (in class exercise)

Are all languages regular ?