CMT107 Visual Computing

Corner Detection

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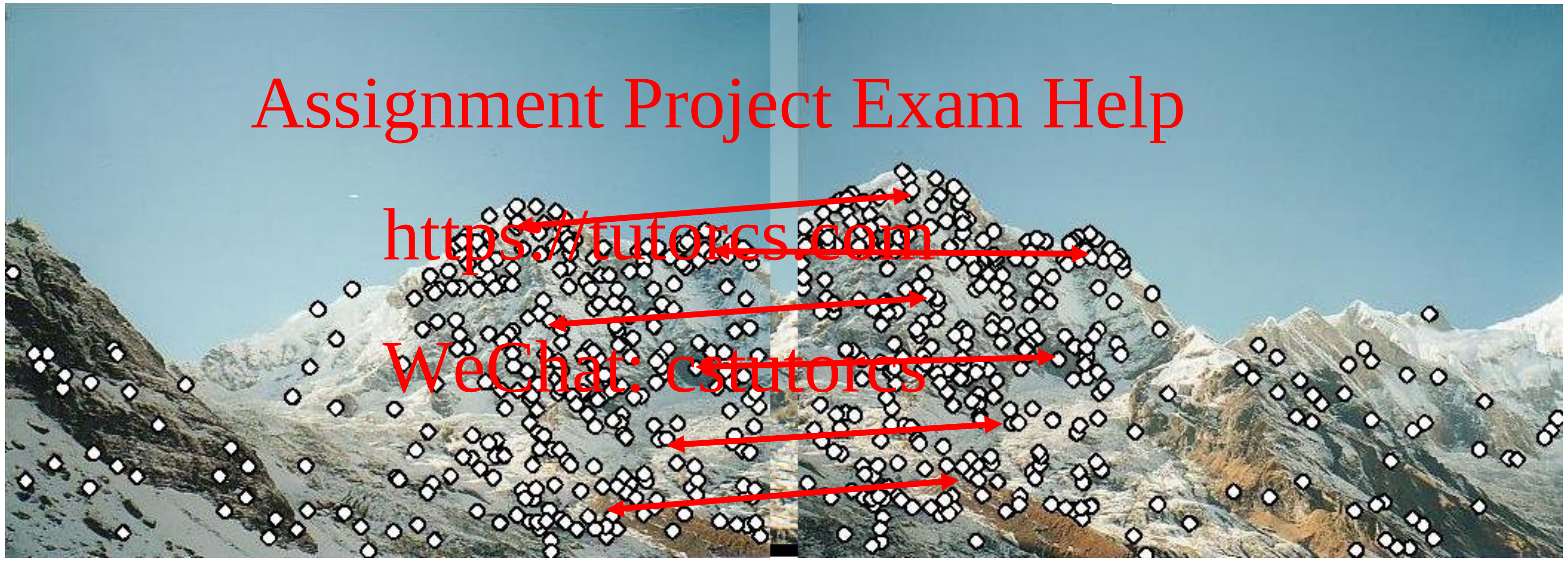
Overview

- Feature Extraction
  - Characteristics of good features
  - Applications
- Corner Detection
  - Basic idea
  - Mathematics
- Harris Detector
- Invariance and Covariance

Feature Extraction: Corners

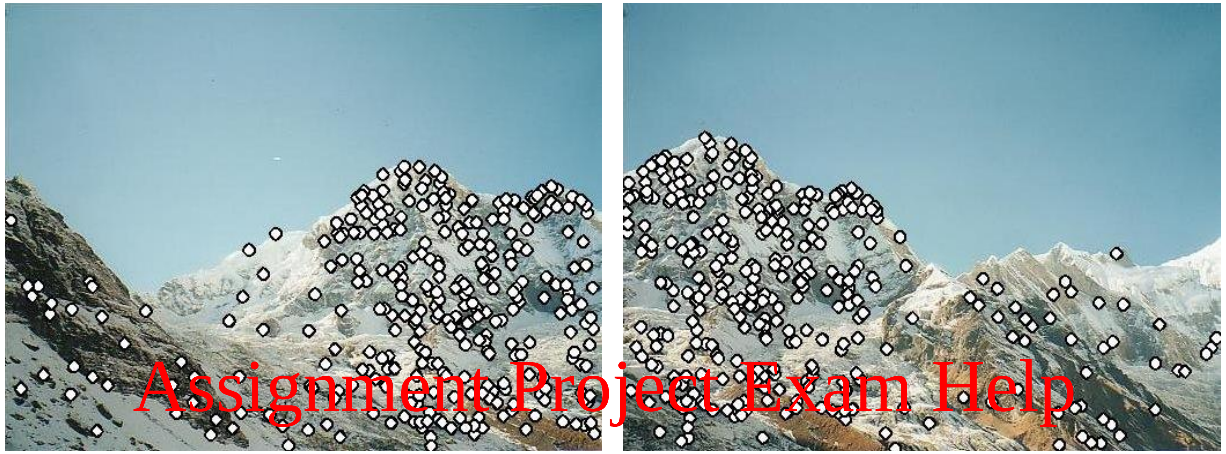
Why Extract Features

- Motivation: panorama stitching
  - We have two images – how do we combine them?



Step 1: extract features
Step 2: match features
Step 3: align images

Characteristics of Good Features

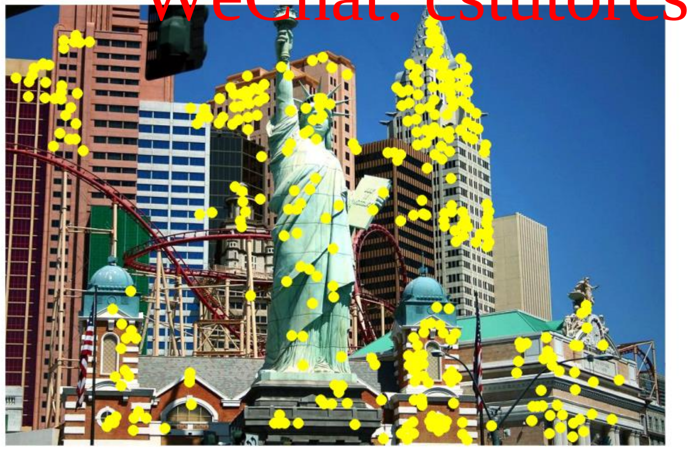


- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations

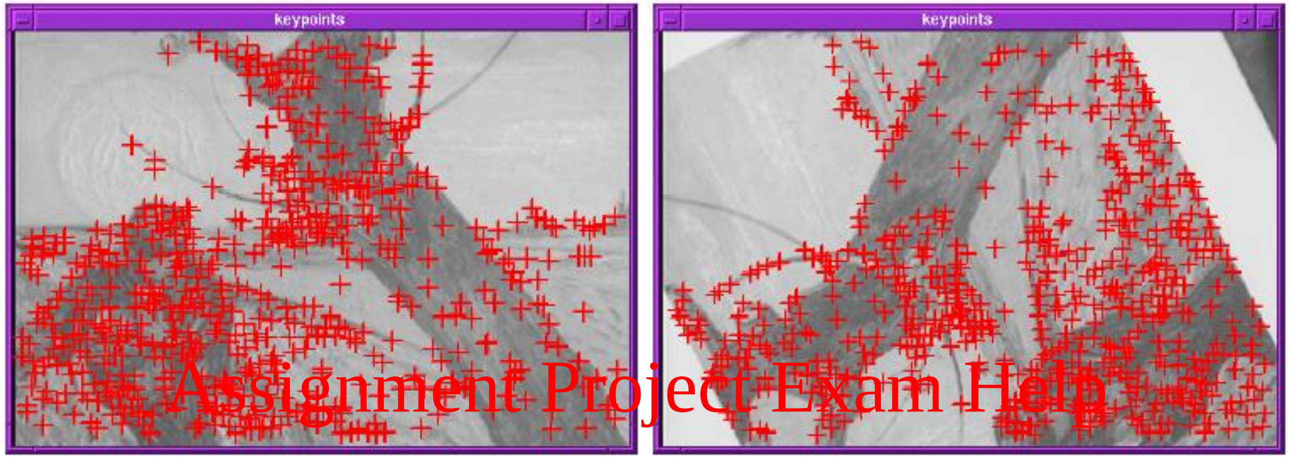
- Saliency
  - Each feature is distinctive
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion

Applications

- Feature points are used for
  - Image alignment
  - 3D reconstruction
  - Motion tracking
  - Robot navigation
  - Indexing and database retrieval
  - Object recognition



Finding Corners



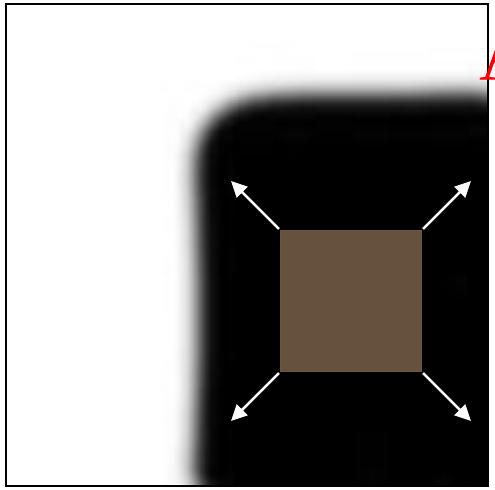
- Key properties: In the region around a corner, image gradient has two or more dominant directions

- Corners are repeatable and distinctive

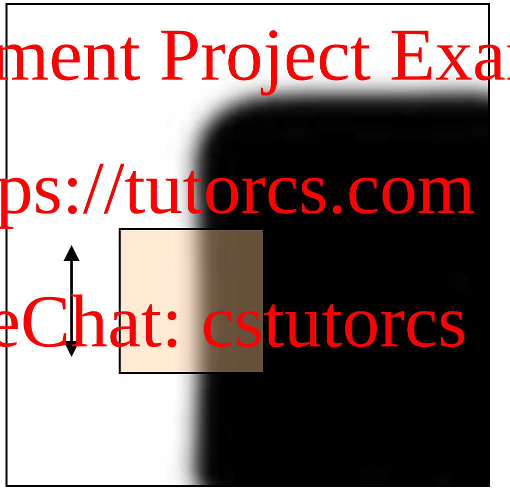
C.Harris and M.Stephens. "A Combined Corner and Edge Detector.“ *Proceedings of the 4th Alvey Vision Conference, 1988*: pages 147--151.

Corner Detection: Basic Idea

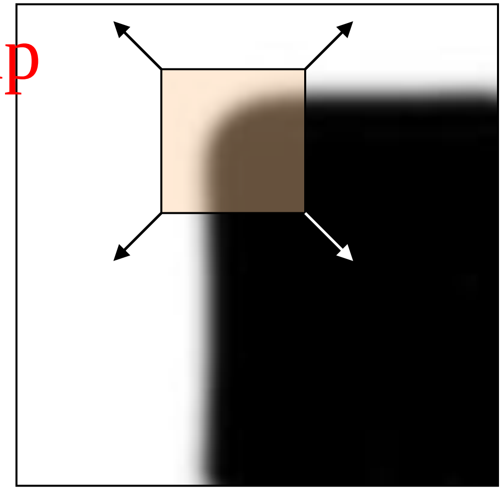
- We can easily recognise the point by looking through a small window
- Shifting a window in any direction should give a large change in intensity



“flat” region:
no change in
all directions



“edge”:
no change along
the edge direction

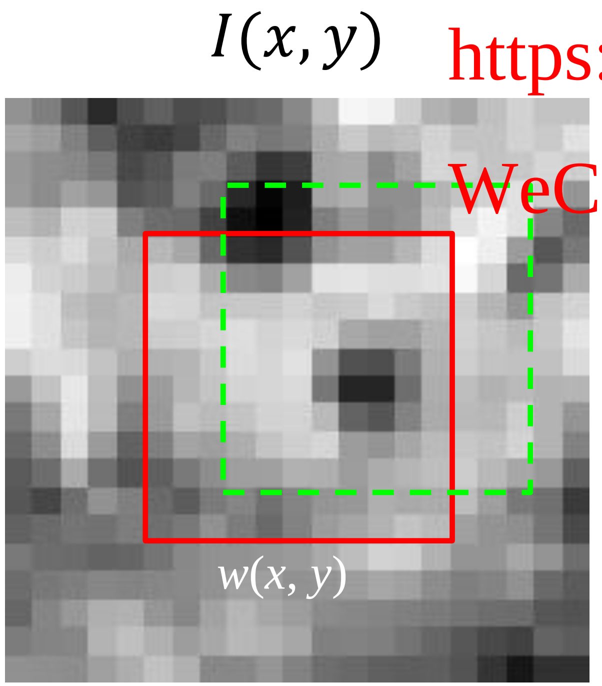


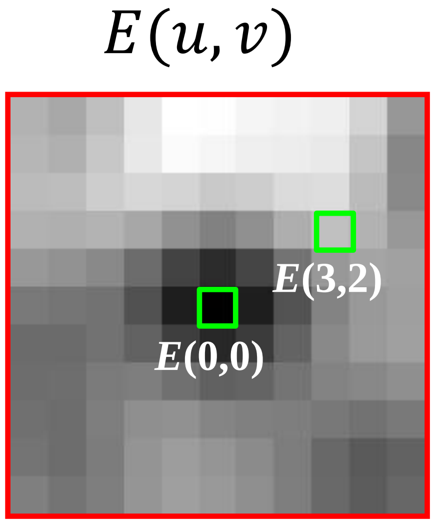
“corner”:
significant change
in all directions

Corner Detection: Mathematics

- Change of intensity for the shift $[u, v]$

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$





Corner Detection: Mathematics

- Change of intensity for the shift $[u, v]$

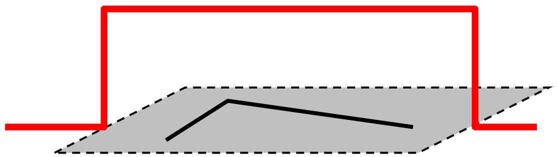
$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

Window function

Shifted intensity

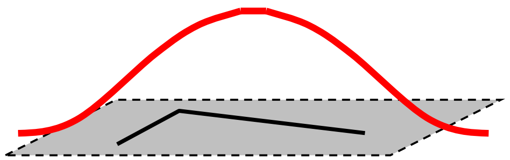
Intensity

- Window function $w(x, y) =$



1 in window, 0 outside

or



Gaussian

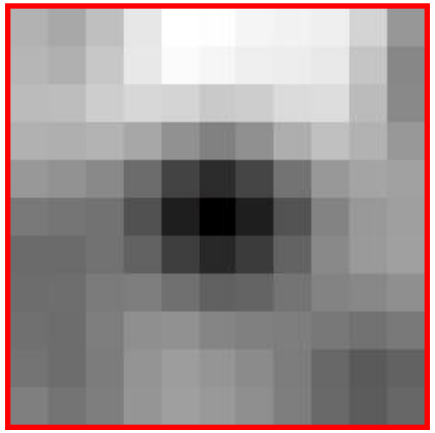
Corner Detection: Mathematics

- Change of intensity for the shift $[u, v]$

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

- We want to find out how this function behaves for small shifts

$E(u,v)$



Corner Detection: Mathematics

- Change of intensity for the shift $[u, v]$

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

- We want to find out how this function behaves for small shifts
- Using *first-order Taylor approximation*:

$I(x+u, y+v) \approx I(x,y) + uI_x(x,y) + vI_y(x,y)$, Then:

$$E(u,v) \approx \sum_{x,y} w(x,y)\left[I(x,y) + uI_x(x,y) + vI_y(x,y) - I(x,y)\right]^2$$

$$= \sum_{x,y} w(x,y)\left[uI_x(x,y) + vI_y(x,y)\right]^2 = \cdots$$

Corner Detection: Mathematics

$$\cdots = \sum_{x,y} w(x,y)\left[u^2 I_x^2 + v^2 I_y^2 + 2uvI_xI_y\right]$$

$$= \sum_{x,y} w(x,y)[u \;\; v]\begin{bmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}$$

$$= [u \;\; v]\underbrace{\left(\sum_{x,y} w(x,y)\begin{bmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{bmatrix}\right)}_{M}\begin{bmatrix} u \\ v \end{bmatrix}$$

Corner Detection: Mathematics

- The approximation simplifies to

$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

- where *M* is a *second moment matrix* computed from image derivatives:

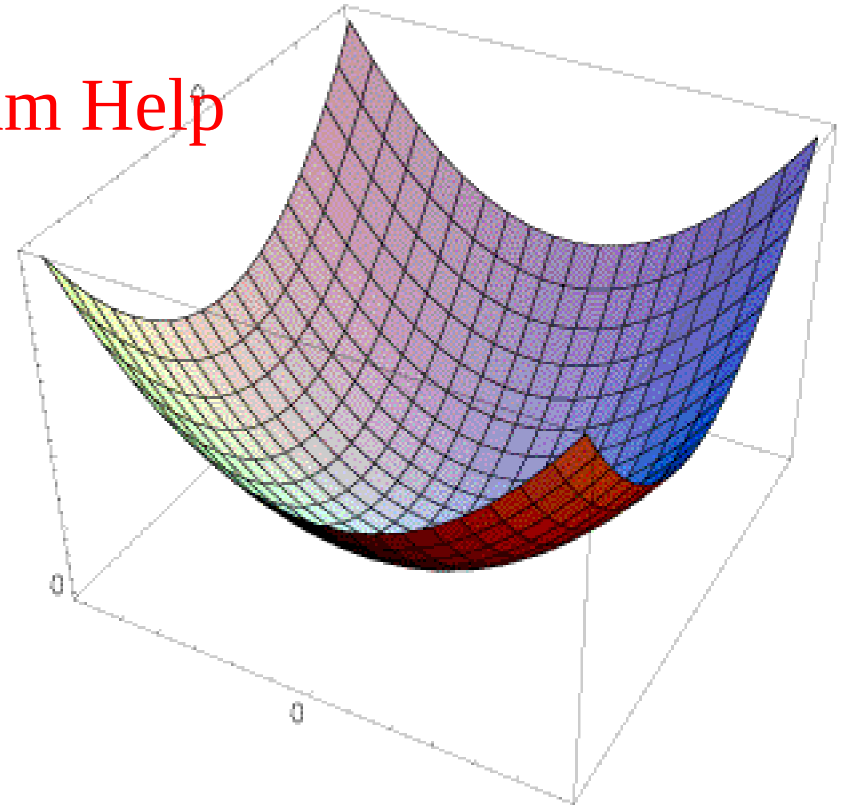
$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Interpreting the Second Moment Matrix

- The surface $E(u, v)$ is locally approximated by a quadratic form. Let's try to understand its shape.

$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Interpreting the Second Moment Matrix

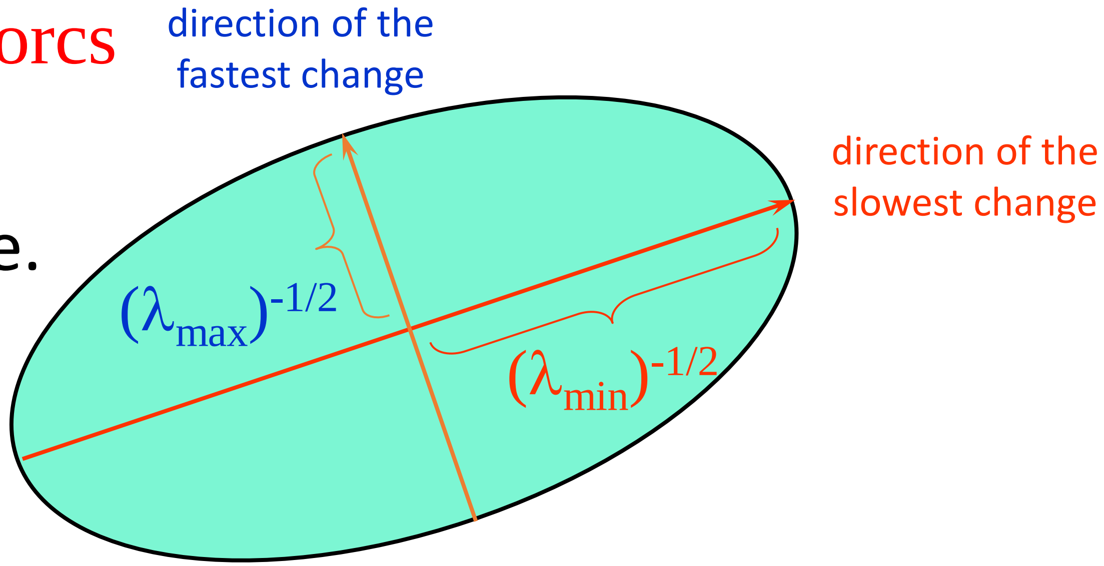
- Consider a horizontal slice of $E(u, v)$: $\quad [u \;\; v]\, M \begin{bmatrix} u \\ v \end{bmatrix} = const$
- This is an equation of an ellipse

Interpreting the Second Moment Matrix

- Consider a horizontal slice of $E(u,v)$: $\quad [u \;\; v]\, M \begin{bmatrix} u \\ v \end{bmatrix} = const$
- This is an equation of an ellipse
- Diagonalization of M: $\quad M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

- The axis lengths are determined by the eigenvalues: λ_1 and λ_2, and the orientation is determined by R

- If either λ is close to 0, then this is not a corner, so look for positions where both are large.



Visualization of Second Moment Matrices

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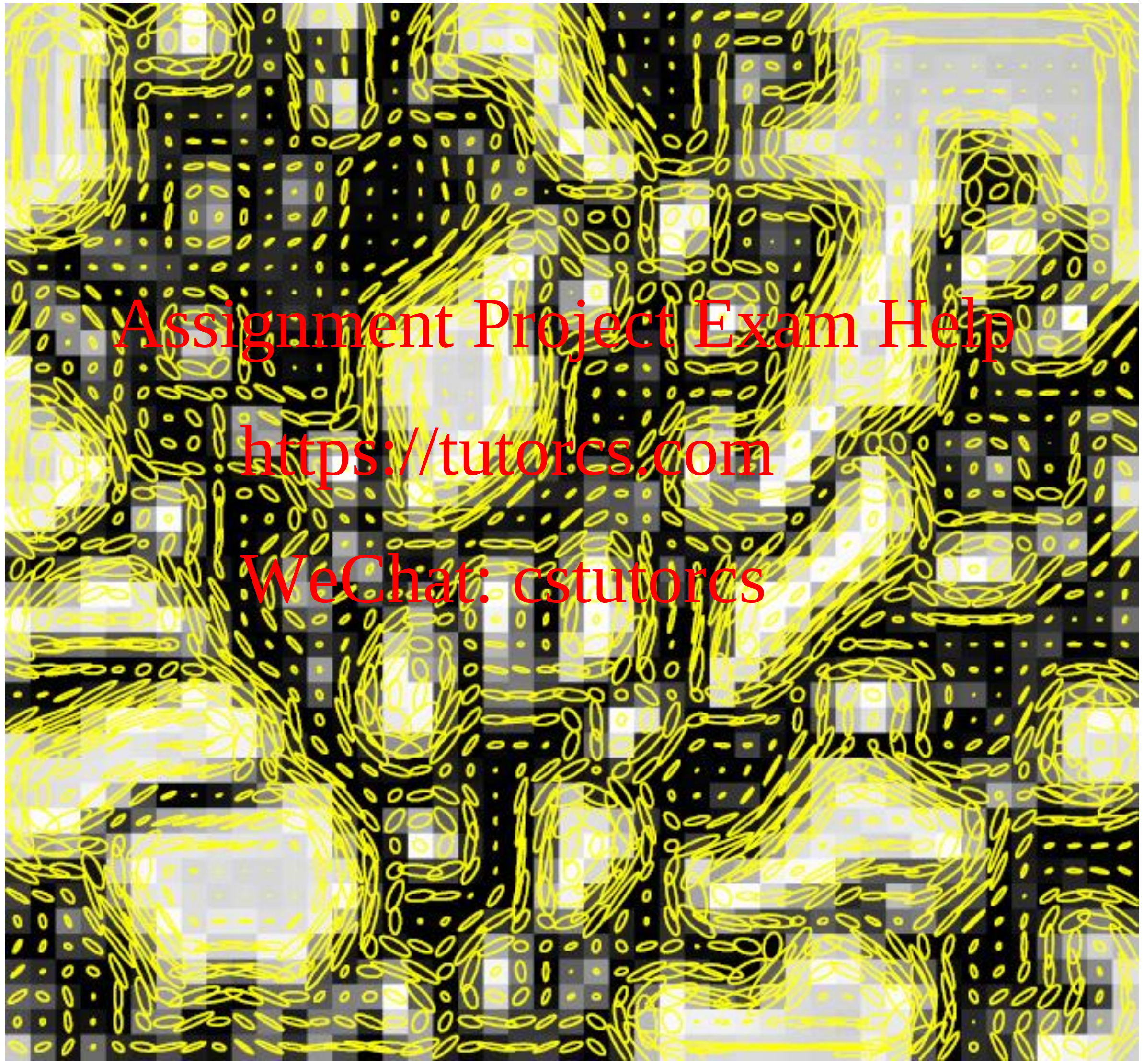
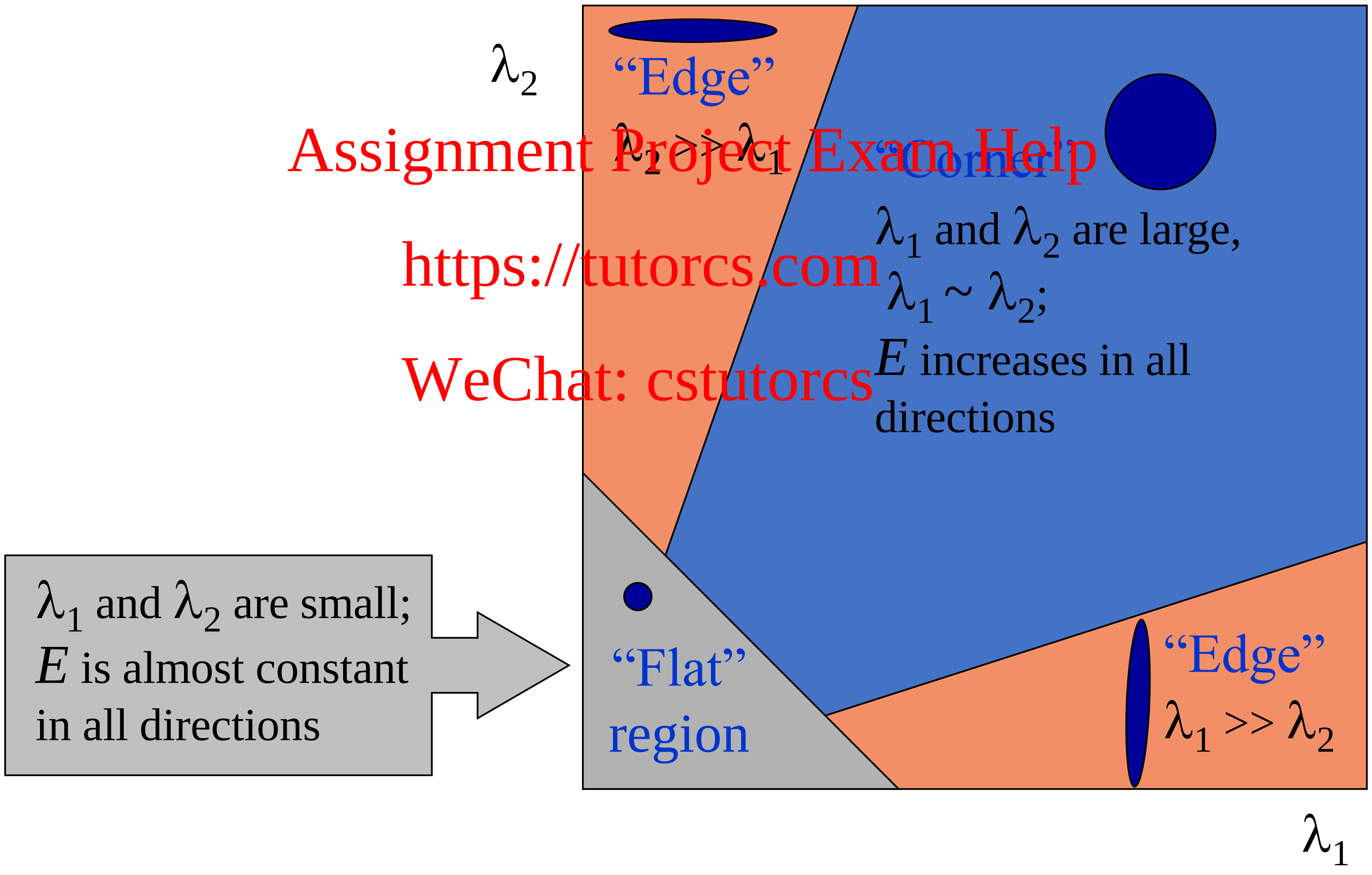


Interpreting the Eigen Values

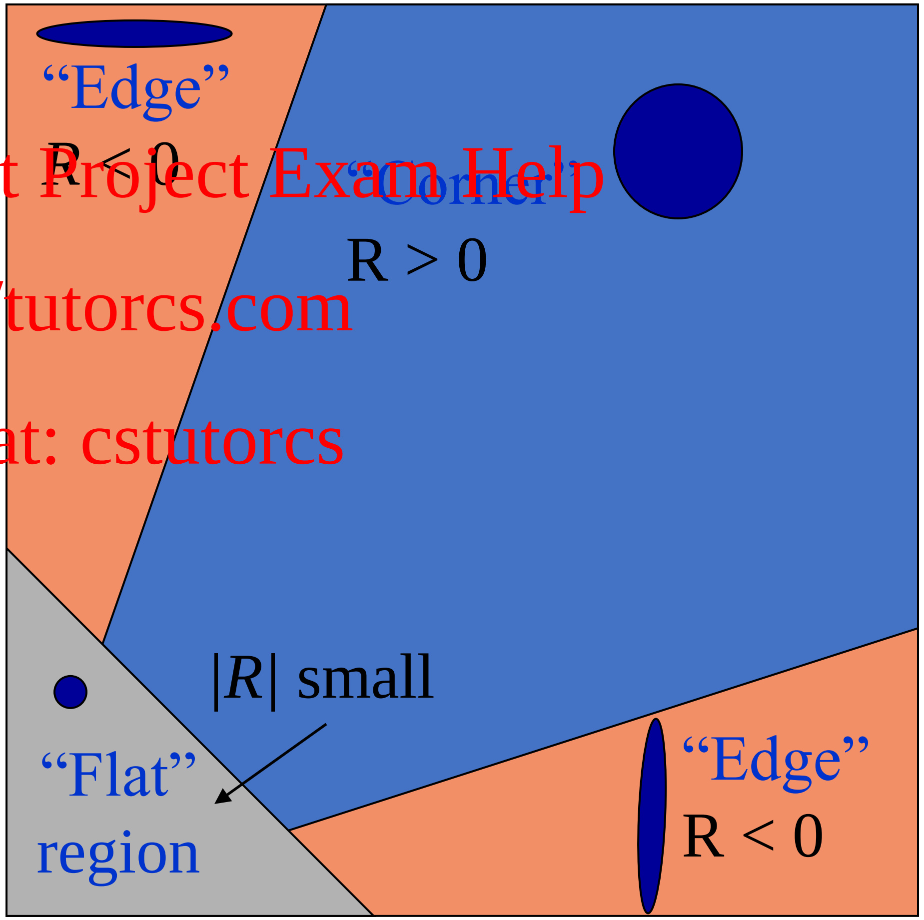
- Classification of image points using eigenvalues of M.



Corner Response function

- $R = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2 = \det(M) - \alpha\,\mathrm{trace}(M)^2$

  α: constant (0.04 to 0.15)



Harris Detector: Steps

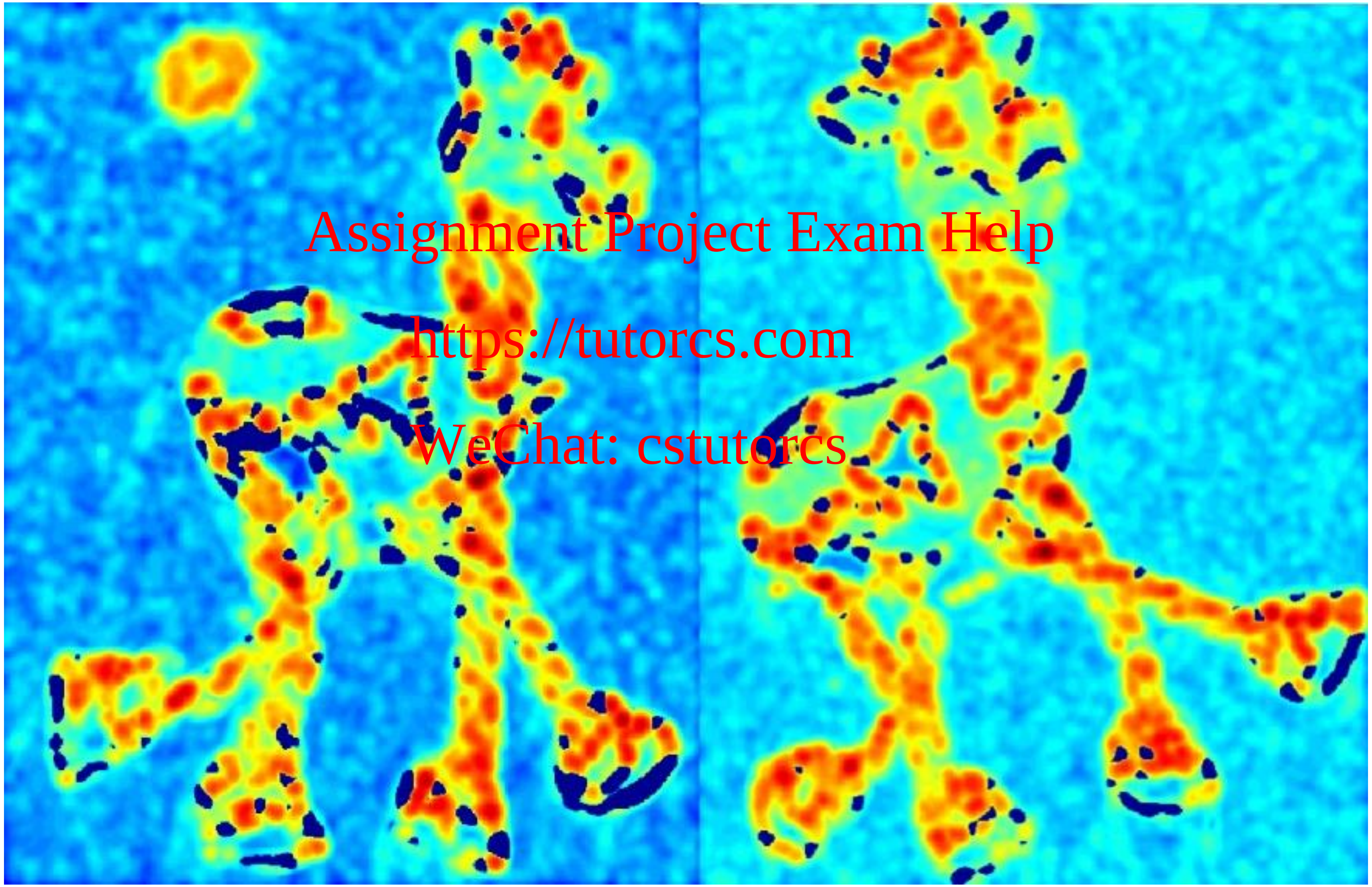
- Compute Gaussian derivatives at each pixel
- Compute second moment matrix M in a Gaussian window around each pixel
- Computer corner response function R
- Threshold R
- Find local maxima of response function (nonmaximum suppression)

C.Harris and M.Stephens. [“A Combined Corner and Edge Detector.”]
Proceedings of the 4th Alvey Vision Conference: pages 147—151, 1988.

Harris Detector: Steps

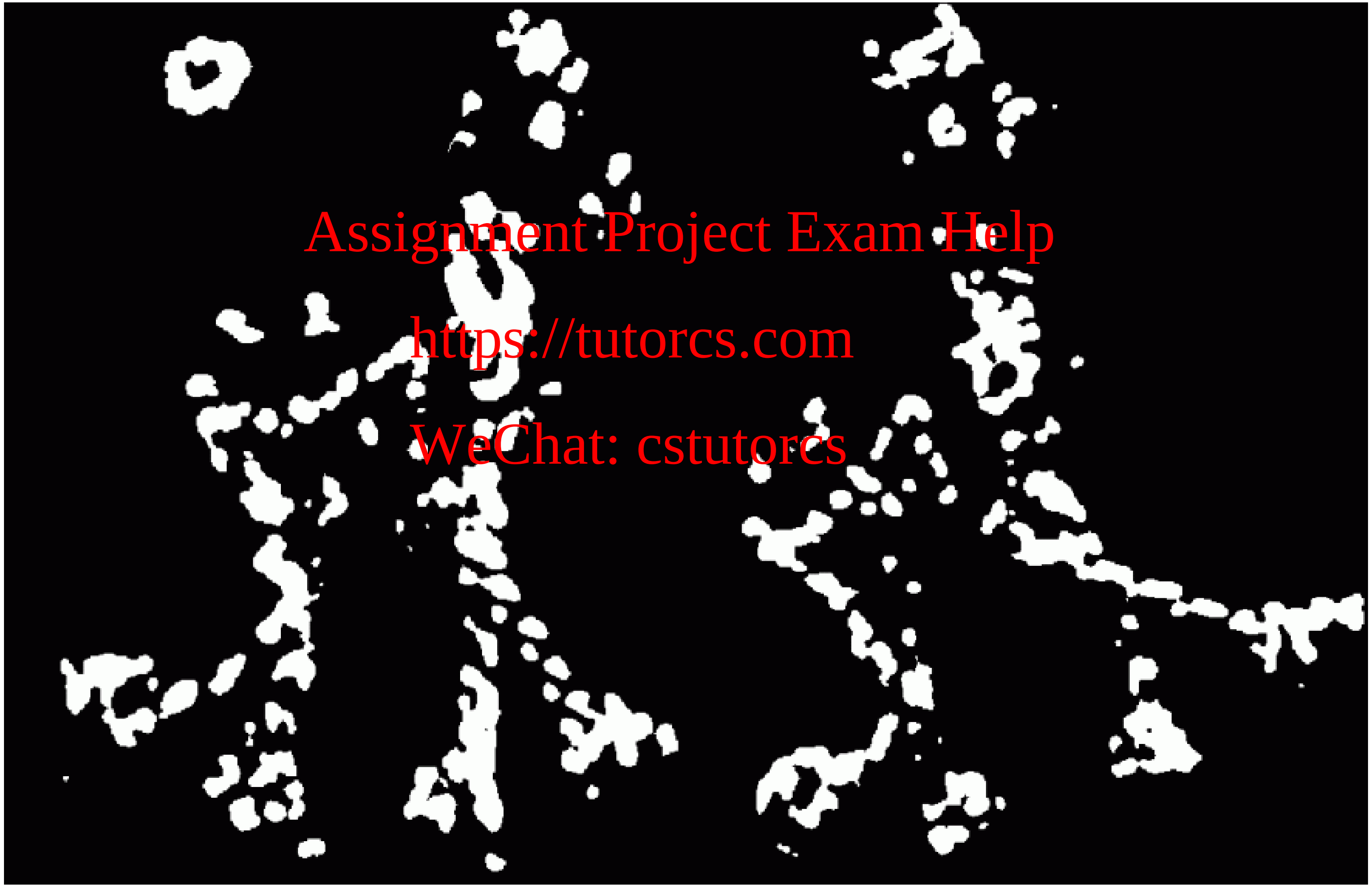
Harris Detector: Steps

- Compute corner response R



Harris Detector: Steps

- Find points with larger corner response: $R > threshold$



Harris Detector: Steps

- Take the points of local maxima of R

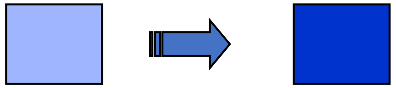
Harris Detector: Steps

Harris Detector: Steps

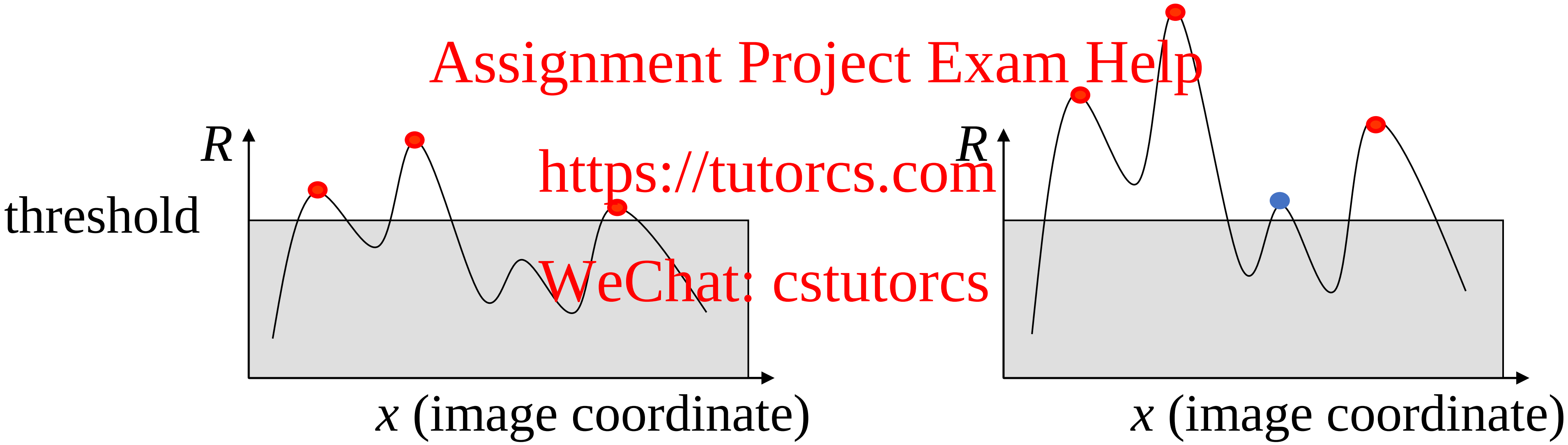
- We want corner locations to be *invariant* to photometric transformations, and *covariant* to geometric transformations.
  - Invariance: image is transformed and corner locations do not change
  - Covariance: if we have two transformed versions of the same image, features should be detected in corresponding locations.

Affine Intensity Change

- $I \rightarrow aI + b$

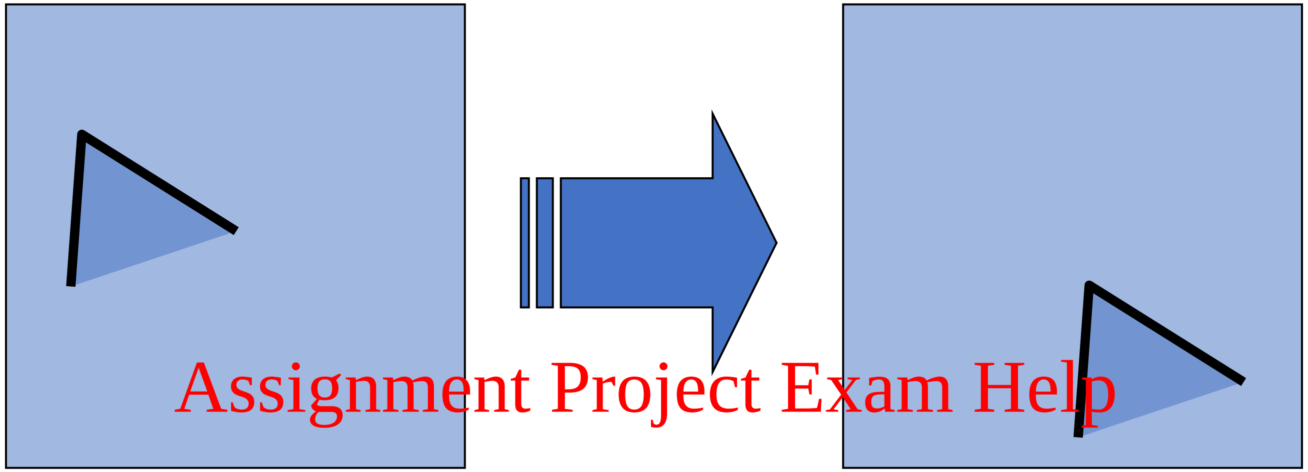


- Only derivatives are used ⇒ invariance to intensity shift $I \rightarrow I + b$
- Intensity scaling: $I \rightarrow aI$



Partially invariant to affine intensity change

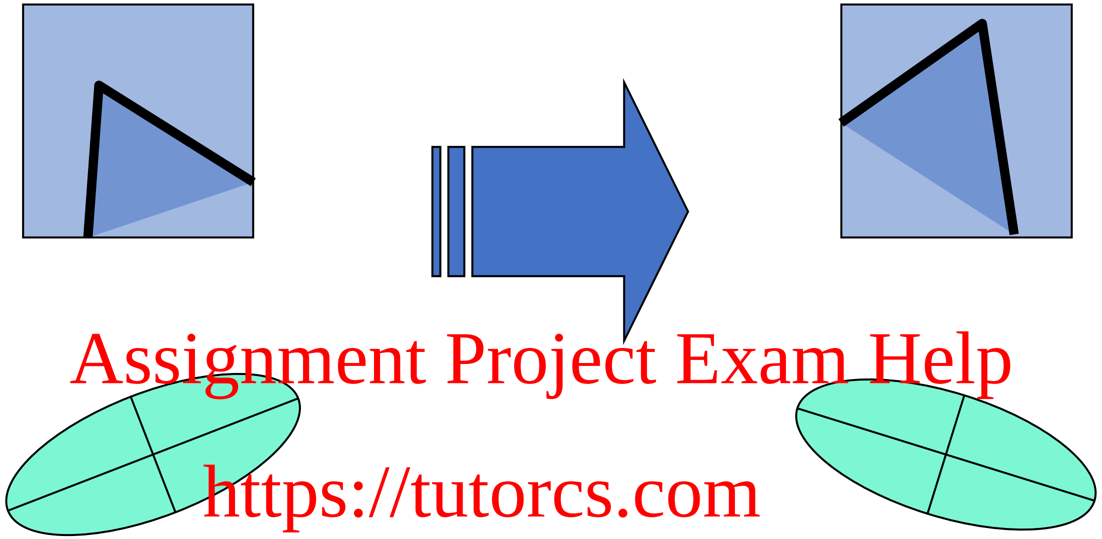
Image Translation



- Derivatives and window function are shift-invariant

Corner location is covariant w.r.t. translation

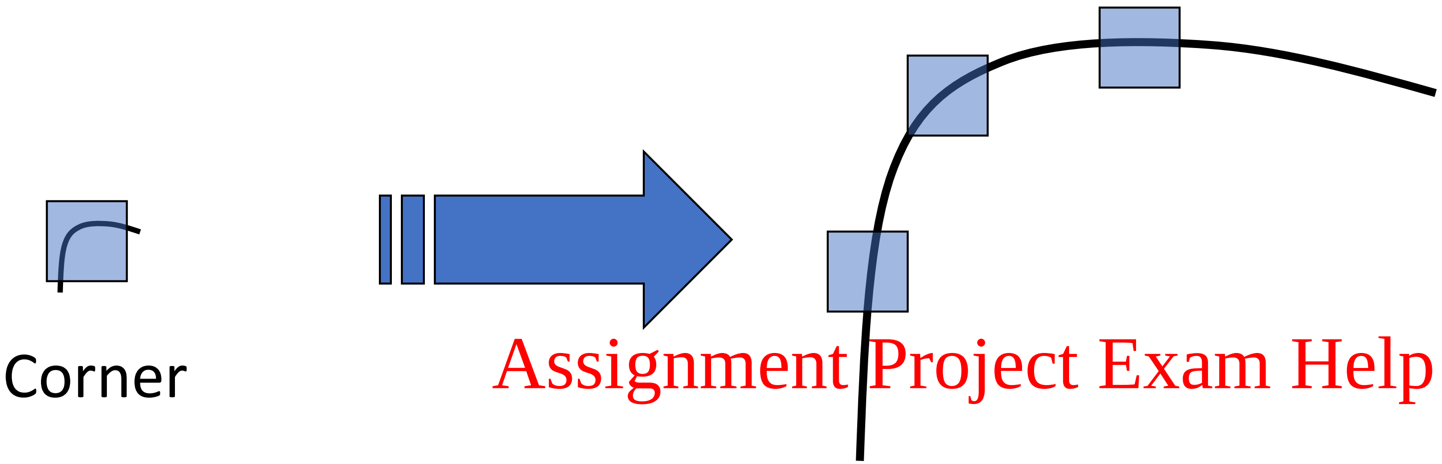
Image Rotation



- Second moment ellipse rotates, but its shape (i.e., eigenvalues) remains the same

Corner location is covariant w.r.t. rotation

Scaling



All points will be classified as edges

Corner location is not covariant to scaling!

Summary

- Why we need feature extraction? What are the applications of feature extraction?
- What are the characteristics of good features?
- Describe the basic idea of corner detection
- How to decide whether a point is in a flat region, on an edge, or a corner according to the two eigenvalues of the second moment matrix?
- Describe the steps of Harris detector
- What is Invariance and Covariance?
- Is affine intensity change invariant? Is image translation, rotation, scaling covariant?