CMT107 Visual Computing

Image Morphology

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Overview

- Morphology
  - Dilation
  - Erosion
  - Duality of Dilation and Erosion
  - Opening
  - Closing
- Hit-or-Miss transformation

Acknowledgement
The majority of the slides in this section are from Punam K Saha at University of Iowa

Morphology

- Morphological operators often take a binary image and a structuring element as input and combine them using a set operator (intersection, union, inclusion, complement).
- The structuring element is shifted over the image. At each pixel of the image, its elements are compared with the set of the underlying pixels.
- If the two sets match the condition defined by the set operator (e.g., if the set of pixels in the structuring element is a subset of the underlying image pixels), the pixel underneath the origin of the structuring element is set to a predefined value (0 or 1 for binary images).
- A morphological operator is defined by its structuring element and the applied set operator.

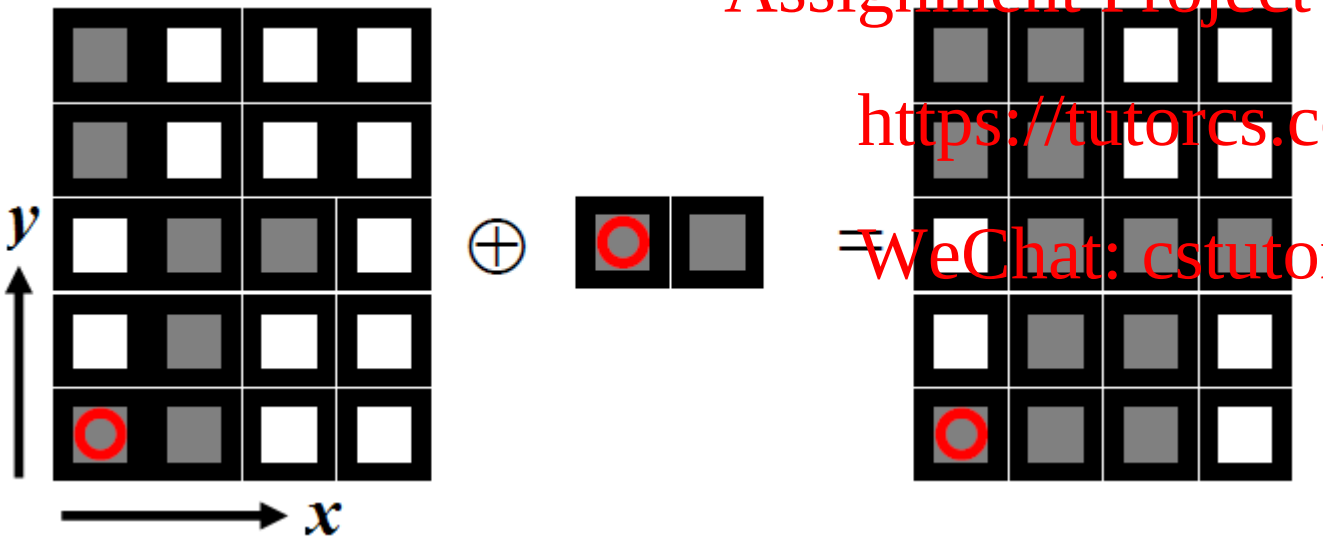
Morphology Applications

- Image pre-processing
  - Noise filtering
  - shape simplification
- Enhancing object structures
  - Skeletonisation
  - Thinning
  - Convex hull
  - Object marking
- Segmentation of the object from background
- Quantitative descriptors of objects
  - Area
  - Perimeter
  - … etc.

Example: Morphological Operation

- Let $\oplus$ denote a morphological operator

$$X \oplus B = \{p \in Z^2 \,|\, p = x + b, x \in X, b \in B\}$$



$B = \{(0,0), (1,0)\}$

$X = \{(0,0), (1,0), (1,1), (1,2), (2,2), (0,3), (0,4)\}$

$(0,0), (1,0) \mid (1,1), (2,1) \mid (2,2), (3,2) \mid (0,4), (1,4)$

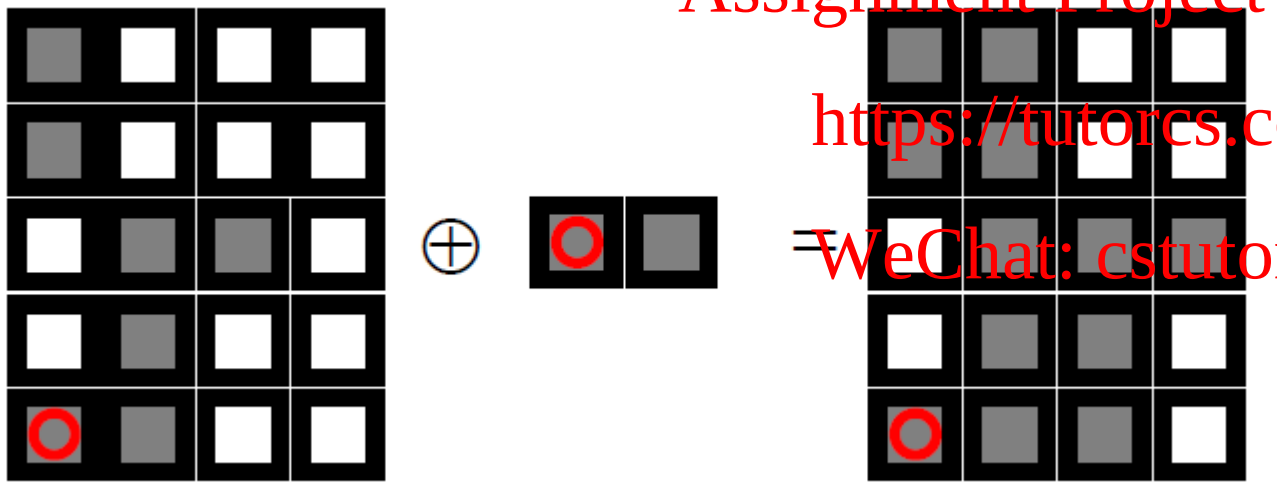
$(1,0), (2,0) \quad (1,2), (2,2) \quad (0,3), (1,3)$

$X \oplus B = \{(0,0), (1,0), (2,0), (1,1), (2,1), (1,2), (2,2), (3,2), (0,3), (1,3), (0,4), (1,4)\}$

Dilation

- Morphological dilation '$\oplus$' combines two sets using vector addition of set elements

$$X \oplus B = \{p \in Z^2 \,|\, p = x + b, x \in X, b \in B\}$$

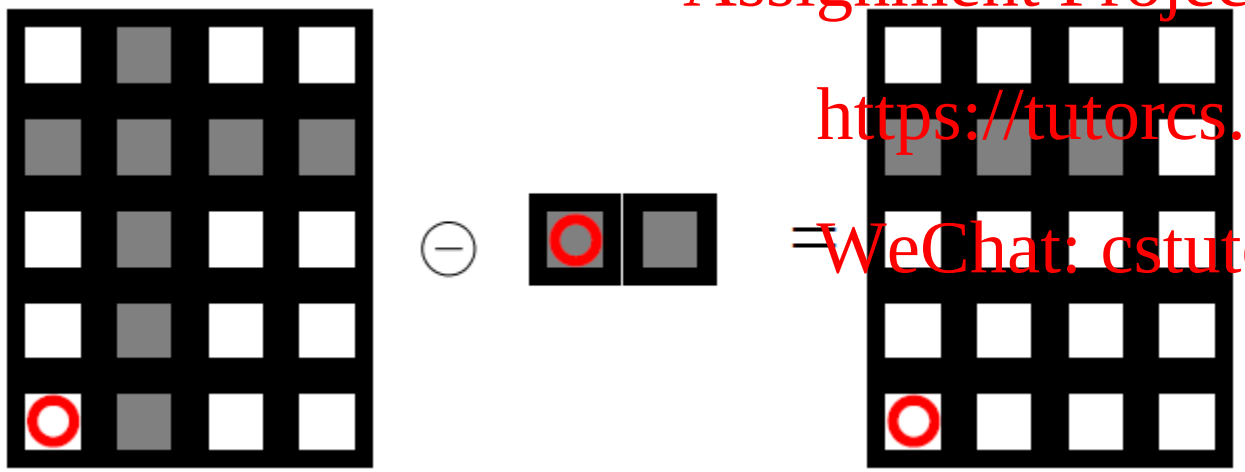


- Commutative:
$$X \oplus B = B \oplus X$$
- Associative:
$$X \oplus B \oplus D = X \oplus (B \oplus D)$$
- Invariant of translation:
$$X_h \oplus B = (X \oplus B)_h$$
$$X_h = \{p \in Z^2 \,|\, p = x + h, x \in X\}$$
- If $X \subseteq Y$, then $X \oplus B \subseteq Y \oplus B$

Erosion

- Morphological erosion '$\ominus$' combines two sets using vector subtraction of set elements, and is a dual operator of dilation

$$X \ominus B = \{p \in Z^2 | \forall b \in B, p + b \in X\}$$

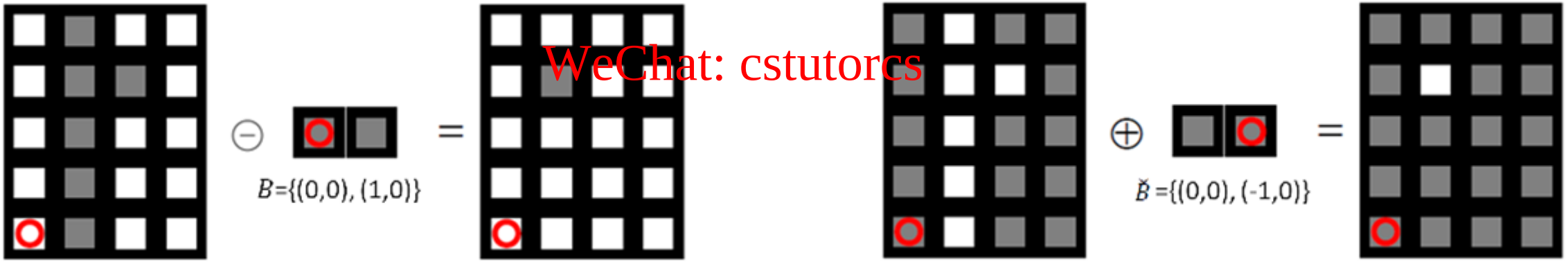


- Not Commutative:
  $X \ominus B \neq B \ominus X$
- Not Associative:
  $X \ominus B \ominus D \neq X \ominus (B \ominus D)$
- Invariant of translation:
  $X_h \ominus B = (X \ominus B)_h$, and
  $X \ominus B_h = (X \ominus B)_{-h}$
- If $X \subseteq Y$, then $X \ominus B \subseteq Y \ominus B$

Duality: Dilation and Erosion

- The transpose $\check{B}$ of a structuring element B is the structuring element mirrored in the origin: $\check{B} = \{-b | b \in B\}$
- Duality between morphological dilation and erosion operators:

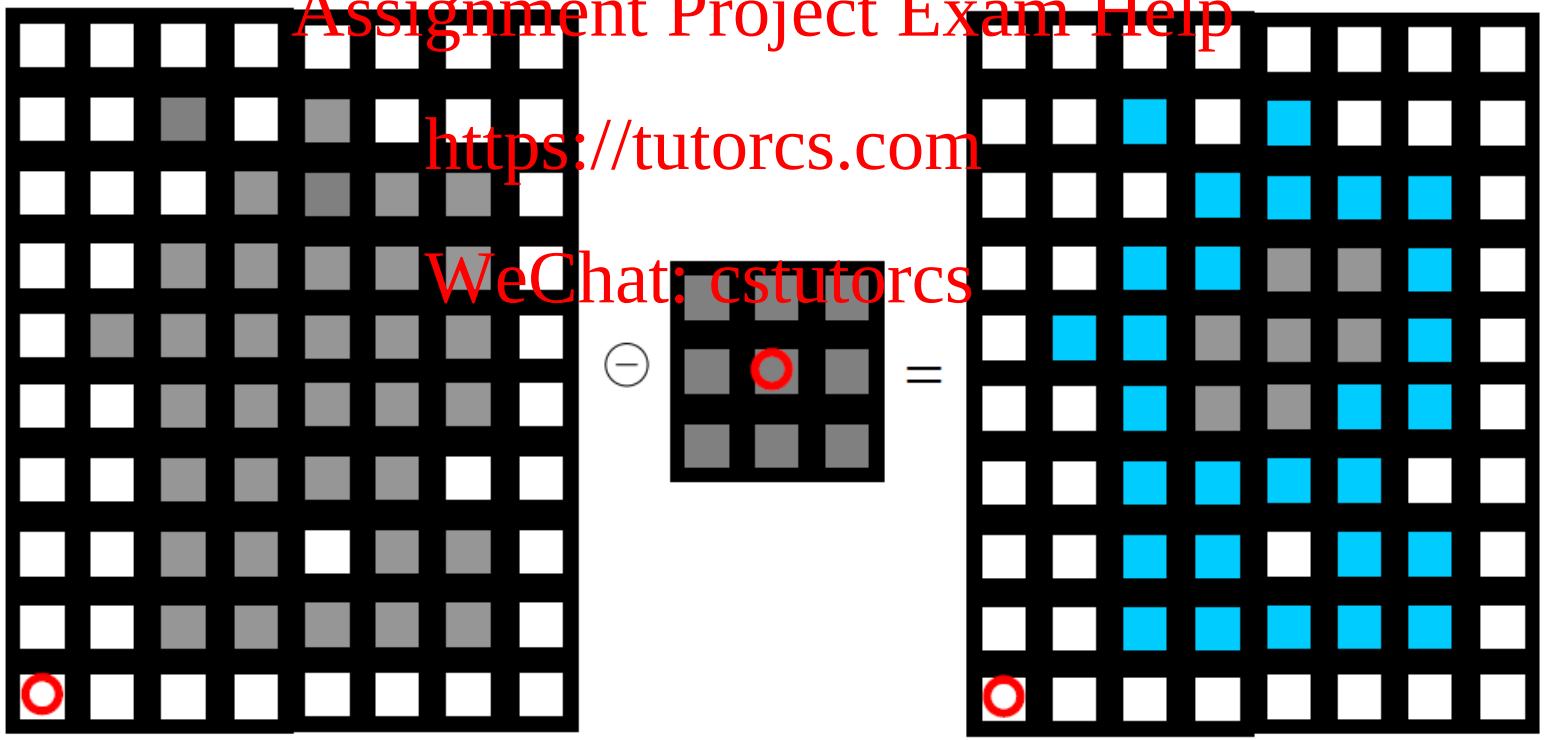
$$(X \ominus B)^c = X^c \oplus \check{B} \quad (c \text{ means complement})$$



Opening

- Erosion and dilation are not inverse transforms. An erosion followed by a dilation leads to an interesting morphological operation, called opening

$$X \circ B = (X \ominus B) \oplus B$$



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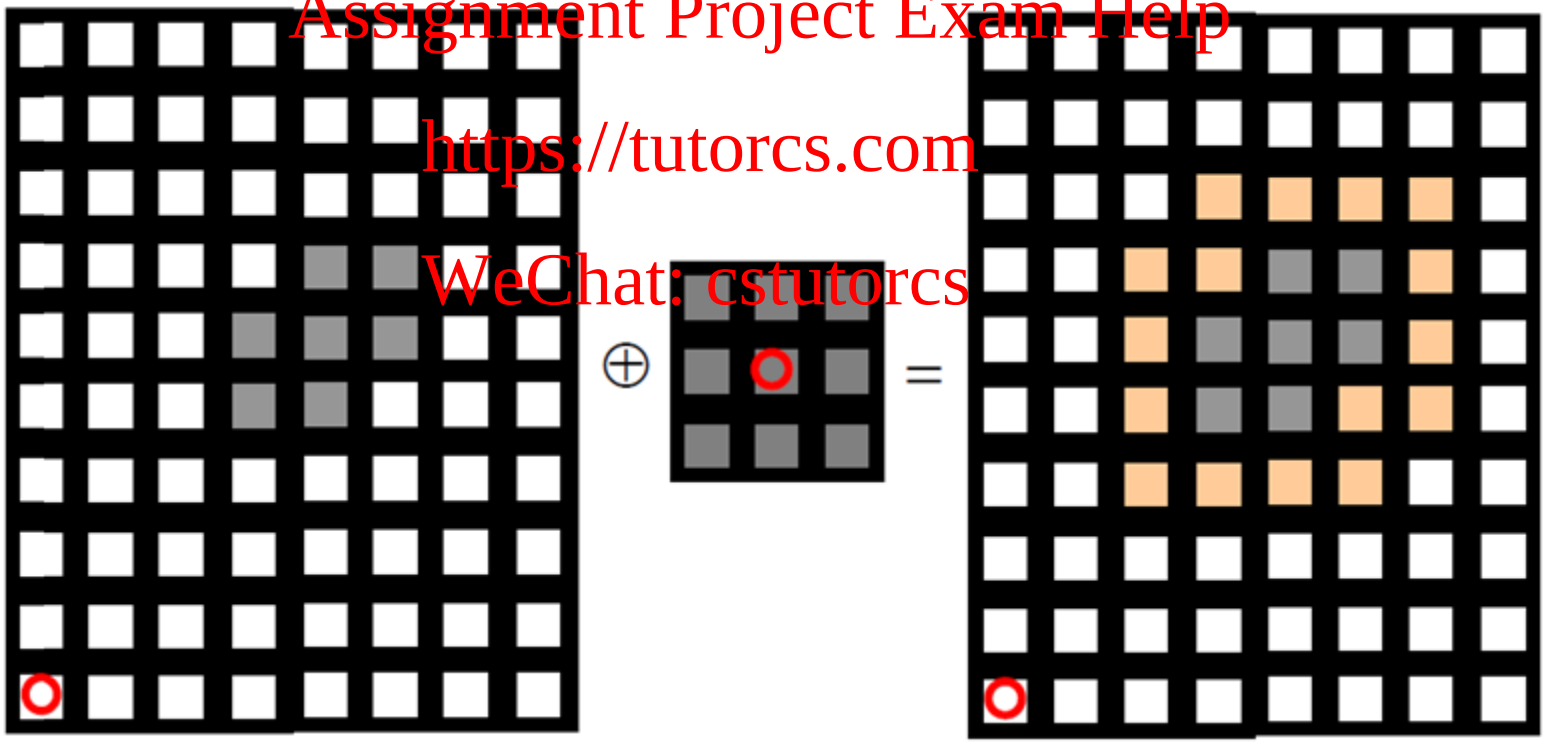

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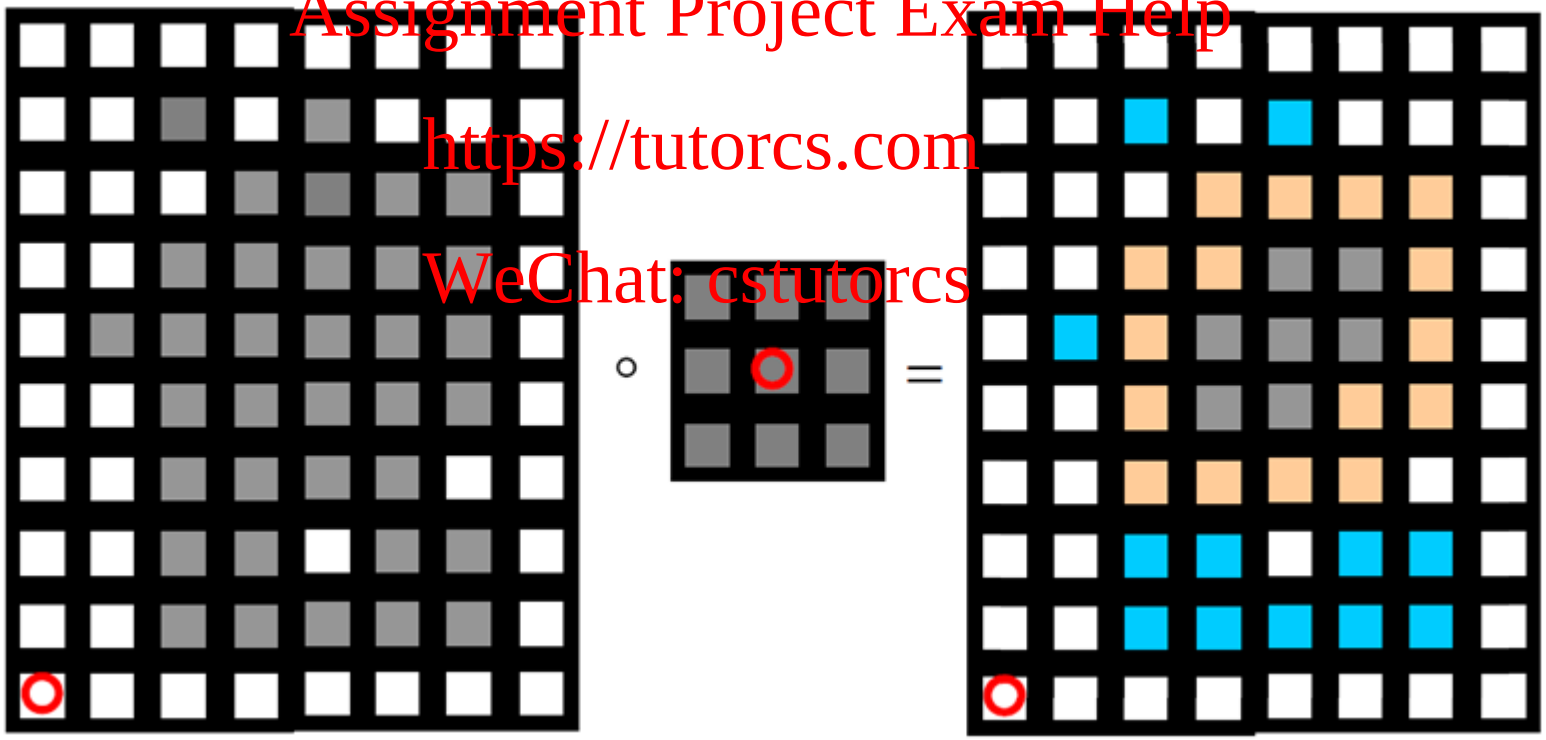

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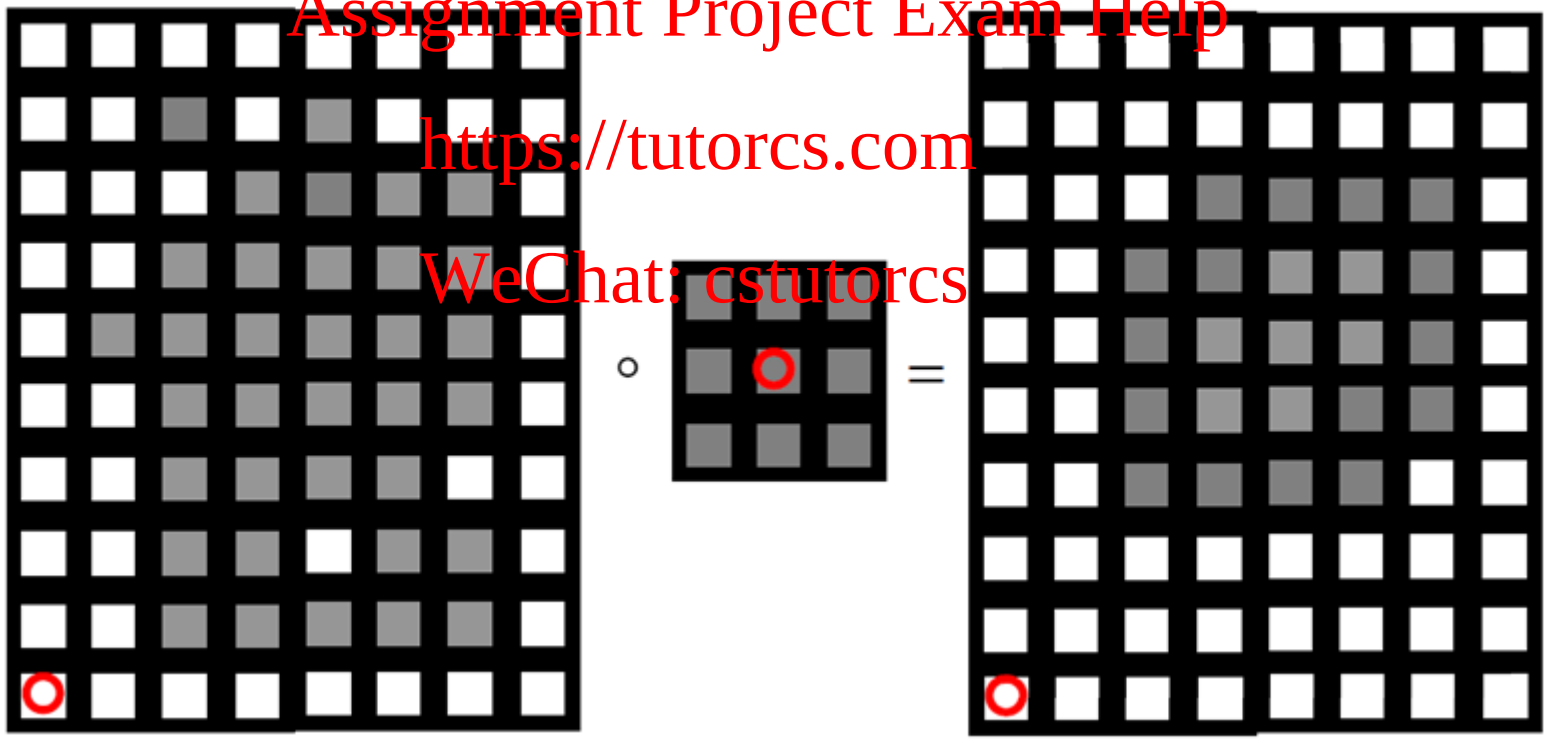
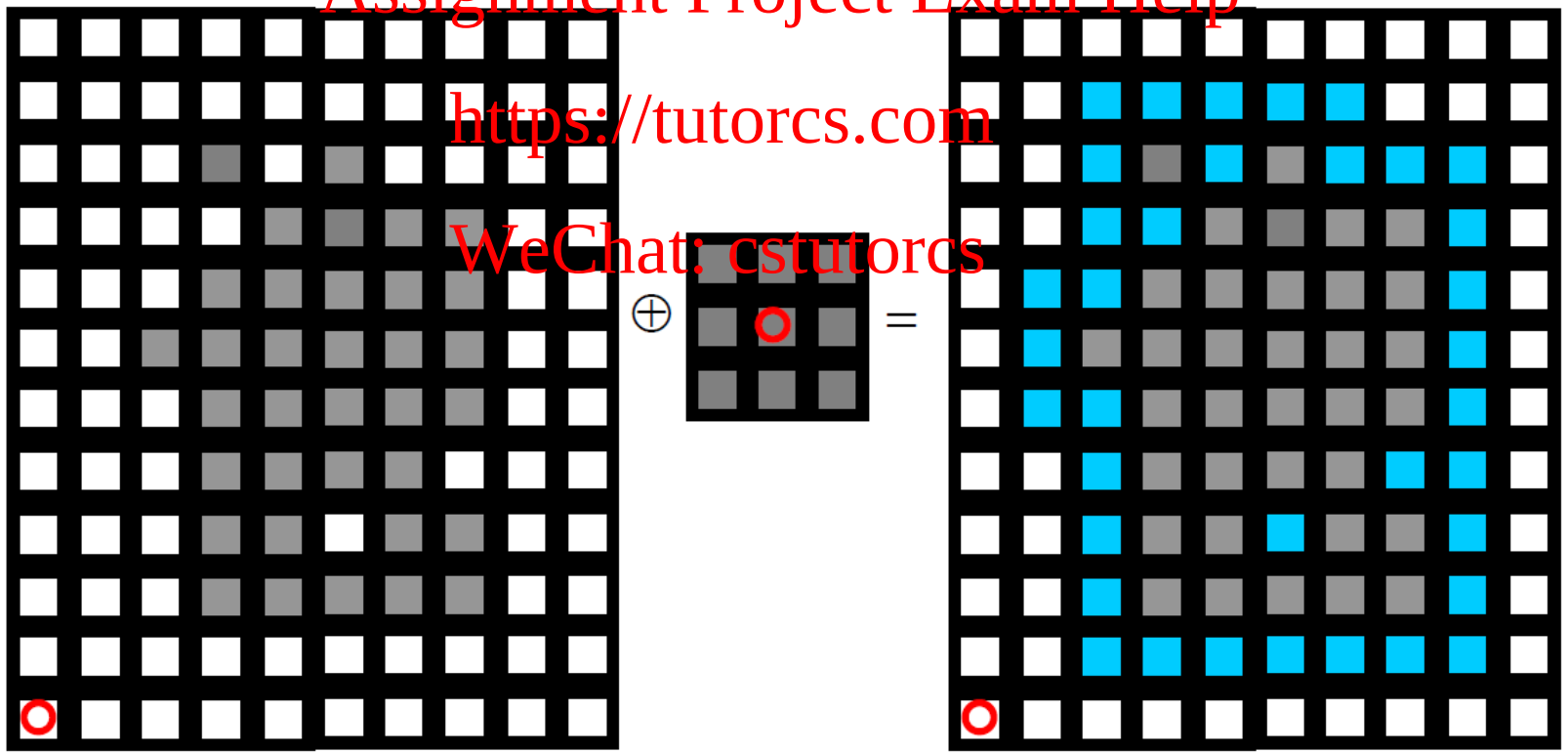


Closing

- A dilation followed by an erosion leads to the interesting morphological operation, called closing

$$X \bullet B = (X \oplus B) \ominus B$$



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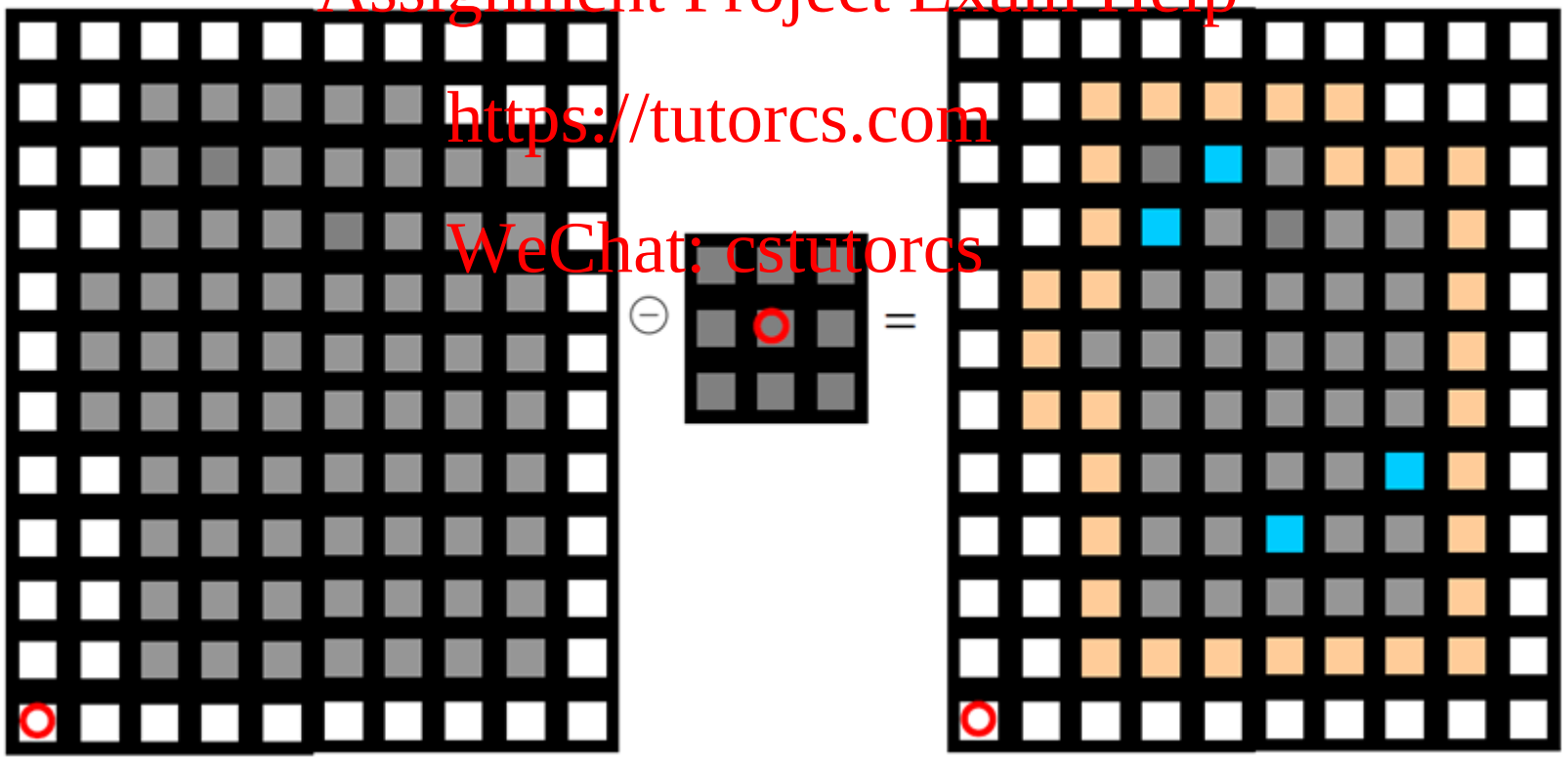

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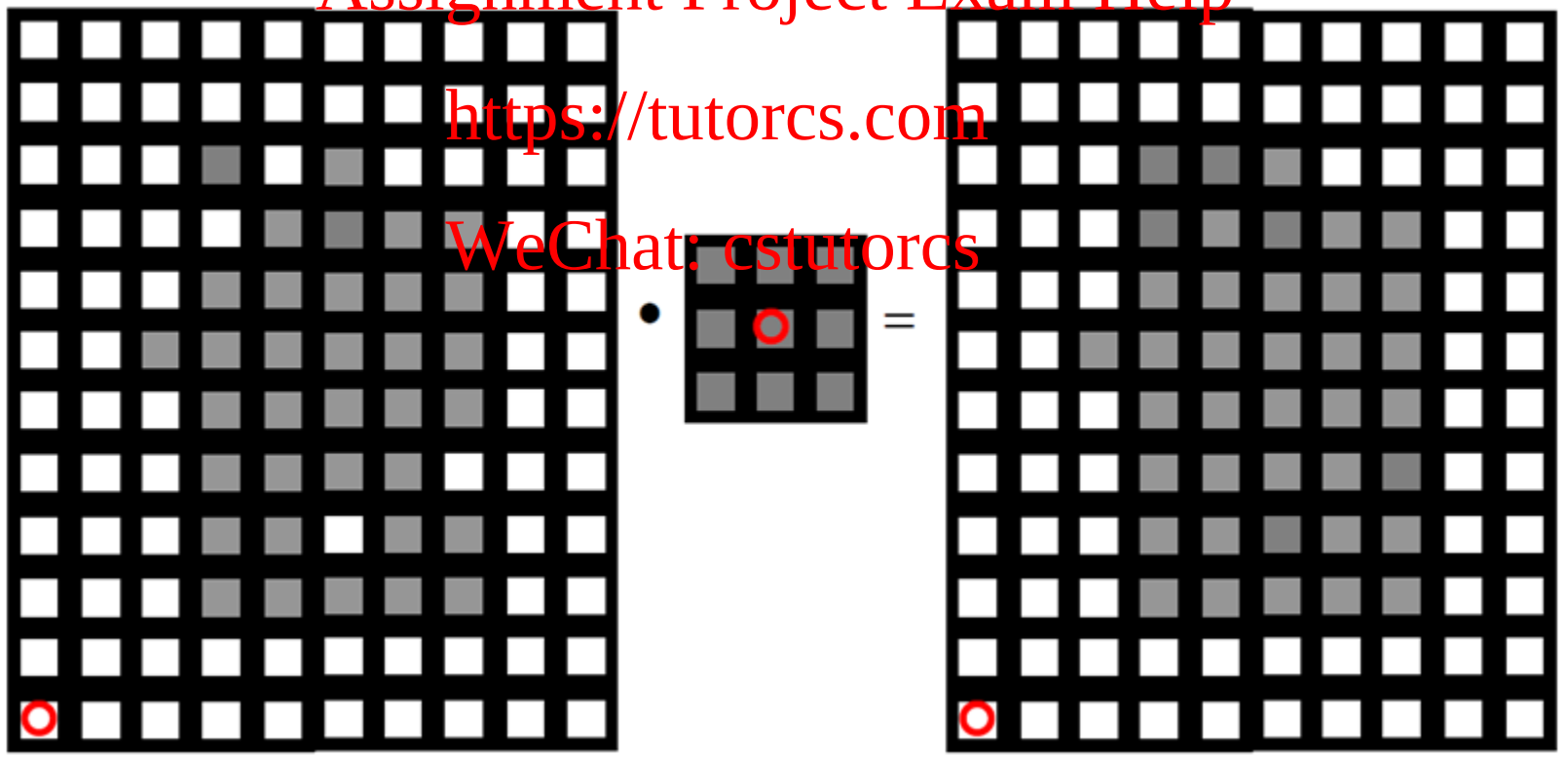


Hit-or-Miss transformation

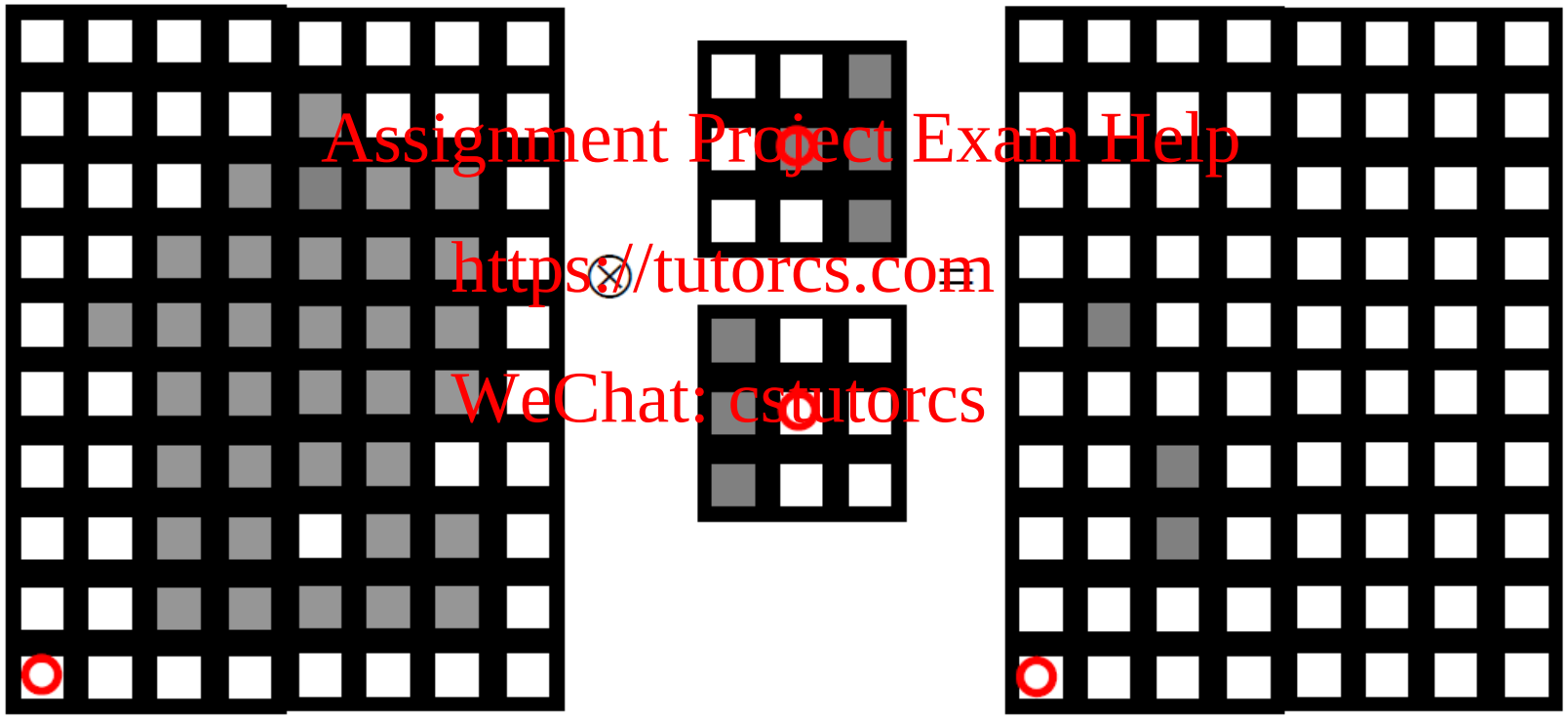
- Hit-or-miss is a morphological operator for finding local patterns of foreground and background pixels. Unlike dilation and erosion, this operation is defined using a composite structuring element $B = (B_1, B_2)$. The hit-or-miss operator is defined as follows

$$X \otimes B = \{x | (B_1)_x \subset X \; and \; (B_2)_x \subset X^c\}$$

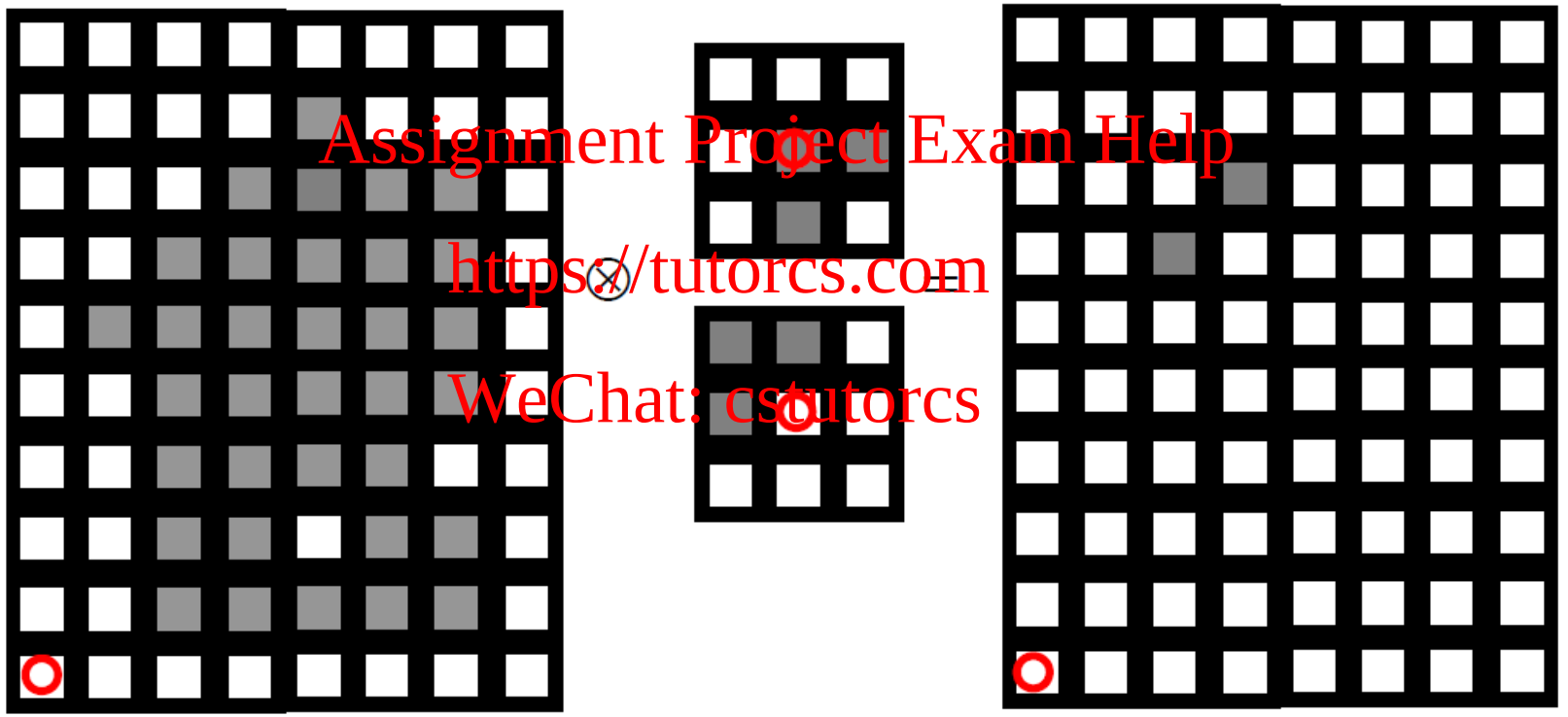
- Relation with erosion

$$X \otimes B = (X \ominus B_1) \cap (X^c \ominus B_2)$$

Hit-or-miss transformation: examples



Hit-or-miss transformation: examples



Summary

- What is morphology? What are the applications of morphology?
- What are the dilation, erosion, opening and closing operators?
- What is hit-or-miss transformation?