Introduction to Database Systems – Part 2

Math Concepts

What are the Math Concepts behind Databases?

- Set
- Tuple

- Cartesian Product of Sets

- Relation

Set Notation

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Set Notation

- We need set notation to represent formal definitions in this course.

- A **set** is a collection of distinct elements.

- Two basic properties of sets
  - The elements in a set have no order.
    e.g., $\{1, 2, 3\} = \{2, 3, 1\}$

  - Each element can not be in the set more than once.
    e.g., $\{$Monday, Monday, Tuesday, Wednesday, Thursday, Friday, Friday$\}$ is Not a set. Note that **Multisets** allow to have duplicate elements.

Set Notation

- **Two ways of specifying a set**
  1. $\{x_1, \ldots, x_n\}$ (i.e., list all the elements in a set)
     - { 2, 3, 4, 5 }
     - {Sydney, Melbourne, Canberra}
     - {} or $\emptyset$, i.e., the *empty* set.

  2. $\{x|\varphi\}$ (i.e., describe the elements that satisfy a property φ)
     - {x | x is a student currently enrolled in COMP7240}
     - {x | x is an integer and x $>$ 0}

Set Operations

- **Membership**: $x \in A$ if x is in the set A; $x \notin A$ if x is not in the set A.

$1 \in \{1,2,3\}$ $\quad$ $3 \in \{1,2,3\}$

$2 \in \{1,2\}$ $\quad$ $3 \notin \{1,2\}$

Set Operations

- **Equality**: If A and B have the same elements, we write $A = B$; otherwise we write $A \neq B$
  - $\{x \mid x$ is an integer, $x > 1$ and $x < 6\} = \{2, 3, 4, 5\}$

- If one set contains some element that is not in the other set, then they are different.

$\{1,2\} \neq \{1,2,3\}$

Set Operations

- **Subset**: A is called a **subset** of B if every element of A is in B and we write $A \subseteq B$
- **Proper subset**: A is called a **proper subset** of B if $A \subseteq B$ and A and B are not equal, and we write $A \subset B$.

$\{1,2\} \subseteq \{1,2,3\}$ $\quad$ $\{1,2\} \subseteq \{1,2\}$

$\{1,2\} \subset \{1,2,3\}$

Set Operations

- **Subset**: A is called a **subset** of B if every element of A is in B and we write $A \subseteq B$
- **Proper subset**: A is called a **proper subset** of B if $A \subseteq B$ and A and B are not equal, and we write $A \subset B$.

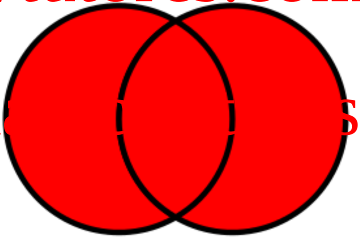
$\subseteq$ means $\subset$ or $=$

$\{1,2\} \subseteq \{1,2,3\}$ $\{1,2\} \subseteq \{1,2\}$

$\{1,2\} \subset \{1,2,3\}$

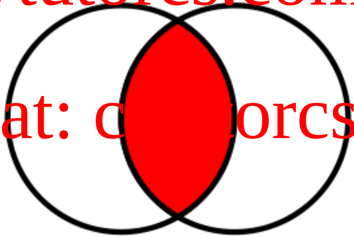
Set Operations

- **Union:** $A \cup B$ for the set containing everything in A and everything in B.
  - $\{3, 4, 5\} \cup \{3, 5, 7, 9\} = \{3, 4, 5, 7, 9\}$.



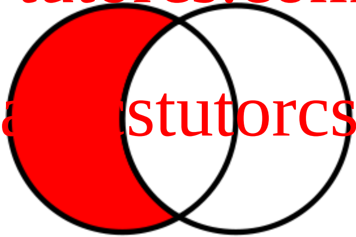
Set Operations

- **Intersection**: $A \cap B$ for the set of elements that are in both A and B
  - $\{3, 4, 5\} \cap \{3, 5, 7, 9\} = \{3, 5\}$.



Set Operations

- **Difference**: $A - B$ is the elements from A that are *not* in B.
  - $\{3, 4, 5\} - \{3, 5, 7, 9\} = \{4\}$.



Set Operations – Exercise

- Let $A = \{1, 2, 3\}$ and $B = \{true, false\}$.
- Which of the following are correct?

1. $\{2\} \in A$ — No! $\{2\} \subset A$ and $2 \in A$
2. $true \subset B$ — No! $true \in B$ and $\{true\} \subset B$
3. $\{2, 3\} \subseteq A \cup B$ — Yes! $A \cup B = \{1, 2, 3, true, false\}$
4. $2 \in A \cap B$ — No! $A \cap B = \{\}$
5. $2 \in A - \{1, 3, 5\}$ — Yes! $A - \{1, 3, 5\} = \{2\}$
6. $\{1, 4\} \subseteq A - B$ — No! $A - B = \{1, 2, 3\}$
7. $\emptyset \cap B = \emptyset$ — Yes! $\emptyset = \{\}$, the empty set

Tuple Notation

Tuple Notation

- A **tuple** is an ordered list of n elements.
  - $(1, 2, 3, 4, 5)$
  - $(Melbourne, Sydney, Canberra)$

- Two tuples are **equal** if they have the same elements in the same order.
  - $(1, 2, 3) \neq (2, 3, 1)$ (i.e., the order does matter!)

- The same element can be in a tuple twice.
  - (Monday, Monday, Tuesday, Wednesday, Thursday, Friday, Friday) is a tuple.
- Ordered pairs are special cases of tuples.

Cartesian Product of Sets

Cartesian Product of Sets

$$\{2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A\}$$

Cartesian Product of Sets

$$\{2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A\}$$

$$\{\spadesuit, \diamondsuit, \clubsuit, \heartsuit\}$$

Cartesian Product of Sets

- The Cartesian product operation takes an ordered list of sets, and returns a set of tuples.

- **Cartesian product** $D_1 \times ... \times D_n$ is the set of all possible combinations of values from the sets $D_1, ..., D_n$.
- It contains all the tuples with the first element from the first set, the second element from the second set, ...

- For example: $A \times B = \{(a, b) \mid a \in A$ and $b \in B\}$.
  If $A = \{2, 3\}$ and $B = \{Clubs, Diamonds, Hearts, Spades\}$
  Then $A \times B = \{(2, Clubs), (2, Diamonds), (2, Hearts), (2, Spades),$
  $(3, Clubs), (3, Diamonds), (3, Hearts), (3, Spades)\}$.
  $(2, Clubs) \in A \times B$, $(Spades, 3) \notin A \times B$, $(4, Hearts) \notin A \times B$
  $\{(3, Clubs), (3, Diamonds), (3, Hearts), (3, Spades)\} \subseteq A \times B$

Relation Notation

$$\{2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A\} \times \{\spadesuit, \diamondsuit, \clubsuit, \heartsuit\}$$

Relation Notation

$$\{2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A\}$$

$$\{\spadesuit, \diamondsuit, \clubsuit, \heartsuit\}$$

Relation Notation

- A **relation** is a subset of a Cartesian product of sets.
- **Example**
  - Let $X = \{Canberra, Paris, Tokyo, Kyoto\}$, and $Y = \{Australia, France, Japan\}$

  - Let $R = \{(a, b) | a \in X, b \in Y \text{ and } a \text{ is a city in } b\}$.
  - It is easy to see that R is a relation

    - $R \subseteq X \times Y$.
    - $(Canberra, Australia) \in R$, $(Paris, France) \in R$ but $(Tokyo, France) \notin R$, $(France, Japan) \notin R$

Relation Notation

- A **relation** is a subset of a Cartesian product of sets.
- **Example**

  - Let $\mathbb{Z} = \{\ldots, -1, 0, 1, 2, \ldots\}$ the set of all integers
  - Let $R = \{(x, y) \mid x \in \mathbb{Z}, y \in \mathbb{Z} \text{ and } x < y\}$.
  - It is easy to see that R is a relation.

    - $R \subseteq \mathbb{Z} \times \mathbb{Z}$.
    - $(0, 1) \in R$, $(-4, -2) \in R$
      but $(0, 0) \notin R$, $(100, -2) \notin R$.