CAPMGEGO

June 24, 2021

1 Calculate the daily log returns of T-Bill, gold, GE stock and market

[92]:
```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
```

[93]:
```python
data = pd.read_csv("C:\\Users\\rluck\\OneDrive\\capm.csv", header=[4])
data
```


[93]:

	DATE	Gold	S&P500	Rf	GE
0	12/08/1975	166.05	87.12	6.40	0.9218
1	13/08/1975	163.50	85.97	6.45	0.9036
2	14/08/1975	163.50	85.60	6.45	0.9036
3	15/08/1975	163.50	86.36	6.42	0.9244
4	18/08/1975	163.50	86.20	6.42	0.9348
…	…	…	…	…	…
10432	6/08/2015	1090.15	2083.56	0.04	26.0300
10433	7/08/2015	1096.85	2077.57	0.06	25.7900
10434	10/08/2015	1103.35	2104.18	0.12	26.2400
10435	11/08/2015	1109.67	2084.07	0.10	25.7100
10436	12/08/2015	1123.85	2086.05	0.10	25.8600

[10437 rows x 5 columns]

#Computing log returns: R_gold = 100*ln(P_g/P_g-1)

Rf = 100/360*ln(1+rf)

[94]:
```python
data['R_gold']=100*np.log(data['Gold']/data['Gold'].shift(1))
data['R_f']= 100/360*np.log(1+data['Rf']/100)
data['R_GE']= 100*np.log(data['GE']/data['GE'].shift(1))
data['R_m']= 100*np.log(data['S&P500']/data['S&P500'].shift(1))
print(data.head())
```

```
         DATE    Gold  S&P500    Rf      GE    R_gold       R_f      R_GE  \
0  12/08/1975  166.05   87.12  6.40  0.9218       NaN  0.017232       NaN
1  13/08/1975  163.50   85.97  6.45  0.9036 -1.547596  0.017363 -1.994150
2  14/08/1975  163.50   85.60  6.45  0.9036  0.000000  0.017363  0.000000
```

```
3  15/08/1975  163.50   86.36  6.42  0.9244  0.000000  0.017284  2.275809
4  18/08/1975  163.50   86.20  6.42  0.9348  0.000000  0.017284  1.118772

        R_m
0       NaN
1 -1.328808
2 -0.431312
3  0.883932
4 -0.185443
```

2 Calculating excess returns for gold and GE

[95]:
```python
data['R_p']= data['R_m']- data['R_f']
data['R_ge']= data['R_GE']-data['R_f']
data['R_go']= data['R_gold']-data['R_f']
data
```

[95]:
```
             DATE     Gold   S&P500    Rf       GE    R_gold       R_f  \
0      12/08/1975   166.05    87.12  6.40   0.9216       NaN  0.017232
1      13/08/1975   163.50    85.97  6.45   0.9036 -1.547596  0.017363
2      14/08/1975   163.50    85.60  6.45   0.9036  0.000000  0.017363
3      15/08/1975   163.50    86.36  6.42   0.9244  0.000000  0.017284
4      18/08/1975   163.50    86.20  6.42   0.9348  0.000000  0.017284
…             …       …       …     …       …        …         …
10432   6/08/2015  1090.15  2083.56  0.04  26.0300  0.415484  0.000111
10433   7/08/2015  1096.85  2077.57  0.06  25.7900  0.612713  0.000167
10434  10/08/2015  1103.35  2104.18  0.12  26.2400  0.590857  0.000333
10435  11/08/2015  1109.67  2084.07  0.10  25.7100  0.571167  0.000278
10436  12/08/2015  1123.85  2086.05  0.10  25.8600  1.269762  0.000278

           R_GE       R_m       R_p      R_ge      R_go
0           NaN       NaN       NaN       NaN       NaN
1     -1.994150 -1.328808 -1.346171 -2.011512 -1.564958
2      0.000000 -0.431312 -0.448674 -0.017363 -0.017363
3      2.275809  0.883932  0.866648  2.258525 -0.017284
4      1.118772 -0.185443 -0.202727  1.101488 -0.017284
…           …        …        …        …        …
10432 -0.268560 -0.778318 -0.778429 -0.268671  0.415373
10433 -0.926290 -0.287903 -0.288069 -0.926457  0.612547
10434  1.729814  1.272690  1.272357  1.729481  0.590524
10435 -2.040494 -0.960313 -0.960591 -2.040772  0.570889
10436  0.581735  0.094961  0.094684  0.581458  1.269484

[10437 rows x 12 columns]
```

3 Data : Remove N/A

[96]:
```python
data = data.dropna(subset=["R_p"])
data.to_csv("C:\\Users\\rluck\\OneDrive\\capm1.csv")
data.head()
```

[96]:
```
         DATE   Gold  S&P500    Rf      GE    R_gold       R_f       R_GE  \
1  13/08/1975  163.5   85.97  6.45  0.9036 -1.547596  0.017363 -1.994150
2  14/08/1975  163.5   85.60  6.45  0.9036  0.000000  0.017363  0.000000
3  15/08/1975  163.5   86.36  6.42  0.9244  0.000000  0.017284  2.275809
4  18/08/1975  163.5   86.20  6.42  0.9348  0.000000  0.017284  1.118772
5  19/08/1975  163.5   84.95  6.47  0.9218  0.000000  0.017415 -1.400432

        R_m       R_p      R_ge      R_go
1 -1.328808 -1.346171 -2.011512 -1.564958
2 -0.431312 -0.448674 -0.017363 -0.017363
3  0.883932  0.866648  2.258525 -0.017284
4 -0.185443 -0.202727  1.101488 -0.017284
5 -1.460733 -1.478148 -1.417847 -0.017415
```


[97]:
```
!pip install sklearn
!pip install statsmodels
```



```
Requirement already satisfied: sklearn in c:\users\rluck\anaconda3\lib\site-
packages (0.0)
Requirement already satisfied: scikit-learn in
c:\users\rluck\anaconda3\lib\site-packages (from sklearn) (0.24.1)
Requirement already satisfied: scipy>=0.19.1 in
c:\users\rluck\anaconda3\lib\site-packages (from scikit-learn->sklearn) (1.6.2)
Requirement already satisfied: joblib>=0.11 in
c:\users\rluck\anaconda3\lib\site-packages (from scikit-learn->sklearn) (1.0.1)
Requirement already satisfied: numpy>=1.13.3 in
c:\users\rluck\anaconda3\lib\site-packages (from scikit-learn->sklearn) (1.20.1)
Requirement already satisfied: threadpoolctl>=2.0.0 in
c:\users\rluck\anaconda3\lib\site-packages (from scikit-learn->sklearn) (2.1.0)
Requirement already satisfied: statsmodels in c:\users\rluck\anaconda3\lib\site-
packages (0.12.2)
Requirement already satisfied: numpy>=1.15 in c:\users\rluck\anaconda3\lib\site-
packages (from statsmodels) (1.20.1)
Requirement already satisfied: scipy>=1.1 in c:\users\rluck\anaconda3\lib\site-
packages (from statsmodels) (1.6.2)
Requirement already satisfied: pandas>=0.21 in
c:\users\rluck\anaconda3\lib\site-packages (from statsmodels) (1.2.4)
Requirement already satisfied: patsy>=0.5 in c:\users\rluck\anaconda3\lib\site-
packages (from statsmodels) (0.5.1)
Requirement already satisfied: python-dateutil>=2.7.3 in
c:\users\rluck\anaconda3\lib\site-packages (from pandas>=0.21->statsmodels)
(2.8.1)
```

```
Requirement already satisfied: pytz>=2017.3 in
c:\users\rluck\anaconda3\lib\site-packages (from pandas>=0.21->statsmodels)
(2021.1)
Requirement already satisfied: six in c:\users\rluck\anaconda3\lib\site-packages
(from patsy>=0.5->statsmodels) (1.15.0)
```

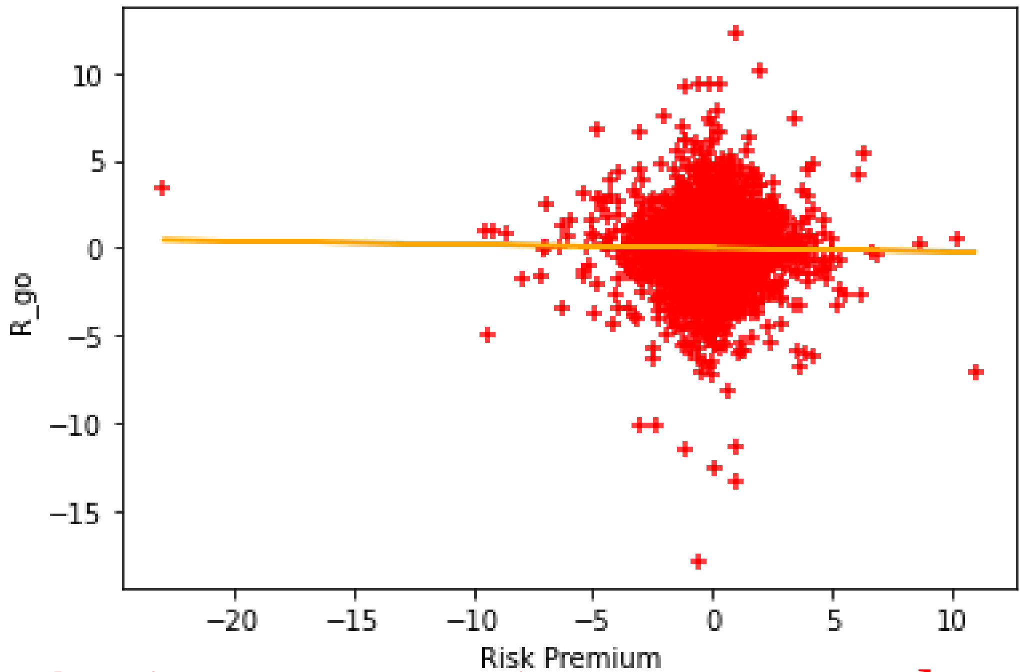
[98]:
```python
%matplotlib inline
import statsmodels.api as sm
import statsmodels.formula.api as smf
from sklearn import linear_model
import matplotlib.pyplot as plt
```

4 I. Plotting Gold excess returns with market excess returns

[99]:
```python
#Regressing excess returns on gold (R_g-Rf) over risk-free rate against the␣
↪excess market return (Rp=Rm-rf)
reg = linear_model.LinearRegression()
X =data[['R_p']].dropna()
y1 =data['R_go'].dropna()
reg.fit(X,y1)
predictions =reg.predict(X)
```

[100]:
```python
plt.xlabel('Risk Premium')
plt.ylabel('R_go')
plt.scatter(data.R_p,data.R_gold,color='red',marker='+')
plt.plot(data.R_p,reg.predict(data[['R_p']]), color='orange')
```

[100]:
```
[<matplotlib.lines.Line2D at 0x18397b4e220>]
```



[101]:
```
#model with intercept
X= sm.add_constant(X)
model = sm.OLS(y1,X).fit()
predictions = model.predict(X)
j= (model.summary())
print(j)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   R_go   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                  0.000
Method:                 Least Squares   F-statistic:                     3.181
Date:                Thu, 24 Jun 2021   Prob (F-statistic):             0.0745
Time:                        13:42:13   Log-Likelihood:                -16959.
No. Observations:               10436   AIC:                         3.392e+04
Df Residuals:                   10434   BIC:                         3.394e+04
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0057      0.012      0.473      0.637      -0.018       0.029
R_p           -0.0201      0.011     -1.784      0.075      -0.042       0.002
==============================================================================
Omnibus:                     2812.110   Durbin-Watson:                   2.071
```

```
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           111926.422
Skew:                          -0.573   Prob(JB):                         0.00
Kurtosis:                      19.003   Cond. No.                         1.07
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

DW-stats of 2.071 is close to 2.0, implying that there is no serial correlation.

Yet, the p-value of the beta coefficient indicates that it is slightly significant at 7.5% significance level and the R-squared is very low, explaining low explanatory power of the model.

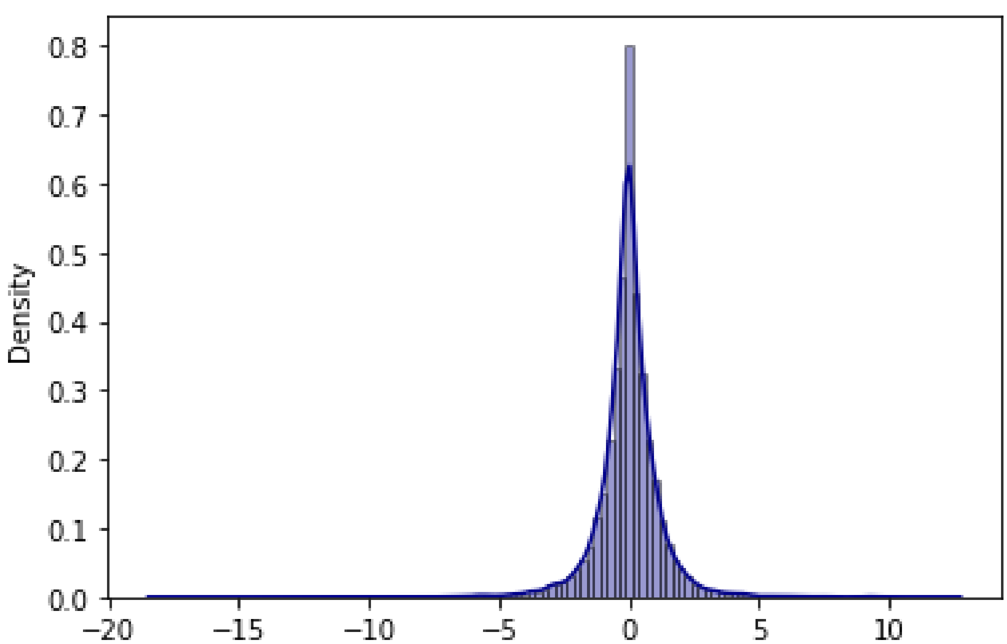
5 Residuals plot for gold

[102]:
```python
residuals_go = model.resid
import seaborn as sns
sns.distplot(residuals_go,hist=True, kde=True, bins=int(120), color='darkblue', hist_kws={'edgecolor':'black'})
```



```
C:\Users\rluck\anaconda3\lib\site-packages\seaborn\distributions.py:2557:
FutureWarning: `distplot` is a deprecated function and will be removed in a
future version. Please adapt your code to use either `displot` (a figure-level
function with similar flexibility) or `histplot` (an axes-level function for
histograms).
  warnings.warn(msg, FutureWarning)
```


[102]: <AxesSubplot:ylabel='Density'>



```
[103]: from scipy import stats
JB_go= stats.jarque_bera(residuals_go)
JB_go
```

```
[103]: Jarque_beraResult(statistic=111926.42195044507, pvalue=0.0)
```

The plot and JB test (p-value <0.05) rejects the null hypothesis of normality. It is clearly a non-normal distribution.

6 Cusum Test for Gold

```
[104]: # endog = data.R_go
Rp = data.R_p
endog = data.R_go
exog = sm.add_constant(Rp)
mod = sm.RecursiveLS(endog,exog)
res_1 = mod.fit()
fig = res_1.plot_cusum(figsize=(10,6));
```



```
C:\Users\rluck\anaconda3\lib\site-
packages\statsmodels\tsa\base\tsa_model.py:578: ValueWarning: An unsupported
index was provided and will be ignored when e.g. forecasting.
  warnings.warn('An unsupported index was provided and will be'
```


Cusum test of stability for gold shows high periods of instability during the early part of the graph (namely before 1980s).Then, the beta stabilises.

7 White Test of Heteroskedasticity for Gold

[105]:
```
from statsmodels.stats.diagnostic import het_white
from statsmodels.compat import lzip
from patsy import dmatrices
```

[106]:
```
expr = 'y1 ~ X'
y1, X = dmatrices(expr, data, return_type='dataframe')
olsr_results = smf.ols(expr, data).fit()
keys = ['Lagrange Multiplier statistic:', 'LM test\'s p-value:', 'F-statistic:
 ↪', 'F-test\'s p-value:']
results = het_white(olsr_results.resid, X)
lzip(keys, results)
```

[106]:
```
[('Lagrange Multiplier statistic:', 36.866651106570274),
 ("LM test's p-value:", 9.874348003656595e-09),
 ('F-statistic:', 18.493356998362814),
 ("F-test's p-value:", 9.608158442586967e-09)]
```


LM test statistic is 36.87 and the corresponding p-value is 0

F-stats = 18.49 and the corresponding p-value is 0

Since the p-value of the both LM and F-stats is less than 0.05, we reject the null hypothesis that there is no heteroskedasticity in the residuals. It infers that the heteroskedasticity exists and the standard errors need to be corrected.

8 Breusch-Godfrey LM test for Gold

[107]:
```
import statsmodels.stats.diagnostic as dg
print (dg.acorr_breusch_godfrey(model, nlags= 2))
```

```
(14.058774886495657, 0.0008854740175917412, 7.036171882380294,
0.0008836668869260258)
```

T-statistic of Chi-squared is 14.0588 and the corresponding p-value is 0.0009

F-statistic is 7.0362 and the corresponding p-value is 0.0009

Since p-value is less than 0.05, we reject the null hypothesis, thus inferring there is some autocorrelation at order less than or equal to 2.0

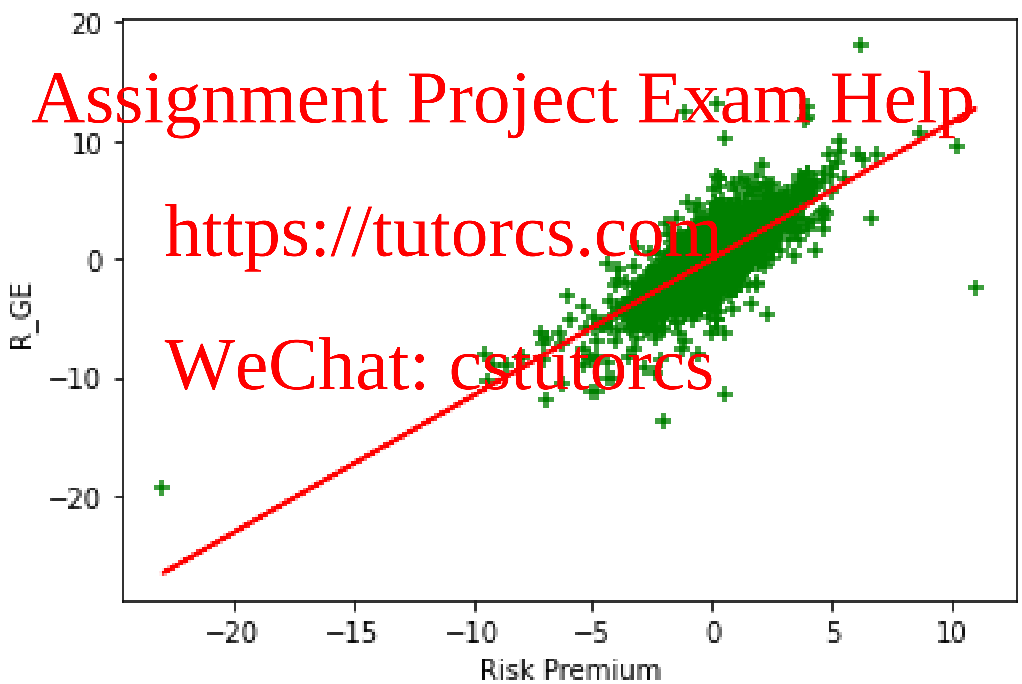
9 II. Plotting GE excess returns with market excess returns

[108]:
```
%matplotlib inline
reg = linear_model.LinearRegression()
X =data[['R_p']]
y =data['R_ge']
reg.fit(X,y)
```

[108]: LinearRegression()

[109]:
```
plt.xlabel('Risk Premium')
plt.ylabel('R_GE')
plt.scatter(data.R_p,data.R_GE,color='green',marker='+')
plt.plot(data.R_p,reg.predict(data[['R_p']]), color='red')
```

[109]: [<matplotlib.lines.Line2D at 0x18397269b20>]



10 Regressing GE excess return with market excess return

[110]:
```
#model with intercept
X =sm.add_constant(X)
model_1 = sm.OLS(y,X).fit()
predictions = model_1.predict(X)
j= (model_1.summary())
```

```python
print(j)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   R_ge   R-squared:                       0.564
Model:                            OLS   Adj. R-squared:                  0.564
Method:                 Least Squares   F-statistic:                 1.351e+04
Date:                Thu, 24 Jun 2021   Prob (F-statistic):               0.00
Time:                        13:42:14   Log-Likelihood:                -15682.
No. Observations:               10436   AIC:                         3.137e+04
Df Residuals:                   10434   BIC:                         3.138e+04
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0012      0.011     -0.114      0.909      -0.022       0.020
R_p            1.1569      0.010    116.224      0.000       1.137       1.176
==============================================================================
Omnibus:                     2325.128   Durbin-Watson:                   1.995
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           109234.319
Skew:                           0.109   Prob(JB):                         0.00
Kurtosis:                      18.848   Cond. No.                         1.07
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```


DW-stats of 1.995 is close to 2.0, implying that there is no serial correlation.

Since p-value of the beta coefficient is less than 0.05, we reject the null hypothesis that beta is zero.

The CAPM equation for GE can be written as follows:

$R_g e = 1.1569 * R_p + Rf$

where $R_g e$ is the return from GE stock, $R_p = Rm - Rf$ is the market risk premium and Rf is the risk free rate of return

If we want to replicate the returns from GE, we can rearrange the above equation:

$R_g e = 1.1569 * Rm + (1 - 1.1569) * Rf$

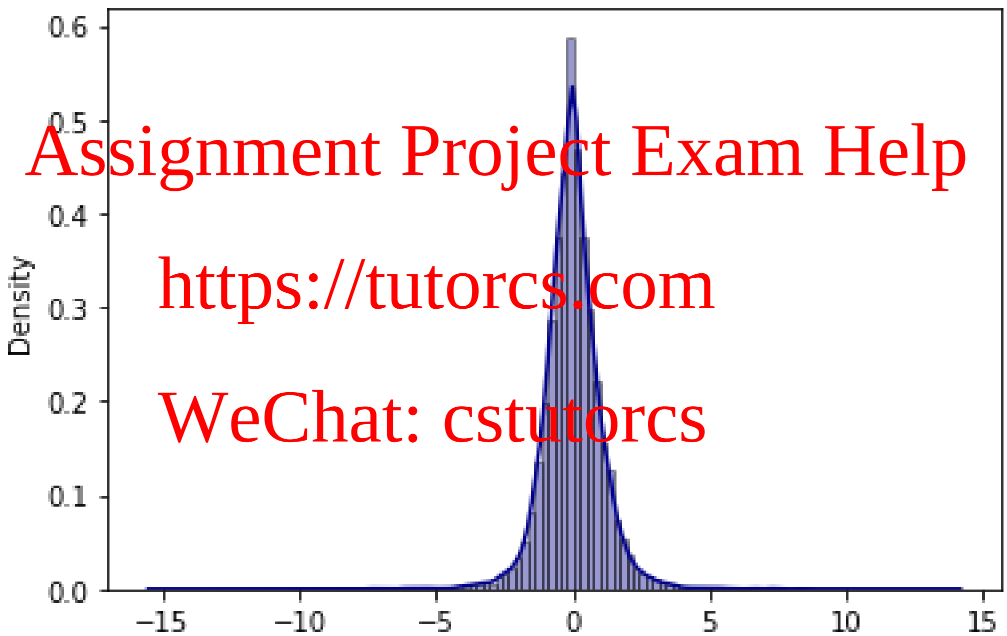
$\Rightarrow$ We can buy 1.1569 of market portfolio (i.e: S&P500 index fund) and then short 0.1569 T-Bill.

11 Residual Plots for GE

[111]:
```python
residuals = model_1.resid
import seaborn as sns
sns.distplot(residuals,hist=True, kde=True, bins=int(120), color=
 'darkblue',hist_kws={'edgecolor':'black'})
```

```
C:\Users\rluck\anaconda3\lib\site-packages\seaborn\distributions.py:2557:
FutureWarning: `distplot` is a deprecated function and will be removed in a
future version. Please adapt your code to use either `displot` (a figure-level
function with similar flexibility) or `histplot` (an axes-level function for
histograms).
  warnings.warn(msg, FutureWarning)
```

[111]:
```
<AxesSubplot:ylabel='Density'>
```



[112]:
```python
from scipy import stats
JB_GE= stats.jarque_bera(residuals)
JB_GE
```

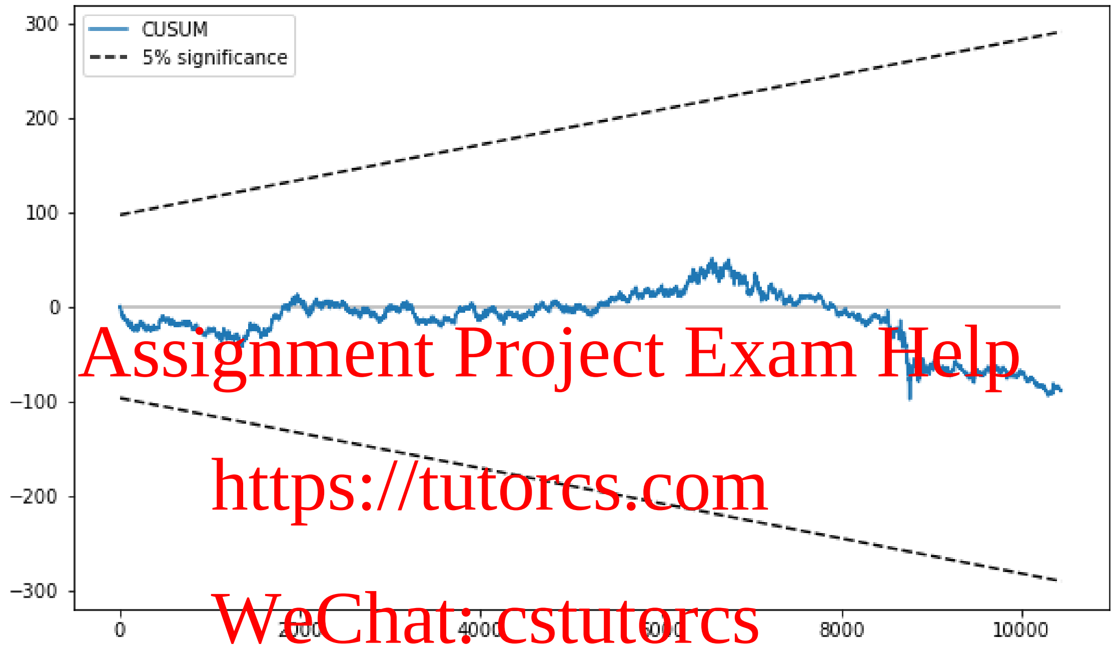
[112]:
```
Jarque_beraResult(statistic=109234.31887176927, pvalue=0.0)
```

The plot and JB test (p-value <0.05) rejects the null hypothesis of normality. It is clearly a non-normal distribution.

[113]:
```python
endog = data.R_ge
Rp = data.R_p
```

```
exog = sm.add_constant(Rp)
mod = sm.RecursiveLS(endog,exog)
res_1 = mod.fit()
fig = res_1.plot_cusum(figsize=(10,6));
```

```
C:\Users\rluck\anaconda3\lib\site-
packages\statsmodels\tsa\base\tsa_model.py:578: ValueWarning: An unsupported
index was provided and will be ignored when e.g. forecasting.
  warnings.warn('An unsupported index was provided and will be'
```



Cusum test of stability for GE shows stability of beta as it is within the 5% significance level band.

12 White Test of Heteroskedasticity for GE

[114]:
```
expr = 'y ~ X'
y, X = dmatrices(expr, data, return_type='dataframe')
olsr_results = smf.ols(expr, data).fit()
keys = ['Lagrange Multiplier statistic:', 'LM test\'s p-value:', 'F-statistic:
 ↪', 'F-test\'s p-value:']
results = het_white(olsr_results.resid, X)
lzip(keys, results)
```

[114]:
```
[('Lagrange Multiplier statistic:', 600.7119211665138),
 ("LM test's p-value:", 3.606315445696676e-131),
 ('F-statistic:', 318.60924780728743),
```

```
("F-test's p-value:", 4.9073934718673876e-135)]
```

LM test statistic is 600.72 and the corresponding p-value is 0

F-stats = 318.61 and the corresponding p-value is 0

Since the p-value of the both LM and F-stats is less than 0.05, we reject the null hypothesis that there is no heteroskedasticity in the residuals. It infers that the heteroskedasticity exists and the standard errors need to be corrected.

[115]:
```python
print (dg.acorr_breusch_godfrey(model_1, nlags= 2))
```

```
(5.174836714176367, 0.07521396525512013, 2.587709781212114, 0.07524031416320724)
```

T-statistic of Chi-squared = 5.1748 and the corresponding p-value = 0.075.

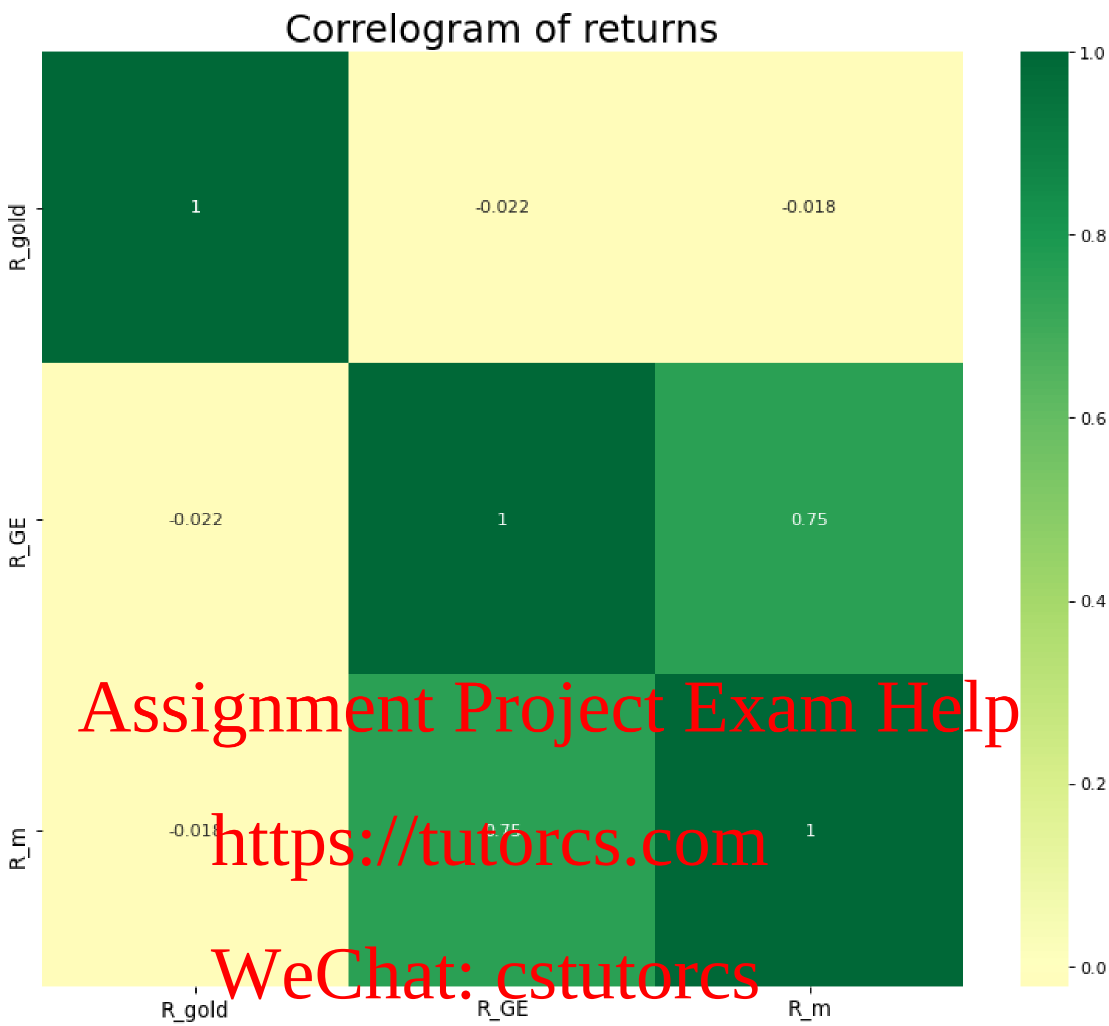
F-statistics = 2.5877 and the corresponding p-value = 0.075

Since p-value exceeds 0.05, we fail to reject the null hypothesis, thus inferring there is no autocorrelation at order less than or equal to 2.0

[ ]:
```python

```


13 Extra: Correlation matrix between returns of gold, GE and market

[116]:
```python
import seaborn as sns
import pandas as pd
# Import Dataset
data = pd.read_csv("C:\\Users\\[illegible]uck\\OneDrive\\capm1.csv",␣
↪usecols=['R_gold','R_m', 'R_GE'])

# Plot
plt.figure(figsize=(12,10), dpi= 80)
sns.heatmap(data.corr(), xticklabels=data.corr().columns, yticklabels=data.
↪corr().columns, cmap='RdYlGn', center=0, annot=True)

# Decorations
plt.title('Correlogram of returns', fontsize=22)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)
plt.show()
```


Correlogram of returns
R_gold
R_GE
R_m
1
-0.022
-0.018
-0.022
1
0.75
-0.018
0.75
1
R_gold
R_GE
R_m
1.0
0.8
0.6
0.4
0.2
0.0


```
[ ]:
```