Financial Econometrics

T2 2021

Sample Answers/Hints to Tutorial 7

1. (Value at Risk)

The VaR is the maximum loss for holding an asset or a portfolio for the given period with the given probability 0.99. Under the stated GARCH(1,1) model, the standardised shock, $v_t = \varepsilon_t/\sigma_t$, is an iid series with mean zero and variance one. Let $q_{0.01}$ be the lower 1% (empirical) quantile of v_t. Then,

$$\text{VaR} = \frac{1}{100}[c + q_{0.01}\sigma_{T+1}] \times 10m = \frac{1}{100}\left[c + q_{0.01}(\alpha_0 + \alpha_1\varepsilon_T^2 + \beta_1\sigma_T^2)^{1/2}\right] \times 10m\ .$$

2. (GARCH-in-mean model)

(a) The rationale for including the conditional variance σ_t^2 (or its square-root) in the mean equation is that a risky investment must be compensated by an expected return that is higher than the risk-free return. The risk premium is the difference between the expected returns of a risky investment and a risk-free investment. According to this rationale, the expected return of a risky asset should be positively related to the expected risk measure, which leads to the GARCH-M model with a positive δ.

(b) The conditional mean is obviously $E(y_t|\Omega_{t-1}) = c + \delta\sigma_t^2$ as the conditional variance is a function of Ω_{t-1}. According the rule of iterated expectations, the unconditional mean is given by $E(y_t) = c + \delta E(\sigma_t^2)$. The variance equation then leads to

$$E(\sigma_t^2) = E(\alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2) = \alpha_0 + \alpha_1 E(\varepsilon_{t-1}^2) + \beta_1 E(\sigma_{t-1}^2) = \alpha_0/(1 - \alpha_1 - \beta_1)$$

because $E(\sigma_t^2) = E(\sigma_{t-1}^2) = E(\varepsilon_{t-1}^2)$ by stationarity and iterated expectations. Therefore $E(y_t) = c + \delta\alpha_0/(1 - \alpha_1 - \beta_1)$.

3. Consider the constant conditional mean - EGARCH model

$$y_t = c + \varepsilon_t, \quad \varepsilon_t|\Omega_{t-1} \sim N(0, \sigma_t^2),$$

$$ln(\sigma_t^2) = \alpha_0 + \alpha_1|v_{t-1}| + \gamma v_{t-1} + \beta_1\, ln(\sigma_{t-1}^2)\,, v_{t-1} = \varepsilon_{t-1}/\sigma_{t-1}$$

(a) The key benefit is that we do not have to impose positivity constraint on the parameters of the model as *exp* transformation ensures that σ_t^2 is positive. In addition the model uses standardized shocks directly and easily allows for leverage effect. Note that for stationarity we would still require that $|\beta_1| < 1$. It is hard to show formally because taking expectation and variance involving logarithmic function is non-trivial, but informally the intuition is similar to AR model. If $|\beta_1| > 1$, the variance becomes explosive as higher and higher variance is added in every period. If $|\beta_1| = 1$ we have something like random walk in variance. Also note that β_1 is expected to be positive to capture volatility clustering (high volatility is followed by high volatility).

(b) Parameter γ characterises the effect of sign of the (standardised) innovation (or news) on the conditional variance or a so-called leverage effect. Empirically we observe higher variance after negative innovations. Hence, γ is expected to be negative.

(c) Compute one-period ahead optimal forecast of y and form 95% confidence bounds.

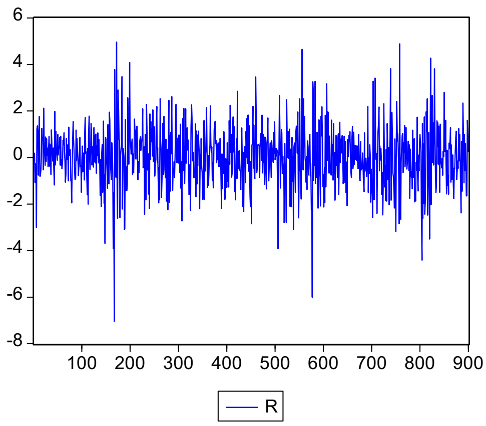
We have showed before that the one-period ahead optimal forecast is equivalent to this conditional expectation $E(y_{t+1}|\Omega_t) = c$. Note that in this specification the conditional mean is modelled just by a constant, c and is constant over time. To form confidence bounds we need to compute conditional variance of the forecast $var(y_{t+1}|\Omega_t) = var(\varepsilon_{t+1}|\Omega_t) = \sigma_{t+1}^2$.

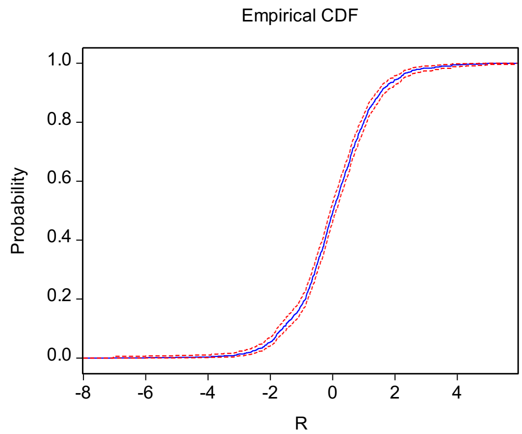
$\sigma_{t+1}^2 = (\sigma_t^2)^{\beta_1} e^{\alpha_0+\alpha_1|v_t|+\gamma v_t}$. Note that all right-hand side variables are in Ω_t and know.

Hence, 95% confidence bound is $c \pm 1.96\sigma_t^{\beta_1} e^{\frac{1}{2}(\alpha_0+\alpha_1|v_t|+\gamma v_t)}$.

4. Computing Exercise

(a) The time series plot of R and its empirical CDF is given below. The lower 1% quantile of R is -3.131.





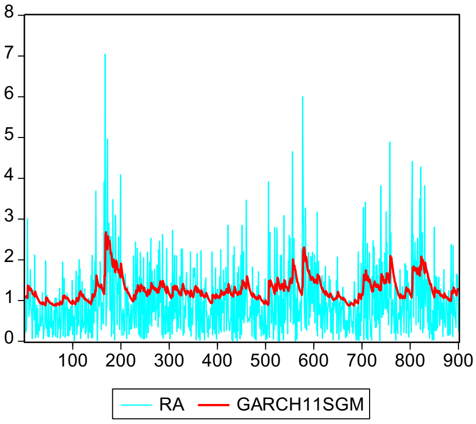
(b-c) The estimation results for GARCH(1,1), GJR and EGARCH are given below. Regarding the σ_t plots, all match the variation patterns in the absolute return (RA) well. The GARCH(1,1) volatility appears to be more persistent than GJR and EGARCH in that its σ_t plot is smoother. For GARCH(1,1) and EGARCH, the restrictions on the parameters are all satisfied (check Slides-07-08). However, there is a violation of restrictions in GJR model: $\hat{\alpha}_1 = -0.044501$, although it is statistically insignificant. Further, the asymmetric effect (ie, a negative ε_{t-1} causes more volatility than a positive one) are confirmed: the γ estimates in GJR and EGARCH models are significantly different from zero. As the GARCH(1,1) does not take into account the asymmetric effect and GJR violates a positivity restriction, the preferred model should be EGARCH. The subsequent answers are all based on the EGARCH.

Dependent Variable: R
Method: ML - ARCH (Marquardt) - Normal distribution

Sample (adjusted): 2 900
Included observations: 899 after adjustments
Convergence achieved after 11 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: ON
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.051250	0.038139	1.343792	0.1790
		Variance Equation		
C	0.068387	0.032560	2.100350	0.0357
RESID(-1)^2	0.087582	0.029300	2.989168	0.0028
GARCH(-1)	0.873860	0.037409	23.35935	0.0000

R-squared	-0.000451	Mean dependent var	0.023630
Adjusted R-squared	-0.003804	S.D. dependent var	1.301407
S.E. of regression	1.303880	Akaike info criterion	3.291679
Sum squared resid	1521.592	Schwarz criterion	3.313042
Log likelihood	-1475.610	Durbin-Watson stat	2.025862



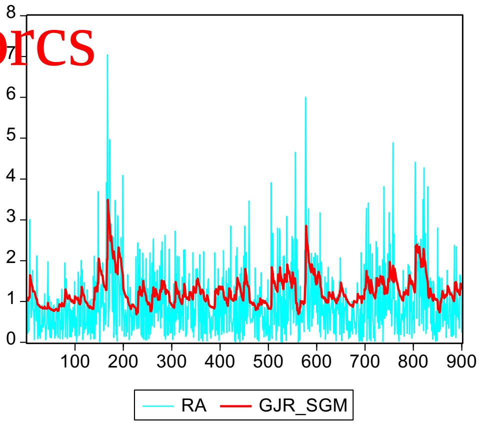
Dependent Variable: R
Method: ML - ARCH (Marquardt) - Normal distribution

Sample (adjusted): 2 900
Included observations: 899 after adjustments
Convergence achieved after 12 iterations
Bollerslev-Wooldrige robust standard errors & covariance

Variance backcast: ON
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0)
+ C(5)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.007760	0.037792	-0.205342	0.8373
		Variance Equation		
C	0.067151	0.023584	2.847359	0.0044
RESID(-1)^2	-0.044501	0.015836	-2.810038	0.0050
RESID(-1)^2*(RESID(-1)<0)	0.213730	0.039618	5.394771	0.0000
GARCH(-1)	0.901058	0.027279	33.03142	0.0000

R-squared	-0.000582	Mean dependent var	0.023630
Adjusted R-squared	-0.005059	S.D. dependent var	1.301407
S.E. of regression	1.304695	Akaike info criterion	3.221998
Sum squared resid	1521.792	Schwarz criterion	3.248702
Log likelihood	-1443.288	Durbin-Watson stat	2.025596

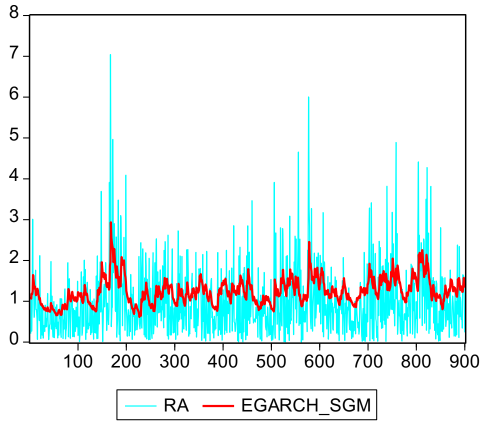


Dependent Variable: R
Method: ML - ARCH (Marquardt) - Normal distribution

Sample (adjusted): 2 900
Included observations: 899 after adjustments
Convergence achieved after 17 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: ON
LOG(GARCH) = C(2) + C(3)*ABS(RESID(-1)/@SQRT(GARCH(-1))) +
C(4)*RESID(-1)/@SQRT(GARCH(-1)) + C(5)*LOG(GARCH(-1))

	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.007582	0.038380	-0.197549	0.8434
	Variance Equation			
C(2)	-0.044614	0.031363	-1.422498	0.1549
C(3)	0.086489	0.040001	2.162175	0.0306
C(4)	-0.170820	0.031777	-5.375648	0.0000
C(5)	0.944635	0.017310	54.57037	0.0000

R-squared	-0.000576	Mean dependent var	0.023630
Adjusted R-squared	-0.005053	S.D. dependent var	1.301407
S.E. of regression	1.304690	Akaike info criterion	3.225155
Sum squared resid	1521.782	Schwarz criterion	3.251859
Log likelihood	-1444.707	Durbin-Watson stat	2.025609



(d) The standardised residuals (or squared) show little autocorrelation, confirming that the autocorrelation in the returns squared is well represented in the EGARCH model. However, the normality is rejected for the standardised residuals. The histograms of the residuals (E) and the standardised residuals (V) show that V has more negative skewness than E while E has more excess kurtosis than V. The lower tail 1% quantile of the standardised residuals is -2.444.

Correlogram of Standardized Residuals

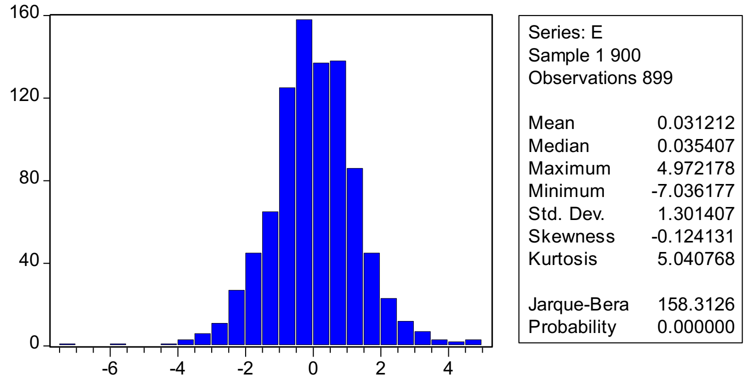
Sample: 2 900
Included observations: 899

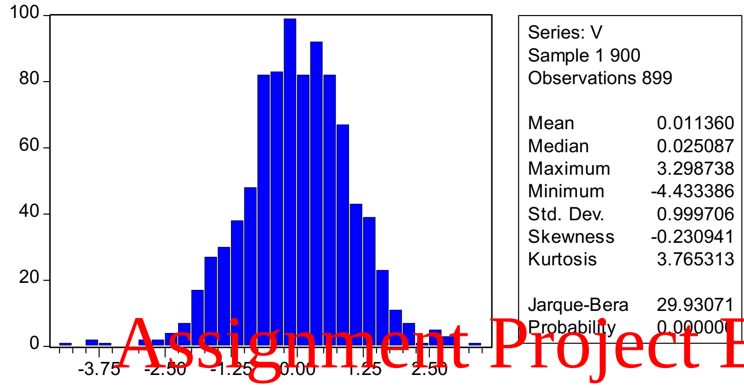
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.024	0.024	0.5006	0.479
		2	0.010	0.009	0.5832	0.747
		3	-0.049	-0.049	2.7188	0.437
		4	-0.023	-0.021	3.2178	0.522
		5	-0.046	-0.044	5.1081	0.403
		6	-0.001	-0.001	5.1088	0.530
		7	-0.040	-0.042	6.5624	0.476
		8	-0.006	-0.009	6.5910	0.581
		9	-0.019	-0.020	6.9098	0.647
		10	0.045	0.040	8.7696	0.554
		11	-0.047	-0.051	10.743	0.465
		12	0.086	0.083	17.566	0.130
		13	0.056	0.055	20.396	0.086
		14	0.004	-0.006	20.410	0.118
		15	0.024	0.033	20.929	0.139
		16	-0.047	-0.045	22.939	0.115

Correlogram of Standardized Residuals Squared

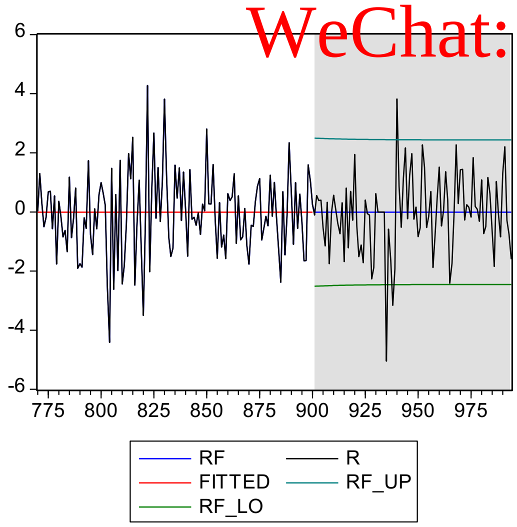
Sample: 2 900
Included observations: 899

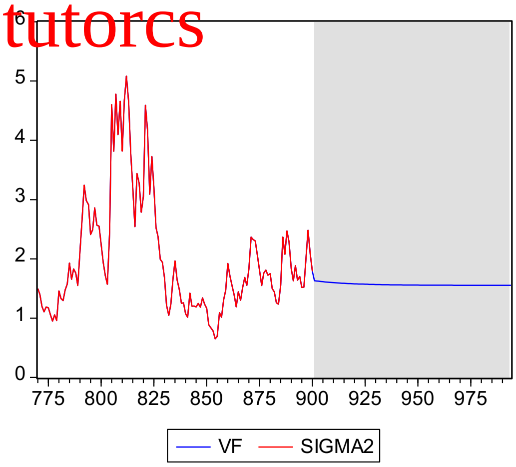
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.039	-0.039	1.3460	0.246
		2	0.031	0.030	2.2389	0.326
		3	-0.012	-0.010	2.3720	0.499
		4	0.017	0.016	2.6416	0.619
		5	0.023	0.025	3.1101	0.683
		6	-0.035	-0.035	4.2409	0.644
		7	0.024	0.021	4.7854	0.686
		8	0.021	0.025	5.1704	0.739
		9	-0.034	-0.036	6.2210	0.718
		10	0.023	0.021	6.7060	0.753
		11	0.008	0.014	6.7709	0.817
		12	-0.010	-0.015	6.8630	0.867
		13	-0.006	-0.005	6.8949	0.907
		14	-0.004	-0.002	6.9128	0.938
		15	-0.027	-0.032	7.5780	0.940
		16	0.040	0.041	9.0456	0.912





(e) The forecasts from the EGARCH model are presented in the graphs below. The conditional variance does exhibits the "mean-reverting" behaviour and converges to the average level rapidly.





(f) The quantities required for computing the conditional VaR are: $T = 900$, $\sigma_{T+1} = 1.278$, $y_{T+1|T} = \hat{c} = -0.0076$, $q_{0.01} = -2.444$. Using the formulae in Q1, we find VaR = $-\$313161$.