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# Financial Econometrics

## Slides-01: Return Properties -Introduction

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# Introduction

Financial time series (FTS) analysis is concerned with theory and practice of asset valuation over time.

Comparison with other Time Series analysis: similarity and difference? Highly related, but with some added uncertainty, because FTS must deal with the ever-changing business & economic environment and the fact that volatility is not directly observed. Objective of the course

- to learn ways to get financial information from web directly and to process the information.
- to provide some basic knowledge of financial time series data such as skewness, heavy tails, and measure of dependence between asset returns
- to introduce some statistical tools & econometric models useful for analyzing these series.

## Examples of kind of problems you can solve

- Testing whether financial markets are weak-form informationally efficient.
- Testing whether the CAPM or APT represent superior models for the determination of returns on risky assets.
- Measuring and forecasting the volatility of bond returns.
- Explaining the determinants of bond credit ratings used by the ratings agencies.
- Modelling long-term relationships between prices and exchange rates
- Testing technical trading rules to determine which makes the most money.
- Testing the hypothesis that earnings or dividend announcements have no effect on stock prices.
- Testing whether spot or futures markets react more rapidly to news.
- Forecasting the correlation between the returns to the stock indices of two countries.

# What are special characteristics of Financial Data?

## ① Frequency & quantity of data

- Stock market prices are measured every time there is a trade or somebody posts a new quote.

## ② Quality

- Recorded asset prices are usually those at which the transaction took place. No possibility for measurement error but financial data are **noisy**.

# Examples of FTS

## Examples of financial time series

1. Daily log returns of Apple stock: 2004 to 2013 (10 years)
2. The VIX index
3. CDS spreads: Daily 3-year CDS spreads of JP Morgan from July 20, 2004 to September 19, 2014.
4. Quarterly earnings of Coca-Cola Company: 1983-2009  
Seasonal time series useful in
  - earning forecasts
  - pricing weather related derivatives (e.g. energy)
  - modeling intraday behavior of asset returns
5. US monthly interest rates (3m & 6m Treasury bills)  
Relations between the two series? Term structure of interest rates
6. Exchange rate between US Dollar vs Euro  
Fixed income, hedging, carry trade
7. Size of insurance claims  
Values of fire insurance claims ( $\times 1000$  Krone) that exceeded 500 from 1972 to 1992.
8. High-frequency financial data:  
Tick-by-tick data of Caterpillars stock: January 04, 2010.

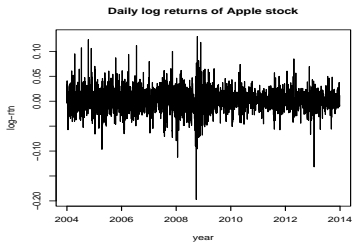


Figure 1: Daily log returns of Apple stock from 2004 to 2013

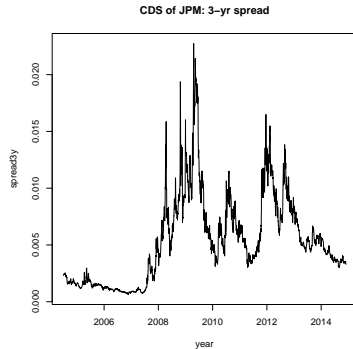


Figure 3: Time plot of daily 3-year CDS spreads of JPM: from July 20, 2004 to September 19, 2014.

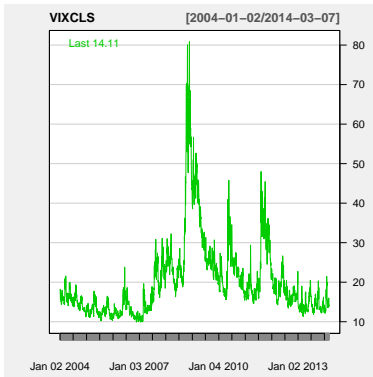


Figure 4: CBOE Vix index: January 2, 2004 to March 7, 2014.

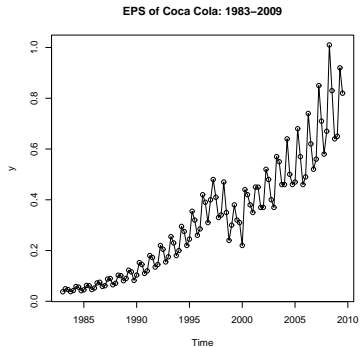


Figure 5: Quarterly earnings per share of Coca-Cola Company





Figure 6: Daily Exchange Rate: Dollars per Euro

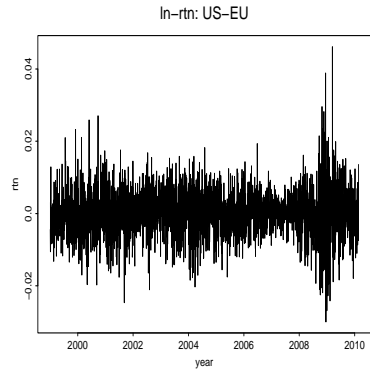


Figure 7: Daily log returns of FX (Dollar vs Euro)

# What Is a Stock?

## What is a stock?

A stock (also known as equity) is a security that represents the ownership of a fraction of a corporation. This entitles the owner of the stock to a proportion of the corporation's assets and profits equal to how much stock they own.

Units of stock are called **shares**.

- Stocks are bought and sold predominantly on stock exchanges, though there can be private sales as well, and they are the foundation of nearly every portfolio.
- Historically, they have outperformed most other investments over the long run. [▶ Historical returns](#)

# Assets and equities

## What is a financial asset?

Financial assets represent investments in the assets and securities of other institutions. Financial assets include stocks, sovereign and corporate bonds, preferred equity, and other hybrid securities. Financial assets are valued depending on how the investment is categorized and the motive behind it.

## What Is Equity?

Equity, typically referred to as shareholders' equity (or owners' equity for privately held companies), represents the amount of money that would be returned to a company's shareholders if all of the assets were liquidated and all of the company's debt was paid off in the case of liquidation. In the case of acquisition, it is the value of company sales minus any liabilities owed by the company not transferred with the sale.

# More Definitions...

## What Is Earnings Per Share (EPS)?

Earnings per share (EPS) is calculated as a company's profit divided by the outstanding shares of its common stock. The resulting number serves as an indicator of a company's profitability. It is common for a company to report EPS that is adjusted for extraordinary items and potential share dilution. The higher a company's EPS, the more profitable it is considered to be.

► Example: Exxon Mobil Investopia

► Example: Exxon Mobil Yahoo Finance

# Asset Returns: Definition

Let  $P_t$  be the price of an asset at time  $t$ , and assume no dividend.

- One-period simple return:

**Gross return**

$$1 + R_t = \frac{P_t}{P_{t-1}}$$

$$P_t = P_{t-1}(1 + R_t)$$

**Simple return:**

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1$$

- Multiperiod simple return

$$\begin{aligned} 1 + R_t(k) &= \frac{P_t}{P_{t-k}} \\ &= (1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1}) \\ &= \prod_{j=0}^{k-1} (1 + R_{t-j}) \end{aligned}$$

# Asset Returns: Example

Table below gives five daily closing prices of Apple stock in December 2011.

| Date      | 12/02  | 12/05  | 12/06  | 12/07  | 12/08  | 12/09  |
|-----------|--------|--------|--------|--------|--------|--------|
| Price(\$) | 389.70 | 393.01 | 390.95 | 389.09 | 390.66 | 393.62 |

- The 1-day **simple** return of holding the stock from 12/8 to 12/9:  
0.76%
- The 3-day **simple** return for holding the stock from 12/02 to 12/07:  
−0.15%
- The 5-day **simple** return for holding the stock from 12/02 to 12/09:  
Answer?

# Annualized Asset Returns

**Time interval** is important! Default is one year.

**Annualized (average) return:**

$$\text{Annualized}[R_t(k)] = \left[ \prod_{j=0}^{k-1} (1 + R_{t-j}) \right]^{1/k} - 1.$$

An approximation:

$$\text{Annualized}[R_t(k)] \approx \frac{1}{k} \sum_{j=0}^{k-1} R_{t-j}.$$

Continuously compounding: Illustration of the power of compounding (int. rate 10% per annum)

| Type         | #(payment) | Int.              | Net       |
|--------------|------------|-------------------|-----------|
| Annual       | 1          | 0.1               | \$1.10000 |
| Semi-Annual  | 2          | 0.05              | \$1.10250 |
| Quarterly    | 4          | 0.025             | \$1.10381 |
| Monthly      | 12         | 0.0083            | \$1.10471 |
| Weekly       | 52         | $\frac{0.1}{52}$  | \$1.10506 |
| Daily        | 365        | $\frac{0.1}{365}$ | \$1.10516 |
| Continuously | $\infty$   |                   | \$1.10517 |

$$A = C \exp[r \times n]$$

where  $r$  is the interest rate per annum,  $C$  is the initial capital,  $n$  is the number of years, and  $\exp$  is the exponential function.

# Annualized Asset Returns

**Present value:**

$$C = A \exp[-r \times n]$$

**Continuously compounded (or log) return**

$$r_t = \ln(1 + R_t) = \ln \frac{P_t}{P_{t-1}} = p_t - p_{t-1},$$

where  $p_t = \ln(P_t)$ .

Multiperiod log return:

$$\begin{aligned} r_t(k) &= \ln[1 + R_t(k)] \\ &= \ln[(1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1})] \\ &= \ln(1 + R_t) + \ln(1 + R_{t-1}) + \cdots + \ln(1 + R_{t-k+1}) \\ &= r_t + r_{t-1} + \cdots + r_{t-k+1}. \end{aligned}$$



# Annualized Asset Returns

Example Consider again the Apple stock price.

- What is the log return from 12/8 to 12/9?

A:

7.5%

- What is the log return from day 12/2 to 12/9?

A:

1%

- What is the log return from day 12/6 to 12/8?

A:

?

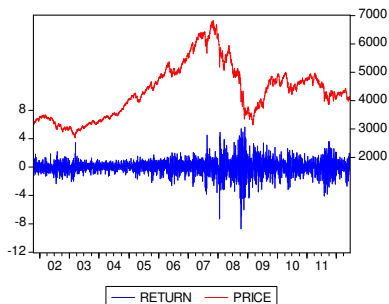
# Market Index and Return

- Market index:  $P_{m,t} = \sum_{i=1}^N w_{it} P_{i,t}$ ,  $t = 1, 2, \dots$   
weight  $w_{it}$  depends on outstanding shares of stock  $i$ , etc
- Log return:

$$r_{m,t} = 100\% \times \ln \left( \frac{P_{m,t}}{P_{m,t-1}} \right)$$

eg. S&P/ASX200 Index and Return

| DATE       | ADJCLOS... | RETURN    |
|------------|------------|-----------|
| 2001-10-17 | 3237.9     | NA        |
| 2001-10-18 | 3199.6     | -1.189917 |
| 2001-10-19 | 3175.4     | -0.759219 |
| 2001-10-23 | 3213.3     | 1.186484  |
| 2001-10-24 | 3246.0     | 1.012502  |
| 2001-10-25 | 3254.7     | 0.267664  |
| 2001-10-26 | 3276.2     | 0.658411  |
| 2001-10-29 | 3256.2     | -0.612334 |
| 2001-10-30 | 3252.8     | -0.104471 |
| 2001-10-31 | 3249.6     | -0.098425 |
| 2001-11-01 | 3248.8     | -0.024621 |
| 2001-11-02 | 3239.9     | -0.274323 |
| 2001-11-05 | 3225.8     | -0.436148 |
| 2001-11-06 | 3285.1     | 1.821611  |
| 2001-11-07 | 3252.6     | -0.994242 |
| 2001-11-08 | 3269.4     | 0.515181  |
| 2001-11-09 | 3286.0     | 0.506454  |



# Portfolio Return

An investor holds stocks of IBM, Microsoft and Citi- Group. Assume that her capital allocation is 30%, 30% and 40%. What is the mean simple return of her stock portfolio?

Assume monthly simple returns for IBM, microsoft and Citi-Group, 1.35%, 2.62% and 1.17% respectively.

**Answer:**

- Portfolio Return:  $R_{p,t} = \sum_{i=1}^N w_{it} R_{i,t}$ ,  $t = 1, 2, \dots$ , where  $N$  is the number of assets held by investor and  $w_{it}$  is wealth allocation.

# Adjusted Returns

## ① Adjusting for dividends (Total Returns)

$$\begin{aligned} r_t &= \ln(1 + R_t) = \ln\left(\frac{P_t + D_t}{P_{t-1}}\right) \\ &= \ln(P_t + D_t) - \ln(P_{t-1}) \end{aligned}$$

## ② Adjusting for inflation (Real Returns)

$$r_t^{Real} = \ln\left(1 + R_t^{Real}\right) = \ln\left(\frac{P_t}{P_{t-1}} \frac{CPI_{t-1}}{CPI_t}\right)$$

## ③ Adjusting for Risk (Excess Returns)

$$\begin{aligned} Z_t &= R_t - R_{ft} \\ z_t &= \ln(Z_t) = \ln(R_t - R_{ft}) \neq r_t - r_{ft} \end{aligned}$$

**BUT** if  $Z_t$  is small:  $z_t \approx r_t - r_{ft}$

# Dividends, Excess returns

Relationship:

$$r_t = \ln(1 + R_t), \quad R_t = e^{r_t} - 1.$$

If the returns are in **percentage**, then

$$r_t = 100 \times \ln\left(1 + \frac{R_t}{100}\right), \quad R_t = [\exp(r_t/100) - 1] \times 100.$$

Temporal aggregation of the returns produces

$$\begin{aligned} 1 + R_t(k) &= (1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1}), \\ r_t(k) &= r_t + r_{t-1} + \cdots + r_{t-k+1}. \end{aligned}$$

These two relations are important in practice, e.g. obtain annual returns from monthly returns.

### Example

If the monthly log returns of an asset are 4.46%, -7.34% and 10.77%, then

- 1 what is the corresponding **quarterly log** return?
- 2 what is the corresponding **quarterly simple** return?

### Answer

- (1) Quarterly log return:
- (2) Quarterly simple return:

# How to invest \$100?

► How much will you invest?

# Recap!

► Slido