Slides-08
Modeling Long Run relationship

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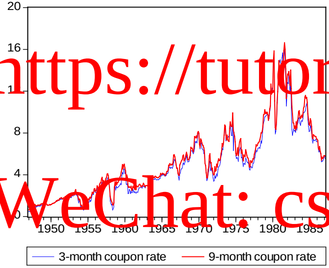
Lecture Plan

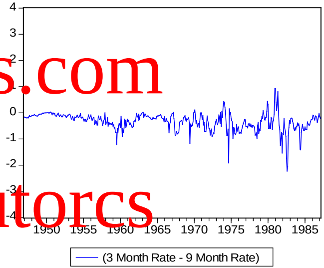
- Long-run relationship: co-movement in trending time series
- Cointegration and common trend
  - Interest rate and inflation
  - Long and short term interest rates
- Regression with $I(1)$ series under cointegration and dynamic OLS
- Spurious regression
- Test for cointegration
- Error correction models
  - Information & price discovery

Long-run relationships

- Co-movement among time series

  eg. US zero coupon rates: 3-month vs 9-month





(1946:12-1987:2, 483 monthly observations)
Both appear non-stationary but move together.

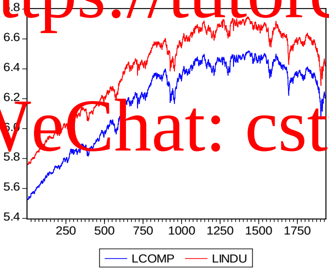
Long-run relationships

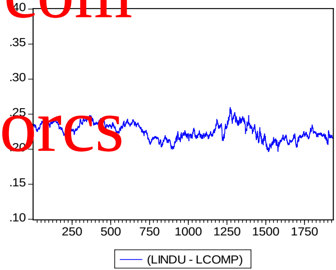
- Co-movement among time series

eg. US zero coupon rates: 3-month vs 9-month

eg. NYSE log Composite & Industrial indices

Both are non-stationary but move together.





Long-run relationships

- Co-movement among time series
  - Two (or more) time series **move together** over time and never depart for long.
  - The time series are individually I(1) and vary a great deal. But their **long-run relationship** appears stable over time.
  - There must be a **common trend** that drives both time series.
  - Important to exploit long-run relationships in finance eg. pairs-trading; rational bubbles; bi-listed stocks
  - We introduce basic facts on modelling long-run relationships, mainly with **bi-variate** cases.

pairs=trading

Pairs trading is a market-neutral trading strategy that matches a long position with a short position in a pair of highly correlated instruments such as two stocks, exchange-traded funds (ETFs), currencies, commodities or options. Pairs traders wait for weakness in the correlation and then go long the under-performer while simultaneously short selling the over-performer, closing the positions as the relationship returns to statistical norms.

The spurious regression problem

- **General result:** a linear combination z_t of a set of variables x_{it}, with order $x_{it} \sim I(1)$, will have an order of integration equal to 1, if there exists a linear combination, $z_t = \sum_{i=1}^{k} \alpha_i x_{it} \sim I(0)$

- Example: consider two series y_t and x_t, with

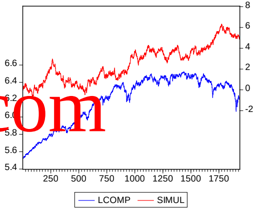
$$y_t \sim I(1);\ x_t \sim I(1)$$

and a linear combination z_t thereof, i.e.

$$z_t = \alpha_0 + \alpha_1 y_t + \alpha_2 x_t \sim I(0)$$

The spurious regression problem

- Example: NYSE log Composite index vs Simulated RW



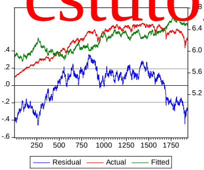
Symptom: the residual looks like RW

Dependent Variable: LCOMP
Method: Least Squares

Sample: 1 1931
Included observations: 1931

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	5.898260	0.007740	762.0372	0.0000
SIMUL	0.117743	0.002540	46.35960	0.0000

R-squared	0.526999	Mean dependent var	6.193532
Adjusted R-squared	0.526753	S.D. dependent var	0.280939
S.E. of regression	0.193266	Akaike info criterion	-0.448462
Sum squared resid	72.05160	Schwarz criterion	-0.442698
Log likelihood	434.9903	F-statistic	2149.212
Durbin-Watson stat	0.006508	Prob(F-statistic)	0.000000



Correlogram of Residuals

Sample: 1 1931
Included observations: 1931

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.994	0.994	1912.0	0.000
		2	0.988	-0.031	3802.0	0.000
		3	0.982	0.013	5670.6	0.000
		4	0.977	0.025	7519.2	0.000
		5	0.971	-0.014	9347.6	0.000
		6	0.966	-0.007	11156.	0.000
		7	0.960	0.012	12944.	0.000
		8	0.954	-0.017	14712.	0.000
		9	0.949	0.002	16460.	0.000
		10	0.943	-0.021	18187.	0.000
		11	0.937	0.023	19896.	0.000
		12	0.932	0.037	21586.	0.000
		13	0.926	-0.076	23256.	0.000
		14	0.920	-0.048	24904.	0.000
		15	0.913	-0.041	26528.	0.000
		16	0.907	0.066	28131.	0.000

Examples of Spurious Regression

- Egyptian infant mortality rate (Y_t), 1971-1990, annual data, on gross aggregate income of American farmers (I_t) and total Honduran money supply (M_t)

$$\begin{array}{rcccccc} \widehat{Y}_t = & 179.9 & - & 0.30 I_t & - & 0.04 M_t \\ & (16.63) & & (-2.32) & & (-4.26) \end{array}$$

$$R^2 = 0.918;\ F = 95.17;\ DW = 0.475$$

- US export index (Y_t), 1960-1990, annual data, on Australian males life expectancy (X_t)

$$\begin{array}{rccc} \widehat{Y}_t = & -2943 & + & 45.80 X_t \\ & (16.70) & & (17.76) \end{array}$$

$$R^2 = 0.916;\ F = 315.2;\ DW = 0.360$$

The spurious regression problem

$$y_t = \beta_1 + \beta_2 x_t + \epsilon_t$$

- The spurious regression problem is characterized by
  - Highly significant value for β_2
  - Fairly high R^2
- Reason: distribution of the conventional test statistics are very different from conventional case (stationary data)
  - OLS estimator does not converge in probability as $T \to \infty$
  - $t-$stats do not have well-defined asymptotic distributions
  - Estimated stdv strongly underestimates true stdv (b/c autocorrelation)
- Sign something is wrong:
  - Highly autocorrelated residuals

Implication

The spurious regression problem implies that when regressing non-stationary variables, the estimation results should not be taken too seriously!!!

- Take first-differences of $I(1)$ variables (GLS correction for autocorrelation)

An important exception arises when the non-stationary series have a common stochastic trend: cointegration.

- Don't take first-differences
  - specification error!
  - advantage of $I(1)$ variables (superconsistency)

Definition cointegration

The k variables of the $k \times 1$ vector $x_t = (x_{1t}, x_{2t}, \cdots, x_{kt})'$ are said to be cointegrated of order **one**, denoted as $x_1 \sim CI(1)$ if

1. All variables in x_t are integrated of the same order **one**, i.e. $x_{it} \sim I(1)$, for all i

2. There exists at least one vector $\beta = (\beta_1, \beta_2, \cdots, \beta_k)'$ of coefficients, called the **cointegrating vector**, such that the linear combination

$$x_t'\beta = (\beta_1 x_{1t} + \beta_2 x_{2t} + \cdots + \beta_k x_{kt})$$

is integrated of a order **zero**, i.e. $x_t \sim I(0)$

Example

In practice, $x_t \sim CI(1)$ is most common.
Consider for instance two variables, y_t and x_t , which are both $I(1)$.
If the residuals ϵ_t of the regression

$$y_t = \beta_1 + \beta_2 x_t + \epsilon_t$$

are $I(0)$, i.e. $\epsilon_t \sim I(0)$, then y_t and x_t are said to be cointegrated of order $CI(1)$ with cointegrating vector $\beta = (1, -\beta_1, -\beta_2)$ as

$$y_t - \beta_1 - \beta_2 x_t = \epsilon_t \sim I(0)$$

- eg. When (9monthRate − 3monthRate) is stationary, they are cointegrated with cointegrating vector $\beta = [1, -1]$.
- eg. When (logIndustrial − 0.98 logComposite) is stationary, they are cointegrated with cointegrating vector $\beta = [1, -0.98]$.

Cointegration & common trend

- Common trend

  eg. A model of interest rates (Fisher equation)

  - Short & long term interest rates (r_t^s, r_t^l) are directly influenced by the inflation (π_t) subject to stationary shocks $(\epsilon_t^s, \epsilon_t^l)$:

$$r_t^s = a^s + \pi_t + \epsilon_t^s,\ r_t^l = a^l + \pi_t + \epsilon_t^l$$

  - Both will be $I(1)$ when the π_t is $I(1)$.
    Here π_t acts as the common trend that represents the trend (non-stationary part) in both r_t^s and r_t^l.
  - (r_t^s, r_t^l) are cointegrated with $\beta = [1, -1]'$ because $r_t^s - r_t^l = a^s - a^l + \epsilon_t^s - \epsilon_t^l$ is $I(0)$.

Economic Interpretation

If two (or more) series are linked to form an **equilibrium** relation

$$y_t = \beta_1 + \beta_2 x_t$$

then even though the series themselves are non-stationary they will nevertheless move closely together over time, i.e. they have a **common trend**, such that deviations from the equilibrium

$$\epsilon_t = y_t - (\beta_1 + \beta_2 x_t)$$

are stationary.

- The concept of cointegration indicates the existence of a long-run equilibrium to which an economic system converges over time and ϵ_t can be interpreted as the equilibrium error, i.e. the distance the system is away from the equilibrium at time t. As equilibrium errors should be temporary, ϵ_t should be stationary.

Economic Interpretation

- The concept of spurious regression indicates that there is no long-run equilibrium relation between y_t and x_t as the error term ϵ_t is non-stationary, implying that deviations from the presumed relation between y_t and x_t are permanent such that this relation is not a long-run equilibrium relation.

Econometric implication

- If non-stationary variables are cointegrated, regression analysis imparts meaningful information about the long-run relationship between the variables.

  In fact, it can be shown that in this case, the OLS estimator $\hat{\beta}$ is even a super consistent estimator for β, i.e. $\hat{\beta}$ converges to β at a much faster rate than with conventional asymptotics (i.e. for stationary variables).

- If non-stationary variables are not cointegrated, regression results are not meaningful, i.e. spurious regression problem.

Cointegration and Error-Correction Mechanisms

The existence of a long-run equilibrium relationship also has its implications for the short-run behaviour of the $I(1)$ variables

- The **Granger representation theorem** states that if a set of variables is cointegrated, there has to be a mechanism that drives the variables back to their long-run equilibrium relationship after the equilibrium has been disturbed by a shock

- This mechanism is called an **error-correction model**

Example of an error-correction model

Consider two variables y_t and x_t which are cointegrated with cointegrating vector $\beta = (1, -\beta_1, -\beta_2)$.
A simple error-correction model (ECM) is given by

$$\begin{aligned} \Delta y_t &= \gamma_1 \Delta x_t - \alpha(y_{t-1} - \beta_1 - \beta_2 x_{t-1}) + \mu_t & (1) \\ &= \gamma_1 \Delta x_t - \alpha \epsilon_{t-1} + \mu_t & (2) \end{aligned}$$

The ECM incorporates both **short-run** and **long-run** effects

- The long-run equilibrium is obtained by imposing the 'no change' condition $\Delta y_t = \Delta x_t = \mu_t = 0$ and solve for y_t

  $$y_t = \beta_1 + \beta_2 x_t$$

  Thus, the long-run impact of x_t on y_t is given by β_2.
- The contemporaneous impact of x_t on y_t is given by γ_1.

Error correction mechanism

- The term $-\alpha\epsilon_{t-1}$ captures the error-correction mechanism. If y_t and x_t are cointegrated, the Granger representation theorem implies that $\alpha > 0$.
  - When y_t is below its equilibrium value implied by x_t, $\epsilon_t < 0$ such that y_t increases back the equilibrium
  - When y_t is above its equilibrium value implied by x_t , $\epsilon_t > 0$ such that y_t decreases back to the equilibrium

Note that α measures the **speed of adjustment** towards the equilibrium. The smaller α (i.e. the closer to zero), the lower this speed of adjustment.

- When y_t and x_t are cointegrated, ϵ_t is the deviation from their long-run equilibrium.
- y_{t+1} and x_{t+1} must move toward eliminating the deviation, or correcting the cointegation error ϵ_t.
- Hence, ϵ_t is useful for predicting Δy_{t+1} and Δx_{t+1} and the models for Δy_{t+1} and Δx_{t+1} should include ϵ_t as an explanatory variable.

Vector Error correction VEC

- Vector error correction (VEC) model:

$$\epsilon_{t-1} = y_{t-1} - \beta_0 - \beta_1 x_{t-1} \quad (3)$$

$$\Delta x_t = c_1 + \alpha_1 \epsilon_{t-1} + \phi_{11} \Delta x_{t-1} + \phi_{12} \Delta y_{t-1} + u_{1t} \quad (4)$$

$$\Delta y_t = c_2 + \alpha_2 \epsilon_{t-1} + \phi_{21} \Delta x_{t-1} + \phi_{22} \Delta y_{t-1} + u_{2t} \quad (5)$$

- Eg, when $\alpha_1 = 0$, the adjustment toward equilibrium is all done by y_t and the common trend is x_t.

  Call α_1 and α_2 adjustment coefficients.

  What happens when both α_1 and α_2 are zero?

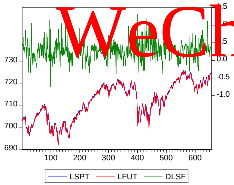
Price discovery in parallel markets

How information is incorporated into prices?

- Examples (usually require intraday price series)
  - Bi-listed stock: which market sets the price?
  - Spot & futures prices: does spot follows futures?
- For two log prices, y_t and x_t, on the same asset, the rule-of-one-price dictates that $\epsilon_t = y_t - x_t$ can only fluctuate around zero.
- Hence, y_t and x_t are cointegrated with $[1, -1]$ being the cointegrating vector. The error correction model is applicable.
- The relative magnitudes of α_1 and α_2 can tell us to what extent x_t acts as price setter, $s_x = \frac{|\alpha_1|}{|\alpha_1|+|\alpha_2|}$

Example: Price discovery in parallel markets

- eg. SP500 spot & futures indices: VEC (20100104-20120810, 656 obs.)
  - The adjustment coefficients:
    - α_{futures} is insignificant (t-stat = 0.46).
    - α_{spot} is significant (t-stat = -2.34).
  - Futures appears to be the price-setter.



Correlogram of DLSF

Sample: 1 656
Included observations: 656

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.20[illegible]	[illegible]	[illegible]	[illegible]
		2	0.173	0.156	46.261	0.000
		3	0.160	0.109	63.301	0.000
		4	0.114	0.049	71.838	0.000
		5	0.096	0.036	77.979	0.000
		6	0.121	0.069	87.753	0.000
		7	0.096	0.037	93.928	0.000
		8	0.075	0.015	97.635	0.000
		9	0.056	0.000	99.733	0.000
		10	0.046	0.000	101.12	0.000
		11	0.029	-0.008	101.70	0.000
		12	0.048	0.019	103.23	0.000
		13	0.070	0.043	106.56	0.000
		14	0.012	-0.027	106.65	0.000
		15	0.021	-0.005	106.95	0.000
		16	-0.051	-0.078	108.73	0.000

Vector Error Correction Estimates

Vector Error Correction Estimates

Sample (adjusted): 3 656
Included observations: 654 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
LSPT(-1)	1.000000	
LFUT(-1)	-1.000254 (0.00170) [-588.083]	
C	-0.097174	
Error Correction:	**D(LSPT)**	**D(LFUT)**
CointEq1	-0.562673 ([illegible]24091) [-2.[illegible]3557]	0.115397 (0.24832) [0.46472]
D(LSPT(-1))	0.333674 (0.19304) [1.72852]	0.450278 (0.19897) [2.26303]
D(LFUT(-1))	-0.421013 (0.19006) [-2.21517]	-0.561507 (0.19590) [-2.86633]
C	0.035489 (0.04754) [0.74656]	0.036533 (0.04900) [0.74561]

Example: US and Canadian 10-years bond yeilds

- Error correction & cointegration

  eg. US and Canadian 10-year bond yields

  Error correction model:

  dca = ca – ca(-1), dus = us – us(-1),

  $e = ca - b_0 - b_1 \cdot us$

Correction is done by CA, not US.

US acts as the common trend.

Dependent Variable: DUS
Method: Least Squares

Sample (adjusted): 3 328
Included observations: 326 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.005204	0.007874	-0.660939	0.5091
E(-1)	0.001231	0.022301	0.055186	0.9560
DUS(-1)	-0.152839	0.083884	-1.822014	0.0694
DCA(-1)	0.008363	0.063225	0.132278	0.8948

R-squared	0.020776	Mean dependent var	-0.004466
Adjusted R-squared	0.011653	S.D. dependent var	0.142895
S.E. of regression	0.142060	Akaike info criterion	-1.052947
Sum squared resid	6.498254	Schwarz criterion	-1.006482
Log likelihood	175.6303	F-statistic	2.277282
Durbin-Watson stat	1.982983	Prob(F-statistic)	0.079572

Dependent Variable: DCA
Method: Least Squares

Sample (adjusted): 3 328
Included observations: 326 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.007500	0.010229	-0.733158	0.4640
E(-1)	-0.068559	0.028973	-2.366334	0.0186
DUS(-1)	-0.279985	0.108980	-2.569134	0.0106
DCA(-1)	0.042490	0.082140	0.517294	0.6053

R-squared	0.045295	Mean dependent var	-0.006380
Adjusted R-squared	0.036400	S.D. dependent var	0.188013
S.E. of regression	0.184560	Akaike info criterion	-0.529493
Sum squared resid	10.96805	Schwarz criterion	-0.483028
Log likelihood	90.30744	F-statistic	5.092300
Durbin-Watson stat	2.028832	Prob(F-statistic)	0.001861

Properties of OLS : Super consistency

Consider two time series y_t and x_t which are both $I(1)$. Estimating the static equation

$$y_t = \beta_1 + \beta_2 x_t + \epsilon_t$$

using OLS yields **super consistent** estimates of the long-run parameters β_1 and β_2 when ϵ_t is $I(0)$.

- Super consistency means that the OLS estimator converges to the true population parameters at a much faster rate than with stationary variables
- This result arises as OLS picks the coefficients $\hat{\beta}$ such that the variance of the estimated residuals $\hat{\epsilon}_t$ is as small as possible. As setting $\hat{\beta} \neq \beta$ implies that $\epsilon_t \sim I(1)$ such that its variance becomes infinitely large when $T \to \infty$, OLS is very efficient in picking the correct β
- The super consistency property of the OLS estimator implies that in estimating the long-run relation between cointegrated variables, dynamics and endogeneity issues can be ignored asymptotically

Properties of OLS : Super consistency

- If two I(1) series y_t and x_t are cointegrated, they may be fitted in the linear regression

  $y_t = \beta_0 + \beta_1 x_t + \varepsilon_t$, ε_t being stationary

  where $[1 \; -\beta_1]$ is the cointegrating vector.

- As long as ε_t is stationary, the OLS estimator of β_1 is consistent, but generally has a non-standard asymptotic distribution.

- To make valid inference about β_1, the "dynamic" OLS estimator of β_1 from

  $y_t = \beta_0 + \beta_1 x_t + \sum_{j=-q}^{q} \psi_j \Delta x_{t-j} + \varepsilon_t$.

Properties of OLS : Super consistency

- The addition of leads and lags removes the deleterious effects that short-run dynamics of the equilibrium process ϵ_t have on the estimate of the cointegrating vector.

- The DOLS estimator is consistent, asymptotically normally distributed, and efficient.
- Asymptotically valid standard errors for the individual elements of the estimated cointegration vector are given by their corresponding HAC (e.g., Newey-West) standard errors.

Testing for cointegration

Consider two time series y_t and x_t.
Suppose we want to estimate the following equation:

$$y_t = \beta_1 + \beta_2 x_t + \epsilon_t$$

Prior to estimation, test the variables for their order of integration

1. If both are $I(0)$: standard regression analysis is valid
2. If they are integrated of a different order, e.g. y_t is $I(1)$ and x_t is $I(0)$: there can be no (long-run) relation between these two variables
3. If both are $I(1)$: use cointegration analysis

Note however that there is almost never certainty about the true order of integration

The Engle-Granger two-step approach

A popular methodology to test for cointegration and to analyse cointegrating relationships is the so-called Engle-Granger two-step approach:

1. Estimate the static model and test for cointegration
2. Estimate an ECM to analyse the short-run dynamics

The Engle-Granger two-step approach

Step 1: estimate static model and test for cointegration

Estimate the model in levels using OLS. Two cases can be distinguished

1. The regression results are spurious if $\varepsilon_t \sim I(1)$
2. OLS is **super consistent** if $\varepsilon_t \sim I(0)$

After estimating a model including non-stationary variables, it is therefore very important to test the order of integration of the estimated residuals $\hat{\varepsilon}_t$. We consider two alternative tests:

1. The cointegrating regression Durbin-Watson (CRDW) test
2. ADF cointegration test

The Engle-Granger two-step approach

1: **Cointegrating Regression Durbin-Watson** (CRDW) test

Tests whether the residuals $\hat{\varepsilon}_t$ are generated by a unit root process:

$$\hat{\varepsilon}_t = \hat{\varepsilon}_{t-1} + v_t$$

against the alternative that $\hat{\varepsilon}_t$ is generated by a stationary AR(1) process:

$$\hat{\varepsilon}_t = \rho\hat{\varepsilon}_t + v_t \quad \text{with} \quad |\rho| < 1$$

using the Durbin-Watson (DW) statistic.

As $DW \approx 2(1 - \hat{\rho})$ this test boils down to testing whether DW is significantly larger than zero.

The Engle-Granger two-step approach

- Formally

$$H_0: \hat{\varepsilon}_t \sim I(1) \text{ corresponds to } \rho = 1 \text{ or } d = 0$$
$$H_1: \hat{\varepsilon}_t \sim I(0) \text{ corresponds to } \rho < 1 \text{ or } d > 0$$

- The 5% critical values for the CRDW test are given by

Number of variables (incl. y_t)	Number of observations 50	100	250
2	0.72	0.38	0.20
3	0.89	0.48	0.25
4	1.05	0.58	0.30
5	1.19	0.68	0.35

- Drawback: the CRDW test is only valid when ε_t follows an AR(1) process as the DW statistic only checks for an AR(1) pattern in the data.

The Engle-Granger two-step approach

2. **ADF cointegration test**

Tests for a unit root in the estimated residuals using the standard DF specification

$$\Delta\hat{\varepsilon}_t = \gamma\hat{\varepsilon}_{t-1} + \sum_{i=1}^{p-1} \alpha_i \Delta\hat{\varepsilon}_{t-i} + \omega_t$$

$$\text{with} \quad H_0 : \ \gamma = 0 \quad \rightarrow \text{no cointegration}$$
$$H_1 : \ \gamma < 0 \quad \rightarrow \text{cointegration}$$

Important notes:

- Deterministic components (i.e. intercept and trend) can be included either in the cointegrating regression or in the ADF test (but not in both!)
- The standard DF critical values are not valid! Reason: the OLS estimator 'picks' β such that the residuals $\hat{\varepsilon}_t$ have the lowest possible variance, i.e. making the residuals appear as stationary as possible even if there is no cointegration (i.e. ε_t is non-stationary).

The Engle-Granger two-step approach

Step 2: Estimate an ECM to analyse the short-run dynamics

Upon finding cointegration, estimate an ECM

$$A(L)\,\Delta y_t = \delta + B(L)\,\Delta x_t + \alpha \hat{\varepsilon}_{t-1} + C(L)\,\mu_t$$

where $\hat{\varepsilon}_{t-1} = y_{t-1} - \hat{\beta}_1 - \hat{\beta}_2 x_{t-1}$.

Since all variables are $I(0)$, this can be done using OLS and statistical inference using standard t- and F-tests is possible.

Example: consumption, income and wealth in the US

- All data in natural logs, sample 1951:Q4-2005:Q4.
- ADF tests show that all series are I(1)
  - Unit root in first differences is rejected
  - Unit root in levels is not rejected
- The null hypothesis of no cointegration can be rejected at the 5% level of significance
  - The CRDW equals 0.31, which is just above the 5% critical value of ≈ 0.30.
  - The ADF test on the residuals of the static regression equals -4.19, which is below the 5% critical value -3.78 $\left(= -3.7429 - 8.352/217 - 13.41/217^2\right)$.
- The error-correction term is significant and shows that consumption is only slowly converting to the long-run equilibrium implied by income and wealth, i.e. every quarter 5.7% of the equilibrium gap is closed.

Example: consumption, income and wealth in the US

Perform unit root tests on levels and first differences

Null Hypothesis: D(CONS) has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic based on SIC, MAXLAG=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-[illegible]0.4[illegible]496[illegible]	0.[illegible]000
Test critical values:	1% level	[illegible]	
	5% level	-2.874804	
	10% level	-2.573917	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: CONS has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic based on SIC, MAXLAG=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.269028	0.6440
Test critical values:	1% level	-3.460739	
	5% level	-2.874804	
	10% level	-2.573917	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(INC) has a unit root
Exogenous: Constant
Lag Length: [illegible] (Automatic based on SIC, MAXLAG=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-14.02696	0.0000
Test critical values:	1% level	-3.460739	
	5% level	-2.874804	
	10% level	-2.573917	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: INC has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic based on SIC, MAXLAG=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.291086	0.6339
Test critical values:	1% level	-3.460596	
	5% level	-2.874741	
	10% level	-2.573883	

*MacKinnon (1996) one-sided p-values.

Example: consumption, income and wealth in the US

Perform Static Regression

Dependent Variable: CONS
Method: Least Squares
Date: 11/08/07 Time: 12:39
Sample: 1951Q4 2005Q4
Included observations: 217

	Coefficient	Std. Error	t-Statistic	Prob.
C	1.246159	0.025660	48.56510	0.0000
INC	0.663406	0.009252	71.70442	0.0000
WEALTH	0.186052	0.008974	20.73252	0.0000

R-squared	0.998660	Mean dependent var	9.587971
Adjusted R-squared	0.998648	S.D. dependent var	0.323944
S.E. of regression	0.011913	Akaike info criterion	-6.008628
Sum squared resid	0.030371	Schwarz criterion	-5.961901
Log likelihood	654.9361	Hannan-Quinn criter.	-5.989752
F-statistic	79750.23	Durbin-Watson stat	0.309370
Prob(F-statistic)	0.000000		

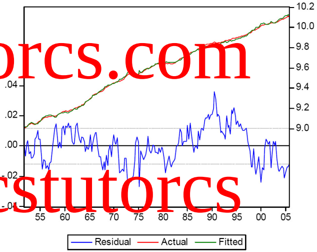
Example: consumption, income and wealth in the US

Perform unit root test on residuals

Null Hypothesis: RES has a unit root
Exogenous: None
Lag Length: 0 (Automatic based on SIC, MAXLAG=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.19[illegible]	0.0000
Test critical values:	1% level	-2[illegible]	
	5% level	-1.94[illegible]	
	10% level	-1.615721	

*MacKinnon (1996) one-sided p-values.



Example: consumption, income and wealth in the US

Estimate the Error Correction Model

Dependent Variable: D(CONS)
Method: Least Squares
Date: 11/08/07 Time: 12:[illegible]4
Sample (adjusted): 1952Q2 2005Q4
Included observations: 215 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.002325	0.000396	5.872706	0.0000
RES(-1)	-0.057[illegible]30	0.022322	-2.563845	0.0111
D(INC)	0.0[illegible]652	0.012665	3.762[illegible]31	0.000[illegible]
D(WEALTH)	0.197[illegible]17	0.0295[illegible]	6.668301	0.0000
D(CONS(-1))	0.115680	0.063686	1.[illegible]16404	0.0707
D(INC(-1))	0.022498	0.013051	1.723821	0.0862
D(WEALTH(-1))	0.098595	0.032804	3.005590	0.0030