

Assignment Project Exam Help

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ECON3206/5206: Financial Econometrics

Week-1: A Review of Random Variables and Distributions

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Random Variables: Definitions

- RV and probability distribution

– Two types of RV

- Discrete RV: it takes a finite or countable number of values.
 - Continuous RV: it may take any value in a interval.

– Probability distribution (how likely the values occur)

- Discrete RV

– Probability mass (pmf):

| | | | |
|-------|-------|-------|-----|
| x_1 | x_2 | x_3 | ... |
| p_1 | p_2 | p_3 | ... |

– Cumulative distribution (cdf): $F(x) = P(X \leq x) = \sum_{x_i \leq x} p_i$

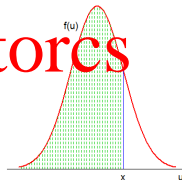
- Continuous RV

– Probability density (pdf): $f(u)$

– Cumulative distribution (cdf):

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(u) du$$

Area under $f(u)$ curve up to x



Random Variables: Unconditional (marginal) Expectations (moments)

- Characterise RVs

- Mean or expected value of X

- The weighted average of all possible values, a measure of location

- Discrete RV: $E(X) = \sum_i x_i P(X = x_i)$

- Continuous RV: $E(X) = \int_{-\infty}^{\infty} u f(u) du$

- Mean of $g(X)$ (eg. X^2, e^X, \dots)

- Discrete RV: $E[g(X)] = \sum_i g(x_i) P(X = x_i)$

- Continuous RV: $E[g(X)] = \int_{-\infty}^{\infty} g(u) f(u) du$

- Variance of X :

- A measure of the amount of variation in all possible values

- $\text{var}(X) = E\{[X - E(X)]^2\} = E(X^2) - [E(X)]^2$

- Covariance between X and Y

- A measure of association

- $\text{Cov}(X, Y) = E\{[X - E(X)][Y - E(Y)]\}$

Random Variables: Conditioning

- Conditional distribution of Y given X
 - The distribution of Y when X is “known”.
 - It depends on the value of X .
 - It is denoted as $Y|X$.

eg. the distribution of wage for age = 20,
the distribution of wage for age = 30, ...
 $wage|age$ depends on age.

$bhp_t|bhp_{t-1}$:
is. of bhp_t
when bhp_{t-1} is
“fixed”

- Conditional expectation $E[g(Y)|X]$
 - It is calculated with the conditional distribution, treating X as “known” or “fixed”.
 - It depends on X and, hence, is also a RV.

Random Variables

– Conditional mean of Y given X

- $E(Y|X)$ generally depends on X .

eg. Linear regression $Y = a + bX + \varepsilon$.

$$E(Y|X) = a + bX + E(\varepsilon|X) = a + bX.$$

– Conditional variance of Y give X

- $\text{Var}(Y|X)$ generally depends on X .

eg. Linear regression $Y = a + bX + \varepsilon$.

$$\text{Var}(Y|X) = \text{Var}(\varepsilon|X).$$

Properties of Expectation operator

Let X and Y be RVs and a be a constant.

– Expectation is a linear operator

$$E(a) = a, \quad \text{Var}(a) = 0;$$

$$E(aX) = aE(X);$$

$$E(X + Y) = E(X) + E(Y);$$

$$E[g(X)|X] = g(X) \text{ for any function } g(\cdot);$$

$$E(Y) = E[E(Y|X)] \text{ (iterated expectations);}$$

$$E(Y|X) = E(Y) \text{ if } Y \text{ is independent of } X;$$

– Variance is a nonlinear operator

$$\text{Var}(aY) = a^2 \text{Var}(Y);$$

$$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X, Y);$$

$$\text{Var}(Y) = E[\text{Var}(Y|X)] + \text{Var}[E(Y|X)].$$

$$E\left(\frac{X}{Y}\right) \neq \frac{E(X)}{E(Y)}$$

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Sample moments

– Suppose we have (time series) observations

$$\{X_t\}_{t=1}^T = \{X_1, X_2, \dots, X_T\} \quad \text{and} \quad \{Y_t\}_{t=1}^T = \{Y_1, Y_2, \dots, Y_T\}$$

– Sample mean

- $\bar{X} = \frac{1}{T} \sum_{t=1}^T X_t$

- a measure of location (central tendency)

- an estimator of population mean $E(X)$

– Sample variance and standard deviation

- $s_X^2 = \frac{1}{T-1} \sum_{t=1}^T (X_t - \bar{X})^2$ $\hat{\sigma}_X = \sqrt{s_X^2}$

- a measure of variation

- an estimator of the population variance $\text{Var}(X)$

Behold the summation operator

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Linear operator:

$$\sum_{t=1}^T (a + bx_t + cy_t) = \sum_{t=1}^T a + b \sum_{t=1}^T x_t + c \sum_{t=1}^T y_t$$

Summation of a constant:

$$\sum_{t=1}^T a = Ta$$

Nonlinear relations:

$$\sum_{t=1}^T x_t y_t \neq \sum_{t=1}^T x_t \sum_{t=1}^T y_t$$

$$\sum_{t=1}^T \frac{x_t}{y_t} \neq \frac{\sum_{t=1}^T x_t}{\sum_{t=1}^T y_t}$$