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Financial Econometrics

ECON3206/5206

Tutorial 8

Sample Answers/Hints to Tutorial 11

1. (Miscellaneous questions)

(a) The conditional variance matrix of a vector of returns is useful for designing “mean-variance efficient” portfolios. A mean-variance efficient portfolio on a given set of assets is one that has the minimum variance for a desired mean return. Consider one-day ahead problem with n assets. Let $r = [r_1, \dots, r_n]'$ be the vector of the 1-day returns for the assets. The mean of r , $\mu = E(r) = [\mu_1, \dots, \mu_n]'$, is also an n dimensional vector. The variance of r ,

$$V = \text{Var}(r) = \begin{bmatrix} V_{11} & \cdots & V_{1n} \\ \vdots & \ddots & \vdots \\ V_{n1} & \cdots & V_{nn} \end{bmatrix} = \begin{bmatrix} \text{Var}(r_1) & \cdots & \text{Cov}(r_1, r_n) \\ \vdots & \ddots & \vdots \\ \text{Cov}(r_n, r_1) & \cdots & \text{Var}(r_n) \end{bmatrix},$$

is an $n \times n$ dimensional matrix. Because the covariances are symmetric $\text{Cov}(r_i, r_j) =$

$\text{Cov}(r_j, r_i)$, the matrix V is symmetric with $V_{ij} = V_{ji}$ for all i and j . Suppose that you invest a portion of your wealth, w_i , in asset i , for $i = 1, \dots, n$, where $\sum_{i=1}^n w_i = 1$. Then your portfolio

is determined by the weight vector $w = [w_1, \dots, w_n]'$ and your portfolio return is given by

$r_p = \sum_{i=1}^n w_i r_i = w'r$ with the mean $\mu_p = E(r_p) = \sum_{i=1}^n w_i \mu_i = w'\mu$ and variance $\sigma_p^2 =$

$\text{Var}(r_p) = w'Vw$. To obtain the mean-variance efficient portfolio, you choose w to minimise

the variance $w'Vw$ for a given mean return μ_p . Because the weights in w can be any set of

numbers with $\sum_{i=1}^n w_i = 1$ and the portfolio return variance σ_p^2 must be non-negative, a

variance matrix V must be such that $w'Vw \geq 0$ for any w with $\sum_{i=1}^n w_i = 1$ (ie, V must be

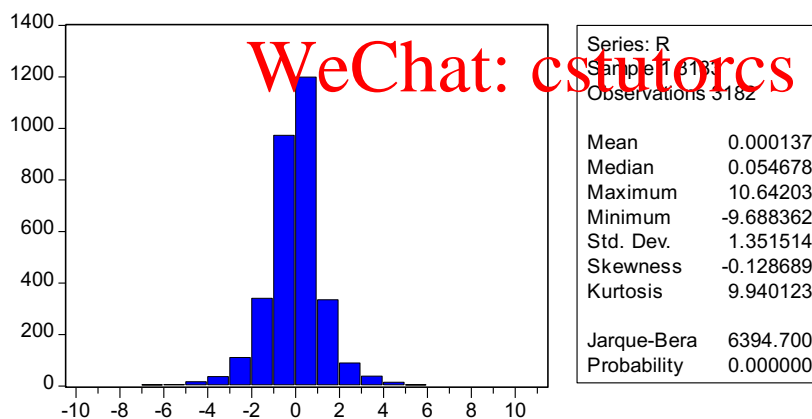
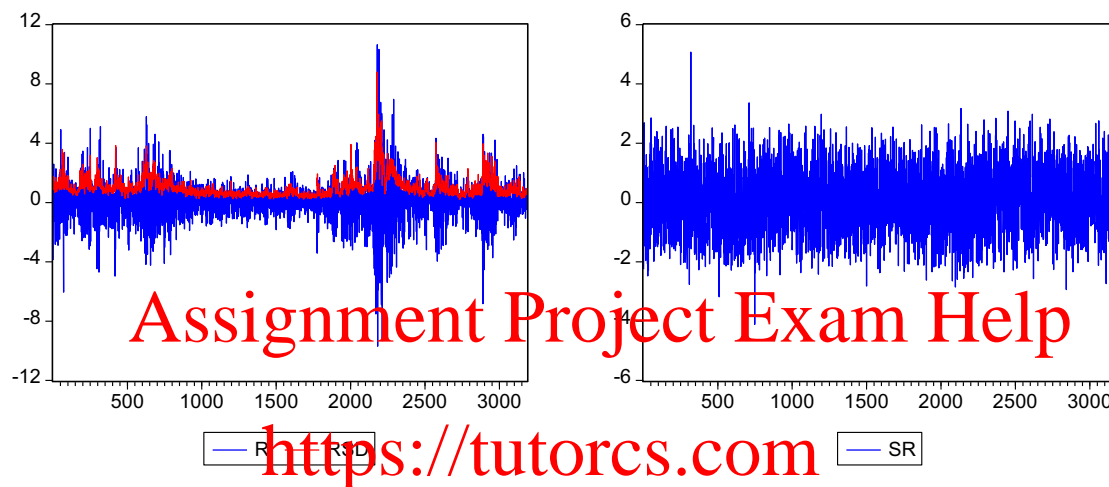
semi-positive definite). Hence a variance matrix V is required to be symmetric and semi-positive definite.

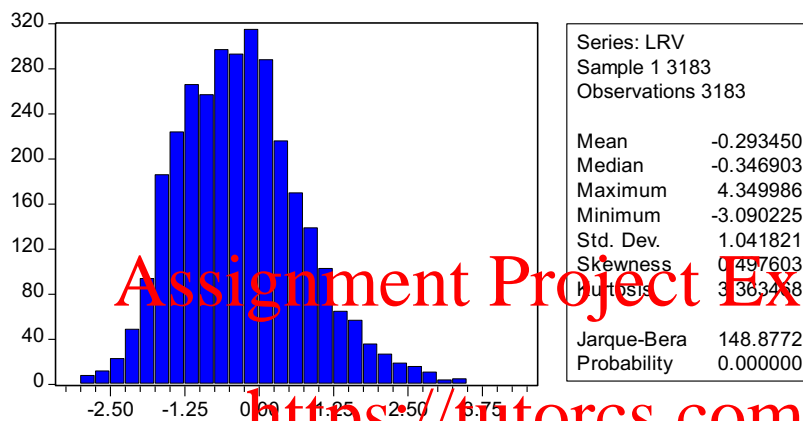
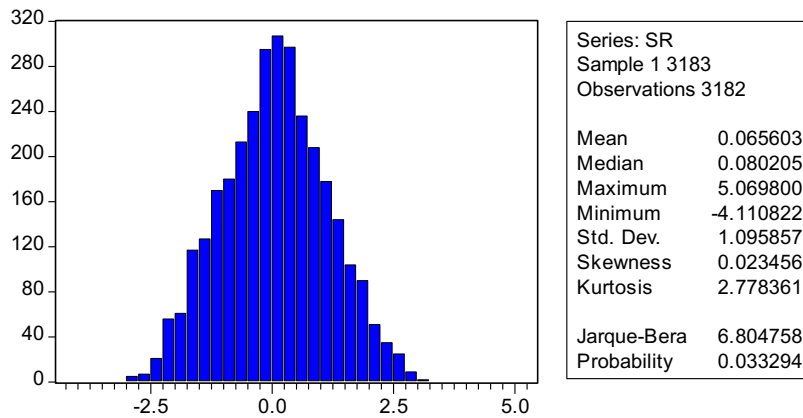
(b) The daily realised variance (RV) of an asset return is constructed from the intraday returns, eg, intraday 5-minute returns. Originally, the RV is computed as the sum of the squared intraday returns. However, there are alternative (and better) ways to compute the RV. The daily RV is an estimate of the integrated variance, which can be regarded as the spot or instantaneous variance of the return.

2. (Realised volatility, data source: <http://realized.oxford-man.ox.ac.uk/data>)

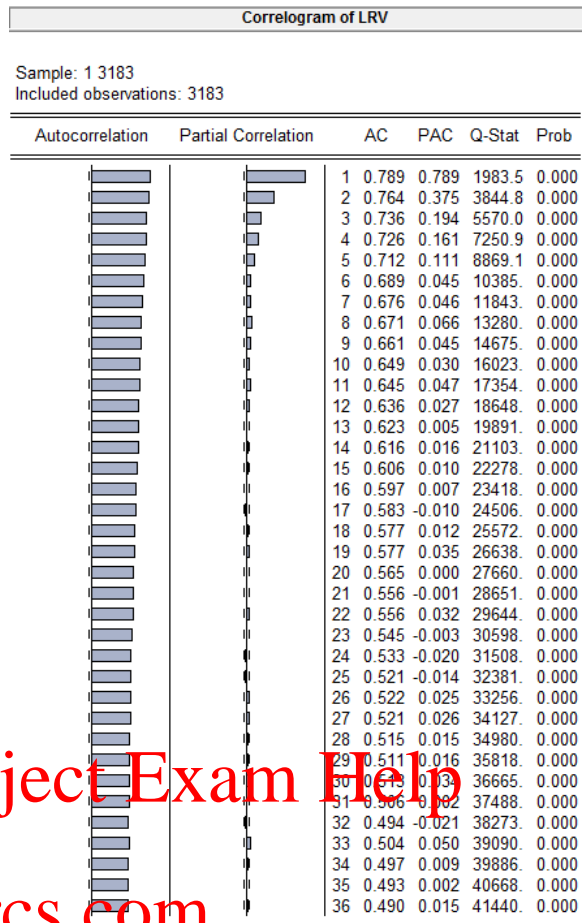
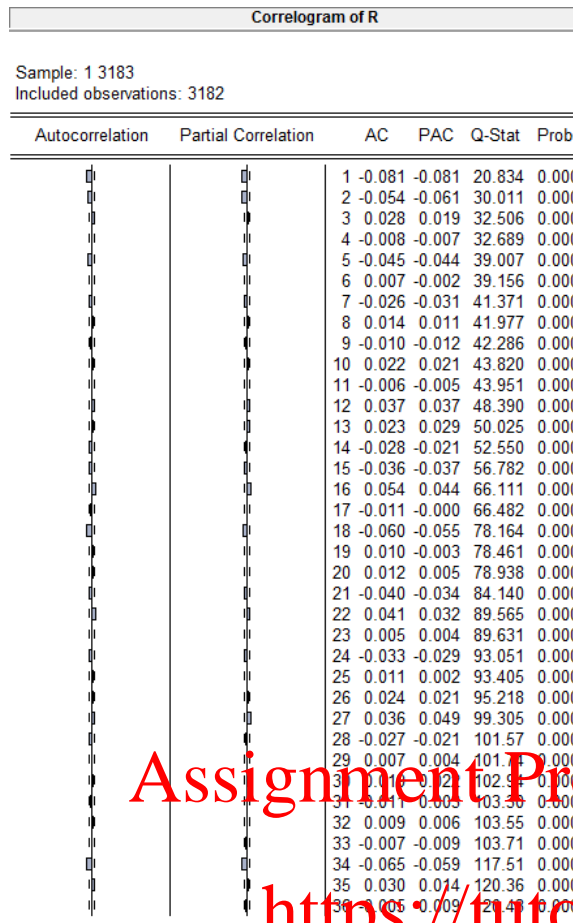
(a) The time series plots show that the variations in R are accurately mirrored in the levels of RSD . The difference between the plots of R and SR provides a sharp contrast: the

clustering in R is mostly attributable to RSD. The histogram and descriptive statistics of R show the usual return characteristics: close-to-zero mean, large standard deviation, negative skewness, large kurtosis, and decisively non-normal. However, the histogram and descriptive statistics of SR are much close to those of a standard normal random variable, although the null hypothesis of normality is still rejected at the 5% level (p-value 0.033). Notice that SR is standardised by using the spot or instantaneous realised variance, which is NOT the conditional variance. The histogram of LRV indicates that it is positively skewed with a kurtosis greater than 3, roughly bell-shaped but non-normal.





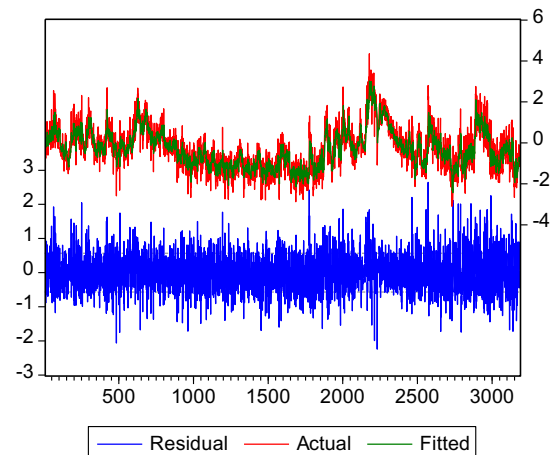
- (b) There are statistically significant (with small magnitudes) autocorrelations in R according to correlograms below. The autocorrelations in LRV are large and long-lasting. In particular, the autocorrelations in LRV do not appear to converge to zero quickly as the lag increases. That is, the decay of the autocorrelations does not appear to be exponential.



(c) The augmented Dickey-Fuller test on LRV is the null hypothesis of a unit-root. Hence, the evidence suggests that LRV is still stationary despite its large and long-lasting autocorrelations. To fit an ARMA model to LRV, the partial autocorrelations suggest that an AR(11) would be a candidate (2 standard error band $2/\sqrt{T} \approx 0.035$, any AC or PAC within the bands are statistically zero). Here, maybe incorrectly, we assume the autocorrelations exponentially decay to zero. The estimation results show that most of the AR coefficients are statistically significant (except lags 6,7,9 and 10). More than 70% of the variations in LRV are explained by the AR(11) model. The actual-fitted-residual plot demonstrates that the model fits the data well. The correlogram of the residuals shows little autocorrelation. Hence the AR(11) model has done a good job in capturing the autocorrelations in LRV, although the true data-generating-process could be a long-memory ARFIMA model. From the residual histogram below, the residual distribution is not normal with positive skewness and heavy tails, although it is roughly bell-shaped. The main point of this part is that the long-lasting autocorrelations of LRV can be approximately captured by an AR(p) with a moderately large p (here we have $p = 11$).

Augmented Dickey-Fuller Unit Root Test on LRV		
Null Hypothesis: LRV has a unit root		
Exogenous: Constant		
Lag Length: 7 (Automatic based on SIC, MAXLAG=28)		
	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-6.074429	0.0000
Test critical values: 1% level	-3.432221	
5% level	-2.862252	
10% level	-2.567193	

*MacKinnon (1996) one-sided p-values.



Correlogram of Residuals

Sample: 12 3183
Included observations: 3172
Q-statistic probabilities adjusted for 11 ARMA term(s)

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
1	-0.002	-0.002	0.0083		
2	-0.001	-0.001	0.0108		
3	-0.002	-0.002	0.0284		
4	-0.003	-0.003	0.0521		
5	-0.004	-0.004	0.1025		
6	-0.003	-0.003	0.1271		
7	-0.007	-0.007	0.2665		
8	-0.008	-0.008	0.4907		
9	-0.008	-0.008	0.7091		
10	-0.015	-0.015	1.4392		
11	-0.026	-0.026	3.6017		
12	0.007	0.006	3.7442	0.053	
13	-0.012	-0.012	4.1815	0.124	
14	0.002	0.001	4.1898	0.242	
15	0.002	0.001	4.1998	0.380	
16	0.002	0.001	4.2101	0.520	
17	-0.028	-0.028	6.6871	0.351	
18	-0.015	-0.016	7.3904	0.389	
19	0.022	0.021	8.9368	0.348	
20	-0.001	-0.002	8.9405	0.443	
21	-0.009	-0.010	9.2105	0.512	
22	0.028	0.028	11.804	0.379	
23	0.007	0.007	11.977	0.448	
24	-0.017	-0.018	12.906	0.455	
25	-0.041	-0.041	18.199	0.198	
26	-0.011	-0.012	18.603	0.232	
27	0.002	0.001	18.614	0.289	
28	-0.006	-0.008	18.741	0.344	
29	-0.009	-0.009	19.011	0.391	
30	0.027	0.028	21.314	0.320	
31	0.005	0.004	21.384	0.375	
32	-0.037	-0.038	25.886	0.211	
33	0.031	0.031	28.908	0.147	
34	0.014	0.012	29.566	0.162	
35	0.003	0.000	29.603	0.198	
36	0.015	0.014	30.335	0.212	

Dependent Variable: LRV
Method: Least Squares

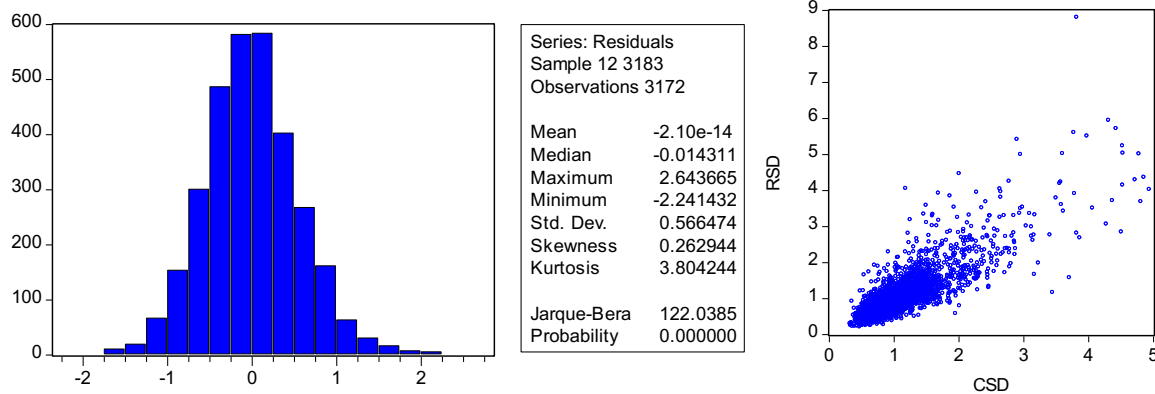
Sample (adjusted): 12 3183
Included observations: 3172 after adjustments
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.312751	0.171904	-1.819333	0.0690
AR(1)	0.352555	0.017757	19.84354	0.0000
AR(2)	0.20517	0.118839	1.72671	0.0890
AR(3)	0.082500	0.019186	4.300025	0.0000
AR(4)	0.092932	0.019229	4.833018	0.0000
AR(5)	0.071244	0.019304	3.690581	0.0002
AR(6)	0.003165	0.019346	0.163608	0.8700
AR(7)	0.005566	0.019303	0.288337	0.7731
AR(8)	0.040333	0.019233	2.097080	0.0361
AR(9)	0.024005	0.019185	1.251233	0.2109
AR(10)	0.014099	0.018835	0.748551	0.4542
AR(11)	0.049803	0.017761	2.804105	0.0051
R-squared	0.704864	Mean dependent var	-0.295525	
Adjusted R-squared	0.703837	S.D. dependent var	1.042723	
S.E. of regression	0.567459	Akaike info criterion	1.708479	
Sum squared resid	1017.551	Schwarz criterion	1.731413	
Log likelihood	-2697.648	F-statistic	686.0859	
Durbin-Watson stat	2.002453	Prob(F-statistic)	0.000000	
Inverted AR Roots	.98	.66-.39i	.66+.39i	.29+.67i
	.29-.67i	-.06+.73i	-.06-.73i	-.50+.56i
	-.50-.56i	-.71+.20i	-.71-.20i	

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(d) The scatter plot above shows the connection and difference between the conditional volatility estimates (CSD) and the spot or instantaneous volatility estimates (RSD). The CSD can be regarded as a point forecast of RSD.

(e) The estimation results of EGARCH(2,1) below shows that the model has adequately represented the clustering in the return R_t , as the ARCH test does not reject the null hypothesis of no ARCH effect in the standardised residuals. The time series and scatter plots show the similarities and differences between the conditional standard deviation from EGARCH and the conditional standard deviation based on LRV. While EGSD and CSD differ markedly, they have extremely strong cross-correlations (see the cross-correlogram below). For example, the static correlation is almost perfect (0.9285).

Dependent Variable: R
Method: ML - ARCH (Marquardt) - Normal distribution

Sample (adjusted): 3 3183
Included observations: 3181 after adjustments
Convergence achieved after 16 iterations
Bollerslev-Wooldridge robust standard errors & covariance
Variance backcast: ON
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) +
C(5)*ABS(RESID(-2)/@SQRT(GARCH(-2))) + C(6)*RESID(-1)
/@SQRT(GARCH(-1)) + C(7)*LOG(GARCH(-1))

	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000747	0.015103	-0.049470	0.9605
AR(1)	-0.051799	0.016694	-3.102858	0.0019

Variance Equation				
C(3)	-0.100653	0.014989	-6.715242	0.0000
C(4)	-0.106205	0.070985	-1.496157	0.1346
C(5)	0.239996	0.067293	3.566442	0.0004
C(6)	-0.136642	0.013501	-10.12120	0.0000
C(7)	0.976230	0.004232	230.7022	0.0000

R-squared	0.005709	Mean dependent var	0.001357
Adjusted R-squared	0.003829	S.D. dependent var	1.349975
S.E. of regression	1.347388	Akaike info criterion	2.923228
Sum squared resid	5762.255	Schwarz criterion	2.936574
Log likelihood	-4642.394	F-statistic	3.037228
Durbin-Watson stat	2.064707	Prob(F-statistic)	0.005791

Inverted AR Roots - .05

ARCH Test:

F-statistic	0.571744	Probability	0.853196
Obs*R-squared	6.300539	Probability	0.852578

Test Equation:
Dependent Variable: STD_RESID^2
Method: Least Squares

Sample (adjusted): 14 3183
Included observations: 3170 after adjustments
White Heteroskedasticity-Consistent Standard Errors & Covariance

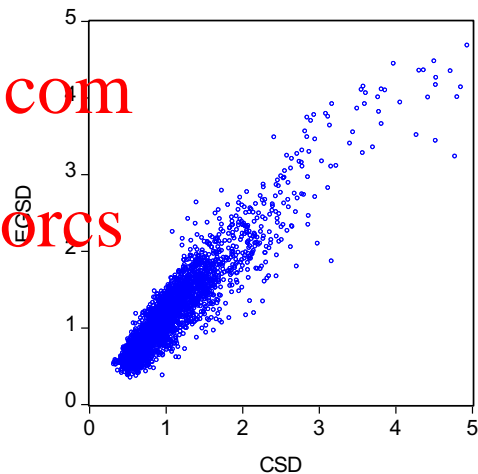
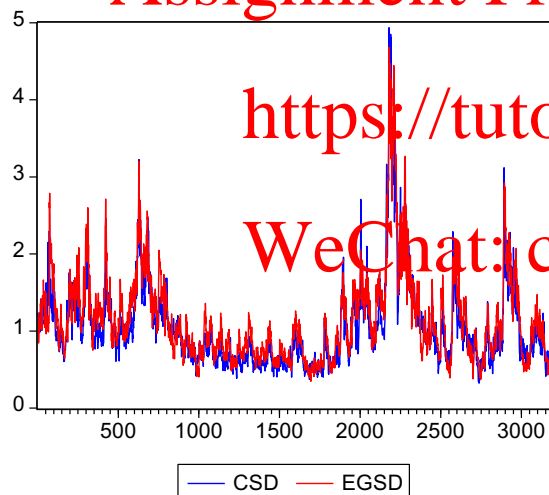
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.943109	0.064534	14.61414	0.0000
STD_RESID^2(-1)	0.005062	0.035266	0.143540	0.8859
STD_RESID^2(-2)	0.008608	0.017323	0.496885	0.6193
STD_RESID^2(-3)	-0.023500	0.013776	-1.705877	0.0881
STD_RESID^2(-4)	-0.003841	0.014189	-0.270678	0.7867
STD_RESID^2(-5)	0.012061	0.015728	0.766866	0.4432
STD_RESID^2(-6)	-0.006952	0.013672	-0.508474	0.6112
STD_RESID^2(-7)	0.023840	0.022081	1.079664	0.2804
STD_RESID^2(-8)	0.002116	0.014556	0.145408	0.8844
STD_RESID^2(-9)	0.009801	0.017094	0.573384	0.5664
STD_RESID^2(-10)	0.018366	0.017243	1.065147	0.2869
STD_RESID^2(-11)	0.012033	0.014944	0.805208	0.4208

R-squared	0.001988	Mean dependent var	1.000805
Adjusted R-squared	-0.001489	S.D. dependent var	1.794991
S.E. of regression	1.796327	Akaike info criterion	4.013143
Sum squared resid	10190.20	Schwarz criterion	4.036089
Log likelihood	-6348.832	F-statistic	0.571744
Durbin-Watson stat	2.000136	Prob(F-statistic)	0.853196

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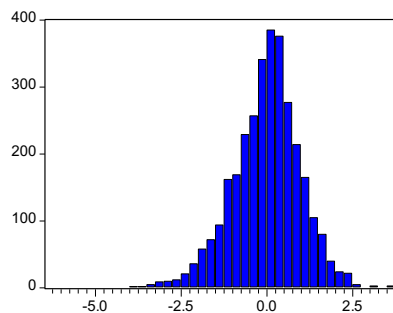
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Cross Correlogram of CSD and EGSD

Sample: 1 3183
Included observations: 3172
Correlations are asymptotically consistent approximations

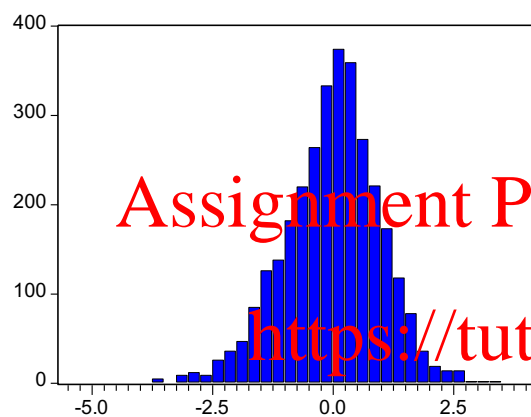
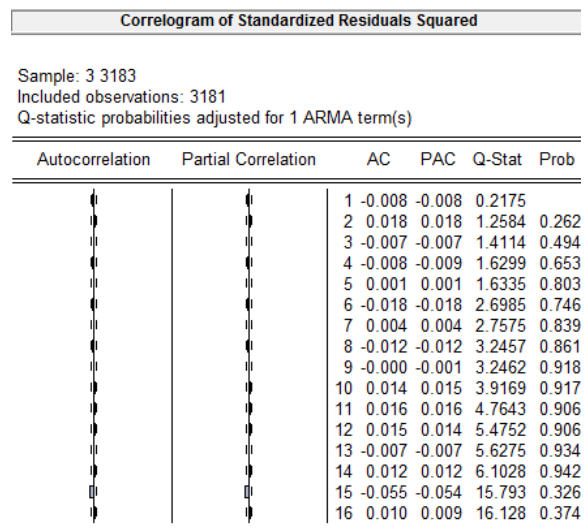
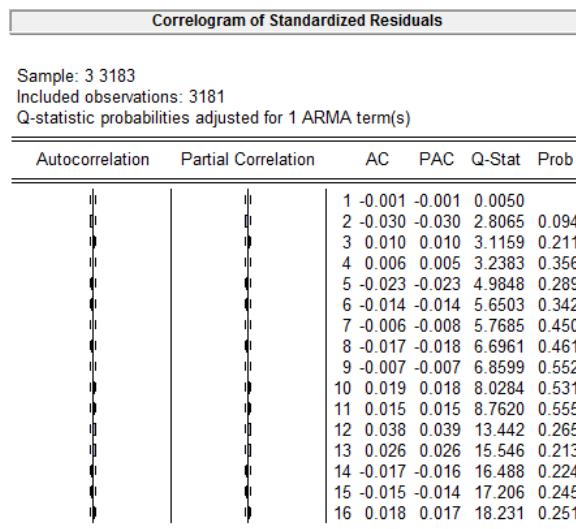
CSD,EGSD(-i)	CSD,EGSD(+i)	i	lag	lead
		0	0.9285	0.9285
		1	0.9256	0.9265
		2	0.9141	0.9139
		3	0.8996	0.9033
		4	0.8846	0.8915
		5	0.8683	0.8783
		6	0.8526	0.8665
		7	0.8385	0.8556
		8	0.8250	0.8442
		9	0.8118	0.8326
		10	0.7985	0.8208
		11	0.7846	0.8086
		12	0.7702	0.7940
		13	0.7566	0.7806
		14	0.7428	0.7661
		15	0.7302	0.7538
		16	0.7180	0.7415



Series: Standardized Residuals	
Sample 3 3183	
Observations 3181	
Mean	0.003251
Median	0.076611
Maximum	3.629484
Minimum	-6.069213
Std. Dev.	1.000745
Skewness	-0.408280
Kurtosis	4.214804
Jarque-Bera	283.9731
Probability	0.000000

(f) The estimation results below confirms that the dependence structure of R is well captured by the extended AR(1)-EGARCH(2,1). No statistically-significant autocorrelations are observed in either the standardised residuals or their squares (see ARCH test as well as correlograms below). The coefficient on LRV(-1) is large and statistically significant, implying that LRV_{t-1} carries useful volatility information that is not available in either v_{t-1} , v_{t-2} or $\ln(\sigma_{t-1}^2)$. The in-sample fit is materially improved with the likelihood ratio being $LR = 2[(-4548.36) - (-4642.39)] = 188.06$ (compared against $\chi^2_{(1)}$ 5% critical value 3.84). The AIC and SIC criteria also favour the inclusion of LRV(-1) in the variance equation. The point estimate of β_1 is 0.7719, much smaller than the estimate 0.9762 in part (e). However, the persistence in the conditional variance of EGARCH that includes LRV(-1) should be measured differently. It should depend on both β_1 and the coefficient on LRV(-1), ψ , probably in a complicated way.

Dependent Variable: R					ARCH Test			
Method: ML - ARCH (Marquardt) Normal distribution					F-statistic	0.418546	Probability	0.948689
					Obs*R-squared	4.614767	Probability	0.948380
Sample (adjusted): 3 3183								
Included observations: 3181 after adjustments								
Convergence achieved after 16 iterations								
Bollerslev-Wooldridge robust standard errors & covariance								
Variance backcast: ON								
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) +								
C(5)*ABS(RESID(-2)/@SQRT(GARCH(-2))) + C(6)*RESID(-1)								
/@SQRT(GARCH(-1)) + C(7)*LOG(GARCH(-1)) + C(8)*LRV(-1)								
Coefficient					Test Equation:			
Std. Error					Dependent Variable: STD_RESID^2			
t-Statistic					Method: Least Squares			
Prob.								
C								
AR(1)								
Variance Equation								
C(3)								
C(4)								
C(5)								
C(6)								
C(7)								
C(8)								
R-squared								
Adjusted R-squared								
S.E. of regression								
Sum squared resid								
Log likelihood								
Durbin-Watson stat								
Inverted AR Roots								
Mean dependent var								
S.D. dependent var								
Akaike info criterion								
Schwarz criterion								
F-statistic								
Prob(F-statistic)								
Variable								
Coefficient								
Std. Error								
t-Statistic								
Prob.								
C								
STD_RESID^2(-1)								
STD_RESID^2(-2)								
STD_RESID^2(-3)								
STD_RESID^2(-4)								
STD_RESID^2(-5)								
STD_RESID^2(-6)								
STD_RESID^2(-7)								
STD_RESID^2(-8)								
STD_RESID^2(-9)								
STD_RESID^2(-10)								
STD_RESID^2(-11)								
R-squared								
Adjusted R-squared								
S.E. of regression								
Sum squared resid								
Log likelihood								
Durbin-Watson stat								
Mean dependent var								
S.D. dependent var								
Akaike info criterion								
Schwarz criterion								
F-statistic								
Prob(F-statistic)								



Series: Standardized Residuals
Sample 3 3183
Observations 3181

Mean	-0.004635
Median	0.071906
Maximum	3.779356
Minimum	-5.438914
Std. Dev.	0.999997
Skewness	-0.327574
Kurtosis	3.727077
Jarque-Bera	20.1552
Probability	0.000000

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