**University of New South Wales**
**School of Economics**
**Financial Econometrics**
**Tutorial 4**

1. Estimating MA

Consider an invertible MA(1) model: $y_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1}$, $\varepsilon_t \sim iid$ WN$(0, \sigma^2)$. Suppose that we know $\varepsilon_0$ (shock at $t = 0$) and $T = 3$ observations on $y_t$, ie, $\{y_1, y_2, y_3\}$. Express $\varepsilon_t$ in terms of the parameters $(\mu, \theta_1)$ and $(\varepsilon_0, y_1, y_2, y_3)$ for $t = 1,2,3$. As we can use $\mu + \theta_1 \varepsilon_{t-1}$, which is in the information set $\Omega_{t-1}$, to forecast $y_t$, what is the interpretation for the shock $\varepsilon_t$? Further, how do you apply the "least squares" principle to estimate the parameters $(\mu, \theta_1)$? Just specify the objective function. The minimisation operation (first order derivatives) is not required. Now assume that the shocks $\varepsilon_t$ are normally distributed. Write down the log-likelihood function in terms of $(\mu, \theta_1)$ and $y_1, y_2, y_3$ and simplify removing all terms not influencing the optimization problem. Compare the objective function for the least squares and the MLE.



2. Find the unconditional variance of ARMA(1,1) model

$y_t = \mu + \theta_1 \varepsilon_{t-1} + \varphi y_{t-1} + \varepsilon_t$, $\varepsilon_t \sim iid$ WN$(0, \sigma^2)$



Note that $\varepsilon_{t-1}$ and $y_{t-1}$ are now linearly dependent. Therefore you also need to consider covariance between these terms when you compute the variance.

3 Computing Exercise. Box-Jenkins methodology

This question is based on the data in the Excel file **fisher_update.XLS**. The file contains 171 quarterly observations, from 1969Q4 to 2012Q2, on the Australian Consumer price Index (P) and on the yield to maturity of 90-day bank accepted bills (R).

(a) Generate the inflation rate as: INF=400*(log(P(1))-log(P)). When we construct the inflation rate this way, we lose the last observation, namely, 2012Q2. We change the sample to 1984Q1 to 2012Q1, which is the post-float period of the exchange rate.

Perform an ADF test for a unit root for inflation over the period 1984Q1-2012Q1. Comment on the results. Would you conclude that INF is stationary?

(b) Generate the correlogram of INF (16 lags). Comment on which ARMA models would fit the data.

(c) Estimate the models you considered in (b). Then select one model by using AIC/BIC.

(d) Perform diagnostic checks on the model of your choice. Comment on whether or not the model fits the data well.

(e) Now you are ready to do a forecasting exercise, using the model you are happy with (the output of (d)). Re-estimate the model for INF over the sample period 1984Q1-2009Q4, thereby keeping the last nine observations for an out-of-sample forecasting exercise.

- First, change the sample period to 1984Q1 2009Q4. Then estimate your model. I
- Then perform (pseudo out of sample) forecast for **2010Q1 2012Q1**; generate both
    - **Static forecast** (meaning 1-step ahead forecast based on most recent available observations for each $t$);
    - **Dynamic Forecast**.
- Compare these forecasts with actuals Inflation series.