**Copyright** © Copyright University of New South Wales 2020. All rights reserved.

## **Course materials subject to Copyright**

UNSW Sydney owns copyright in these materials (unless stated otherwise). The material is subject to copyright under Australian law and overseas under international treaties. The materials are provided for use by enrolled UNSW students. The materials, or any part, may not be copied, shared or distributed, in print or digitally, outside the course without permission. Students may only copy a reasonable portion of the material for personal research or study or for criticism or review. Under no circumstances may these materials be copied or reproduced for sale or commercial purposes without prior written permission of UNSW Sydney.

### Statement on class recording

To ensure the free and open discussion of ideas, students may not record, by any means, classroom lectures, discussion and/or activities without the advance written permission of the instructor, and any such recording proper state and advance can be used the view the state of the proper state as a such recording proper state and any such recording proper state as a such recording proper state

WARNING: Your failure to comply with these conditions may lead to disciplinary action, and may give rise to a civil action or a criminal offence under the law.

https://tutorcs.com
The above information must not be removed from this material.

WeChat: cstutorcs

#### ECON3206/ECON5206 Financial Econometrics

# Sample Answers/Hints to Tutorial 2

- 1. Linear regression models are linear in parameters. With a log transformation, (1) becomes  $\ln(y_t) = \alpha + \beta \ln(x_t) + u_t$ , a linear regression model. Hence both (1) and (3) are linear regression and can be estimated by OLS. However, (2) is not linear in parameters and OLS is not applicable. Its parameters cannot be properly estimated at all because the parameters are not identified in the sense that  $\beta$  and  $\gamma$  are observationally equivalent to  $\beta c$  and  $\gamma/c$  for any non-zero constant c. In this case, we can only use OLS to estimate the product  $\beta \gamma$  but cannot estimate  $\beta$  and  $\gamma$  separately.
- 2. Population (true) model  $Y_t = \alpha + u_t$ , assume  $E(u_t) = 0$ Regression  $Y_t = \grave{\alpha} + e_t$ , t = 1...T

# sse Assignment Project Exam Help

By definition ordinary least squares estimator is defined as  $\dot{a} = \arg\min SS$  ttps://tutorcs.com

FOC:

$$\frac{\partial SSE}{\partial \dot{\boldsymbol{\alpha}}} = -2\sum (Y_t - \dot{\boldsymbol{\alpha}}) = -2\sum Y_t + 2T \, \dot{\boldsymbol{\alpha}} \equiv 0$$

$$\dot{\mathbf{a}} = \frac{1}{T} \sum Y_t$$
, which is just (not surprisingly) a sample mean estimator

SOC: (just to make sure)

$$\frac{\partial^2 SSE}{\partial \hat{\alpha}^2} = 2T > 0 \rightarrow \text{true minimum}$$

We can check if à is unbiased:

$$E(\grave{\alpha}) = \frac{1}{T} \sum E(Y_t) = \frac{1}{T} \sum E(\alpha + u_t) = \frac{T\alpha}{T} + 0 = \alpha \rightarrow unbiased$$

3. The OLS estimator for the assumed (omitted variable) regression is  $\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$ . To find the bias compute

$$E(\mathbf{b}) = E_{\mathbf{X}}(E[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y} \mid \mathbf{X}]) = E_{\mathbf{X}}(E[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{X}\boldsymbol{\beta} + (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Z}\boldsymbol{\gamma} + (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{u} \mid \mathbf{X}])$$
using  $\mathbf{E}[\mathbf{u} \mid \mathbf{X}, \mathbf{Z}] = 0$ 

$$= \boldsymbol{\beta} + \gamma E_{\mathbf{X}}(E[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Z} \mid \mathbf{X}) = \boldsymbol{\beta} + \gamma \underbrace{E[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Z})}_{\text{bias}}$$

Hence, omitted variable estimator is typically biased, unless  $E(\mathbf{Z} \mid \mathbf{X}) = 0$ , the omitted variable **Z** is independent of the included variable **X**.

4. The yield (to maturity) is the rate of return on a bond (expressed as an annual rate) if purchased at the current market price and held until the Maturity Date. The annualised yield will vary through time with changes in the price and remaining term to maturity of the bond. In turn, the Coupon Interest Rate is set when the bond is first issued and remains fixed for the life of the bond. If the price of bond increases the (implied) yield goes down because you pay more for the same Coupon Interest Rate and terminal (face) value of the bond. The time to maturity is reported on the horizontal axes. The corresponding yields on the vertical axes form a yield curve. Different colours indicate the evolution of the yield curve comparing the current yield curve (almost current: 1 July 2015) with the curve one month ago, and one year ago. From the figure we see the yield curve is flattening. This indicates that the investors expect that the long term interest rates will be lower relatively to what they expected a month or a year ago. However, the expectation is still the rate exist rise in the future. Using the Fishen model the expectation about the future enterest rates depend on the expectations about the future inflation. The RBA as the other central banks targets (regulates) inflation setting the rates. Higher inflation in the future typically signals economic growth. Flattening of the yield curve indicates that the rivestors expedit the growns ville grown ville confirmed by Glenn Stevens, the governor of the RBA. http://www.afr.com/news/economy/monetary-policy/rbas-glenn-stevens-says-growth-

below-3pc-is-nev norm 1-20150722-gihunt the speaks the yields, the exchange rates, the shares move.

Durbin-Watson statistics is given by as

$$DW = \frac{\sum_{t=2}^{T} (e_t - e_{t-1})^2}{\sum_{t=1}^{T} e_t^2} = \frac{\sum_{t=2}^{T} e_t^2}{\sum_{t=1}^{T} e_t^2} + \frac{\sum_{t=2}^{T} e_{t-1}^2}{\sum_{t=1}^{T} e_t^2} - \frac{2\sum_{t=2}^{T} e_t e_{t-1}}{\sum_{t=1}^{T} e_t^2} \approx 1 + 1 - 2\hat{\rho}$$
$$= 2(1 - \hat{\rho})$$

As  $-1 \le \hat{\rho} \le 1$ ,  $0 \le DW \le 4$ , when  $\hat{\rho}$  is small DW is close to 2.

6. Show that in the linear regression sum of squares total SST =  $\sum_{t=1}^{T} [Y_t - \overline{Y}]^2$  can be decomposed into sum of squares explained  $SSE = \sum_{t=1}^{T} [\hat{Y}_t - \overline{Y}]^2$  and sum of squared residuals SSR =  $\sum_{t=1}^{T} [Y_t - \hat{Y}_t]^2$ .

$$SST = \sum_{t=1}^{T} [Y_t - \overline{Y}]^2 = \sum_{t=1}^{T} [Y_t - \hat{Y}_t + \hat{Y}_t - \overline{Y}]^2 = \sum_{t=1}^{T} [Y_t - \hat{Y}_t]^2 + \sum_{t=1}^{T} [\hat{Y}_t - \overline{Y}]^2 + \sum_{t=1}^{T} 2[\hat{Y}_t - \overline{Y}][Y_t - \hat{Y}]$$

Next look at the last term and show that this term equals zero in the least squares regression.

$$\sum_{t=1}^{T} 2[\hat{Y}_{t} - \bar{Y}][Y_{t} - \hat{Y}] = 2\sum_{t=1}^{T} [\hat{Y}_{t} - \bar{Y}]e_{t} = 2\sum_{t=1}^{T} \hat{Y}_{t}e_{t} + 2\bar{Y}\sum_{t=1}^{T} e_{t}$$

$$\sum_{t=1}^{T} \hat{Y}_{t}e_{t} = \sum_{t=1}^{T} (\hat{\beta}_{1}X_{1t} + \hat{\beta}_{2}X_{2t} + ... + \hat{\beta}_{K}X_{Kt})e_{t} = \hat{\beta}_{1}\sum_{t=1}^{T} X_{1t}e_{t} + \hat{\beta}_{2}\sum_{t=1}^{T} X_{2t}e_{t} + ... \hat{\beta}_{K}\sum_{t=1}^{T} X_{Kt}e_{t}$$

The first order condition of the least squares minimization problem implies that  $\mathbf{X}'\mathbf{e} = 0$ . Recall that the first column of  $\mathbf{X}$  is a vector of 1. Using the rules of matrix-vector multiplication on  $\mathbf{X}'\mathbf{e} = 0$  it is apparent that  $\sum_{t=1}^T e_t = 0$  and  $\sum_{t=1}^T X_{kt} e_t = 0$  for all k = 1...K

7. Show how F statistics,  $F = \frac{(SSR_r - SSR_u)/j}{SSR_u/(T - K - 1)}$  can be expressed in terms of R<sup>2</sup>. This follows trivially from the definition of R<sup>2</sup>:  $R^2 = \frac{SSE}{SST} = 1 - \frac{SSR}{SST}$ .

For the restricted model  $R_r^2 = 1 - \frac{SSR_r}{SST}$ , so that,  $SSR_r = (1 - R_r^2)SST$ , so and for the

unrestricted model  $R_u^2 = 1 - \frac{SSR_u}{SST}$ , so that  $SSR_u = (1 - R_u^2)SST$ . Note that SST is the same for both models we are the same than  $R_u = 1 - \frac{SSR_u}{SST}$ .

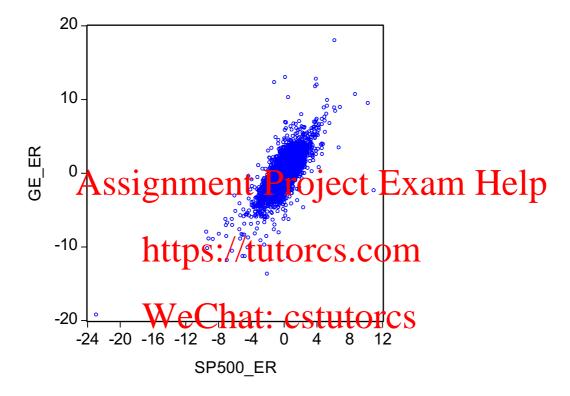
$$F = \frac{(\text{SSR}_{r} - \text{SSR}_{u}) / j}{\text{SSR}_{u} / (T - K)} = \frac{((1 - R_{r}^{2})\text{SST} - (1 - R_{u}^{2})\text{SST}) / j}{\text{SST} / (T - K)} = \frac{(R_{u}^{2} - R_{r}^{2}) / j}{\text{SST} / (T - K - 1)}.$$

Intuitively if the restrictions are true the fit of the restricted model (measured by  $R^2$ ) is close to the fit of the unrestricted model and F is small Also because  $R_u^2 > R_r^2$  F is always positive.

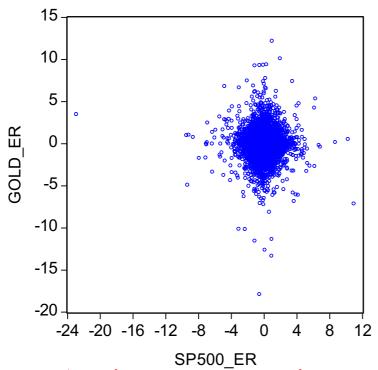
# **Computing Exercises**

### 1. Estimating the CAPM

- (a)-(c) See the **capm.wf1** file to check if your output is correct.
- (d) The plot indicates an upward-sloping relationship between GE\_ER (excess return of GE) and SP500\_ER (excess return of the market). Note the outlier (more than -20% fall in one day). This is Black Monday, October 19, 1987, when stock markets around the world crashed, shedding a huge value in a very short time.



(e) The plot indicates no specific slope between GOLD\_ER (excess return of GOLD) and SP500\_ER (excess return of the market). Note that GOLD appreciated during the Black Friday when S&P500 fell more than -20% in one day.



Assignment Project Exam Help
While GOLD is considered a safe asset, the range of fluctuations of its returns is not

particularly narrow. The remarkable drop in price is recorded on the 22<sup>nd</sup> of January, 1980 (outlier of about -2020) the after the process of Sociological the previous several days. Here is what news around this time says

# WeChat: cstutorcs

Business/Economy news – January 21, 1980

Consumer prices for 1979 soar to 13.3% - the largest gain in 33 years. The gain in the previous year (1978) was a mere 9%. Inflation jumped to its highest in 1946 - when the government lifted WWII price controls and prices soared 18.2% for the year.

On inflation fears - gold continues to rise - reaching \$845 an ounce this week - rising \$100 in a single day. Profit takers drove the price down to \$808. The price of gold has soared more than \$640 an ounce from a year ago.

(f)-(g) The estimation output using GE indicates that the CAPM is supported by data ( $\alpha = 0$  cannot be rejected). The beta coefficient is the % increase in the expected excess return of GE associated with 1% increase in the expected excess return of "market". The estimate of beta is more than 1 (at 1.156882) and highly significant indicating that the GE may is a bit more risky than the market). The adjusted R-squared is 0.564154, indicating that the explanatory

power of the model is relatively good (In time series setting we rarely see high R-squared). The Durbin-Watson statistic is 1.995, very close to 2. Hence there is no strong evidence for autocorrelation in the error term.

Dependent Variable: GE\_ER Method: Least Squares Date: 08/15/15 Time: 17:23

Sample (adjusted): 8/13/1975 8/12/2015 Included observations: 10436 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.	
C SPECO ED	-0.001219 1.156882	0.010646 0.009954	-0.114470 116.2238	0.9089 0.0000	
SP500_ER	1.150002			0.0000	
R-squared	0.564196	Mean depend	dent var	0.018960	
Adjusted R-squared	0.564154	S.D. depende	ent var	1.647184	
S.E. of regression	1.087447	Akaike info c	riterion	3.005735	
Sum squared resid	12338.64	Schwarz crite	erion	3.007125	
Log likelihood	-15681.93	F-statistic	_	13507.97	
Durbin-Wattonstat Q	nimen	Prob(F-statis	ect Ex	X020ppq0	Hel

The estimation output using GOLD also indicates that the CAPM is supported by data ( $\alpha = 0$  cannot be rejected). The estimate of beta is less than 0 (at -0.02) and marginally significant at 7.5% significance level indicating that the GOLD has small negative correlation with the market. Note that the asset with active Cells the transfer They are useful for portfolio diversification as the move against the market. The adjusted R-squared is very small, indicating that the explanatory power of the model is relatively poor. The Durbin-Watson statistic is 2.07, very close to 2. Hence there is no strong evidence for autocorrelation in the error term.

Dependent Variable: GOLD ER

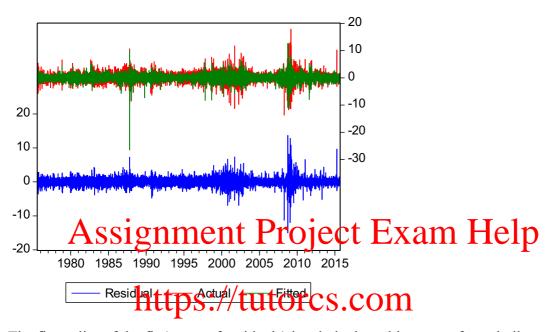
Method: Least Squares Date: 08/15/15 Time: 17:31

Sample (adjusted): 8/13/1975 8/12/2015 Included observations: 10436 after adjustments

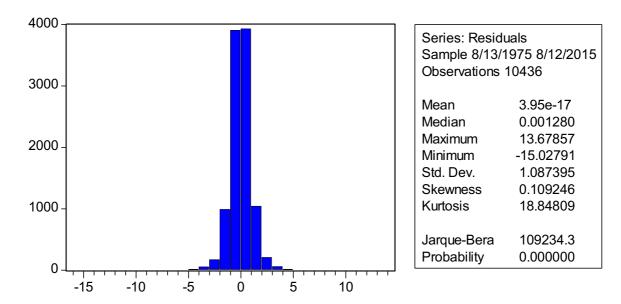
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C SP500_ER	0.005685 -0.020065	0.012033 0.011250	0.472438 -1.783554	0.6366 0.0745
R-squared Adjusted R-squared S.E. of regression	0.000305 0.000209 1.229062	Mean dependent var S.D. dependent var Akaike info criterion		0.005335 1.229190 3.250570

Sum squared resid	15761.52	Schwarz criterion	3.251960
Log likelihood	-16959.48	F-statistic	3.181064
Durbin-Watson stat	2.071244	Prob(F-statistic)	0.074525

### (h) Actual-Fitted-Residuals plot for GE.

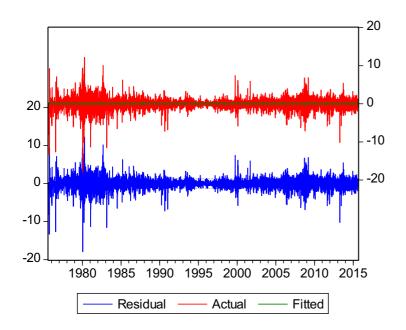


The fit quality of the fit (range of residuals) is relatively stable except for volatile periods of early 2000s recession to the late burst state the region of the GFC.

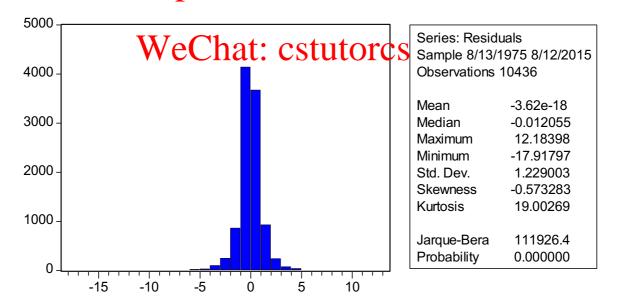


The normality is overwhelmingly rejected.

### Actual-Fitted-Residuals plot for GOLD.



The residuals plot for GOLD shows relatively poor fit in the 1980, the periods of high inflations and high forculations of gold prices. The high fluctuations may be explained by abandoning of the gold standard (in October 1976 US dollar was no longer tight to Gold) and subsequent deregular to the gold trial tenergy is a subseque



The normality is also rejected.

© Copyright University of New South Wales 2020. All rights reserved. This copyright notice must not be removed from this material.

(i)

#### For GE:

#### White Heteroskedasticity Test:

F-statistic	318.6093	Probability	0.000000
Obs*R-squared	600.7120	Probability	0.000000

The homoskedasticity is rejected and hence standard errors need to be corrected (using White or HAC standard errors).

#### For GOLD:

#### White Heteroskedasticity Test:

F-statistic	18.49337	Probability	0.000000
Obs*R-squared	36.86668	Probability	0.000000

# Similarly for Soliganment Project Exam Help rors need to be corrected (using White or HAC standard errors).

# https://tutorcs.com

(j)

# Breusch-Godfrey Serial Correlation Lattest: CStutorcs

F-statistic	2.587708	Probability	0.075240
Obs*R-squared	5.174834	Probability	0.075214

**Test Equation:** 

Dependent Variable: RESID Method: Least Squares Date: 08/15/15 Time: 18:00

Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-5.97E-06	0.010645	-0.000561	0.9996
SP500_ER	0.000405	0.009959	0.040645	0.9676
RESID(-1)	0.002741	0.009793	0.279895	0.7796
RESID(-2)	-0.022111	0.009791	-2.258246	0.0240
R-squared Adjusted R-squared S.E. of regression	0.000496	Mean dependent var		3.95E-17
	0.000208	S.D. dependent var		1.087395
	1.087282	Akaike info criterion		3.005622

Sum squared resid	12332.53	Schwarz criterion	3.008402
Log likelihood	-15679.34	F-statistic	1.725139
Durbin-Watson stat	2.001699	Prob(F-statistic)	0.159464

There is no strong evidence for autocorrelation in the residual as the p-value is large.

#### For GOLD

#### Breusch-Godfrey Serial Correlation LM Test:

F-statistic	7.036154	Probability	0.000884
Obs*R-squared	14.05874	Probability	0.000885

Test Equation:

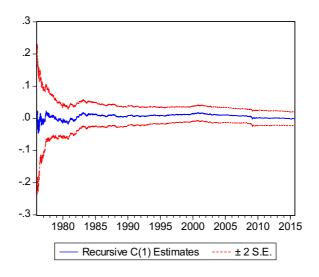
Dependent Variable: RESID Method: Least Squares Date: 08/15/15 Time: 18:09

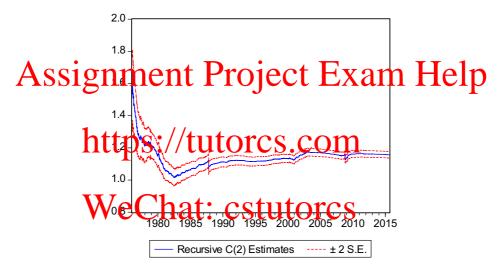
Presample missing value lagged residuals set to zero.

Variable S	gnerren	t Project	tatictic X 2 rob
C	-4.45E-06		0.9997
SP500_ER RESID(-1)	9.12E-05 https://	0.011245 0.0 tppppggcs3.0	0.9935 2001 0.0003
RESID(-2)	0.008292	0.009792 0.8	346781 0.3971
R-squared	W 0.004347	Mean dependent v	ar -3.62E-18
Adjusted R-squared S.E. of regression	1.228351	Akaike info criterio	
Sum squared resid Log likelihood	15740.29 -16952.44	Schwarz criterion F-statistic	3.252386 4.690769
Durbin-Watson stat		Prob(F-statistic)	0.002820

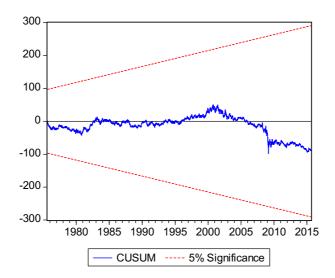
There is some evidence for autocorrelation in the residual as the p-value is small. Note that the conclusions are different from Durbin-Watson test commented on earlier. The reason for difference is that DW test looks only on lag 1, while Breusch-Godfrey Serial Correlation LM Test includes lags of higher order and it more general. HAC standard errors should be in this case.

### (k) Stability for GE



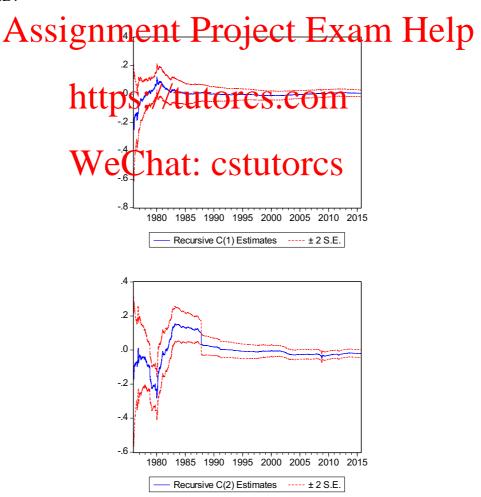


The recursive coefficients show that generally the coefficients are stable (especially after 90es). The coefficients are somewhat noisy in the beginning as relatively small samples are used. It is reasonable to assume higher beta in the beginning of the sample.

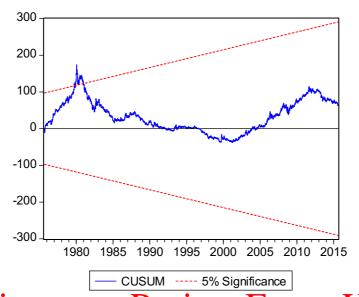


The model stability is formally tested via the CUSUM test. As the CUSUM does not go outside the 95% probability bands, there is no evidence for instability.

For GOLD:

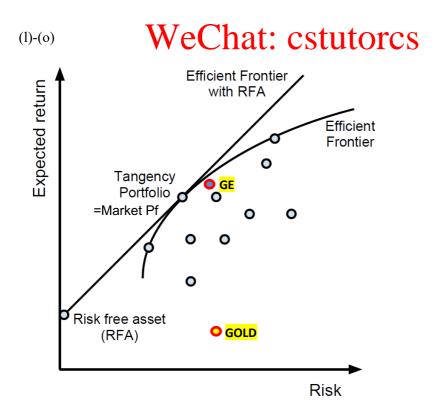


We observe relatively high parameter instability around 80ies. Before 80is GOLD seem to have relatively large negative beta. After 80is the beta increase and become positive and there was a structural jump around 1987 a sharp reduction in Gold's beta.



Assignment Project Exam Help CUSUM test also signals some instability. Hence it may be useful to separate different periods using the dummy variables.

# https://tutorcs.com



The expected return of GE can be replicated by combining market portfolio with the risk-free asset.

© Copyright University of New South Wales 2020. All rights reserved. This copyright notice must not be removed from this material.

Generally this portfolio will be  $\beta$  fraction of market portfolio and  $(1-\beta)$  risk-free asset. We discussed in class why this it optimal. Because  $\beta$  is greater than one  $(1-\beta)$  is negative indicating that we need to short sell Tbill (borrow money at risk-free rate) and buy S&P500 (market) index.

The portfolio combination: 1.156882\*S&P500 - 0.156882\*Tbill

To replicate Gold return we will do something rather bizarre: short-sell a small fraction of the market portfolio and buy secure Tbills.

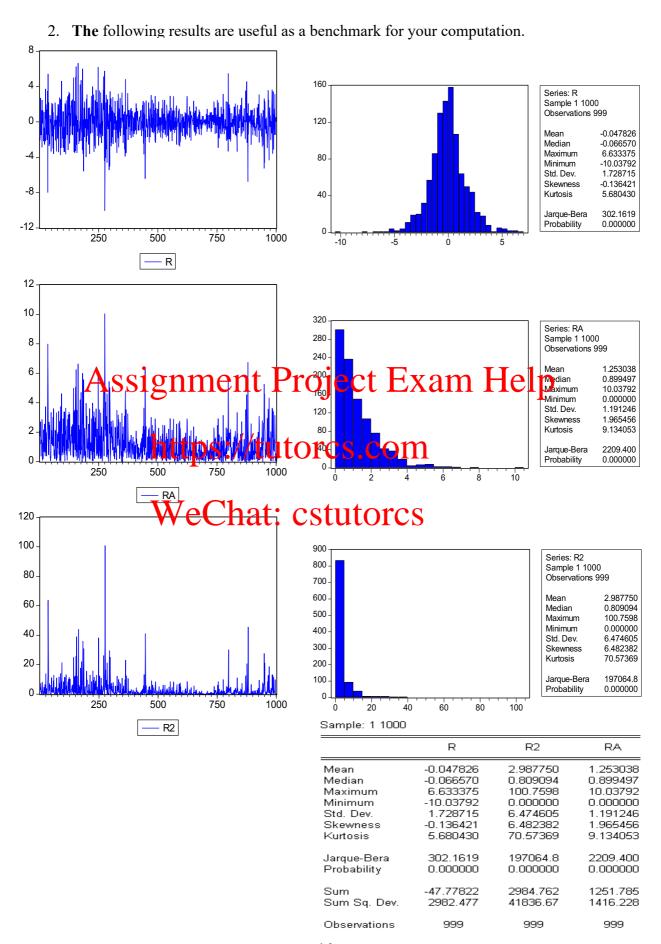
The portfolio combination: -0.02\*S&P500 + 1.02\*Tbill

	GE_R	GE_EQUIV_P	GOLD_R	GOLD_EUIV_P	SP500_R
Mean	0.031948	0.033167	0.018323	0.012900	0.030431
Std. Dev.	1.647229	1.237171	1.229045	0.023489	1.069396

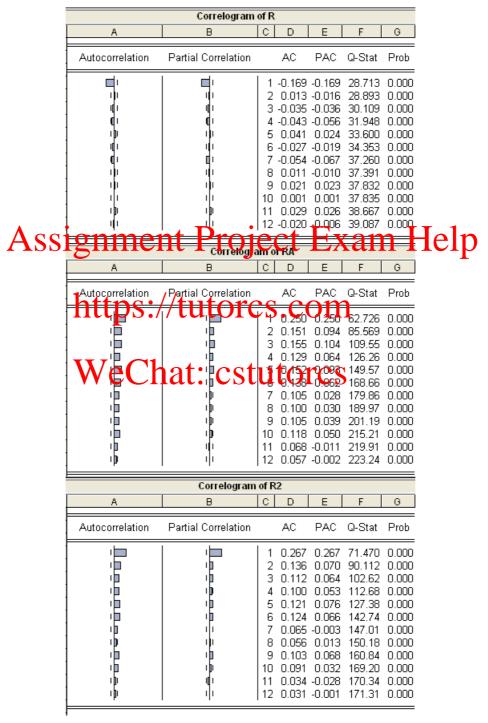
The replicated expected returns are close the expected returns of the underlying assets. The risk the replicated portfolio is smaller than of the underlying asset (especially in the case of gold).

The market A valling of it wind and with the expected eturn only the systematic rest. In he case of Gold the risk premium is negative (as the gold can be used as a hedge, you have to pay to hedge the risk). Difference between the risk (st.dev.) of the underlying asset and the corresponding portfolio can be viewed as the idiosyntratic risk (which is not rewarded by expected return): about 0.31 for GE and 1.2 for Gold.

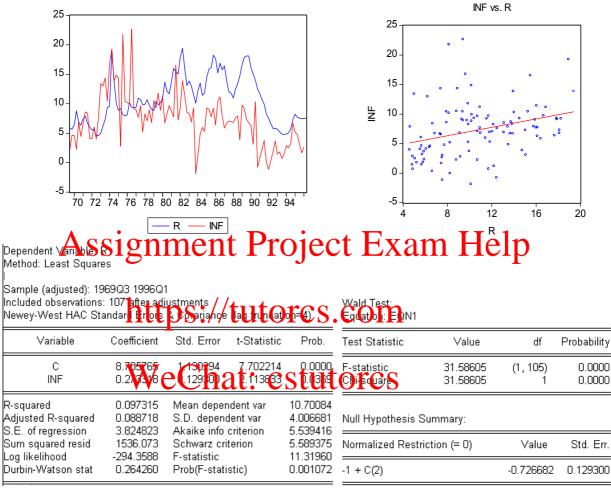
WeChat: cstutorcs

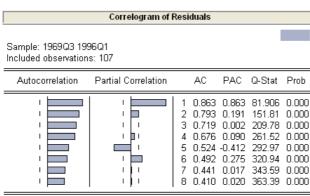


(a) , (b), (c): You should obtain results that are the same as the above. The "Histogram" from Eviews delivers the required empirical distribution and statistics. Clearly, rc2 and rca are non-negative variables and are nowhere near a "bell-shaped" distribution (see their histograms). Their time-series plots illustrate the variability in rc to some extent (as the average of rc2 is approximately the variance of rc). For rc, its JB statistic convincingly reject the normality hypothesis since it is negatively skewed and with a kurtosis as large as 5.68 (comparing to 3 in a normal distribution).

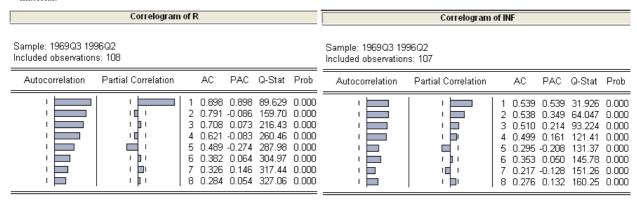


- (d) According to the above correlograms, there is a statistically significant first order autocorrelation in rc, which (-0.169) is outside the Bartlett bands (dashed lines). Because of this, the log price of Copper does not behave like a random walk and hence the efficient market hypothesis is statistically rejected. For rc2 and rca, the autocorrelations are much stronger than for rc, indicating the predictability of volatility.
- 3. The sample answers are based on the following computation results.





© Copyright University of New South Wales 2020. All rights reserved. This copyright notice must not be removed from this material



- (a) and (b) are straightforward.
- (c) When  $\beta_1 = 1$ , the intercept is in fact the *real* interest rate. The hypothesis  $\beta_1 = 1$  is rejected, see the p-value for F-test or z = (0.273318 1)/0.1293 = -5.62, which is far too negative in comparison to the critical value -1.96 at the 5% level of significance.
- (d) From the correlograms, both R and INF have strong autocorrelations (both AC and PAC are outside the bands). The autocorrelation in R is mainly caused by the first-order PAC, although a smallish fifth-order PAC is also statistically significant. In this case,  $R_{t-2}$  is not directly correlated with  $R_t$  but affects  $R_t$  via  $R_{t-1}$ . Further, we note that the residual from the regression model is autocorrelated. This is a vio align of the basic assumption that error term is uncorrelated with one another.

https://tutorcs.com

WeChat: cstutorcs