Financial Econometrics

Slides-12: Further Issues for GARCH & Realized Volatility

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Lecture Plan

- Asymmetric GARCH: Leverage effect
- Quantify the effect of standardised shock and avoid positivity restrictions: EGARCH
- Measure the risk premium effect: GARCH-M model

GARCH Extensions

Asymmetric GARCH models

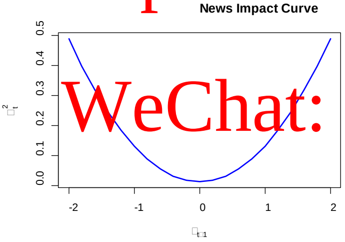
- Motivation: a negative shock to financial time series is likely to cause volatility to rise by more than a positive shock of the same magnitude

- This is due to **leverage effects**, i.e. a fall in the value of a firm's stock causes the firm's debt to equity ratio to rise, which makes the future stream of dividends more volatile
- Standard GARCH models assume a symmetric response of volatility to positive and negative shocks since by squaring the lagged error term the sign is lost:

  In GARCH(1,1): $\sigma_t^2 = \alpha_0 + \alpha_1 \mu_{t-1}^2 + \beta_1 \sigma_{t-1}^2$, the impact μ_{t-1} on σ_t^2 is **symmetric**.

Asymmetric GARCH: Motivation

- In equity markets, however, bad news (-ve shock) tends to cause more volatility than good news (+ve shock), aka “asymmetric effect” or “leverage effect”.
- Desirable to allow for asymmetric effect in GARCH.



Asymmetric GARCH

- **The Threshold GARCH (TGARCH) model.** Glosten, Jagannathan and Runkle [JF, 1993, 48(5), p1779-1801] propose a so-called TGARCH model (GJR) in which the conditional variance equation is given by

$$\sigma_t^2 = \alpha_0 + \alpha_1 \mu_{t-1}^2 + \gamma \mu_{t-1}^2 I_{t-1} + \beta_1 \sigma_{t-1}^2,$$

where I_{t-1} is a dummy variable: $I_{t-1} = 1$ if $\mu_{t-1} < 0$ and $I_{t-1} = 0$ otherwise. If leverage effects are present $\gamma > 0$

- If $\mu_{t-1} < 0$, its effect on σ_t^2 is $\alpha_1 + \gamma$

  If $\mu_{t-1} \geq 0$, its effect on σ_t^2 is α_1

- The asymmetric effect exists if and only if $\gamma > 0$. Reduced back to GARCH if $\gamma = 0$.

Example: GJR/TGARCH

Example estimates TGARCH/GJR model for returns for S&P500 index with robust standard errors

Dependent Variable: RSP500
Method: ML - ARCH
Date: 12/15/06 Time: 15:51
Sample(adjusted): 3 2610
Included observations: 2608 after adjusting endpoints
Convergence achieved after [illegible] iterations
Bollerslev-Wooldrige robust standard errors & covariance

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000384	0.000148	2.593854	0.0095
AR(1)	0.067818	0.019481	3.481313	0.0005
Variance Equation				
C	[illegible]	[illegible]	[illegible]	[illegible]
ARCH(1)	[illegible]	[illegible]	[illegible]	0.3300
(RESID<0)*ARCH(1)	[illegible]	[illegible]	[illegible]	[illegible]
GARCH(1)	0.901285	0.016959	53.14402	0.0000

R-squared	-0.001759	Mean dependent var	0.000529
Adjusted R-squared	-0.003684	S.D. dependent var	0.008696
S.E. of regression	0.008712	Akaike info criterion	-6.861454
Sum squared resid	0.197479	Schwarz criterion	-6.847958
Log likelihood	8953.336	Durbin-Watson stat	2.086341
Inverted AR Roots	.07		

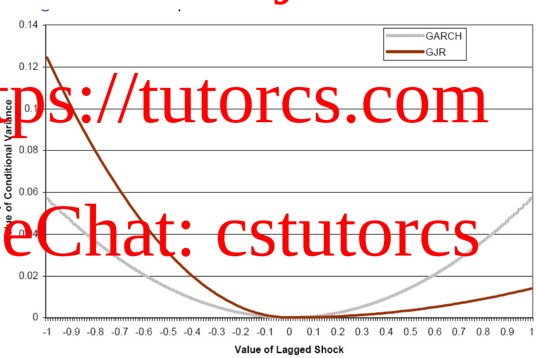
News impact curve

- Graphical representation of the degree of asymmetry of volatility to positive and negative shocks: the curves are drawn by using the estimated conditional variance equation of the model under consideration.

- Calculate the values of the conditional variance σ_t over a range of past error terms. Set the lagged conditional variance at the **unconditional** variance
- Example: News impact curve from estimates TGARCH model for returns for S&P500 index

Example: GJR/TGARCH

News impact curve from estimates TGARCH model



Properties of the TGARCH/GJR model

- **Unconditional variance:**

$$\mu_t|\Omega_{t-1} \sim N(0, \sigma_t^2),\ \sigma_t^2 = \alpha_0 + \alpha_1\mu_{t-1}^2 + \gamma\mu_{t-1}^2 I_{t-1} + \beta_1\sigma_{t-1}^2$$

  - $E(\mu_{t-1}^2) = E\left(\sigma_{t-1}^2\right)$
  - [illegible]

$$\begin{aligned} E(I_{t-1}\mu_{t-1}^2) &= E\left[E\left(I_{t-1}\mu_{t-1}^2|\Omega_{t-2}\right)\right] \\ &= E\left[\frac{1}{2}E\left(\mu_{t-1}^2|\Omega_{t-2}\right)\right] = \frac{1}{2}E\left(\sigma_{t-1}^2\right) \end{aligned}$$

  - $E(\sigma_t^2) = \alpha_0 + \left(\alpha_1 + \beta_1 + \frac{1}{2}\gamma\right)E\left(\sigma_{t-1}^2\right)$
  - Stationarity: $E\left(\sigma_t^2\right) = E\left(\sigma_{t-1}^2\right) = \alpha_0/\left[1 - \left(\alpha_1 + \beta_1 + \frac{1}{2}\gamma\right)\right]$
  - The above is valid when the conditional distribution of $\mu_t|\Omega_{t-1}$ is **symmetric**.

Properties of TGARCH/GJR: persistence

- Let $\omega_t = \mu_t^2 - \sigma_t^2$, then μ_t^2 has a representation:

$$\mu_t^2 = \alpha_0 + (\alpha_1 + \beta_1 + \gamma I_{t-1})\mu_{t-1}^2 + \omega_t - \beta_1 \omega_{t-1}$$

- When the shocks are zero, ie, $\omega_t = 0$ for all t, by substitution,

$$\mu_t^2 \approx \Pi_{\tau=0}^{t-1}(\alpha_1 + \beta_1 + \gamma I_\tau)\mu_0^2.$$

- $E(I_\tau|\Omega_{\tau-1}) = \frac{1}{2}$ by symmetry.
- On average, the impact of μ_0^2 on μ_t^2 is

$$\begin{aligned} & E\left\{\Pi_{\tau=0}^{t-1}(\alpha_1 + \beta_1 + \gamma I_\tau)\right\} \\ = & E\left\{(\alpha_1 + \beta_1 + \gamma E\left[I_{t-1}|\Omega_{t-2}\right])\Pi_{\tau=0}^{t-2}(\alpha_1 + \beta_1 + \gamma I_\tau)\right\} \\ = & (\alpha_1 + \beta_1 + \gamma/2)\, E\left\{\Pi_{\tau=0}^{t-2}(\alpha_1 + \beta_1 + \gamma I_\tau)\right\} \\ = & \cdots = (\alpha_1 + \beta_1 + \gamma/2)^t\,. \end{aligned}$$

- Half-life time, t_H, is defined as $t_H = \frac{\ln(1/2)}{\ln(\alpha_1+\beta_1+\gamma/2)}$

Example.

eg. NYSE composite return: $\hat{\gamma} = 0.1977$, significant $\hat{\alpha}_1$ negative, insignificant

Dependent Variable: R[illegible]
Method: ML - ARCH (Marquardt) - Normal distribution

Sample (adjusted): 3 1931
Included observations: 1929 after adjustments
Convergence achieved after 17 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: ON
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-1)^2*(RESID(-1)<0) + C(6)*GARCH(-1)

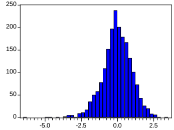
	Coefficient	Std. Error	z-Statistic	Prob.
C	[illegible]	[illegible]	[illegible]	[illegible]
AR(1)	[illegible]	[illegible]	4.842711	[illegible]
Variance Equation				
C	0.018550	0.003814	4.863488	0.0000
RESID(-1)^2	-0.006923	0.017268	-0.400937	0.6885
RESID(-1)^2*(RESID(-1)<0)	0.197665	0.035273	5.603918	0.0000
GARCH(-1)	0.892814	0.018492	48.28240	0.0000

R-squared	0.002458	Mean dependent var	0.035168
Adjusted R-squared	-0.000135	S.D. dependent var	1.006452
S.E. of regression	1.006520	Akaike info criterion	2.572041
Sum squared resid	1948.156	Schwarz criterion	2.589350
Log likelihood	-2474.733	F-statistic	0.947847
Durbin-Watson stat	2.081856	Prob(F-statistic)	0.448800
Inverted AR Roots	.12		

Correlogram of Standardized Residuals Squared

Sample: 3 1931
Included observations: 1929
Q-statistic probabilities adjusted for 1 ARMA term(s)

	AC	PAC	Q-Stat	Prob
1	-0.008	-0.008	0.1359	
2	0.022	0.022	1.0841	0.298
3	-0.025	-0.025	2.2815	0.320
4	-0.009	-0.010	2.4360	0.487
5	-0.028	-0.027	3.9213	0.417
6	-0.002	-0.003	3.9314	0.559
7	0.007	0.008	4.0353	0.672
8	-0.004	-0.005	4.0624	0.773
9	-0.000	-0.001	4.0624	0.851
10	-0.018	-0.018	4.7003	0.860
11	-0.002	-0.002	4.7051	0.910
12	-0.011	-0.010	4.9418	0.934
13	0.005	0.004	4.9859	0.958
14	-0.011	-0.011	5.2180	0.970
15	-0.016	-0.017	5.6856	0.974
16	-0.008	-0.008	5.8236	0.983



Series: Standardized Residuals
Sample 3 1931
Observations 1929

Mean	-0.015283
Median	-0.007326
Maximum	3.437926
Minimum	-6.279817
Std. Dev.	0.999823
Skewness	-0.465783
Kurtosis	4.617748
Jarque-Bera	280.1008
Probability	0.000000

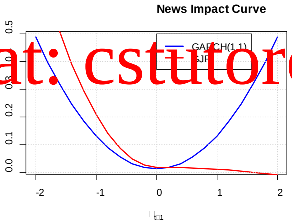
Example: Test for asymmetry

eg. NYSE composite return: Asymmetric news impact. GJR is preferred by AIC/SIC

Test for asymmetry

$$LR = 2\left(\mathrm{logL}_U - \mathrm{logL}_R\right) = 2\left[(-2472.7) - (-2523.6)\right] = 97.8$$

	log Likelihood	AIC	SIC
AR(1)-GARCH(1,1)	-2523.6	2.622	2.636
AR(1)-GJR	-2474.7	2.572	2.589

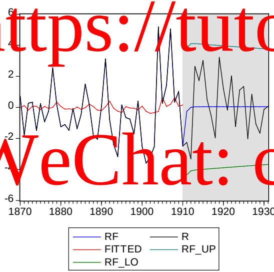


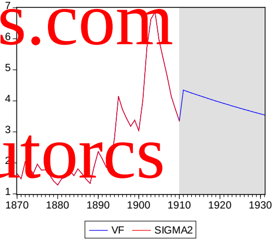
Example: Forecasts

eg. NYSE composite return: forecasts

σ_t^2 is still persistent, but less than GARCH(1,1).

$\alpha_1 + \beta_1 + \frac{1}{2}\gamma = 0.985$, $t_H = 45.9$ (days)





Example: VaR

eg. NYSE composite return: VaR
Portfolio valued at $\$1m$ at $T = 2002-08-29$.
AR(1)-GJR : $\sigma_{T+1} = 1.577, y_{T+1|T} = 0.0185$.
The 1% quantile of ν_t: $Q_{0.01} = -2.678$

$$VaR = \frac{1}{100}(y_{T+1|T} - 2.678\sigma_{T+1}) \times \$1m$$

	σ_{T+1}	$y_{T+1\|T}$	$q_{0.01}$	VaR
AR(1)-ARCH(5)	1.253	0.050	−2.774	−34260
AR(1)-GARCH(1,1)	1.642	0.051	−2.873	−46660
AR(1)-GJR	1.577	0.019	−2.678	−42048

Exponential GARCH

- In GARCH, positivity restrictions on parameters make the ML estimation difficult. Why not exponential?
- In GARCH, new info is incorporated via the term
$$\alpha_1 \mu_{t-1}^2 = \alpha_1 \nu_{t-1}^2 \sigma_{t-1}^2$$
Why not separate the news ν_{t-1}^2 from non-news σ_{t-1}^2?
- EGARCH (Nelson, 1991, Econometrica, 59(2), p347-370)
  - Exponential functional form: no need to worry about positivity;
  - Separation of the effect of pure news;
  - Incorporation of asymmetric effect.

Exponential GARCH

- Model: $\mu_t|\Omega_{t-1} \sim N(0, \sigma_t^2)$,

$$\ln(\sigma_t^2) = \alpha_0 + \alpha_1|\nu_{t-1}| + \gamma\nu_{t-1} + \beta_1 \ln(\sigma_{t-1}^2),$$

  $-1 < \beta_1 < 1, \nu_{t-1} = \mu_{t-1}/\sigma_{t-1}$

  - if $\nu_{t-1} < 0$, its effect on $\ln(\sigma_t^2)$ is $(\alpha_1 - \gamma)|\nu_{t-1}|$.
    if $\nu_{t-1} \geq 0$, its effect on $\ln(\sigma_t^2)$ is $(\alpha_1 + \gamma)|\nu_{t-1}|$.

  - Negative shocks cause more volatility if and only if $\gamma < 0$.
    Reduced to symmetry if $\gamma = 0$.
  - $\sigma_t^2 = (\sigma_{t-1}^2)^{\beta_1} \exp\{\alpha_0 + \alpha_1|\nu_{t-1}| + \gamma\nu_{t-1}\}$

Exponential GARCH: persistence

- $\mu_t|\Omega_{t-1} \sim N(0, \sigma_t^2)$,

$$\ln(\sigma_t^2) = \alpha_0 + \alpha_1|\nu_{t-1}| + \gamma\nu_{t-1} + \beta_1 \ln(\sigma_{t-1}^2),$$

$-1 < \beta_1 < 1$, $\nu_{t-1} = \mu_{t-1}/\sigma_{t-1}$

  - By substitution, $\ln(\sigma_t^2) \approx \beta_1^{t-1}(\alpha_1|\nu_0| + \gamma\nu_0)$.
    Initial impact of the shock ν_0 on $\ln(\sigma_1^2)$: $(\alpha_1|\nu_0| + \gamma\nu_0)$.
  - The time for the initial impact to halve:

$$\beta_1^{t_H-1}(\alpha_1|\nu_0| + \gamma\nu_0) = \frac{1}{2}(\alpha_1|\nu_0| + \gamma\nu_0)$$

  - Half-life time: $t_H = \frac{\ln(1/2)}{\ln(\beta_1)} + 1$.

Example: EGARCH

eg. NYSE composite return: AR(1)-EGARCH $\hat{\gamma} = -0.1573$, significant

Dependent Variable: R
Method: ML - ARCH (Marquardt) - Normal distribution

Sample (adjusted): 3 1931
Included observations: 1929 after adjustments
Convergence achieved after 16 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: ON
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

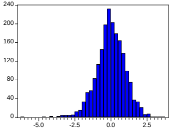
	Coefficient	Std. Error	z-Statistic	Prob.
C	[illegible]031749	0.020669	1.536066	0.1245
AR(1)	[illegible]	[illegible]	[illegible]	[illegible]
Variance Equation				
C(3)	-0.106711	0.021744	-4.907600	0.0000
C(4)	0.126534	0.028093	4.504123	0.0000
C(5)	-0.157348	0.023806	-6.609664	0.0000
C(6)	0.964484	0.006308	152.9036	0.0000

R-squared	0.002270	Mean dependent var	0.035168
Adjusted R-squared	-0.000324	S.D. dependent var	1.006452
S.E. of regression	1.006615	Akaike info criterion	2.557613
Sum squared resid	1948.524	Schwarz criterion	2.574922
Log likelihood	-2460.818	F-statistic	0.875099
Durbin-Watson stat	2.085495	Prob(F-statistic)	0.496926
Inverted AR Roots	.12		

Correlogram of Standardized Residuals Squared

Sample: 3 1931
Included observations: 1929
Q-statistic probabilities adjusted for 1 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.010	0.010	0.1833	
		2	0.014	0.014	0.5709	0.450
		3	[illegible]	[illegible]	1.2413	0.538
		4	[illegible]	[illegible]	1.2875	0.732
		5	[illegible]	[illegible]	2.0488	0.727
		6	-0.007	-0.007	2.1358	0.830
		7	0.010	0.010	2.3119	0.889
		8	0.001	-0.000	2.3128	0.941
		9	-0.006	-0.007	2.3923	0.967
		10	-0.013	-0.013	2.7264	0.974
		11	0.006	0.007	2.8066	0.986
		12	-0.003	-0.003	2.8231	0.993
		13	-0.004	-0.004	2.8484	0.997
		14	-0.001	-0.001	2.8501	0.998
		15	0.002	0.002	2.8613	0.999
		16	0.002	0.002	2.8689	1.000



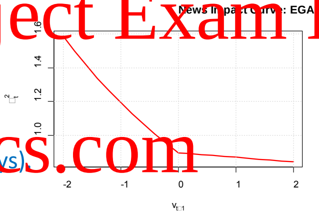
Series: Standardized Residuals
Sample 3 1931
Observations 1929

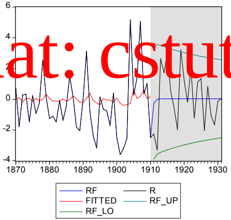
Mean	-0.005748
Median	0.000146
Maximum	3.521650
Minimum	-6.035894
Std. Dev.	1.002586
Skewness	-0.377514
Kurtosis	4.385068
Jarque-Bera	200.0118
Probability	0.000000

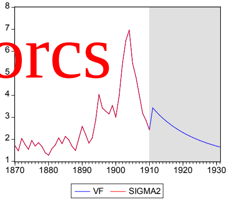
Example: EGARCH

eg. NYSE composite return:

- Asymmetric news impact.
- $\hat{\beta}_1 = 0.9645$, $t_\gamma = 20.2$ (days).
- Revert to mean quickly.







Example: EGARCH

eg. NYSE composite return:VaR.
Portfolio valued at $\$1m$ at $T = 2002 - 08 - 29$.
AR(1)-EGARCH:$\sigma_{T+1} = 1.482$, $y_{T+1|T} = 0.0124$
The 1% quantile of ν_t : $Q_{0.01} = -2.678$

$$VaR = \frac{1}{T}(y_{T+1|T} - 2.678\sigma_{T+1}) \times \$1m = -39,565$$

	σ_{T+1}	$y_{T+1\|T}$	$q_{0.01}$	VaR
AR(1)-ARCH(5)	1.253	0.050	−2.774	−34260
AR(1)-GARCH(1,1)	1.642	0.051	−2.873	−46660
AR(1)-GJR	1.577	0.019	−2.678	−42048
AR(1)-EGARCH	1.482	0.012	−2.678	−39565

GARCH in mean

- Risk premium effect: investing in a riskier asset should be rewarded by a higher expected return.

- In the context of a market index: investing in a riskier (more volatile) period should be rewarded by a higher expected return.
  - In AR(1) GARCH, the mean equation $y_t = c + \phi y_{t-1} + \mu_t$: implies the **expected return** $= y_t = c + \phi y_{t-1}$, which is unrelated to the volatility or risk measure σ_t.
  - Motivation: investors should be rewarded for taking additional risk by obtaining a higher return

- **GARCH-M** is used to account for the risk premium

$$y_t = c + \delta\sigma_{t-1} + \mu_t \qquad \mu_t|\Omega_{t-1} \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1\mu_{t-1}^2 + \beta_1\sigma_{t-1}^2,$$

where δ **measures the risk premium effect.**
(See Lundblad (2007, JFE, p123-150) among others.)

Example.

eg. NYSE composite return No evidence for the “risk premium” effect in any of GARCH(1,1), TGARCH/GJR and EGARCH

GARCH(1,1)

Dependent Variable: RC
Method: ML - ARCH (Marquardt) - Normal distribution

Sample (adjusted): 3 1931
Included observations: 1929 after adjustments
Convergence achieved after 16 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: OFF
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	0.076352	0.063905	1.194770	0.2322
C	0.016878	0.050651	0.333226	0.7390
AR(1)	0.103027	0.025309	4.070805	0.0000
Variance Equation				
C	0.013845	0.004594	3.013633	0.0026
RESID(-1)^2	0.120280	0.025125	4.787328	0.0000
GARCH(-1)	0.875387	0.021393	40.91840	0.0000

GJR

Dependent Variable: RC
Method: ML - ARCH (Marquardt) - Normal distribution

Sample (adjusted): 3 1931
Included observations: 1929 after adjustments
Convergence achieved after 17 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: OFF
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*RESID(-1)^2*(RESID(-1)<0)
+ C(7)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	0.002775	0.065764	0.042196	0.9663
C	0.035056	0.049298	0.711105	0.4770
AR(1)	0.114460	0.024099	4.749571	0.0000
Variance Equation				
C	0.020172	0.004755	4.242354	0.0000
RESID(-1)^2	-0.003509	0.018165	-0.193186	0.8468
RESID(-1)^2*(RESID(-1)<0)	0.210087	0.036850	5.701133	0.0000
GARCH(-1)	0.882810	0.019965	44.21804	0.0000

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Method: I
Sample (
Included
Converge
Bollerslev
Variance
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Summary

- We completed the ARCH/GARCH extensions that capture:
  - Leverage effect/Asymmetry in the returns volatility
  - Positivity of the volatility and the impossibility constraints
- Next... how about structural change in volatility?