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# The Effect of Asymmetries on Optimal Hedge Ratios*



There is widespread evidence that the volatility of stock returns displays an asymmetric response to good and bad news. This article considers the impact of asymmetry on time-varying hedges for financial futures. An asymmetric model that allows forecasts of cash and futures return volatility to respond differently to positive and negative return innovations gives superior in-sample hedging performance. However, the simpler symmetric model is not inferior in a hold-out sample. A method for evaluating the models in a modern risk-management framework is presented, highlighting the importance of allowing optimal hedge ratios to be both time-varying and asymmetric.

## I. Introduction

Over the past two decades, increases in the availability and usage of derivative securities has allowed agents who face price risk the opportunity to reduce their exposure. Although there are many techniques available for reducing and managing risk, the simplest and perhaps the most widely used is hedging with futures contracts. A hedge is achieved by taking opposite positions in spot and futures markets simultaneously, so that any loss sustained from an adverse price movement in one market should, to some degree, be offset by a favorable price movement in the other. The ratio of the number of units of the futures asset that are purchased relative to the number of units of the spot asset is known as the hedge ratio.



Since risk in this context is usually measured as the volatility of portfolio returns, an intuitively plausible strategy might be to choose the hedge ratio that min-

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imizes the variance of the returns of a portfolio containing the spot and futures position; this is known as the optimal hedge ratio. There has been much empirical research into the calculation of optimal hedge ratios (see, e.g., Cecchetti, Cumby, and Figlewski 1988; Myers and Thompson 1989; Baillie and Myers 1991; Kroner and Sultan 1991; Lien and Luo 1993; Lin, Najand, and Yung 1994; Strong and Dickinson 1994; Park and Switzer 1995).

The general consensus is that the use of multivariate generalized autoregressive conditionally heteroscedastic (MGARCH) models yields superior performances, evidenced by lower portfolio volatilities, than either time-invariant or rolling ordinary least squares (OLS) hedges. Cecchetti et al. (1988), Myers and Thompson (1989), and Baillie and Myers (1991), for example, argue that commodity prices are characterized by time-varying covariance matrices. As news about spot and futures prices arrives to the market, the conditional covariance matrix and, hence, the optimal hedging ratio become time-varying. Baillie and Myers (1991) and Kroner and Sultan (1991), inter alia, employ MGARCH models to capture time variation in the covariance matrix and the resulting hedge ratio.



On the other hand, there is also evidence that the benefits of a time-varying hedge are substantially diminished as the duration of the hedge is increased (e.g., Lin et al. 1994). Moreover, there is evidence that the use of volatility forecasts implied by options prices can further improve hedging effectiveness (Strong and Dickinson 1994).



This article has three main aims. First, we link the concept of the optimal hedge with Kroner and Ng's (1998) notion of the "news impact surface." The hedging surface of the OLS model is independent of news arriving to the market and therefore could be suboptimal. Second, we extend the models of Cecchetti et al. (1988), Myers and Thompson (1989), and Baillie and Myers (1991) to allow for time variation and asymmetry across the entire variance-covariance matrix of returns. This means that the hedge ratio will be sensitive to the size and sign of the change in prices resulting from information arrival. Third, we adapt the methods used by Hsieh (1993) to show how the effectiveness of hedges can be evaluated by the calculation of the minimum capital risk requirements (MCRRs). Such a procedure allows the hedging performance of the various models to be assessed using a relevant economic loss function as well as on purely statistical grounds.



The article is laid out in six sections. Section II presents the theoretical framework for deriving the hedge ratios, while Section III describes the data. Section IV presents the empirical evidence on the performance of each hedging model, while Section V outlines the bootstrap methodology used to calculate the MCRR for each of the portfolios. Section VI concludes.

## II. The Derivation of Optimal Hedge Ratios

Let $C_t$ and $F_t$ represent the logarithms of the stock index and stock index futures prices, respectively. The actual return on a spot position held from time $t-1$ to $t$ is $\Delta C_t = C_t - C_{t-1}$; similarly, the actual return on a futures position is $\Delta F_t = F_t - F_{t-1}$. However, at time $t-1$, the expected return, $E_{t-1}(R_t)$, of the portfolio comprising one unit of the stock index and $\beta$ units of the futures contract may be written as

$$E_{t-1}(R_t) = E_{t-1}(\Delta C_t) - \beta_{t-1} E_{t-1}(\Delta F_t), \tag{1}$$

where $\beta_{t-1}$ is the hedge ratio determined at time $t-1$, for employment in period $t$.[1] The variance of the portfolio may be written as

$$h_{p,t} = h_{C,t} + \beta_{t-1}^2 h_{F,t} - 2\beta_{t-1} h_{CF,t}, \tag{2}$$

where $h_{p,t}$, $h_{F,t}$, and $h_{C,t}$ represent the conditional variances of the portfolio and the spot and futures positions, respectively, and $h_{CF,t}$ represents the conditional covariance between the spot and futures position. If the agent has the two-moment utility function,



$$U(E_{t-1}R_t, h_{p,t}) = E_{t-1}(R_t) - \psi h_{p,t}, \tag{3}$$

then the utility maximizing agent with degree of risk aversion $\psi$ seeks to solve



$$\begin{aligned} &\max U(E_{t-1}R_t, h_{p,t}) \\ &= E_{t-1}(\Delta C_t) - \beta_{t-1} E_{t-t}(\Delta F) - \psi(h_{C,t} + \beta_{t-1}^2 h_{F,t} - 2\beta_{t-1} h_{CF,t}). \end{aligned} \tag{4}$$

Solving equation (4) with respect to $\beta$ under the assumption that $F_t$ is a martingale process such that $E_{t-1}(\Delta F_t) = E_{t-1}(F_t) - F_{t-1} = F_{t-1} - F_{t-1} = 0$ yields $\beta_{t-1}^*$, the optimal number of futures contracts in the investor's portfolio,



$$\beta_{t-1}^* = -\frac{h_{CF,t}}{h_{F,t}}. \tag{5}$$

If the conditional variance–covariance matrix is time-invariant (and if $C_t$ and $F_t$ are not cointegrated), then an estimate of $\beta^*$, the constant optimal hedge ratio, may be obtained from the estimated slope coefficient $b$ in the regression

$$\Delta C_t = a + b\Delta F_t + u_t. \tag{6}$$

The OLS estimate of $b = h_{CF}/h_F$ is also valid for the multiperiod hedge in the case where the investors' utility function is time separable.

However, it has been shown by numerous studies (see Sec. I above) that the data do not support the assumption that the variance-covariance matrix of returns is constant over time. Therefore, we follow recent literature by employing a bivariate generalized autoregressive conditional heteroscedasticity (GARCH) model, which allows the conditional variances and covariances used as inputs to the hedge ratio to be time-varying.

1. Note that we are not requiring at this stage that the hedge ratio, $\beta_{t-1}$, be time-varying but, rather, that it is determined using information to time $t-1$.

In the absence of transactions costs, market microstructure effects, or other impediments to their free operation, the efficient markets hypothesis and the absence of arbitrage opportunities imply that the spot and corresponding futures markets react contemporaneously and identically to new information. There has been some debate in the literature as to whether this implies that the two markets must be cointegrated. Ghosh (1993), for example, suggests that market efficiency should imply that cash and futures are cointegrated, while Baillie and Myers (1991) suggest that, as a consequence of possible nonstationarity of the risk-free proxy employed in the cost-of-carry model, this need not be the case. We do not wish to enter into this debate from a theoretical viewpoint, but suffice to say that in all of our ensuing analysis, we allow for, but do not impose, a $[-1, 1]$ cointegrating vector for the two series. The conditional mean equations of the model employed in this article are a bivariate Vector Error Correction Mechanism (VECM), which may be written as



$$\Delta Y_t = \mu + \sum_{i=1}^{4} \Gamma_i \Delta Y_{t-i} + \Pi v_{t-1} + \varepsilon_t$$

$$Y_t = \begin{bmatrix} F_t \\ C_t \end{bmatrix}; \mu = \begin{bmatrix} \mu_F \\ \mu_C \end{bmatrix}; \Gamma_i = \begin{bmatrix} \Gamma_{i,F}^{(F)} & \Gamma_{i,C}^{(F)} \\ \Gamma_{i,F}^{(C)} & \Gamma_{i,C}^{(C)} \end{bmatrix}; \Pi = \begin{bmatrix} \pi_F \\ \pi_C \end{bmatrix}; \varepsilon_t = \begin{bmatrix} \varepsilon_{F,t} \\ \varepsilon_{C,t} \end{bmatrix}. \tag{7}$$



Under the assumption $\varepsilon_t | \Omega_t \sim (0, H_t)$, where $\epsilon_t$ represents the innovation vector in equation (6) and $v_{t-1}$ represents an error correction term, and by defining $h_t$ as $vech(H_t)$, where $vech(.)$ denotes the vector-half operator that stacks the lower triangular elements of an $N \times N$ matrix into an $[N(N+1)/2] \times 1$ vector, the bivariate VECM($p$) GARCH(1,1) *vech* model may be written as



$$vec(H_t) = h_t = \begin{bmatrix} h_{C,\ t} \\ h_{CF,\ t} \\ h_{F,\ t} \end{bmatrix} = C_0 + A_1 vec(\varepsilon_{t-1}\varepsilon_{t-1}') + B_1 h_{t-1}, \tag{8}$$

where

$$c_0 \begin{bmatrix} c_{11} \\ c_{12} \\ c_{22} \end{bmatrix}; A_1 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}; B_1 = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}.$$

Restricting the matrices $A_1$ and $B_1$ to be diagonal gives the model proposed by Bollerslev, Engle, and Wooldridge (1988), where each element of the conditional variance-covariance matrix $h_{ij,\ t}$ depends on past values of itself and past values of $\varepsilon_{t-1}\varepsilon_{t-1'}$. There are 21 parameters in the conditional variance-covariance structure of the bivariate $GARCH$(1,1) *vech* model (eq. [8]) to be estimated, subject to the requirement that $H_t$ be positive-definite for all values of $\varepsilon_t$ in the sample. The difficulty of checking, let alone imposing such a restriction, led Engle and Kroner (1995) to propose the Bollerslev, Engle, Kroner, and Kraft (BEKK) parameterization:

$$H_t = C_0^{*\prime} C_0^* + A_{11}^{*\prime} \varepsilon_{t-1} \varepsilon_{t-1}' A_{11}^* + B_{11}^{*\prime} H_{t-1} B_{11}^*. \tag{9}$$

The BEKK parameterization requires estimation of only 11 parameters in the conditional variance-covariance structure and guarantees $H_t$ to be positive definite. It is important to note that the BEKK and *vec* models imply that only the magnitude of past return innovations is important in determining current conditional variances and covariances. This assumption of symmetric time-varying variance-covariance matrices must be considered tenuous given the existing body of evidence documenting the asymmetric response of equity volatility to positive and negative innovations of equal magnitude (see Engle and Ng 1993; Glosten, Jagannathan, and Runkle 1993; Kroner and Ng 1998, inter alia).

Defining $\xi_{i,t} = \min\{\varepsilon_i, 0\}$, for $i$ = futures, cash, the BEKK model in equation (9) may be extended to allow for asymmetric responses as



$$H_t = C_0^{*\prime} C_0^* + A_{11}^{*\prime} \varepsilon_{t-1} \varepsilon_{t-1}' A_{11}^* + B_{11}^{*\prime} H_{t-1} B_{11}^* + D_{11}^{*\prime} \xi_{t-1} \xi_{t-1}' D_{11}^*, \tag{10}$$

where



$$C_0^* = \begin{bmatrix} c_{11}^* & c_{12}^* \\ 0 & c_{22}^* \end{bmatrix}; A_{11}^* = \begin{bmatrix} \alpha_{11}^* & \alpha_{12}^* \\ \alpha_{21}^* & \alpha_{22}^* \end{bmatrix};$$



$$B_{11}^* = \begin{bmatrix} \beta_{11}^* & \beta_{12}^* \\ \beta_{21}^* & \beta_{22}^* \end{bmatrix}; D_{11}^* = \begin{bmatrix} \delta_{11}^* & \delta_{12}^* \\ \delta_{21}^* & \delta_{22}^* \end{bmatrix}; \text{ and } \xi_t = \begin{bmatrix} \xi_{F,t} \\ \xi_{C,t} \end{bmatrix}. \tag{11}$$

The symmetric BEKK model (eq. [9]) is given as a special case of equation (10) for $\delta_{m,n} = 0$, for all values of $m$ and $n$.

## III. Data Description

The data employed in this study comprise 3,580 daily observations on the FTSE 100 stock index and stock index futures contract spanning the period January 1, 1985–April 9, 1999.[2] Days corresponding to U.K. public holidays are removed from the series to avoid the incorporation of spurious zero returns.

The FTSE 100 comprises the 100 U.K. companies, quoted on the London Stock Exchange, with the largest market capitalization and accounting for 73.2% of the market value of the FTSE All Share Index on December 29, 1995 (Sutcliffe 1997). The FTSE 100 futures contracts are quoted in the same units as the underlying index, except that the decimal is rounded to the nearest

2. Since these contracts expire four times per year—March, June, September, and December—we obtain a continuous time series by using the closest-to-maturity contract unless the next closest has greater volume, in which case we switch to this contract.

0.5.[3] The price of a futures contract (contract size) is the quoted number (measured in index points) multiplied by the contract multiplier, which is £25 for the contract. There are four delivery months: March, June, September, and December. Trading takes place in the 3 nearest delivery months, although volume in the "far" contract is very small. Each contract is therefore traded for 9 months. The FTSE100 futures contracts are cash-settled as opposed to physical delivery of the underlying. All contracts are marked to market on the last trading day, which is the third Friday in the delivery month, at which point all positions are deemed closed. For the FTSE100 futures contract, the settlement price on the last trading day is calculated as an average of minute-by-minute observations between 10:10 A.M. and 10:30 A.M., rounded to the nearest 0.5.

Summary statistics for the data are displayed in panel A of table 1. Using Dickey Fuller unit root tests, it is not possible to reject the null hypothesis of nonstationarity for the cash and futures price series. This nonstationarity of the price series is consistent with weak-form efficiency of the cash and futures markets. The return series are calculated as 100 × ($C_t/C_{t-1}$) and 100 × ($F_t/F_{t-1}$), respectively. The returns are skewed to the left, leptokurtic, and stationary. These features are entirely in accordance with expectations and results presented elsewhere. In the absence of a long-run relationship between $C_t$ and $F_t$, optimal inference based on asymptotic theory requires the use of returns rather than price data in calculating the estimation of dynamic hedge ratios.

Results for both Engle and Granger (1987) and Johansen (1988) tests for cointegration are displayed in table 1. The Engle and Granger results of table 1, panel B, clearly demonstrate that the null of nonstationarity in the residuals of the cointegrating regression is strongly rejected, for the test both with and without a constant term. Moreover, the estimated slope coefficient is very close to unity, whether the spot or futures price is the dependent variable. Similarly, the Johansen test statistics, for both the trace and the max forms, reject the null of no cointegrating vector but do not reject the null of one cointegrating vector. A restriction of the cointegrating relationship between the series to be [1, −1] was marginally rejected at the 5% level. However, after normalizing the estimated cointegration vector on $C_t$, the estimated coefficient on $F_t$ was −1.006, suggesting that this rejection may not be economically important. On close examination of the short-run components of the VECM, it appears that the futures prices are weakly exogenous. A likelihood ratio test supports this restriction. Thus, while the cointegrating equilibrium is defined by both cash and futures prices, equilibrium is restored through the cash markets. A test of the joint hypothesis that futures prices are weakly exogenous and that the parameters of the cointegration vector are [1,−1] was not rejected at the 5% level of significance. Baillie and Myers

3. The reason for this is that the minimum price movement (known as tick) for the futures contract is £12.50, i.e., a change of 0.5 in the index.

**TABLE 2** **Estimates of the Multivariate Asymmetric GARCH Model**

Conditional Mean Equations

$$\Delta Y_t = \mu + \sum_{i=1}^{4} \Gamma_i \Delta Y_{t-i} + \Pi v_{t-1} + \varepsilon_t$$

$$Y_t = \begin{bmatrix} F_t \\ C_t \end{bmatrix}; \mu = \begin{bmatrix} \mu_F \\ \mu_C \end{bmatrix}; \Gamma_i = \begin{bmatrix} \Gamma_{i,F}^{(F)} & \Gamma_{i,C}^{(F)} \\ \Gamma_{i,F}^{(C)} & \Gamma_{i,C}^{(C)} \end{bmatrix};$$

$$\Pi = \begin{bmatrix} \pi_F \\ \pi_C \end{bmatrix}; \varepsilon_t = \begin{bmatrix} \varepsilon_{F,t} \\ \varepsilon_{C,t} \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} -.0078 & -.0225 \\ (.0060) & (.0072) \\ & \\ .0759 & .0257 \\ (.0053) & (.0061) \end{bmatrix} \qquad \Gamma_2 = \begin{bmatrix} -.1499 & .1399 \\ (.0089) & (.0110) \\ & \\ .0272 & .0238 \\ (.0080) & (.0092) \end{bmatrix}$$

$$\Gamma_3 = \begin{bmatrix} -.1225 & .1083 \\ (.0111) & (.0149) \\ & \\ -.0352 & .0293 \\ (.0114) & (.0117) \end{bmatrix} \qquad \Gamma_4 = \begin{bmatrix} -.0699 & .0084 \\ (.0227) & (.0256) \\ & \\ .0141 & -.0032 \\ (.0182) & (.0232) \end{bmatrix}$$

$$\mu = \begin{bmatrix} .0518 \\ (.0057) \\ \\ .0523 \\ (.0050) \end{bmatrix} \qquad \Pi = \begin{bmatrix} .1636 \\ (.0195) \\ \\ -.1719 \\ (.0142) \end{bmatrix}$$

$$\kappa \quad \begin{matrix} 6.0606 \\ (.2454) \end{matrix}$$



NOTE.—SEs are displayed in parentheses.

(1991) argue that a perfect one-to-one association does not exist in a commodity futures hedge because of the cost of carry, although this does not preclude some other cointegrating relationship from existing. On balance, the data appear to be cointegrated with a [1,−1] cointegrating vector.

## IV. Hedging Model Estimates, Tests, and Performance

Given the evidence of a long-run or cointegrating relationship between $C_t$ and $F_t$, the conditional mean equations are parameterized as a VECM rather than a vector autoregression (VAR) to avoid the loss of long-run information.

The parameter estimates and associated residual diagnostics for the multivariate asymmetric GARCH model are presented in tables 2–4. Again, the factor loading associated with the futures prices is positive, indicating that the return to equilibrium is achieved via the cash markets. A high degree of persistence in variance is indicated in both markets. The persistence is measured by $\alpha_{ii}^2 + \beta_{ii}^2$ for $i = 1, 2$. The statistical significance of the elements of the $D_{11}^*$ matrix indicates the presence of asymmetries in the variance-covariance matrix.

Kroner and Ng (1998) analyze the asymmetric properties of time-varying covariance matrix models, identifying three possible forms of asymmetric behavior. First, the covariance matrix displays "own variance asymmetry" if

**TABLE 3** **Estimates of the Multivariate Asymmetric GARCH Model**

| | Residual Diagnostics | | | | | |
|---|---|---|---|---|---|---|
| | Mean | Variance | Skewness | Excess Kurtosis | $Q(10)$ | $Q^2(10)$ |
| $\varepsilon_{F,t}$ | −.0023 | 1.0790 | −.9077 | 12.7237 | 13.3361 | 2.1991 |
| | | | [.0000] | [.0000] | [.2055] | [.9946] |
| $\varepsilon_{C,t}$ | −.0079 | 1.0438 | .4578 | 5.9459 | 12.0461 | 7.6730 |
| | | | [.0000] | [.0000] | [.2820] | [.6607] |

NOTE.—Marginal significance levels are displayed in brackets. The $Q(10)$ and $Q^2(10)$ are Ljung Box tests for tenth-order serial correlation in $z_{j,t}$ and $z_{j,t}^2$, respectively, for $j = F, C$.

$h_{C,t}(h_{F,t})$, the conditional variance of $C_t(F_t)$, is affected by the sign of the innovation in $C_t(F_t)$. Second, the covariance matrix displays cross "variance asymmetry" if the conditional variance of $C_t(F_t)$ is affected by the sign of the innovation in $F_t(C_t)$. Finally, if the covariance of returns $h_{CF,t}$ is sensitive to the sign of the innovation in return for either $C_t$ or $F_t$, then the model is said to display covariance asymmetry.

The residual diagnostics indicate that the model was able to capture all of the dependence on past values in both the conditional mean and conditional variances for both the spot and futures equations. The coefficients of skewness and excess kurtosis are much reduced relative to their values on the raw data, again indicating a reasonable fit of the model to the two series. The robust likelihood ratio tests suggested by Kroner and Ng (1998) to detect such asymmetry in MGARCH models indicate that the asymmetric model provides a superior data characterization to the symmetric MGARCH(1, 1). The final row of table 4 tests the restriction of the asymmetric model to be symmetric; that is, a restriction that good and bad news affect the volatility of the spot and futures markets equally. This restriction is clearly rejected, suggesting that the pursuit of an asymmetric model is important and may yield superior



**TABLE 4** **Estimates of the Multivariate Asymmetric GARCH Model**

Conditional Variance—Covariance Structure

$$H_t = C_0^{*\prime}C_0^* + A_{11}^{*\prime}\epsilon_{t-1}\epsilon_{t-1}^{\prime}A_{11}^* + B_{11}^{*\prime}H_{t-1}B_{11}^* + D_{11}^*\xi_{t-1}\xi_{t-1}^{\prime}D_{11}^*$$

$$\varepsilon_{t-1} = \begin{bmatrix}\varepsilon_{F,t-1}\\ \varepsilon_{C,t-1}\end{bmatrix}; \xi_{t-1} = \begin{bmatrix}\min(\varepsilon_{F,t-1}, 0)\\ \min(\varepsilon_{C,t-1}, 0)\end{bmatrix}$$

$$C_0^* = \begin{bmatrix} \underset{(.0184)}{.1680} & \underset{(.0151)}{.1488} \\ 0 & \underset{(.0036)}{-.0131} \end{bmatrix} \qquad B_{11}^* = \begin{bmatrix} \underset{(.0077)}{.9785} & \underset{(.0067)}{-.0031} \\ \underset{(.0080)}{-.0217} & \underset{(.0072)}{.9633} \end{bmatrix}$$

$$A_{11}^* = \begin{bmatrix} \underset{(.0208)}{-.1198} & \underset{(.00170)}{.0305} \\ \underset{(.0289)}{-.0611} & \underset{(.0239)}{-.2144} \end{bmatrix} \qquad D_{11}^* = \begin{bmatrix} \underset{(.0885)}{.3685} & \underset{(.0717)}{.3528} \\ \underset{(.1048)}{-.2172} & \underset{(.0892)}{.2576} \end{bmatrix}$$

$H_0$: $\delta_{m,n} = 0$ for $m, n = 1, 2$ 30.7106 [.0000]

NOTE.—SEs are displayed in parentheses. Marginal significance levels are displayed in brackets.

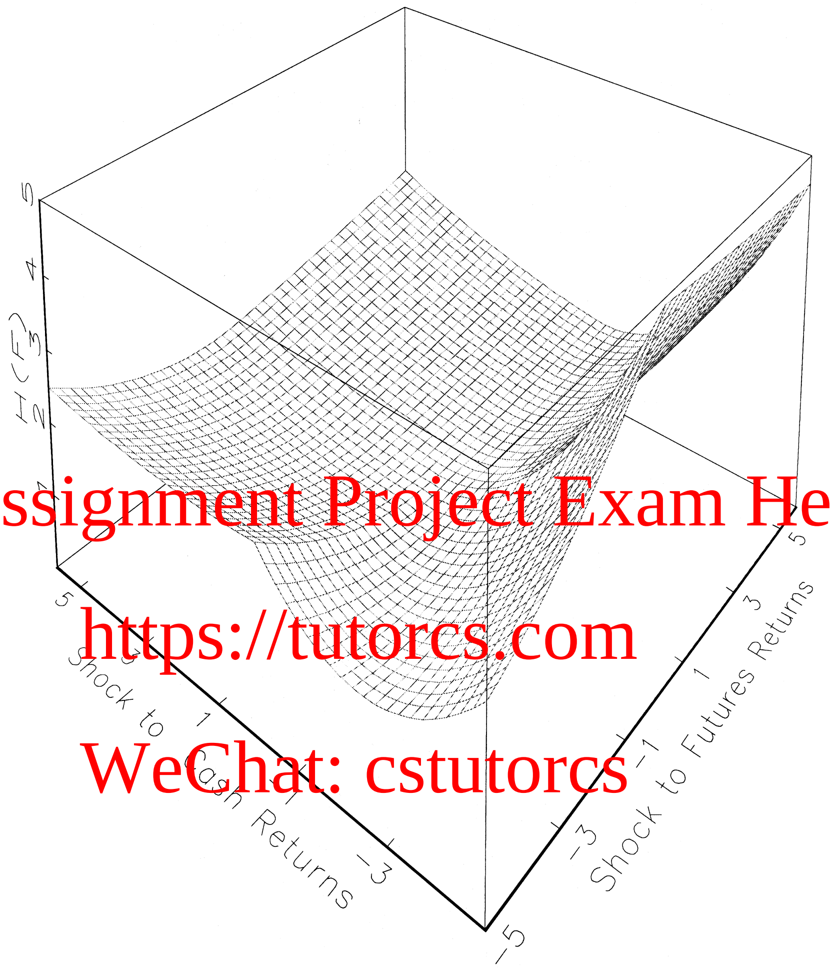




FIG. 1.—News impact surface for futures market volatility

hedging performance relative to a model that ignores this feature, which is manifest in the data.

The price innovations, $C_t - C_{t-1} = \varepsilon_{C,t}$ and $F_t - F_{t-1} = \varepsilon_{F,t}$, represent changes in information available to the market (ceteris paribus). Kroner and Ng (1998) treat such innovations as a collective measure of news arriving to market $j$ between the close of trade on period $t - 1$ and the close of trade on period $t$. They define the relationship between innovations in return and the conditional variance-covariance structure as the “news impact surface,” a multivariate form of the news impact curve of Engle and Ng (1993). Figures 1–3 display the variance and covariance news impact surfaces from the estimates

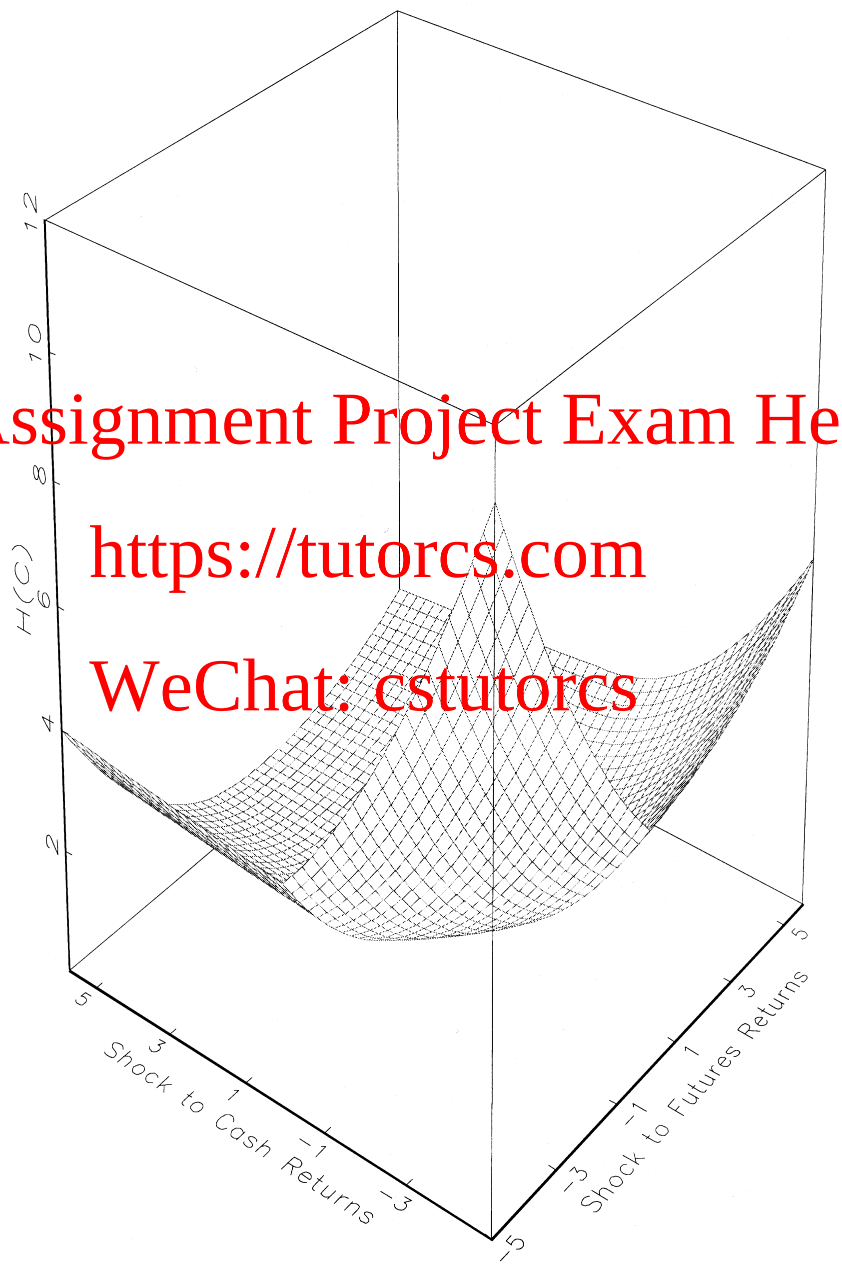




FIG. 2.—News impact surface for cash market volatility

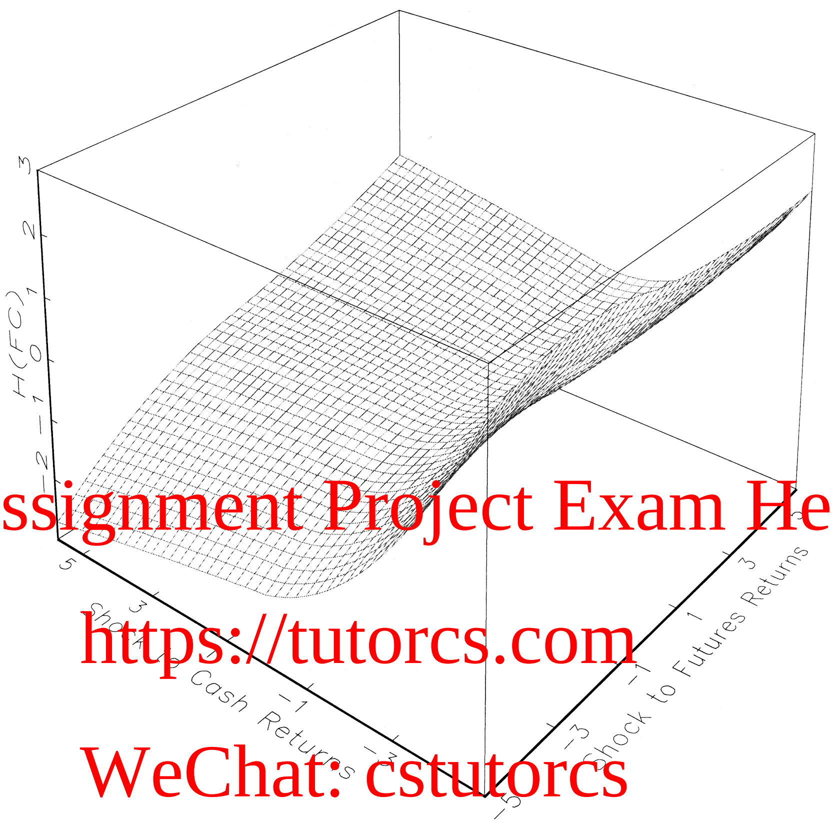




FIG. 3.—Covariance news impact surface

displayed in tables 2–4. Following Kroner and Ng (1998) and Engle and Ng (1993), each surface is evaluated in the region $\varepsilon_{j,t} = [-5, 5]$ for $j =$ futures, cash. There are relatively few extreme outliers in the data, which suggests that some caution should be exercised in interpreting the news impact surfaces for larger values of $\varepsilon_{j,t}$. Despite this caveat, the asymmetry in variance and covariance is clear from each figure.

The returns and variances for the various hedging strategies are presented in table 5. The simplest approach, presented in the second column, is that of no hedge at all. In this case, the portfolio simply comprises a long position in the cash market. Such an approach is able to achieve significant positive returns in sample, but with a large variability of portfolio returns. Although none of the alternative strategies generate returns that are significantly different from zero, either in sample or out of sample, it is clear from columns 3–5 of table 5 that any hedge generates significantly less return variability than none at all.

**TABLE 5** **Portfolio Returns**

| | Unhedged $\beta = 0$ | Naive Hedge $\beta = -1$ (C.I. HEDGE) | Symmetric Time-Varying Hedge $\beta^*_{t-1} = h_{FC,t}/h_{F,t}$ | Asymmetric Time-Varying Hedge $\beta^*_{t-1} = h_{FC,t}/h_{F,t}$ |
|---|---|---|---|---|
| In sample: | | | | |
| Return | .0389 | −.0003 | .0061 | .0060 |
| | (2.3713) | (−.0351) | (.9562) | (.9580) |
| Variance | .8286 | .1718 | .1240 | .1211 |
| Out of sample: | | | | |
| Return | .0819 | −.0004 | .0120 | .0140 |
| | (1.4958) | (.0216) | (.7761) | (.9083) |
| Variance | 1.4972 | .1696 | .1186 | .1188 |

NOTE.—$t$-ratios displayed in parentheses.

The naive or cointegrating hedge, which takes one short futures contract for every spot unit but does not allow the hedge to time-vary, generates a reduction in variance of the order of 80% in sample and nearly 90% out of sample relative to the unhedged position. Allowing the hedge ratio to be time-varying and determined from a symmetric multivariate GARCH model leads to a further reduction as a proportion of the unhedged variance of 5% and 2% on the in- and hold-out samples, respectively. Allowing for an asymmetric response of the conditional variance to positive and negative shocks yields a very modest reduction in variance (a further 0.5% of the initial value) in sample and virtually no change out of sample.

Figure 4 graphs the time-varying hedge ratio from the symmetric and asymmetric MGARCH models. The optimal hedge ratio is never greater than 0.9586 futures contracts per index contract, with an average value of 0.8177 futures contracts sold per long index contract. The variance of the estimated optimal hedge ratio is 0.0019. Moreover, the optimal hedge ratio series obtained through the estimation of the asymmetric GARCH model appears stationary. An Augmented Dickey Fuller (ADF) test (see, e.g., Fuller 1976) of the null hypothesis $\beta^*_{t-1} \sim I(1)$ was strongly rejected by the data (ADF $= -5.7215$, 5% critical value $= -2.8630$). The time-varying hedge requires the sale (purchase) of fewer futures contracts per long (short) index contract.[4]

The optimal hedge ratio, $\beta^*_{t-1}$, may be linked to the arrival of news to the market using equation (5) and the relevant futures price and covariance news impact surfaces. Evaluating $\beta^*_{t-1}$ in the range $\varepsilon_{j,t} = [-5, 5]$ for $j =$ futures, cash as before gives us the response of the optimal hedge to news. Note that the surface is drawn under the assumption that the portfolio comprises a long position in the stock index and that the optimal hedge ratio is written in terms of the number of futures contracts to sell. A negative optimal hedge ratio thus implies the purchase of futures contracts. Figure 5 graphs the response of $\beta^*_{t-1}$ to news.

4. Although, of course, a time-varying hedge may result in considerably increased transactions costs in the likely event that such a hedge requires daily adjustments of the futures position. We therefore cannot state categorically that the time-varying hedge would be cheaper.

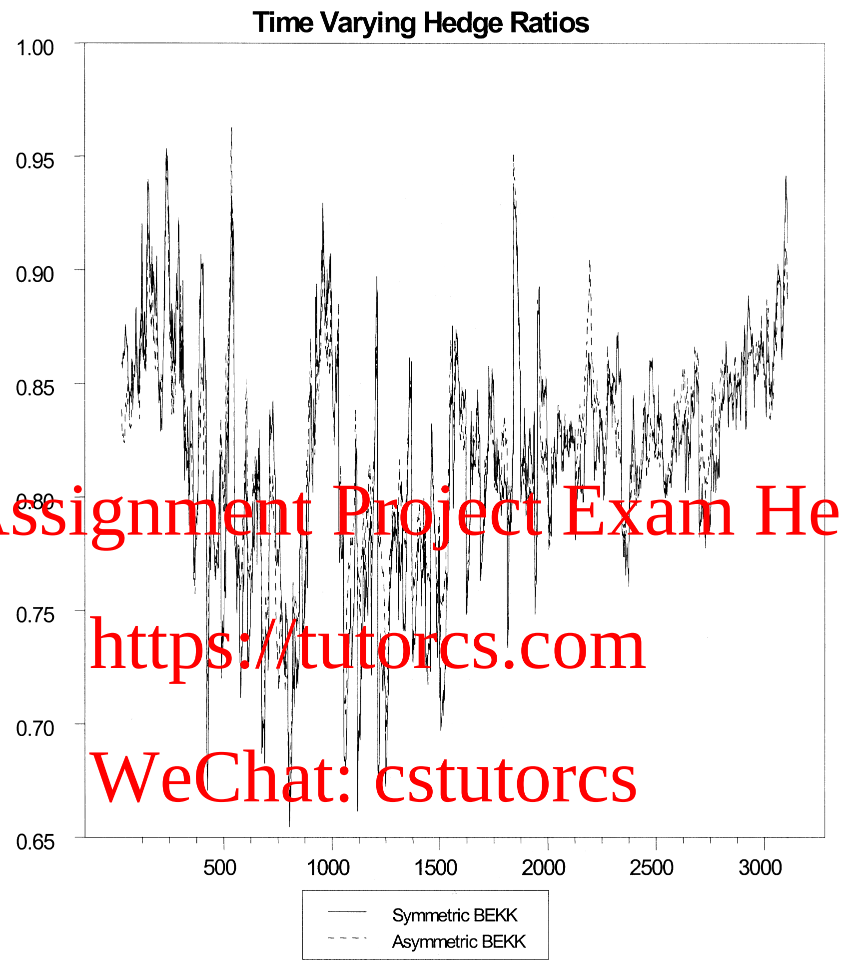




FIG. 4.—The optimal dynamic hedge ratio, $\beta^*_{t-1}$

It is worth noting that $\beta^*_{t-1}$ responds far more dramatically to bad news about the cash market index than to news about the future price. Negative innovations in the cash price cause the optimal hedge ratio to increase in magnitude toward one. Large positive innovations in the cash price suggest a negative hedge ratio. This might appear counterintuitive; however, the surface is drawn holding past information constant. The implication of the asymmetry is that the hedge has very low value in bull market situations. In contrast, the cointegrating hedge implies that the hedging surface is a plane at $\beta^*_{t-1} = \beta = 1$. One possible interpretation of the better performance of the dynamic strategies over the naive hedge is that the dynamic hedge uses short-run information, while the cointegrating hedge is driven by long-run consid-

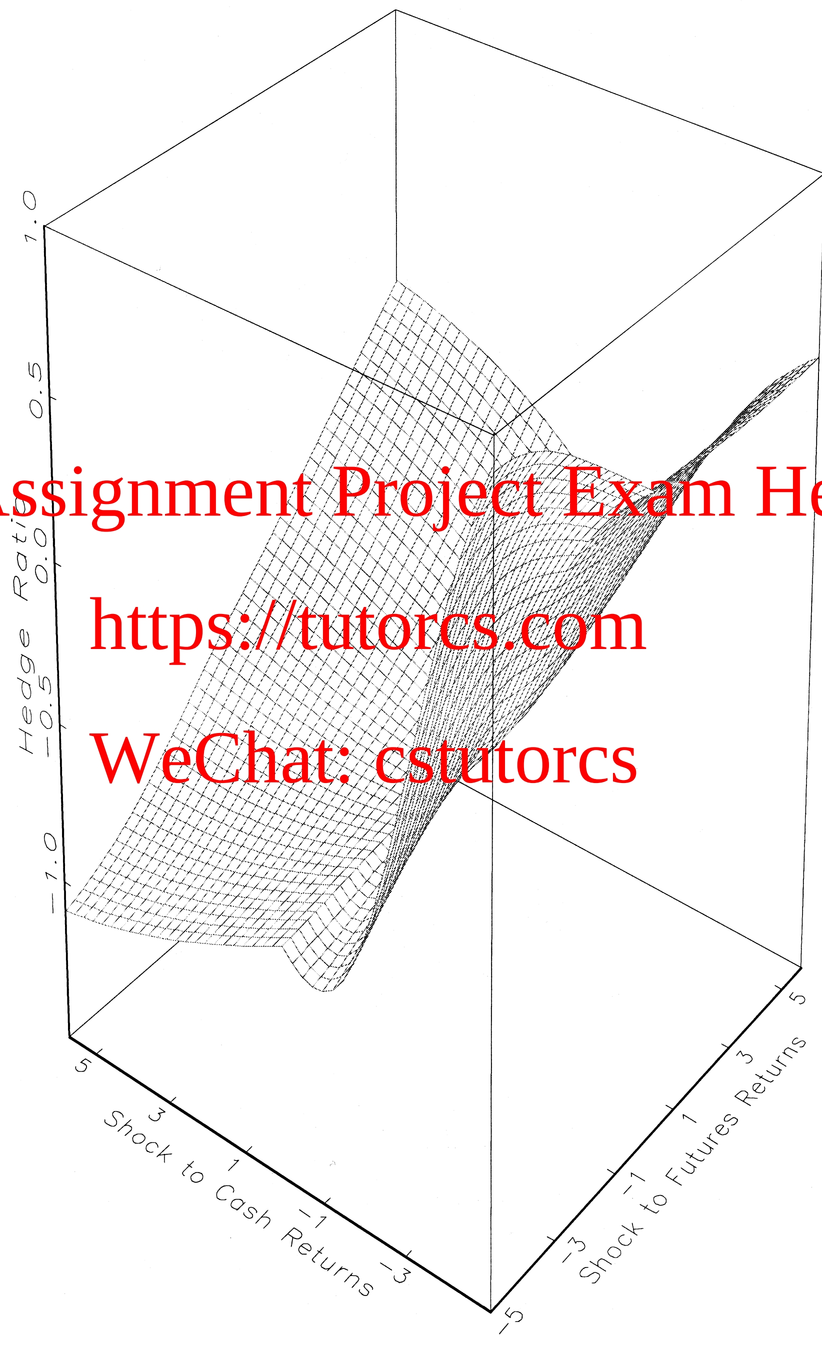




FIG. 5.—Hedging surface: The response of $\beta_t^*$ to news

erations. The performance evaluation in table 5 is in terms of 1-day-ahead hedges. In Section V, we use a new criterion to judge hedging over various horizons, including the 1-day horizon.

## V. Evaluating Hedging Effectiveness by Calculating Minimum Capital Risk Requirements

Ensuring that banks hold sufficient capital to meet possible future losses has been a topic of great import for regulators and risk managers in recent years. A very popular approach involves the calculation of the institution's value at risk (VaR) inherent in its trading book positions. The VaR is an estimation of the probability of likely losses that might occur from changes in market prices from a particular securities position, and the minimum capital risk requirement (MCRR) is defined as the minimum amount of capital required to absorb all but a prespecified percentage of these possible losses. We address an approach to the calculation of MCRRs that is similar in spirit to the approach adopted in many Internal Risk Management Models (IRMM), proposed by Hsieh (1993).[5]



Capital risk requirements are estimated for 1-day, 10-day, 30-day, 3-month, and 6-month investment horizons by simulating the conditional densities of price changes, using Efron's (1982) bootstrapping methodology, which is based on the multivariate GARCH(1, 1) model presented in equations (7) and (9), both with and without asymmetries, for comparison. The simulated errors are generated by drawing randomly, with replacement, from the standardized residuals, and, hence, a path of future $\Delta Y_t$'s can be generated, using the estimates of $\mu$, $\Gamma$, $\Pi$, $C_0$, $A_{11}$, and $B_{11}$, from the sample and multistep-ahead forecasts of $H_t$.



A securities firm wishing to calculate the VaR of a portfolio containing the cash and futures assets would have to simulate the price of the assets when it initially opened the position.[6] To calculate the appropriate capital risk requirement, it would then have to estimate the maximum loss that the position might experience over the proposed holding period.[7] For example, by tracking the daily value of a long cash and short futures position and by recording its lowest value over the sample period, the firm can report its maximum loss for this particular simulated path of cash and futures prices. Repeating this

5. See also Brooks, Clare, and Persand (2000) for a more detailed description of this methodology and issues in its implementation.

6. See Dimson and Marsh (1997) for a discussion of a number of potential issues that a financial institution may face when calculating appropriate levels of capital for multiple positions during periods of stress.

7. The current BIS rules state that the MCRR should be the higher of the (i) average MCRR over the previous 60 days, or (ii) the previous trading days' MCRR.

procedure for 20,000 simulated paths generates an empirical distribution of the maximum loss. This maximum loss ($Q$) is given by

$$Q = (x_0 - x_1), \tag{12}$$

where $x_0$ is the initial value of the portfolio and $x_1$ is the lowest simulated value of the portfolio (for a long futures position) or the highest simulated value (for a short futures position) over the holding period. We can express the maximum loss as a proportion of the initial value of the portfolio as follows:

$$\frac{Q}{x_0} = \left(1 - \frac{x_1}{x_0}\right). \tag{13}$$

In this case, since $x_0$ is a constant, the distribution of $Q$ will depend on the distribution of $x_1$.

From equation (13), it can be seen that the distribution of $Q/x_0$ will depend on the distribution of $x_1/x_0$. Hence, the first step is to find the fifth quantile of $\ln(x_1/x_0)$:



$$\frac{\ln(x_1/x_0) - m}{\mathrm{SD}} = \pm\alpha, \tag{14}$$



where $\alpha$ is the fifth quantile from a standard normal distribution, $m$ is the mean of $\ln(x_1/x_0)$, and SD is the standard deviation of $\ln(x_1/x_0)$. Cross-multiplying and taking the exponential,

$$x_1 = x_0 \times \text{Exponential}[(\pm\alpha \times Sd) + m]; \tag{15}$$



therefore,

$$Q = x_0 \times \{1 - \text{Exponential}[(\pm\alpha \times Sd) + m]\}. \tag{16}$$

In this article, we compare the MCRRs generated by the portfolios constructed using the hedge ratios derived from the models described above. The asymmetric multivariate GARCH model appears well specified and able to capture the salient features of the data. Given this, we now determine what would be an appropriate amount of capital to cover the cash and futures portfolio derived from the hedge ratio as implied by the model. In particular, we consider whether this portfolio minimizes the need for capital, given that all such capital is tied up in an unproductive and unprofitable fashion.

The estimated minimum capital risk requirements are presented in tables 6 and 7 for each of the models—ignoring and allowing for asymmetries, respectively—and are given in units of index points.[8] Panel A of tables 6 and 7 present the MCRR for a short hedge (long cash, short futures), while panel B of these tables presents the results for a long hedge (long futures, short

8. See Sec. III. Although Hsieh (1993) and Brooks et al. (2000) measure MCRRs as a proportion of the initial value of the position, this is not sensible in our case since by definition an appropriately hedged portfolio will have a zero value.

**TABLE 6 MCRR Estimates—Symmetric Hedging Models**

| Days | Unhedged | Naive Hedge | Time-Varying Hedge |
|---|---|---|---|
| A. Long cash and short futures: | | | |
| 1 | 27.851 | 22.175 | 11.763 |
| 10 | 211.210 | 99.819 | 96.308 |
| 20 | 234.215 | 197.217 | 124.214 |
| 30 | 358.872 | 238.632 | 167.297 |
| 60 | 411.058 | 425.661 | 245.312 |
| 90 | 513.368 | 499.756 | 293.263 |
| 180 | 651.402 | 569.952 | 378.451 |
| B. Short cash and long futures: | | | |
| 1 | 49.525 | 25.783 | 16.294 |
| 10 | 260.847 | 147.355 | 84.773 |
| 20 | 385.323 | 217.493 | 176.856 |
| 30 | 414.618 | 258.481 | 216.965 |
| 60 | 667.067 | 320.512 | 290.489 |
| 90 | 943.051 | 567.666 | 348.487 |
| 180 | 1,290.627 | 761.248 | 537.951 |



cash). The most important feature of the results is that any type of hedge, even a naive hedge, is better than a naked exposure. Moreover, at short investment horizons, there are large gains to be made by allowing the hedge to vary over time. For example, the short hedge portfolio MCRR is 22.2 index points for a naive hedge but only 11.8 for a Multivariate GARCH hedge. The long hedge positions seem to be more risky overall over our out-of-sample period, generating higher values at risk than the corresponding short hedges.





The gain from the use of an asymmetric model, as opposed to a constrained symmetric model, which does not allow good and bad news to effect returns differently, is large at short time horizons. For example, for the symmetric time-varying short hedge, the portfolio MCRR is 11.8, while modeling the asymmetries reduces this to 2.0. However, the benefits of these more complex asymmetric and time-varying hedges and, moreover, the benefits of hedging per se are considerably reduced as the time horizon is extended beyond 1 month. For example, the MCRR for a long hedge calculated using asymmetric MGARCH is less than 10% of that using no hedge at the 1-day horizon but rises to more than 25% over a 6-month hedging period. This result is in agreement with the findings of Lin et al. (1994).

## VI. Conclusions

This article seeks to advance the extant literature in this field by considering the impact of asymmetries on the hedging of stock-index positions using financial futures contracts.[9] We find that asymmetric models, which allow

9. However, the methodology could, of course, be equally applied to hedging a position in any financial asset using futures contracts.

**TABLE 7 MCRR Estimates—Asymmetric Hedging Models**

| Days | Unhedged | Naive Hedge | Time-Varying Hedge |
|---|---|---|---|
| A. Long cash and short futures: | | | |
| 1 | 20.792 | 2.356 | 2.003 |
| 10 | 196.812 | 83.475 | 74.268 |
| 20 | 237.567 | 182.852 | 96.776 |
| 30 | 370.988 | 228.123 | 155.325 |
| 60 | 416.221 | 416.632 | 229.875 |
| 90 | 529.219 | 484.566 | 292.852 |
| 180 | 746.852 | 549.633 | 354.685 |
| B. Short cash and long futures: | | | |
| 1 | 46.852 | 8.511 | 3.321 |
| 10 | 228.562 | 120.256 | 83.523 |
| 20 | 415.785 | 176.118 | 105.963 |
| 30 | 507.952 | 213.963 | 153.523 |
| 60 | 717.633 | 315.784 | 221.541 |
| 90 | 1,004.159 | 644.935 | 273.965 |
| 180 | 1,462.774 | 743.226 | 381.522 |



positive and negative price innovations to affect volatility forecasts differently, yield improvements in forecast accuracy in sample, but not out of sample, when evaluated using the traditional variance of realized returns metric.



The article also demonstrates how such hedging methodologies can be evaluated in a modern risk management context, using a technique based on the estimation of value at risk. Our primary finding is that allowing for asymmetries leads to considerably reduced portfolio risk at the shortest forecasting horizons and modest benefits when the duration of the hedge is increased.



Our results have at least two important implications for those financial market transactors who wish to reduce their exposure to risk using futures contracts, and for further research in this area. First, hedge ratios that are determined taking into account asymmetries in volatility are expected, in general, to be more effective than those that do not. Second, since recent changes in legislation in Europe have allowed market risk to be determined using value at risk technologies under the second European Community Capital Adequacy Directive (CAD II), it is surely desirable for hedgers to measure the risk inherent in their hedged portfolios in a similar fashion. Such procedures are already now in widespread use in Europe as well as the United States. The value-at-risk approach is (or soon will be) used to assess the risk of the books of securities firms as a whole. The use of traditional methods for assessing hedging effectiveness, such as portfolio return variances, could be incompatible with, and give very different results to, those based on value-at-risk methods.

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