

## Examples ARMA( $p, q$ ) process

Consider the following ARMA(1,1) process:

$$y_t = 2 + 0.5y_{t-1} + 0.9\varepsilon_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, 1), \quad T = 100$$

This is a stationary series as  $0.5 < 1$ .

Properties

- ▶ The expected value is given by

$$E(y_t) = 2 / (1 - 0.5) = 4$$

- ▶ The variance is given by

$$\gamma_0 = \frac{1 + \beta_1^2 + 2\alpha_1\beta_1}{1 - \alpha_1^2} \sigma^2 = \frac{1 + 0.9^2 + 2 \times 0.5 \times 0.9}{1 - 0.5^2} = 3.6133$$

- ▶ The ACF is given by

$$\rho_1 = \frac{(1 + \alpha_1 \beta_1)(\alpha_1 + \beta_1)}{1 + \beta_1^2 + 2\alpha_1 \beta_1} = \frac{(1 + 0.5 \times 0.9)(0.5 + 0.9)}{1 + 0.9^2 + 2 \times 0.5 \times 0.9} = 0.7491$$

$$\rho_2 = \alpha_1 \rho_1 = 0.5 \times 0.7491 = 0.3745$$

$$\rho_3 = \alpha_1 \rho_2 = 0.5 \times 0.3745 = 0.1873$$

$$\rho_4 = \alpha_1 \rho_3 = 0.5 \times 0.1873 = 0.0936$$

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Figure 46 : Theoretical ACF and PACF of generated ARMA(1,1) process

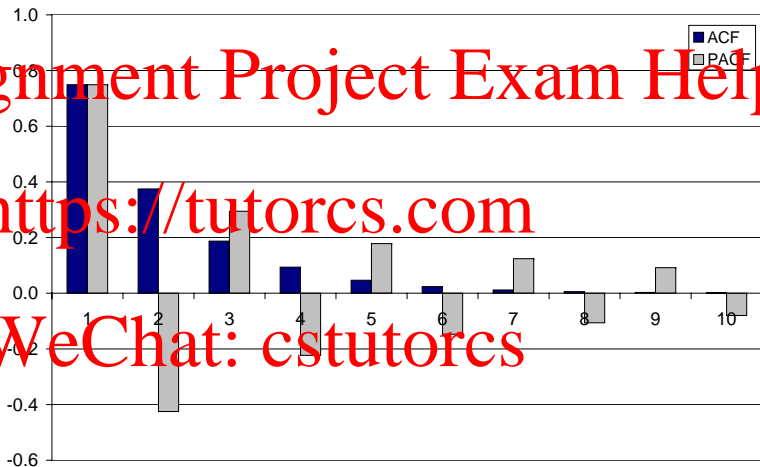


Figure 47 : Dynamic impact of a shock  $\varepsilon_t$  on  $y$

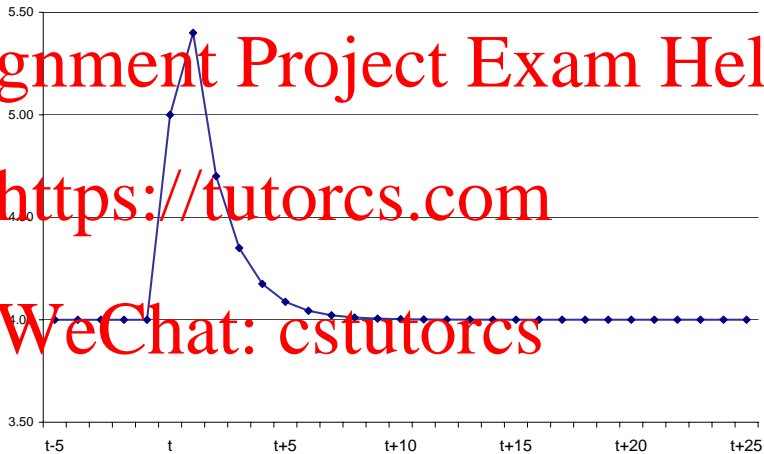


Figure 48 : A generated ARMA(1,1) process



Figure 49 : Sample ACF and PACF of generated ARMA(1,1) process

