INF-Cointegration Series

July 16, 2021

1 Importing packages

[1]:
```
#importing packages
import statsmodels.api as sm
from statsmodels.tsa.stattools import adfuller
import pandas as pd
import numpy as np
import statsmodels.formula.api as smf
from sklearn import linear_model
import matplotlib.pyplot as plt
```


2 Reading Excel file saved in hard drive

[2]:
```
#reading the file
df = pd.read_excel("C:\\Users\\rluck\\OneDrive\\fisher_update.xlsx")
df
```


[2]:

	DATE	P	R
0	1969-12-01	17.1	5.65
1	1970-03-01	17.3	7.15
2	1970-06-01	17.5	8.70
3	1970-09-01	17.6	6.35
4	1970-12-01	17.9	6.50
..	…	…	…
166	2011-06-01	178.3	4.99
167	2011-09-01	179.4	4.81
168	2011-12-01	179.4	4.51
169	2012-03-01	179.5	4.44
170	2012-06-01	180.4	3.49

[171 rows x 3 columns]

3 Calculating annual inflation from quarterly CPI

Quarterly CPI:

$$INF_{qtr} = 100 * ln(P_t/P_{t-1})$$

Annual CPI

$$INF_{qtr} = 400 * ln(P_t/P_{t-1})$$

[3]:
```
#computing the inflation rate
df['INF'] = 400*np.log(df['P']/df['P'].shift(1))
df
```

[3]:

	DATE	P	R	INF
0	1969-12-01	17.1	5.65	NaN
1	1970-03-01	17.3	7.15	4.651215
2	1970-06-01	17.5	8.70	4.597752
3	1970-09-01	17.6	6.35	2.279208
4	1970-12-01	17.9	6.50	6.760724
..	…	…	…	…
166	2011-06-01	178.3	4.99	3.605658
167	2011-09-01	179.4	4.81	2.460170
168	2011-12-01	179.4	4.51	0.000000
169	2012-03-01	179.5	4.44	0.222903
170	2012-06-01	180.4	3.49	2.000560

[171 rows x 4 columns]

[4]:
```
# Generating integrating differences series
df['DINF'] = df['INF'].diff(1).dropna()
df['DINF1'] = df['DINF'].shift(1).dropna()
df['DINF2'] = df['DINF'].shift(2).dropna()
df['DINF3'] = df['DINF'].shift(3).dropna()
df['DINF4'] = df['DINF'].shift(4).dropna()
df['DR'] = df['R'].diff(1).dropna()
df['DR1'] = df['DR'].shift(1).dropna()
df['DR2'] = df['DR'].shift(2).dropna()
df['DR3'] = df['DR'].shift(3).dropna()
df['DR4'] = df['DR'].shift(4).dropna()
df.head(60)
```


[4]:

	DATE	P	R	INF	DINF	DINF1	DINF2
0	1969-12-01	17.1	5.65	NaN	NaN	NaN	NaN
1	1970-03-01	17.3	7.15	4.651215	NaN	NaN	NaN
2	1970-06-01	17.5	8.70	4.597752	-0.053463	NaN	NaN
3	1970-09-01	17.6	6.35	2.279208	-2.318543	-0.053463	NaN
4	1970-12-01	17.9	6.50	6.760724	4.481516	-2.318543	-0.053463
5	1971-03-01	18.1	8.00	4.444490	-2.316234	4.481516	-2.318543
6	1971-06-01	18.4	8.15	6.575491	2.131000	-2.316234	4.481516
7	1971-09-01	18.8	6.45	8.602482	2.026992	2.131000	-2.316234
8	1971-12-01	19.2	5.90	8.421364	-0.181118	2.026992	2.131000
9	1972-03-01	19.4	5.50	4.145115	-4.276249	-0.181118	2.026992
10	1972-06-01	19.6	5.75	4.102600	-0.042515	-4.276249	-0.181118

11	1972-09-01	19.9	4.50	6.076066	1.973466	-0.042515	-4.276249
12	1972-12-01	20.1	4.45	4.000033	-2.076033	1.973466	-0.042515
13	1973-03-01	20.5	5.45	7.882028	3.881995	-2.076033	1.973466
14	1973-06-01	21.2	6.40	13.430518	5.548490	3.881995	-2.076033
15	1973-09-01	21.9	9.25	12.994182	-0.436336	5.548490	3.881995
16	1973-12-01	22.7	9.25	14.351315	1.357133	-0.436336	5.548490
17	1974-03-01	23.3	10.10	10.435374	-3.915941	1.357133	-0.436336
18	1974-06-01	24.3	18.80	16.809196	6.373821	-3.915941	1.357133
19	1974-09-01	25.5	12.60	19.280841	2.471645	6.373821	-3.915941
20	1974-12-01	26.4	9.75	13.874223	-5.406618	2.471645	6.373821
21	1975-03-01	27.4	8.75	14.871601	0.997378	-5.406618	2.471645
22	1975-06-01	28.4	8.80	14.338453	-0.533149	0.997378	-5.406618
23	1975-09-01	28.6	8.10	2.807029	-11.531424	-0.533149	0.997378
24	1975-12-01	30.2	7.70	21.774083	18.967054	-11.531424	-0.533149
25	1976-03-01	31.0	8.40	10.458112	-11.315971	18.967054	-11.531424
26	1976-06-01	31.8	10.27	10.191634	-0.266478	-11.315971	18.967054
27	1976-09-01	32.6	9.31	9.938399	-0.253235	-0.266478	-11.315971
28	1976-12-01	34.5	9.44	22.658814	12.720415	-0.253235	-0.266478
29	1977-03-01	35.3	9.73	9.169456	-13.489358	12.720415	-0.253235
30	1977-06-01	36.1	10.09	8.963961	-0.205495	-13.489358	12.720415
31	1977-09-01	36.8	10.43	7.681992	-1.281969	-0.205495	-13.489358
32	1977-12-01	37.7	9.75	9.664900	1.982908	-1.281969	-0.205495
33	1978-03-01	38.2	10.04	5.270168	-4.394731	1.982908	-1.281969
34	1978-06-01	39.0	10.63	8.290452	3.020284	-4.394731	1.982908
35	1978-09-01	39.7	9.72	7.115817	-1.174636	3.020284	-4.394731
36	1978-12-01	40.6	8.76	8.966752	1.850935	-1.174636	3.020284
37	1979-03-01	41.3	9.16	6.837773	-2.128978	1.850935	-1.174636
38	1979-06-01	42.4	10.26	10.514345	3.676572	-2.128978	1.850935
39	1979-09-01	43.4	9.87	9.324432	-1.189913	3.676572	-2.128978
40	1979-12-01	44.7	10.12	11.805624	2.481193	-1.189913	3.676572
41	1980-03-01	45.7	11.47	8.849919	-2.955706	2.481193	-1.189913
42	1980-06-01	47.0	13.83	11.219722	2.369803	-2.955706	2.481193
43	1980-09-01	47.8	11.69	6.751215	-4.468506	2.369803	-2.955706
44	1980-12-01	48.8	12.45	8.281869	1.530654	-4.468506	2.369803
45	1981-03-01	50.0	14.63	9.717077	1.435208	1.530654	-4.468506
46	1981-06-01	51.1	15.58	8.704597	-1.012480	1.435208	1.530654
47	1981-09-01	52.1	15.35	7.752181	-0.952416	-1.012480	1.435208
48	1981-12-01	54.3	15.54	16.543711	8.791531	-0.952416	-1.012480
49	1982-03-01	55.3	18.89	7.299473	-9.244239	8.791531	-0.952416
50	1982-06-01	56.6	18.57	9.294431	1.994958	-9.244239	8.791531
51	1982-09-01	58.6	15.46	13.890285	4.595854	1.994958	-9.244239
52	1982-12-01	60.3	12.13	11.438963	-2.451322	4.595854	1.994958
53	1983-03-01	61.6	15.26	8.531907	-2.907056	-2.451322	4.595854
54	1983-06-01	62.9	14.24	8.353717	-0.178189	-2.907056	-2.451322
55	1983-09-01	64.0	11.06	6.934768	-1.418949	-0.178189	-2.907056
56	1983-12-01	65.5	8.89	9.266824	2.332056	-1.418949	-0.178189
57	1984-03-01	65.2	13.77	-1.836269	-11.103093	2.332056	-1.418949

```
58 1984-06-01  65.4  12.81   1.225116   3.061385 -11.103093   2.332056
59 1984-09-01  66.2  10.53   4.863282   3.638166   3.061385 -11.103093

        DINF3      DINF4    DR   DR1   DR2   DR3   DR4
0         NaN        NaN   NaN   NaN   NaN   NaN   NaN
1         NaN        NaN  1.50   NaN   NaN   NaN   NaN
2         NaN        NaN  1.55  1.50   NaN   NaN   NaN
3         NaN        NaN -2.35  1.55  1.50   NaN   NaN
4         NaN        NaN  0.15 -2.35  1.55  1.50   NaN
5   -0.053463        NaN  1.50  0.15 -2.35  1.55  1.50
6   -2.318543  -0.053463  0.15  1.50  0.15 -2.35  1.55
7    4.481516  -2.318543 -1.70  0.15  1.50  0.15 -2.35
8   -2.316234   4.481516 -0.55 -1.70  0.15  1.50  0.15
9    2.131000  -2.316234 -0.40 -0.55 -1.70  0.15  1.50
10   2.026992   2.131000  0.25 -0.40 -0.55 -1.70  0.15
11  -0.181118   2.026992 -1.25  0.25 -0.40 -0.55 -1.70
12  -4.276249  -0.181118 -0.05 -1.25  0.25 -0.40 -0.55
13  -0.042515  -4.276249  1.00 -0.05 -1.25  0.25 -0.40
14   1.973466  -0.042515  0.95  1.00 -0.05 -1.25  0.25
15  -2.076033   1.973466  2.85  0.95  1.00 -0.05 -1.25
16   3.881995  -2.076033  0.00  2.85  0.95  1.00 -0.05
17   5.548490   3.881995  0.85  0.00  2.85  0.95  1.00
18  -0.436336   5.548490  8.70  0.85  0.00  2.85  0.95
19   1.357133  -0.436336 -6.20  8.70  0.85  0.00  2.85
20  -3.915941   1.357133 -2.85 -6.20  8.70  0.85  0.00
21   6.373821  -3.915941 -1.00 -2.85 -6.20  8.70  0.85
22   2.471645   6.373821  0.05 -1.00 -2.85 -6.20  8.70
23  -5.406618   2.471645 -0.70  0.05 -1.00 -2.85 -6.20
24   0.997378  -5.406618 -0.40 -0.70  0.05 -1.00 -2.85
25  -0.533149   0.997378  0.70 -0.40 -0.70  0.05 -1.00
26 -11.531424  -0.533149  1.87  0.70 -0.40 -0.70  0.05
27  18.967054 -11.531424 -0.96  1.87  0.70 -0.40 -0.70
28 -11.315971  18.967054  0.13 -0.96  1.87  0.70 -0.40
29  -0.266478 -11.315971  0.29  0.13 -0.96  1.87  0.70
30  -0.253235  -0.266478  1.22  0.29  0.13 -0.96  1.87
31  12.720415  -0.253235 -0.52  1.22  0.29  0.13 -0.96
32 -13.489358  12.720415 -0.68 -0.52  1.22  0.29  0.13
33  -0.205495 -13.489358  0.29 -0.68 -0.52  1.22  0.29
34  -1.281969  -0.205495  0.59  0.29 -0.68 -0.52  1.22
35   1.982908  -1.281969 -0.91  0.59  0.29 -0.68 -0.52
36  -4.394731   1.982908 -0.96 -0.91  0.59  0.29 -0.68
37   3.020284  -4.394731  0.40 -0.96 -0.91  0.59  0.29
38  -1.174636   3.020284  1.10  0.40 -0.96 -0.91  0.59
39   1.850935  -1.174636 -0.39  1.10  0.40 -0.96 -0.91
40  -2.128978   1.850935  0.25 -0.39  1.10  0.40 -0.96
41   3.676572  -2.128978  1.35  0.25 -0.39  1.10  0.40
42  -1.189913   3.676572  2.36  1.35  0.25 -0.39  1.10
```

```
43   2.481193  -1.189913 -2.14  2.36  1.35  0.25 -0.39
44  -2.955706   2.481193  0.76 -2.14  2.36  1.35  0.25
45   2.369803  -2.955706  2.18  0.76 -2.14  2.36  1.35
46  -4.468506   2.369803  0.95  2.18  0.76 -2.14  2.36
47   1.530654  -4.468506 -0.23  0.95  2.18  0.76 -2.14
48   1.435208   1.530654  0.19 -0.23  0.95  2.18  0.76
49  -1.012480   1.435208  3.35  0.19 -0.23  0.95  2.18
50  -0.952416  -1.012480 -0.32  3.35  0.19 -0.23  0.95
51   8.791531  -0.952416 -3.11 -0.32  3.35  0.19 -0.23
52  -9.244239   8.791531 -3.33 -3.11 -0.32  3.35  0.19
53   1.994958  -9.244239  3.13 -3.33 -3.11 -0.32  3.35
54   4.595854   1.994958 -1.02  3.13 -3.33 -3.11 -0.32
55  -2.451322   4.595854 -3.18 -1.02  3.13 -3.33 -3.11
56  -2.907056  -2.451322 -2.17 -3.18 -1.02  3.13 -3.33
57  -0.178189  -2.907056  4.88 -2.17 -3.18 -1.02  3.13
58  -1.418949  -0.178189 -0.96  4.88 -2.17 -3.18 -1.02
59   2.332056  -1.418949 -2.28 -0.96  4.88 -2.17 -3.18
```

4 Selecting sample data from row 57: Qtr 1 1984 to row 170: Qtr 2012

[5]:
```
#Selecting the sample from
dta =df.iloc[57:170].dropna()
dta
```

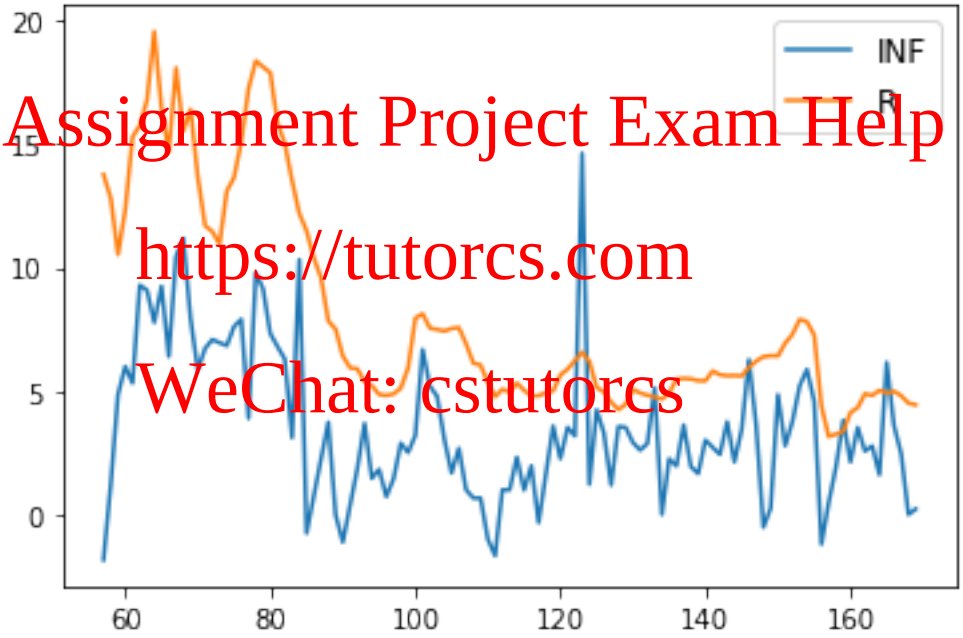

[5]:


```
          DATE     P     R       INF      DINF     DINF1     DINF2  \
57  1984-03-01  65.2  13.77 -1.886269 -11.103093  2.332056 -1.418949
58  1984-06-01  65.4  12.81  1.225116   3.061385 -11.103093  2.332056
59  1984-09-01  66.2  10.53  4.863282   3.638166   3.061385 -11.103093
60  1984-12-01  67.2  12.34  5.997114   1.133832   3.638166   3.061385
61  1985-03-01  68.1  15.29  5.321586  -0.675528   1.133832   3.638166
..         …     …     …       …         …         …         …
165 2011-03-01  176.7   4.92  6.159232   4.546791  -1.166956   0.214539
166 2011-06-01  178.3   4.99  3.605658  -2.553574   4.546791  -1.166956
167 2011-09-01  179.4   4.81  2.460170  -1.145488  -2.553574   4.546791
168 2011-12-01  179.4   4.51  0.000000  -2.460170  -1.145488  -2.553574
169 2012-03-01  179.5   4.44  0.222903   0.222903  -2.460170  -1.145488

         DINF3     DINF4    DR   DR1   DR2   DR3   DR4
57   -0.178189 -2.907056  4.88 -2.17 -3.18 -1.02  3.13
58   -1.418949 -0.178189 -0.96  4.88 -2.17 -3.18 -1.02
59    2.332056 -1.418949 -2.28 -0.96  4.88 -2.17 -3.18
60  -11.103093  2.332056  1.81 -2.28 -0.96  4.88 -2.17
61    3.061385 -11.103093  2.95  1.81 -2.28 -0.96  4.88
..         …        …    …    …    …    …    …
165  -0.959393  1.394699 -0.11  0.21 -0.07  0.56  0.20
```

```
166   0.214539  -0.959393  0.07 -0.11  0.21 -0.07  0.56
167  -1.166956   0.214539 -0.18  0.07 -0.11  0.21 -0.07
168   4.546791  -1.166956 -0.30 -0.18  0.07 -0.11  0.21
169  -2.553574   4.546791 -0.07 -0.30 -0.18  0.07 -0.11

[113 rows x 14 columns]
```

5 Plotting the time series: Inflation

[6]:
```
#plotting the series
plt.plot(dta['INF'],label='INF')
plt.plot(dta['R'],label='R')
plt.legend(loc='best', fontsize='large')
plt.show()
```

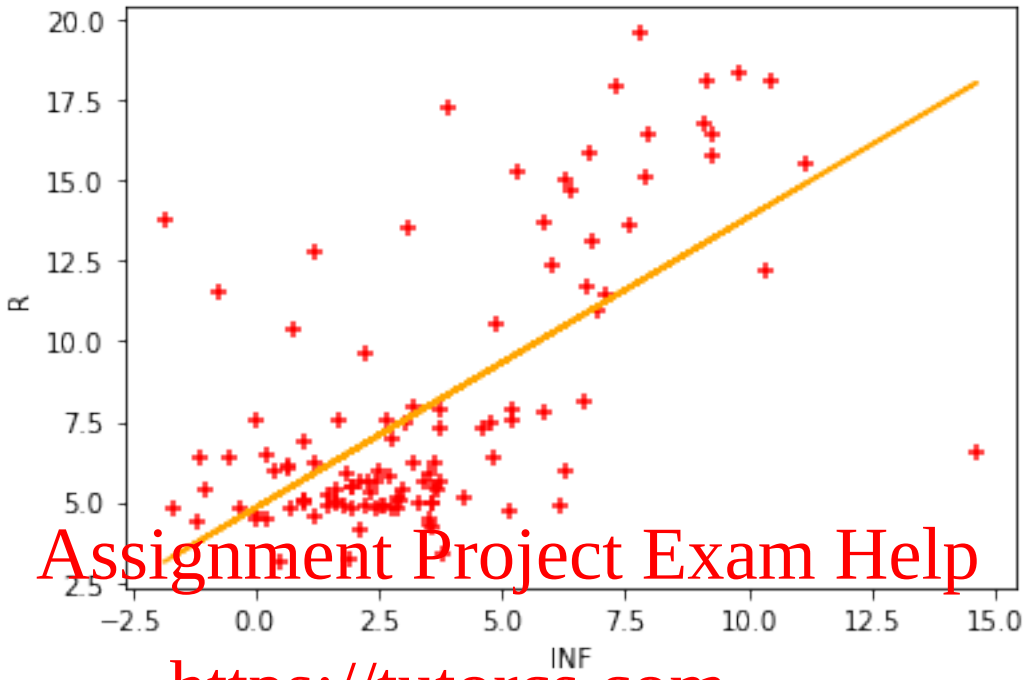


[7]:
```
#Regressing Interest Rate (Y=R) against the Inflation rate (X= INF)
reg = linear_model.LinearRegression()
X =dta[['INF']].dropna()
y =dta['R'].dropna()
reg.fit(X,y)
predictions =reg.predict(X)
```

[8]:
```
plt.xlabel('INF')
plt.ylabel('R')
plt.scatter(dta.INF,dta.R,color='red',marker='+')
```

```
plt.plot(dta.INF,reg.predict(dta[['INF']]), color='orange')
```

[8]: [<matplotlib.lines.Line2D at 0x2850cff1940>]



[9]:
```
#model with intercept
X = dta.INF
y = dta.R
X= sm.add_constant(X)
model= sm.OLS(y,X).fit()
predictions = model.predict(X)
G= (model.summary())
print(G)
```

OLS Regression Results

Dep. Variable:	R	R-squared:	0.412
Model:	OLS	Adj. R-squared:	0.407
Method:	Least Squares	F-statistic:	77.80
Date:	Fri, 16 Jul 2021	Prob (F-statistic):	1.83e-14
Time:	21:22:16	Log-Likelihood:	-294.55
No. Observations:	113	AIC:	593.1
Df Residuals:	111	BIC:	598.5
Df Model:	1		
Covariance Type:	nonrobust		

```
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          4.8120      0.480     10.020      0.000       3.860       5.764
INF            0.9039      0.102      8.821      0.000       0.701       1.107
==============================================================================
Omnibus:                       11.514   Durbin-Watson:                   0.699
Prob(Omnibus):                  0.003   Jarque-Bera (JB):               14.994
Skew:                           0.546   Prob(JB):                     0.000555
Kurtosis:                       4.411   Cond. No.                         7.42
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

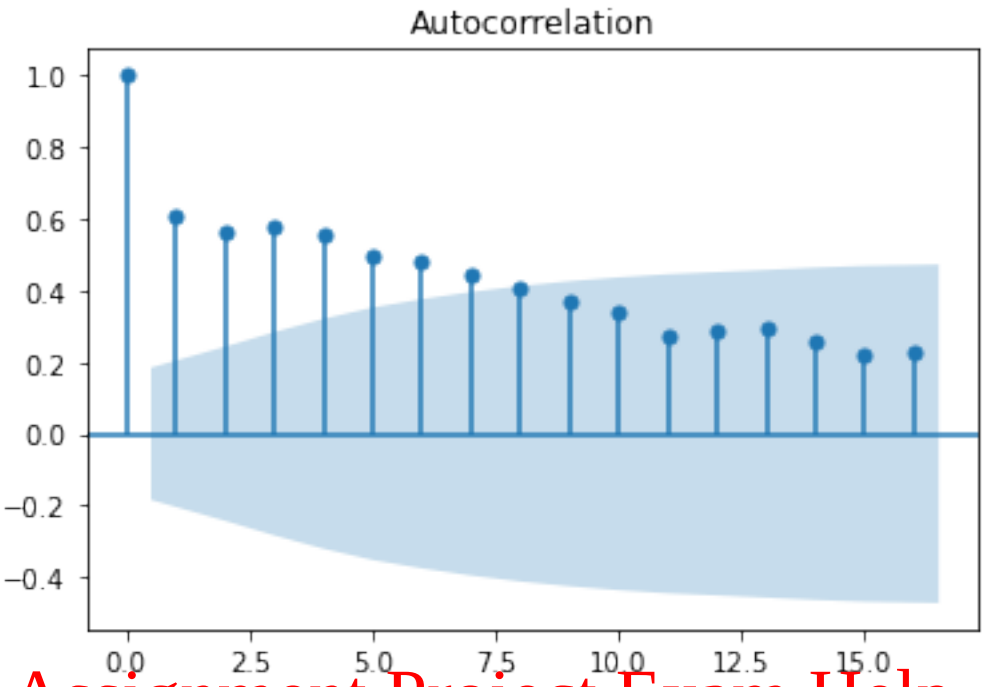
6 Correlogram of Residuals: ACF and PACF

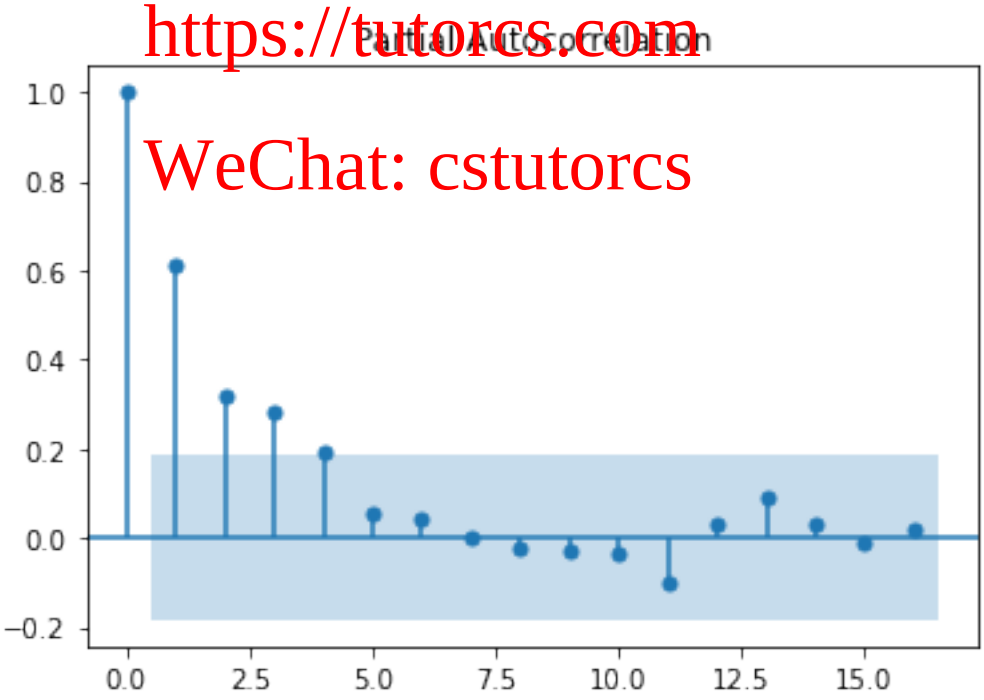
$\epsilon_t = y_t - (\beta * X_t + \alpha)$

[10]:
```
dtr = model_resid
dtr
```


[10]:
```
57     10.617794
58      6.890682
59      1.322232
60      2.107389
61      5.667982
          ...
165    -5.459146
166    -3.081032
167    -2.225653
168    -0.301965
169    -0.573442
Length: 113, dtype: float64
```


[11]:
```
#running ACF and PACF
sm.graphics.tsa.plot_acf(dtr.values.squeeze(),lags=16)
sm.graphics.tsa.plot_pacf(dtr.values.squeeze(),lags=16)
plt.show()
```





[12]:
```
# Generating the Q tables
import numpy as np
r,q,p = sm.tsa.acf(dtr.values.squeeze(), qstat=True)
data = np.c_[range(1,41), r[1:], q, p]
table = pd.DataFrame(data, columns=['lag', "AC", "Q", "Prob(>Q)"])
print (table.set_index('lag'))
```

```
            AC           Q      Prob(>Q)
lag
1.0   0.604164   42.351394  7.626197e-11
2.0   0.561012   79.197947  6.344308e-18
3.0   0.574160  118.142798  1.938169e-25
4.0   0.558060  155.271704  1.509379e-32
5.0   0.498832  185.212371  4.120822e-38
6.0   0.477251  212.874547  3.437139e-43
7.0   0.444019  237.044412  1.579656e-47
8.0   0.404992  257.343619  4.775363e-51
9.0   0.370545  274.499942  6.605085e-54
10.0  0.337784  288.895138  3.451093e-56
11.0  0.275816  298.567198  1.738371e-57
12.0  0.286480  309.146676  5.627670e-59
13.0  0.296186  320.546711  1.193303e-60
14.0  0.259174  329.363744  8.672219e-62
15.0  0.223192  335.969290  1.795166e-62
16.0  0.226696  342.854116  3.193704e-63
17.0  0.255196  351.669754  2.217145e-64
18.0  0.223284  358.489525  3.951410e-65
19.0  0.298762  370.829082  4.992993e-67
20.0  0.321913  385.309124  2.265168e-69
21.0  0.282732  396.600261  4.685822e-71
22.0  0.298276  409.305178  4.926778e-73
23.0  0.234833  417.267758  4.917945e-74
24.0  0.236777  425.453665  4.382705e-75
25.0  0.229401  433.224808  4.722116e-76
26.0  0.174816  437.789603  2.301703e-76
27.0  0.162803  441.794612  1.445240e-76
28.0  0.223076  449.402502  1.643747e-77
29.0  0.149866  452.877109  1.299917e-77
30.0  0.154104  456.595241  9.069163e-78
31.0  0.098993  458.148250  1.727635e-77
32.0  0.100837  459.779524  3.130430e-77
33.0  0.027877  459.905761  1.130134e-76
34.0  0.021439  459.981366  4.113627e-76
35.0  0.014447  460.016136  1.503308e-75
36.0 -0.023361  460.108237  5.272135e-75
37.0 -0.053373  460.595316  1.518910e-74
38.0 -0.062695  461.276367  3.950474e-74
39.0 -0.003679  461.278743  1.387193e-73
```

```
40.0 -0.076177  462.311755  2.995337e-73

C:\Users\rluck\anaconda3\lib\site-packages\statsmodels\tsa\stattools.py:657:
FutureWarning: The default number of lags is changing from 40 tomin(int(10 *
np.log10(nobs)), nobs - 1) after 0.12is released. Set the number of lags to an
integer to  silence this warning.
  warnings.warn(
C:\Users\rluck\anaconda3\lib\site-packages\statsmodels\tsa\stattools.py:667:
FutureWarning: fft=True will become the default after the release of the 0.12
release of statsmodels. To suppress this warning, explicitly set fft=False.
  warnings.warn(
```

[ ]:

7 ADF test of stationarity and unit root

[13]:
```
residuals = model.resid
residuals
```

[13]:
```
57     10.617794
58      6.890682
59      1.322232
60      2.107389
61      5.667982
          …
165    -5.459146
166    -3.081032
167    -2.225653
168    -0.301965
169    -0.573442
Length: 113, dtype: float64
```


[14]:
```
dtr
```

[14]:
```
57     10.617794
58      6.890682
59      1.322232
60      2.107389
61      5.667982
          …
165    -5.459146
166    -3.081032
167    -2.225653
168    -0.301965
169    -0.573442
Length: 113, dtype: float64
```

[15]:
```
#ADF Tests
from arch.unitroot import ADF
ADF(residuals,trend="n",lags=1, max_lags=12, method='BIC')
```

[15]:
```
<class 'arch.unitroot.unitroot.ADF'>
"""
   Augmented Dickey-Fuller Results
=====================================
Test Statistic                 -3.854
P-value                         0.000
Lags                                1
-------------------------------------

Trend: No Trend
Critical Values: -2.59 (1%), -1.94 (5%), -1.61 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
"""
```


8 Engle Granger Cointegration Test

[16]:
```
from arch.unitroot import engle_granger
engle_test = engle_granger(y,X, trend="n",lags=1)
engle_test
```


[16]:
```
Engle-Granger Cointegration Test
Statistic: -3.8538026730006763
P-value: 0.010421142946669465
Null: No Cointegration, Alternative: Cointegration
ADF Lag length: 1
Trend: c
Estimated Root   ( +1): 0.7108844276270996
Distribution Order: 2
ID: 0x2850f984a90
```

9 DR Regressed against DR & DINF with lags up to 4

$\epsilon_t = y_t - (\beta x_t + \alpha)$

[17]:
```
dta['resid'] = y -model.predict(X)
#Residual series by lag 1
dta['resid_1']= dta['resid'].shift(1)
dta = dta.dropna(subset=['resid_1'])
dta
```

[17]:
```
           DATE      P      R       INF      DINF      DINF1      DINF2  \
58   1984-06-01   65.4  12.81  1.225116  3.061385 -11.103093   2.332056
59   1984-09-01   66.2  10.53  4.863282  3.638166   3.061385 -11.103093
60   1984-12-01   67.2  12.34  5.997114  1.133832   3.638166   3.061385
61   1985-03-01   68.1  15.29  5.321586 -0.675528   1.133832   3.638166
62   1985-06-01   69.7  15.75  9.289242  3.967656  -0.675528   1.133832
..          ...    ...    ...       ...       ...        ...        ...
165  2011-03-01  176.7   4.92  6.159232  4.546791  -1.166956   0.214539
166  2011-06-01  178.3   4.99  3.605658 -2.553574   4.546791  -1.166956
167  2011-09-01  179.4   4.81  2.460170 -1.145488  -2.553574   4.546791
168  2011-12-01  179.4   4.51  0.000000 -2.460170  -1.145488  -2.553574
169  2012-03-01  179.5   4.44  0.222903  0.222903  -2.460170  -1.145488

         DINF3      DINF4    DR   DR1   DR2   DR3   DR4     resid    resid_1
58   -1.418949  -0.178189 -0.96  4.88 -2.17 -3.18 -1.02  6.890682  10.617794
59    2.332056  -1.418949 -2.28 -0.96  4.88 -2.17 -3.18  1.322232   6.890682
60  -11.103093   2.332056  1.81 -2.28 -0.96  4.88 -2.17  2.107389   1.322232
61    3.061385 -11.103093  2.95  1.81 -2.28 -0.96  4.88  5.667982   2.107389
62    3.638166   3.061385  0.46  2.95  1.81 -2.28 -0.96  2.541714   5.667982
..         ...        ...   ...   ...   ...   ...   ...       ...        ...
165  -0.959393   1.394699 -0.11  0.21 -0.07  0.56  0.20 -5.459146  -1.239412
166   0.214539  -0.959393  0.07 -0.11  0.21 -0.07  0.56 -3.081032  -5.459146
167  -1.166956   0.214539 -0.18  0.07 -0.11  0.21 -0.07 -2.225653  -3.081032
168   4.546791  -1.166956 -0.30 -0.18  0.07 -0.11  0.21 -0.301965  -2.225653
169  -2.553574   4.546791 -0.07 -0.30 -0.18  0.07 -0.11 -0.573442  -0.301965

[112 rows x 16 columns]
```


Multiple Regression

[18]:
```
#model with intercept
x_1 = dta[['resid_1','DR1','DR2','DR3','DR4','DINF1','DINF2','DINF3','DINF4']]
y_1 = dta['DR']
x_1= sm.add_constant(x_1)
model_1 = sm.OLS(y_1,x_1).fit()
predictions = model_1.predict(x_1)
h= (model_1.summary())
print(h)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                     DR   R-squared:                       0.189
Model:                            OLS   Adj. R-squared:                  0.117
Method:                 Least Squares   F-statistic:                     2.633
Date:                Fri, 16 Jul 2021   Prob (F-statistic):            0.00881
Time:                        21:22:17   Log-Likelihood:                -151.97
No. Observations:                 112   AIC:                             323.9
Df Residuals:                     102   BIC:                             351.1
```

```
Df Model:                           9
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0631      0.094     -0.674      0.502      -0.249       0.123
resid_1       -0.0579      0.035     -1.631      0.106      -0.128       0.012
DR1            0.1464      0.091      1.613      0.110      -0.034       0.327
DR2            0.0222      0.091      0.243      0.808      -0.159       0.203
DR3            0.2249      0.085      2.653      0.009       0.057       0.393
DR4           -0.0686      0.082     -0.833      0.407      -0.232       0.095
DINF1          0.0264      0.047      0.558      0.578      -0.067       0.120
DINF2          0.0365      0.051      0.717      0.475      -0.064       0.137
DINF3         -0.0415      0.049     -0.843      0.401      -0.139       0.056
DINF4         -0.0570      0.040     -1.425      0.157      -0.136       0.022
==============================================================================
Omnibus:                       15.294   Durbin-Watson:                   2.004
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               59.082
Skew:                           0.087   Prob(JB):                     1.48e-13
Kurtosis:                       6.554   Cond. No.                         5.07
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```


10 DINF Regressed against DR & DINF with lags up to 4

[19]:
```
#model with intercept
x_1 = dta[['resid_1','DR1','DR2','DR3','DR4','DINF1','DINF2','DINF3','DINF4']]
y_2 = dta['DINF']
x_1= sm.add_constant(x_1)
model_2 = sm.OLS(y_2,x_1).fit()
predictions = model_2.predict(x_1)
I= (model_2.summary())
print(I)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   DINF   R-squared:                       0.367
Model:                            OLS   Adj. R-squared:                  0.311
Method:                 Least Squares   F-statistic:                     6.574
Date:                Fri, 16 Jul 2021   Prob (F-statistic):           2.36e-07
Time:                        21:22:17   Log-Likelihood:                -252.89
No. Observations:                 112   AIC:                             525.8
Df Residuals:                     102   BIC:                             553.0
Df Model:                           9
```

```
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0106      0.231     -0.046      0.964      -0.468       0.447
resid_1        0.0622      0.087      0.712      0.478      -0.111       0.235
DR1            0.5445      0.224      2.436      0.017       0.101       0.988
DR2            0.4540      0.225      2.021      0.046       0.008       0.900
DR3            0.1029      0.209      0.493      0.623      -0.311       0.517
DR4            0.0865      0.203      0.427      0.670      -0.316       0.489
DINF1         -0.6278      0.116     -5.392      0.000      -0.859      -0.397
DINF2         -0.4934      0.125     -3.938      0.000      -0.742      -0.245
DINF3         -0.3980      0.121     -3.279      0.001      -0.639      -0.157
DINF4         -0.1518      0.098     -1.542      0.126      -0.347       0.043
==============================================================================
Omnibus:                       25.323   Durbin-Watson:                   2.106
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              113.849
Skew:                           0.558   Prob(JB):                     1.90e-25
Kurtosis:                       7.811   Cond. No.                         5.07
==============================================================================
```


Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Wald Tests

[20]:
```
#Wald test om model_1: Restricting coefficient 5 to 10 (if p >0,05,␣
 ↪restrictions cannot be rejected)
R = np.eye(len(model_1.params))[5:10]
model_1.wald_test(R)
```

[20]:
```
<class 'statsmodels.stats.contrast.ContrastResults'>
<F test: F=array([[1.29567193]]), p=0.2716573127484845, df_denom=102, df_num=5>
```

[21]:
```
#Wald test om model_1: Restricting coefficient 5 and 6 (if p >0,05,␣
 ↪Restrictions cannot be rejected)
R = np.eye(len(model_2.params))[4:6]
model_2.wald_test(R)
```

[21]:
```
<class 'statsmodels.stats.contrast.ContrastResults'>
<F test: F=array([[0.2258903]]), p=0.7982036497279359, df_denom=102, df_num=2>
```

11 DR Regressed against DR with lags up to 4

[22]:
```
x_3 = dta[['resid_1','DR1','DR2','DR3','DR4']]
y_3 = dta['DR']
x_3= sm.add_constant(x_3)
model_3 = sm.OLS(y_3,x_3).fit()
predictions = model_3.predict(x_3)
J= (model_3.summary())
print(J)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                     DR   R-squared:                       0.144
Model:                            OLS   Adj. R-squared:                  0.104
Method:                 Least Squares   F-statistic:                     3.578
Date:                Fri, 16 Jul 2021   Prob (F-statistic):            0.00498
Time:                        21:22:17   Log-Likelihood:                -154.93
No. Observations:                 112   AIC:                             321.9
Df Residuals:                     106   BIC:                             338.2
Df Model:                           5
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0653      0.094     -0.693      0.490      -0.252       0.122
resid_1       -0.0732      0.029     -2.537      0.013      -0.130      -0.016
DR1            0.1638      0.085      1.926      0.057      -0.005       0.333
DR2           -0.0158      0.081     -0.195      0.846      -0.177       0.145
DR3            0.2324      0.079      2.926      0.004       0.075       0.390
DR4           -0.0773      0.081     -0.958      0.340      -0.237       0.083
==============================================================================
Omnibus:                       19.421   Durbin-Watson:                   1.978
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               88.039
Skew:                           0.279   Prob(JB):                     7.63e-20
Kurtosis:                       7.307   Cond. No.                         3.37
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```


12 DINF Regressed against DINF with lags up to 4

[23]:
```
x_4 = dta[['resid_1','DR1','DR2','DINF1','DINF2','DINF3','DINF4']]
y_4 = dta['DINF']
x_4= sm.add_constant(x_4)
model = sm.OLS(y_4,x_4).fit()
```

```
predictions = model.predict(x_4)
K= (model.summary())
print(K)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   DINF   R-squared:                       0.364
Model:                            OLS   Adj. R-squared:                  0.322
Method:                 Least Squares   F-statistic:                     8.515
Date:                Fri, 16 Jul 2021   Prob (F-statistic):           3.26e-08
Time:                        21:22:17   Log-Likelihood:                -253.14
No. Observations:                 112   AIC:                             522.3
Df Residuals:                     104   BIC:                             544.0
Df Model:                           7
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0244      0.228     -0.107      0.915      -0.477       0.428
resid_1        0.0794      0.096      0.834      0.406      -0.110       0.268
DR1            0.5336      0.216      2.465      0.015       0.104       0.963
DR2            0.4309      0.220      1.960      0.053      -0.005       0.867
DINF1         -0.6069      0.111     -5.487      0.000      -0.826      -0.388
DINF2         -0.4627      0.115     -4.016      0.000      -0.691      -0.234
DINF3         -0.3719      0.114     -3.259      0.002      -0.598      -0.146
DINF4         -0.1343      0.094     -1.433      0.155      -0.320       0.052
==============================================================================
Omnibus:                       25.561   Durbin-Watson:                   2.122
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              122.519
Skew:                           0.537   Prob(JB):                     2.48e-27
Kurtosis:                       8.010   Cond. No.                         4.64
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```


[24]:
```
#Wald test om model_3: Restricting coefficient 4 (if p >0,05, restrictions␣
↪cannot be rejected )
R = np.eye(len(model_3.params))[3:4]
model_3.wald_test(R)
```

[24]:
```
<class 'statsmodels.stats.contrast.ContrastResults'>
<F test: F=array([[0.03790856]]), p=0.8459995477076325, df_denom=106, df_num=1>
```

[25]:
```
#Model 5
x_5 = dta[['resid_1','DR1','DR3','DR4']]
y_5 = dta['DR']
```

```
x_5= sm.add_constant(x_5)
model_5 = sm.OLS(y_5,x_5).fit()
predictions = model_5.predict(x_5)
k= (model_5.summary())
print(k)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                     DR   R-squared:                       0.144
Model:                            OLS   Adj. R-squared:                  0.112
Method:                 Least Squares   F-statistic:                     4.503
Date:                Fri, 16 Jul 2021   Prob (F-statistic):            0.00211
Time:                        21:22:17   Log-Likelihood:                -154.95
No. Observations:                 112   AIC:                             319.9
Df Residuals:                     107   BIC:                             333.5
Df Model:                           4
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0644      0.094     -0.687      0.494      -0.250       0.122
resid_1       -0.0733      0.029     -2.553      0.012      -0.130      -0.016
DR1            0.1631      0.085      1.928      0.057      -0.005       0.331
DR3            0.2313      0.079      2.933      0.004       0.075       0.388
DR4           -0.0758      0.080     -0.948      0.345      -0.234       0.083
==============================================================================
Omnibus:                       19.887   Durbin-Watson:                   1.976
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               90.139
Skew:                           0.307   Prob(JB):                     2.67e-20
Kurtosis:                       7.352   Cond. No.                         3.35
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

[26]:
```
#Wald test om model_5: Restricting coefficient 5 (if p >0,05, restrictions␣
↪cannot be rejected )
R = np.eye(len(model_5.params))[4:5]
model_5.wald_test(R)
```

[26]:
```
<class 'statsmodels.stats.contrast.ContrastResults'>
<F test: F=array([[0.89881769]]), p=0.345235217373811, df_denom=107, df_num=1>
```

[27]:
```
# Model 6
x_6 = dta[['resid_1','DR1','DR3']]
y_6 = dta['DR']
x_6= sm.add_constant(x_6)
```

```
model_6 = sm.OLS(y_6,x_6).fit()
predictions = model_6.predict(x_6)
k= (model_6.summary())
print(k)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                     DR   R-squared:                       0.137
Model:                            OLS   Adj. R-squared:                  0.113
Method:                 Least Squares   F-statistic:                     5.710
Date:                Fri, 16 Jul 2021   Prob (F-statistic):            0.00115
Time:                        21:22:17   Log-Likelihood:                -155.42
No. Observations:                 112   AIC:                             318.8
Df Residuals:                     108   BIC:                             329.7
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0587      0.094      0.628      0.532      -0.127       0.244
resid_1       -0.0749      0.029     -2.613      0.010      -0.132      -0.018
DR1            0.1467      0.083      1.772      0.079      -0.017       0.311
DR3            0.2235      0.078      2.851      0.005       0.068       0.379
==============================================================================
Omnibus:                       16.578   Durbin-Watson:                   1.925
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               66.108
Skew:                           0.188   Prob(JB):                     4.41e-15
Kurtosis:                       6.745   Cond. No.                         3.35
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

[ ]: