Financial Econometrics

Slides-02
Linear Regression
Review and Applications in Finance

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Economics[1]

Linear Regression

- A model where one variable Y_t is linearly explained by a group of variables $(X_{1t}, \cdots, X_{kt}), t = 1, \cdots, T$
  - Easy to Implement
  - Versatile for financial data analysis
  - Foundation for more advanced models
- General formulation
  - $Y_t = \beta_1 + \beta_2 X_{t1} + \beta_3 X_{t2} + \cdots + \beta_K X_{tK} + \mu_t, t = 1 \cdots, T$
  - Y_t: dependent variable
  - $X_{t1}, \cdots X_{tK}$: explanatory variables, regressors
  - $\beta_1, \beta_2, \cdots, \beta_K$: parameters (to be estimated)
  - μ_t: error term
  - T: number of observations

Application 1: Capital Asset Pricing Model *aka CAPM*

One of the most important problems of modern financial economics is the **quantification of the tradeoff between risk and expected return**. Common sense suggests risky investments (stock market) will generally yield higher returns than investments free of risk!

- Markowitz (1995) casts the investor's portfolio selection problem in terms of expected return and variance of the return.

$\rightarrow$ Investors optimally hold a mean-variance efficient portfolio: a portfolio with the highest expected return for a given level of variance.

$\Longrightarrow$ **The Efficient Frontier & Capital Market Line**

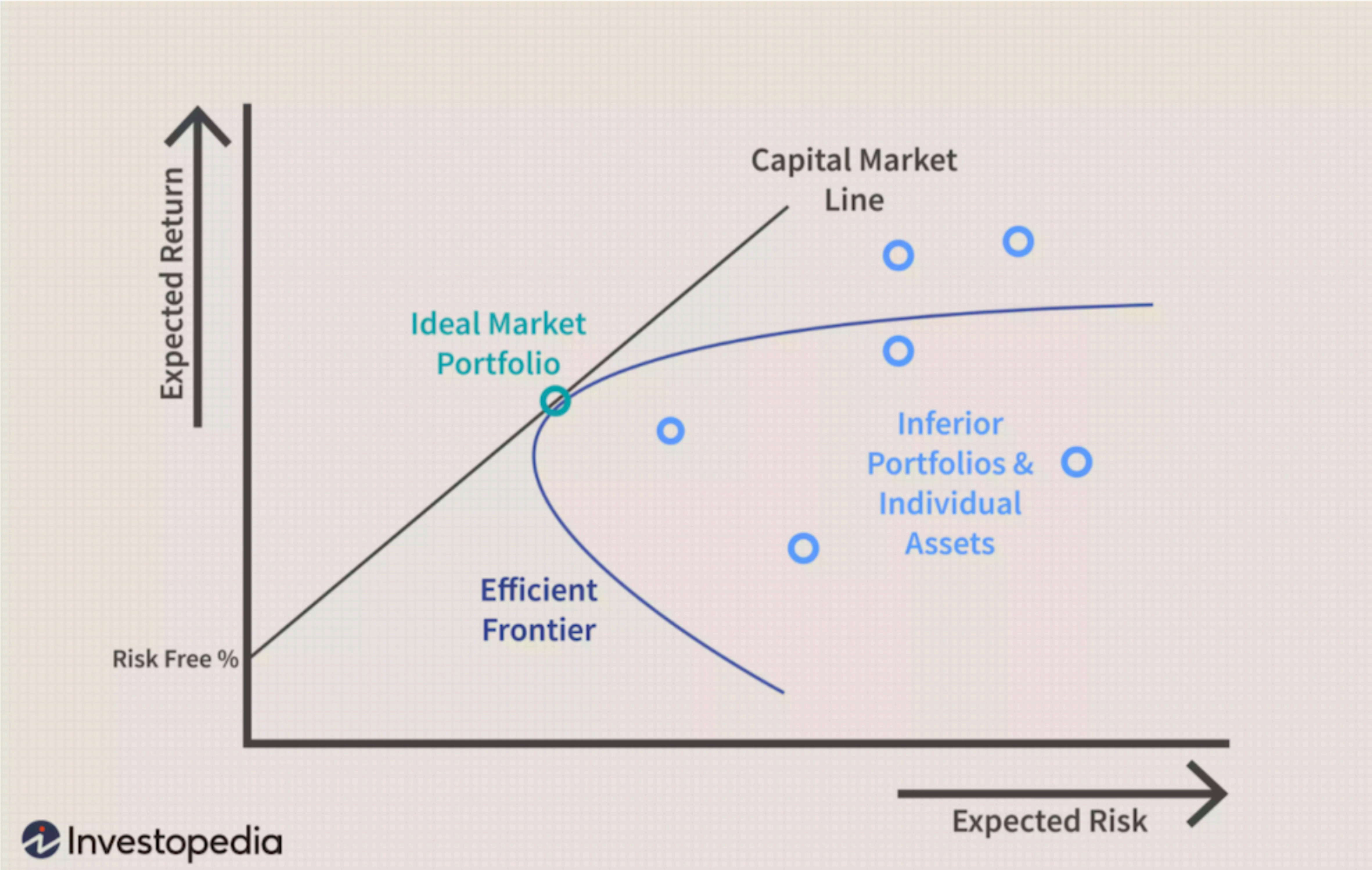
- Capital Asset Pricing Model is concerned with the pricing of assets in equilibrium. In equilibrium, all assets must be held by someone.

$\longrightarrow$ How investors determine the expected returns—and thereby asset prices—as a function of risk.

$\Longrightarrow$ **The Security Market Line**

CAPM

- Given that: some risk can be diversified, diversification is easy and costless, and rational investors diversify
- There should be no premium associated with diversifiable risk.
- The question becomes: What is the equilibrium relation between systematic risk and expected return in the capital markets?
- The CAPM is the best-known and most-widely used equilibrium model of the risk/return (systematic risk/return) relation.



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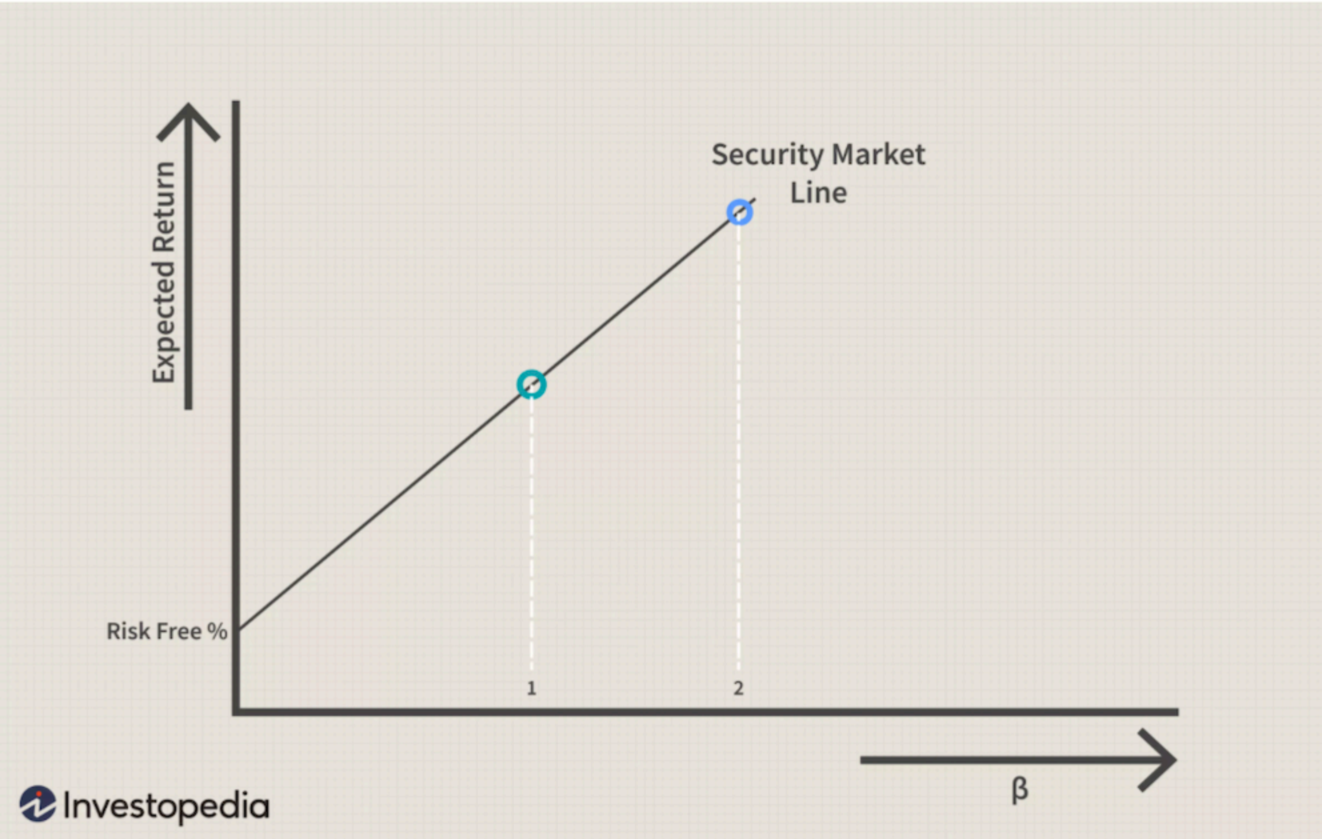

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CAPM Intuition

What would be a "fair" expected return on any stock?

- $E(R_{it}) = R_{ft}$ (risk free) + Risk Premium
- Risk free assets earn the risk-free rate (think of this as a rental rate on capital). The risk free compensate for time.
- If the asset is risky, we need to add a risk premium.
- The size of the risk premium depends on the amount of systematic risk for the asset (stock, bond, or investment project) and the price per unit risk.
- $R_{it} - R_{ft}$: Excess return

CAPM Intuition Formalized

$$E\left[R_{it}\right] = R_{ft} + \frac{Cov\left(R_{it}, R_{mt}\right)}{Var\left(R_{mt}\right)}\left[E\left[R_{mt}\right] - R_{ft}\right]$$
$$E\left[R_{it}\right] = R_{ft} + \beta_i\left[E\left[R_{mt}\right] - R_{ft}\right]$$

The expression above is referred to as the "Security Market Line".

- $E\left[R_{mt}\right] - R_{ft}$ Market Risk premiun (compensation for risk) or the price per unit of risk
- β_i: number of units of systematic risk
  - $\beta_i > 1$ (or < 1): the asset is more (less) risky than the market portfolio
  - $\beta_i < 0$: the asset is a hedge against the market portfolio
  - β_i how sensitive the asset to **market movement**

CAPM Formalized

Three inputs are required:

1. An estimate of the risk free interest rate. The current yield on short term treasury bills is one proxy. Practitioners tend to favor the current yield on longer-term treasury bonds but this may be a fix for a problem we don't fully understand.
2. An estimate of the market risk premium, $E\left[R_{mt}\right] - R_{ft}$. Expectations are not observable. Generally use a historically estimated value.

   The market is defined as a portfolio of all wealth including real estate, human capital, etc. In practice, a broad based stock index, such as the S&P 500 or the portfolio of all NYSE stocks, is generally used.
3. An estimate of beta.

CAPM: Econometric model

Let $X_{mt} = R_{mt} - R_{ft}$ and $X_{it} = R_{it} - R_{ft}$ and consider the econometric model:

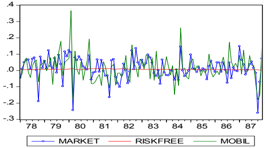
$$X_{it} = \alpha_i + \beta_i X_{mt} + \mu_{it}$$

- The CAPM can be examined by testing $H_0 : \alpha_i = 0$
- If $\alpha_i > 0$, asset i beats the market by earning more than $\beta_i E\left[X_{mt}\right]$
- This has been used to test the performance of mutual funds (application in the Brooks textbook)

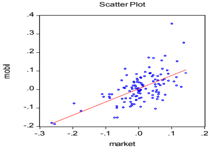
CAPM: Application

What detremines the expected return of an asset?

Example: Mobil (a US petroleum firm), 1978:01-1987:12 with $T = 120$.



Dependent Variable: E_MOBIL				
Method: Least Squares				
Sample: 1978M01 1987M12				
Included observations: 120				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.004241	0.005881	0.721087	0.4723
E_MARKET	0.714695	0.085615	8.347761	0.0000
R-squared	0.371287	Mean dependent var		0.009353
Adjusted R-squared	0.365959	S.D. dependent var		0.080468
S.E. of regression	0.064074	Akaike info criterion		-2.641019
Sum squared resid	0.484452	Schwarz criterion		-2.594561
Log likelihood	160.4612	F-statistic		69.68511
Durbin-Watson stat	2.087124	Prob(F-statistic)		0.000000



Application 2: The term structure of interest rates

- Interest rates are the price of money and, in equilibrium, interest rates equate the amount of borrowing to the amount of saving.
- The Term Structure of Interest Rates shows the relation between interest rates for different term-to-maturity loans.
- In the most basic sense, theories to explain the term structure are still based on interest rates equating the supply and demand for loanable funds.
- Different rates may exist over different terms because of expectations of changing inflation and differing preferences regarding longer-term vs. shorter-term saving.

The term structure of interest rate

The relation of long and short bonds?

- Monthly return on the $n-$month bond at time n:$R_{n,t}$, $n = 1, 3$

  Rule of one price (Expectations Hypothesis)

$$(1 + R_{3,t})^3 = (1 + R_{1,t})(1 + E_t R_{1,t+1})(1 + E_t R_{1,t+2})$$

$$R_{3,t} \approx [R_{3,t} + E_t R_{1,t+1} + E_t R_{1,t+2}]/3$$

  where E_t is expectation formed at time t

- If $R_{1,t}$ follows a random walk: $R_{1,t+1} = R_{1,t} + v_{t+1}$ with $E_t v_{t+1} = 0$ then $E_t R_{1,t+1} = E_t R_{1,t+2} = R_{1,t}$ and

$$R_{3,t} \approx R_{1,t}$$

- Test the null $H_0 : \beta_0 = 0, \beta_1 = 1$ in $R_{3,t} = \beta_0 + \beta_1 R_{1,t} + u_t$.

Term structure of interest rate: example

- Consider that given expectations for inflation over the next year, investors require 4% for a one year loan.
- Suppose investors currently expect inflation for the next year (the second year) to be higher so that they expect to require 6% for a one year loan (starting one year from now).
- Then, the Pure-Expectations Hypothesis, is consistent with the current 2-year spot rate defined as follows:

$$(1 + R_{2,t})^2 = (1 + R_{1,t})(1 + E_t[R_{1,t+1}) = (1.04)(1.06)$$

so $R_{2,t} = 4.995238\%$

- Restated, if we observe $R_{1,t} = 4\%$ and $R_{2,t} = 4.995238\%$, then, under the Pure-Expectations Hypothesis, we would have $E_t[R_{1,t+1}]$ to be 6%.

Term Structure

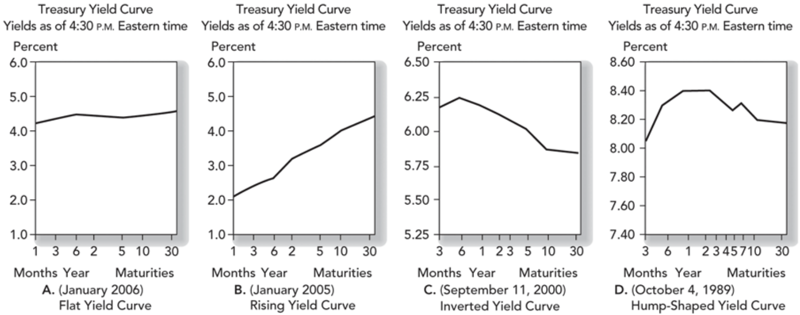


FIGURE 15.1 Treasury yield curves

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Application 3: Present Value (Gordon) Model

- The price of the asset today is a discounted sum of all possible future cash flows (or dividends D)

$$P_t = \sum_{j}^{\infty} \frac{E_t\left(D_{t+j}\right)}{(1+R)^j}$$

$$E_t\left(D_{t+j}\right) = D_t$$

Present Value (Gordon) Model

$$
\begin{aligned}
P_t &= D_t \left[\frac{1}{1+R} + \frac{1}{(1+R)^2} + K \right] \\
&= \frac{D_t}{1+R} \left[1 + \frac{1}{1+R} + \frac{1}{(1+R)^2} + K \right] \\
&= \frac{D_t}{1+R} \left[\frac{1}{1 - 1/(1+R)} \right] \\
&= \frac{D_t}{R}
\end{aligned}
$$

- we used the property of the infinite converging geometric progression series: $\sum_{k=0}^{\infty} a^k = 1/(1-a)$

Present Value (Gordon) Model

- the model is still nonlinear, but we may take the logs: $\log(P_t) = -\log(R_t) + \log(D_t)$
- And once again use OLS to test whether the model is correct

$$\log(P_t) = \alpha + \beta \log(D_t) + u_t$$

Test the null hypothesis $H_0 : \beta = 1$