ECON3206/ECON5206 Financial Econometrics

Sample Answers/Hints to Tutorial 1

1. The Taylor expansion of $f(x) = \ln(1+x)$ around $x_0 = 0$ is simply $f(x) \approx f(0) + f'(0)(x-0) = x$. Then $r_t = \ln\left(1 + \frac{P_t - P_{t-1}}{P_{t-1}}\right) \approx \frac{P_t - P_{t-1}}{P_{t-1}} = R_t$.
2. The return from the end of day 1 to the end of day 3 is $\ln\left(\frac{P_3}{P_1}\right) = \left[\ln\left(\frac{P_2}{P_1}\right) + \ln\left(\frac{P_3}{P_2}\right)\right] = r_2 + r_3 = .02\ or\ 2\%$. In general, a 5-day return is the sum of 5 daily returns: $\ln\left(\frac{P_5}{P_0}\right) = \left[\ln\left(\frac{P_1}{P_0}\right) + \ln\left(\frac{P_2}{P_1}\right) + \ln\left(\frac{P_3}{P_2}\right) + \ln\left(\frac{P_4}{P_3}\right) + \ln\left(\frac{P_5}{P_4}\right)\right]$. It is easy to see that an n-day return is the sum of n daily returns.
3.
   (a) Only 0, 1 and 2, called support of the discrete distribution.

   (b) 0.25

   (c) $E(X) = 0\times0.25 + 1\times0.5 + 2\times0.25 = 1$

   For Binomial distribution, $E(X) = n\times p = 2\times0.5 = 1$

   $Var(X) = (0-1)^2\times0.25 + (1-1)^2\times0.5 + (2-1)^2\times0.25 = 0.5$

   For Binomial distribution $Var(X) = n\times p(1-p) = .5$

   (d) Zero, we can only talk about the probability of a range of values for continuous distributions.

   (e) $P(X < 180) = 0.5$

   (f) (g) (h) These are important notions. While definitions are given in the slides or elsewhere, you must make sure that you are able to understand and explain these notions in your own words.
4. In the lecture slides, an example (involving *wage* and *age*) for the notion of conditional distribution is given. You should be able to find an example of your own. Here is another example. Consider the end-of-day trading prices of BHP share: P_{t-1} for today and P_t for tomorrow. The function (typically pdf or pmf) that describes the likelihoods of the possible values of P_t when P_{t-1} is known (or fixed) is the conditional distribution of P_t given P_{t-1}, which differs from the (unconditional) distribution of P_t. The variance of the latter is generally much larger than the former

(imagine the range of possible values for P_t without knowing P_{t-1}). In general, the conditional distribution of P_t given P_{t-1} depends on what we observe for P_{t-1}. For instance, the conditional distribution of P_t given $P_{t-1} = 30$ may differ from the conditional distribution of P_t given $P_{t-1} = 33$. Consequently, the conditional mean and the conditional variance of P_t given P_{t-1} depend on P_{t-1} and are functions of P_{t-1}.

5. (a) $E(\bar{X}) = \frac{1}{n}E(\sum_i^n X_i) = \frac{1}{n}\sum_i^n E(X_i) = \mu$

(b) $\mathrm{Var}(\bar{X}) = \frac{1}{n^2}\mathrm{Var}(\sum_1^n X_i) = \frac{1}{n^2}\sum_1^n \mathrm{Var}(X_i) = \frac{\sigma^2}{n}$ (when RVs are uncorrelated).

(c) $E\big(f(\bar{X})\big) = f(E(\bar{X}))$ only in a very special case when $f(x)$ is a *linear* function, i.e., $f(x) = c + ax$, where *c* and *a* are some constants. Otherwise, this equality does not hold, e.g., $\big(ln(\bar{X})\big) \neq ln(E(\bar{X}))$ and there is no easy solution.

(d) Yes, but when the RVs are correlated, things get more complicated. We can still work it out from the basics. First, we use an algebraic formula for the square of the sum of *n* numbers (you do not have to remember it, but an analogy of the square of the sum of two, three numbers should be familiar) to write $\left(\frac{1}{n}\sum_i^n X_i - \mu\right)^2 = \frac{1}{n^2}(\sum_i^n (X_i - \mu))^2 = \frac{1}{n^2}\sum_{i=1}^n (X_i - \mu)^2 + \frac{2}{n^2}\sum_{i=1}^{n-1}\sum_{j=i+1}^n (X_i - \mu)(X_j - \mu)$. Then, after taking expectations, we find $\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{n} + \frac{2}{n^2}\sum_{i=1}^{n-1}\sum_{j=i+1}^n \mathrm{Cov}(X_i, X_j)$. You may try the cases for $n = 2$ and 3 to practice.

(e) As n gets larger ($n \to \infty$), $Var(\bar{X}) \to 0$, which implies that $\bar{X}$ gets more and more concentrated around its mean μ. Essentially, this is what the law of large numbers states. *[Technically this result is equivalent to the *weak* law of large numbers, stated in the lecture slides. The *weak* law of large numbers uses *convergence in probability*, for small ε

$$\lim_{n\to\infty} \Pr\Big(|\overline{X}_n - \mu| > \varepsilon \Big) = 0.$$

The *strong* law of large numbers states *almost sure convergence*

$$\Pr\Big(\lim_{n\to\infty} \overline{X}_n = \mu \Big) = 1.$$

We will not emphasise the difference during this course] Remember that *[ ] is optional information.

(a)-(e) see the attached excel file

(f) While the exact values are different, you should find that the stylised facts summarise in the lecture (what are these?) largely hold for this data set. We note that this data set covers the volatile period of the global financial crisis, a reason why the standard deviations here are larger than the data set used in the lecture. We also see more pronounced "clustering"- large (small) variations followed by large (small) variations, see the plots of returns.