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Financial Econometrics

Slides-05: Time Series Analysis using ARMA models Part 2

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Plan.

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Time Series Models (Mainly Theoretical Aspects)

- MA process
- AR process
  - Wold Decomposition
  - AF and PACF patterns
  - Impulse response function

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- Moving average models
  - In Wold decomposition, $b_i \to 0$ as $i \to \infty$. A simple approximation to the GLP is to restricting
    $$b_i = 0 \text{ for all } i > q.$$
  - The result is $MA(q)$ model:
    $$y_t = \mu + \epsilon_t + \theta_1\epsilon_{t-1} + \cdots + \theta_q\epsilon_{t-q}, \epsilon_t \sim \text{ i.i.d } WN(0, \sigma^2),$$
    where y_t is the "average" of the current shock and its q recent lags. The shock ϵ_t and its lags are unobservable.
  - Use lag operator L: $Lz_t = z_{t-1}$ to write $MA(q)$:
    $$y_t = \mu + \Theta(L)\epsilon_t,$$
    where
    $$\Theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \cdots + \theta_q L^q.$$

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- $MA(1)$ model
  - $MA(1)$ model (as a data generating process)

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$$y_t = \mu + \epsilon_t + \theta_1 \epsilon_{t-1}, \epsilon_t \sim \text{i.i.d } WN(0, \sigma^2),$$

  - $MA(1)$:

$$y_t = \mu + \Theta(L)\epsilon_t,$$

where $\Theta(L) = 1 + \theta_1 L$.

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MA(1) model: Unconditional moments

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- $MA(1)$ model: Characteristics
  - It is always stationary.
    $E(y_t) = \mu$, $Var(y_t) = (1+\theta_1^2)\sigma^2$,

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$$\gamma_j = Cov(y_t, y_{t-j}) = \begin{Bmatrix} \theta_1\sigma^2, & j = 1 \\ 0, & j > 1 \end{Bmatrix}$$

$$\rho_j = \frac{\gamma_j}{\gamma_0} = \begin{Bmatrix} \theta_1/(1+\theta^2), & j = 1 \\ 0, & j > 1 \end{Bmatrix} \text{(AC cutoff at } j = 1\text{).}$$

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  - If the estimated $\hat{\rho}_j$ has a cutoff at $j = 1$, the time series may be fitted in an $MA(1)$ model.

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- $MA(1)$ model: Conditional moments
  - Conditional on $\Omega_t = \{\epsilon_t, \epsilon_{t-1}, \cdots ; y_t, y_{t-1}, \cdots\}$

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$$E\left(y_{t+h}|\Omega_t\right) = \left\{ \begin{array}{ll} \mu + \theta_1 \epsilon_t, & h = 1 \\ \mu, & h > 1 \end{array} \right\},$$

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$$Var\left(y_{t+h}|\Omega_t\right) = \left\{ \begin{array}{ll} \sigma^2, & h = 1 \\ (1+\theta_1^2)\sigma^2, & h > 1 \end{array} \right\}.$$

- Conditional variance $\leq$unconditional variance (why?)

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MA(1) model: Dynamic Behavior

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- $MA(1)$ model: Impulse response function
  - the effect on y_{t+h} of a one-std-deviation increase in ϵ_t:

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$$\sigma \frac{\delta y_{t+h}}{\delta \epsilon_t} = \left\{ \begin{array}{ll} \sigma\theta_1, & h = 1 \\ 0, & h > 1 \end{array} \right\}.$$

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- $MA(1)$ model: Invertibility
  - Can we back out a unique θ_1 from:

$$\rho_j = \frac{\gamma_j}{\gamma_0} = \begin{cases} \theta_1/(1+\theta_1^2), & j = 1 \\ 0, & j > 1 \end{cases}.$$

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  Can we get to know $\{\epsilon_t, \epsilon_{t-1}, \cdots\}$ based on $\{y_t, y_{t-1}, \cdots\}$?
  - Yes if MA is *invertible*

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- The $MA(q)$ process $y_t = \mu + \Theta(L)\epsilon_t$ is invertible if the roots of $\Theta(z) = 0$ are all outside the unit circle.
- For $MA(1)$, the root of $1 + \theta_1 z = 0$ is $z = -1/\theta_1$. Hence, $MA(1)$ is invertible when $|-1/\theta_1| > 1$ or $|\theta_1| < 1$.
- **Invertible** in the sense that $\Theta(L)^{-1}$ exists properly.

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- $MA(1)$ model: Invertible
  - When MA is invertible, the shock may be recovered from the observable: $\epsilon_t = \Theta(L)^{-1}(y_t - \mu)$. For $MA(1)$, when invertible,

$$\Theta(L)^{-1} = (1+\theta_1 L)^{-1} = 1 + (-\theta_1)L + (-\theta_1)^2 L^2 + \cdots, \quad (1)$$

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$$\epsilon_t = y_t - \mu + \sum_{i=1}^{\infty}(-\theta_1)^i (y_{t-i} - \mu) \quad (2)$$

$$= y_t + \sum_{i=1}^{\infty}(-\theta_1)^i y_{t-i} - \mu/(1+\theta_1). \quad (3)$$

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Hint. Use expansion: $1/(1-x) = 1 + x + x^2 + \cdots$

  - Parameters can be estimated by minimizing $\sum_{t=1}^{T} \epsilon_t^2$
  - The alternative expression: $y_t = \mu/(1+\theta_1) - \sum_{i=1}^{\infty}(-\theta_1)^i y_{t-i} + \epsilon_t$ indicates that the PAC function of invertible $MA(1)$ has no cutoffs and decays exponentially.

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$MA(1)$ model: Example

$MA(1)$: simulated and fitted

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eg. time series plots of simulated MA(1)

$\rho_1 = \theta_1/(1+\theta_1^2)$

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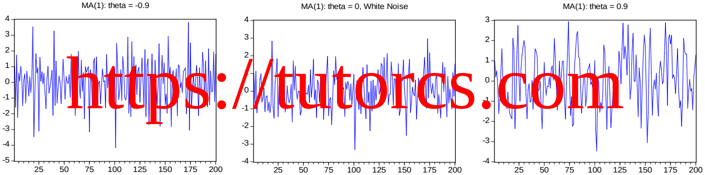

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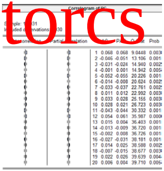
eg. NYSE comp return

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.035311	0.02457	1.43729	0.1508
MA(1)	0.075177	0.02271	3.31031	0.0009



	AC	PAC	Q-Stat	Prob
1	0.068	0.068	9.0448	0.003
2	-0.046	-0.051	13.106	0.001
3	-0.031	-0.024	14.940	0.002
4	-0.001	0.001	14.942	0.005
5	-0.052	-0.055	20.226	0.001
6	-0.014	-0.008	20.624	0.002
7	-0.033	-0.037	22.761	0.002
8	0.011	0.012	22.992	0.003
9	0.033	0.028	25.155	0.003
10	0.028	0.021	26.723	0.003
11	-0.043	-0.044	30.332	0.001
12	0.054	0.061	35.987	0.000
13	0.015	0.004	36.403	0.001
14	-0.013	-0.009	36.720	0.001
15	-0.002	0.008	36.726	0.001
16	-0.027	-0.031	38.181	0.001
17	0.014	0.025	38.588	0.002
18	-0.007	-0.015	38.677	0.003
19	0.022	0.026	39.639	0.004
20	0.006	0.004	39.710	0.005

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■Dynamic Behaviour of a Moving Average Process $MA(q)$

An MA process is simply a linear combination of white noise error terms ?. These error terms can be seen as **impulses** or **innovations** or **shocks** while the MA model describes the **dynamic impact** of these shocks on the series y_t.

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The **impulse response function**, i.e. the dynamic impact of an impulse ϵ_t on $y_t, y_{t+1}, \cdots$ is given by

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$$
\begin{aligned}
\delta y_t / \delta \epsilon_t &= 1 \\
\delta y_t / \delta \epsilon_t &= \theta_1 \\
&\ldots \\
\delta y_{t+q} / \delta \epsilon_t &= \theta_q \\
\delta y_{t+q+k} / \delta \epsilon_t &= 0, \text{ for } k > 0
\end{aligned}
$$

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■ **General Properties of a Moving Average Process $MA(q)$**

- $E(y_t) = \mu$

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- $\gamma_0 = (1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2)\sigma^2$
- The ACF:

$$\begin{aligned}\gamma_k &= (\theta_k + \theta_{k+1}\theta_1 + \theta_{k+2}\theta_2 + \cdots + \theta_q\theta_{q-k})\sigma^2, \text{ fpr } k = 1, \cdots, q. \\ \gamma_k &= 0, \text{ for } k > q.\end{aligned}$$

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- The PACF? $p_k \neq 0 \; \forall k$ dies out slowly

■ **Stationarity conditions** for an MA process:

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- γ_0 is finite
- γ_k is finite

$\Longrightarrow$ a finite order MA process will always be stationary.

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$MA(q)$ Conclusions

- As the **ACF cuts off after q lags**, the order of an MA process can be determined from an inspection of the sample ACF.
- It can be shown (see below) that the **PACF dies out slowly**.
- A finite order MA process is **stationary by construction**, as it is a weighted sum of a fixed number of white noise processes, i.e. the mean, variance and autocovariances don't depend on time!

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Defining an Autoregressive Process

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Let ε_t be a white noise process. Then:

$$\begin{aligned} y_t &= \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \ldots + \alpha_p y_{t-p} + \varepsilon_t \qquad (12) \\ &= \alpha_0 + \sum\nolimits_{i=1}^{p} \alpha_i y_{t-i} + \varepsilon_t \end{aligned}$$

is an **autoregressive process of order** p, denoted AR(p).

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$\rightarrow$ y_t depends on its own lagged values and on the current value of a white noise disturbance term ε_t.

The model can conveniently be rewritten in so-called **lag operator notation** as

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$$y_t = \alpha_0 + \sum\nolimits_{i=1}^{p} \alpha_i L^i y_t + \varepsilon_t \quad \text{with} \quad L^i y_t = y_{t-i}$$

$$\alpha(L)\, y_t = \alpha_0 + \varepsilon_t \qquad (13)$$

where $\alpha(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \ldots - \alpha_p L^p$ is a lag polynomial of order p

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In an AR process, the value for y_t is simply a linear combination of past values plus a white noise error term ε_t. Again, these error terms can be seen as **impulses** or **innovations** or **shocks** while the AR model describes the **dynamic impact** of these shocks on the series y_t.

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In order to trace out the dynamic impact of an impulse ε_t on $y_t, y_{t+1}, \ldots$, it is very convenient to first 'solve' the AR model in terms of the ε sequence. For notational convenience, first consider an AR(1) process

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$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \varepsilon_t$$

where ε_t is a white noise process.

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The easiest way to express y_t as a function of the ε sequence is b **backward substitution**. This implies substituting

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$$y_{t-1} = \alpha_0 + \alpha_1 y_{t-2} + \varepsilon_{t-1}$$

in the equation for y_t to obtain

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$$\begin{aligned} y_t &= \alpha_0 + \alpha_1 \left(\alpha_0 + \alpha_1 y_{t-2} + \varepsilon_{t-1}\right) + \varepsilon_t \\ &= \left(1 + \alpha_1\right) \alpha_0 + \alpha_1^2 y_{t-2} + \alpha_1 \varepsilon_{t-1} + \varepsilon_t \end{aligned}$$

Next substitute

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$$y_{t-2} = \alpha_0 + \alpha_1 y_{t-3} + \varepsilon_{t-2}$$

in the equation for y_t to obtain

$$y_t = \left(1 + \alpha_1 + \alpha_1^2\right) \alpha_0 + \alpha_1^3 y_{t-3} + \alpha_1^2 \varepsilon_{t-2} + \alpha_1 \varepsilon_{t-1} + \varepsilon_t$$

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AR Process: Impulse response function

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After repeating this $t-1$ times, we obtain

$$y_t = \left(1 + \alpha_1 + \ldots + \alpha_1^{t-1}\right)\alpha_0 + \alpha_1^t y_0 + \alpha_1^{t-1}\varepsilon_1 + \ldots + \alpha_1\varepsilon_{t-1} + \varepsilon_t$$
$$= \alpha_0 \sum_{i=0}^{t-1} \alpha_1^i + \alpha_1^t y_0 + \sum_{i=0}^{t-1} \alpha_1^i \varepsilon_{t-i} \qquad (14)$$

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where y_0 is the initial condition or the value for y in period 0.

The **impulse response function** can now easily be obtained

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$$\begin{aligned} dy_t/d\varepsilon_t &= \alpha_1^0 = 1 \\ dy_{t+1}/d\varepsilon_t &= \alpha_1 \\ dy_{t+2}/d\varepsilon_t &= \alpha_1^2 \\ dy_{t+3}/d\varepsilon_t &= \alpha_1^3 \\ &\vdots \end{aligned}$$

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AR Process: Convergence

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Note that whether an AR(1) series is mean-reverting after being hit by a shock depends on the particular value for α_1. Two cases can be distinguished:

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- The **convergence case** $|\alpha_1| < 1$
  A shock affects all future observations but with a decreasing effect, i.e. the AR(1) process is mean-reverting.
- The **non-convergence case** $|\alpha_1| \geq 1$
  A shock affects all future observations but with an equal impact ($\alpha_1 = 1$) or with an increasing impact ($\alpha_1 > 1$), i.e. the AR(1) series is not mean-reverting.

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Properties of an AR(1) Process

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Let $t \to \infty$ in eq. (14):

$$y_t = \alpha_0 \sum_{i=0}^{\infty} \alpha_1^i + \alpha_1^{\infty} y_0 + \sum_{i=0}^{\infty} \alpha_1^i \varepsilon_{t-i} \qquad (15)$$

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- The expected value of y_t is given by

$$E(y_t) = E\left(\left(1 + \alpha_1 + \alpha_1^2 + \dots\right)\alpha_0 + \alpha_1^{\infty} y_0 + \sum_{i=0}^{\infty} \alpha_1^i \varepsilon_{t-i}\right)$$

$$= E\left(\left(1 + \alpha_1 + \alpha_1^2 + \dots\right)\alpha_0 + \alpha_1^{\infty} y_0\right)$$

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$\rightarrow$ if $|\alpha_1| < 1$: $E(y_t)$ converges to $\dfrac{\alpha_0}{(1-\alpha_1)}$

$\rightarrow$ if $|\alpha_1| \geq 1$: $E(y_t)$ is time-dependent

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- The variance of y_t is given by

$$
\begin{aligned}
V(y_t) &= E\left(y_t - E(y_t)\right)^2 \\
&= E\left(\sum_{i=0}^{\infty} \alpha_1^i \varepsilon_{t-i}\right)^2 \\
&= E\left(\varepsilon_t^2 + \alpha_1^2 \varepsilon_{t-1}^2 + \alpha_1^4 \varepsilon_{t-2}^2 + \cdots + \text{cross products}\right) \\
&= E\left(\varepsilon_t^2\right) + \alpha_1^2 E\left(\varepsilon_{t-1}^2\right) + \alpha_1^4 E\left(\varepsilon_{t-2}^2\right) + \ldots \\
&= \left(1 + \alpha_1^2 + \alpha_1^4 + \ldots\right) \sigma^2
\end{aligned}
$$

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$\rightarrow$ if $|\alpha_1| < 1$: $V(y_t)$ converges to $\dfrac{\sigma^2}{(1-\alpha_1^2)}$

$\rightarrow$ if $|\alpha_1| \geq 1$: $V(y_t)$ is time-dependent

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Properties of $AR(1)$ Process: ACF

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- The autocovariances γ_k are given by

$$\gamma_1 = cov\left(y_t, y_{t-1}\right) = E\left(\left(y_t - E\left(y_t\right)\right)\left(y_{t-1} - E\left(y_{t-1}\right)\right)\right)$$

$$= E\left(\left(\varepsilon_t + \alpha_1\varepsilon_{t-1} + \alpha_1^2\varepsilon_{t-2} + \ldots\right)\left(\varepsilon_{t-1} + \alpha_1\varepsilon_{t-2} + \alpha_1^2\varepsilon_{t-3} + .\right.\right.$$

$$= E\left(\alpha_1\varepsilon_{t-1}^2 + \alpha_1^3\varepsilon_{t-2}^2 + \alpha_1^5\varepsilon_{t-3}^2 + \ldots + \text{cross-products}\right)$$

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$$= \alpha_1 E\left(\varepsilon_{t-1}^2\right) + \alpha_1^3 E\left(\varepsilon_{t-2}^2\right) + \alpha_1^5 E\left(\varepsilon_{t-3}^2\right) + \ldots$$

$$= \left(\alpha_1 + \alpha_1^3 + \alpha_1^5 + \ldots\right)\sigma^2$$

$$= \alpha_1\left(1 + \alpha_1^2 + \alpha_1^4 + \ldots\right)\sigma^2$$

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$\rightarrow$ if $|\alpha_1| < 1$: $\quad \gamma_1$ converges to $\alpha_1 \dfrac{\sigma^2}{(1-\alpha_1^2)}$

$\rightarrow$ if $|\alpha_1| \geq 1$: $\quad \gamma_1$ is time-dependent

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$$
\begin{aligned}
\gamma_2 &= cov(y_t, y_{t-2}) = E\left((y_t - E(y_t))(y_{t-2} - E(y_{t-2}))\right) \\
&= E\left(\left(\varepsilon_t + \alpha_1\varepsilon_{t-1} + \alpha_1^2\varepsilon_{t-2} + \ldots\right)\left(\varepsilon_{t-2} + \alpha_1\varepsilon_{t-3} + \alpha_1^2\varepsilon_{t-4} + \ldots\right)\right) \\
&= E\left(\alpha_1^2\varepsilon_{t-2}^2 + \alpha_1^4\varepsilon_{t-3}^2 + \alpha_1^6\varepsilon_{t-4}^2 + \ldots + \text{cross-products}\right) \\
&= \alpha_1^2 E\left(\varepsilon_{t-2}^2\right) + \alpha_1^4 E\left(\varepsilon_{t-3}^2\right) + \alpha_1^6 E\left(\varepsilon_{t-4}^2\right) + \ldots \\
&= \left(\alpha_1^2 + \alpha_1^4 + \alpha_1^6 + \ldots\right)\sigma^2 \\
&= \alpha_1^2\left(1 + \alpha_1^2 + \alpha_1^4 + \ldots\right)\sigma^2
\end{aligned}
$$

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$\rightarrow$ if $|\alpha_1| < 1$: γ_2 converges to $\alpha_1^2 \dfrac{\sigma^2}{(1-\alpha_1^2)}$

$\rightarrow$ if $|\alpha_1| \geq 1$: γ_2 is time-dependent

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$$\gamma_k = cov(y_t, y_{t-k}) = E((y_t - E(y_t))(y_{t-k} - E(y_{t-k})))$$

$$\rightarrow \text{if } |\alpha_1| < 1 : \quad \gamma_k \quad \text{converges to} \quad \alpha_1^k \frac{\sigma^2}{(1 - \alpha_1^2)}$$

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$$\rightarrow \text{if } |\alpha_1| \geq 1 : \quad \gamma_k \quad \text{is time-dependent}$$

- The ACF (for stationary series!) is given by

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$$\rho_1 = \gamma_1 / \gamma_0 = \alpha_1$$
$$\rho_2 = \gamma_2 / \gamma_0 = \alpha_1^2$$
$$\vdots$$
$$\rho_k = \gamma_k / \gamma_0 = \alpha_1^k$$

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Stationarity conditions for an AR(1) process

- $\alpha_1^\infty = 0$
- $(1 + \alpha_1 + \alpha_1^2 + ...)$ is finite
- $(1 + \alpha_1^2 + \alpha_1^4 + \ldots)$ is finite
- $\alpha_1 \left(1 + \alpha_1^2 + \alpha_1^4 + \ldots\right)$ is finite
- $\ldots$

$\rightarrow$ an AR(1) process is stationary is $|\alpha_1| < 1$

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- The PACF cuts off after 1 lag.
- The ACF is infinite in extent (but dies out for covariance stationary processes).
- The properties of an AR(1) process crucially depend on the value for α_1
  - If $|\alpha_1| < 1$ the AR(1) process can be written as a stable infinite MA process (the so-called **MA representation**):

$$y_t = \frac{\alpha_0}{1-\alpha_1} + \sum\nolimits_{i=0}^{\infty} \alpha_1^i \varepsilon_{t-i}.$$

    In this case the series is **stationary** as it has finite constant mean, variance and autocovariances.
  - If $|\alpha_1| \geq 1$ no stable MA representation exists. In this case the series is **non-stationary** as the mean, variance and autocovariances are time-varying.

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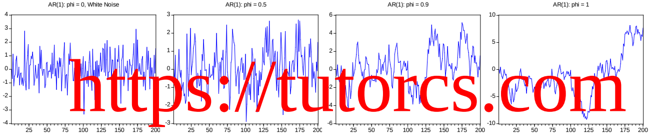
eg. time series plots of simulated AR(1)

$\rho_j = \phi_1^j$



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eg. NYSE comp return: 'c' below is in fact $\mu = c/(1-\phi_1)$

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Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.035159	0.024547	1.43235	0.1522
AR(1)	0.068401	0.022727	3.00976	0.0026

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