Consider the following AR(2) process:

$$y_t = 2 - 0.5y_{t-1} + 0.3y_{t-2} + \varepsilon_t, \qquad \varepsilon_t \sim N(0,1), \qquad T = 100$$

The characteristic equation is given by

$$1 + 0.5z - 0.3z^2 = 0$$

The characteristic roots are

$$z_1 = \frac{-0.5 + \sqrt{0.5^2 + 4 \times 0.3}}{-2 \times 0.3} = -1.17$$

$$z_2 = \frac{-0.5 - \sqrt{0.5^2 + 4 \times 0.3}}{-2 \times 0.3} = 2.84$$

This is a stationary series as the characteristic roots are larger than 1 in absolute value.

Note that stationarity could also be concluded from

- $\sum_{i=1}^{p} \alpha_i = -0.5 + 0.3 = -0.2 < 1$
- $\sum_{i=1}^{p} |\alpha_i| = 0.5 + 0.3 = 0.8 < 1$

Properties

- The expected value of the series is given by

$$E(y_t) = 2/(1 + 0.5 - 0.3) = 1.67$$

- The variance is given by

$$V(y_t) = \frac{(1 - 0.3)}{(1 + 0.3)(1 + 0.5 - 0.3)(1 - 0.5 - 0.3)} = 2.2436$$

- The ACF is given by

$$\rho_1 = -0.5/(1-0.3) = -0.7143$$
$$\rho_2 = 0.5^2/(1-0.3) + 0.3 = 0.6571$$
$$\rho_3 = 0.5 \times 0.6571 + 0.3 \times 0.7143 = -0.5429$$
$$\rho_4 = 0.5 \times 0.5429 + 0.3 \times 0.6571 = 0.4686$$
$$\vdots$$

- The PACF is given by

$$\tau_{11} = -0.7143$$
$$\tau_{22} = 0.3$$
$$\tau_{kk} = 0 \quad \forall k > 2$$

Figure 37 : Theoretical ACF and PACF of generated AR(2) process

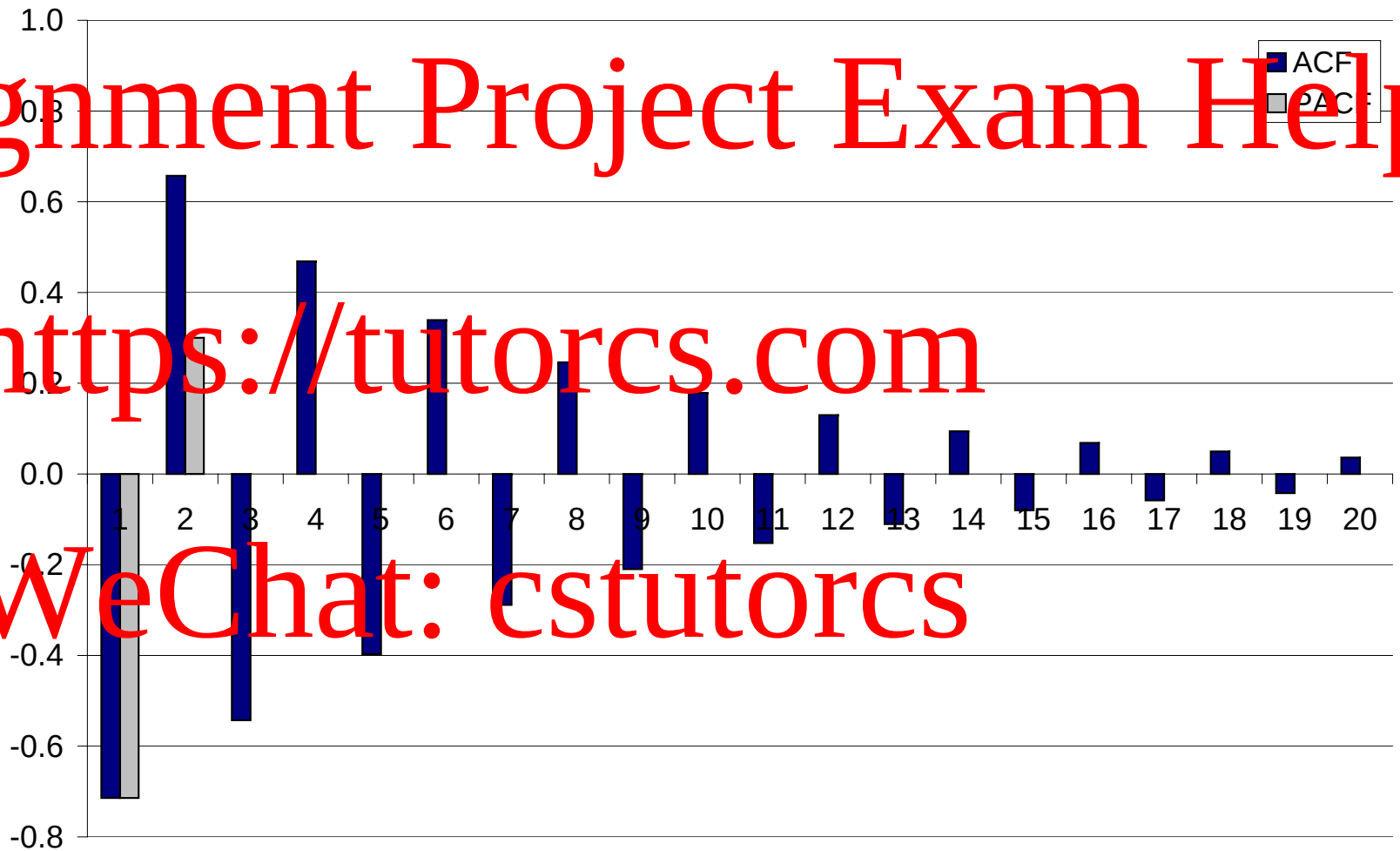

Figure 38 : Dynamic impact of a shock ε_t on y