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University of New South Wales
School of Economics
Financial Econometrics
Tutorial 6

1. (ARCH model characteristics)

Consider the following AR(1)-ARCH(2) model,

$$y_t = c + \phi_1 y_{t-1} + \varepsilon_t, \quad |\phi_1| < 1, \quad \varepsilon_t | \Omega_{t-1} \sim N(0, \sigma_t^2),$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2, \quad \alpha_0 > 0, \quad \alpha_1 \geq 0, \quad \alpha_2 \geq 0, \quad \alpha_1 + \alpha_2 < 1,$$

where Ω_t is the information set at the end of period t.

(a) Find $E(\varepsilon_t | \Omega_{t-1})$, $E(y_t | \Omega_{t-1})$ and their unconditional counterparts.

(b) Find $\text{Var}(\varepsilon_t | \Omega_{t-1})$, $\text{Var}(y_t | \Omega_{t-1})$ and their unconditional counterparts.

(c) Is ε_t a white noise process? Is it an independent (or iid) WN process? Verify your answer.

(d) In what fundamental way does the ARCH model differ from the standard (homoscedastic) ARMA models? What is the purpose of the variance equation?

(e) Ceteris paribus, what is the change in σ_t^2 caused by a one-unit change in ε_{t-1}^2?

(f) Suppose $\alpha_2 = 0$, $\alpha_1^2 < 1/3$ and ε_t is strictly stationary. Find $E(\varepsilon_t^4)$ and the unconditional kurtosis of ε_t. Comment on its implication for the tails of the unconditional distribution of ε_t .

[Hint: for a zero-mean normal random variable $Z \sim N(0, \omega^2)$, $E(Z^4) = 3\omega^4$.]

2. (GARCH model characteristics)

Consider the following AR(1)-GARCH(1,1) model,

$$y_t = c + \phi_1 y_{t-1} + \varepsilon_t, \quad |\phi_1| < 1, \quad \varepsilon_t | \Omega_{t-1} \sim N(0, \sigma_t^2),$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \quad \alpha_0 > 0, \quad \alpha_1 \geq 0, \quad \beta_1 \geq 0, \quad \alpha_1 + \beta_1 < 1,$$

where Ω_t is the information set at the end of period t.

(a) Find $E(\varepsilon_t | \Omega_{t-1})$, $E(y_t | \Omega_{t-1})$ and their unconditional counterparts.

(b) Find $\text{Var}(\varepsilon_t | \Omega_{t-1})$, $\text{Var}(y_t | \Omega_{t-1})$ and their unconditional counterparts.

(c) Is ε_t a white noise process? Is it an independent (or iid) WN process? Verify your answer.

(d) Ceteris paribus, what is the change in σ_t^2 caused by a one-unit change in ε_{t-1}^2?

(e) Let $w_t = \varepsilon_t^2 - \sigma_t^2$. Show (i) w_t has no autocorrelation; (ii) ε_t^2 has an ARMA(1,1) representation with w_t being the shock.

(f) Some researchers prefer to write $\alpha_0 = \omega(1 - \alpha_1 - \beta_1)$, where ω is a free parameter (the unconditional variance of ε_t). For the an *integrated* GARCH(1,1), where $\alpha_1 + \beta_1 = 1$, show that the integrated GARCH(1,1) is in fact an EWMA of ε_t^2.

COMPUTING EXERCISES

3. (Estimation of ARCH)

This question is based on the data contained in the Excel file *SHARE.XLS*. The file contains daily data on the S&P500 from the 2nd of January, 1998 to the 10th of December, 2001 comprising a total of 994 observations. The S&P500 index is designated *PRICE* in the file. Generate the series for the percentage log return as: *R*=100*(log(*PRICE*)-log(*PRICE*(*-1*)))

(a) Perform the Jarque-Bera test for normality and show the empirical histogram for the returns. Also show the correlogram for the returns and interpret your results.

(b) Generate the series for the squared return as *R2*=*R***R* and create time series plot of *R2*. Also show the correlogram for squared returns and interpret your results.

(c) Assume the mean equation for returns is: $R_t = c + \varepsilon_t$. Perform an LM test for ARCH effects on the residuals from the regression. Interpret the testing results.

(d) Assume the mean equation for returns is: $R_t = c + \varepsilon_t$. Estimate an ARCH(5) model given the mean equation specified above. Interpret your results. Are the restrictions for the ARCH parameters satisfied? Extract and plot σ_t^2 from the estimated equation. Inspect and comment on the plot. Perform an LM test for ARCH effect on the standardised residual series and comment.

(e) Compare the histograms of residuals and standardised residuasls; and the correlograms of squared residuals and squared standardised-residuals from the model in (d) and comment.

(f) How would you choose the lags in the variance equation [why ARCH(5)?]? Would a more sophisticated mean equation help? Try some of your suggestions and comment on your results.

4. (Estimation of GARCH)

This question is based on the data contained in the Excel file *SHARE.XLS*. The file contains daily data on the S&P500 index from the 2nd of January, 1998 to the 10th of December, 2001 comprising a total of 994 observations. The S&P500 index is designated *PRICE* in the file. Generate the series for the percentage log return as: *R*=100*(log(*PRICE*) − log(*PRICE*(-1)))

(a) Assume the mean equation for returns is

$$R_t = c + \varepsilon_t$$

and that the variance equation for returns is a GARCH(1,1). Estimate the model and interpret your results. Are the sign restrictions for the GARCH specification satisfied?

(b) Extract and plot σ_t^2 from the estimated equation, and make a comparison to the same plot from ARCH(5).

(c) Perform an LM test on the standardized residuals from this GARCH(1,1) model. Interpret your results.

(d) Report the Jarque-Bera test for normality on the standardized residuals. Report the correlogram of the squared standardised-residuals. Comment on your results.

(e) Re-estimate the GARCH(1,1) model but do not select heteroscedastic-consistent standard errors. Compare the results with those in (a) and comment.

(f) Estimate GARCH(2,1), GARCH(1,2) and GARCH(2,2) models. Inspect and comment on the estimation results.