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University of New South Wales School of Economics Financial Econometrics Tutorial 6

1. (ARCH model characteristics)

- 1. (ARCH model characteristics)
- (a) The specification $\varepsilon_t | \Omega_{t-1} \sim N(0, \sigma_t^2)$ implies that the conditional mean of ε_t is 0. The conditional mean of y_t is $c + \phi_1 y_{t-1}$. The unconditional mean of ε_t is 0, by iterated expectations. The unconditional mean of y_t is also obtained by Rule 5: $E(y_t) = c + \phi_1 E(y_{t-1})$ and stationarity $E(y_t) = E(y_{t-1}) = c/(1 \phi_1)$.
- The specification $\varepsilon_t | \Omega_{t-1} \sim N(0, \sigma_t^2)$ implies that the conditional variance of ε_t is σ_t^2 . The conditional variance of y_t is the same as that of ε_t , σ_t^2 , because the conditional mean $c + \phi_1 y_{t-1}$ of y_t is fixed for given information set $\Omega_t t_1$. Define conditional theorem in variance of ε_t is obtained by $E(\varepsilon_t^2) = E\{E(\varepsilon_t^2 | \Omega_{t-1})\} = E(\sigma_t^2) = \alpha_0 + \alpha_1 E(\varepsilon_{t-1}^2) + \alpha_2 E(\varepsilon_{t-2}^2)$ and stantionarity $E(\sigma_t^2) = E(\varepsilon_{t-1}^2) = E(\varepsilon_{t-2}^2) = \alpha_0/(1 \alpha_1 \alpha_2)$.

 For the unconditional variance of y_t , we find $Var(y_t) = \phi_1 Var(y_{t-1}) + Var(\varepsilon_t)$ because ε_t

For the unconditional variance of y_t , we find $\operatorname{Var}(y_t) = \phi_1^2 \operatorname{Var}(y_{t-1}) + \operatorname{Var}(\varepsilon_t)$ because ε_t is uncorrelated with y_{t-1} . Then, by stationarity, $\operatorname{Var}(y_t) = \operatorname{Var}(y_{t-1}) = \operatorname{Var}(\varepsilon_t)/(1-\phi_1^2)$. Finally, $\operatorname{Var}(\varepsilon_t) = E(\sigma_t^2) = \alpha \operatorname{Var}(\varepsilon_t^2) + \alpha$

(c) Yes, ε_t is a WN process because its variance is finite, mean is zero (verified in Part (a) and (b)) and autocovariances are zero for any j > 0: (iterated expectations)

$$\gamma_{j} = \operatorname{Cov}(\varepsilon_{t}, \varepsilon_{t-j}) = E(\varepsilon_{t}\varepsilon_{t-j}) = E\{E(\varepsilon_{t}\varepsilon_{t-j}|\Omega_{t-1})\} = E\{E(\varepsilon_{t}|\Omega_{t-1})\varepsilon_{t-j}\} = E\{0\} = 0.$$

However, ε_t is NOT an independent WN process because the conditional variance of ε_t is a function of ε_{t-1} and ε_{t-2} , by definition.

- (d) The ARCH model differ from the standard homoscedastic model in that the conditional variance of the shock (or error term) is a function of lagged shocks whereas the conditional variance of the shock in any standard ARMA model is a constant. The variance equation in the ARCH model is designed to capture "clustering" or the dependence structure in squared shocks.
- (e) It is easily seen from the variance equation: $\partial \sigma_t^2 / \partial \varepsilon_{t-1}^2 = \alpha_1$.

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(f) Now the variance equation is simplified to $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2$. First, we find $E(\varepsilon_t^4) = E\{E(\varepsilon_t^4 | \Omega_{t-1})\} = E\{3\sigma_t^4\} = 3E\{\alpha_0^2 + 2\alpha_0\alpha_1\varepsilon_{t-1}^2 + \alpha_1^2\varepsilon_{t-1}^4\}$. Because $E(\varepsilon_{t-1}^2) = \alpha_0/(1-\alpha_1)$ by part (b) and $E(\varepsilon_t^4) = E(\varepsilon_{t-1}^4)$ by stationarity, it then follows that

$$E(\varepsilon_t^4) = 3\{\alpha_0^2 + 2\alpha_0\alpha_1[\alpha_0/(1-\alpha_1)]\}/(1-3\alpha_1^2) = 3\alpha_0^2(1+\alpha_1)/[(1-\alpha_1)(1-3\alpha_1^2)].$$

Finally, we find the unconditional kurtosis,

Kurtosis =
$$\frac{E([\varepsilon_t - E(\varepsilon_t)]^4)}{(\operatorname{Var}(\varepsilon_t))^2} = \frac{E(\varepsilon_t^4)}{[E(\varepsilon_t^2)]^2} = 3 \frac{1 - \alpha_1^2}{1 - 3\alpha_1^2} > 3,$$

and conclude that the unconditional distribution of ε_t is non-normal with heavy tails (kurtosis > 3), noting that its conditional distribution is normal.

2. (GARCH model characteristics)

(a-b) From $\varepsilon_t | \Omega_{t-1} \sim N(0, \sigma_t^2)$, it is clear that $E(\varepsilon_t | \Omega_{t-1}) = 0$ and $Var(\varepsilon_t | \Omega_{t-1}) = \sigma_t^2$. It follows that $E(y_t | \Omega_{t-1}) = c + \phi_1 y_{t-1}$ and $Var(y_t | \Omega_{t-1}) = \sigma_t^2$. The unconditional means are obtained by iterated expectations:

$E(\varepsilon_t) = A(S(s) + Project Exam Help)$

$$E(y_t) = E\{E(y_t|\Omega_{t-1})\} = c + \phi_1 E(y_{t-1})$$
 and

$$E(y_t) = E(y_{t-1}) https//tutorcs.com$$

Because the (conditional) mean of ε_t is zero, we find

$$Var(\varepsilon_t) = E\{Var(A) \cap \{0\}\} + E\{d_t^2\} + C\{d_t^2\} + C\{$$

and, by stationarity,

$$Var(\varepsilon_t) = E\{\sigma_t^2\} = E(\varepsilon_{t-1}^2) = E(\sigma_{t-1}^2) = \alpha_0/(1 - \alpha_1 - \beta_1).$$

Further $Var(y_t) = \phi_1^2 Var(y_{t-1}) + Var(\varepsilon_t)$ because ε_t is uncorrelated with y_{t-1} . Again, by stationarity, we find $Var(y_t) = Var(y_{t-1}) = Var(\varepsilon_t)/(1 - \phi_1^2)$.

(c) Yes, ε_t is a WN process because its variance is finite, mean is zero (verified in Part (a) and (b)) and autocovariances are zero for any j > 0:

$$\gamma_{j} = \operatorname{Cov}(\varepsilon_{t}, \varepsilon_{t-j}) = E(\varepsilon_{t}\varepsilon_{t-j}) = E\{E(\varepsilon_{t}\varepsilon_{t-j}|\Omega_{t-1})\} = E\{E(\varepsilon_{t}|\Omega_{t-1})\varepsilon_{t-j}\} = E\{0\} = 0.$$

However, ε_t is NOT an independent WN process because the conditional variance of ε_t is a function of ε_{t-1} , by definition.

(d) It is easily seen from the variance equation: $\partial \sigma_t^2/\partial \varepsilon_{t-1}^2 = \alpha_1$. Note that σ_{t-1}^2 in the variance equation is a function of Ω_{t-2} .

- (e-i) We show that $w_t = \varepsilon_t^2 \sigma_t^2$ has a zero mean and zero autocorrelations. First, because $E(\varepsilon_t|\Omega_{t-1}) = 0$, $E(w_t|\Omega_{t-1}) = E(\varepsilon_t^2|\Omega_{t-1}) \sigma_t^2 = \sigma_t^2 \sigma_t^2 = 0$, implying $E(w_t) = 0$. Second, $Cov(w_t, w_{t-j}) = E(w_t w_{t-j}) = E\{E(w_t|\Omega_{t-1})w_{t-j}\} = E\{0\} = 0$, for all $j \ge 1$.
- (e-ii) It only involves some substitutions:

$$\begin{split} \varepsilon_t^2 &= \sigma_t^2 + w_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + w_t \\ &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 (\varepsilon_{t-1}^2 - w_{t-1}) + w_t \\ &= \alpha_0 + (\alpha_1 + \beta_1) \varepsilon_{t-1}^2 + w_t - \beta_1 w_{t-1}, \end{split}$$

i.e., an ARMA(1,1) for ε_t^2 .

(f) When $\alpha_1 + \beta_1 = 1$, the variance equation

$$\sigma_t^2 = \omega(1 - \alpha_1 - \beta_1) + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 = (1 - \beta_1) \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

becomes an EWMA of ε_t^2 .

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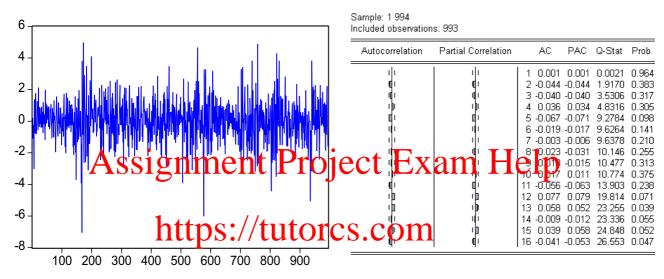
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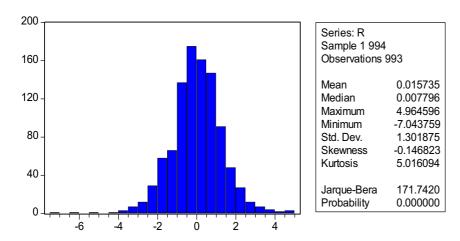
COMPUTING EXERCISES

3. (Estimation of ARCH)

(a) The time series plot, histogram and correlogram of the return series are given below. The correlogram shows little autocorrelation in the return (large p-values for Ljung-Box Q-statistics). The data distribution in the histogram is roughly "bell shaped" but has a negative skewness (-0.147) and large kurtosis (5.016). The normality is rejected (JB = 171.74 with a tiny p-value).



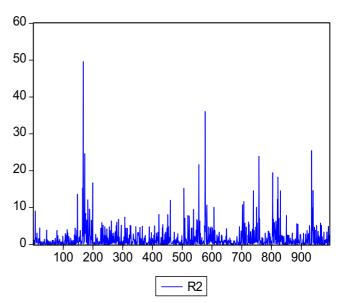
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(b) For the squared return series, the clustering is prominent in the time series plot and the correlogram reveals strong autocorrelations (tiny p-values for the Q-statistics).

Sample: 1 994 Included observations: 993

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
	ıb	1	0.116	0.116	13.515	0.000
ı		2	0.138	0.126	32.447	0.000
ıþ	l ilji	3	0.045	0.016	34.436	0.000
ıþ	1 1	4	0.043	0.020	36.296	0.000
ı <u> </u>	· =	5	0.162	0.153	62.675	0.000
ıþ	ļ iļi	6	0.056	0.017	65.776	0.000
ıþ	1	7	0.089	0.043	73.631	0.000
ıþ	1	8	0.072	0.047	78.819	0.000
ıþı		9	0.026	-0.008	79.521	0.000
ıþ	ļ iļi	10	0.050	0.010	82.082	0.000
ıþ	1	11	0.064	0.047	86.265	0.000
ıþ		12	0.042	0.004	88.038	0.000
ı l ı	į di	13	-0.014	-0.055	88.249	0.000
ı j ı		14	0.010	0.002	88.358	0.000
ı ı		15	0.001	-0.009	88.358	0.000
		16	0.093	0.077	97.092	0.000



Assignment Project Exam Help The LM test for ARCH effect rejects the null hypothesis of no ARCH effect (tirty p-val

We note that the auxiliary equation in the test have large coefficients at lag 1,2 and 5. We conclude that the ARCH effect in the test have large coefficients at lag 1,2 and 5. We conclude that the ARCH effect in the test have large coefficients at lag 1,2 and 5. We conclude that the ARCH effect in the test have large coefficients at lag 1,2 and 5. We conclude that the ARCH effect in the test have large coefficients at lag 1,2 and 5. We conclude that the ARCH effect in the test have large coefficients at lag 1,2 and 5. We conclude that the ARCH effect in the test have large coefficients at lag 1,2 and 5. We conclude that the ARCH effect in the test have large coefficients at lag 1,2 and 5.

ARCH Test:

WeChat: cstuber diaes 11.13600 Probability 0.000000 0.000000

Test Equation:
Dependent Variable: RESID*2
Method: Least Squares

Sample (adjusted): 7 994

Included observations: 988 after adjustments

Dependent Variable: R Method: Least Squares

Sample (adjusted): 2 994

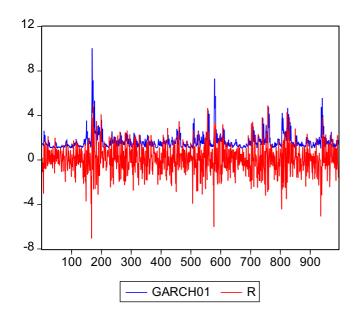
Included observations: 993 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
С	0.015735	0.041314	0.380854	0.7034
R-squared Adjusted R-squared S.E. of regression Sum squared resid Log likelihood	0.000000 0.000000 1.301875 1681.319 -1670.464	Mean deper S.D. depend Akaike info Schwarz cri Durbin-Wats	lent var criterion terion	0.015735 1.301875 3.366494 3.371430 1.994052

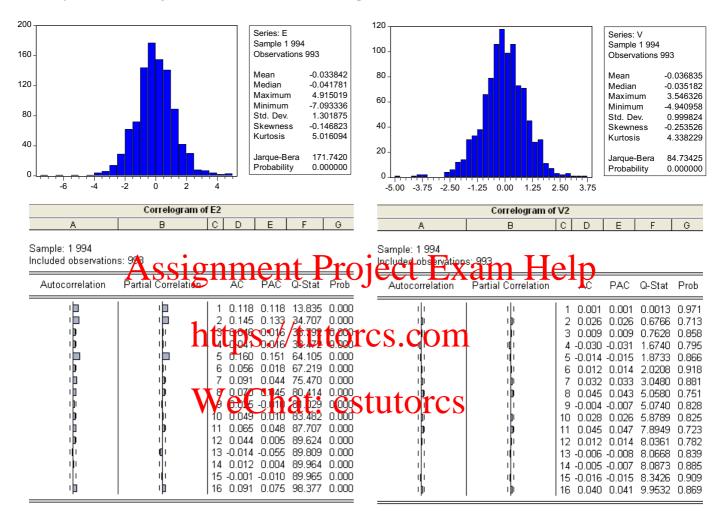
Variable	Coefficient	Std. Error	t-Statistic	Prob.	
C RESID*2(-1) RESID*2(-2) RESID*2(-3) RESID*2(-4) RESID*2(-5)	1.058869 0.096896 0.122086 -0.004176 0.004833 0.152973	0.144610 0.031449 0.031600 0.031837 0.031617 0.031463	7.322252 3.081046 3.863463 -0.131154 0.152863 4.861981	0.0000 0.0021 0.0001 0.8957 0.8785 0.0000	
R-squared Adjusted R-squared S.E. of regression Sum squared resid Log likelihood Durbin-Watson stat	djusted R-squared 0.048840 E. of regression 3.310230 um squared resid 10760.38 og likelihood -2581.556		Mean dependent var S.D. dependent var Akaike info criterion Schwarz criterion F-statistic Prob(F-statistic)		

(d) From the ARCH(5) estimation results, $\hat{\alpha}_0$ (1.029) is significantly positive. The point estimates of $(\alpha_1, ..., \alpha_5)$ are all non-negative. The point estimate of $\alpha_1 + \cdots + \alpha_5$ is positive and less than one. Hence the restrictions on the ARCH parameters are satisfied. We plot the return over the plot of σ_t^2 (called GARCH01 in EViews). It is visually clear that the conditional variance follows the variations in the return and the clustering closely. The LM test for ARCH effect cannot reject the null of no ARCH effect in the standardised residual series (large p-value).

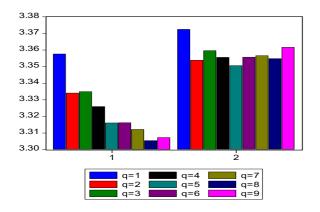
Dependent Variable: R ARCH Test: Method: ML - ARCH (Marquardt) - Normal distribution Probability 0.366277 0.871806 F-statistic Sample (adjusted): 2 994 Obs*R-squared 1.839143 Probability 0.870925 Included observations: 993 after adjustments Convergence achieved after 12 iterations Bollersley-Wooldrige robust standard errors & covariance Test Equation: Variance backcast: ON Dependent Variable: STD_RESID*2 $GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-2)^2 + C(5)*RESID(-1)^2 + C(5)^2 + C$ Method: Least Squares -3)^2 + C(6)*RESID(-4)^2 + C(7)*RESID(-5)^2 Sample (adjusted): 7 994 Proh Coefficient z-Statistic Included observations: 988 after adjustments Std. Error White Heteroskedasticity-Consistent Standard Errors & Covariance С 0.049577 0.037379 1.326328 0.1847 Variable Coefficient Std. Error t-Statistic Prob. Variance Equation 0.998874 0.097108 0.000010 28624 STD_HESID/2(4 STD_HESID/2(4 0.1690 C 1.028249 **0.0**012**2**6 0.041550 0.9669 dolgb48 7**43**81 RESID(-1)² **00604** 0. **1**277 **8**0 0.510229 0.6100 RESID(-2)/2 0.141911 1.952344 0.0509 STD RESID^2(-3) 0.072688 0.010317 0.036485 0.282764 0.7774 RESID(-3)/2 STD RESID/2(-4) 0.027673 0.041101 0.673291 0.5008 0.0214360.1784-0.028865 -1.346571 0.049543 0.0387671.277979 RESID(-4)^2 0.2013 STD_RESID^2(-5) -0.013435 0.024427 -0.5500020.5824 RESID(-5)*2 0.1040780,041422 2.5/2608 0.0120 Mean Jependeni var R-squared 1111 Adjusted R-squared 0.001861 0.995998 Mean dependent var -0.000676 -0.003221 R-squared 1.829877 S.D. dependent var Adjusted R-squared -0.006766 S.D. dependent var 1.301875 S.E. of regression 1.832821 Akaike info criterion 4.055644 3.316074 S.E. of regression 1.306271 Akaike info criterion Sum squared resid 3298.768 Schwarz criterion 4.085375 Sum squared resid 1682.456 Schwarz criterion 3.350621 Log likelihood -1997.488 F-statistic 0.366277 Log likelihood -1639.431 1,992704 1.998431 urbin-W itson st urbin-Watson stat Prob(F-statistic) 0.871806



(e) The histograms indicate that the distributions of both the residuals (E) and the standardised residuals (V) are non-normal (large JB statistics and tiny p-values). The correlograms show that the squared residuals (E2) have strong autocorrelations but the squared standardised residuals (V2) have little. This is consistent with the LM test results in parts (c) and (d). The ARCH(5) model does a good job in accounting for the autocorrelations in the squared residuals.



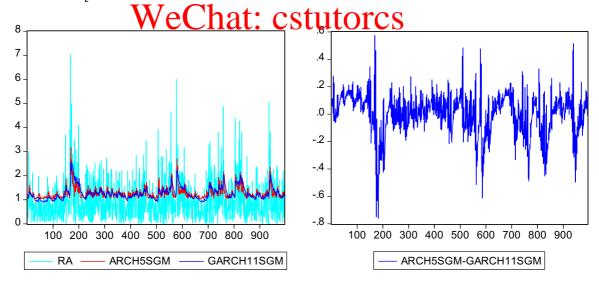
(f) We may use AIC or SIC to choose the number of lags in ARCH models. For ARCH(q), the AIC and SIC for q = 1,2,...,9 are displayed in the bar chart below, where AICs are in group 1 on the left and SICs are in group 2 on the right. Clearly, AIC selects q = 8 but SIC selects q = 5.



Based on SIC and the principle of parsimony, the ARCH(5) model stays for the variance equation. For the mean equation, as the returns have little autocorrelation, a more sophisticated ARMA specification is unlikely to improve the model's fit.

4. (Estimation of GARCH)

- (a) The estimation results at Part (e) below indicate that all restriction on the parameters are satisfied: $\alpha_0 > 0$, $\alpha_1 \ge 0$, $\beta_1 \ge 0$ and $\alpha_1 + \beta_1 < 1$. We also note that the β_1 estimate is about 0.9, the α_1 estimate is about 0.1 The α_1 + β_1 estimate is very dole to be about 0.9. The α_1 estimate is about 0.1 The α_1 estimate is very dole to be about 0.9.
- (b) The plot below contain the absolute return series (RA) and σ_t series from ARCH(5) and GARCH(1,1) respectively. Both σ_t series match the variations in the return. However the two σ_t series differ markedly in a number of places. The GARCH σ_t appears smoother than the ARCH σ_t .



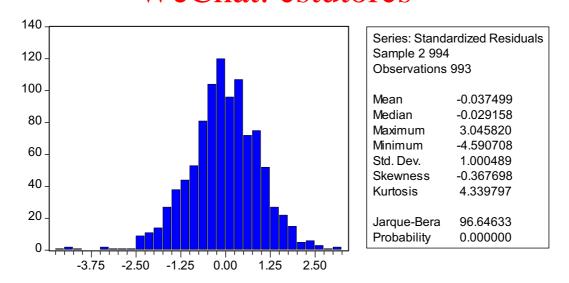
(c) The LM test for ARCH effect (see below) cannot reject that the null hypothesis that there is no ARCH effect in the standardised residuals, as the p-value is quite large (0.17). The correlogram of the squared standardised residuals also indicate that there are no

autocorrelations. Hence the GARCH variance equation has adequately captured the clustering in the error term ε_t .

ARCH Test:										
F-statistic Obs*R-squared	1.553443 7.753346	Probability Probability		0.170654 0.170363						
Test Equation:					Correl	ogram of Standardize	d Residual	s Square	ed	
Dependent Variable: \$ Method: Least Square Sample (adjusted): 7	es	!			Sample: 2 994 Included observation	ıs: 993				
Included observations White Heteroskedasti	: 988 after adj		rrors & Cova	riance	Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
Variable	Coefficient	Std. Error	t-Statistic	Prob.	<u></u>	1 1		-0.013		
Variable	Coefficient	Std. Ellol	t-Statistic	P100.	T.	1 7	2 0.069		4.9755 5.3764	
С	0.968768	0.094300	10.27322	0.0000	ili.	1 7	4 -0.038			
STD RESID^2(-1)	-0.010058	0.027461	-0.366253	0.7143	i)i	1 1	5 0.021	0.0.0		0.110
STD RESID^2(-2)	0.073431	0.065495	1.121177	0.2625	ı(ı	1 1/1	6 -0.016	-0.011	7.5290	0.275
STD_RESID^2(-3)	-0.020112	0.030708	-0.654941	0.5127	ı)ı	l di	7 0.021	0.016	7.9864	0.334
STD_RESID^2(-4)	-0.040896	0.022349	-1.829879	0.0676	ı ı		8 0.005	0.007	8.0145	0.432
STD_RESID^2(-5)	0.025099	0.032189	0.779727	0.4357	ψ.	1 10			8.4264	
						1 1			8.5389	
R-squared	0.007848	Mean deper		0.996322	<u>"</u>	<u> </u>	11 0.007	0.0.0		
Adjusted R-squared	0.002796	S.D. depend		1.834917	191	<u> </u>	12 -0.026			
S.E. of regression	1.832350	Akaike info		4.055130	11:	1 !!			9.3559	
Sum squared resid	3297.071	Schwarz cri	terion	4.084861	#!	1 !!			9.7581	
Log likelihood	-1997.234	F-statistic	-47-3	1.553443	У.	1 !!!		-0.023		
Durbin-Watson stat	1.998554	Prob(F-stati	stic)	0.170654	ılı.	III	16 0.002	0.001	10.309	0.850

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(d) The histogram of the standardised residual show that the null hypothesis of normality is strongly rejected with Sero pour Conference of the Rutosis is much smaller than the kurtosis in the return series, which confirms that the variance equation can partially explain the excess kurosis in the return series tutorcs



(e) The estimation results with or without the quasi ML robust standard errors are given below. They are different and may lead to different conclusions. Generally, the robust standard errors in the variance equation are larger.

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Dependent Variable: R

Method: ML - ARCH (Marquardt) - Normal distribution

Sample (adjusted): 2 994

Included observations: 993 after adjustments Convergence achieved after 11 iterations

Bollerslev-Wooldrige robust standard errors & covariance

Variance backcast: ON

 $GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)$

Dependent Variable: R

Method: ML - ARCH (Marquardt) - Normal distribution

Sample (adjusted): 2 994

Included observations: 993 after adjustments Convergence achieved after 11 iterations

Variance backcast: ON

 $GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)$

	Coefficient	Std. Error	z-Statistic	Prob.		Coefficient	Std. Error	z-Statistic	Prob.
С	0.042397	0.036383	1.165292	0.2439	С	0.042397	0.040573	1.044951	0.2960
Variance Equation				Variance Equation					
C RESID(-1)^2 GARCH(-1)	0.073836 0.080407 0.877380	0.032445 0.026746 0.036232	2.275744 3.006251 24.21561	0.0229 0.0026 0.0000	C RESID(-1)^2 GARCH(-1)	0.073836 0.080407 0.877380	0.025873 0.015193 0.026154	2.853797 5.292469 33.54611	0.0043 0.0000 0.0000
R-squared Adjusted R-squared S.E. of regression Sum squared resid Log likelihood	-0.000420 -0.003454 1.304121 1682.025 -1634.734	S.D. dependent var Akaike info criterion Schwarz criterion		0.015735 1.301875 3.300572 3.320313 1.993215	R-squared Adjusted R-squared S.E. of regression Sum squared resid Log likelihood	-0.000420 -0.003454 1.304121 1682.025 -1634.734	Mean deper S.D. depend Akaike info Schwarz cri Durbin-Wat	dent var criterion iterion	0.015735 1.301875 3.300572 3.320313 1.993215

(f) Assignment Project Exam Help The GARCIE models can be easily estimated to obtain AIC and SIC in Exicus. The

AIC and SIC for GARCH(1,1), GARCH(2,1), GARCH(1,2) are GARCH(2,2) are presented in the bar chart below (At the late of the lat

