ARCH_share

July 22, 2021

Importing packages

```
[61]: #importing packages
      import statsmodels.api as sm
      from statsmodels.tsa.stattools import adfuller
      import pandas as pd
      import numpy as np
      import statsmodels.formula.api as smf
      from sklearn import linear_model
      import matplotlib.pyplot as plt
```

Reading Excel file saved in hard drive

```
[62]: #reading the file
      df = pd.read_excel("C:\\Users\\fluck\\OneDrive\\share.xlsx")
      df
```

```
[62]:      OBS    PRICE
      0      1   975.04
      1      2   977.07
      2      3   966.58
      3      4   964.00
      4      5   956.05
      ..   …        …
      989  990  1144.80
      990  991  1170.35
      991  992  1167.10
      992  993  1158.31
      993  994  1139.93

      [994 rows x 2 columns]
```

Calculating daily returns and daily squared returns from SP500

Daily returns (R)

$$R = 100 * ln(P_t/P_{t-1})$$

Daily squared returns (R^2)

$$R = R^2$$

[63]:

```
#computing the inflation rate
df['R'] = 100*np.log(df['PRICE']/df['PRICE'].shift(1))
df['R_squared'] = df['R']**2
df = df.dropna(subset=["R"])
df
```

[63]:

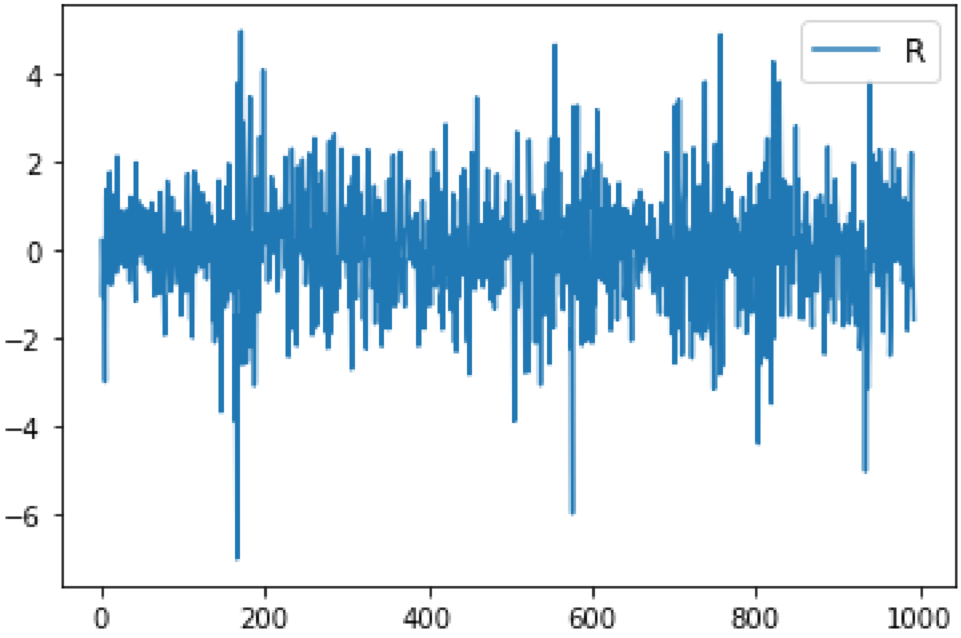
```
     OBS    PRICE         R  R_squared
1      2   977.07  0.207980   0.043256
2      3   966.58 -1.079423   1.165154
3      4   964.00 -0.267277   0.071437
4      5   956.05 -0.828108   0.685763
5      6   927.69 -3.011259   9.067679
..   …       …        …         …
989  990  1144.80  1.310082   1.716314
990  991  1170.35  2.207290   4.872129
991  992  1167.10 -0.278081   0.077329
992  993  1158.31 -0.755999   0.571535
993  994  1139.93 -1.599519   2.558461

[993 rows x 4 columns]
```

$**Plotting the time series: R and** $R^{2}$$

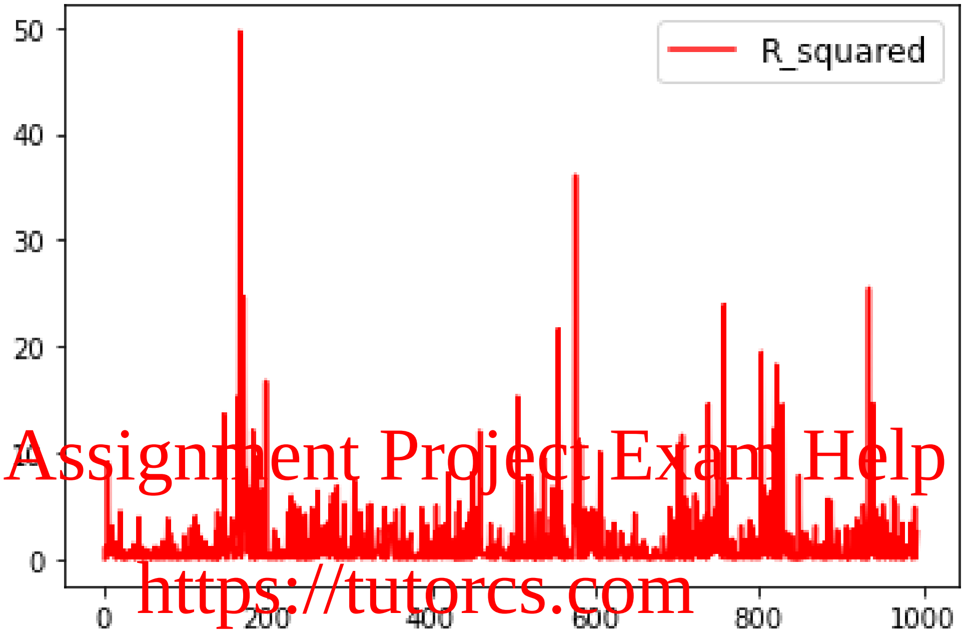
[64]:

```
#plotting the R series
plt.plot(df['R'],label='R')
plt.legend(loc='best', fontsize='large')
plt.show()
```



[65]:

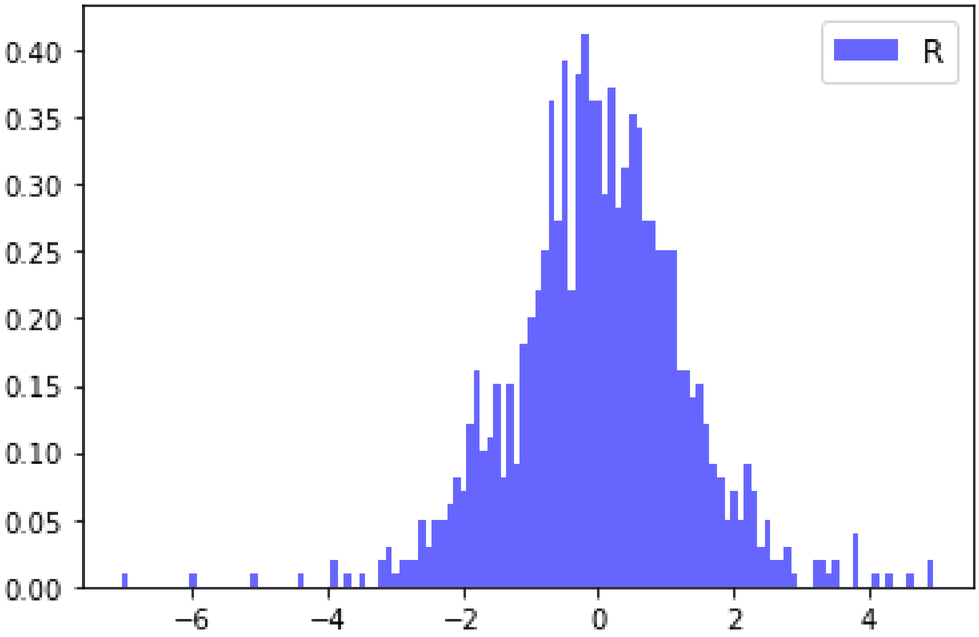
```python
#plotting the R_squared series
plt.plot(df['R_squared'],label='R_squared',color='Red')
plt.legend(loc='best', fontsize='large')
plt.show()
```



Histogram and descriptive statistics

[66]:

```python
#Plot histogram of R
plt.hist(df['R'],bins=120,label='R', density=True, alpha=0.6, color='b')
plt.legend(loc='best', fontsize='large')
plt.show()
```



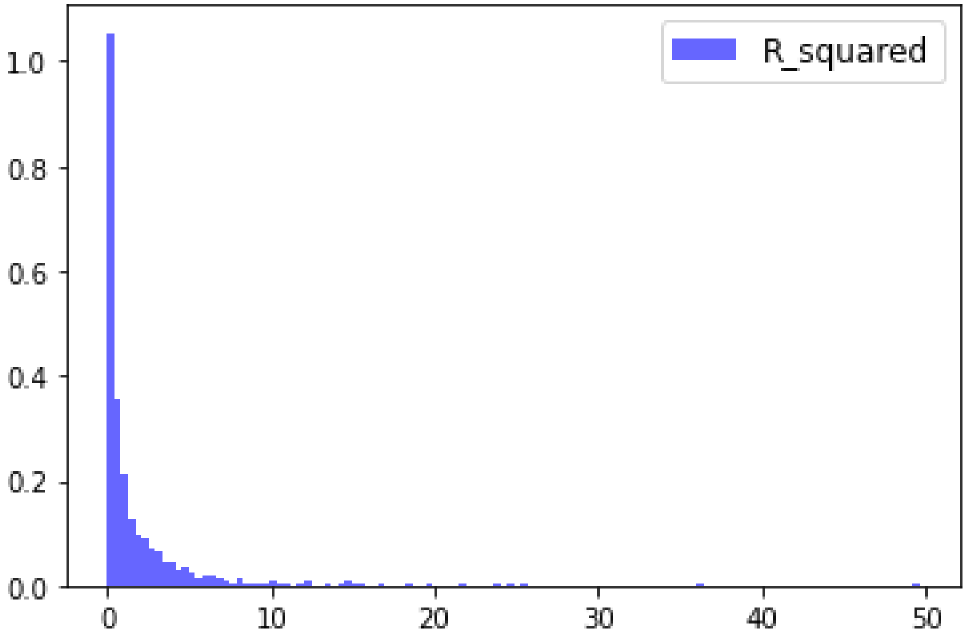
```
[67]: from scipy import stats
      stats.describe(df['R'])
```

```
[67]: DescribeResult(nobs=993, minmax=(-7.043759037302043, 4.964596183505854),
      mean=0.015734505558862576, variance=1.694877827267905,
      skewness=-0.1468232170367387, kurtosis=2.016094075647234)
```

```
[68]: stats.jarque_bera(df['R'])
```

```
[68]: Jarque_beraResult(statistic=171.7419793855507, pvalue=0.0)
```

```
[69]: #Plot histogram of R_squared
      plt.hist(df['R_squared'],bins=120,label='R_squared', density=True, alpha=0.6,␣
       ↪color='b')
      plt.legend(loc='best', fontsize='large')
      plt.show()
```



[70]:
```
stats.describe(df['R_squared'])
```

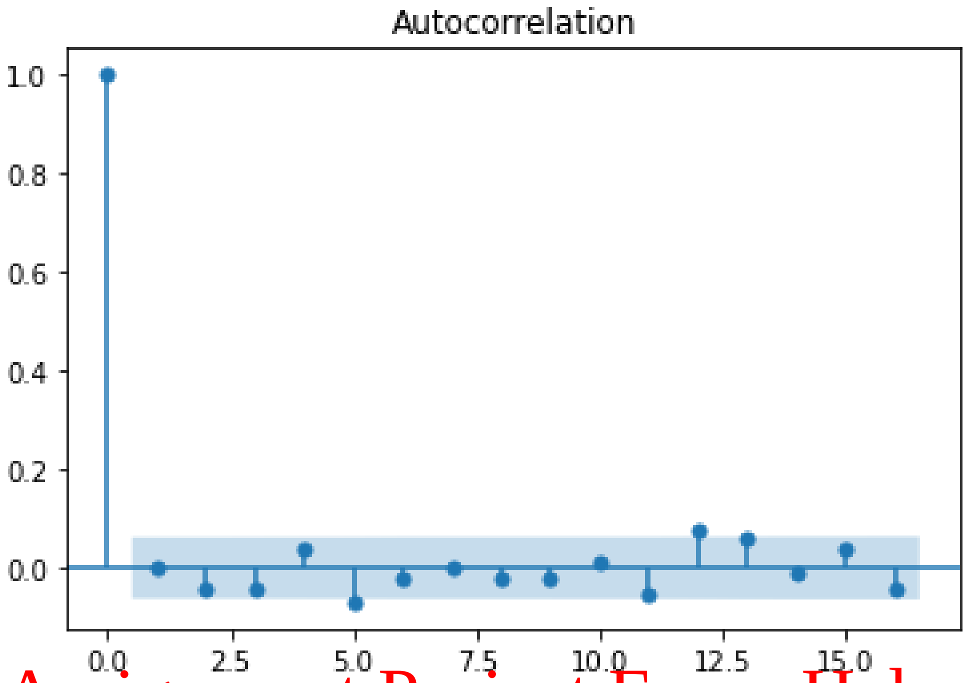
[70]:
```
DescribeResult(nobs=993, minmax=(0.0, 49.614541375574206),
mean=1.693418576326566, variance=11.506356201573688, skewness=6.16939777089506,
kurtosis=59.64692480452559)
```

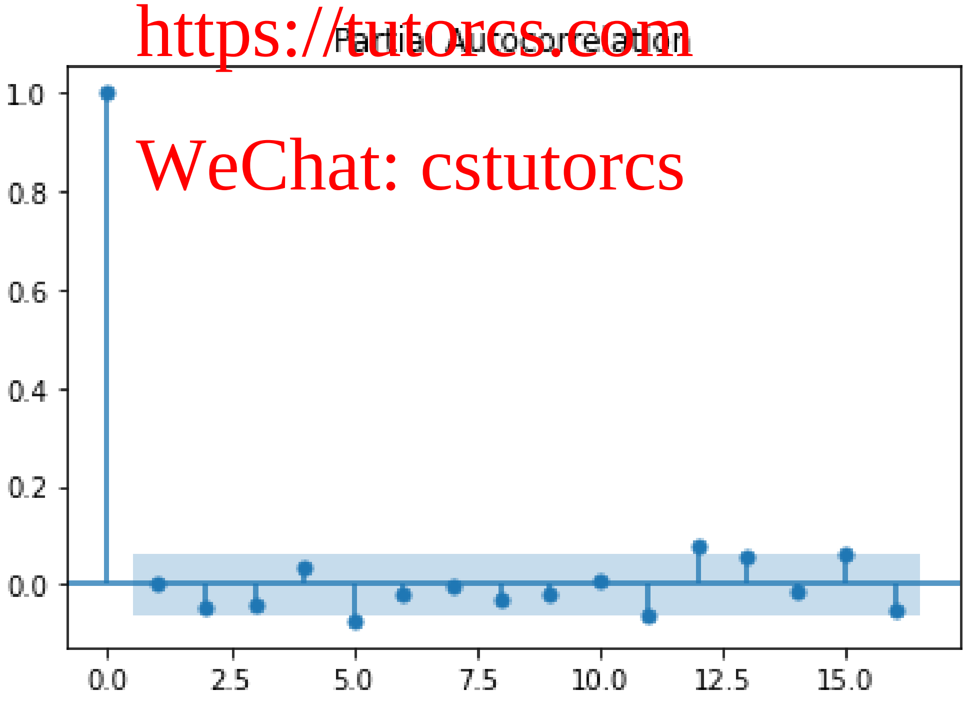
[71]:
```
stats.jarque_bera(df['R_squared'])
```

[71]:
```
Jarque_beraResult(statistic=153501.31264418407, pvalue=0.0)
```

ACF , PACF of R

[72]:
```
from statsmodels.graphics import tsaplots
fig =tsaplots.plot_acf(df['R'],lags=16)
fig =tsaplots.plot_pacf(df['R'],lags=16)
plt.show()
```


Autocorrelation
1.0
0.8
0.6
0.4
0.2
0.0
0.0
2.5
5.0
7.5
10.0
12.5
15.0


Partial Autocorrelation
1.0
0.8
0.6
0.4
0.2
0.0
0.0
2.5
5.0
7.5
10.0
12.5
15.0

[73]:

```
# Generating the Q tables
import numpy as np
r,q,p = sm.tsa.acf(df['R'].values.squeeze(), qstat=True)
data = np.c_[range(1,41), r[1:], q, p]
table = pd.DataFrame(data, columns=['lag', "AC", "Q", "Prob(>Q)"])
print (table.set_index('lag'))
```

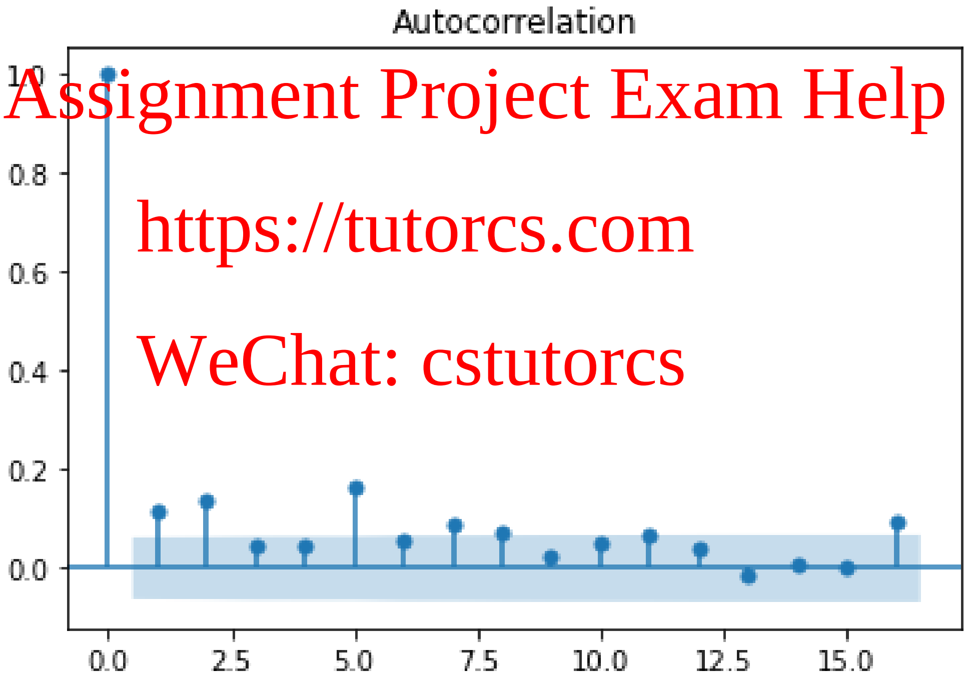
```
            AC          Q  Prob(>Q)
lag
1.0   0.002187   0.004765  0.944967
2.0  -0.044549   1.983469  0.370933
3.0  -0.042759   3.808159  0.282939
4.0   0.036575   5.144571  0.272783
5.0  -0.068961   9.900359  0.078108
6.0  -0.019748  10.290740  0.112930
7.0  -0.001411  10.292735  0.172582
8.0  -0.019274  10.665353  0.221391
9.0  -0.019877  11.062067  0.271479
10.0  0.014334  11.268578  0.336980
11.0 -0.054077  14.210889  0.221543
12.0  0.077208  20.214671  0.063133
13.0  0.059358  23.766889  0.033343
14.0 -0.008254  23.835639  0.047960
15.0  0.039619  25.421426  0.044562
16.0 -0.040201  27.055817  0.040867
17.0  0.014268  27.261915  0.054337
18.0 -0.057071  30.562506  0.032324
19.0  0.021465  31.029898  0.040069
20.0 -0.010472  31.141257  0.053353
21.0 -0.054597  34.171259  0.034738
22.0 -0.015008  34.400460  0.044705
23.0  0.028022  35.200283  0.049681
24.0  0.036951  36.592496  0.048029
25.0 -0.033563  37.742296  0.049009
26.0  0.017079  38.040350  0.060041
27.0  0.082436  44.991090  0.016296
28.0  0.001189  44.992538  0.022101
29.0  0.017297  45.299169  0.027513
30.0 -0.000220  45.299219  0.036196
31.0  0.004336  45.318524  0.046688
32.0 -0.053556  48.267405  0.032500
33.0 -0.040998  49.997358  0.029234
34.0 -0.089707  58.288305  0.005893
35.0 -0.037434  59.733514  0.005697
36.0  0.025398  60.399493  0.006620
37.0 -0.010012  60.503099  0.008705
38.0 -0.001130  60.504421  0.011575
39.0  0.057283  63.902786  0.007182
```

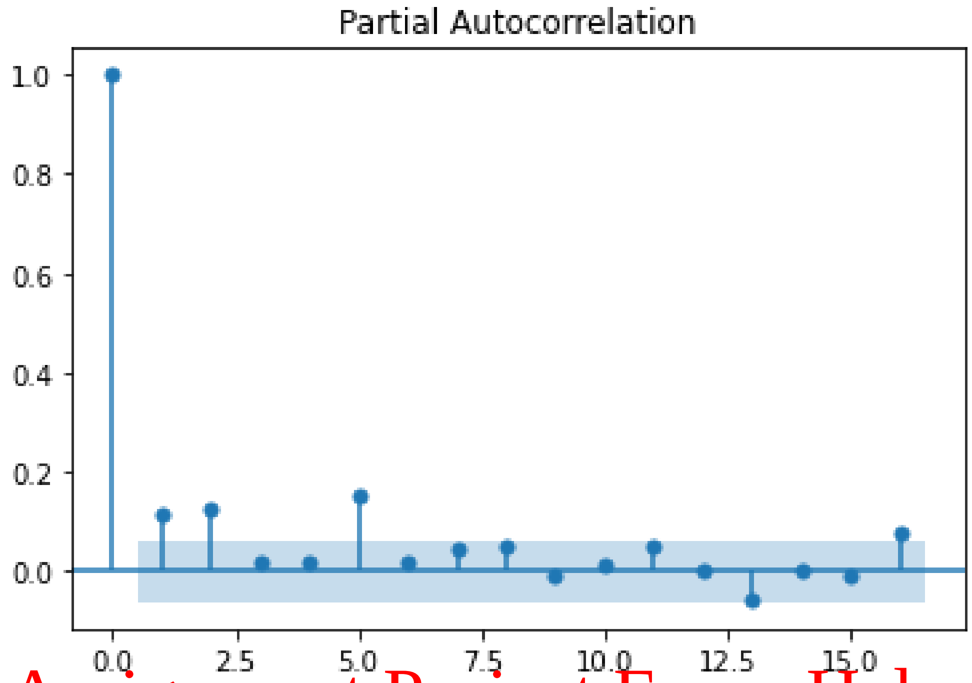
```
40.0 -0.073371  69.484020  0.002633
```

```
C:\Users\rluck\anaconda3\lib\site-packages\statsmodels\tsa\stattools.py:657:
FutureWarning: The default number of lags is changing from 40 tomin(int(10 *
np.log10(nobs)), nobs - 1) after 0.12is released. Set the number of lags to an
integer to  silence this warning.
  warnings.warn(
C:\Users\rluck\anaconda3\lib\site-packages\statsmodels\tsa\stattools.py:667:
FutureWarning: fft=True will become the default after the release of the 0.12
release of statsmodels. To suppress this warning, explicitly set fft=False.
  warnings.warn(
```

$**ACF , PACF of** $R^2$$

[74]:
```
fig =tsaplots.plot_acf(df['R_squared'],lags=16)
fig =tsaplots.plot_pacf(df['R_squared'],lags=16)
plt.show()
```





[75]:

```
# Generating the Q tables
import numpy as np
r,q,p = sm.tsa.acf(df['R_squared'].values.squeeze(), qstat=True)
data = np.c_[range(1,41), r[1:], q, p]
table = pd.DataFrame(data, columns=['lag', "AC", "Q", "Prob(>Q)"])
print (table.set_index('lag'))
```

```
            AC          Q      Prob(>Q)
lag
1.0   0.116401  13.495026  2.391966e-04
2.0   0.137366  32.307886  9.647874e-08
3.0   0.045022  34.330823  1.686854e-07
4.0   0.043314  36.205092  2.625763e-07
5.0   0.163064  62.795866  3.208813e-12
6.0   0.055033  65.827678  2.922824e-12
7.0   0.088429  73.663464  2.678057e-13
8.0   0.072519  78.938732  7.992968e-14
9.0   0.026248  79.630526  1.913609e-13
10.0  0.051198  82.265203  1.803486e-13
11.0  0.065028  86.519895  7.988149e-14
12.0  0.041676  88.269223  1.067025e-13
13.0 -0.014482  88.480668  2.726470e-13
14.0  0.010197  88.585610  7.015944e-13
15.0  0.001769  88.588769  1.817136e-12
16.0  0.091960  97.140862  1.186384e-13
```

```
17.0  0.036990   98.525997  1.667347e-13
18.0  0.072813  103.898575  4.253523e-14
19.0  0.076362  109.813725  8.627403e-15
20.0  0.020351  110.234269  1.780030e-14
21.0  0.110912  122.738581  2.240937e-16
22.0  0.050038  125.286355  1.894182e-16
23.0 -0.001192  125.287803  4.611835e-16
24.0  0.044396  127.297499  4.775153e-16
25.0  0.003592  127.310670  1.115576e-15
26.0  0.028843  128.160663  1.814862e-15
27.0  0.082465  135.116334  2.425735e-16
28.0  0.026399  135.829855  4.136894e-16
29.0 -0.004892  135.854381  9.184485e-16
30.0 -0.007907  135.918535  1.972095e-15
31.0  0.009156  136.004633  4.128389e-15
32.0  0.079173  142.449347  6.937586e-16
33.0  0.006087  142.487482  1.465969e-15
34.0 -0.039022  144.056309  1.678684e-15
35.0 -0.018729  144.418079  3.052000e-15
36.0 -0.038257  145.929177  3.521024e-15
37.0 -0.025003  146.575300  5.624595e-15
38.0 -0.043604  148.542369  5.383179e-15
39.0 -0.014147  148.749640  9.993128e-15
40.0 -0.042500  150.622304  9.812920e-15

C:\Users\rluck\anaconda3\lib\site-packages\statsmodels\tsa\stattools.py:657:
FutureWarning: The default number of lags is changing from 40 tomin(int(10 *
np.log10(nobs)), nobs - 1) after 0.12is released. Set the number of lags to an
integer to  silence this warning.
  warnings.warn(
C:\Users\rluck\anaconda3\lib\site-packages\statsmodels\tsa\stattools.py:667:
FutureWarning: fft=True will become the default after the release of the 0.12
release of statsmodels. To suppress this warning, explicitly set fft=False.
  warnings.warn(
```

ARCH(5)

[76]:
```python
from arch import arch_model
```

[77]:
```python
dt = df['R']
model = arch_model(dt,mean ='Constant', vol ='ARCH', q=5)
x_5 =model.fit(update_freq=0)
x_5
```

```
Optimization terminated successfully    (Exit mode 0)
            Current function value: 1664.047869085027
            Iterations: 5
            Function evaluations: 28
            Gradient evaluations: 5
```

[77]:

```
                         Constant Mean - ARCH Model Results
==============================================================================
Dep. Variable:                      R   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                       ARCH   Log-Likelihood:               -1664.05
Distribution:                  Normal   AIC:                           3334.10
Method:            Maximum Likelihood   BIC:                           3348.80
                                        No. Observations:                  993
Date:                Thu, Jul 22 2021   Df Residuals:                      992
Time:                        20:04:12   Df Model:                            1
                               Mean Model
===========================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
---------------------------------------------------------------------------
mu             0.0293  4.132e-02      0.709      0.478 [-5.170e-02,  0.110]
                            Volatility Model
===========================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
---------------------------------------------------------------------------
omega          1.5288      0.104     14.654  1.263e-48     [  1.324,  1.733]
alpha[1]       0.0963  4.954e-02      1.944  5.194e-02 [-8.088e-04,  0.193]
===========================================================================

Covariance estimator: robust
ARCHModelResult, id: 0x21914bdbe20
```

[78]:

```
#Aligning AIC, BIC from Python with Stata's AIC, BIC
n =993
name = ['AIC_stata','BIC_stata']
stata=[x_5.aic/n,x_5.bic/n]
lzip(name, stata)
```

[78]:

```
[('AIC_stata', 3.357598930684848), ('BIC_stata', 3.3724047635067365)]
```

ARCH Test

[79]:

```
from statsmodels.stats.diagnostic import het_arch
from statsmodels.compat import lzip
res = het_arch(dt.values, nlags=5)
name =['lm','lm_pval','fval','f_pval']
lzip(name,res)
```

[79]:

```
[('lm', 52.586810853630816),
 ('lm_pval', 4.0887029149590036e-10),
 ('fval', 11.041163168843253),
 ('f_pval', 2.3131694315533897e-10)]
```

ARCH Test of Standardised Residuals

[80]:

```
std_resid = x_5.resid/x_5.conditional_volatility
res = het_arch(std_resid, nlags=5)
name =['lm','lm_pval','fval','f_pval']
lzip(name,res)
```

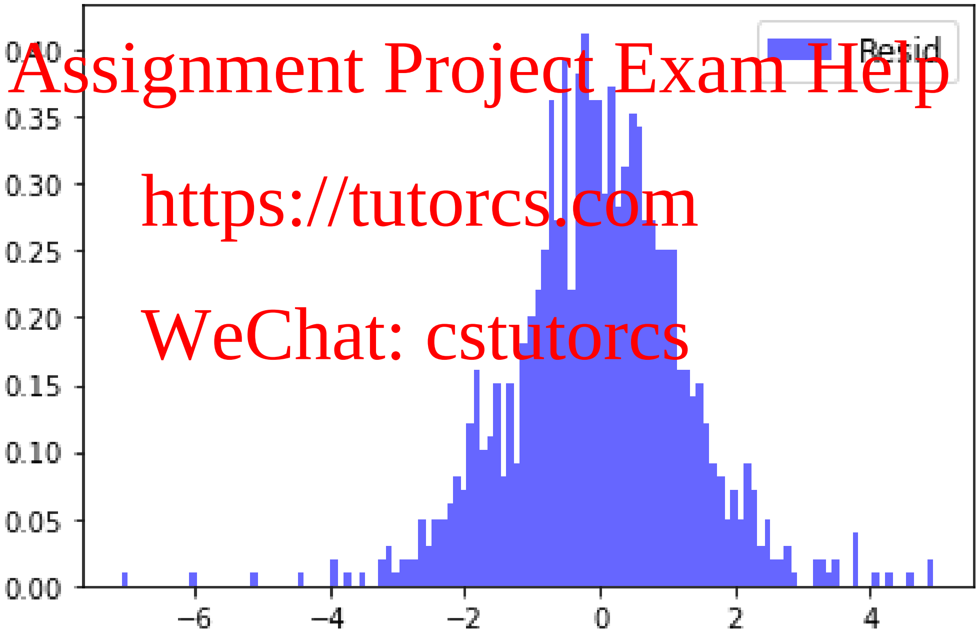
[80]:

```
[('lm', 44.61654794255047),
 ('lm_pval', 1.7360584950680533e-08),
 ('fval', 9.2885771918154),
 ('f_pval', 1.1768118600559285e-08)]
```

Histogram of Residuals

[81]:

```
#Historgram of residuals
resid = x_5.resid
plt.hist(resid,bins=120,label='Resid', density=True, alpha=0.6, color='b')
plt.legend(loc='best', fontsize='large')
plt.show()
```



[82]:

```
stats.describe(resid)
```

[82]:

```
DescribeResult(nobs=993, minmax=(-7.073049296466991, 4.9353059243409065),
mean=-0.013555753606322416, variance=1.694877827267905,
skewness=-0.1468232170367387, kurtosis=2.016094075647234)
```

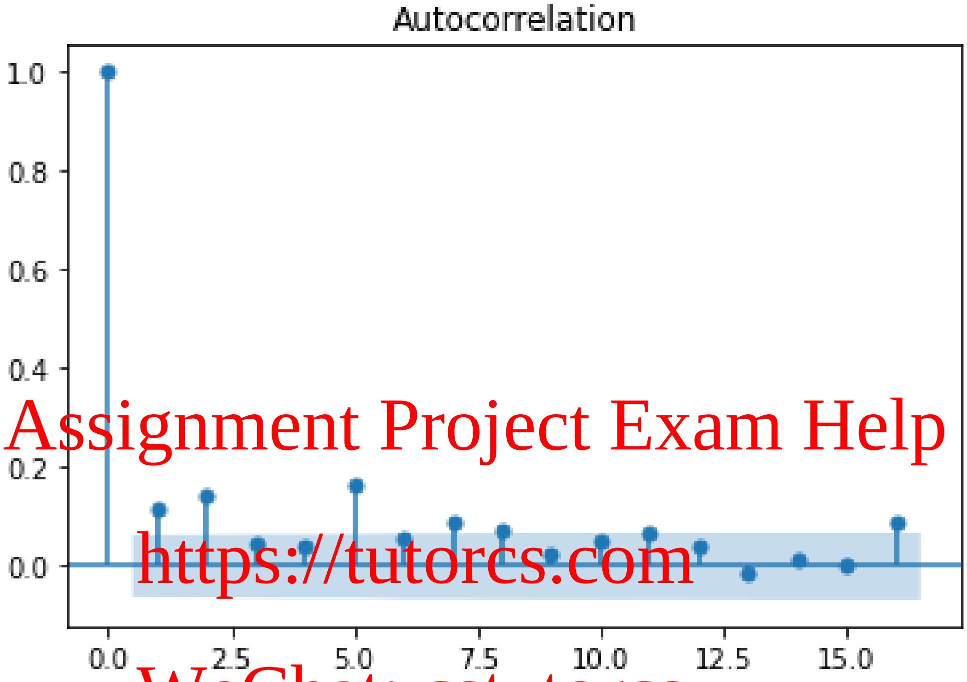
[83]:

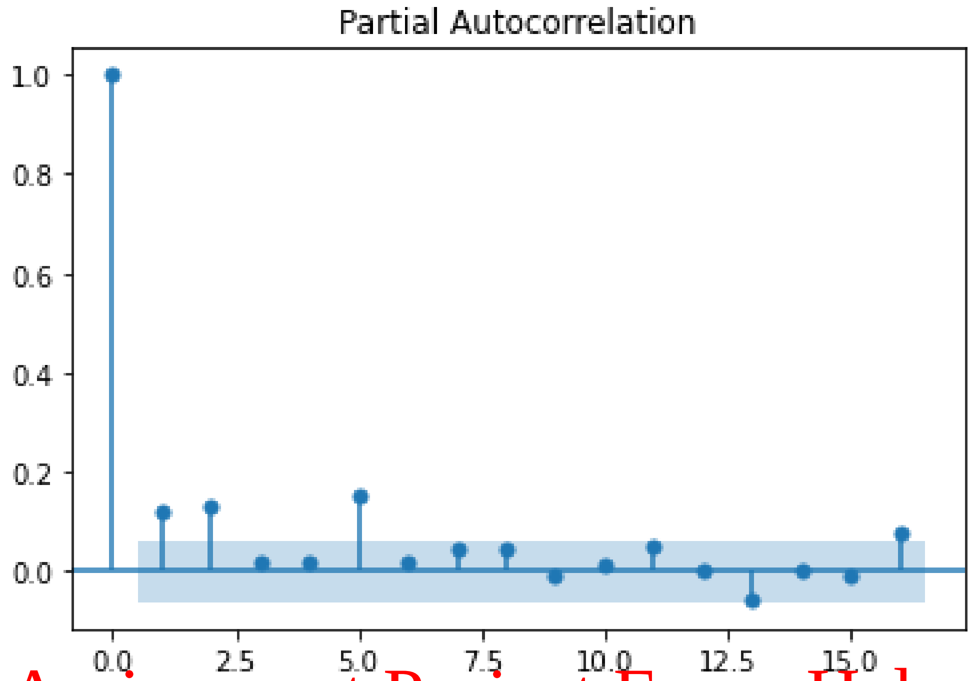
```
stats.jarque_bera(resid)
```

```
[83]: Jarque_beraResult(statistic=171.7419793855507, pvalue=0.0)
```

ACF , PACF of Squared Residuals

```
[84]: fig =tsaplots.plot_acf(resid**2,lags=16)
      fig =tsaplots.plot_pacf(resid**2,lags=16)
      plt.show()
```

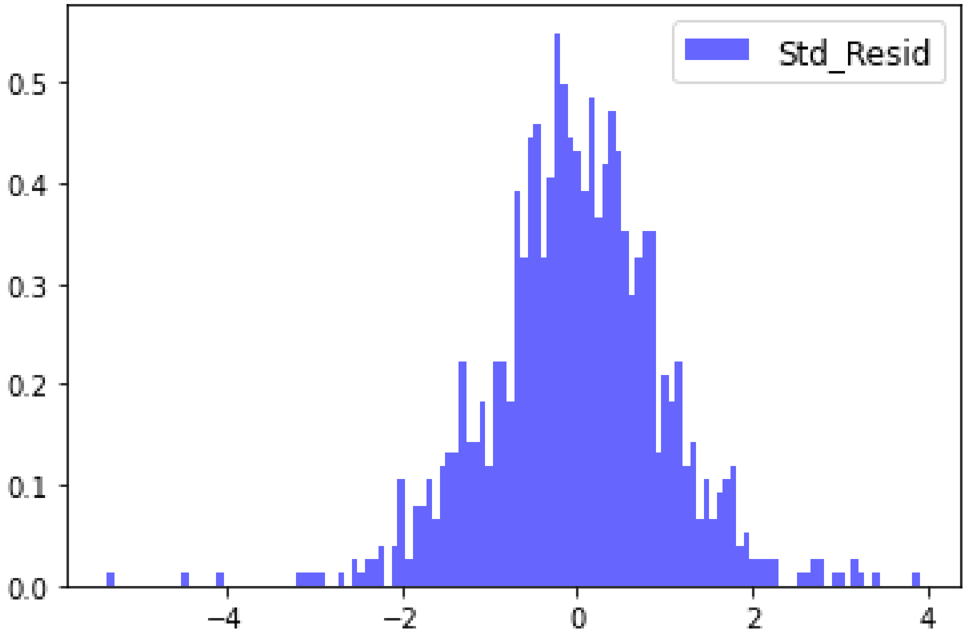




Histogram of Standardised Residuals

[85]:

```
#Historgram of std residuals
plt.hist(std_resid,bins=120,label='Std_Resid', density=True, alpha=0.6,␣
 ↪color='b')
plt.legend(loc='best', fontsize='large')
plt.show()
```



[86]:
```
stats.describe(std_resid)
```

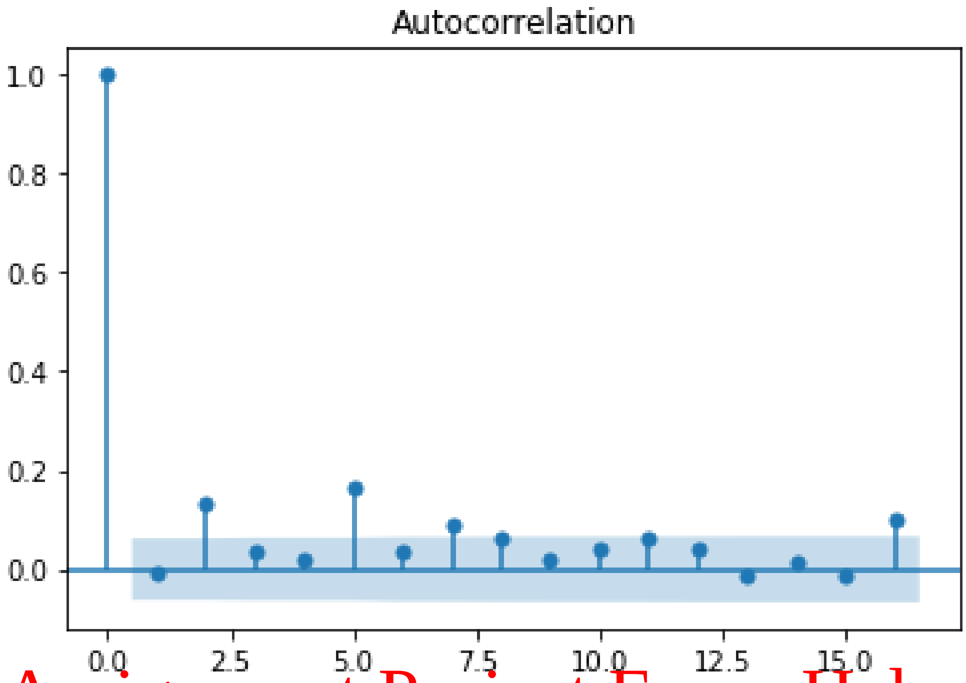
[86]:
```
DescribeResult(nobs=993, minmax=(-5.343797790419605, 3.8965888804733484),
mean=-0.014086543542439298, variance=1.0008098057706427,
skewness=-0.17614610401990488, kurtosis=1.8124982770599578)
```

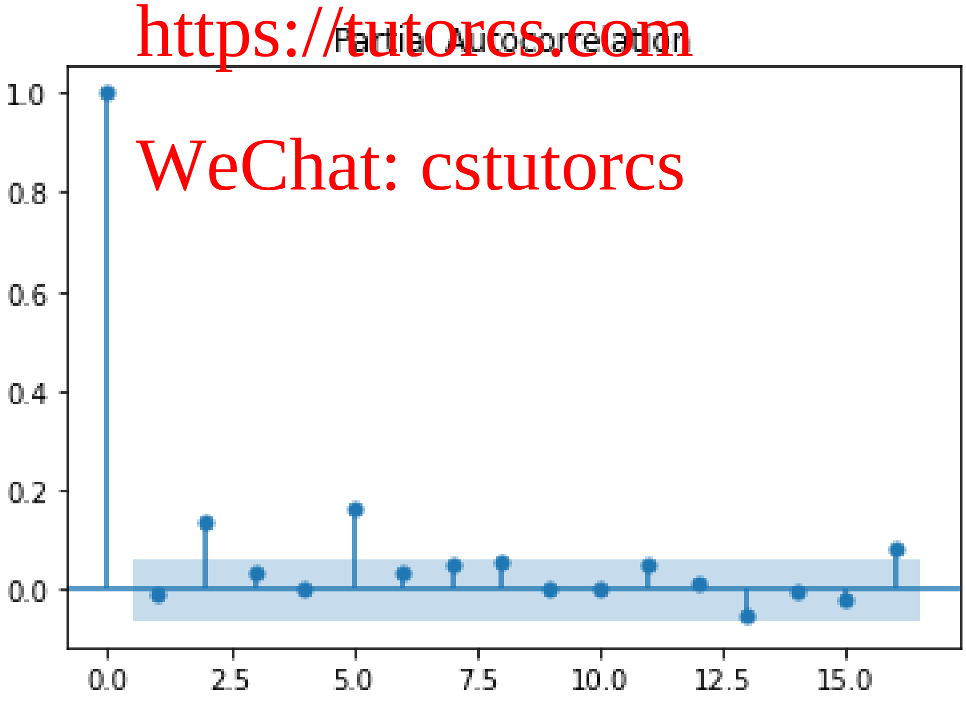
[87]:
```
stats.jarque_bera(std_resid)
```

[87]:
```
Jarque_beraResult(statistic=141.05812439839767, pvalue=0.0)
```

ACF , PACF of Squared Std Residuals

[88]:
```
fig =tsaplots.plot_acf(std_resid**2,lags=16)
fig =tsaplots.plot_pacf(std_resid**2,lags=16)
plt.show()
```





Choosing the ARCH lags

[89]:

```
#Running ARCH from p=1 to p =9
model_1 = arch_model(dt,mean ='Constant', vol ='ARCH', p=1)
model_2 = arch_model(dt,mean ='Constant', vol ='ARCH', p=2)
model_3 = arch_model(dt,mean ='Constant', vol ='ARCH', p=3)
model_4 = arch_model(dt,mean ='Constant', vol ='ARCH', p=4)
model_5 = arch_model(dt,mean ='Constant', vol ='ARCH', p=5)
model_6 = arch_model(dt,mean ='Constant', vol ='ARCH', p=6)
model_7 = arch_model(dt,mean ='Constant', vol ='ARCH', p=7)
model_8 = arch_model(dt,mean ='Constant', vol ='ARCH', p=8)
model_9 = arch_model(dt,mean ='Constant', vol ='ARCH', p=9)
x_1= model_1.fit(update_freq=0)
x_2= model_2.fit(update_freq=0)
x_3= model_3.fit(update_freq=0)
x_4= model_4.fit(update_freq=0)
x_5= model_5.fit(update_freq=0)
x_6= model_6.fit(update_freq=0)
x_7= model_7.fit(update_freq=0)
x_8= model_8.fit(update_freq=0)
x_9= model_9.fit(update_freq=0)
```

```
Optimization terminated successfully    (Exit mode 0)
            Current function value: 1664.047869085027
            Iterations: 5
            Function evaluations: 28
            Gradient evaluations: 5
Optimization terminated successfully    (Exit mode 0)
            Current function value: 1651.3667967409056
            Iterations: 8
            Function evaluations: 49
            Gradient evaluations: 8
Optimization terminated successfully    (Exit mode 0)
            Current function value: 1650.803109203952
            Iterations: 12
            Function evaluations: 83
            Gradient evaluations: 12
Optimization terminated successfully    (Exit mode 0)
            Current function value: 1645.3224422301862
            Iterations: 13
            Function evaluations: 104
            Gradient evaluations: 13
Optimization terminated successfully    (Exit mode 0)
            Current function value: 1639.433689366865
            Iterations: 15
            Function evaluations: 135
            Gradient evaluations: 15
Optimization terminated successfully    (Exit mode 0)
            Current function value: 1638.5159772755142
            Iterations: 15
            Function evaluations: 153
            Gradient evaluations: 15
Optimization terminated successfully    (Exit mode 0)
            Current function value: 1635.51431460451
            Iterations: 15
            Function evaluations: 170
            Gradient evaluations: 15
Optimization terminated successfully    (Exit mode 0)
            Current function value: 1631.19776764798
            Iterations: 17
            Function evaluations: 209
            Gradient evaluations: 17
Optimization terminated successfully    (Exit mode 0)
            Current function value: 1631.1128849200488
            Iterations: 20
            Function evaluations: 264
            Gradient evaluations: 20
```

[90]:

```
#Computing the AIC  (AIC_stata= AIC_Python/ no of obs)
n = 993
aic=[x_1.aic/n,x_2.aic/n,x_3.aic/n,x_4.aic/n,x_5.aic/n,x_6.aic/n,x_7.aic/n,x_8.
 ↪aic/n,x_9.aic/n]
bic= [x_1.bic/n,x_2.bic/n,x_3.bic/n,x_4.bic/n,x_5.bic/n,x_6.bic/n,x_7.bic/n,x_8.
 ↪bic/n,x_9.bic/n]
name␣
 ↪=['ARCH_1','ARCH_2','ARCH_3','ARCH_4','ARCH_5','ARCH_6','ARCH_7','ARCH_8','ARCH_9']
lzip(name,aic, bic)
```

[90]:

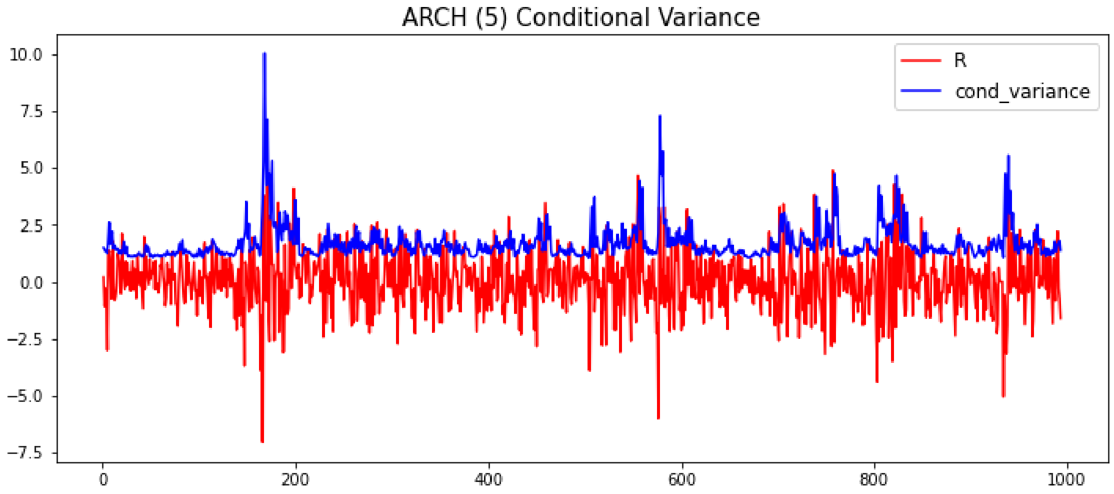
```
[('ARCH_1', 3.357598930684848, 3.3724047635067365),
 ('ARCH_2', 3.334072098168994, 3.3538132085981793),
 ('ARCH_3', 3.3349508745296115, 3.359627262566093),
 ('ARCH_4', 3.325926369043678, 3.3555380346874557),
 ('ARCH_5', 3.316079938301843, 3.350626881552917),
 ('ARCH_6', 3.3162456742709248, 3.3557278951292946),
 ('ARCH_7', 3.31221412810576, 3.3566316265714264),
 ('ARCH_8', 3.305534275222518, 3.35488705129548),
 ('ARCH_9', 3.3073774117221526, 3.361665465402411)]
```

[91]:

```
#Defining Conditional_Variance as squared conditional volatility from ARCH(5)␣
 ↪which was defined as x_5 beforehand
conditional_variance = x_5.conditional_volatility**2
```

[92]:

```
#Plotting the R and Conditional Variance from ARCH(5)
dt.plot(figsize=(12,5), color ='red',label ='R')
conditional_variance.plot(figsize=(12,5),color='blue',label='cond_variance')
plt.title('ARCH (5) Conditional Variance', size=15)
plt.legend(loc='best', fontsize='large')
plt.show()
```


ARCH (5) Conditional Variance
R
cond_variance
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
0
200
400
600
800
1000

```
[ ]:
```