

# ECON7350: Applied Econometrics for Macroeconomics and Finance

## Tutorial 4: Dynamic Relationships

At the end of this tutorial you should be able to:

- derive the ECM representation of an  $ARDL(p, l, s)$  model;
- create a function in R;
- estimate IRFs to permanent and one-off shocks as well as LRMs;
- construct confidence intervals for IRFs and LRMs;
- Select an adequate set of ARDL models and draw inference on dynamic relationships.

## Problems

<https://tutorcs.com>

1. Derive the ECM representation of the following  $ARDL(1, 1, 2)$  model:

$$y_t = a_0 + a_1(y_{t-1}) + b_1x_t + b_2x_{t-1} + c_0w_t + c_1w_{t-1} + c_2w_{t-2} + \epsilon_{y,t}.$$

Which parameter(s) in the resulting ECM are long-run multiplier(s) and adjustment parameter(s)?

2. Create a function in R to compute coefficients  $\theta_0, \dots, \theta_h$  in

$$\theta(L) = b(L)/a(L) = \theta_0 + \theta_1L + \dots + \theta_hL^h + \dots,$$

where  $a(L) = a_0 + a_1L + \dots + a_pL^p$  and  $b(L) = b_0 + b_1L + \dots + b_qL^q$ .

3. Create a function in R to compute IRFs (to both one-off and permanent shocks) up to horizon  $h$  as well as the LRMs for the  $ARDL(p, l, s)$ :

$$a(L)y_t = a_0 + b(L)x_t + c(L)w_t + \epsilon_{y,t}.$$

4. The file `wealth.csv` contains observations on:

- $c_t$ : the log of total real per capita expenditures on durables, nondurables and services;

- $a_t$ : the log of a measure of real per capita household net worth (including all financial and household wealth); and
- $y_t$ : the log of after-tax labour income.

The sample period from 1952Q2 through 2006Q2 (see Koop, G., S. Potter and R. W. Strachan (2008) “Re-examining the consumption-wealth relationship: The role of uncertainty” *Journal of Money, Credit and Banking*, Vol. 40, No. 2.3, 341-367).

- Estimate an ARDL(1, 2, 2) specified for  $c_t$  and use the functions created in Questions 2 and 3 to obtain the estimated IRFs to permanent shocks in  $a_t$  and  $y_t$  as well the LRMs. Hint: to estimate the ARDL parameters, try the `ardl` function that is provided by the `ARDL` package.
- Estimate the ECM representation of the ARDL(1, 2, 2) and report the results. How do the LRMs in the estimated ECM compare to those computed in part (a)? Hint: use the `recm` and `multipliers` functions to convert the output produced by `ardl`.
- Use the function `ardl_irfs_ci` that is provided in the file `ardl_irfs_ci.R` to construct 68% confidence intervals for the IRFs obtained in part (a).

The function `ardl_irfs_ci` takes the following inputs:

- `ardl_est`: this is the output of `ardl`;
- `h`: the maximum IRF horizon (default is 40);
- `cumirf`: whether to compute IRFs to a permanent shock (default is `TRUE`);
- `conf`: the confidence level of the intervals (default is 0.95).

It returns the following outputs:

- `lb`: an  $h \times k$  matrix of lower-bounds for confidence intervals;
- `md`: an  $h \times k$  matrix of mid-points for confidence intervals;
- `ub`: an  $h \times k$  matrix of upper-bounds for confidence intervals.

Note that  $k$  is the number of independent variables in the ARDL, so that column  $j$  of `lb`, `md` and `ub` is related the confidence intervals for IRFs to a shock in the  $j$ th independent variable.

- Compare the values in `md` to the IRFs estimates obtained in part (a).
- Use the LRM estimates and standard deviations obtained in part (b) to construct 68% confidence intervals for the LRMs, assuming the sampling distributions of the LRM estimators are approximately normal. How do they compare to the IRF confidence intervals obtained in part (c)?
- Construct an adequate set of ARDL( $p, l, s$ ) models for  $c_t$ .
- Draw inference about the dynamic relationship between expenditures and wealth.