

HW1 Solutions

Problem 1

Classical outputs	Quantum outputs	Probability
00	$ 00\rangle$	$1/2$
01	$ 01\rangle$	$1/16$
10	$ 10\rangle$	$1/4$
11	$ 11\rangle$	$3/16$

Problem 2

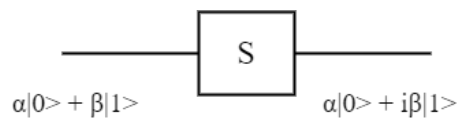
Classical outputs	Quantum outputs	Probability
00	$\gamma 00\rangle + \frac{1}{\sqrt{2}} 01\rangle + \frac{i}{4} 10\rangle$, $\gamma = \frac{1}{2} = \frac{16}{9}$	$9/16$
10	$\Rightarrow 00\rangle \left(\frac{2\sqrt{2}}{3} 00\rangle + \frac{i}{3} 01\rangle \right)$	
01	$\left(-\frac{2}{\sqrt{7}} 10\rangle + \frac{\sqrt{3}i}{\sqrt{7}} 11\rangle \right)$	$7/16$
11	—	0
	—	0

If $j_1 = 1$ and $j_2 = 0$ then the state of qubits 3 and 4 is:

$$-\frac{2}{\sqrt{7}}|10\rangle + \frac{\sqrt{3}i}{\sqrt{7}}|11\rangle$$

Problem 3

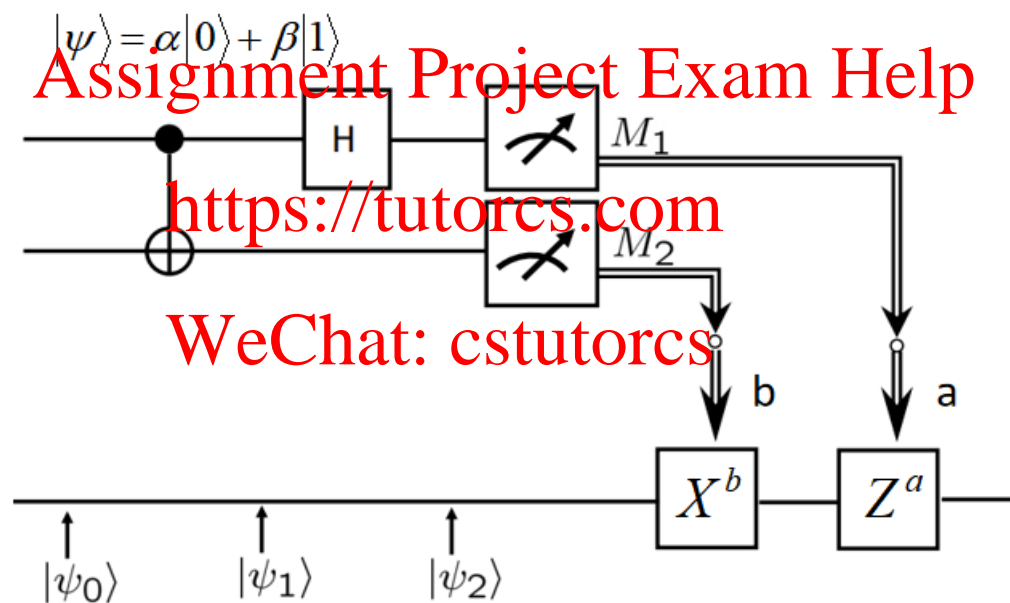
$$U_S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \Rightarrow$$



Problem 4

$$U_{\text{Toffoli}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Problem 5



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\begin{aligned} |\psi_0\rangle &= \frac{1}{\sqrt{2}}\alpha|0\rangle(|01\rangle - |10\rangle) + \frac{1}{\sqrt{2}}\beta|1\rangle(|01\rangle - |10\rangle) \\ &= \frac{1}{\sqrt{2}}(\alpha|001\rangle - \alpha|010\rangle + \beta|101\rangle - \beta|110\rangle) \end{aligned}$$

$$\begin{aligned} |\psi_1\rangle &= \frac{1}{\sqrt{2}}(\alpha|001\rangle - \alpha|010\rangle + \beta|111\rangle - \beta|100\rangle) \\ &= \frac{1}{\sqrt{2}}[\alpha|0\rangle(|01\rangle - |10\rangle) + \beta|1\rangle(|11\rangle - |00\rangle)] \end{aligned}$$

$$\begin{aligned} |\psi_2\rangle &= \frac{1}{2}[\alpha(|0\rangle + |1\rangle)(|01\rangle - |10\rangle) + \beta(|0\rangle - |1\rangle)(|11\rangle - |00\rangle)] \\ &= \frac{1}{2}[|00\rangle(\alpha|1\rangle - \beta|0\rangle) + |01\rangle(-\alpha|0\rangle + \beta|1\rangle) + |10\rangle(\alpha|1\rangle + \beta|0\rangle) + |11\rangle(-\alpha|0\rangle - \beta|1\rangle)] \end{aligned}$$

Classical outputs	Quantum outputs	Probability
00	$ 00\rangle(\alpha 1\rangle - \beta 0\rangle)$	1/4
01	$ 01\rangle(-\alpha 0\rangle + \beta 1\rangle)$	1/4
10	$ 10\rangle(\alpha 1\rangle + \beta 0\rangle)$	1/4
11	$ 11\rangle(-\alpha 0\rangle - \beta 1\rangle)$	1/4

Value of M_1, M_2	Bob's Unitary Operator
00	ZX
01	Z
10	X
11	I_4