SEC204

Computer Architecture and Low Level Programming

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Outline

- Positional Numbering Systems
- Converting Positional Numbering Systems
- Basic Binary Arithmetic Operations
- Signed Integer Representation
- Floating Point Representation
- Character Codes

What to do…

- In this file you will find the Lecture slides together with **some extra slides**
- **These extra slides are examples you need to understand and solve on your own**

- **Study all the examples that follow and try to solve them on your own…**

- **Pay attention to the following slides**
  - ***15-21***
  - ***24-27***
  - ***35-42***
  - ***48-49***

Basics (1)

- The bit is the most basic unit of information in a computer
  - Switching activity 0 or 1
- A Byte is a group of 8 bits
  - A byte is the smallest possible addressable unit of computer storage
  - The term, "addressable," means that a particular byte can be retrieved according to its location in memory
- A word is a contiguous group of bytes, e.g., an integer uses 4 bytes
- Word sizes of 4 or 8 bytes are most common

Basics (2)

Kilo- (K) = 1 thousand = $\mathbf{10^3}$ and 2^{10}

Mega- (M) = 1 million = $\mathbf{10^6}$ and 2^{20}

Giga- (G) = 1 billion = $\mathbf{10^9}$ and 2^{30}

Tera- (T) = 1 trillion = $\mathbf{10^{12}}$ and 2^{40}

Peta- (P) = 1 quadrillion = $\mathbf{10^{15}}$ and 2^{50}

Exa- (E) = 1 quintillion = $\mathbf{10^{18}}$ and 2^{60}

Zetta- (Z) = 1 sextillion = $\mathbf{10^{21}}$ and 2^{70}

Yotta- (Y) = 1 septillion = $\mathbf{10^{24}}$ and 2^{80}

Normally, powers of 2 are used for measuring capacity

Milli- (m) = 1 thousandth = $\mathbf{10^{-3}}$

Micro- (μ) = 1 millionth = $\mathbf{10^{-6}}$

Nano- (n) = 1 billionth = $\mathbf{10^{-9}}$

Pico- (p) = 1 trillionth = $\mathbf{10^{-12}}$

Basics (3)

- Hertz = clock cycles per second (frequency)
  - 1MHz = 1,000,000Hz
  - Processor speeds are measured in MHz or GHz
- Byte = a unit of storage
  - 1KB = 2^{10} = 1024 Bytes
  - 1MB = 2^{20} = 1,048,576 Bytes
  - 1GB = 2^{30} = 1,099,511,627,776 Bytes
- Main memory (RAM) is measured in GB
- Disk storage is measured in GB for small systems, TB (2^{40}) for large systems

Think Pair Share activity

- How many milliseconds (ms) are in 1 second?

- How many nanoseconds (ns) are in 1 millisecond?

- How many kilobytes (KB) are in 1 gigabyte (GB)?

- How many bytes are in 20 megabytes?

POSITIONAL NUMBERING SYSTEMS

- Positional numbering systems are systems in which the placement of a digit in connection to its intrinsic value determines its actual meaning in a numeral string
- The organization of any computer depends considerably on how it represents numbers, characters, and control information
  - **There are several positional numbering systems such as Decimal, Binary, Octal, Hexadecimal etc**
- The positioning system is provided as a subscript, e.g., 14_{10}, 10101_{2}, 82_{16}
- Our decimal system is the base-10 system. It uses powers of 10 for each position in a number
- The binary system is also called the base-2 system
- The hexadecimal system is the base-16 system
- The Mayan and other Mesoamerican cultures used a number system based in a base-20 system

Decimal System

- **Decimal system**: Our well known and used system.
  - **It uses 10 different digits: 0,1,2,3,4,5,6,7,8,9**
  - Our decimal system is the base-10 system. It uses powers of 10 for each position in a number.
  - For example, the decimal number 947 in powers of 10 is

    947 =

    $=9 \times 100 + 4 \times 10 + 7 \times 1 =$

    $=9 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$

  - $70216 = 7 \times 10000 + 0 \times 1000 + 2 \times 100 + 1 \times 10 + 6 \times 1 =$

    $= 7 \times 10^4 + 0 \times 10^3 + 2 \times 10^2 + 1 \times 10^1 + 6 \times 10^0$

- The decimal number 3812.46 in powers of 10 is ($3 \times 10^3 + 8 \times 10^2 + 1 \times 10^1 + 2 \times 10^0 + 4 \times 10^{-1} + 6 \times 10^{-2}$)

Binary System (1)

- A binary number is a number expressed in the base-2 numeral system or binary numeral system, which uses only two symbols: typically 0 (zero) and 1 (one)
- The base is 2

- **2 different digits are used: 0,1**

- For example, $101_2 = 1\mathbf{x}2^2 + 0\mathbf{x}2^1 + 1\mathbf{x}2^0$

  $= 1\mathbf{x}4 + 0\mathbf{x}2 + 1\mathbf{x}1$

  $= 5_{10}$

- The binary number 11001 in powers of 2 is: $1\text{x}2^4 + 1\text{x}2^3 + 0\text{x}2^2 + 0\text{x}2^1 + 1\text{x}2^0 = 16 + 8 + 0 + 0 + 1 = 25_{10}$
- $1011.101_2 =$

  $= 1\mathbf{x}2^3 + 0\mathbf{x}2^2 + 1\mathbf{x}2^1 + 1\mathbf{x}2^0 + 1\mathbf{x}2^{-1} + 0\mathbf{x}2^{-2} + 1\mathbf{x}2^{-3} =$

  $= 1\mathbf{x}8 + 0\mathbf{x}4 + 1\mathbf{x}2 + 1\mathbf{x}1 + 1\mathbf{x}0.5 + 0\mathbf{x}0.25 + 1\mathbf{x}0.125$

  $= 11.625_{10}$

Binary System (2)

2^n representation	2^{10}	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
number	1024	16	8	4	2	1	0.5	0.25	0.125

Convert the following binary number 1101.101 to decimal

$$\mathbf{1101.101}_{\,2} = 1x2^3 + 1x2^2 + 0x2^1 + 1x2^0 + 1x2^{-1} + 0x2^{-2} + 1x2^{-3}$$
$$= 8 + 4 + 0 + 1 + 0.5 + 0 + 0.125 = 13.625_{10}$$

Octal system

- The base is 8
- **8 different digits are used only: 0,1,2,3,4,5,6,7**
- For example: $436_8 = 4\mathbf{x}8^2 + 3\mathbf{x}8^1 + 6\mathbf{x}8^0$

  $= 4\mathbf{x}64 + 3\mathbf{x}8 + 6\mathbf{x}1$

  $= 286_{10}$

Convert the following octal number 205.24_8, to decimal:

$205.24_8 = 2x8^2 + 0\mathbf{x}8^1 + 5\mathbf{x}8^0 + 2x8^{-1} + 4x8^{-2}$

$= 2x64 + 0 + 5 + 2x0.125 + 4x0.015625$

$= \mathbf{133.3125_{10}}$

Hexadecimal system

- The base is 16
- **16 different digits are used: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F**

(we do not use numbers with 2 digits like 10,11,12,...), but

A instead of 10, B instead of 11, C instead of 12, etc)

- Example: $3B1_{16} = 3\mathbf{x}16^2 + 11\mathbf{x}16^1 + 1\mathbf{x}16^0$

$= 3\mathbf{x}256 + 11\mathbf{x}16 + 1 =$

$= 768 + 176 + 1 =$

$= 945_{10}$

Convert the following hexadecimal number $20C.2_{16}$ to decimal

$\mathbf{20C.2_{16}} = 2x16^2 + 0x16^1 + 12x16^0 + 2x16^{-1} =$

$= 2x256 + 0 + 12x1 + 2x0.0625 =$

$= 512 + 12 + 0.125 =$

$= \mathbf{524.125_{10}}$

Positional Numbering Systems - General case

- Base: r
- **Uses r different digits: 0,1,2,3,..r-1**

- $N\mathbf{r} = A_{n-1}\ A_{n-2} \ldots A_1\ A_0\ A_{-1}\ A_{-2} \ldots A_{-(m-1)} A_{-m}$

  $N_r = A_{n-1} \times r^{n-1} + A_{n-2} \times r^{n-2} + \ldots + A_1 \times r^1 + A_0 \times r^0 + A_{-1} \times r^{-1} + A_{-2} \times r^{-2} + \ldots + A_{-(m-1)} \times r^{-(m-1)} + A_{-m} \times r^{-m}$

For example if $234.03_5 = ?_{10}$ then $n=3$, $m=2$ and $r=5$

- The left most digit (A**n-1**) is called Most Significant Bit-(MSB) while the right most (A**-m**) Least Significant Bit-(LSB)

Converting Positional Numbering Systems

From Decimal to Binary

The easiest method of converting integers from decimal to some other base uses division

37 / 2 = 18 remainder 1

Base Quotient Remainder

Procedure:

a. Divide the decimal with 2

b. Write down the quotient and the remainder

c. Divide quotient with 2

d. Write down the quotient and the remainder

e. Repeat the process (c)-(d) until the quotient becomes zero

f. Write down the binary number from bottom (MSB) to top (LSB)

From Decimal to Binary, an example

$83_{10} = ?_2$

$83 \div 2 = 41$ remainder **1** **(LSB)**

$41 \div 2 = 20$ remainder **1**

$20 \div 2 = 10$ remainder **0**

$10 \div 2 = 5$ remainder **0**

$5 \div 2 = 2$ remainder **1**

$2 \div 2 = 1$ remainder **0**

$1 \div 2 = 0$ remainder **1** **(MSB)**

$83_{10} = 1010011_2$

If the decimal number is lower than 1, e.g., 0.25, or contains a fractional part the procedure applied is different

Our result, is the remainders in reverse order (reading from bottom to top)

Convert From Decimal To Another Base

- We follow the same procedure as in the previous slide (from decimal to binary) but instead of using 2-base we use the r-base.

- **Convert the decimal 524_{10} to hexadecimal**

524:16= 32 remainder 12 ($12_{10}=C_{16}$)

32:16=2 remainder 0

2:16=0 remainder 2

Thus, **$524_{10}= 20C_{16}$**

$66_{10} = ?_2$

$128_{10}= ?_{16}$

Convert from binary to octal

Procedure:

1. ***To convert a binary number of octal***, *we group all the 1's and 0's in the binary number in* ***sets of three***, *starting from the far right*

2. *Start from the right to make your groups*
3. *Add zeros to the left of the last digit if you don't have enough digits to make a set of three*

4. *Write down the decimal representation of every group*

Binary			Octal
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

101011_2 = ¡$_8$

=101 011

= 5 3

$101011_2 = 53_8$

10011011_2 = ¡$_8$

10 011 011

010 011 011

2 3 3

$10011011_2 = 233_8$

Convert from Octal to binary

- Converting from octal to binary is as easy as converting from binary to octal. Simply look up each octal digit to obtain the equivalent group of three binary digits

$317.2_8 = ?_2$

$= 011\ 001\ 111.010$

$= 11001111.01_2$

Binary			Octal
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

Convert Binary to Hexadecimal

Procedure:

1. *Cut your string of binary numbers* ***into groups of four,*** *starting from the right*
2. *Add extra zeros to the front of the first number if it is not four digits*
3. *Convert one 4-digit group at a time. To convert between Binary and Hexadecimal, you simply replace each Hex digit with its 4-bit binary equivalent and vice versa*

$10001101011_2 = ¿_{16}$

$= 100 \quad 0110 \quad 1011$

$= 0100 \quad 0110 \quad 1011$

$= 4 \quad 6 \quad B$

$10001101011_2 = 46B_{16}$

$0100_2 = 0+4+0+0 = 4_{16}$

$0110_2 = 0+4+2+0 = 6_{16}$

$1011_2 = 8+0+2+1 = 11 = B_{16}$

Hexadecimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Convert Hexadecimal to Binary

- Likewise from Octal to Binary
- Simply look up each hexadecimal digit to obtain the equivalent group of **four binary digits**

$8B.2_{16} = 1000\ 1011.0010_2$

Hexadecimal	Binary	Decimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
A	1010	10
B	1011	11
C	1100	12
D	1101	13
E	1110	14
F	1111	15

- That's why we use hexadecimal in computer systems! Humans can still understand it, and computers can calculate Hex faster than decimal values!

Basic arithmetic operations

- The basic arithmetic operations are applied to all the previous numerical systems. There are:
  - Addition
  - Subtraction
  - Multiplication
  - Division

- For the reminder of this lecture we will focus on the binary system

Binary Addition (1)

Binary addition is like decimal addition:

1. Put the numbers in a vertical column, aligning the decimal points
2. Add each column of digits, starting on the right and working left. If the sum of a column is more than ten, "carry" digits to the next column on the left.

457 +
148
605
(011 Carry)

- In the example above we add 8+7 and write 5 instead of 15. We propagate 10 (which is the base) to the left and we write the remainder
- For every propagation, we add 1 carry to the next addition (left)
- **The same holds when adding different numerical system numbers too**

Binary Addition (2)

Binary addition:

```
  1 0 1 1 0 1 1 1 +
    1 1 1 0 1 1 0
  -----------------
1 0 0 1 0 1 1 0 1
( 0 1 1 1 1 0 1 1 0 carry)
```

```
   1 1 1 1 1 0 1 1 +
   1 1 1 0 1 0 1 1
   ---------------
 1 1 1 1 0 0 1 1 0
(  1 1 1 1 1 0 1 1   carry)
```


Note that: 0+0= 0 and carry 0

1+0=0+1= 1 and carry 0

1+1= 0 and carry 1, as $1 + 1 = 10_2 = 2_{10}$

1+1+1= 1 and carry 1, as $1 + 1 + 1 = 11_2 = 3_{10}$

Binary Subtraction (1)

Binary subtraction is like decimal subtraction:

1. Put the numbers in a vertical column, aligning the decimal points.
2. Subtract each column, starting on the right and working left. If the digit being subtracted in a column is larger than the digit above it, "borrow" a digit from the next column to the left. When we borrow a carry, we add 1 to the subtrahend of the next subtraction

$$
\begin{array}{r}
4\ 3\ 5\ 6 \\
-\underline{2\ 1\ 8\ 9} \\
2\ 1\ 6\ 7
\end{array}
$$

(0 0 1 1 borrow carry from the left)

Binary Subtraction (2)

For binary subtraction:

$0 - 0 = 0$

$1 - 0 = 1$

$1 - 1 = 0$

$10 - 1 = 1$, as $10_2 = 2_{10}$

The first three are the same as in decimal. The fourth fact is the only new one; it is the borrow case. It applies when the "top" digit in a column is 0 and the "bottom" digit is 1

```
 0 0   0
 1 1 0 1 0 1 1
-  1 1 0 1 1 0
 0 1 1 0 1 0 1
 ( 1 1 0 1 0 0  carry)
```

```
            0 ->2
 0 ->2  0 ->2   0 ->2
          ->
 1  0  1  0  1. 1  0  1
 -  1  0  1  1. 1  1
 0  1  0  0  1. 1  1  1
```

Binary Subtraction (3)

- The procedure explained in the previous slides **holds only when we subtract X-Y where X≥Y**
- If X<Y then a different method is used which is out of the scope of this module
- In the next slides you will be taught another method (two's complement, slide 37)

Binary Multiplication

- As in the decimal system:


```
       110111 X
         1101
      110111
     000000
    110111    +
   110111____
  1011001011
(0111221100) Carry
```

Note that 1+1+1+1= 0 and carry 2, as 1+1+1+1= $100_2 = 4_{10}$

Signed integer representation

Introduction

- In practice we have to use negative binary numbers too. **We need to define signed binary numbers**

- ✓ **There are three ways in which signed binary integers may be expressed:**

1. **Signed magnitude**
2. **One's complement**
3. **Two's complement**

Signed Magnitude Representation (1)

- **Allocate the high-order (leftmost) bit to indicate the sign of a number**
  - The high-order bit is the leftmost bit. It is also called the most significant bit
  - 0 is used to indicate a positive number; 1 indicates a negative number
- The remaining bits contain the value of the number
- Note that we also pay attention to the number of bits used to represent signed binary numbers
  - i.e. if using 4 bit numbers, then we use 0001_2 rather than 1_2
- In an 8-bit word, signed magnitude representation places the absolute value of the number in the 7 bits to the right of the sign bit

For example:

+3 is: **0**0000011

- 3 is: **1**0000011

Signed Magnitude Representation (2)

- The "binary addition algorithm" does NOT work with sign-magnitude

0 0 1 $1_2 = 3_{10}$

1 1 0 $0_2 = -4_{10}$


```
0     0 1 1
1  +  1 0 0
-----------
1     1 1 1
```

this is wrong

Signed Magnitude: intuitive for humans, difficult for computers

- Signed magnitude representation is easy for people to understand, but it requires complicated computer hardware

- Also it allows two different representations for zero: positive zero and negative zero

- As such, computer systems employ **complement systems** for signed number representation

Signed Integer Representation
Complement Systems

- In binary systems, these are:
  - **One's Complement.** To represent negative values, invert all the bits in the binary representation of the number (swapping 0s for 1s and vice versa)
    - 1 becomes 0 and 0 becomes 1
    - To represent positive numbers no change is applied

    For example, using 8-bit one's complement representation

    + 3 is: 00000011

    - 3 is: 11111100

    More examples

    X=11011100, 1C(X)=00100011

    X=1011, 1C(X)=?

    - One's complement still has the disadvantage of having two different representations for zero: positive zero and negative zero
    - In addition positive and negative integers need to be processed separately
    - Two's complement solves this problem
  - **Two's complement**
    - One's Complement add 1

Signed Integer Representation
Two's Complement

Two's complement 2C(X)

- You represent positive numbers, just like the unsigned numbers
- To represent negative values, start with the corresponding positive number, invert all the bits. Then add 1

- For example, using 8-bit two's complement representation:

+ 3 is: 00000011	*1.Start with positive number*
11111100	*2.Invert bits*
+ 1	*3. Add 1*
- 3 is: 11111101	

-3 in 8-bit Two's Complement Representation is 11111101

✓Negative numbers must always start with '1'

✓Both positive and negative numbers must have the same number of bits

Signed Integer Representation
Two's Complement – Example1

- Example: X=01001 ($+9_{10}$), n=5 bits -> Y=2C(X)= 10111 = -9_{10}

Check: X+Y=

$$\begin{array}{r} 01001 \\ +10111 \\ \hline 1\,00000 = 00000 \end{array}$$

The carry is discarded as the result must be 5 bits

- We can always check whether the two's complement result is correct by adding the two numbers. The result has to be zero. **Note that the result of the addition must be of the n bits, where n is the number of bits of the inputs**
- *Find the negative binary number (two's complement) of the following positive number with 7bits 0101101 (45_{10})*

  Answer 1010011

Signed Integer Representation
Two's Complement – Example2

Find the negative binary number (-12_{10})

- Write down the positive $12_{10} = 01100_2$
- Decide the number of the bits. We can use 5 or more. Let say 8 bits
- Find the two's complement as follows

$12_{10} = 00001100_2$, $-12_{10} = 11110011 + 1 =$ **11110100**

-12_{10} =11110100 (negative number)

Wrap up

Given a positive binary number, we find its negative binary number by following the procedure:

1. *We decide the number of bits of the positive number. At least one '0' has to appear on its left.*
2. *We find its two's complement*
3. *If the MSB (the left most) is not '1' then we made a mistake…*

Subtraction with two's complement

- We know that A-B=A+(-B)
- So, instead of applying a subtraction we can make an addition with the opposite number, i.e., the two's complement. *The procedure follows:*
  1. *Find –B, i.e., its two's complement*
  2. *Add A with B's two complement*
  3. ***The result has as many bits as the inputs***

Make the subtraction Z=12-5 (use 5 digits)

$12_{10}=01100_2$, $5_{10}=00101_2$, $-5_{10}=11011_2$

Z=01100 + 11011=100111, but we need 5 bits thus $Z=00111_2=7_{10}$

Z=00111

Make the subtraction 9-12 (use 5 digits)

$9_{10}=01001_2$, $12_{10}=01100_2$, $-12_{10}=10100_2$

Z=01001 + 10100=11101 and thus Z=11101 (-3_{10})

Z=11101

Unsigned and Signed Integer Representation

- **Both signed and unsigned numbers are useful**
  - For example, memory addresses are always unsigned
- Using the same number of bits, unsigned integers can express twice as many "positive" values as signed numbers.
  - For example, the range of values that can be represented in **4-bits** is:
    - 0000_2 to 1111_2 (or 0 to 15) as unsigned
    - 0111_2 to 1111_2 (or +7 to -7) as signed magnitude
    - 0111_2 to 1000_2 (or +7 to -8) as two's complement

Example #1

What decimal value does the 8-bit binary number 10011110 have if

a) it is interpreted as an unsigned number?

b) it is on a computer using signed-magnitude representation?

c) it is on a computer using one's complement representation?

d) it is on a computer using two's complement representation?

Answer:

a. $\mathbf{10011110_2 = 1x2^7 + 1x2^4 + 1x2^3 + 1x2^2 + 1x2^1 = 128+16+8+4+2=158_{10}}$

b. $\mathbf{10011110_2}$ = **1 (negative)** $\mathbf{0011110_2 =- 1x2^4 + 1x2^3 + 1x2^2 + 1x2^1 = -30_{10}}$

c. Find the positive of $\mathbf{10011110_2}$;invert the bits of $\mathbf{10011110_2}$ and therefore $\mathbf{01100001_2}$. $\mathbf{01100001_2 =1x2^6 + 1x2^5 + 1x2^0=64+32+1=97_{10}}$. Since the original number was negative, the number is $\mathbf{-97_{10}}$

d. Find the positive of $\mathbf{10011110_2}$;invert the bits of $\mathbf{10011110_2}$ **and add 1; therefore** $\mathbf{01100010_2}$. $\mathbf{01100010_2 =1x2^6 + 1x2^5 + 1x2^1=64+32+2=98_{10}}$. Since the original number was negative, the number is $\mathbf{-98_{10}}$

Example #2

What decimal value does the 8-bit binary number 00010001 have if

a) it is interpreted as an unsigned number?

b) it is on a computer using signed-magnitude representation?

c) it is on a computer using one's complement representation?

d) it is on a computer using two's complement representation?

Answer:

a. $00010001_2 = 1\text{x}2^4 + 1\text{x}2^0 = 16+1=17_{10}$

b. $00010001_2 =$ **0 (positive)** $0010001_2 = 1\text{x}2^4 + 1\text{x}2^0 = 16+1=17_{10}$

c. $00010001_2 = 1\text{x}2^4 + 1\text{x}2^0 = 16+1=17_{10}$

d. $00010001_2 = 1\text{x}2^4 + 1\text{x}2^0 = 16+1=17_{10}$

Example #3

Perform the following binary subtraction using two's complement representation: Z=8 – 6

Answer:

1. Instead of subtraction we do addition: 8 - 6 =8 + (-6)

2. Choose the number of bits to represent 8 and 6 decimal numbers to binary. **At least one zero has to appear on the left.** Thus, we need 5 bits or more

   $8_{10}=01000_2$

   $6_{10}=00110_2$

3. Find - 6_{10} (two's complement) – 6_{10} = 11010_2

3. Make the addition (***The result has 5 bits***)

$$\begin{array}{r} 0\,1\,0\,0\,0 \\ 1\,1\,0\,1\,0 \\ \hline 1\,\boxed{0\,0\,0\,1\,0} \end{array}$$

-> $00010_2=2_{10}$

Signed Integer Representation Multiplication and Division by 2 (1)

- Binary multiplication and division by 2 is very easy. (as easy as it is to multiply and divide by 10 in the decimal system)
- Simply use an arithmetic shift operation

$$1_2 = 1_{10}$$

$$10_2 = 2_{10}$$

$$100_2 = 4_{10}$$

$$1000_2 = 8_{10}$$

$$10000_2 = 16_{10}$$

- A left arithmetic shift inserts a 0 in for the rightmost bit and shifts everything else left one bit; in effect, it multiplies by 2
- A right arithmetic shift shifts everything one bit to the right, but copies the sign bit; it divides by 2

Signed Integer Representation Multiplication and Division by 2 (2)

- Multiplication by 2
  - Shift left by one place
  - E.g. to calculate 2 * 7

Binary	Decimal
0000 0111	+7
0000 1110	+14

- Division by 2
  - Shift right by one place
  - E.g., to calculate 14/2

- To multiply by 4, we perform a left shift twice
- To divide by 4, we perform a right shift twice

- Using arithmetic shifting, perform the following:

  a) double the value 00010101_2

  b) quadruple the value 00110111_2

  c) divide the value 11001010_2 in half

Floating-Point Representation (1)

- To represent real numbers with fractional values, floating-point representation is used
- Floating-point numbers are often expressed in scientific notation
  - For example: $0.125 = 1.25 \times 10^{-1}$

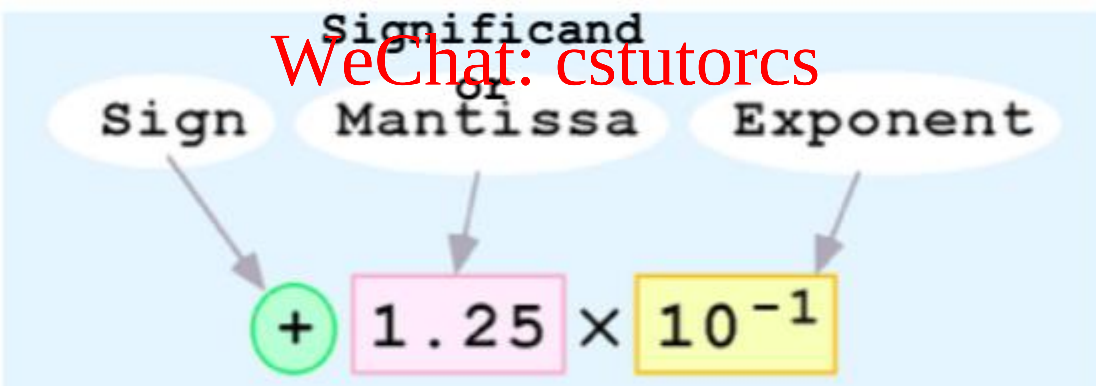
- Remember that when a number is multiplied by its base, e.g., 10, then we add a zero or we move the '.' by one position to the right

  - $235 \times 10 = 2350$

  - $1.345 \times 10 = 13.45$
  - $110_2 \times 2 = 1100_2$ $(6 \times 2 = 12_{10})$
  - $101.11_2 \times 2 = 1011.1$ $(5.75 \times 2 = 11.5_{10})$

Floating-Point Representation (2)

- Computers use a form of scientific notation for floating-point representation
  - Single Precision floating point format 32-bit
  - Double Precision floating point format 64-bit
- Numbers written in scientific notation have three components:



Single precision Floating-Point format (1)

A binary number is represented in FP format as follows:

1. We write the number using only a single non-zero digit before the radix point :

   e.g., $1011010010001 = 1.011010010001 \times 2^{12}$

   $1101.10111 = 1.10110111 \times 2^{3}$

2. Then we transform the number to the following format using 32 bits

$$N = (-1)^S (1+F)(2^{E-127})$$

Sign-S	Exponent-E	Mantissa (Fraction) - F
1-bit	8 - bits	23 - bits

S: Sign, 0/1 for positives/negatives, respectively

E: Exponent. E-127=exp, where exp is the corresponding exponent

F: Significant or Mantissa. We write the fractional part in 23 bits

E=127+exp in order to avoid using negative numbers. exp=[-127,128] and therefore E=[0,255] – 255 needs 8 bits

Single precision Floating-Point format (2)

Convert the positive number N=1011010010001 in Floating point format

Step1: 1011010010001= 1.011010010001 x 2^{12}

Step2: $N = (-1)^S (1+F)(2^{E-127})$

S = 0 (positive number)

E - 127 = 12, and thus E = 139_{10} and E = 10001011_2

F = 01101001000100000000000

Therefore N in FP format is:

0	10001011	01101001000100000000000

Single precision Floating-Point format (3)

Suppose that the 32-bit floating-point representation pattern is the following. Find the binary number

1	10010001	10001110001000000000000

S is 1 and thus the number is negative

E is 10010001 = 145_{10}, and thus the exponent is exp=E-127=145-127=18

F= 10001110001000000000000

$N = (-1)^S\,(1+F)(2^{E-127})$

N is $(-1)^1 \times 1.10001110001000000000000 \times 2^{18}$ or

N = - 1100011100010000000

Floating-Point Representation (1)

- No matter how many bits we use in a FP representation, the model is finite
  - The real number system is, of course, infinite, so our models can give nothing more than an approximation of a real value
  - e.g., how to represent 33.33333333333333333333?
- At some point, every model breaks down, introducing errors into our calculations
  - By using a greater number of bits in our model, we can reduce these errors, but we can never totally eliminate them

Why is 0.1+0.2 not equal to 0.3 in most programming languages?

- computers use a binary floating point format that cannot accurately represent a number like 0.1_{10}
- 0.1_{10} is already rounded to the nearest number in that format
- 0.1_{10} doesn't exist in the FP representation
- 0.1_{10} is already rounded to the nearest number in that format, which results in a small rounding error
- This means that 0.1_{10} is converted to a binary number that's just very close to 0.1_{10}
- The error is tiny since 0.1_{10} is 0.1000000000000000055511151231257827
- The constants 0.2_{10} and 0.3_{10} are also approximations to their true values
- So, $0.1_{10} + 0.2_{10} == 0.30000000000000004440892098500 6_{10}$

Character Codes

- So far, we have learnt how to represent numbers. How about text?
- To represent text characters, we use character codes
  - Essentially, we assign a number for each character we want to represent

- As computers have evolved, character codes have evolved. Larger computer memories and storage devices permit richer character codes

- Some of the character codes are
  1. BCD
  2. ASCII (American Standard Code for Information Interchange) (7 bits)
  3. Extended ASCII (8-bits)
  4. Unicode
  5. and others

- A binary number of n bits gives 2^n different codes
  - For n=2 there are 2^2 =4 different codes, i.e., bit combinations {00, 01, 10, 11}

Binary Coded Decimal (BCD) code

- when numbers, letters or words are represented by a specific group of symbols, it is said that the number, letter or word is being encoded. The group of symbols is called as a code

- **Binary Coded Decimal (BCD) code**
  - In this code each decimal digit is represented by a 4-bit binary number
  - BCD is a way to express each of the decimal digits with a binary code
  - In the BCD, with four bits we can represent sixteen numbers (0000 to 1111)

$256_{10} = 0010\ 0101\ 0110_{BCD}$

And vise versa

$0011\ 1000\ 1001_{BCD} = 389_{10}$

ASCII Code

- The most widely accepted code is called the American Standard Code for Information Interchange (ASCII).
- The ASCII code associates an integer value for each symbol in the character set, such as letters, digits, punctuation marks, special characters, and control characters
- The ASCII table has 128 characters, with values from 0 through 127. Thus, 7 bits are sufficient to represent a character in ASCII

ASCII Code

Dec	Hx	Oct	Char	Dec	Hx	Oct	Html	Chr	Dec	Hx	Oct	Html	Chr	Dec	Hx	Oct	Html	Chr
0	0	000	NUL (null)	32	20	040		Space	64	40	100	@	@	96	60	140	`	`
1	1	001	SOH (start of heading)	33	21	041	!	!	65	41	101	A	A	97	61	141	a	a
2	2	002	STX (start of text)	34	22	042	"	"	66	42	102	B	B	98	62	142	b	b
3	3	003	ETX (end of text)	35	23	043	#	#	67	43	103	C	C	99	63	143	c	c
4	4	004	EOT (end of transmission)	36	24	044	$	$	68	44	104	D	D	100	64	144	d	d
5	5	005	ENQ (enquiry)	37	25	045	%	%	69	45	105	E	E	101	65	145	e	e
6	6	006	ACK (acknowledge)	38	26	046	&	&	70	46	106	F	F	102	66	146	f	f
7	7	007	BEL (bell)	39	27	047	'	'	71	47	107	G	G	103	67	147	g	g
8	8	010	BS (backspace)	40	28	050	(	(	72	48	110	H	H	104	68	150	h	h
9	9	011	TAB (horizontal tab)	41	29	051	)	)	73	49	111	I	I	105	69	151	i	i
10	A	012	LF (NL line feed, new line)	42	2A	052	*	*	74	4A	112	J	J	106	6A	152	j	j
11	B	013	VT (vertical tab)	43	2B	053	+	+	75	4B	113	K	K	107	6B	153	k	k
12	C	014	FF (NP form feed, new page)	44	2C	054	,	,	76	4C	114	L	L	108	6C	154	l	l
13	D	015	CR (carriage return)	45	2D	055	-	-	77	4D	115	M	M	109	6D	155	m	m
14	E	016	SO (shift out)	46	2E	056	.	.	78	4E	116	N	N	110	6E	156	n	n
15	F	017	SI (shift in)	47	2F	057	/	/	79	4F	117	O	O	111	6F	157	o	o
16	10	020	DLE (data link escape)	48	30	060	0	0	80	50	120	P	P	112	70	160	p	p
17	11	021	DC1 (device control 1)	49	31	061	1	1	81	51	121	Q	Q	113	71	161	q	q
18	12	022	DC2 (device control 2)	50	32	062	2	2	82	52	122	R	R	114	72	162	r	r
19	13	023	DC3 (device control 3)	51	33	063	3	3	83	53	123	S	S	115	73	163	s	s
20	14	024	DC4 (device control 4)	52	34	064	4	4	84	54	124	T	T	116	74	164	t	t
21	15	025	NAK (negative acknowledge)	53	35	065	5	5	85	55	125	U	U	117	75	165	u	u
22	16	026	SYN (synchronous idle)	54	36	066	6	6	86	56	126	V	V	118	76	166	v	v
23	17	027	ETB (end of trans. block)	55	37	067	7	7	87	57	127	W	W	119	77	167	w	w
24	18	030	CAN (cancel)	56	38	070	8	8	88	58	130	X	X	120	78	170	x	x
25	19	031	EM (end of medium)	57	39	071	9	9	89	59	131	Y	Y	121	79	171	y	y
26	1A	032	SUB (substitute)	58	3A	072	:	:	90	5A	132	Z	Z	122	7A	172	z	z
27	1B	033	ESC (escape)	59	3B	073	;	;	91	5B	133	[	[	123	7B	173	{	{
28	1C	034	FS (file separator)	60	3C	074	<	<	92	5C	134	\	\	124	7C	174			|
29	1D	035	GS (group separator)	61	3D	075	=	=	93	5D	135	]	]	125	7D	175	}	}
30	1E	036	RS (record separator)	62	3E	076	>	>	94	5E	136	^	^	126	7E	176	~	~
31	1F	037	US (unit separator)	63	3F	077	?	?	95	5F	137	_	_	127	7F	177	&#127;	DEL

Source: www.LookupTables.com

Extended ASCII Characters

- ASCII was designed in the 1960s for teleprinters and telegraphy, and some computing
- The number of printable characters was deliberately kept small, to keep teleprinters and line printers inexpensive
- When computers and peripherals standardized on eight-bit bytes, it became obvious that computers and software could handle text that uses 256-character sets at almost no additional cost in programming, and no additional cost for storage
- An eight-bit character set (using one byte per character) encodes 256 characters, so it can include ASCII plus 128 more characters
- The extra characters represent characters from foreign languages and special symbols for drawing pictures

A set of codes that extends the basic ASCII set. The extended ASCII character set uses 8 bits, which gives it an additional 128 characters

128	Ç	144	É	160	á	176	░	192	└	208	╨	224	α	240	≡
129	ü	145	æ	161	í	177	▒	193	┴	209	╤	225	ß	241	±
130	é	146	Æ	162	ó	178	▓	194	┬	210	╥	226	Γ	242	≥
131	â	147	ô	163	ú	179	│	195	├	211	╙	227	π	243	≤
132	ä	148	ö	164	ñ	180	┤	196	─	212	╘	228	Σ	244	⌠
133	à	149	ò	165	Ñ	181	╡	197	┼	213	╒	229	σ	245	⌡
134	å	150	û	166	ª	182	╢	198	╞	214	╓	230	µ	246	÷
135	ç	151	ù	167	º	183	╖	199	╟	215	╫	231	τ	247	≈
136	ê	152	ÿ	168	¿	184	╕	200	╚	216	╪	232	Φ	248	°
137	ë	153	Ö	169	⌐	185	╣	201	╔	217	┘	233	Θ	249	∙
138	è	154	Ü	170	¬	186	║	202	╩	218	┌	234	Ω	250	·
139	ï	155	¢	171	½	187	╗	203	╦	219	█	235	δ	251	√
140	î	156	£	172	¼	188	╝	204	╠	220	▄	236	∞	252	ⁿ
141	ì	157	¥	173	¡	189	╜	205	═	221	▌	237	φ	253	²
142	Ä	158	₧	174	«	190	╛	206	╬	222	▐	238	ε	254	■
143	Å	159	ƒ	175	»	191	┐	207	╧	223	▀	239	∩	255	

Source: www.LookupTables.com

UNICODE

- Many of today's systems embrace Unicode that can encode the characters of every language in the world
  - The Java programming language, and some operating systems now use Unicode as their default character code
    - UTF-8 (8-bits: essentially the extended ASCII Table)
    - UTF-16 (16 bits: Most spoken languages in the world, widely used)
    - UTF-32 (32 bits: includes past languages, space inefficient)

Any questions?