No electronic/communication devices are permitted.

Students may take exam question paper away after the exam.

Mathematics and Statistics

EXAMINATION

End-of-year Examinations, 2020

STAT317 / ECON323 -20S2 (C) Time Series Methods

STAT 456 / ECON614-20S2 (C) Time Series and Stochastic Processes

Examination Duration: 120 minutes

Exam Conditions:

Restricted Book exam: Approved materials only.

Calculators with a 'UC' sticker approved.

Materials Permitted in the Exam Venue:

Restricted Book exam materials.

Students may bring in one A4, double sided, handwritten page of notes.

Materials to be Supplied to Students:

1 x Standard 16-page UC answer book.

Instructions to Students:

Use black or blue ink only (not pencil).

Students in STAT456 and ECON614 have to work on ALL 6 questions.

Students in STAT317 and ECON323 have to CHOOSE 5 out of 6 questions.

Show ALL working.

If you use additional paper this must be tied within the exam booklet. Remember to write your name and student number on it.

Questions Start on Page 3

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Note

Students in **STAT456** and **ECON614** have to work on ALL 6 questions.

Students in **STAT317** and **ECON323** have to CHOOSE 5 out of 6 questions. Only 5 questions will be marked.

Each question is worth 20 marks.

Q.1 Time series decomposition [20 marks]

(a) Name and briefly discuss the three time series components that a time series is typically decomposed into.   [4 marks]

(b) Why are these components termed *unobserved components*?   [4 marks]

(c) You have the time series of monthly spending by New Zealanders at retail stores in the past twenty years. For each of the three unobserved components give an example of consumer spending behaviour or changes in the New Zealand retail environment that primarily affect that component's month-to-month change.
                                                                          [4 marks]

(d) The relationship between the measured value and its three components can be written either as an additive model or a multiplicative model. Write down examples of the 2 equations. Also sketch a plot of the two types of series.
                                                                          [4 marks]

(e) Can a periodogram be used in identifying the existence of any of the unobserved components?   [4 marks]

Q.2 Basic Concepts [20 marks]

(a) You have been asked by a client to develop a system to forecast a number of their time series. In less than one page outline the questions you might ask the client before you start developing your time series models. Explain why you asked each question.   [5 marks]

(b) Discuss briefly why you should plot a time series before analysing it. [4 marks]

(c) Describe the meaning of the following terms as they apply to time series   [4 marks]

    i. Stochastic process

    ii. Stationarity

    iii. Data generating process

(d) What would you expect the residuals or errors from your time series model to be like?   [3 marks]

(e) Sketch what the time series plot of the residuals would be like if you had a level shift (i.e. an abrupt change in the mean level) in the original time series if you fitted a time series model ignoring the level shift?   [4 marks]
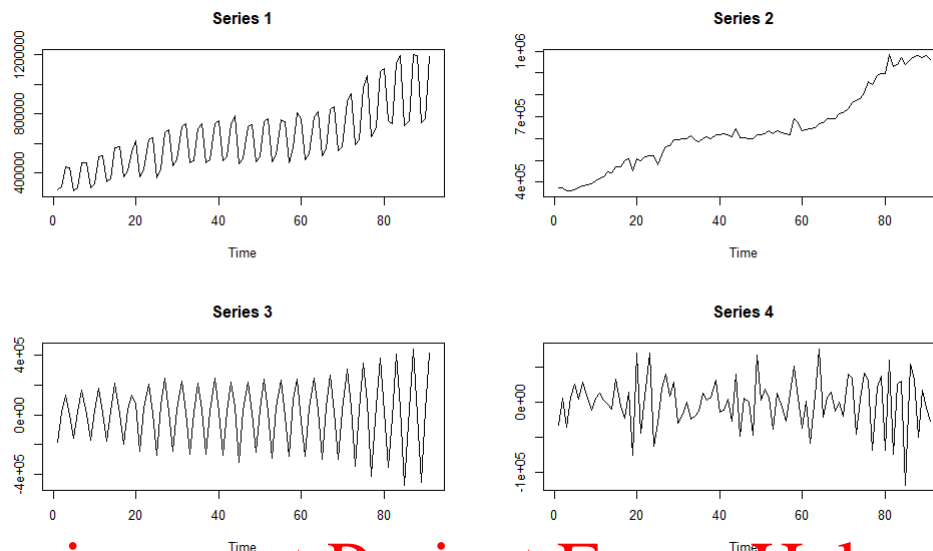
Q.3 Autocovariance and Autocorrelation [20 marks]

(a) For each of the four series   [5 marks]

choose which of the following four ACF graphs is from that series
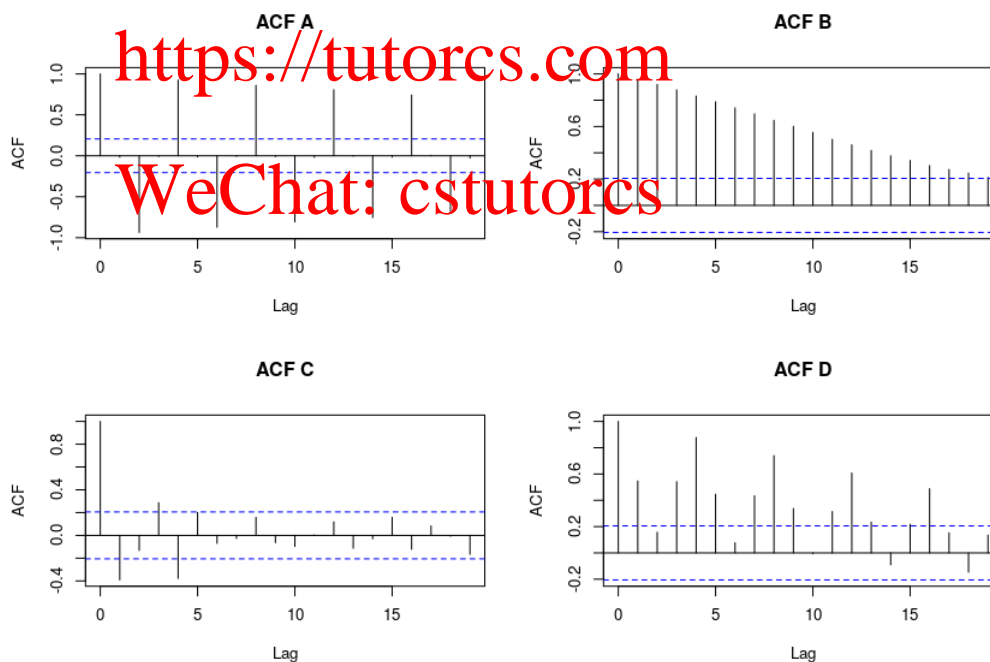
Clearly explain for **Series 2** why you chose that ACF graph.

(b) Describe what each of the terms in the exponential smoothing model represents.
             [3 marks]

$$\hat{y}_{t+1} = \alpha y_t + (1 - \alpha)\hat{y}_t$$

(c) What is the range of values that α can take?   [3 marks]

(d) In what respect is the model different when α is at its minimum value? And when α is at its maximum?   [3 marks]

(e) Assume we have observations $y_1, y_2, \ldots, y_n$ of a stationary time series. Write out the formula used to calculate the estimator for the autocovariance from the sample.

   If your formula involves other estimators, then also explain these. It must be clear in the end how the estimator of the autocovariance function is computed from the sample. [4 marks]

(f) Assume the value of the autocovariance function, $\gamma(h)$, is 0.07. What is the value of $\gamma(-h)$. [2 marks]

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Q.4 Periodogram [20 marks]

(a) A monthly time series X_t for $t = 1, \ldots, n$ is simulated from the following model:

$$X_t = 7\sin\left(\frac{2\pi t}{24} + \frac{\pi}{3}\right) + W_t$$

where $W_t \sim WN(0, \sigma_W^2)$. What are the following quantities:

i. frequency;   [2 marks]

ii. period of the cycle;   [2 marks]

iii. amplitude;   [2 marks]

iv. phase in radians; and   [2 marks]

v. phase in time units.   [3 marks]

(b) Consider the following periodogram. Identify the frequencies and the periods of possible cycles in the underlying time series.   [4 marks]

Series: x
Raw Periodogram

frequency
bandwidth = 0.00144
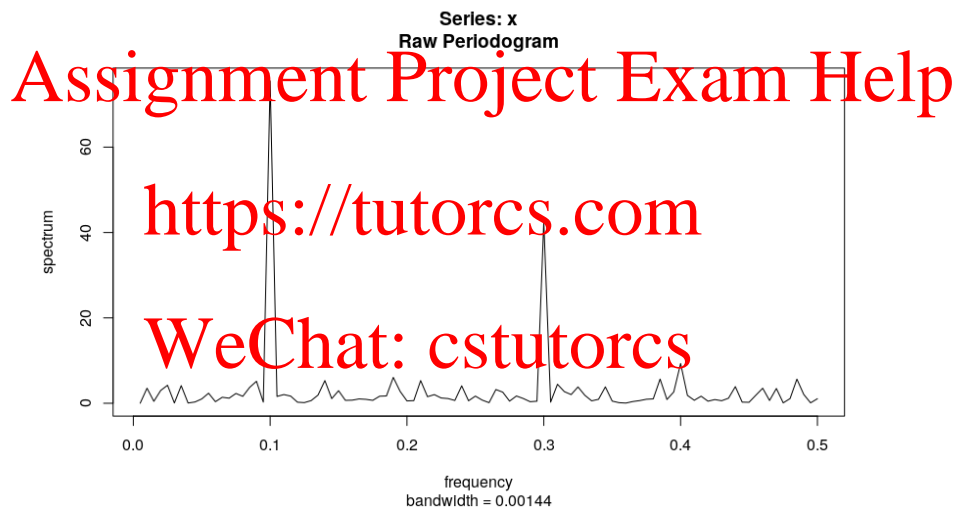
(c) The Nyquist frequency is the highest frequency represented in the periodogram.

i. What is the Nyquist frequency for equidistant observations $x_1, x_2, \ldots, x_n$?   [2 marks]

ii. How does a cosine wave behave at the Nyquist frequency?   [3 marks]

Q.5 Wold Decomposition [20 marks]

(a) Which kind of time series have a Wold decomposition?   [2 marks]

(b) Explain the Wold decomposition for a time series X_t. Provide the formula as part of your explanation.   [3 marks]

(c) Which condition holds for the coefficients of the Wold decomposition? [3 marks]

(d) Explain the relationship between the Wold decomposition and ARMA models.   [6 marks]

(e) Show how a mean zero AR(1) $X_t = \phi_1 X_{t-1} + W_t$ process can be seen as an infinite MA(∞) process. Which condition on ϕ_1 must hold?   [6 marks]
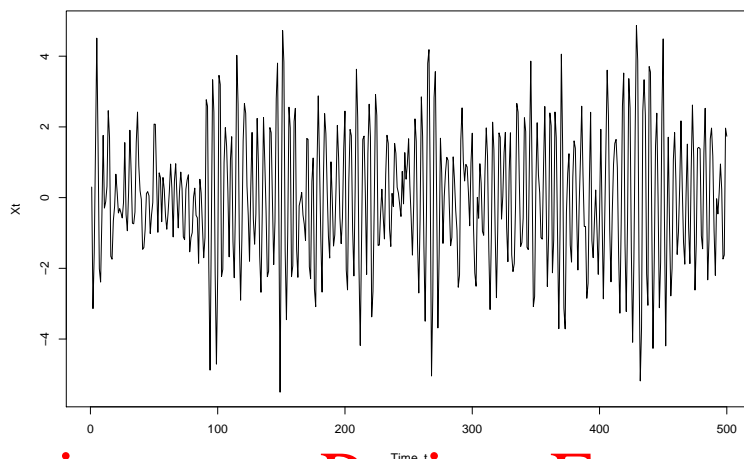
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Q.6 Model Selection [20 marks]

(a) Explain the key features of the dependence observed in the following plot of a times series. [5 marks]

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(b) Explain how the features mentioned in part (a) are shown in the sample auto-correlation and partial autocorrelation functions below. [4 marks]

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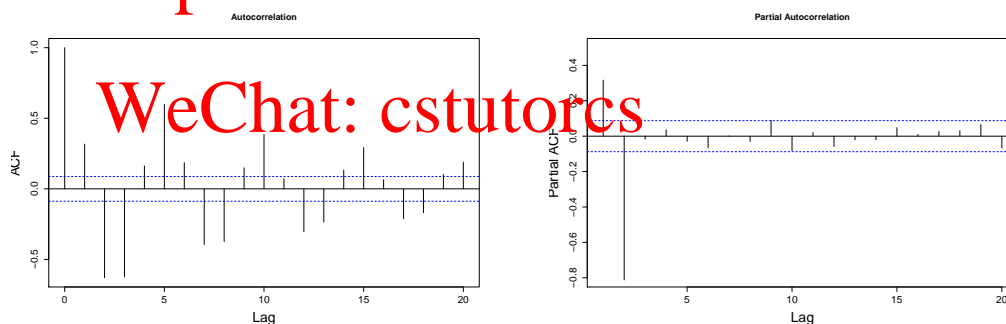
(c) Use these plots to identify whether a suitable model could be an AR(p), MA(q) or mixed ARMA(p,q). Explain your choice and suggest the order of the model. [4 marks]

(d) Write down the backshift (or characteristic) polynomials for the ARMA(p,q) model:

$$X_t - \phi_1 X_{t-1} - \cdots - \phi_p X_{t-p} = W_t + \theta_1 W_{t-1} + \cdots + \theta_q W_{t-q}$$

[2 marks]

(e) What are the conditions for invertibility and stationarity for an ARMA process? [3 marks]

(f) What condition is needed to avoid parameter redundancy for an ARMA process. [2 marks]