University of Canterbury

End-of-year Examinations 2015

Prescription Number(s): STAT317-15S2 / ECON323-15S2
STAT456-15S2 / ECON614-15S2

Paper Title(s): Time Series Methods
Time Series and Stochastic Processes

Time Allowed: 2 hours

Number of Pages: 4

Please read these instructions carefully:

- This is an open book exam
- Any calculator can be used
- Attempt all four questions
- Each of the 4 questions is worth the same amount

QUESTIONS START ON PAGE 3

Question 1

a) Name and briefly describe the three typical components that a time series is usually decomposed into.

b) For a time series of spending by New Zealanders at retail stores give one example for each of the three components of how the data generating process may contribute to that component.

c) The three components from question 1a can be either an additive or a multiplicative model. Write down examples of these two model variants. Include a sketch of a plot giving an example of both of these model variants.

d) Why are these three components termed "*unobserved components"?*

e) Can a periodogram be used in identifying the existence of any of the unobserved components?

Question 2

a) You are asked to develop a system to forecast a number of time series for a client. In less than 2 pages outline the questions you might ask the client before you start developing your time series models. Explain why you asked each question.

b) Discuss briefly why you should plot a time series before analysing it.

c) Describe the meaning of following terms as they apply to time series:
   i. stochastic process;
   ii. stationarity; and
   iii. data generating process;

d) Why is it important that a times series is stationary for ARMA modelling? Include a brief description of the consequences if an ARMA model is used for non-stationary time series.

e) What behaviour would you expect of the residuals/errors from your time series model?

f) Sketch an example of a time series plot of the residuals if you had a level shift (i.e. an abrupt change in the mean level) in the original time series, if you fitted a stationary time series model.

Question 3

a) Show that the weights (α) used in single exponential smoothing forecasting (i.e. no trend or seasonal) as applied to <u>all</u> the data in the actual time series sum to one.

You may find the following geometric sum useful:

$$\sum_{k=1}^{n} ar^{k-1} = \frac{a(1-r^n)}{(1-r)}$$

b) Show that a single exponential smoothed model is equivalent to an ARIMA(0,1,1) model.

c) Below is a table of the last ten values of a time series $\{r_t\}$, and the innovations $\{a_t\}$.

Time (t)	1	2	3	4	5	6	7	8	9	10
r_t	1	2	0	-2	1	0	-1	-2	4	0
a_t	0	-1	-1	0	0	2	-1	1	1	-1

Use these observed values to forecast the next two values of the series. If the series is

a. An MA(1) model with θ=0.6

b. An AR(1) model with φ= -0.3
c. An ARMA(1,1) model with θ=0.3 and φ= -0.4

Question 4

a) Given the moving average process

$$x_t = z_t - 0.6z_{t-1} - 0.4z_{t-2}$$
$$z_t \sim N(0, \sigma^2)$$

Find the values of the autocorrelation function $\rho(l)$ for lags l = 0, 1, 2 and 3.

b) Write out the full equation for r_t for a SARIMA(1,0,2)x(0,1,1)$_4$ model. As part of writing out the equation define each term you use in your model.

c) Show that an invertible MA(q) model for any integer value of q is equivalent to an AR of infinite order.

<u>**END OF PAPER**</u>