Family Name _____

First Name _____

Student Number |__|__|__|__|__|__|__|__|

Venue _____

Seat Number _____

UNIVERSITY OF CANTERBURY
Te Whare Wānanga o Waitaha
CHRISTCHURCH NEW ZEALAND

No electronic/communication devices are permitted.

Students may take exam question paper away after the exam.

Mathematics and Statistics

EXAMINATION

End-of-year Examinations, 2018

STAT317-18S2 (C) Time Series Methods

STAT456-18S2 (C) Time Series and Stochastic Processes

ECON323-18S2 (C) Time Series Methods

ECON614-18S2 (C) Time Series and Stochastic Processes

Examination Duration: 120 minutes

Exam Conditions:

Restricted Book exam: Approved materials only.

Any scientific/graphics/basic calculator is permitted.

Materials Permitted in the Exam Venue:

One A4 double sided, handwritten page of notes and formulas

Materials to be Supplied to Students:

1 x Write-on question paper/answer book

Instructions to Students:

This is a closed book examination

Use black or blue ink only (not pencil)

Attempt ALL 6 questions. Show ALL working

If you use additional paper this must be tied within the exam booklet and remember to write your name and student number on it.

For Examiner Use Only

Question	Mark
Q1	
Q2	
Q3	
Q4	
Q5	
Q6	

Total _____

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Q.1 Basic Concepts [16 marks]

(a) Give the definition of strong white noise. [2 marks]

(b) Name a statistical test that can give evidence for strong white noise. [1 mark]

(c) Explain what a cycle of length s in a time series X_t, $t = 1, 2, 3, \ldots, n$ is. [4 marks]

(d) Explain the growth rate of a time series X_t, $t = 1, 2, 3, \ldots, n$ and give a formula for how it is computed. [4 marks]

(e) Give the definition of the autocorrelation function of an arbitrary time series X_t, when X_t is not necessarily stationary. [1 marks]

(f) Define an estimator of the autocorrelation function given a sample $x_1, x_2, \ldots, x_n$ when X_t is stationary.

If your definition involves other estimators then also explain these. It must be clear in the end how the estimator of the autocorrelation function is computed from the sample. [4 marks]

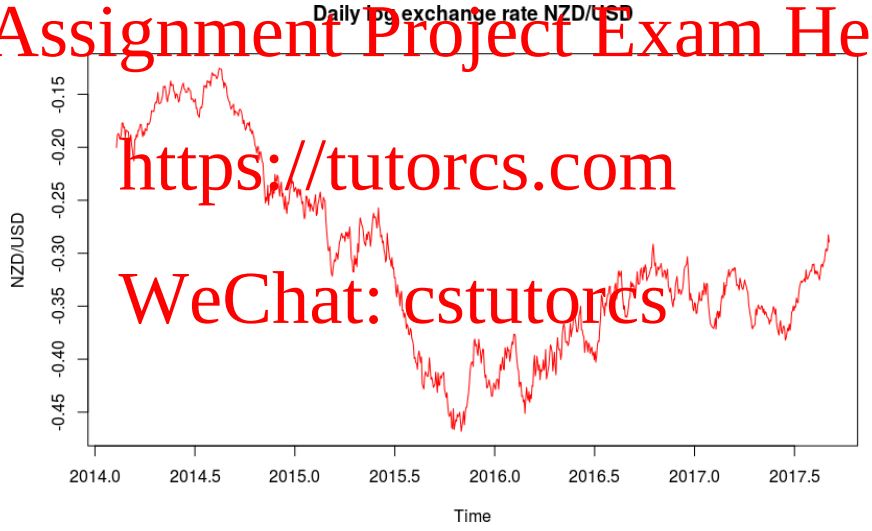
Q.2 Random Walk [16 marks]

(a) Give the definition for a random walk X_t, $t = 1, 2, 3, \ldots, n$ with drift δ, volatility σ, and initial value $X_0 = 0$. [4 marks]

(b) Derive from the definition of the random walk the expectation $E(X_t)$ and the variance $Var(X_t)$. [4 marks]

(c) Is a random walk stationary if $\delta = 0$? Give reasons for your answer. [4 marks]

(d) Give at least one arguments for and one argument against the suggestion to model the log-exchange rate of NZD and USD by a random walk. [4 marks]

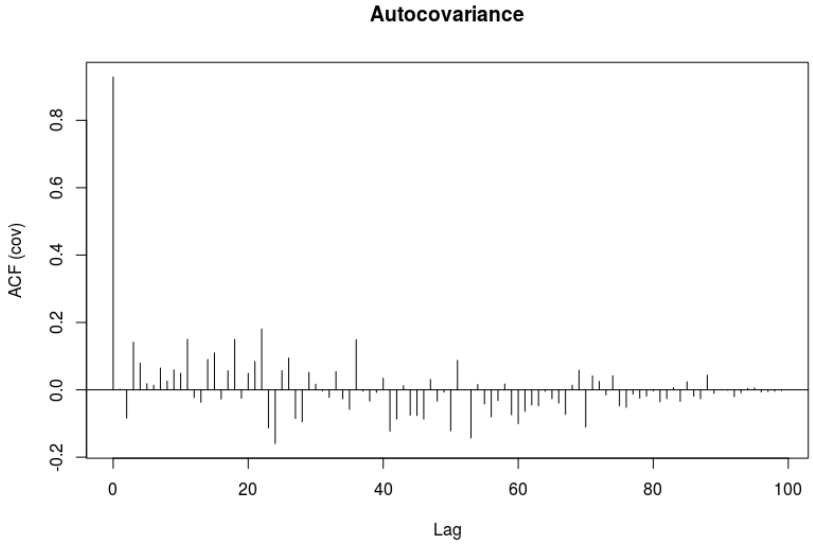


Q.3 Autocovariance [16 marks]

(a) Give the formula for the autocovariance function of a white noise process with variance σ. [4 marks]

(b) Explain in one or two sentences why the function takes these values. [4 marks]

(c) The following plot shows an estimated autocovariance function for a white noise process using the estimator for stationary time series.



Explain why the estimation errors become smaller at the right hand side of the plot. Use the formula of the estimator in your argument. [4 marks]

(d) What can be learned from the autocovariance function of a time series and how is that different from the autocorrelation function? [4 marks]

Q.4 ARMA Models [16 marks]

(a) Write down the backshift (or characteristic) polynomials for the ARMA(p, q) model:

$$X_t - \phi_1 X_{t-1} - \cdots - \phi_p X_{t-p} = \epsilon_t + \theta_1 \epsilon_{t-1} + \cdots + \theta_q \epsilon_{t-q}$$

[2 marks]

(b) What are the conditions for invertibility and causal stationarity for an ARMA process? [2 marks]

(c) What condition is needed to avoid parameter redundancy for an ARMA process.[1 mark]

(d) Identify the order of the following ARMA(p, q) models and check they satisfy these three conditions.

You may find it useful to know that the roots of the quadratic $az^2 + bz + c$ are given by $z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

i. $X_t = 1.1X_{t-1} - 0.3X_{t-2} + \epsilon_t + 0.4\epsilon_{t-1}$ [4 marks]

ii. $X_t = 0.9X_{t-1} - 0.2X_{t-2} + \epsilon_t - \epsilon_{t-1}$ [4 marks]

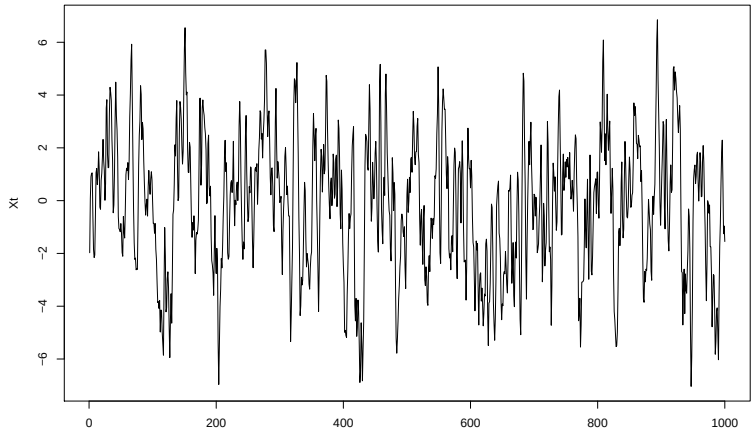
(e) The following ARMA(2,1) is overparameterised, write down a simplified version of this model:

$$X_t - 1.3X_{t-1} + 0.4X_{t-2} = \epsilon_t - 0.5\epsilon_{t-1}.$$

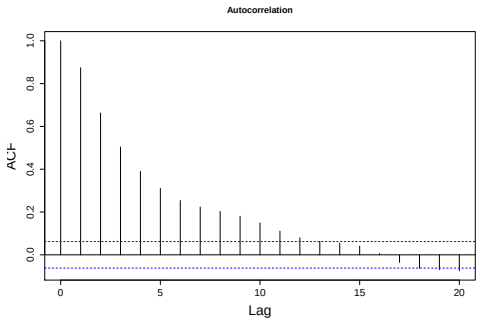
[3 marks]

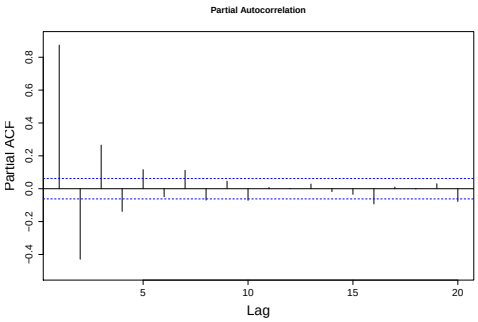
Q.5 Model Selection [16 marks]

(a) Explain the key features of the dependence observed in the following plot of a times series. [2 marks]



(b) Explain how the features mentioned in part (a) are shown in the sample autocorrelation and partial autocorrelation functions below. [2 marks]



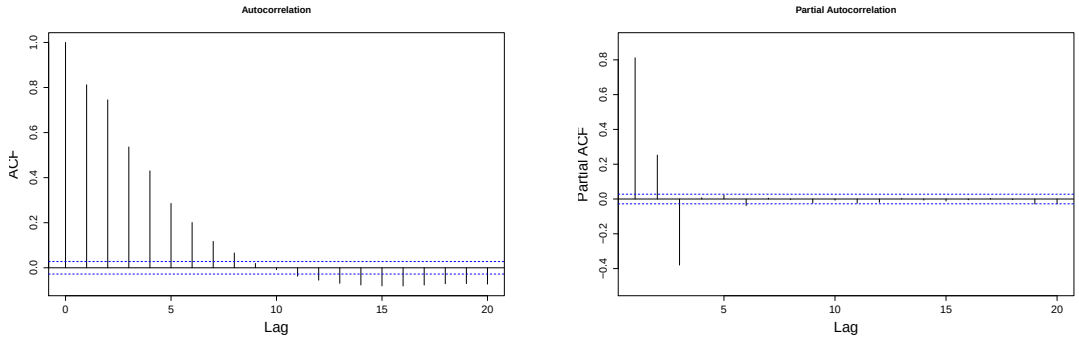


(c) Use these plots to identify whether a suitable model could be an AR(p), MA(q) or mixed ARMA(p, q). Explain your choice and suggest the order of the model. [3 marks]

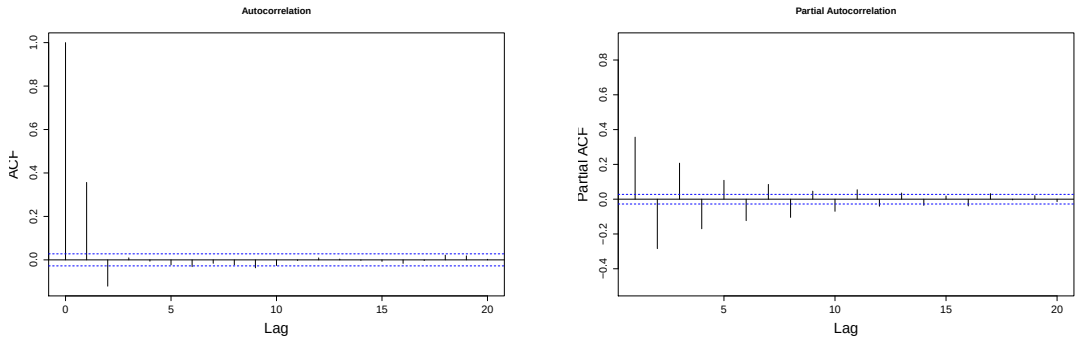
(d) Consider the following pairs of autocorrelation and partial autocorrelation plots of from **pure AR and MA models only**.

Explain the key features of the dependence shown in these plots, identify whether it is an AR or MA and it's order. Also identify whether the coefficients will be positive or negative at each lag.

i. [3 marks]



ii. [3 marks]



iii. Explain how a model fit statistic like the Akaike Information Criterion (AIC) could be used to determine the model order. [3 marks]

Q.6 AR(1) and MA(1) Models [16 marks]

(a) A zero mean AR(1) model can be written as

$$X_t = \phi X_{t-1} + \epsilon_t.$$

i. Under the assumption of stationarity, derive the variance of X_t. [2 marks]

ii. Under the assumption of stationarity, derive the autocovariance function of X_t. [3 marks]

iii. Calculate the autocovariance for an AR(1) with $\phi = 0.5$ for lags 0, 1 and 2. [3 marks]

iv. State a condition on the values of ϕ that will make the AR(1) a causal stationary model. [2 marks]

(b) An MA(1) model can be written as

$$X_t = \mu + \theta\epsilon_{t-1} + \epsilon_t.$$

i. Derive the mean and variance of X_t. [2 marks]

ii. Derive the autocovariance of X_t. [2 marks]

iii. What happens to an MA(1) model if $|\theta| > 1$? Explain why this is a problem. [2 marks]

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