AMSC 460 Homework Set 2 **Due: 21.04.01**

Note:

- Due: 2pm, April 01, 2021. Late submission is subject to automatic 20% reduction.
- When submitting, make sure you upload your matlab or python codes, in addition to your answer sheets to analytical questions.

Topic: Newton Interpolation with Divided Difference

Consider a polynomial interpolation problem $p_n(x)$ of an (unknown) function $f(x)$,

$$p_n(x) = \sum_{j=0}^{n} c_j \varphi_j(x) \qquad (1)$$

where c_j and $\varphi_j(x)$ are the j-th interpolating constant and function, respectively. Using $(n+1)$ data sample $\{(x_j, y_j=f(x_j))\}$ for $j=0,\ldots,n$, Newton interpolation using divided difference can be written as

$$c_j = f[x_0, \ldots, x_j]$$
$$\varphi_j(x) = \prod_{i=0}^{j-1}(x - x_i) \qquad (2)$$

where

$$f[x_i] = y_i$$
$$f[x_0, \ldots, x_j] = \frac{f[x_1, \ldots, x_j] - f[x_0, \ldots, x_{j-1}]}{x_j - x_0}$$

Main Problems

1. Show that the formula (2) is consistent with the data sample $\{(x_j, y_j)\}$

2. Write a pseudocode, as computationally efficient as possible (i.e., minimize number of operations)

3. Write a matlab or python code based on the pseudocode

4. For a data sample {(1,3),(5,11),(2,2),(4,12)},
   a. Obtain analytical solution, i.e., obtain c_j and $\varphi_j(x)$ by hand
   b. Show that the analytical solution is consistent with the data sample
   c. Predict $f(x^*)$ at $x^*=3$, i.e., compute it by hand

5. Using the same data sample
   a. Obtain c_j and $\varphi_j(x)$ by your matlab or python code, verify against your analytical solution obtained in Question 4 above
   b. Verify that the code reproduces the sample data
   c. Verify that the code reproduces $f(x^*=3)$ obtained in Question 4 above

Bonus points

B. Derive the formula (2) analytically.