MAT101

Obligatorisk øving 2

Gruppe:
145157 Pål V. Gjerde
145161 Henriette Lie
142054 Marlene Sætevik

Q1

a) No. 13 mod 6 = 1, while 45 mod 6 = 3.
b) Yes. -2 mod 6 = 4 and 28 mod 6 = 4.
c) Yes. 123 mod 6 = 3 and 21 mod 3 = 3.
d) No. -128 mod 6 = 4 and -21 mod 6 = 3.

Q2

An equivalence relation is one that is reflexive, symmetric and transitive.
Reflexive: A number is necessarily congruent with itself mod r (a = a mod r).
Symmetric: Two numbers that have the same remainder r when divided by the same number are congruent mod r. This is true regardless of which number is first.
Transitive: If the remainder (r) is the same between three numbers, they are all congruent mod r, which means the relation has elements connecting all of them. This is a transitive relation.
Since the relation is reflexive, symmetric and transitive, it is an equivalence relation.

Q3

r is 4 and k is 3.

$$a^i = 4 * 3^i$$

Q4

a)

$$\sum_{i=0}^{4} 2i - 2 = (0-2) + (2-2) + (4-2) + (6-2) + (8-2) = -2+0+2+4+6 = 10$$

b)

$$\sum_{n=1}^{3} n^3 = 1 + 8 + 27 = 36$$

c)

$$\sum_{n=1}^{10} n = 1+2+3+4+5+6+7+8+9+10 = 55$$

Q5

$$\sum_{i=m+1}^{n} rk^i = \sum_{i=0}^{n} rk^i - \sum_{i=0}^{m} rk^i$$

Since r and k have not been given, I will choose r=5 and k=3.

$$\sum_{i=3}^{4} 5*3^i = 5*27 + 5*64 = 135 + 320 = 455$$

$$\sum_{i=0}^{4} 5*3^i - \sum_{i=0}^{2} 5*3^i = (5*1+5*3+5*9+5*27+5*64) – (5*1+5*3+5*9) = 520–65 = 455$$

Q6

a)
Original series:
1 + 1/2 + 1/3 + 1/4 + 1/5 + 1/6 + 1/7 + 1/8 +
1/9 + 1/10 + 1/11 + 1/12 + 1/13 + 1/14 + 1/15 + 1/16 ≈ 3.3

Rounded down:
1 + 1/2 + 1/4 + 1/4 + 1/8 + 1/8 + 1/8 + 1/8 +
1/16 + 1/16 + 1/16 + 1/16 + 1/16 + 1/16 + 1/16 + 1/16 = 3

This gives a lower bound of 3.

b)
$m(n) = ceil(\log_2 n)$

c)
$2^m/2$ terms

d)
For the lower bound series, you can always add 1/2 by doubling the number of terms. This can therefore be simplified as:
1 + 1/2 + 1/2 + ...

Specifically, the lower bounds for the finite sum $1+1/2+1/3+...+1/2^m$, for the first few m are:
0: 1 (First value only)
1: 1,5 (First two values, 1+1/2)
2: 2 (First four values, lower bound 1+1/2+1/2 = 2)
3: 2,5 (First eight values, lower bound 1+1/2+1/2+1/2)
4: 3 (First sixteen values, lower bound 1+1/2+1/2+1/2+1/2)
From this it can be seen that the lower bound for the sum up to $1/2^m$ is $f(m)=(m+2)/2$.

e) The lower bound series, therefore, has no limit. The original series' terms are always greater than or equal to the lower bound's terms, so the sum of that will be greater than the sum of the lower bound. Given that fact and the fact that the lower bound series will diverge to infinity, the original series will also diverge to infinity.