Derivatives

1. Introduction

1) "transform" the performance of underlying
- ① risk allocation. transfer. management.
- ② Information → price. volatility
- ④ Operational Advantages: 低成本、高流动性、易short
- ⑤ better market efficiency
- ⑥ 与 financial traders 相互联系

2) OTC vs. Exchange-trade
- OTC ↓ Default/counterparty risk
- Exchange-trade ↓ no default risk.
- "most" transactions to be reported to the regulators. (2008年之后)

3) Commitment vs. Contingent.
- (双方均可违约) Commitment ↓ Forward/Future/swap
- (仅对持可违约) Contingent ↓ options/CDS
- [total return swap/credit-linked not (CLN)]

【交割方式】

Settle { Delivery / Exchange of cash — economic affects 相同

↓ 期权远期期货都有两种方式交割 但期货通常CASH settle

Non-deliverable Forwards (NDFs)
= cash-settled forwards
= Exchange of cash

【优势与风险】

优: · adjust risk exposure to a desired level.

劣:
- · financial Contagion (蔓延)
- · speculators on leverage
- · complex nature → more mathematicians/physicists

【应用】

1) Issuer:
- ① cash-flow hedging → swaps
- ② Fair-value hedging → Future/option
- ③ net invest hedging → currency swap

2) Investor { Speculate / Arbitrage. / Hedge.

[Forward & Future]

1) 特点
- ① Forward: agree-upon prices
- ② Future.
  - • agree-upon price + price limit (涨跌板) { limit up / limit down } → 交易所控制 credit risk
  - • mark-to-market: daily settlement — 以当日收盘平均价 (每日收益)
  - interest-rate 与 Future value 正corr, $P_{future} > P_{forward}$
  - • Margin
    - Initial → at "most" 10% of future price
    - Maintenance: margin call, 加回 Initial margin
- ③ Forward Rate Agreement (FRA) "押注"未来的利率涨跌
  - a×b FRA — 0 — 30天 — 120天
  - ↳ 2×5 FRA
  - Real FRA
  - Synthetic FRA: Long 120days Eurodollar + short 30days Eurodollar

2) Pricing → $F_0(T)$ 本质是无风险套利

price在0至T之间不会改变

price就是一开始合同内约定的远期结算价

① Forward:

$$F_0(T) = S_0 \times (1+r_f)^T - FV(I) + FV(C)$$

(I: convenience yield; C: 成本 → 机会成本 / 其他持有成本)

⇕

$$F_0(T) = S_0 \times e^{(r_f + c - i) \times T}$$

② Exchange-forward

$$F_{A/B} = S_{A/B} \times \frac{(1+r_A)^T}{(1+r_B)^T}$$

⇕

$$F_{A/B} = S_{A/B} \times e^{(r_A - r_B) \times T}$$

③ Forward rate agreement (FRA)

Implied Forward Rate: $IFR_{A/B-A}$:

$$(1+Z_A)^A \times (1+IFR_{A,B-A})^{B-A} = (1+Z_B)^B$$

④ Future.
- r_f 与 F_{future} 正相关 ⇒ $F_0(t)_{future} > F_0(t)_{forward}$
- r_f 与 F_{future} 负相关 ⇒ $F_0(t)_{future} < F_0(t)_{forward}$

3) Valuation · 合约对双方的执行价值.

$V_L + V_S = 0$ 在整个0到T内不断动态变动

① Forward: S_t 产生I、C. S_T

0 ——— t ——— T

$V_L = V_S = 0$

$\begin{cases} V_{L,t} = S_t - PV_t(I) + PV_t(C) - F_0(T)/(1+r_f)^{T-t} \\ V_{S,t} = -V_{L,t} \end{cases}$

$\begin{cases} V_L = S_T - F_0(T) \\ V_S = -V_{L,t} \end{cases}$

② Exchange-currency forward

0 ——— t ——— T

$V_L = V_S = 0$

$\begin{cases} V_{L,t} = S_t - F_0(T)/e^{(r_{fd}-r_{ff})(T-t)} \\ V_{S,t} = -V_{L,t} \end{cases}$

$\begin{cases} V_L = S_T - F_0(T) \\ V_S = -V_L \end{cases}$

③ FRA

$S_A(B-A)$

0 ——— A ——— B

★ $\text{Cash Settle} = NP \times \dfrac{[S_A(B-A) - IFR_{A,B-A}] \times \frac{T}{360}}{1 + S_A(B-A) \times \frac{T}{360}}$

[Swaps] is a series of (off-market) forwards

1° customized/private agreements

2° OTC → largely unregulated.

3° "off market": 假想的 & 市场上并不存在的Forward

对每一个Forward created with a contract Price that gives it an nonzero value at initiation

但对整体swap: sum of the PVs = 0.

4° Default/counterparty risk.

5° difficult to alter/terminate.

6° Pricing: Fixed Price in swap contract

7° Value: $\begin{cases} \text{initial: } V_{fix} = V_{float} = 0 \\ \text{Expected future Price}\uparrow \rightarrow V_{fix} > V_{float} \\ \text{Expected future Price}\downarrow \rightarrow V_{float} > V_{fix} \end{cases}$

★常见swaps★

1) interest rate swaps
- nominal amount/principal 一直不变
  - ↳用于算两个利息的差异
- interest payments are netted.
- Floating paid in arrears

2) equity swaps → 固定支出 + Index回报

3) Currency swaps
两币种 + 两个本金 + 两个利息金额

[Options]

- in-the-money
- at-the-money
- out-of-the-money

1) Pay-offs.

2) Value

Total Value = Exercise Value + Time Value.

Call: Exercise Value $= \max(S_t - X/(1+r_f)^{T-t}, 0)$

put, Exercise Value $= \max(X/(1+r_f)^{T-t} - S_t, 0)$

3) Value Limits

Call: $\max(S_t - X/(1+r_f)^{T-t}, 0) \le C_t \le S_t$

put: $X \ge p_t \ge \max(X/(1+r_f)^{T-t} - S_t, 0)$

4) 影响因素

$C = e^{-\delta \times T} \times S \times N(d_1) - e^{-r \times T} \times K \times N(d_2)$

$\begin{cases} d_1 = \dfrac{\ln(S/K)}{\sigma\sqrt{T}} + \dfrac{(r - \delta + \frac{\sigma^2}{2}) \times T}{\sigma\sqrt{T}} \\ d_2 = d_1 - \sigma\sqrt{T} \end{cases}$

1° T: 正相关

2° S: 正相关 (对put相反)

3° K: 负相关 (对put正相关)

4° r_f: 正相关 (对put负相关)

6° σ. 正相关

7° δ 负相关 (对put正相关)

欧式看跌: deep-in-the-money. 预计价格上涨 or r_f 太高
↳ T与 P_t 负相关的特例

5) Put-Call Parity

等价 ① Fiduciary Call: $C_0 + \dfrac{K}{(1+r_f)^T}$

② protective put: $P_0 + S_0$

"p + s = c + k"

③ Forward-put-call parity

$\dfrac{F_0(T)}{(1+r_f)^t} + P_0 = C_0 + \dfrac{K}{(1+r_f)^t}$

6) Binomial Model

$S_0 \rightarrow S_0 \times u \rightarrow S_0 \times u^2$

$S_0 \times u \rightarrow S_0 \times u \times d$

$S_0 \rightarrow S_0 \times d \rightarrow S_0 \times u \times d$

$S_0 \times d \rightarrow S_0 \times d^2$

risk-neutral probability $\begin{cases} p = \dfrac{(1+r_f)^t - d}{u - d} \\ 1 - p = \dfrac{u - (1+r_f)^t}{u - d} \end{cases}$

7) Hedge

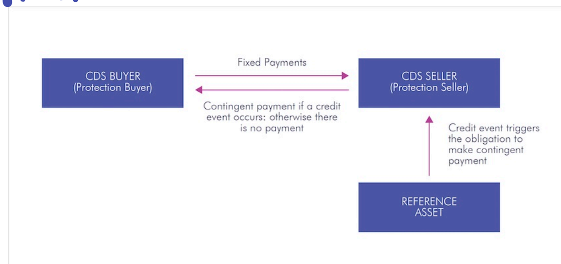
① Hedge ratio: $h_c = \dfrac{C_1^u - C_1^d}{S_0 \times u - S_0 \times d}$ ($h_c > 0$)

$h_p = \dfrac{P_1^u - P_1^d}{S_0 \times u - S_0 \times d}$ ($h_p < 0$)

② Hedge portfolio

【其他期权】

1) CDS



2) Equity swap (option) → 欧式看涨

以D为X，以V_E为S_T.

$$E_t = \max(V_{E_t} - D, 0)$$