Assignment 1: Sines, impulse response, and continuous-time systems

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November 2024

1 Task 1

1.1 a

The relationship between the period T and the natural frequency f can be explained by Equation 1.

$$T = \frac{1}{f} \quad [s] \tag{1}$$

The angular frequency ω is determined by Equation 2.

$$\omega = 2 * \pi * f \quad [rad/s] \tag{2}$$

The results for both signals are summarized in Table 1.

Signal	Period (s)	Natural Frequency (Hz)	Angular Frequency (rad/s)	Function
x_1	0.01	100	200π	$\sin(200\pi * t)$
x_2	0.001	1000	2000π	$\sin(2000\pi * t)$

Table 1: Results of frequencies, periods and the resulting signal functions

1.2 b

The signals $x1$ and $x2$, plotted in Figure 1, are two sine signals with different periods, natural and angular frequencies.

The figure illustrates the direct relationship between the (natural and angular) frequency as well as the period. Signal $x1$, with a lower frequency (100 Hz), has a longer period. The pattern of the sine curve repeats after a longer time. Signal $x2$, with a higher frequency (1000 Hz), has a shorter period, resulting in a denser waveform with more oscillations over the same time interval.

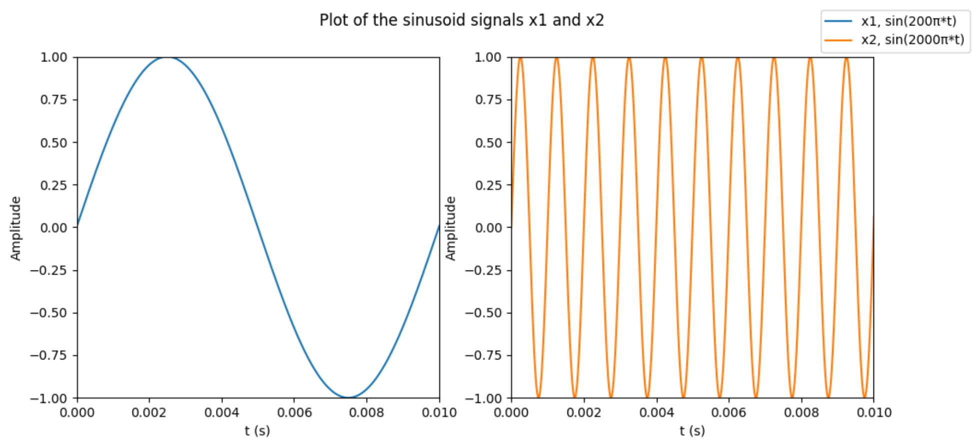

Figure 1: x1 and x2 plotted over 0.01s

1.3 c

The signal $x1$ (100Hz) resembles a low electric hum, for example the sound created by appliances such as refrigerators or florescent lights/tubes.

Signal $x2$ (1000Hz) resembles a high pitch tone or beep. It's similar to sounds of morse code responses or a dial tone.

2 Task 2

The impulse response function of the LTI system can be seen in Equation 3 and the corresponding graph is shown in Figure 2.

$$h(t) = \alpha^2 t e^{-\alpha t} u(t) \tag{3}$$

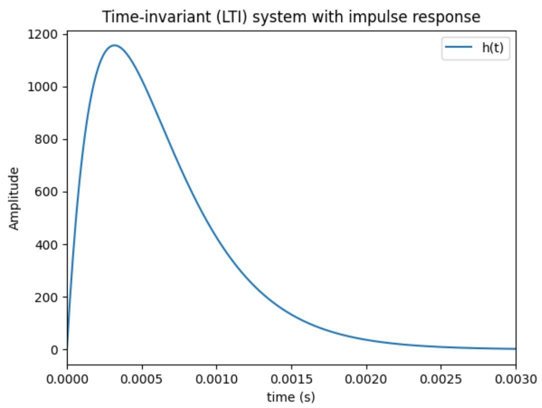

Figure 2: Time-invariant (LTI) system with impulse response

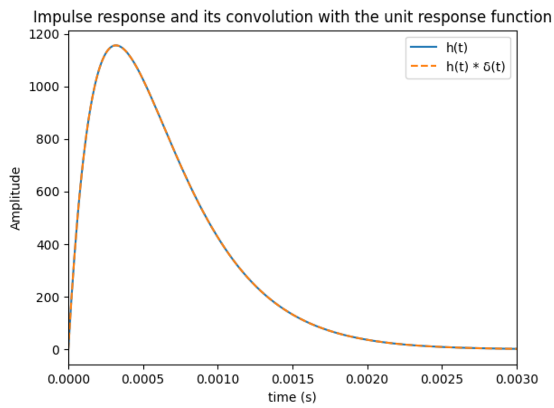

Figure 3: Impulse response and convolution with the unit response function

When convoluting a function, say $x(t)$, with the Dirac delta function, $\delta(t)$, the result will equal the original function $x(t)$. This is because the Dirac delta function is the identity for convolution. This can also be seen in Figure 3, where the graphs for the impulse response of the LTI system, $h(t)$, is the same as its convolution with $\delta(t)$.

3 Task 3

The first input, $x1$, applied to the system has its output affected as a decrease of the amplitude. The output signal, $y1$, does not quite reach amplitude 1, as the input signal does, which is seen in Figure 4.

The second output signal, $y2$, also has a lower amplitude than the corresponding input signal $x2$, just like the output of $x1$. The major difference between the two signals being a larger decrease in amplitude, when the frequency increases.

The natural and angular frequency, f and ω, of the system's input and output signals is not subject to any change through the convolution. This occurs because the system is a time invariant (LTI) system, and such systems don't affect the frequency of sinusoidal signals.

The results indicate that the system decreases the amplitude of an input signal proportionally to the frequency of said signal. It resembles the characteristics of a low pass filter. Meaning lower frequency signals are passed through without any major change, while higher frequency signals will be repressed.

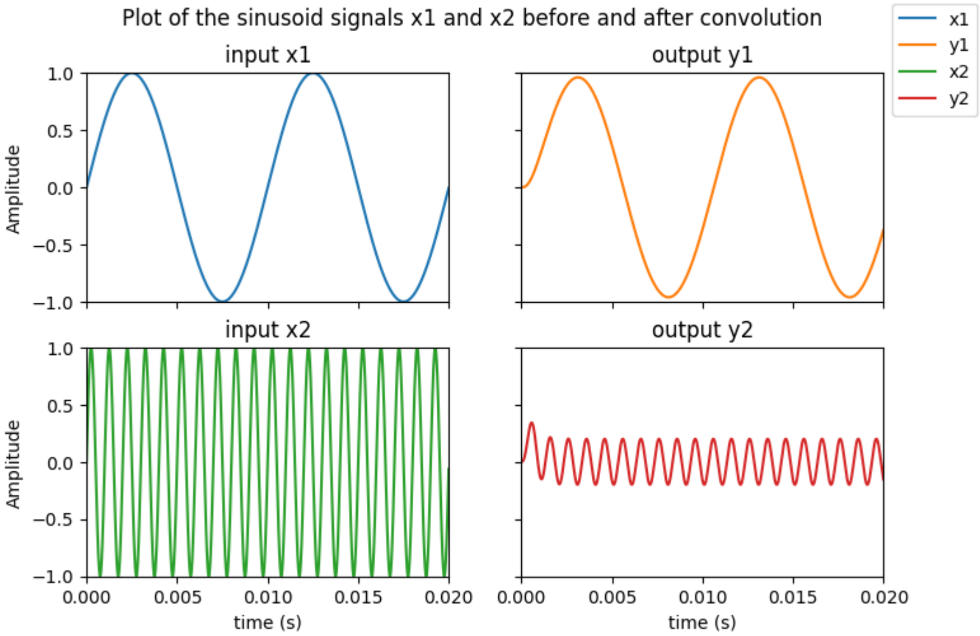

Figure 4: Signals x1 and x2, as well as the system output given x1 and x2 as input

4 Task 4

Given the sum of two signals as input, the output of a linear system will be the same as the sum of the two output signals, given the input signals individually. This happens because of the superposition principle, which holds for every linear system and can be seen in Equation (4).

$$\begin{aligned} &\text{If} \quad \mathcal{H}\{x_1(t)\} = y_1(t) \quad \text{and} \quad \mathcal{H}\{x_2(t)\} = y_2(t), \\ &\text{then} \quad \mathcal{H}\{ax_1(t) + bx_2(t)\} = ay_1(t) + by_2(t). \end{aligned} \tag{4}$$

By analyzing the graphs from this task, seen in Figure 5, we can see that the two plots are equal, meaning that the superposition principle holds in this case. This would suggest that the system is linear, but we cannot actually be sure, since it's possible for the superposition principle to hold for certain inputs even in non-linear systems. We therefore need to check that the principle holds for all inputs to confirm that the system indeed is linear.

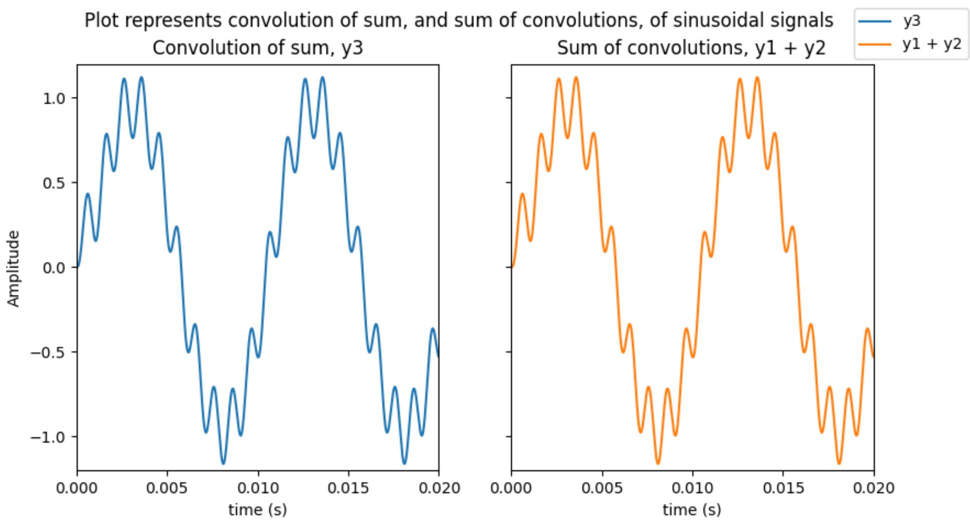

Figure 5: System output, y3, given the sum of the signals x1 and x2 as input, and the sum of the outputs y1 and y2