Proceedings of Machine Learning Research 266:1–3, 2025

Physics-Aware Conformal Prediction for Deep Learning-based Wheelchair Local Navigation

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Editor: Khuong An Nguyen, Zhiyuan Luo, Harris Papadopoulos, Tuwe Löfström, Lars Carlsson and Henrik Boström

Abstract

When dealing with conformal prediction for real-world artificial intelligence applications, it is necessary to ensure its physical feasibility. In this work, we propose to tackle this problem for an autonomous wheelchair guided by a deep neural network for local navigation. We adapt the conformal sets to be compliant with the wheelchair's kinematics, enhancing their efficiency while preserving coverage guarantees.

Keywords: Conformal Prediction, Cyber-Physical Systems, Social Navigation

1. Introduction

Ambient Assisted Living (AAL) benefits from Artificial Intelligence (AI)-driven systems, thanks to the advent of mobile robots equipped with advanced sensors (Xiao et al., 2022). Ensuring AI safety, e.g., via conformal prediction (CP) (Shafer and Vovk, 2007), is critical to avoid harmful consequences. However, integrating physical constraints into CP remains a key, underexplored, challenge. This work focuses on a *Deep Neural Network-based Local Navigation Approach* (DNN-LNA) for the local path planning of an autonomous wheelchair, and constitutes a first attempt of providing *physics-aware* CP sets that meet the wheelchair's kinematics.

2. Physics-aware conformalization of the DNN-LNA in REXASI-PRO

The Horizon Europe project REXASI-PRO[1] aims to release a novel framework, based on Trustworthy AI principles, for social navigation of smart wheelchairs (Mandel et al., 2018).

1. REliable & eXplainable Swarm Intelligence for People with Reduced mObility https://rexasi-pro.spindoxlabs.com/

DNN-LNA model. The local planner is a neural network, DNN-LNA, defined as:

$$\hat{f} : (\mathbb{R}^{p\times p}, \mathbb{R}^d, \mathbb{R}^{d_g}) \to \mathbb{R}^{5\times n_s}, \quad (m, \mathcal{P}_0, \mathcal{P}_g) \mapsto \hat{\mathbf{P}} \doteq \left[\hat{X}_{1:n_s}, \hat{Y}_{1:n_s}, \hat{\theta}_{1:n_s}, \hat{v}_{1:n_s}, \hat{\omega}_{1:n_s}\right]^T \tag{1}$$

where m is a $p\times p$ *occupancy map*; $\mathcal{P}_0 = (\theta_0, v_0, \omega_0)$ is the *initial odometry vector*, comprising the yaw θ_0, translational velocity v_0, and rotational velocity ω_0; $\mathcal{P}_g = (X_g, Y_g, \theta_g, v_g, \omega_g)$ is the *goal odometry vector*. $\hat{\mathbf{P}}$ is the matrix of predicted states over n_s time steps, where the j-th row $\hat{\mathbf{P}}_{j,:}$ refers to a component (among X, Y, θ, v, ω) and each column to a time step. We also refer to the *flattened* version of $\hat{\mathbf{P}}$ as $\hat{\boldsymbol{p}}$. Likewise, we denote the ground-truth trajectory matrix and its flattened form by $\mathbf{P}$ and $\boldsymbol{p}$, respectively. For conciseness, we identify $\boldsymbol{x} \doteq (m, \mathcal{P}_0, \mathcal{P}_g)$ and $\hat{f}(\boldsymbol{x}) = \hat{\boldsymbol{p}}$.

Quantile Adjustment. We design a score function measuring the nonconformity of each trajectory point (i.e., each entry of $\boldsymbol{p}$):

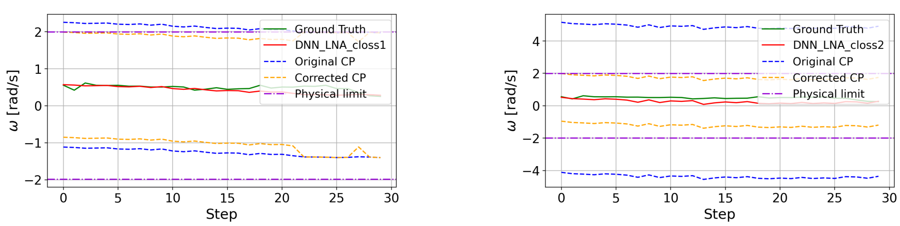
$$s(\boldsymbol{x}, \boldsymbol{p}) = \frac{|\hat{f}(\boldsymbol{x}) - \boldsymbol{p}|}{\mathrm{MSE}(\boldsymbol{x}, \boldsymbol{p})}, \tag{2}$$

with $\mathrm{MSE}(\boldsymbol{x}, \boldsymbol{p})$ being the global mean squared error. We aim at designing *corrected conformal sets* $\tilde{\mathcal{C}}(\boldsymbol{x})$ that meet the physical constraints of the wheelchair (Eq. 4). To this end, we define a physical quantile $\tilde{Q}_{1-\alpha}(\boldsymbol{s}) \doteq Q_{1-\alpha}(\boldsymbol{s}) - \delta$, where $\delta \in \mathbb{R}$ is the minimal correction of the empirical $(1-\alpha)$-quantile needed to bring the conformal set into feasible ranges. It is found by solving:

$$\begin{cases} \hat{\mathbf{P}}_{j,:} + \mathrm{MSE}(\boldsymbol{x}, \boldsymbol{p})\tilde{Q}_{1-\alpha}(\boldsymbol{s}) \leq \max \mathbf{P}_{j,:} \\ \hat{\mathbf{P}}_{j,:} - \mathrm{MSE}(\boldsymbol{x}, \boldsymbol{p})\tilde{Q}_{1-\alpha}(\boldsymbol{s}) \geq \min \mathbf{P}_{j,:} \end{cases} \qquad j = 1, ..., 5 \tag{3}$$

$$\tilde{\mathcal{C}}(\boldsymbol{x}) = [\hat{f}(\boldsymbol{x}) - \mathrm{MSE}(\boldsymbol{x}, \boldsymbol{p})\tilde{Q}_{1-\alpha}(\boldsymbol{s}),\ \hat{f}(\boldsymbol{x}) + \mathrm{MSE}(\boldsymbol{x}, \boldsymbol{p})\tilde{Q}_{1-\alpha}(\boldsymbol{s})] \tag{4}$$

Preliminary Results. We decided to focus on $\omega \in [-1.99, +1.99]$ rad/s (5-th row of $\hat{\mathbf{P}}$) only, being a crucial factor to ensure a smooth and stable trajectory. The CP set correction was effective across different DNN-LNA versions (Fig. 1).



(a) DNN_LNA_closs1 (b) DNN_LNA_closs2

Figure 1: DNN-LNA prediction for ω, with original (blue dashed lines) and corrected (orange dashed lines) conformal sets satisfying the physical bounds (purple lines).

Acknowledgments

This work was partially supported by REXASI-PRO H-EU project, call HORIZON-CL4-2021-HUMAN-01-01, Grant agreement ID: 101070028. The work was also supported by Future Artificial Intelligence Research (FAIR) project, Italian Recovery and Resilience Plan (PNRR), Spoke 3 - Resilient AI.

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