

Distillation, quantization, and pruning

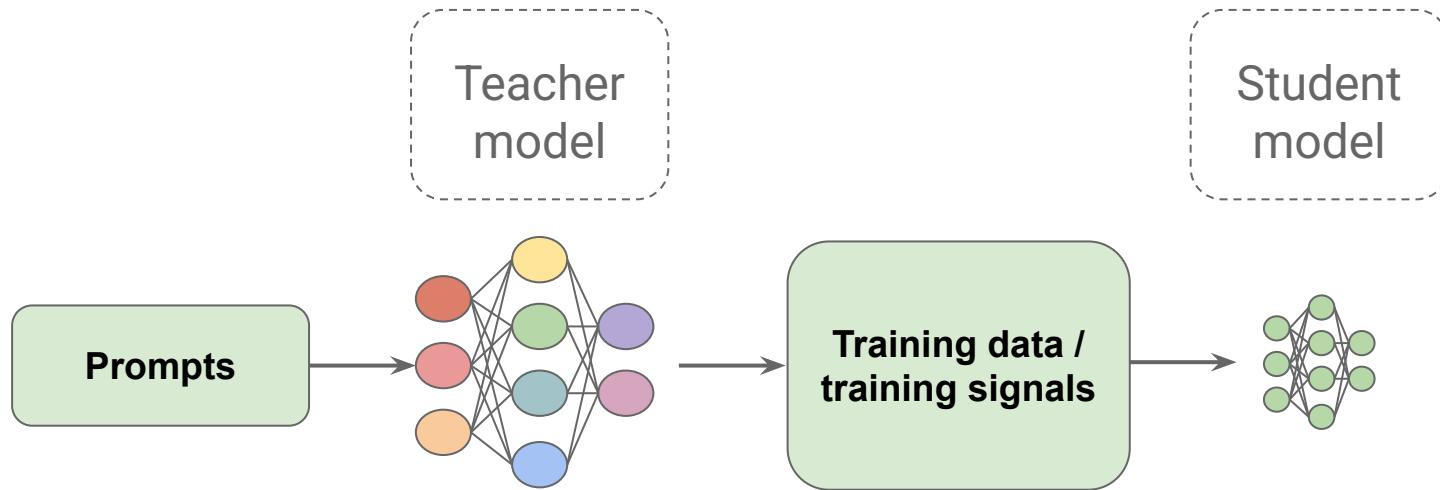
CS 5624: Natural Language Processing
Spring 2025

<https://tuvllms.github.io/nlp-spring-2025>

Tu Vu



Knowledge distillation



Pros and cons of knowledge distillation

Distilling the Knowledge in a Neural Network

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DistilBERT, a distilled version of BERT: smaller, faster, cheaper and lighter

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$$\text{Loss} = \lambda_{\text{ce}} \cdot \mathcal{L}_{\text{ce}} + \lambda_{\text{kd}} \cdot \mathcal{L}_{\text{kd}}$$

$$\text{Loss} = \lambda_{\text{ce}} \cdot \left(- \sum_{i=1}^N y_i \log(p_i) \right) + \lambda_{\text{kd}} \cdot D_{\text{KL}}(q_{\text{teacher}}(x) \| q_{\text{student}}(x))$$

Where:

- y_i is the true label for token i ,
- p_i is the predicted probability for the correct token for token i ,
- N is the number of tokens,
- $D_{\text{KL}}(q_{\text{teacher}}(x) \| q_{\text{student}}(x))$ is the Kullback-Leibler divergence between the teacher and student models' probability distributions,
- $q_{\text{teacher}}(x)$ and $q_{\text{student}}(x)$ are the output probability distributions from the teacher and student models, respectively,
- λ_{ce} and λ_{kd} are the weighting hyperparameters for the cross-entropy and knowledge distillation losses, respectively.

Assume two different distributions for predicting the next word:

- P (from Model 1):
 - $mat \rightarrow 0.7$
 - $floor \rightarrow 0.2$
 - $chair \rightarrow 0.1$
- Q (from Model 2):
 - $mat \rightarrow 0.5$
 - $floor \rightarrow 0.3$
 - $chair \rightarrow 0.2$

Kullback–Leibler (KL) Divergence Calculation

KL divergence measures how much P diverges from Q :

$$D_{KL}(P||Q) = \sum_i P(i) \log \frac{P(i)}{Q(i)}$$

Substituting the values:

$$D_{KL}(P||Q) = 0.7 \log \frac{0.7}{0.5} + 0.2 \log \frac{0.2}{0.3} + 0.1 \log \frac{0.1}{0.2}$$

Training loss The student is trained with a distillation loss over the soft target probabilities of the teacher: $L_{ce} = \sum_i t_i * \log(s_i)$ where t_i (resp. s_i) is a probability estimated by the teacher (resp. the student). This objective results in a rich training signal by leveraging the full teacher distribution. Following Hinton et al. [2015] we used a *softmax-temperature*: $p_i = \frac{\exp(z_i/T)}{\sum_j \exp(z_j/T)}$ where T controls the smoothness of the output distribution and z_i is the model score for the class i . The same temperature T is applied to the student and the teacher at training time, while at inference, T is set to 1 to recover a standard *softmax*.

The final training objective is a linear combination of the distillation loss L_{ce} with the supervised training loss, in our case the *masked language modeling* loss L_{mlm} [Devlin et al., 2018]. We found it beneficial to add a *cosine embedding* loss (L_{cos}) which will tend to align the directions of the student and teacher hidden states vectors.

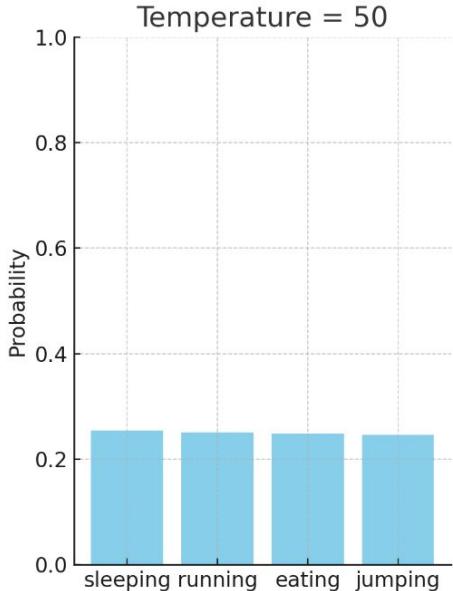
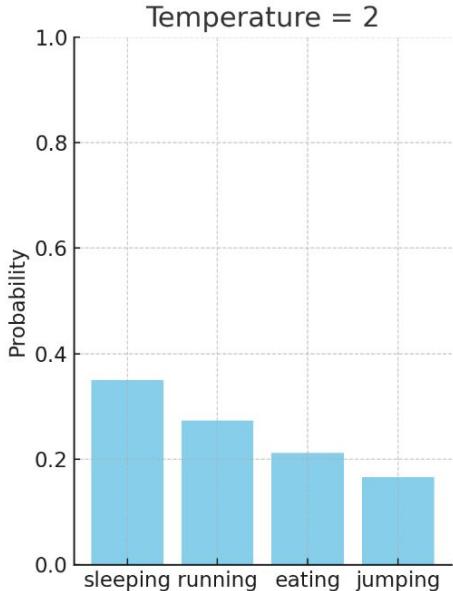
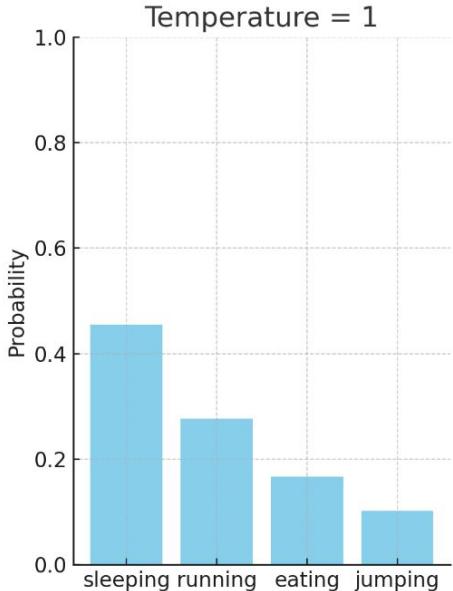
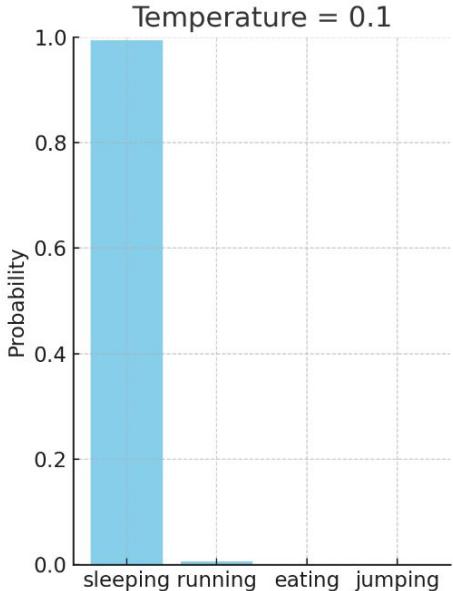
Temperature

$$P(y_i|\mathbf{x}) = \frac{\exp\left(\frac{z_i}{T}\right)}{\sum_j \exp\left(\frac{z_j}{T}\right)}$$

where:

- $P(y_i|\mathbf{x})$ is the probability of token y_i given the input \mathbf{x}
- z_i is the logit (raw score before softmax) for token y_i
- T is the temperature (where $T = 1$ is the default, and $T < 1$ reduces randomness while $T > 1$ increases randomness)
- The summation in the denominator is over all possible tokens j

Temperature (cont'd)



peaked distribution
(more deterministic)

flatter distribution
(more randomness)

DistilBERT reduces BERT's size by 40%, while retaining 97% of its performance and being 60% faster

Model	Score	CoLA	MNLI	MRPC	QNLI	QQP	RTE	SST-2	STS-B	WNLI
ELMo	68.7	44.1	68.6	76.6	71.1	86.2	53.4	91.5	70.4	56.3
BERT-base	79.5	56.3	86.7	88.6	91.8	89.6	69.3	92.7	89.0	53.5
DistilBERT	77.0	51.3	82.2	87.5	89.2	88.5	59.9	91.3	86.9	56.3

Model	IMDb (acc.)	SQuAD (EM/F1)
BERT-base	93.46	81.2/88.5
DistilBERT	92.82	77.7/85.8
DistilBERT (D)	-	79.1/86.9

Model	# param. (Millions)	Inf. time (seconds)
ELMo	180	895
BERT-base	110	668
DistilBERT	66	410

Distilling Step-by-Step! Outperforming Larger Language Models with Less Training Data and Smaller Model Sizes

**Cheng-Yu Hsieh^{1*}, Chun-Liang Li², Chih-Kuan Yeh³, Hootan Nakhost²,
Yasuhiba Fujii³, Alexander Ratner¹, Ranjay Krishna¹, Chen-Yu Lee², Tomas Pfister²**

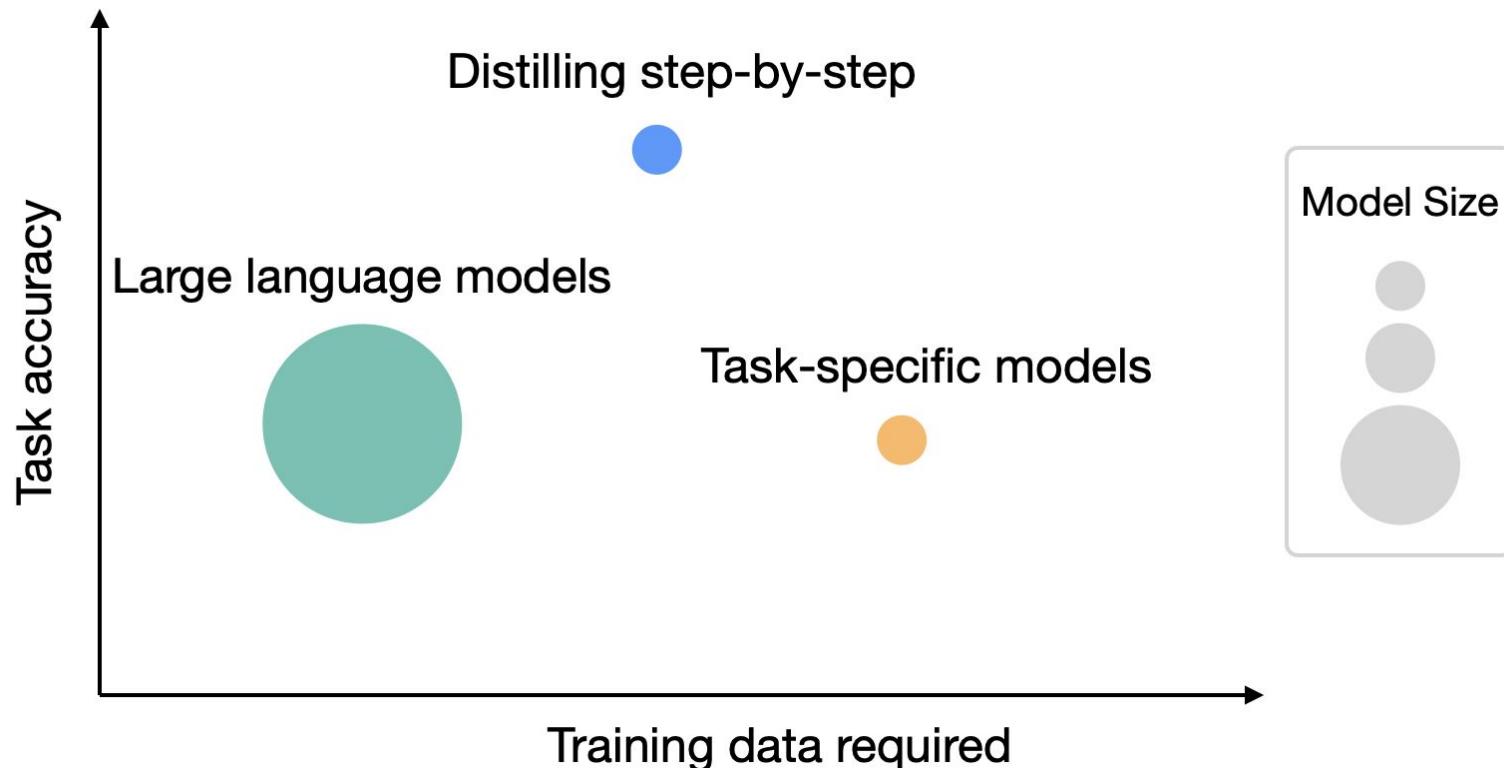
¹University of Washington, ²Google Cloud AI Research, ³Google Research
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Distilling Step-by-Step! Outperforming Larger Language Models with Less Training Data and Smaller Model Sizes

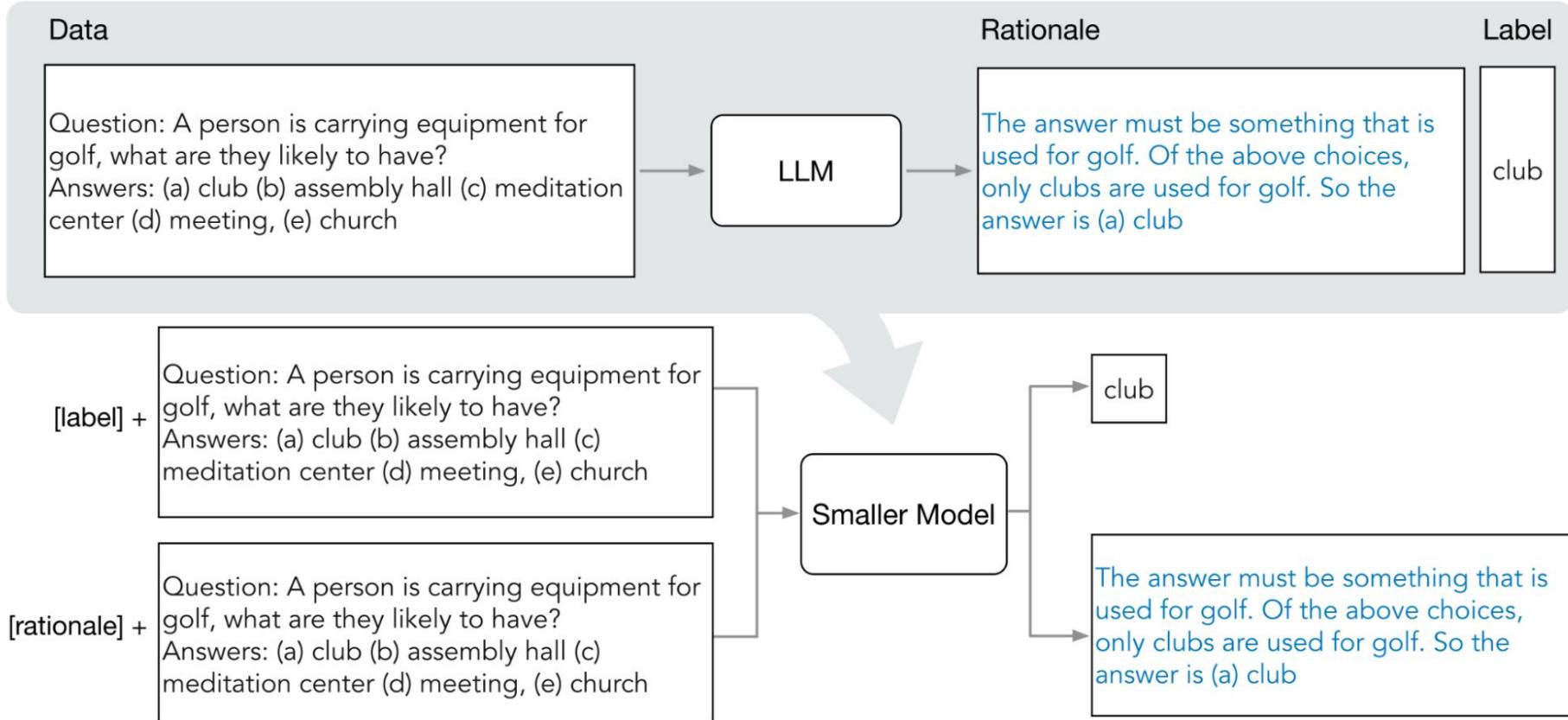
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Yasuhiba Fujii³, Alexander Ratner¹, Ranjay Krishna¹, Chen-Yu Lee², Tomas Pfister²**

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Enabling a 770M parameter T5 model to outperform the few-shot prompted 540B PaLM model



Distilling step-by-step



Leveraging few-shot CoT prompting to extract rationales from LLMs

Few-shot CoT

Question: Sammy wanted to go to where the people are. Where might he go?
Answer Choices: (a) populated areas, (b) race track, (c) desert, (d) apartment, (e) roadblock

Answer: The answer must be a place with a lot of people. Of the above choices, only populated areas have a lot of people. So the answer is (a) populated areas.

Input

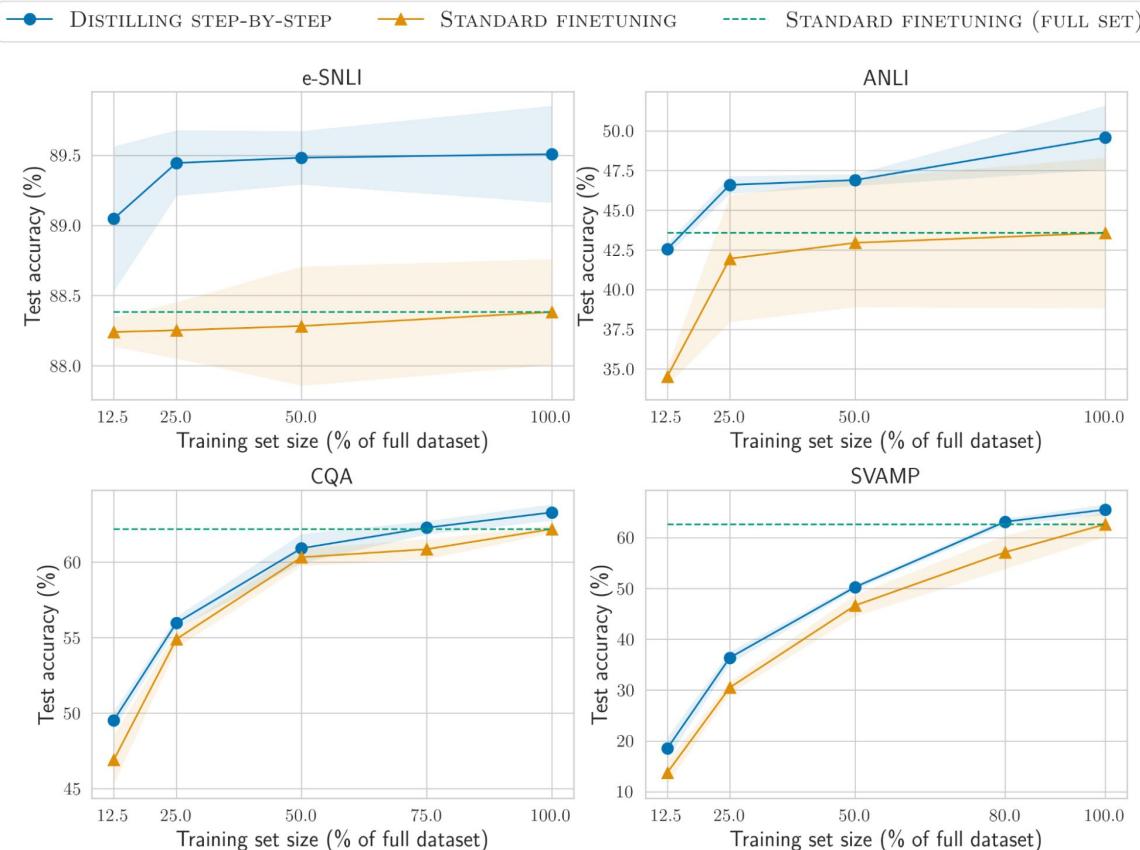
Question: A person is carrying equipment for golf. What are they likely to have?
Answer Choices: (a) club, (b) assembly hall, (c) meditation center, (d) meeting, (e) church

Answer:

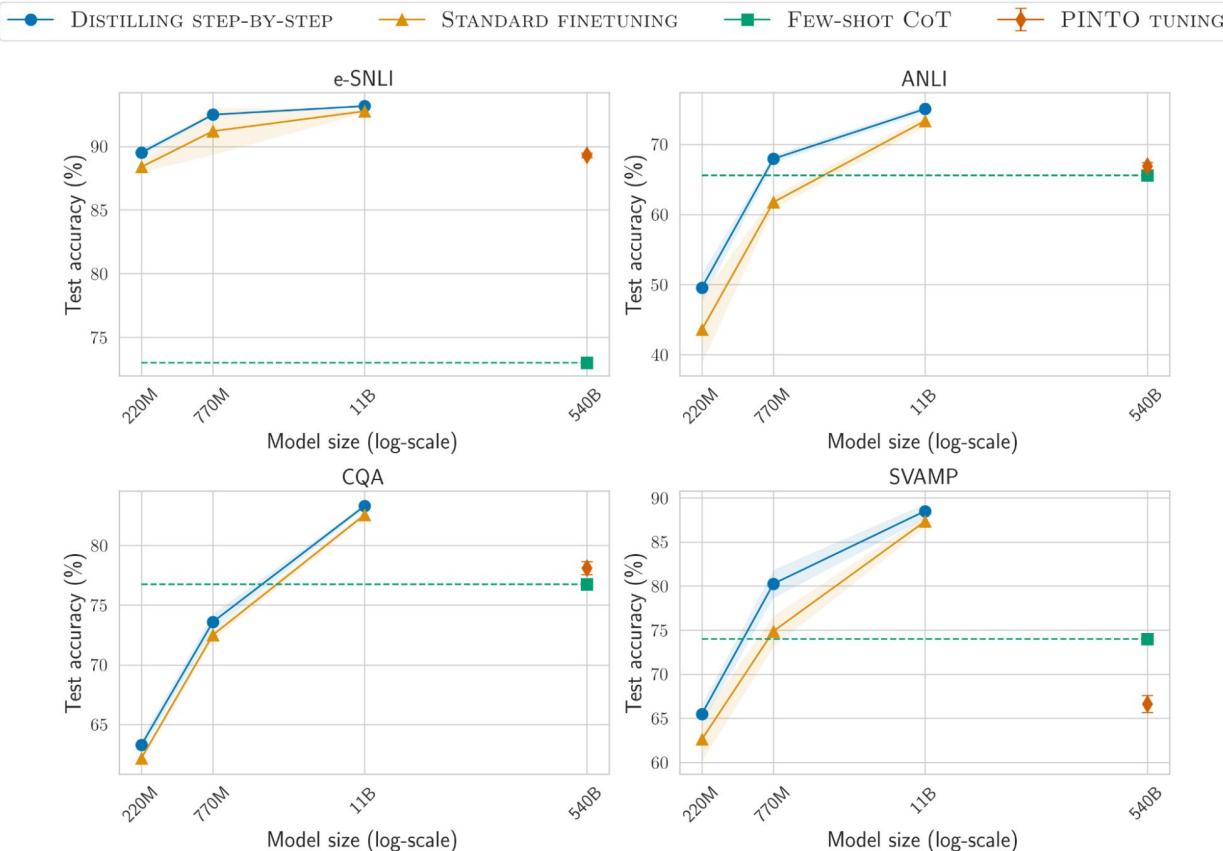
Output

The answer must be something that is used for golf. Of the above choices, only clubs are used for golf. So the answer is (a) club.

Less training data

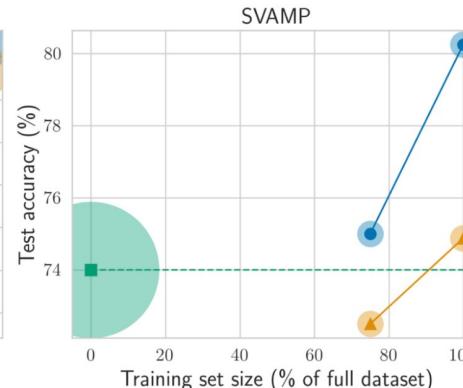
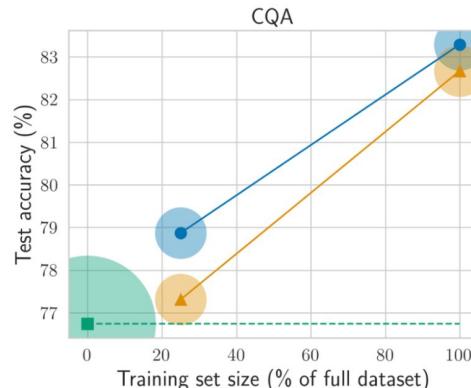
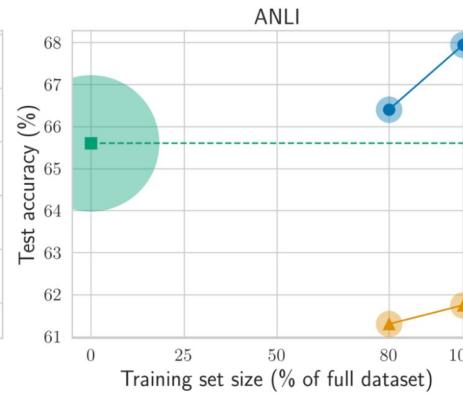
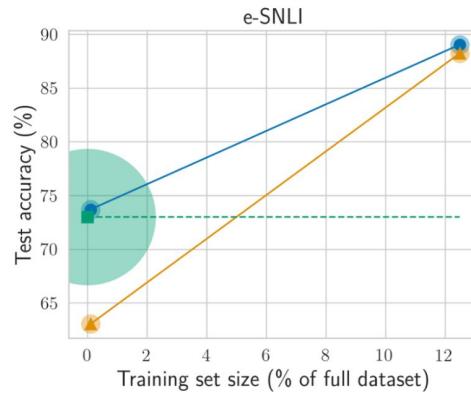


Smaller deployed model size

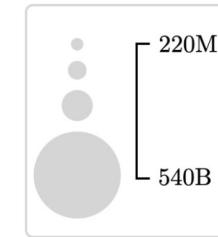


Distilling step-by-step outperforms few-shot LLMs with smaller models using less data

● DISTILLING STEP-BY-STEP ▲ STANDARD FINETUNING ■ FEW-SHOT CoT



MODEL SIZE





DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning

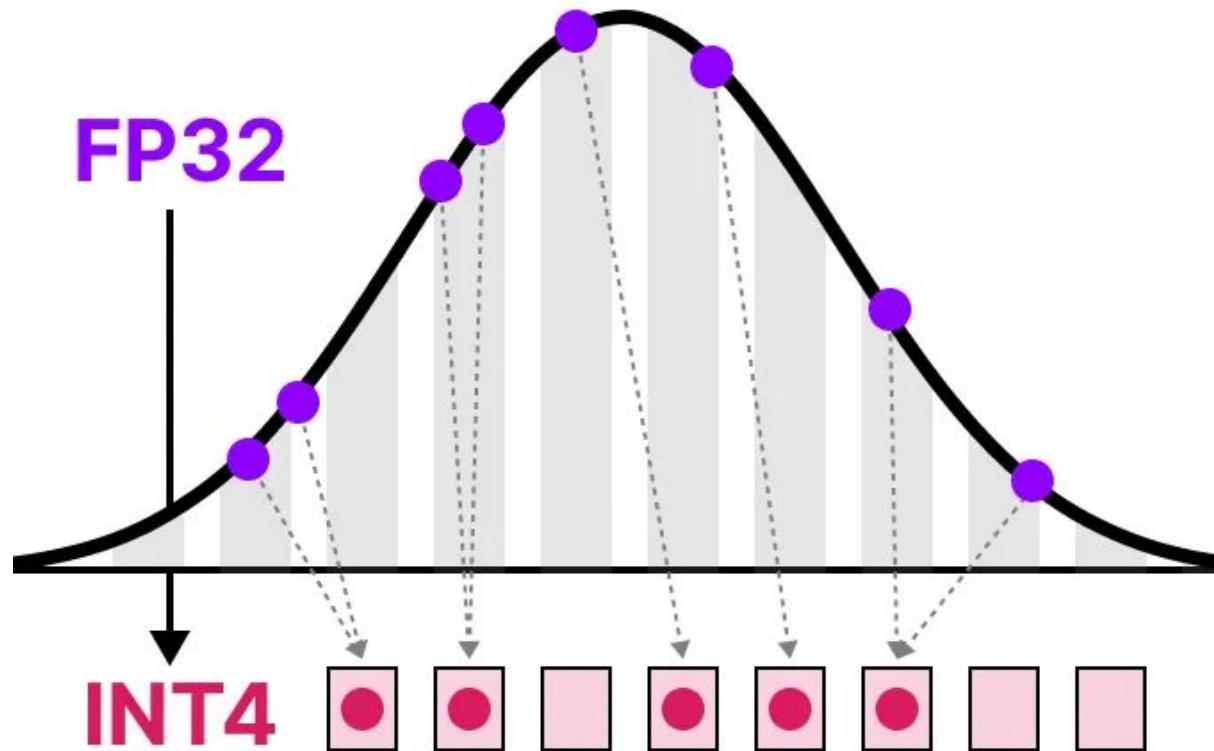
DeepSeek-AI

research@deepseek.com

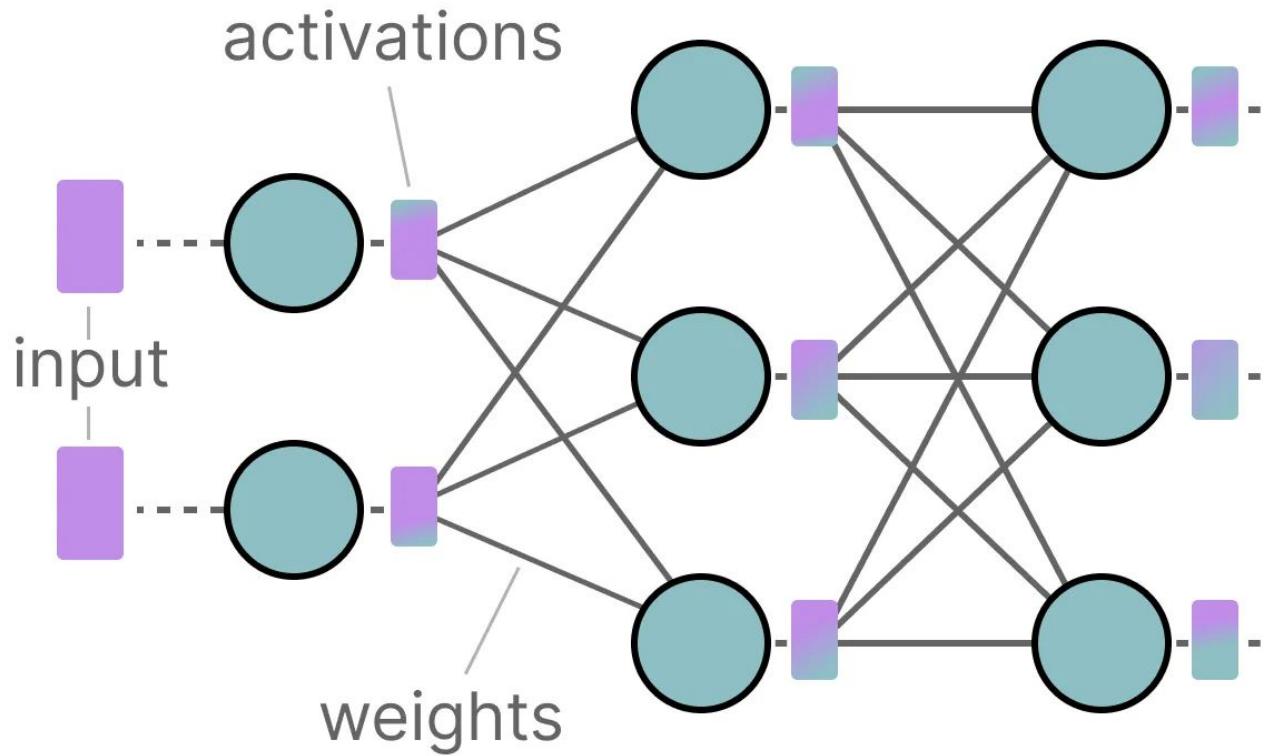
Model	AIME 2024		MATH-500	GPQA Diamond	LiveCode Bench	CodeForces
	pass@1	cons@64	pass@1	pass@1	pass@1	rating
GPT-4o-0513	9.3	13.4	74.6	49.9	32.9	759
Claude-3.5-Sonnet-1022	16.0	26.7	78.3	65.0	38.9	717
OpenAI-o1-mini	63.6	80.0	90.0	60.0	53.8	1820
QwQ-32B-Preview	50.0	60.0	90.6	54.5	41.9	1316
DeepSeek-R1-Distill-Qwen-1.5B	28.9	52.7	83.9	33.8	16.9	954
DeepSeek-R1-Distill-Qwen-7B	55.5	83.3	92.8	49.1	37.6	1189
DeepSeek-R1-Distill-Qwen-14B	69.7	80.0	93.9	59.1	53.1	1481
DeepSeek-R1-Distill-Qwen-32B	72.6	83.3	94.3	62.1	57.2	1691
DeepSeek-R1-Distill-Llama-8B	50.4	80.0	89.1	49.0	39.6	1205
DeepSeek-R1-Distill-Llama-70B	70.0	86.7	94.5	65.2	57.5	1633

Table 5 | Comparison of DeepSeek-R1 distilled models and other comparable models on reasoning-related benchmarks.

Quantization

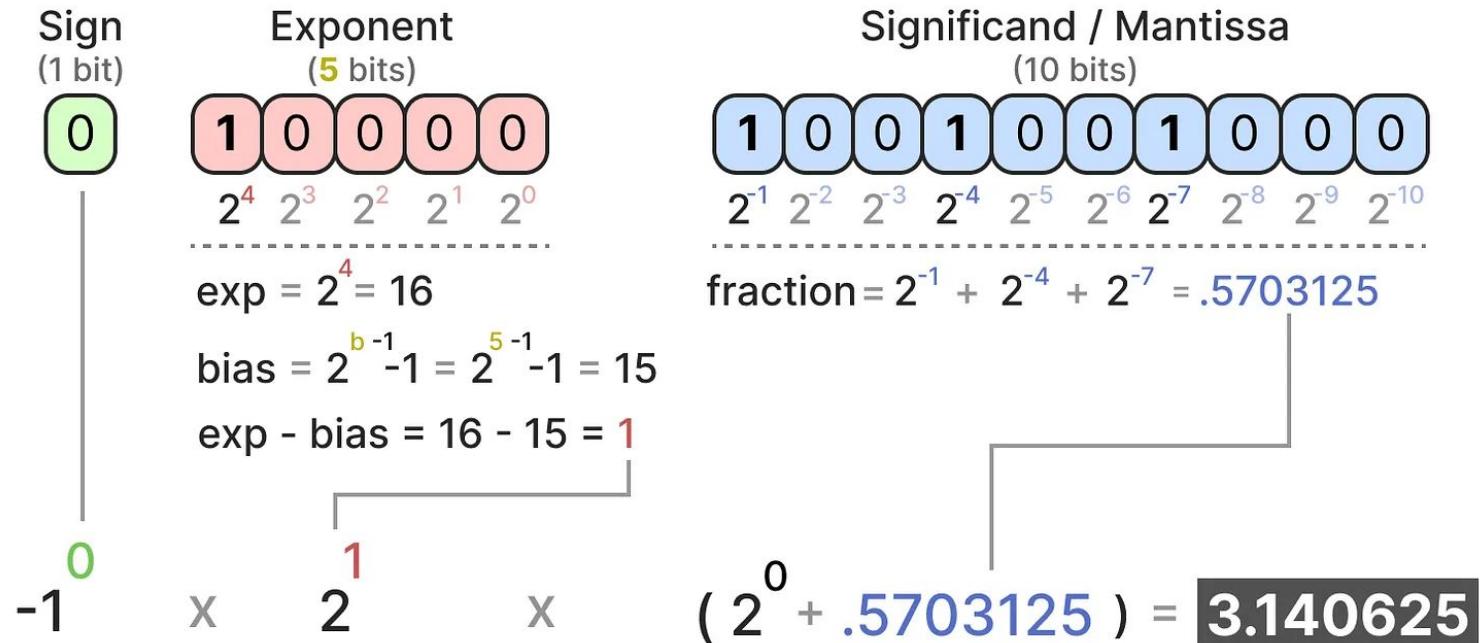


Quantizing both the weights and activations



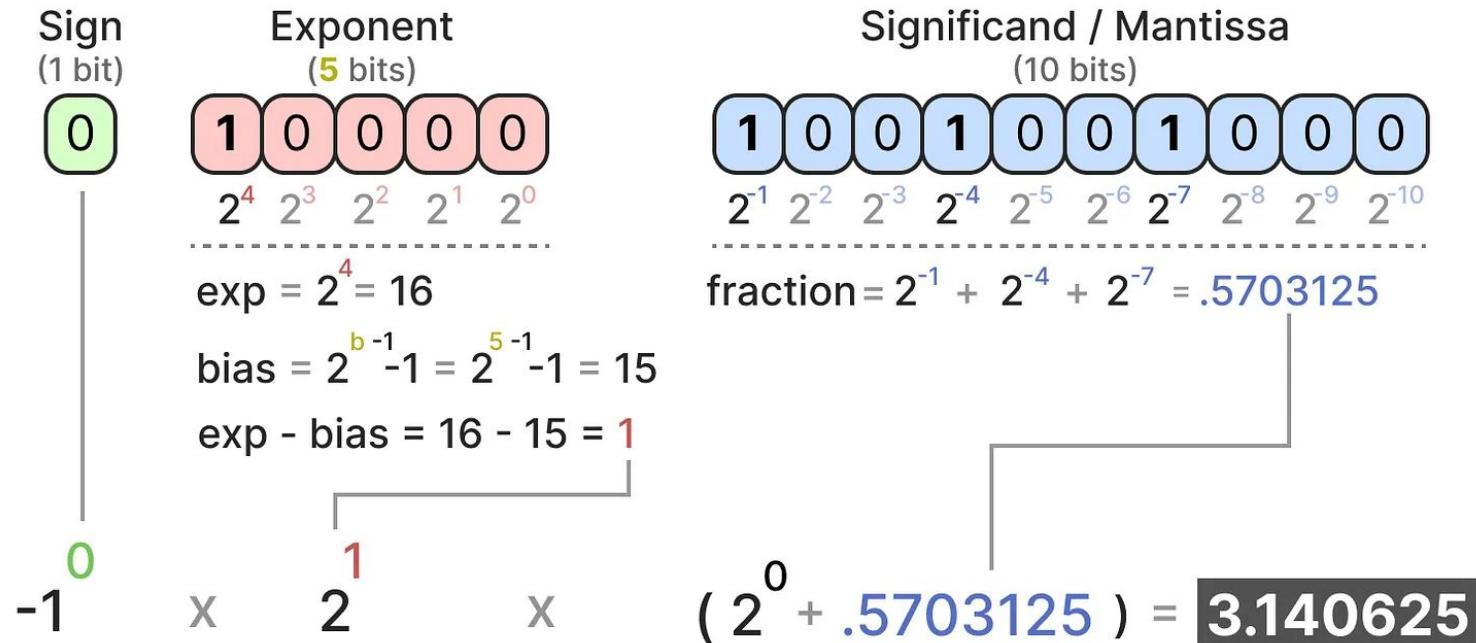
How to represent numerical values

Float 16-bit (FP16)



How to represent numerical values (cont'd)

Float 16-bit (FP16)



How to represent numerical values (cont'd)

Float 32-bit (FP32)

0 10000000 1001001000011111011011

$$(-1)^0 \times 2^1 \times 1.5707964 = 3.1415927410125732$$

higher precision

Float 16-bit (FP16)

0 10000 1001001000

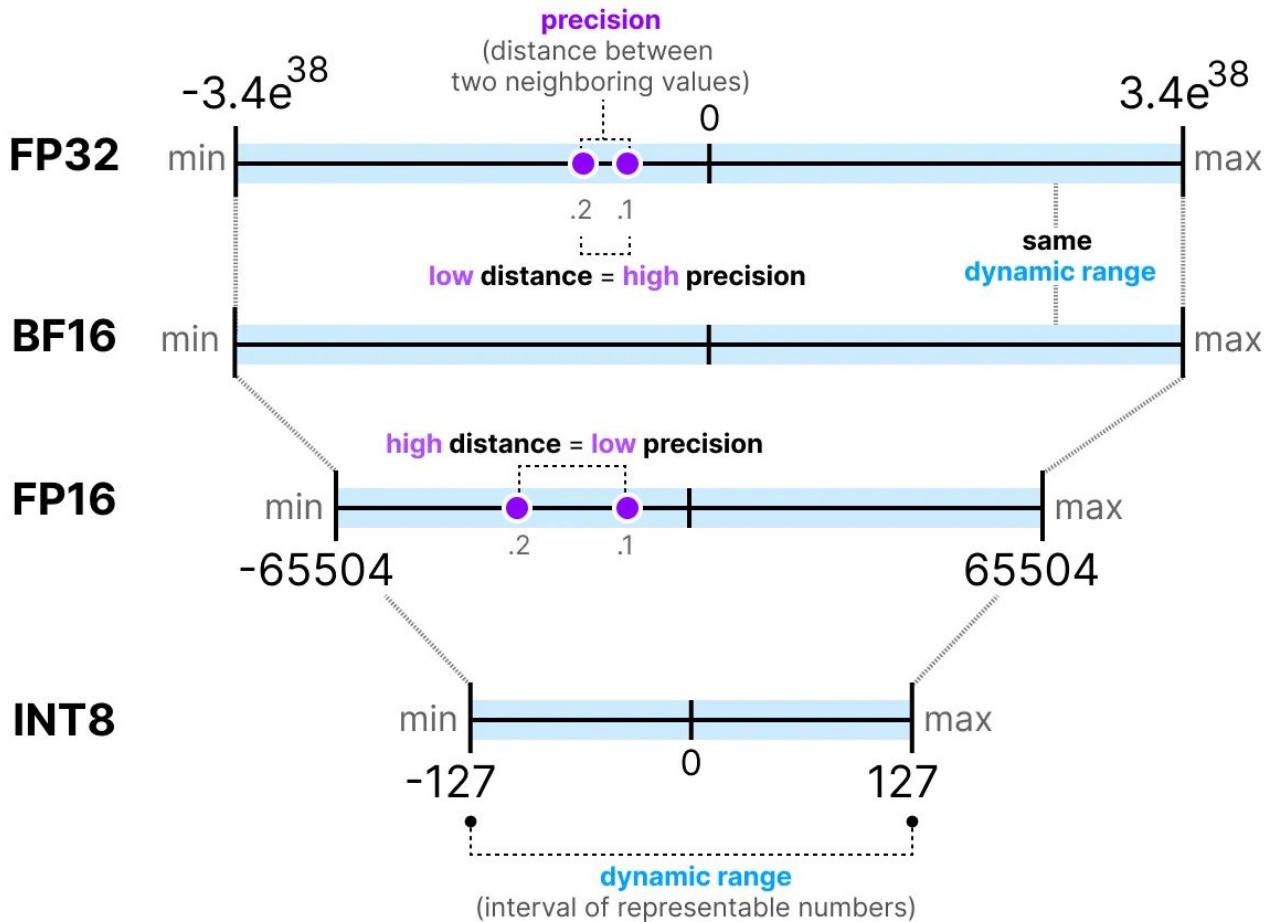
$$(-1)^0 \times 2^1 \times 1.5703125 = 3.140625$$

lower precision

original value

3.1415927

Memory constraints



Memory constraints (cont'd)

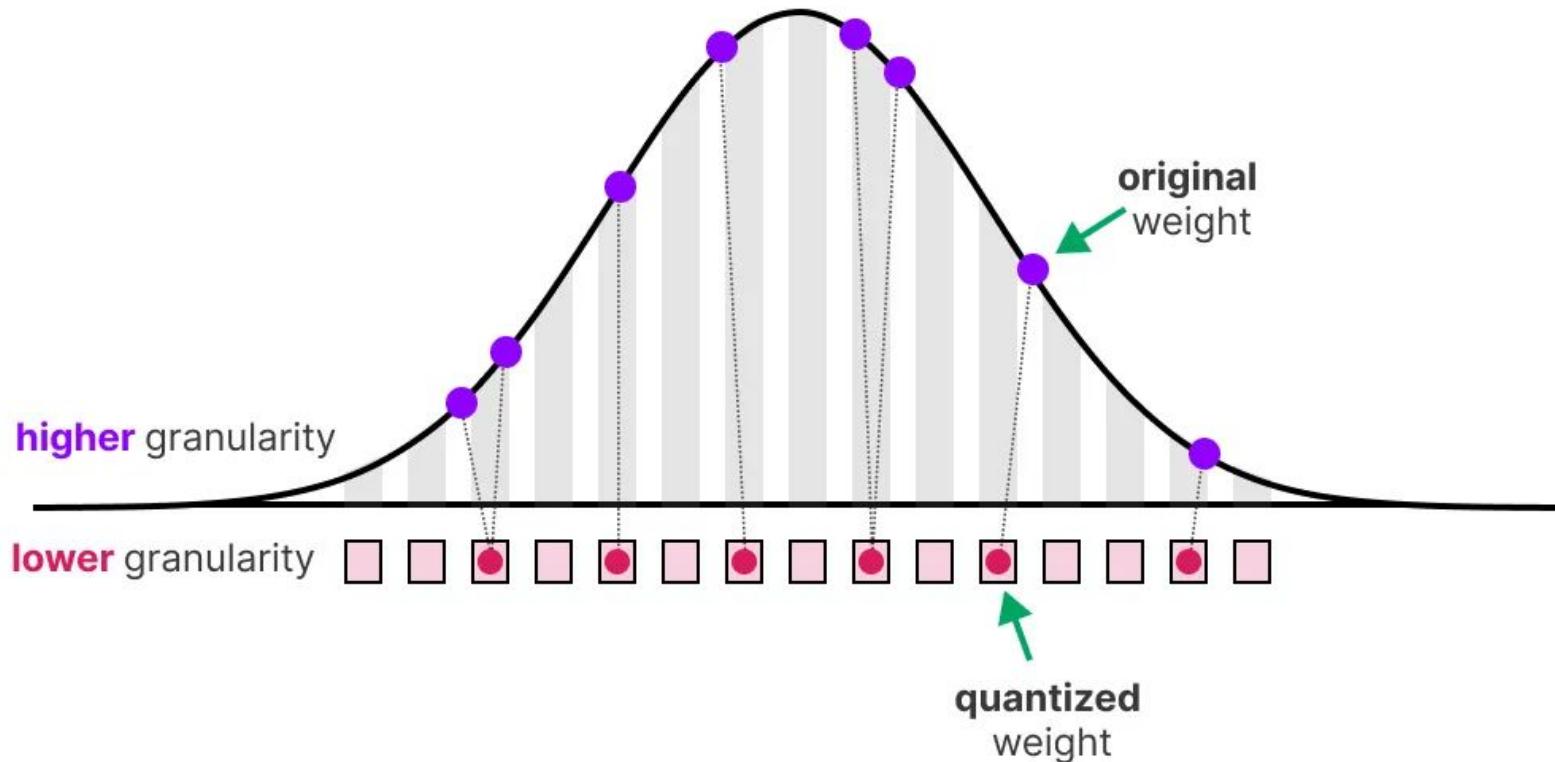
$$\text{memory} = \frac{\text{nr_bits}}{8} \times \text{nr_params}$$

$$\mathbf{64\text{-bits}} = \frac{64}{8} \times 70\text{B} \approx \mathbf{560 \text{ GB}}$$

$$\mathbf{32\text{-bits}} = \frac{32}{8} \times 70\text{B} \approx \mathbf{280 \text{ GB}}$$

$$\mathbf{16\text{-bits}} = \frac{16}{8} \times 70\text{B} \approx \mathbf{140 \text{ GB}}$$

Quantization



Quantization

Original Image

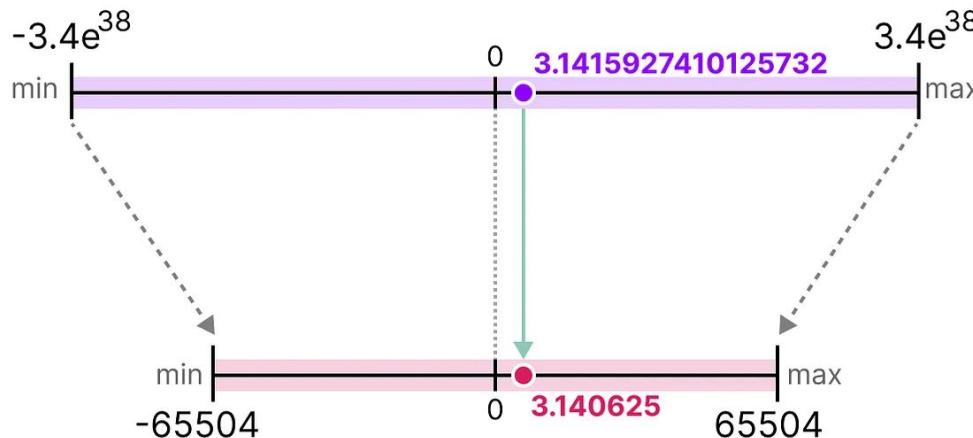


“Quantized” Image



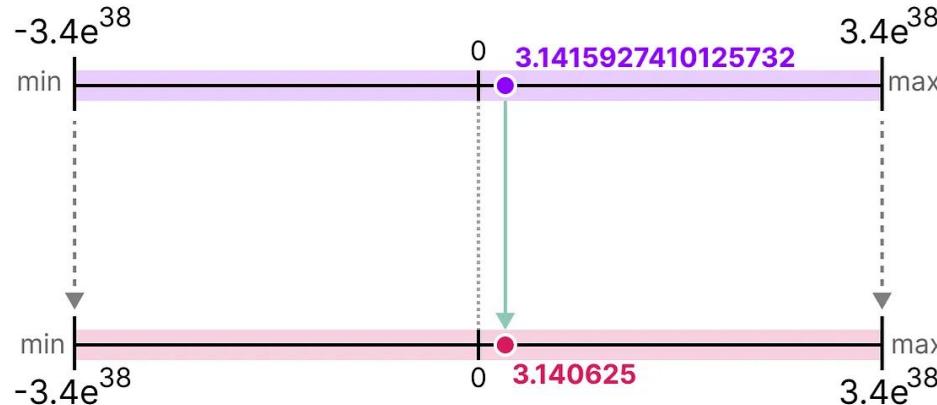
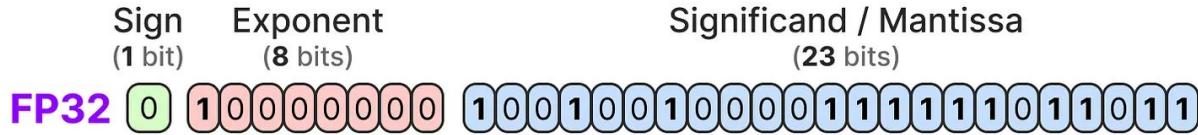
Common data types: FP16 (half precision)

Sign (1 bit)	Exponent (8 bits)	Significand / Mantissa (23 bits)
FP32	0 10000000	1001001000011111011011



FP16	Sign (1 bit)	Exponent (5 bits)	Significand / Mantissa (10 bits)
0 10000 1001001000	0	10000	1001001000

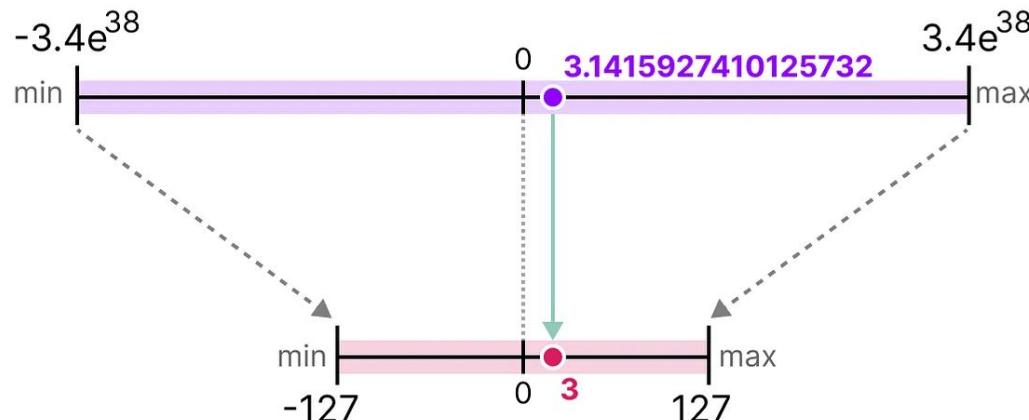
Common data types: BF16



Common data types: INT8

FP32 Sign (1 bit) Exponent (8 bits) Significand / Mantissa (23 bits)

0 10000000 1001001000011111011011

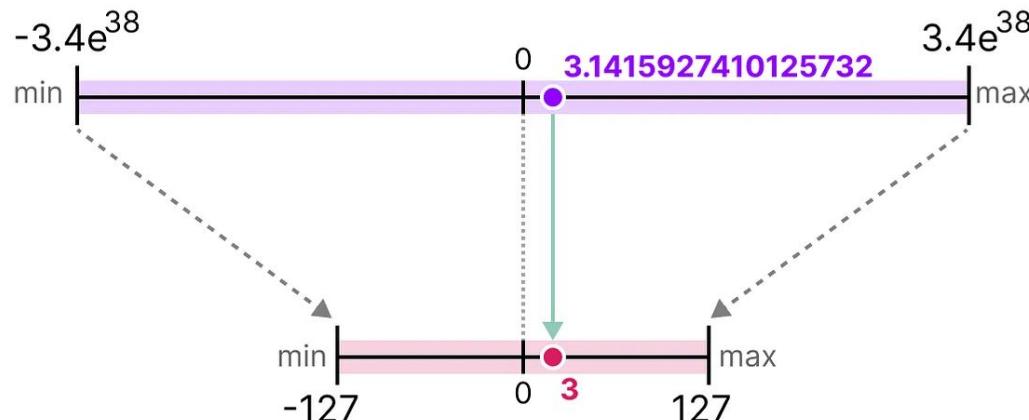


(signed) INT8 0 1001000
(1 bit) (7 bits)

Common data types: INT8

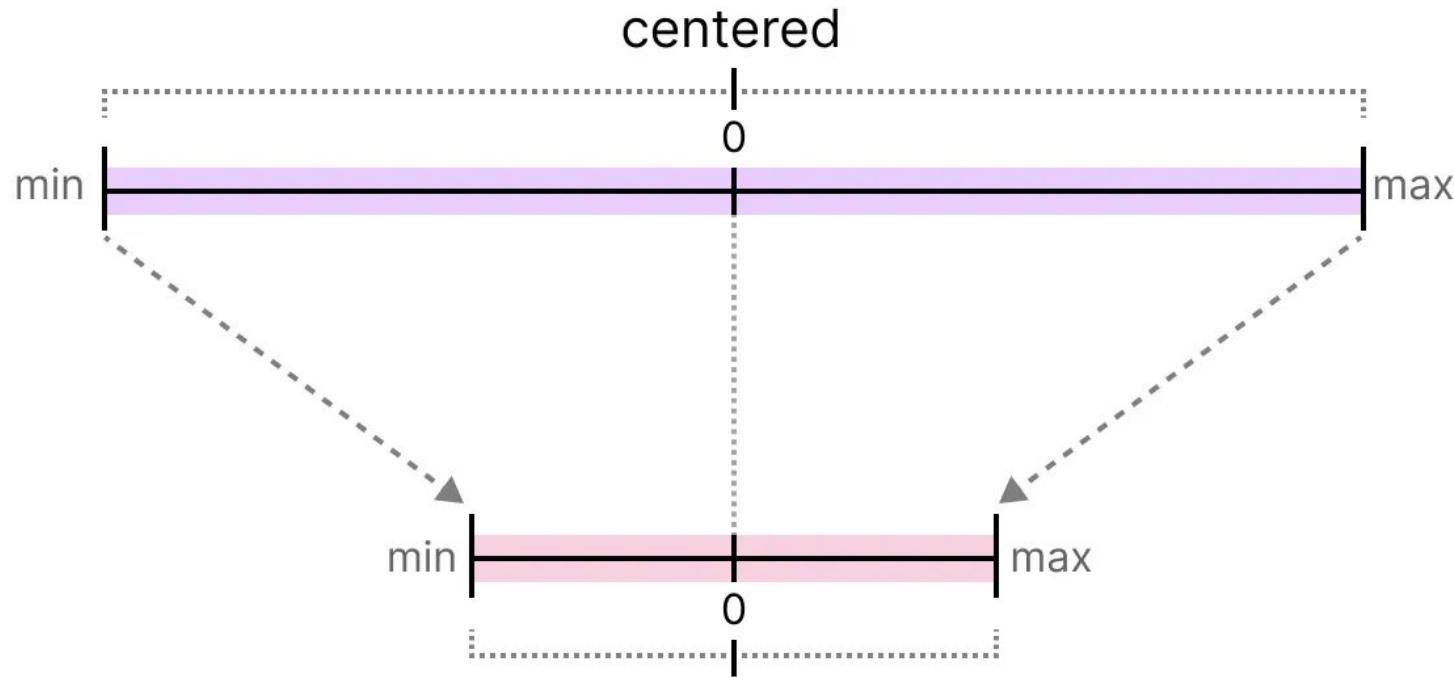
FP32 Sign (1 bit) Exponent (8 bits) Significand / Mantissa (23 bits)

0 10000000 1001001000011111011011



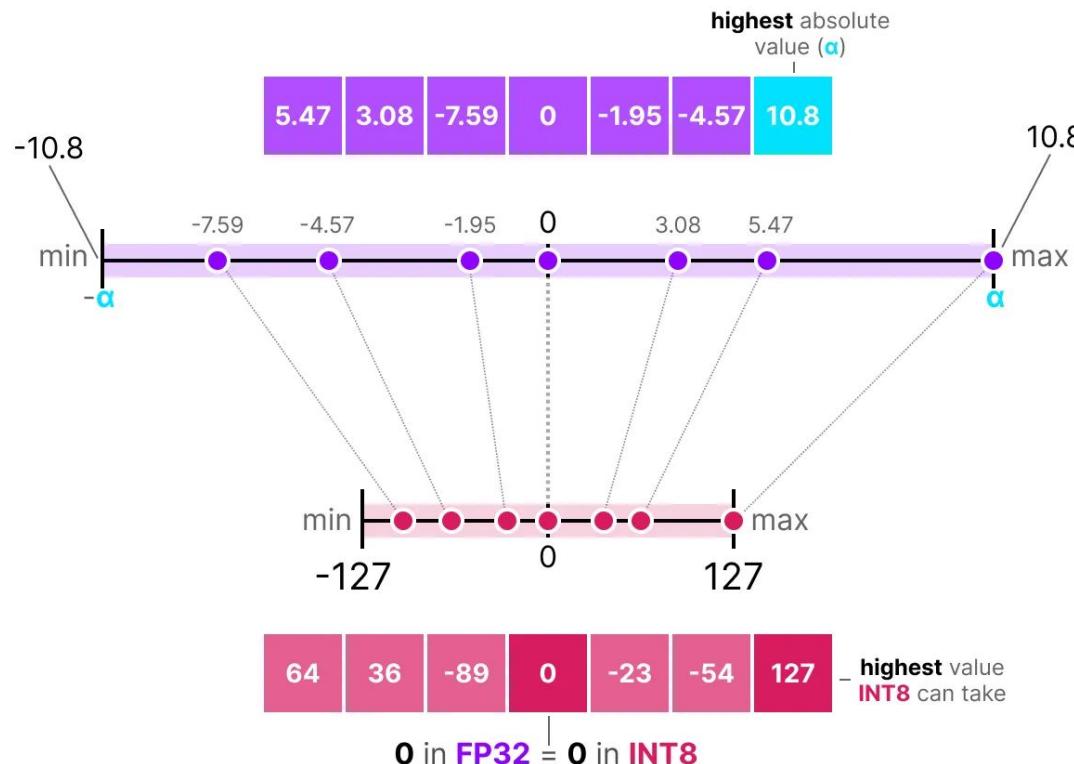
(signed) INT8 0 1001000
(1 bit) (7 bits)

Symmetric quantization



0 in FP32 = 0 in INT8

Absolute maximum (absmax) quantization



Absolute maximum (absmax) quantization

We first calculate a scale factor (s) using:

- b is the number of bytes that we want to quantize to (8),
- α is the *highest* absolute value,

Then, we use the s to quantize the input x :

$$s = \frac{2^{b-1} - 1}{\alpha} \quad (\text{scale factor})$$

$$x_{\text{quantized}} = \text{round}(s \cdot x) \quad (\text{quantization})$$

Filling in the values would then give us the following:

$$s = \frac{127}{10.8} = 11.76 \quad (\text{scale factor})$$

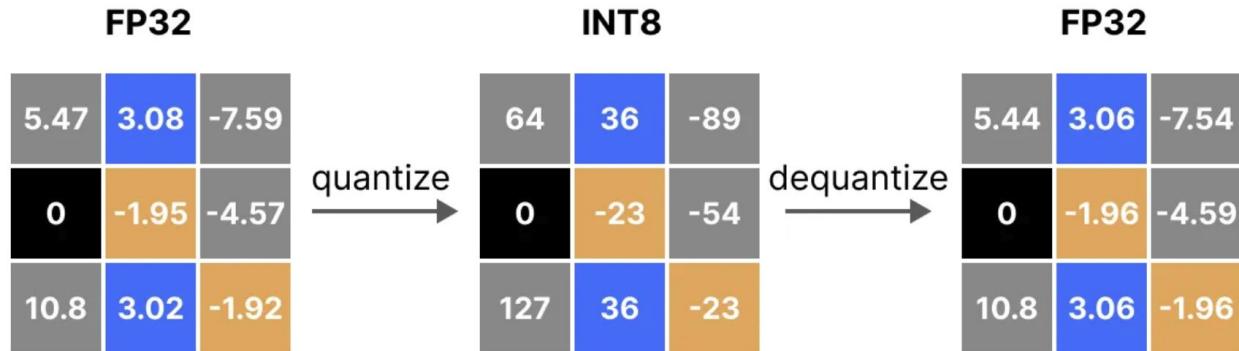
$$x_{\text{quantized}} = \text{round}(11.76 \cdot \text{█████}) \quad (\text{quantization})$$

Dequantization

To retrieve the original FP32 values, we can use the previously calculated *scaling factor* (s) to *dequantize* the quantized values.

$$X_{\text{dequantized}} = \frac{\text{█}}{s} \quad (\text{dequantize})$$

Applying the quantization and then dequantization process to retrieve the original looks as follows:



Dequantization

FP32 (original)		
5.47	3.08	-7.59
0	-1.95	-4.57
10.8	3.02	-1.92

FP32 (dequantized)		
5.44	3.06	-7.54
0	-1.96	-4.59
10.8	3.06	-1.96

Quantization error		
.03	.02	.05
0	-.01	-.02
0	-.04	-.04

-

=

Dequantization

FP32 (original)		
5.47	3.08	-7.59
0	-1.95	-4.57
10.8	3.02	-1.92

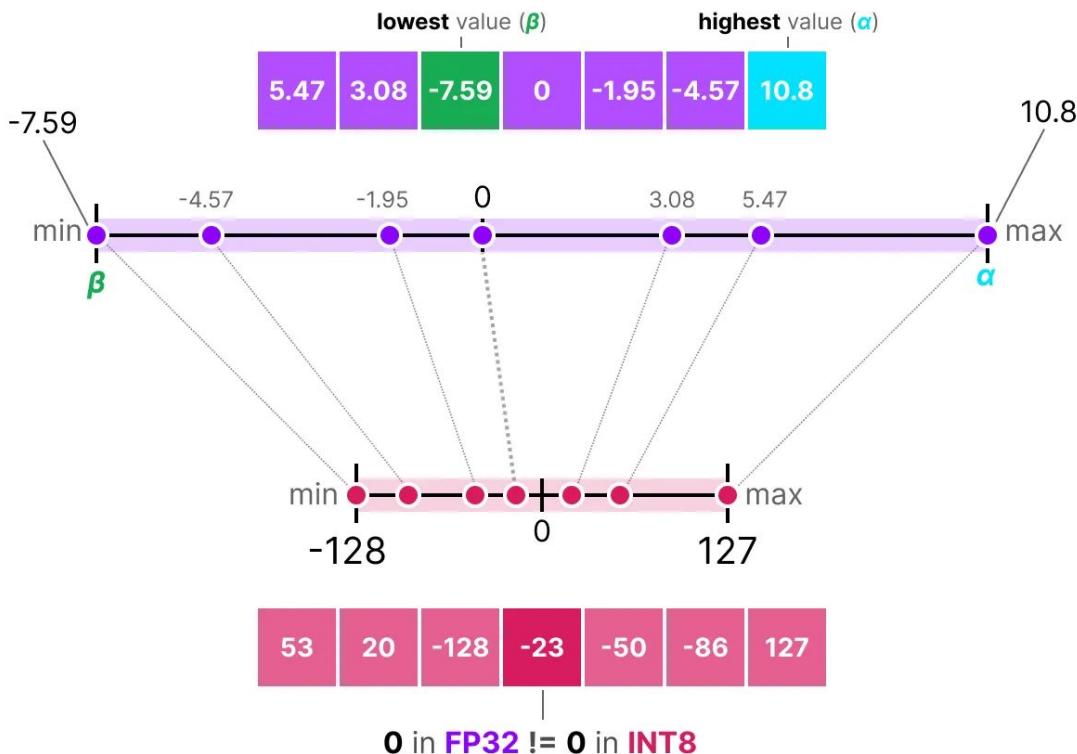
FP32 (dequantized)		
5.44	3.06	-7.54
0	-1.96	-4.59
10.8	3.06	-1.96

Quantization error		
.03	.02	.05
0	-.01	-.02
0	-.04	-.04

-

=

Asymmetric quantization



Asymmetric quantization (cont'd)

$$S = \frac{128 - -127}{\alpha - \beta} \quad (\text{scale factor})$$

$$Z = \text{round}(-S \cdot \beta) - 2^{b-1} \quad (\text{zeropoint})$$

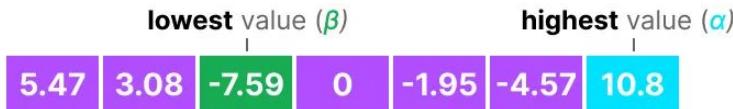
$$X_{\text{quantized}} = \text{round}(S \cdot X + Z) \quad (\text{quantization})$$

Asymmetric quantization (cont'd)

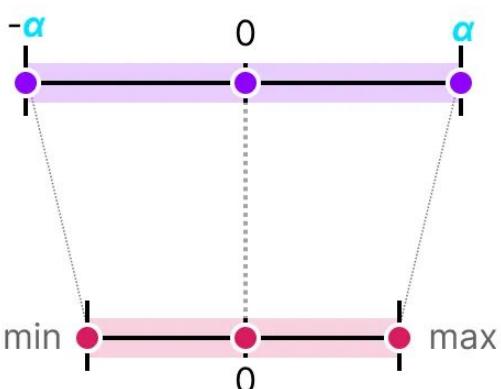
$$x_{\text{dequantized}} = \frac{\text{[pink bar]}}{s} - z$$

(dequantize)

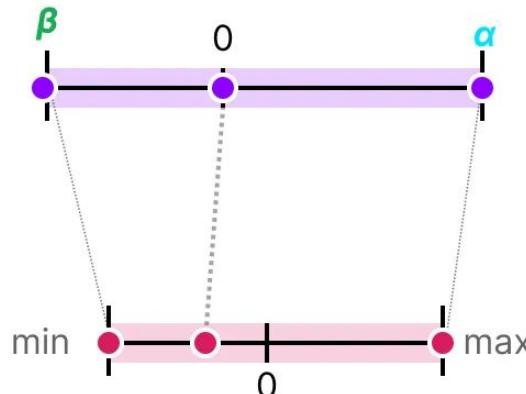
Symmetric vs. Asymmetric quantization



Symmetric
[-10.8, 10.8]



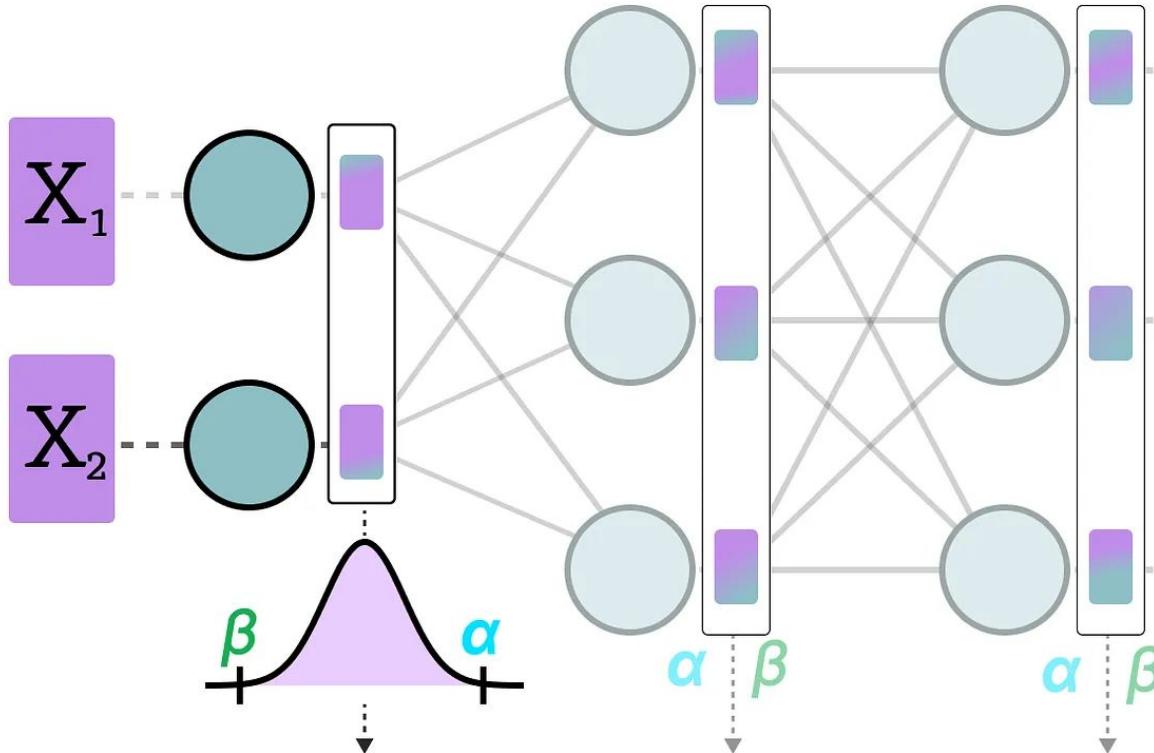
Asymmetric
[-7.59, 10.8]



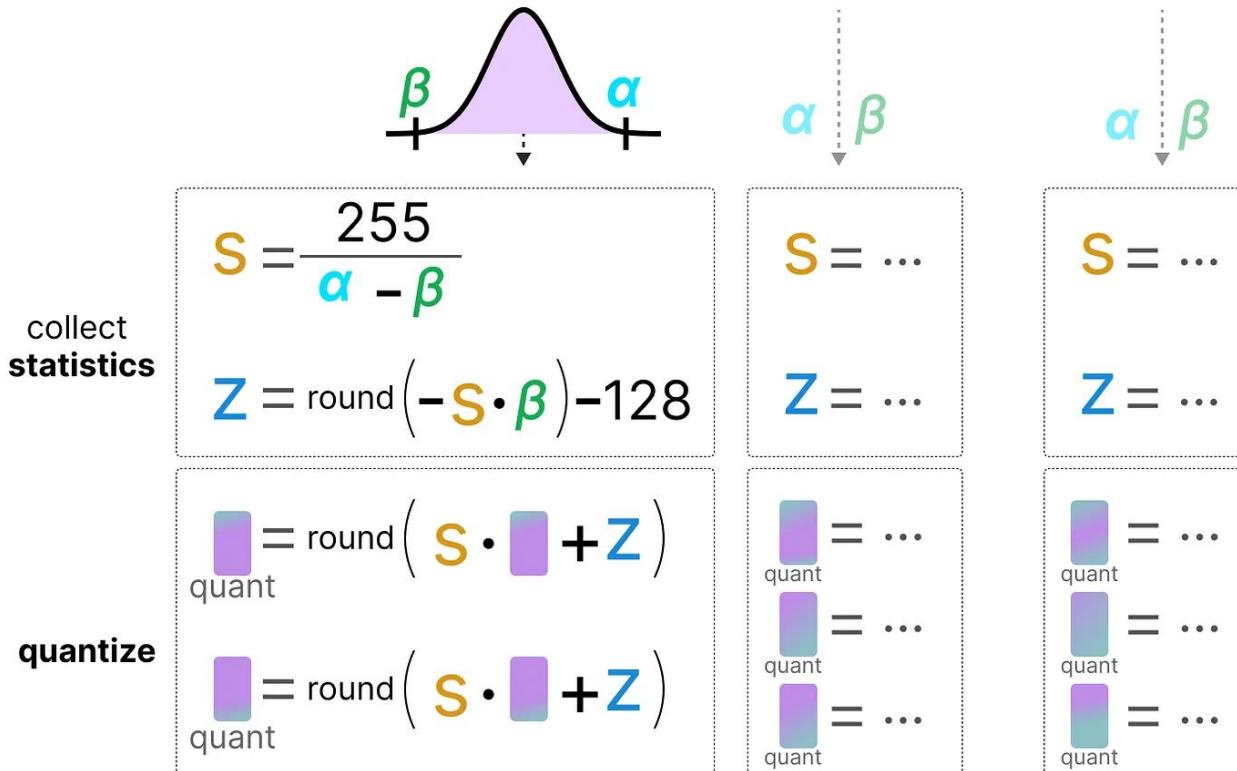
Post-training quantization

- Dynamic Quantization
- Static Quantization

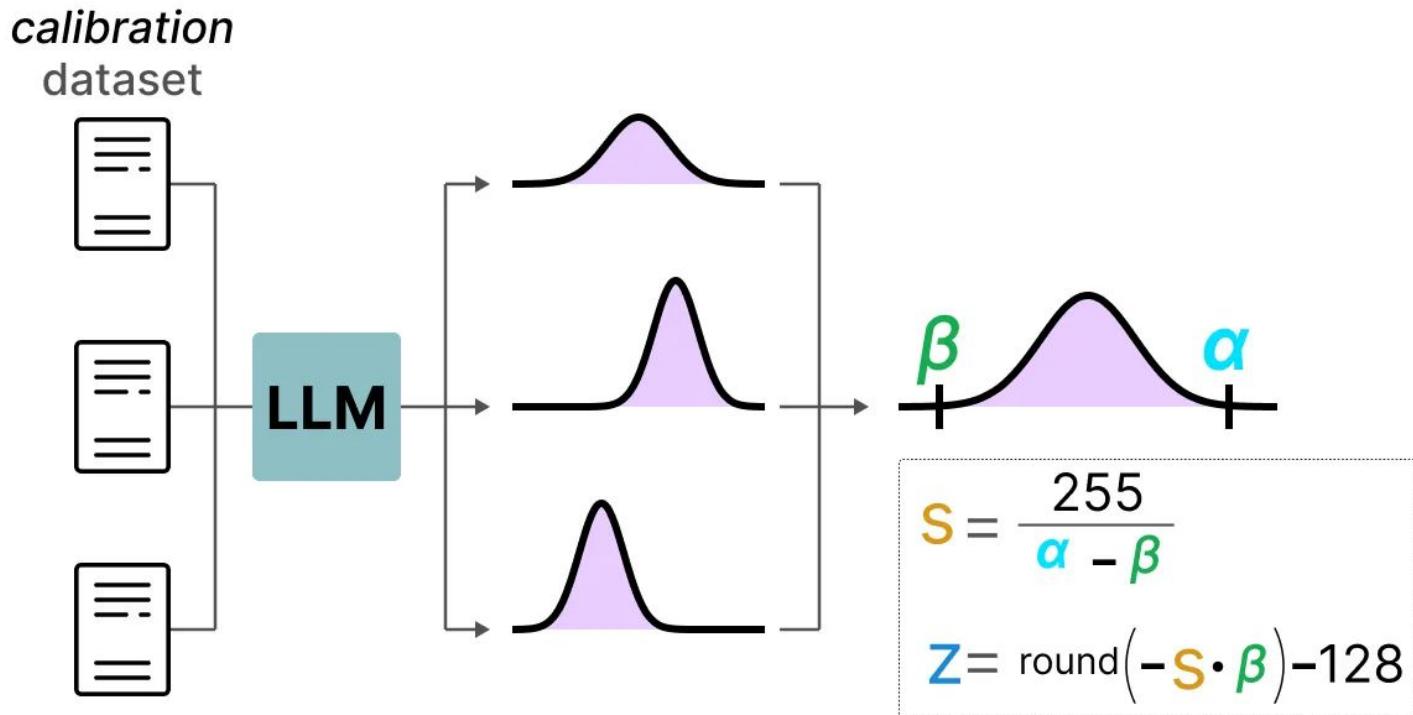
Dynamic quantization



Dynamic quantization (cont'd)



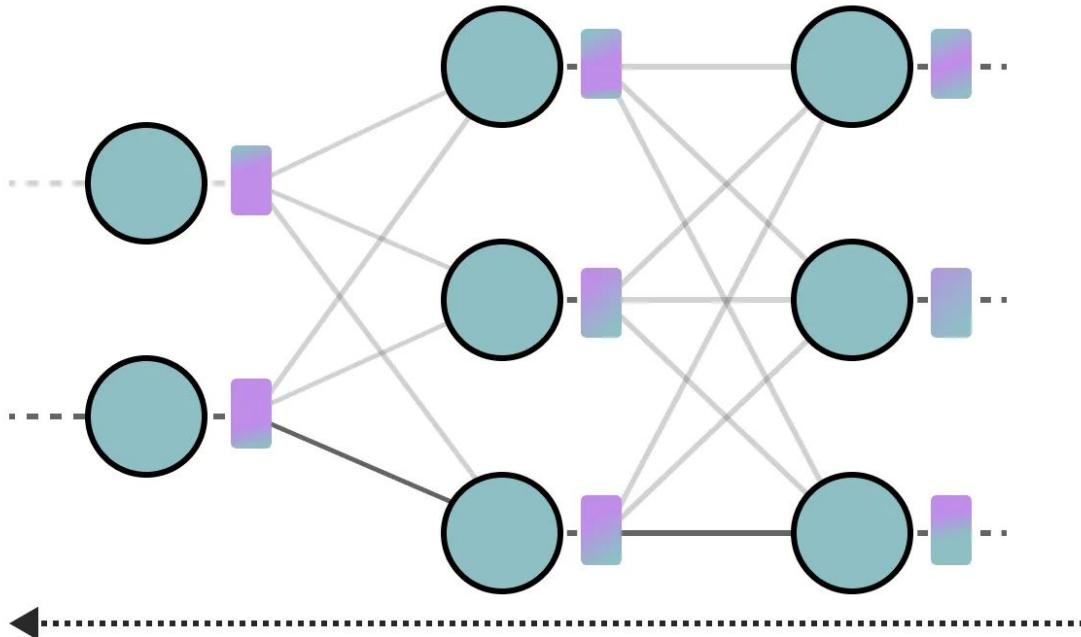
Static quantization



The realm of 4-bit quantization

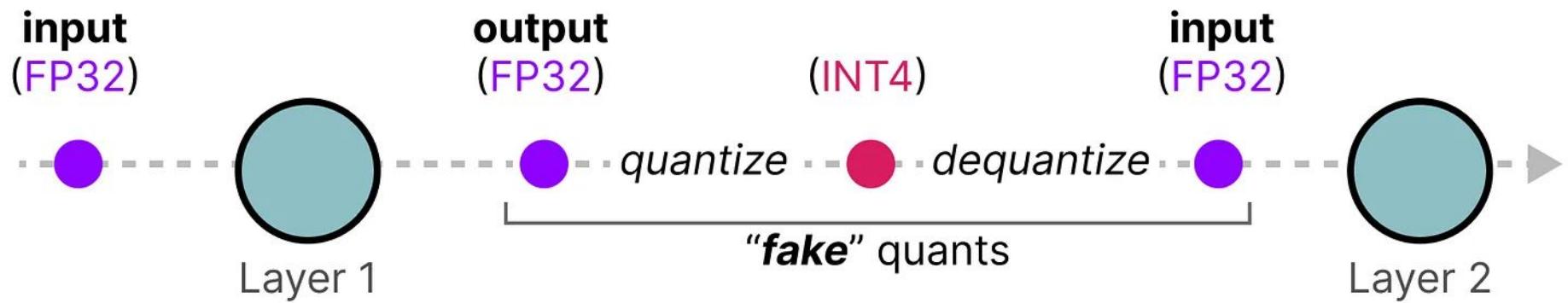
- GPTQ (full model on GPU)
- GGUF (potentially offload layers on the CPU)

Quantization aware training



Learn **quantization parameters** (s , α , β , z)
during **backward pass**

Quantization aware training (cont'd)



QLoRA: Efficient Finetuning of Quantized LLMs

Tim Dettmers*

Artidoro Pagnoni*

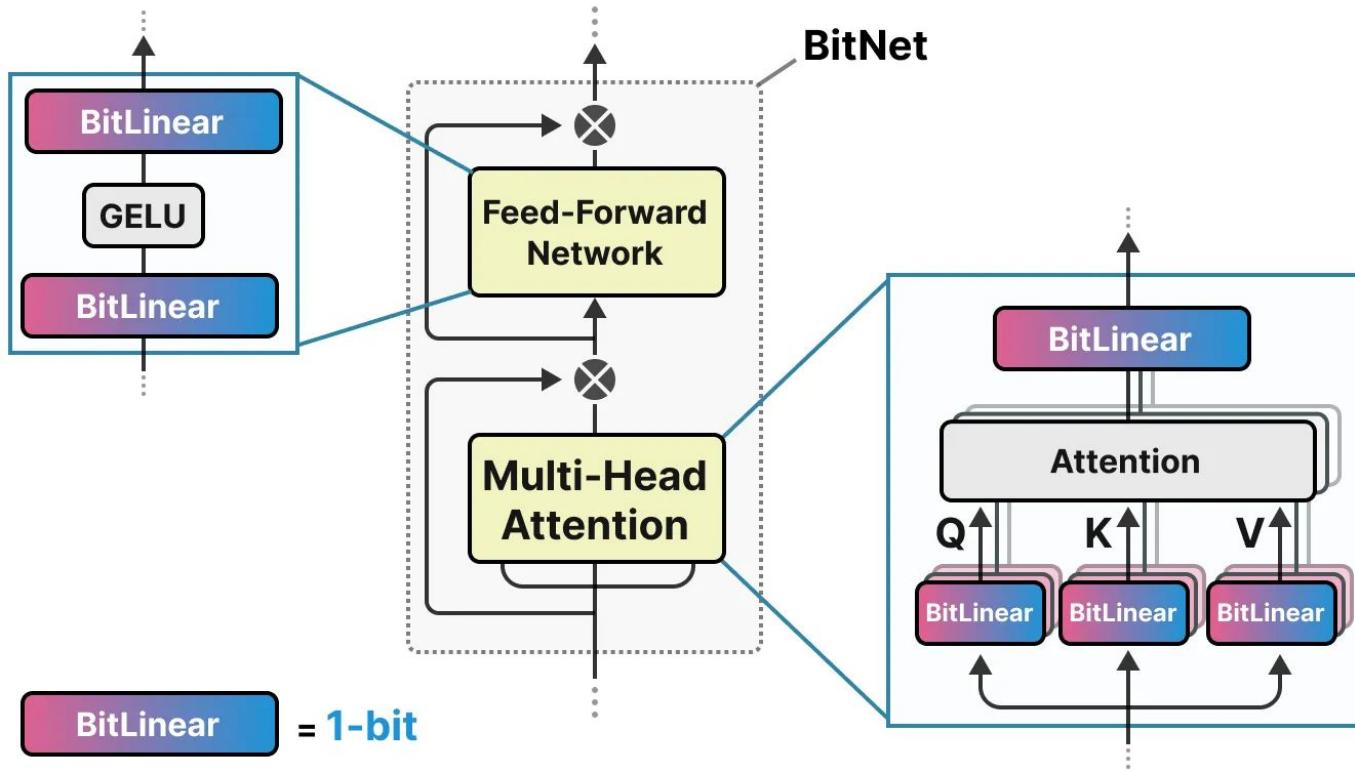
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The era of 1-bit LLMs: BitNet



Pruning

- Remove parameters from the model after training

Published as a conference paper at ICLR 2024

A SIMPLE AND EFFECTIVE PRUNING APPROACH FOR LARGE LANGUAGE MODELS

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Zhuang Liu^{2*}

Anna Bair¹

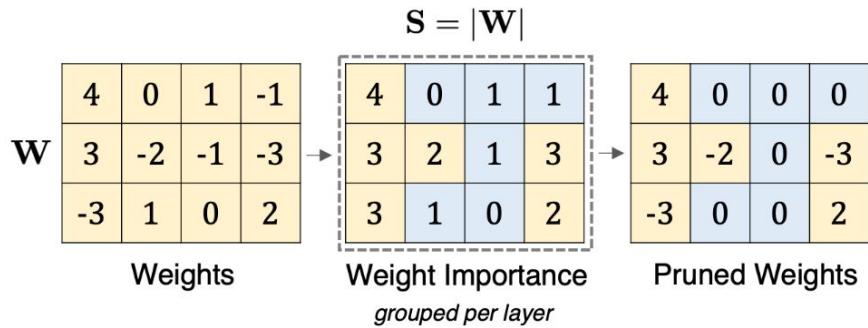
J. Zico Kolter^{1,3}

¹Carnegie Mellon University

²Meta AI Research

³Bosch Center for AI

Magnitude Pruning



Wanda

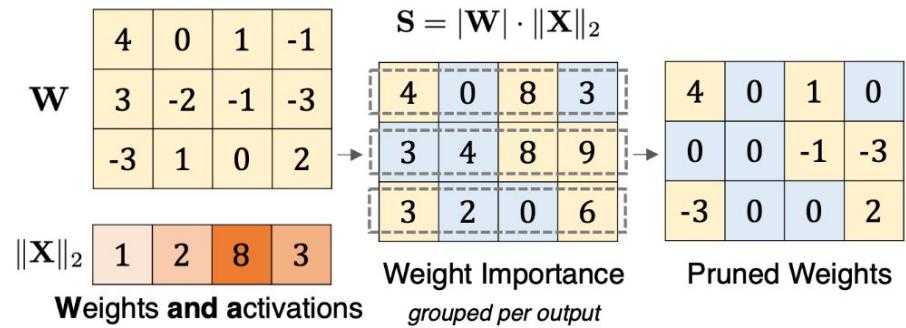


Figure 1: Illustration of our proposed method Wanda (Pruning by **Weights and activations**), compared with the magnitude pruning approach. Given a weight matrix \mathbf{W} and input feature activations \mathbf{X} , we compute the weight importance as the elementwise product between the weight magnitude and the norm of input activations ($|\mathbf{W}| \cdot \|\mathbf{X}\|_2$). Weight importance scores are compared on a *per-output* basis (within each row in \mathbf{W}), rather than globally across the entire matrix.

Are Sixteen Heads Really Better than One?

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Published as a conference paper at ICLR 2019

THE LOTTERY TICKET HYPOTHESIS: FINDING SPARSE, TRAINABLE NEURAL NETWORKS

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Training a pruned randomly-initialized networks can be better than training the full randomly-initialized network

Thank you!