

# Transformers

**CS 4804: Introduction to AI**

*Fall 2025*

<https://tuvllms.github.io/ai-fall-2025/>

**Tu Vu**



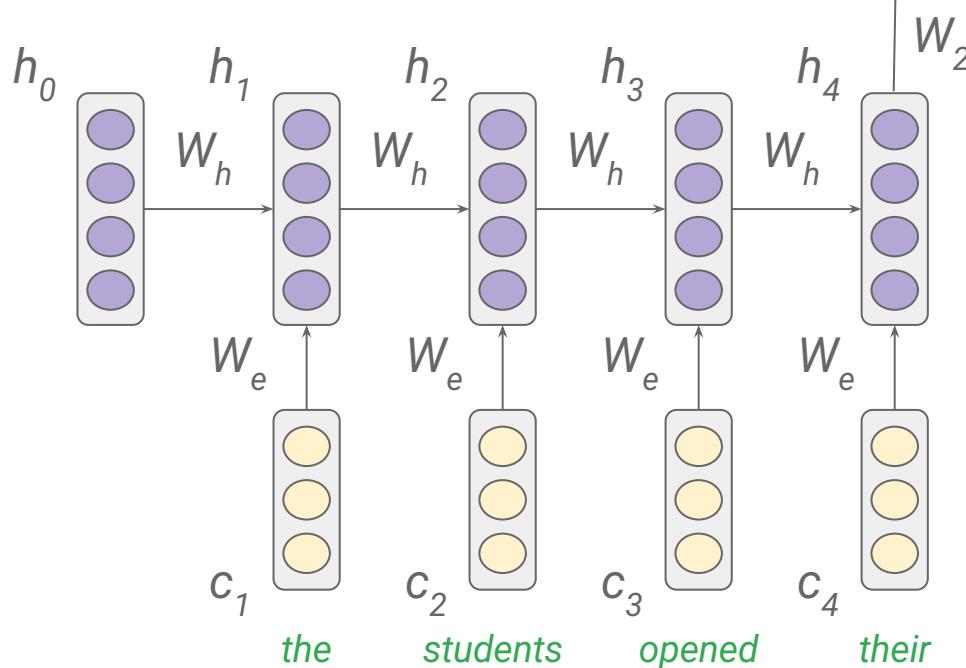
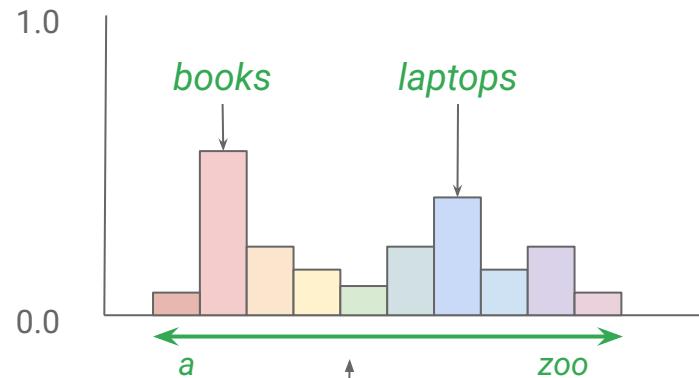
# Logistics

- Homework 0 (due September 16<sup>th</sup>)
  - accuracy
- Quiz 0 (due tomorrow)
  - genuine attempt
- Final project group information (tomorrow)

# Recurrent neural networks (RNNs)

## hidden states

$$h^{(t)} = f(W_h h^{(t-1)} + W_e c^t)$$



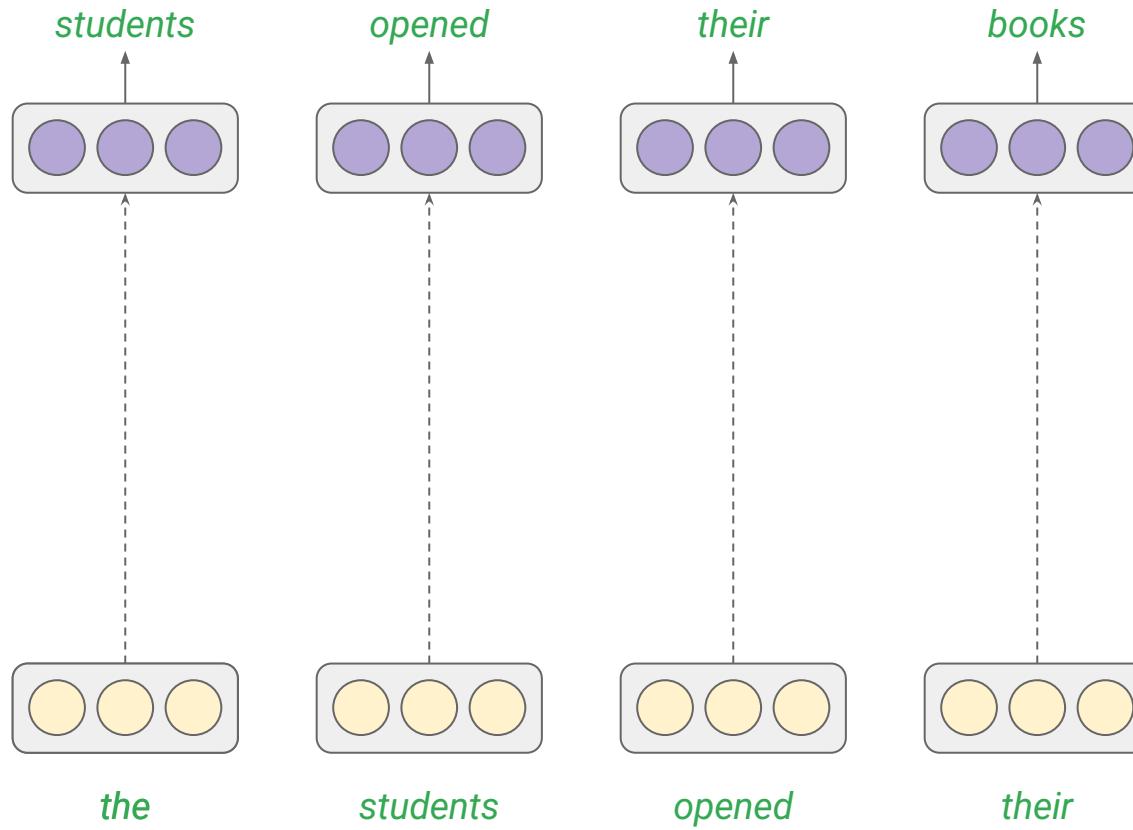
## output distribution

$$\hat{y} = \text{softmax}(W_2 h^{(n-1)})$$

# Problems with RNNs

- Bottleneck representation issue
- Lack of parallelism

# Seq2Seq



# Transformers

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## Attention Is All You Need

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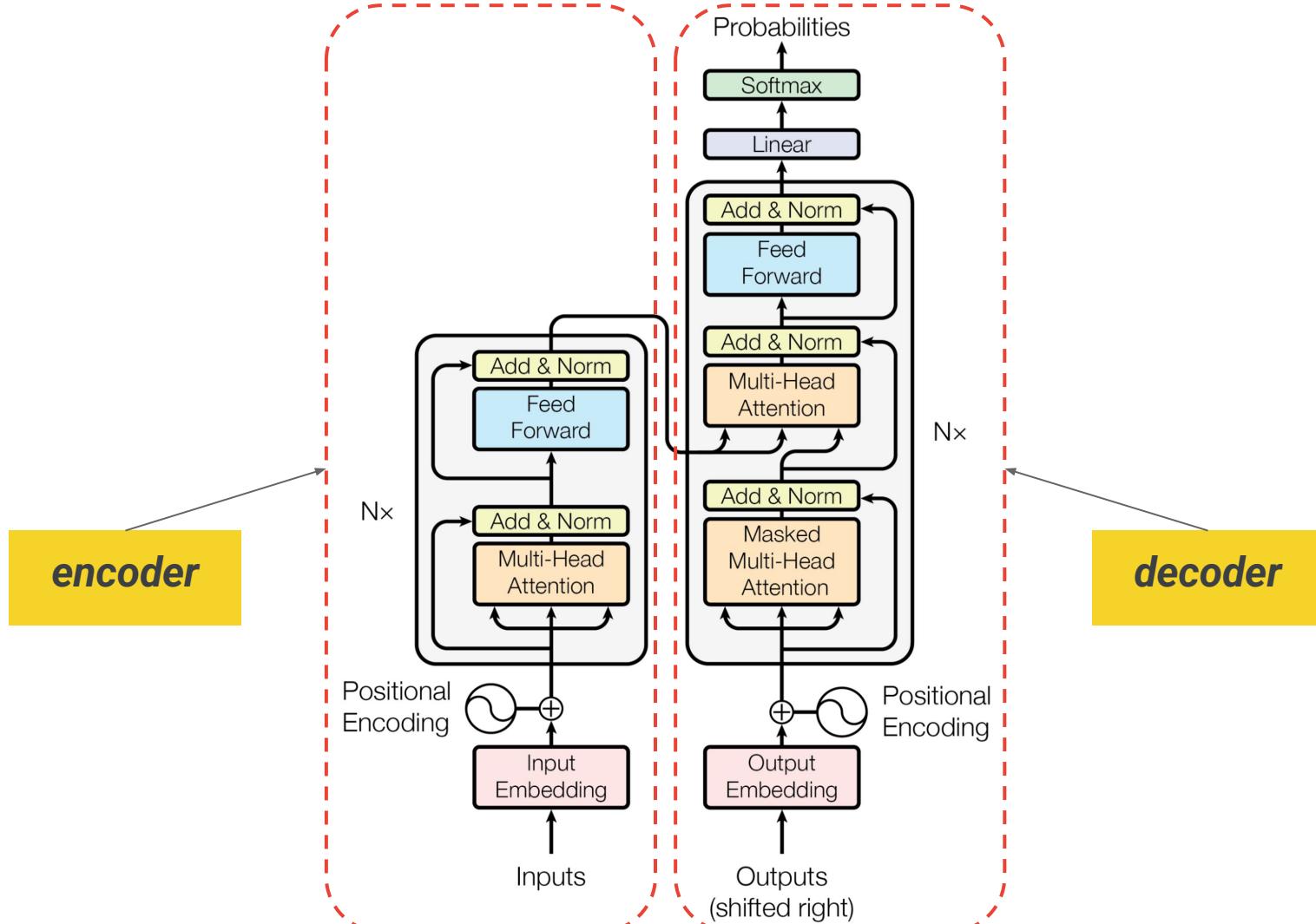
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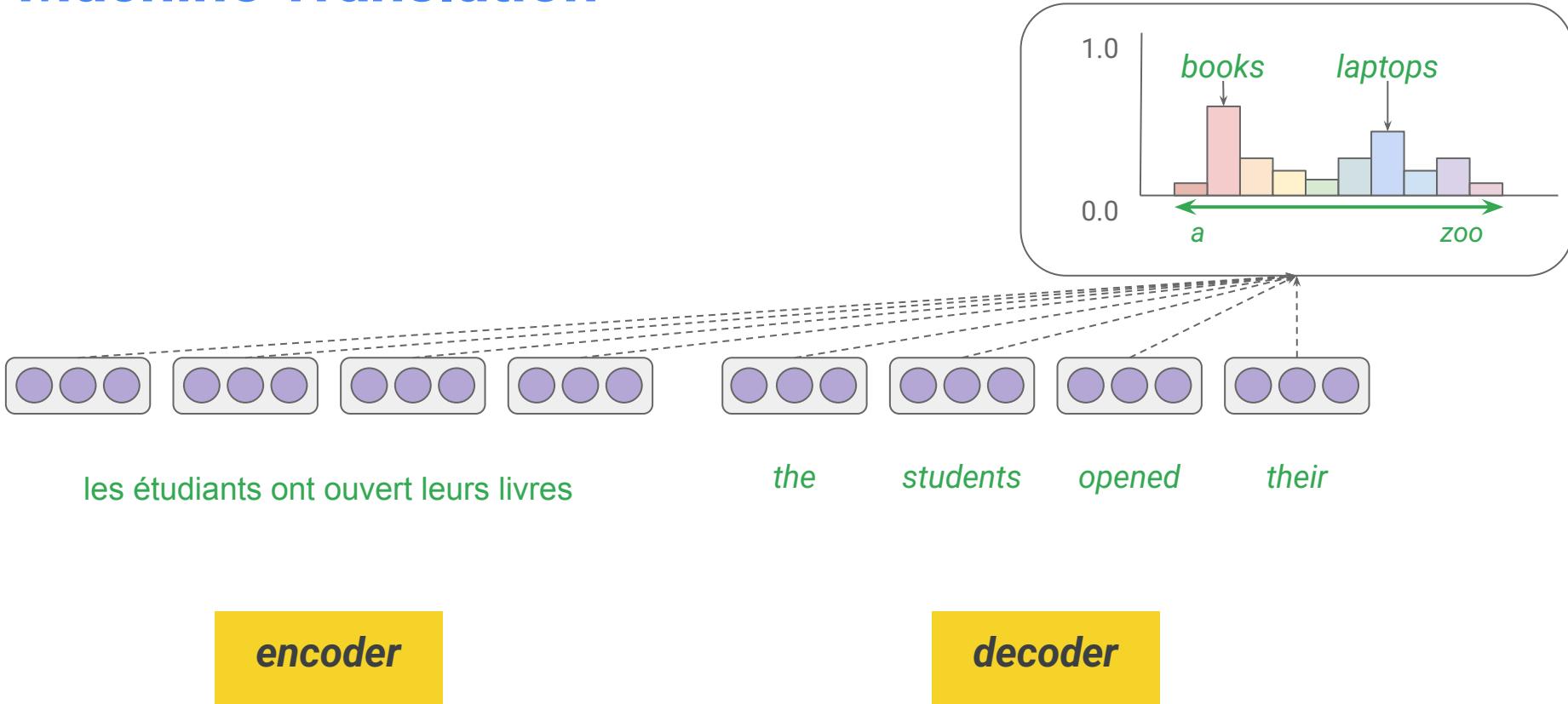


# Transformers

- Before 2017
  - Recurrent neural networks (RNNs)
    - LSTM (Long Short-Term Memory)
  - Convolutional neural networks (CNNs)
- These days
  - Transformers



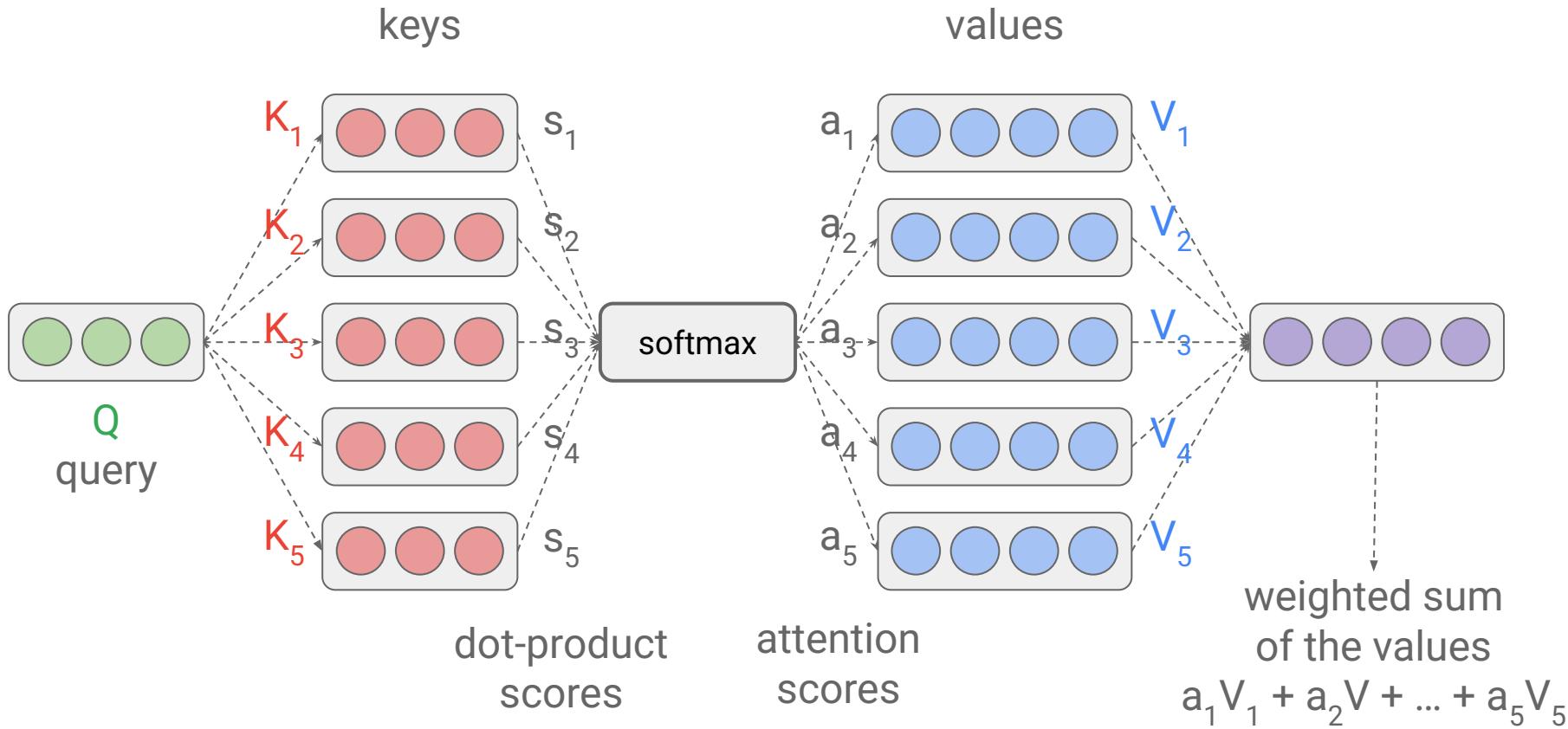
# Machine Translation



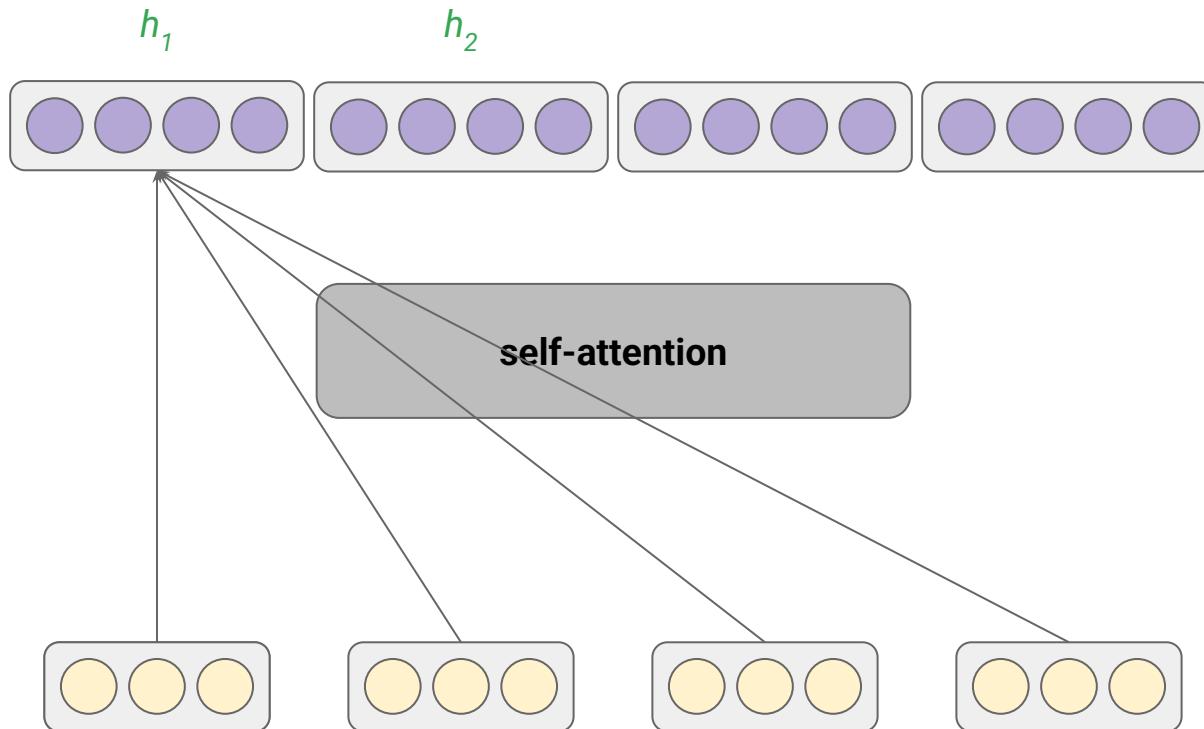
# Different model architectures

- Encoder-only
  - BERT
- Encoder-decoder
  - T5
- Decoder-only
  - GPT

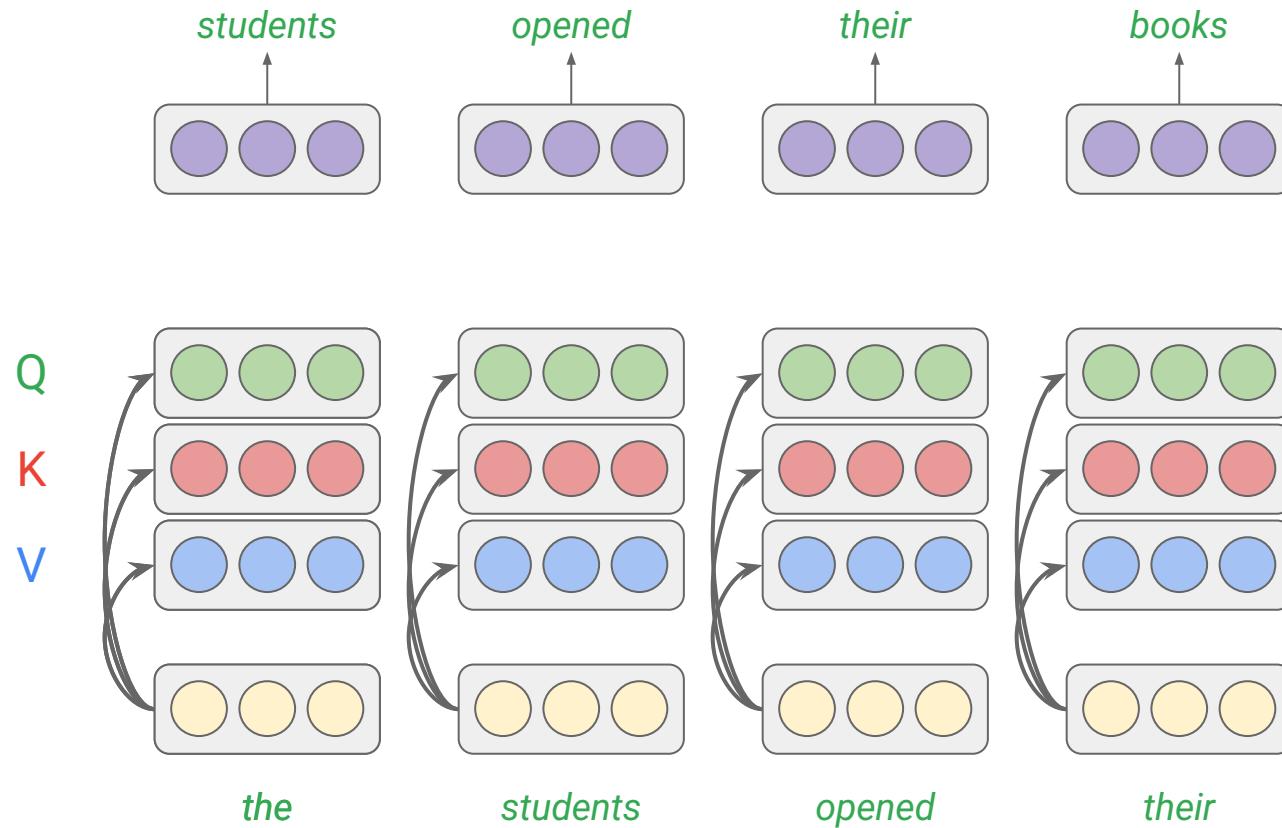
# Attention mechanism



# Self-attention



# Self-attention (cont'd)



$$Q = X \cdot W_Q$$

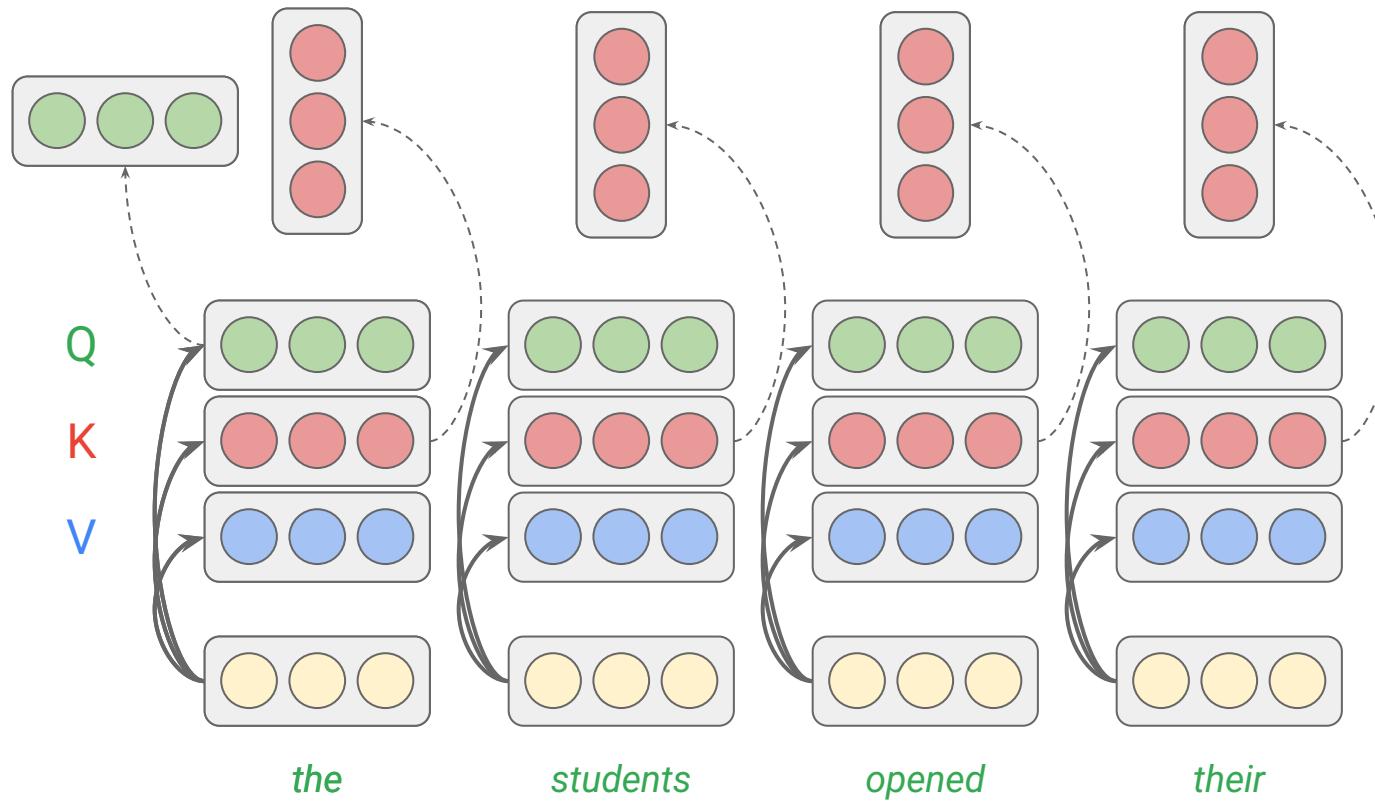
$$K = X \cdot W_K$$

$$V = X \cdot W_V$$

linear  
projections

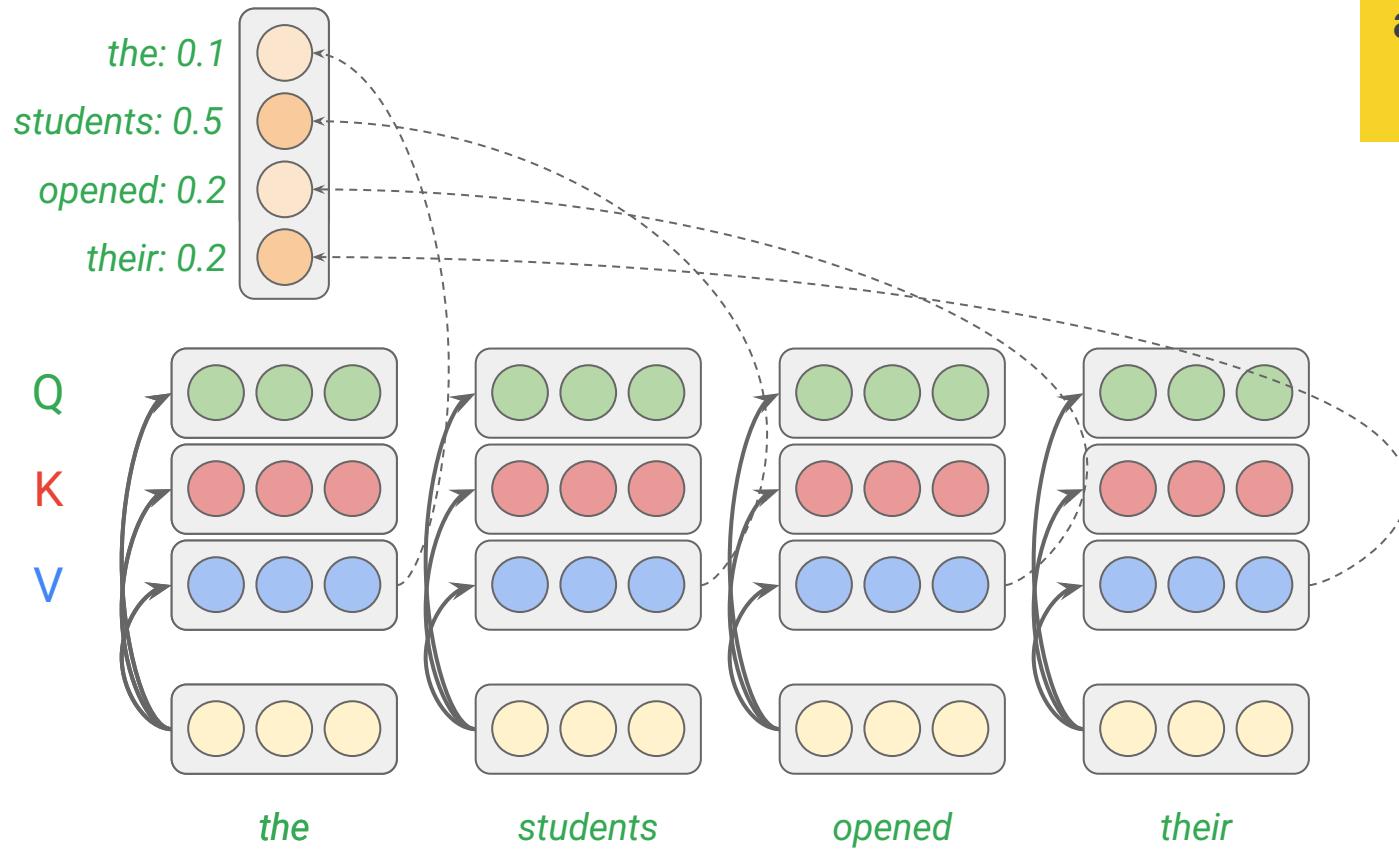
# Self-attention (cont'd)

all computations  
are parallelized



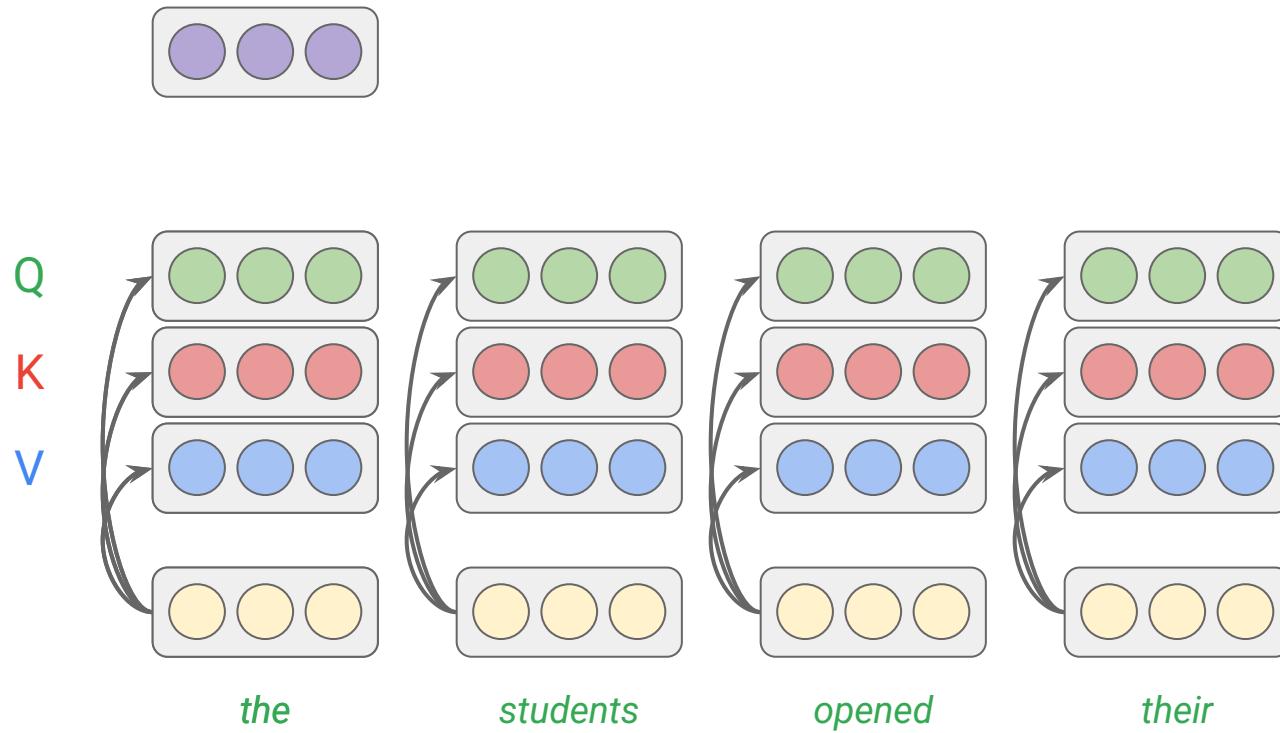
# Self-attention (cont'd)

all computations  
are parallelized



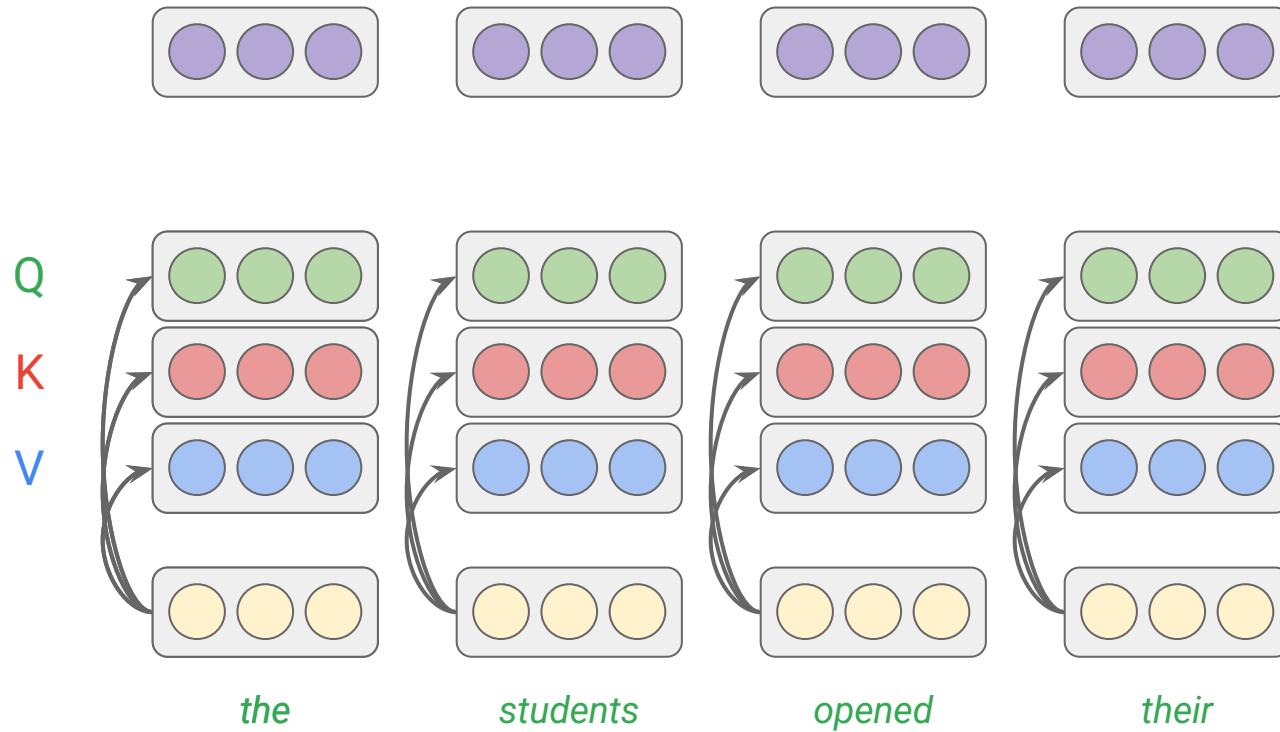
# Self-attention (cont'd)

all computations  
are parallelized



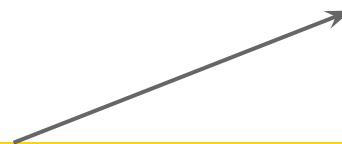
# Self-attention (cont'd)

**all computations  
are parallelized  
during training  
and sequential  
during inference**



# All computations are parallelized

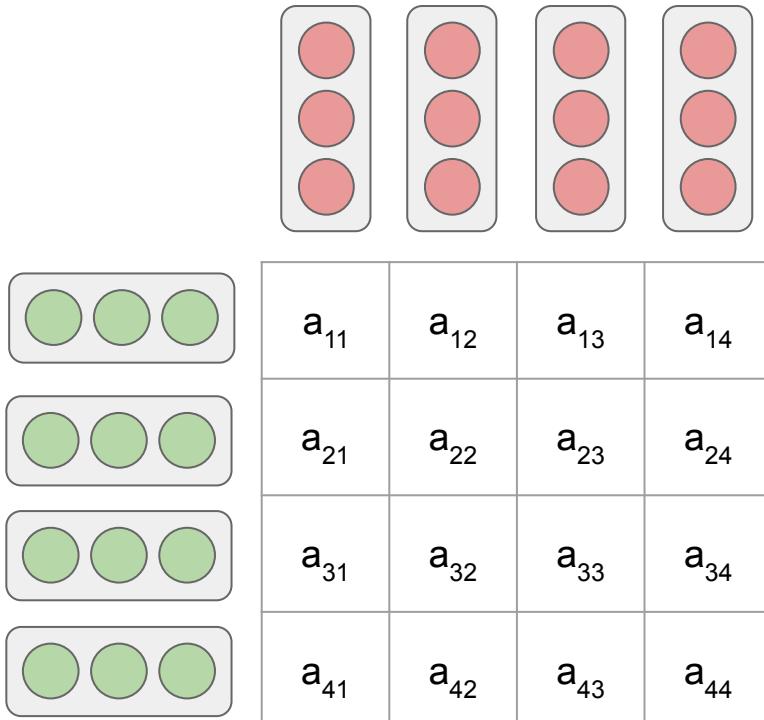
$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V$$



*$d_k$ : scaling factor*

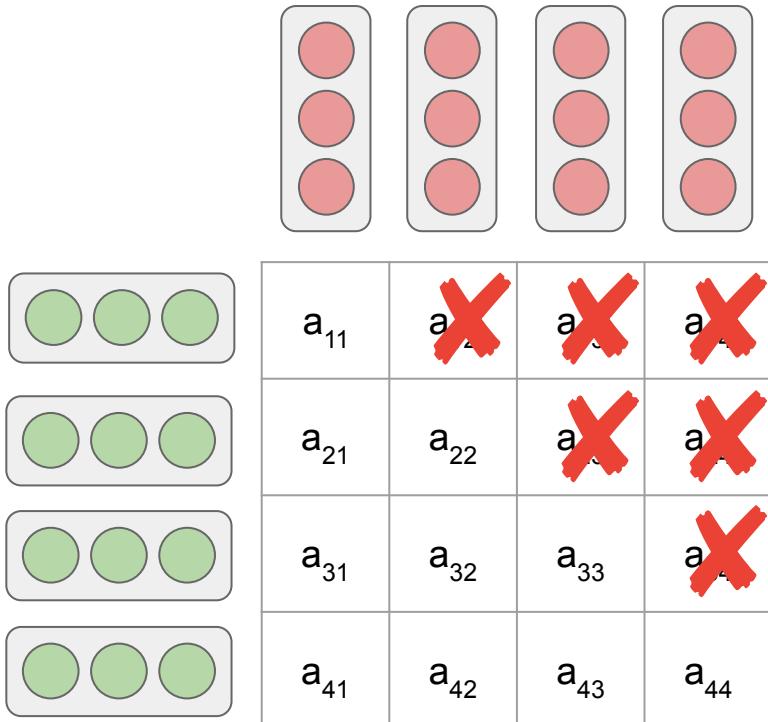
*large products push the softmax function into regions where it has extremely small gradients*

# Quadratic complexity



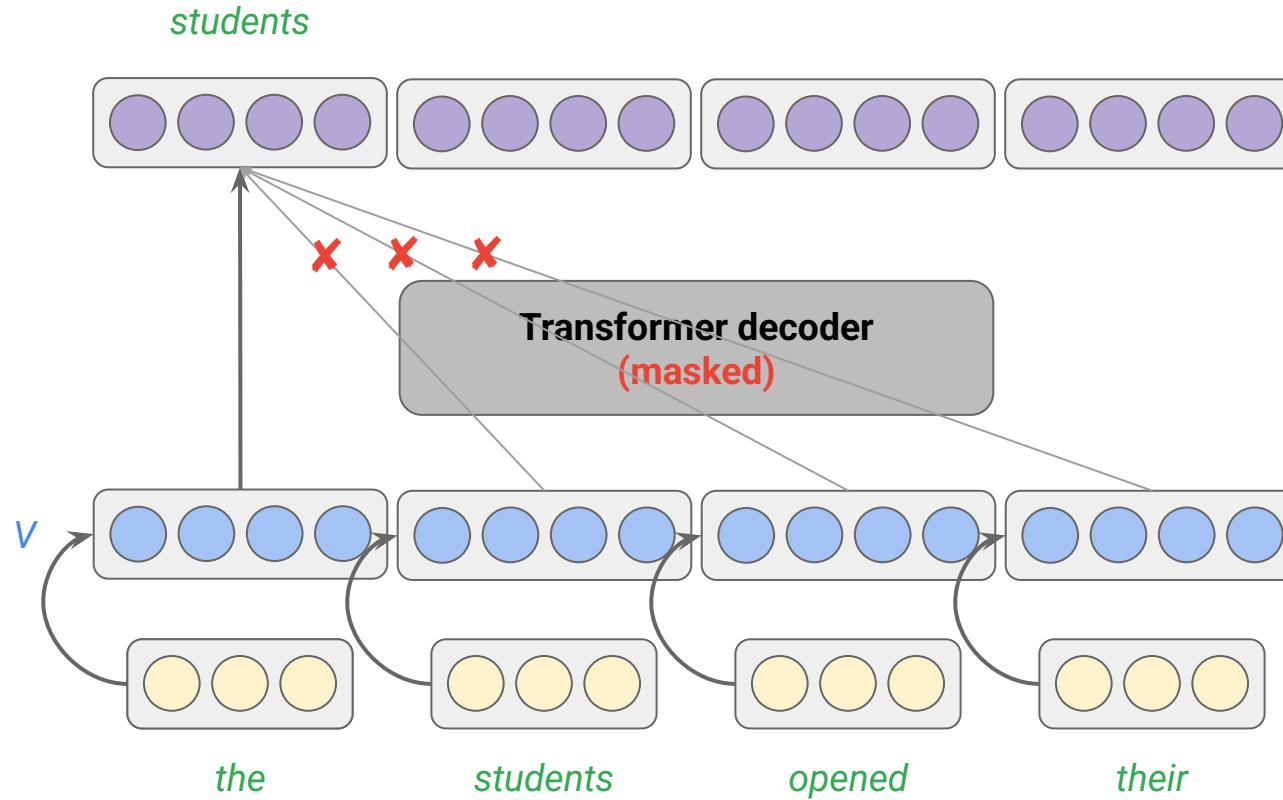
*The time complexity of self-attention is quadratic in the input length  $O(n^2)$*

# Self-attention in the decoder

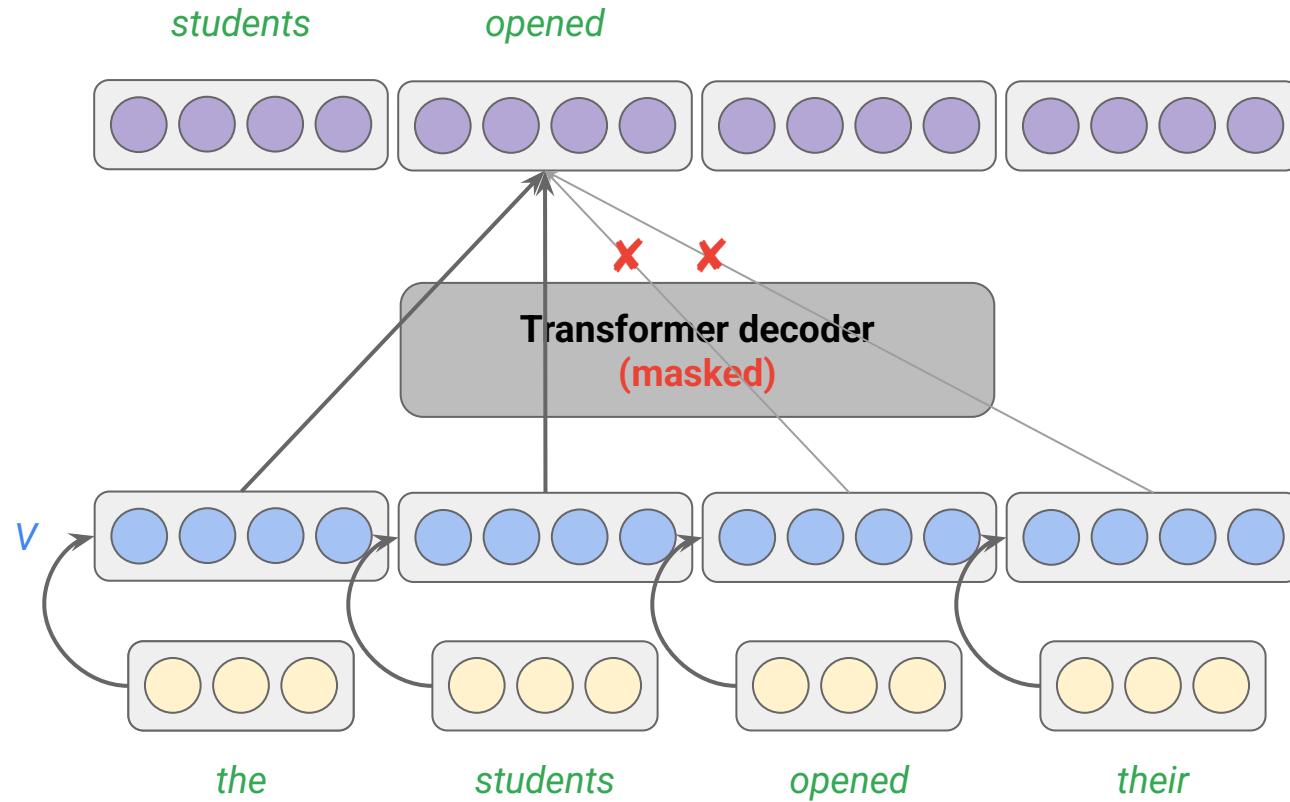


*masking out all values in  
the input of the softmax  
which correspond to  
illegal connections*

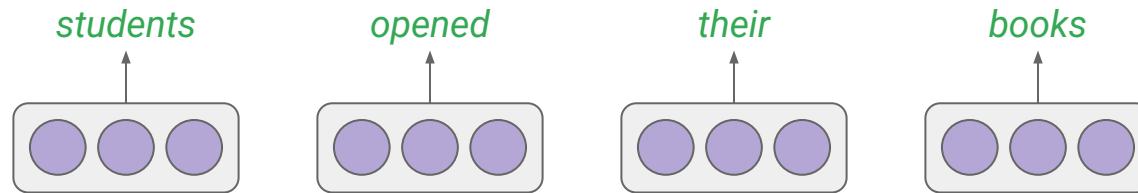
# Transformer decoder



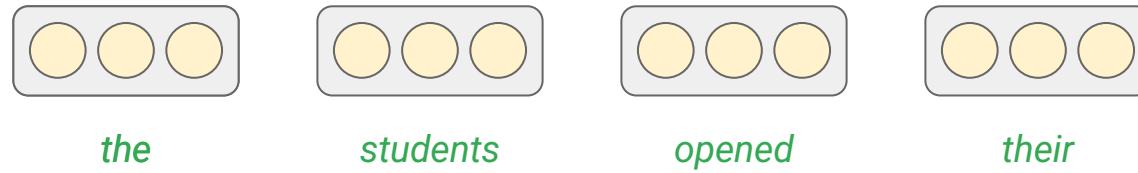
# Transformer decoder (cont'd)



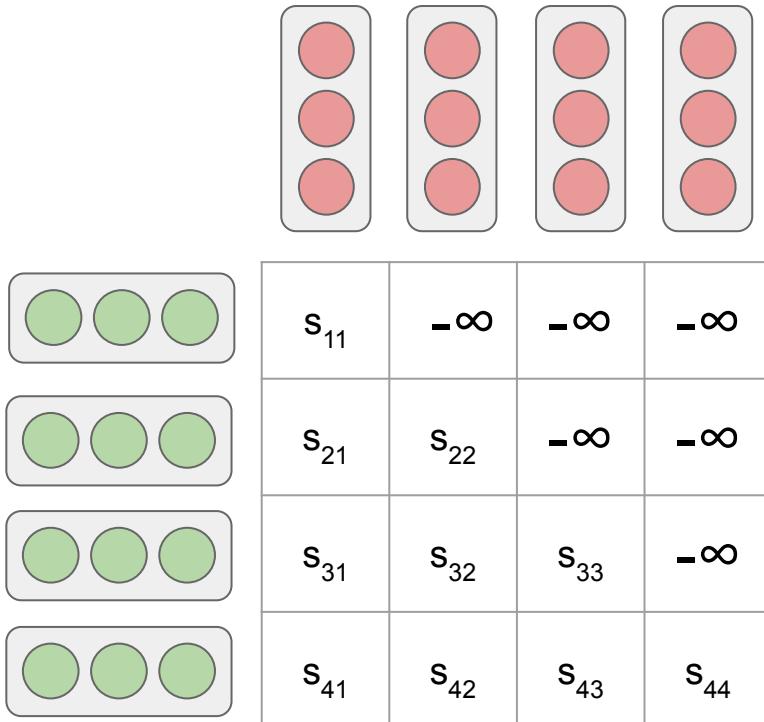
# Transformer decoder (cont'd)



Transformer decoder  
(masked)

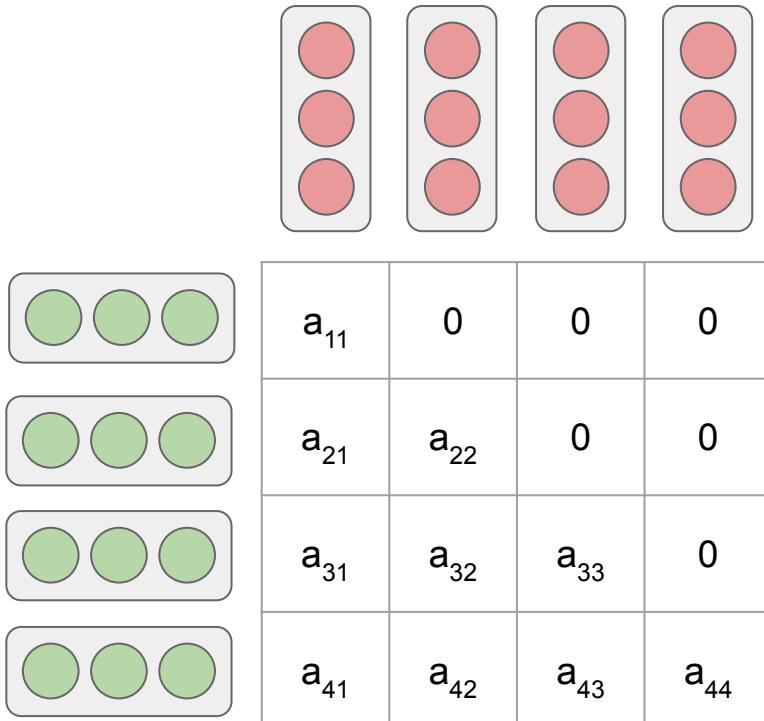


# Self-attention in the decoder



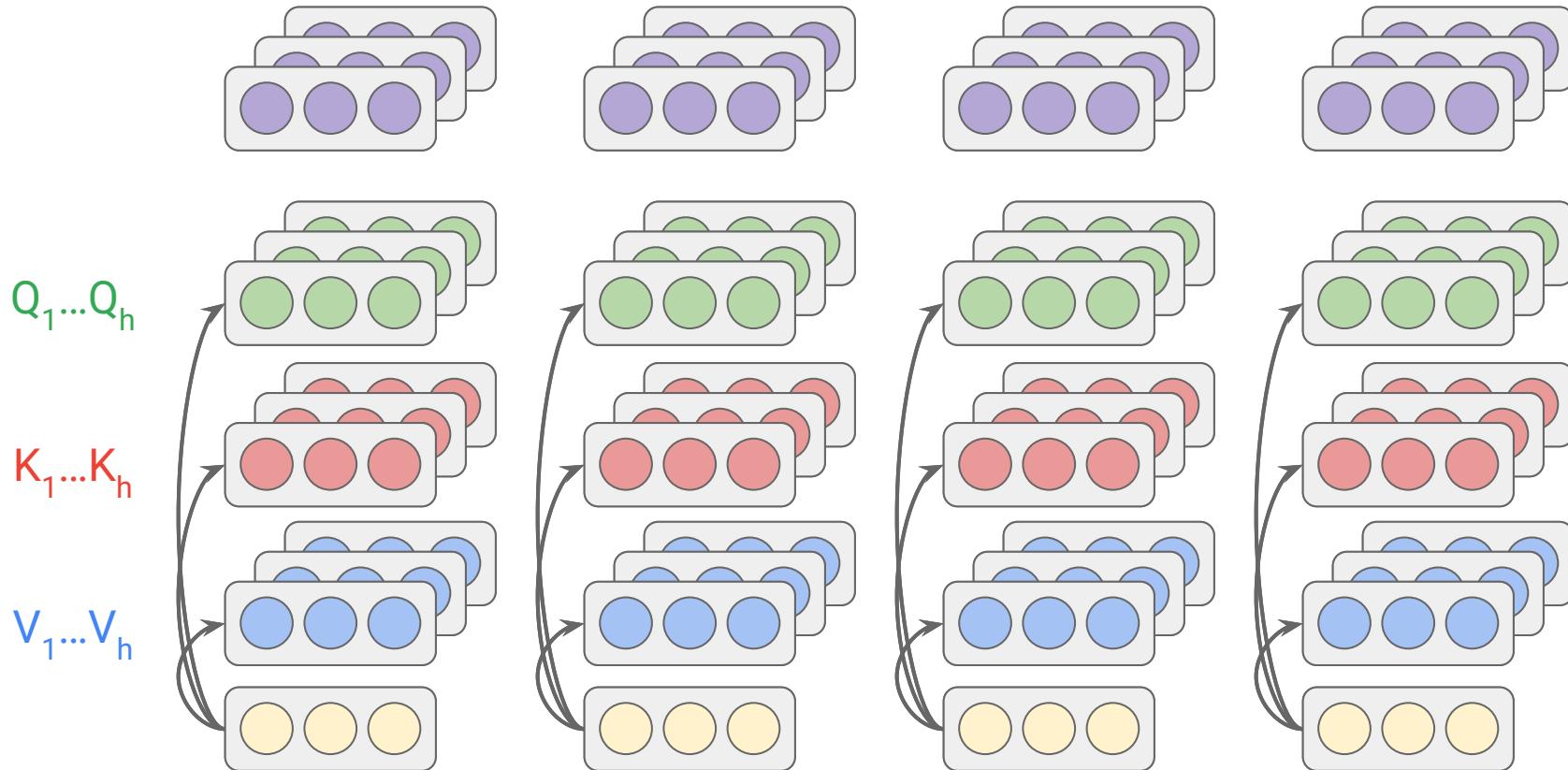
***masking out (setting to  $-\infty$ ) all values in the input of the softmax which correspond to illegal connections***

# Self-attention in the decoder (cont'd)

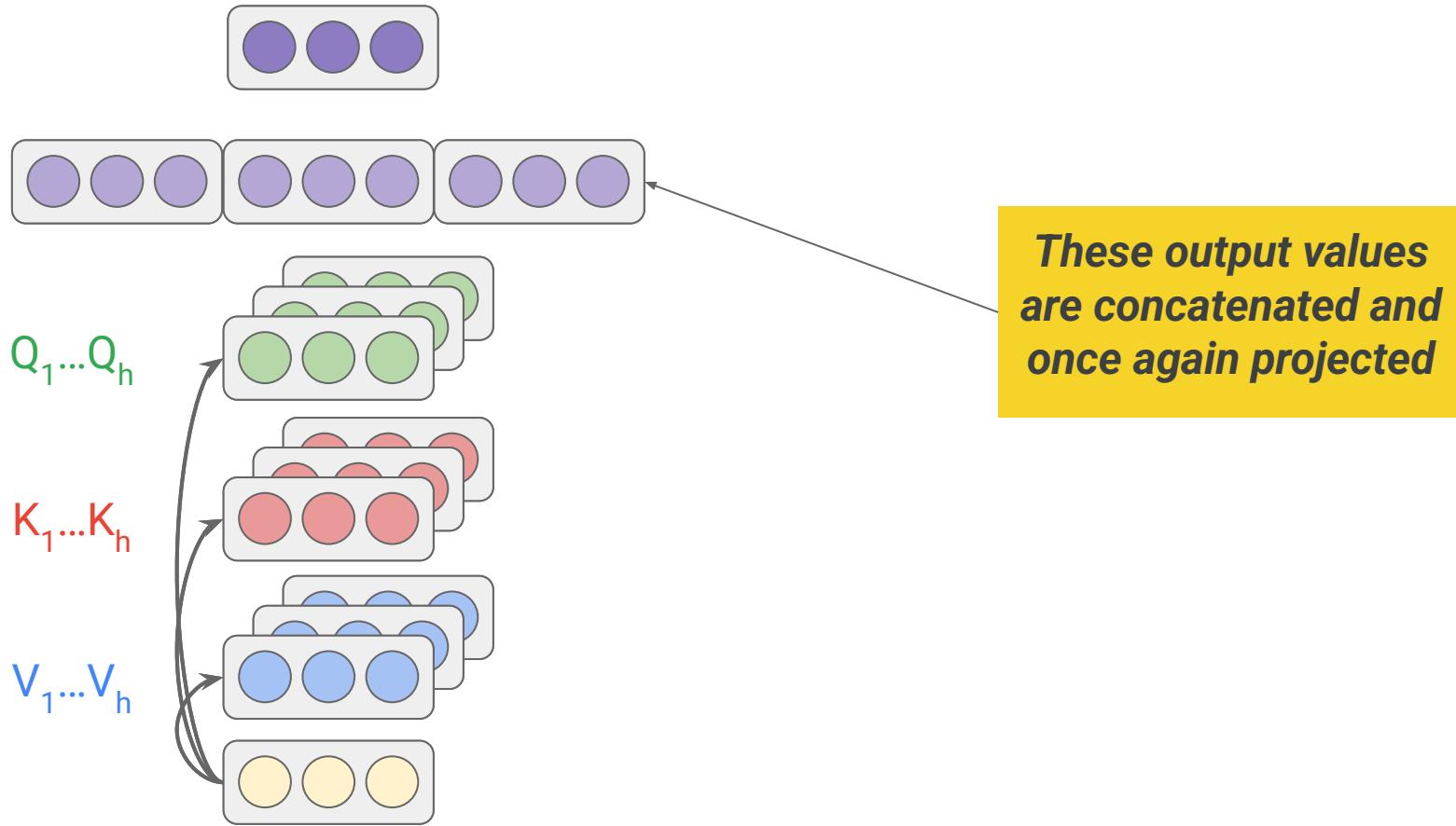


*masking out all values in  
the input of the softmax  
which correspond to  
illegal connections*

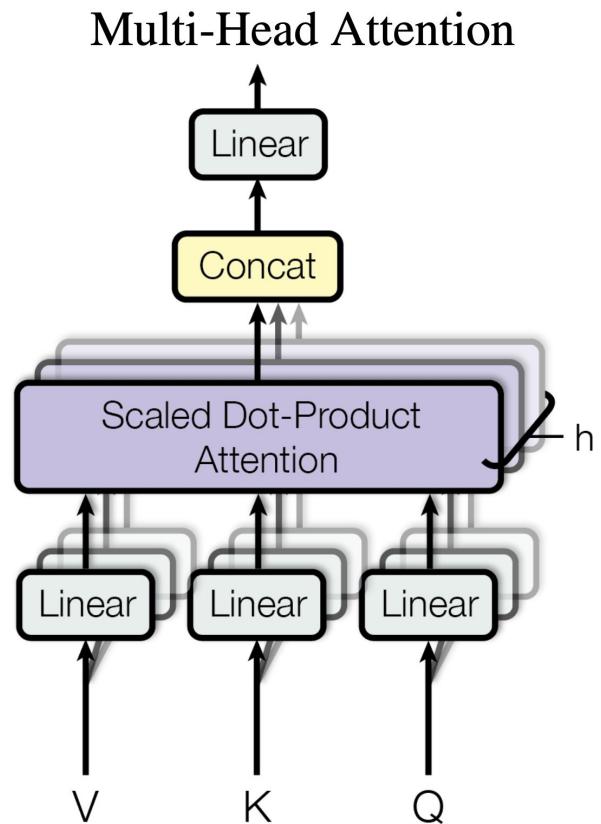
# Multi-head attention



# Multi-head attention (cont'd)

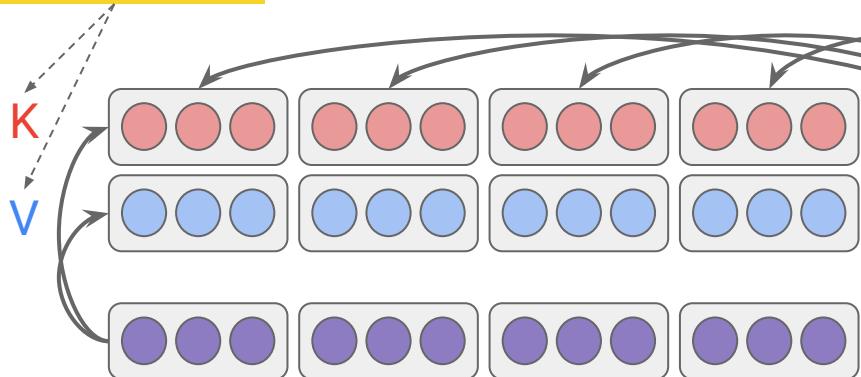


# Multi-head attention (cont'd)



# Cross-attention in the decoder

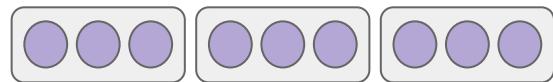
linear  
projections



Multi-head Attention  
(unmasked)



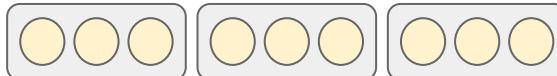
encoder



Multi-head cross-attention  
(unmasked)



Multi-head Attention  
(masked)

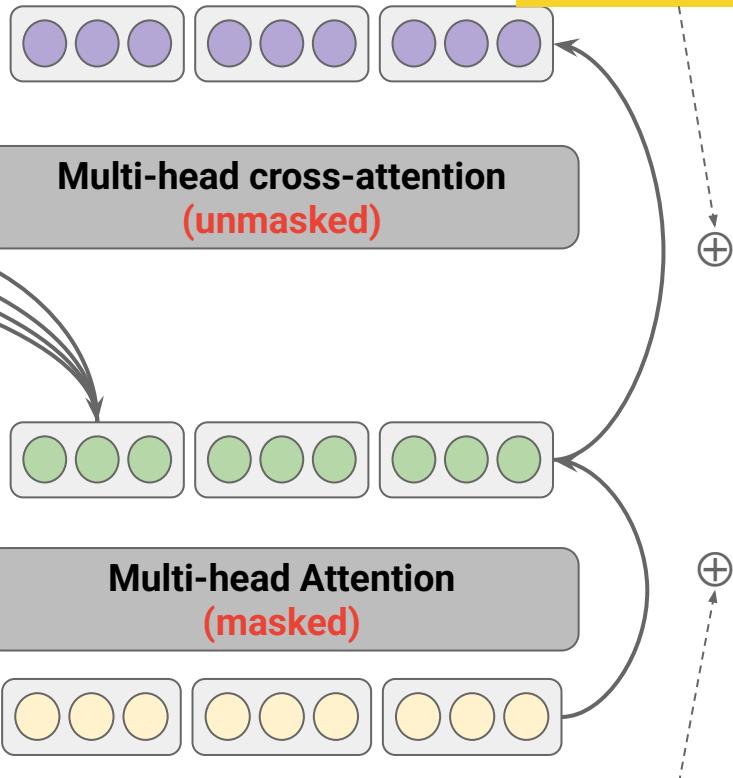
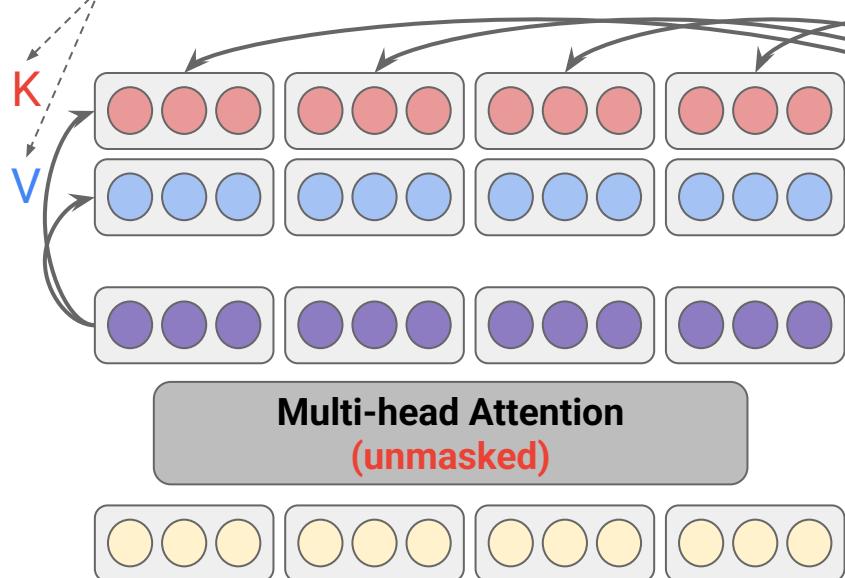


decoder

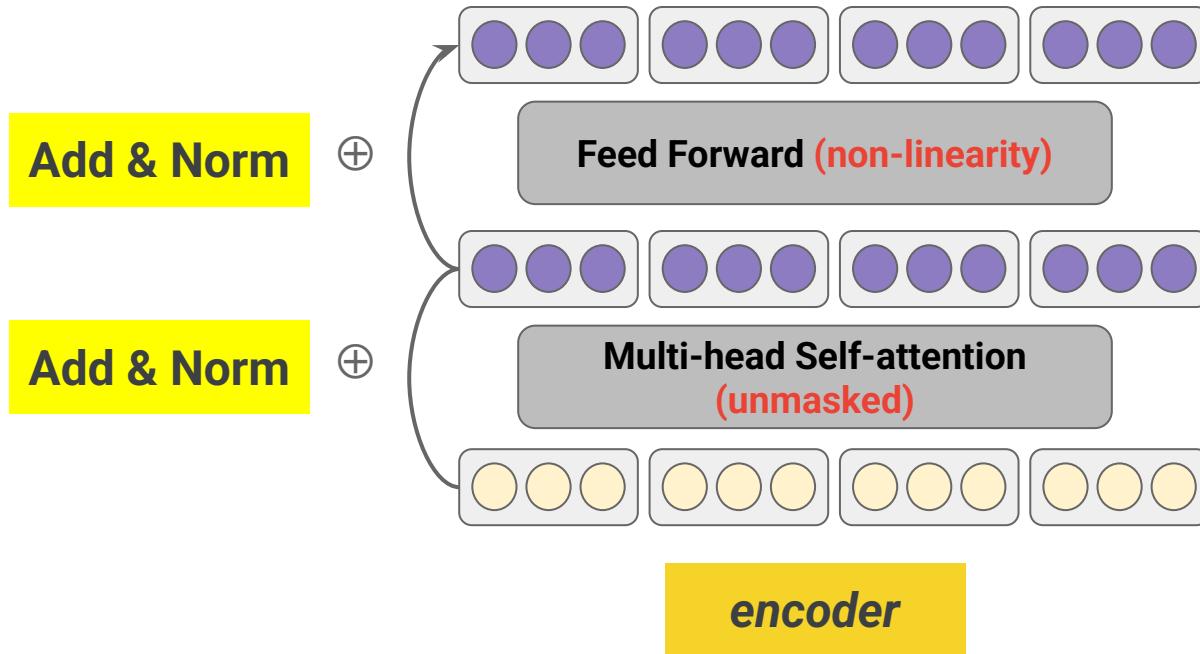
# Cross-attention in the decoder (cont'd)

residual connections

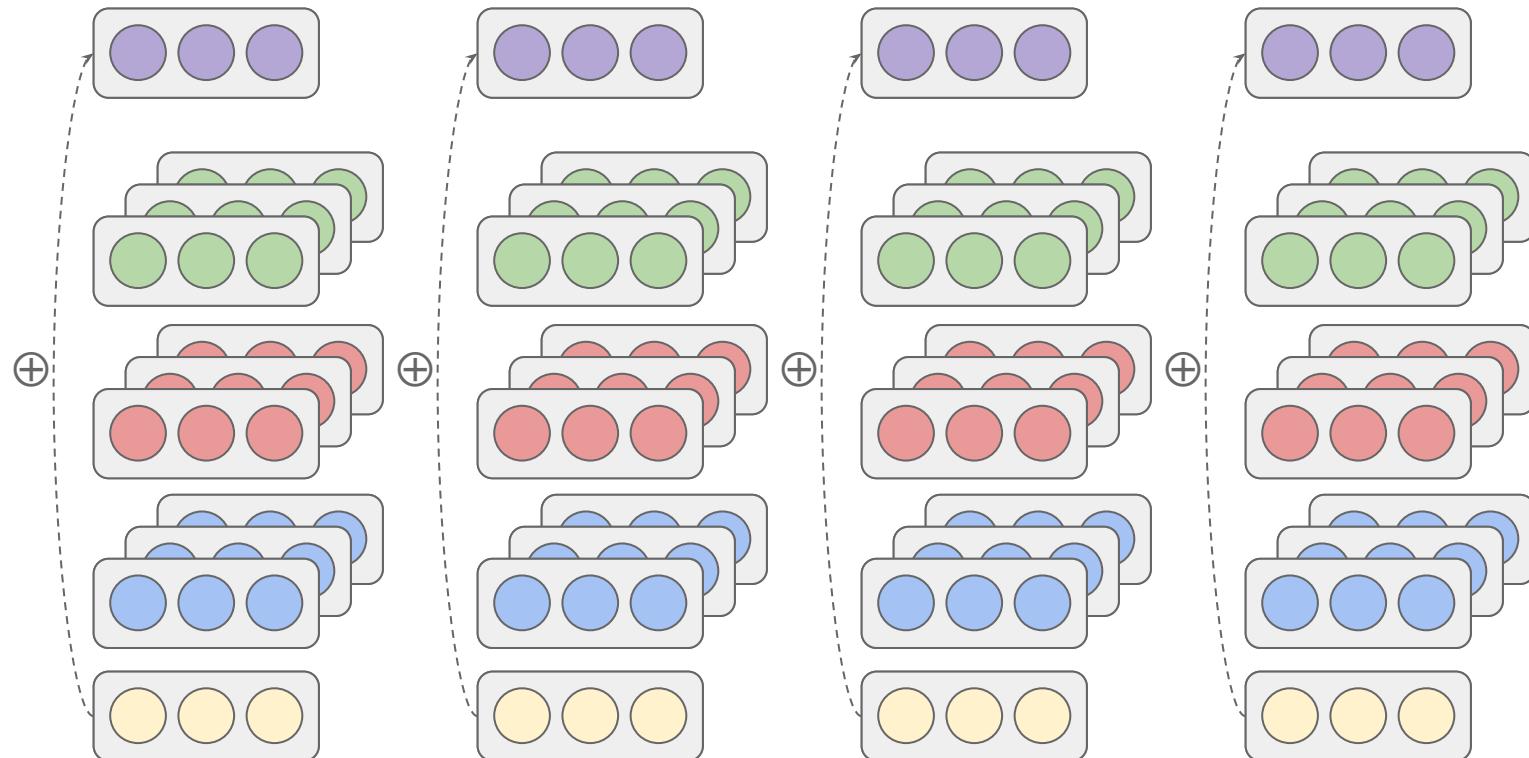
linear projections



# Encoder (one layer)



# Residual connection



## Residual connection

$$\text{output} = \text{sublayer}(x) + x$$

# Layer normalization

$$\text{Norm}(z) = \frac{z - \mu}{\sigma} \cdot \gamma + \beta$$

where  $\mu$  and  $\sigma$  are the mean and standard deviation of the activations, and  $\gamma, \beta$  are learnable parameters.

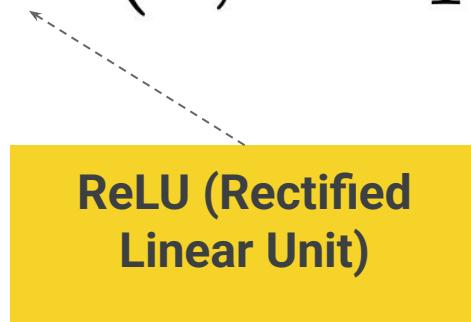
*Each activation vector is normalized so that its components have mean 0 and variance 1. This prevents activations from becoming too large or too small as they propagate through the network.*

# Residual connection and layer normalization

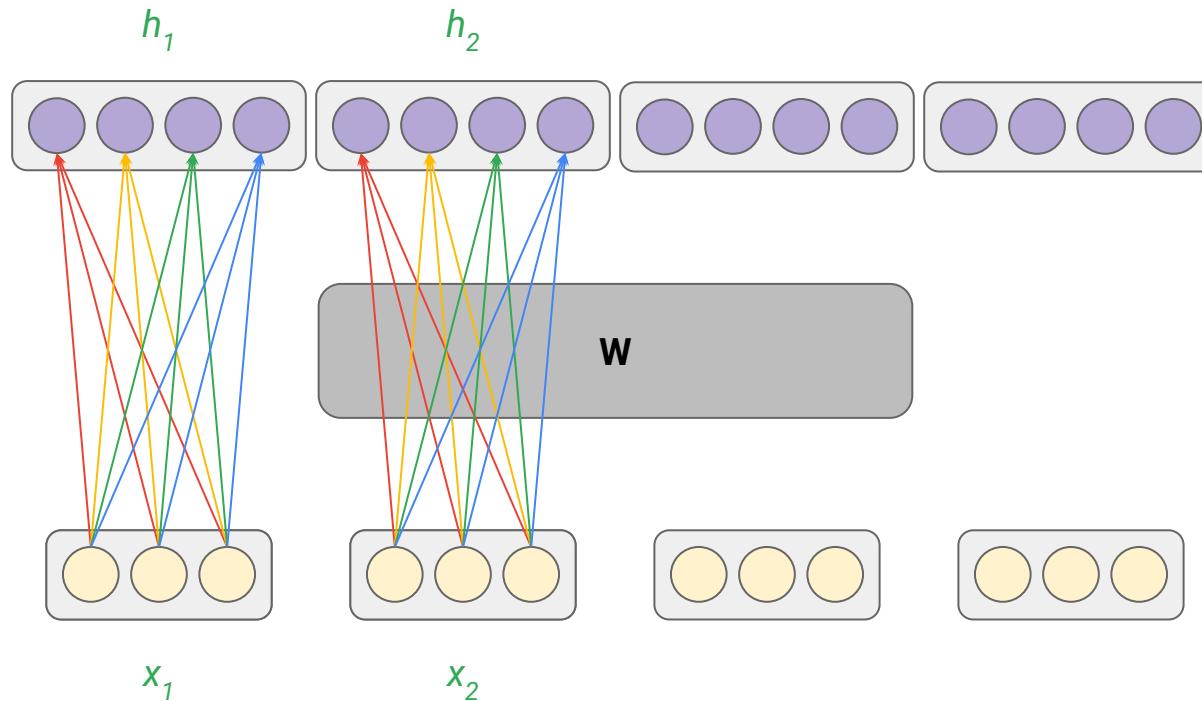
$$\text{LayerNorm}(x + \text{Sublayer}(x))$$

# Position-wise Feed-Forward Networks

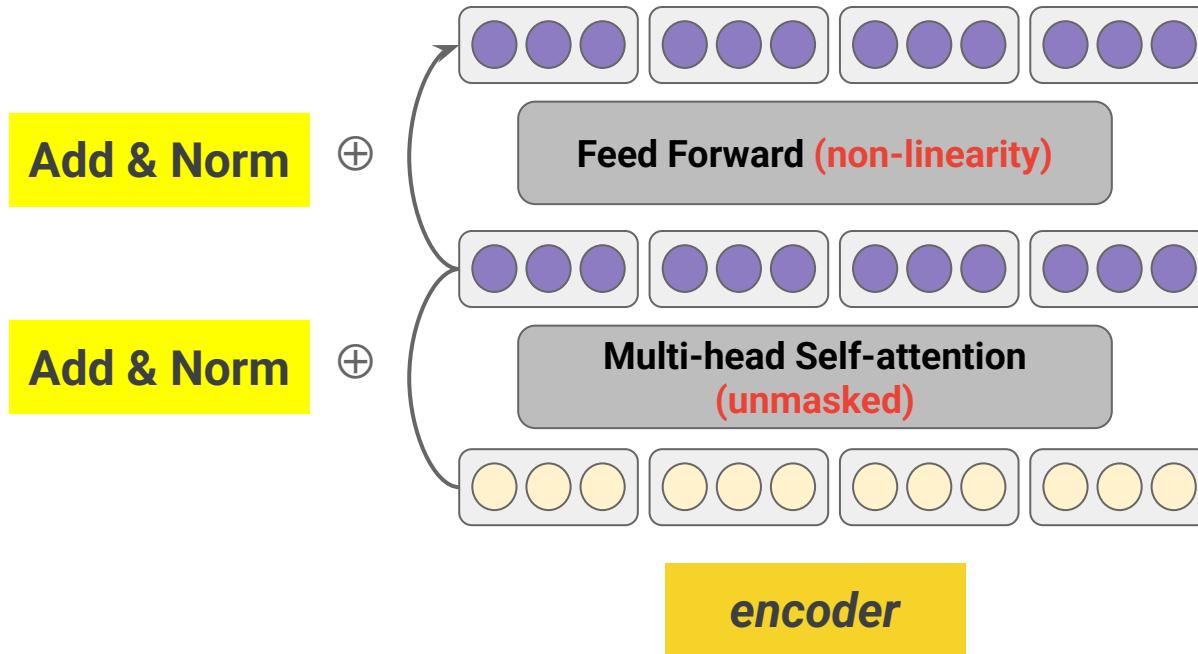
$$\text{FFN}(x) = \max(0, xW_1 + b_1)W_2 + b_2$$



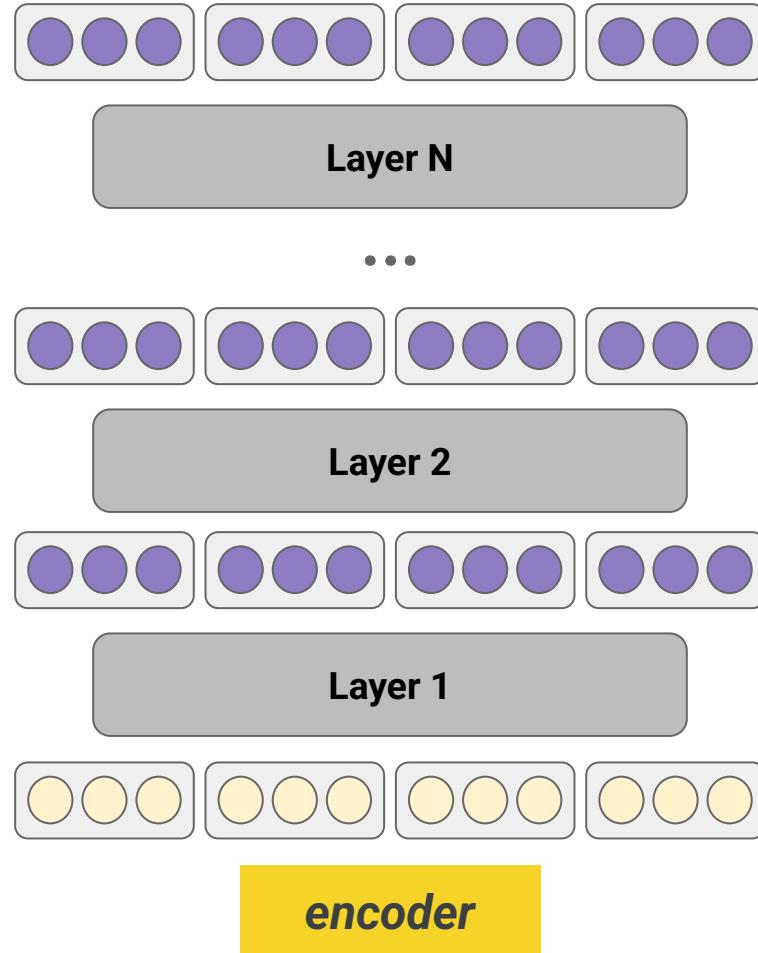
# Position-wise Feed-Forward Networks (cont'd)



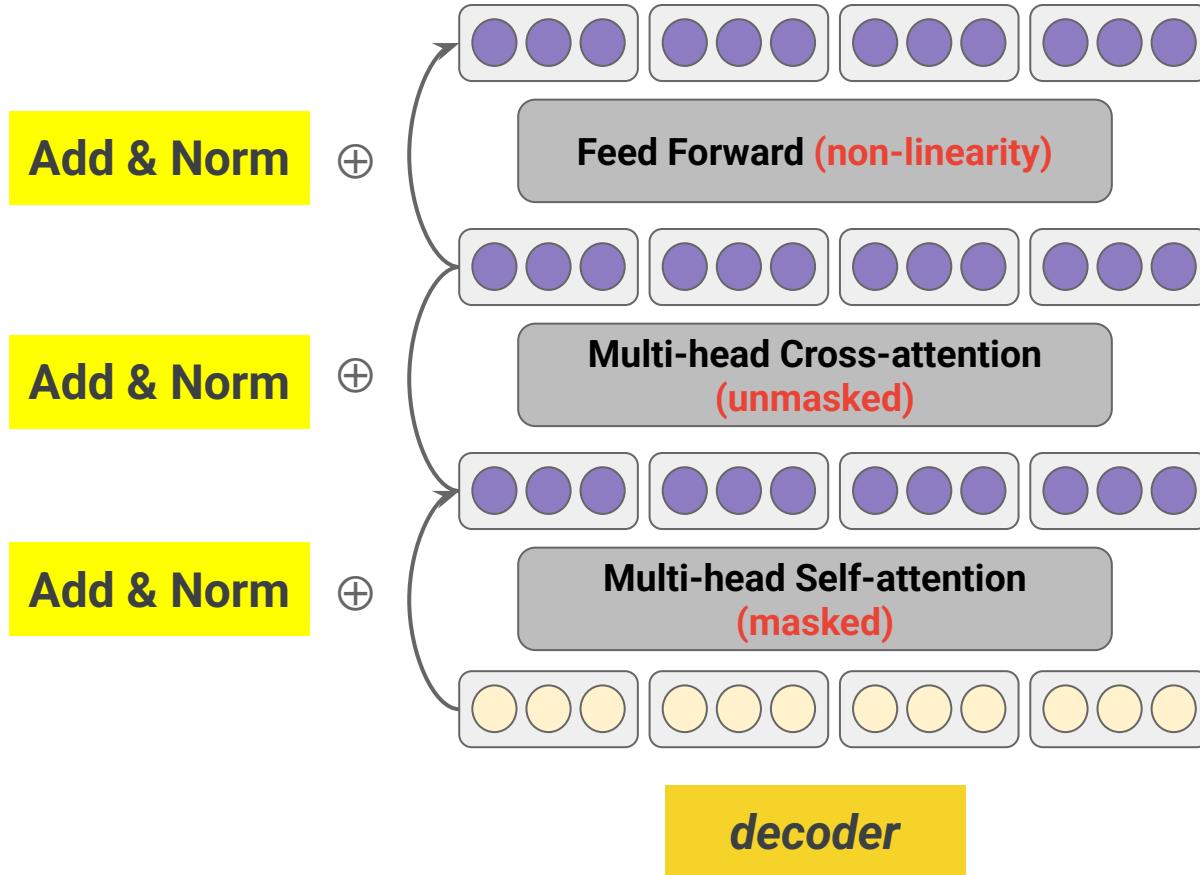
# Encoder (one layer)



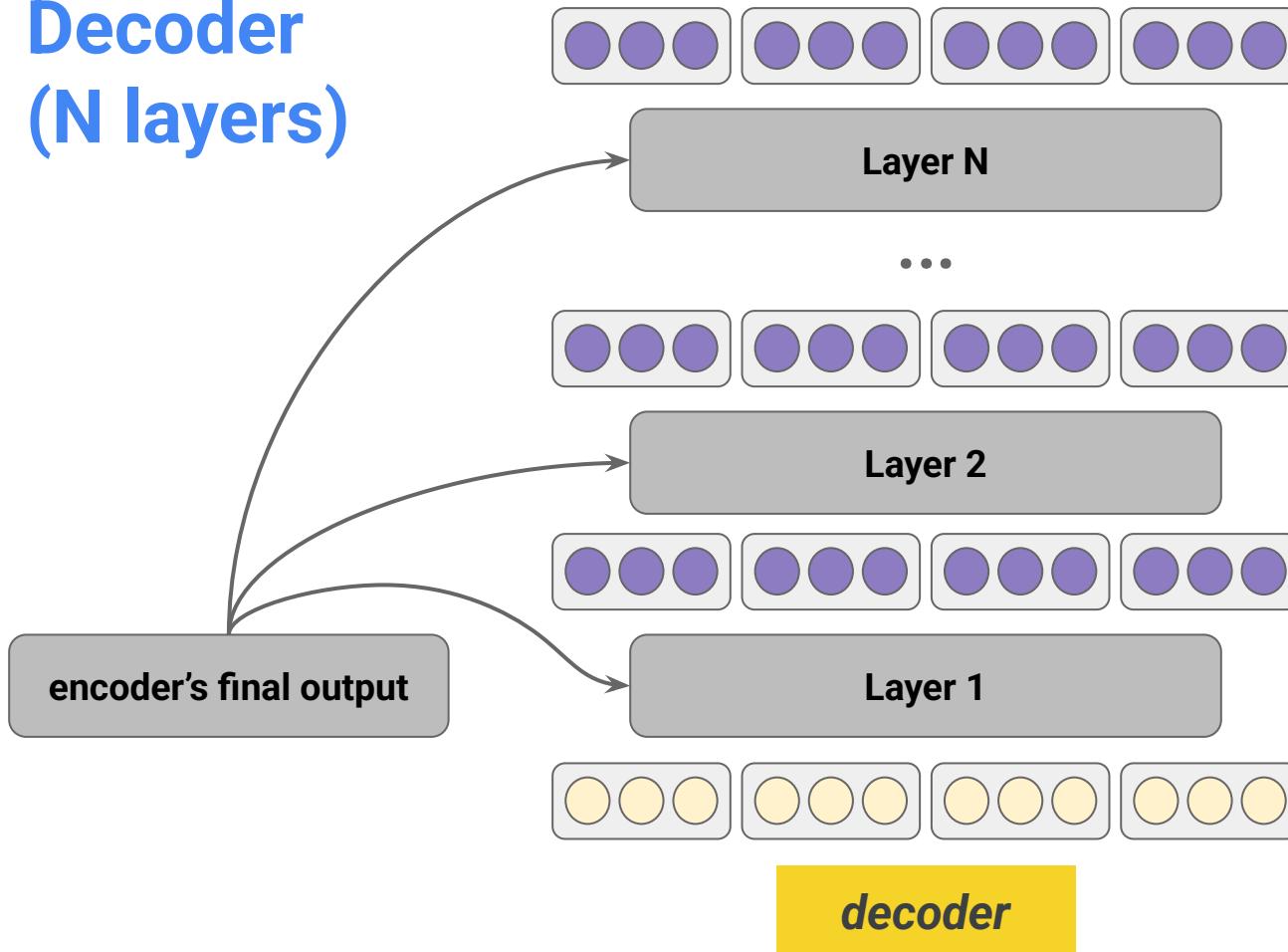
# Encoder (N layers)



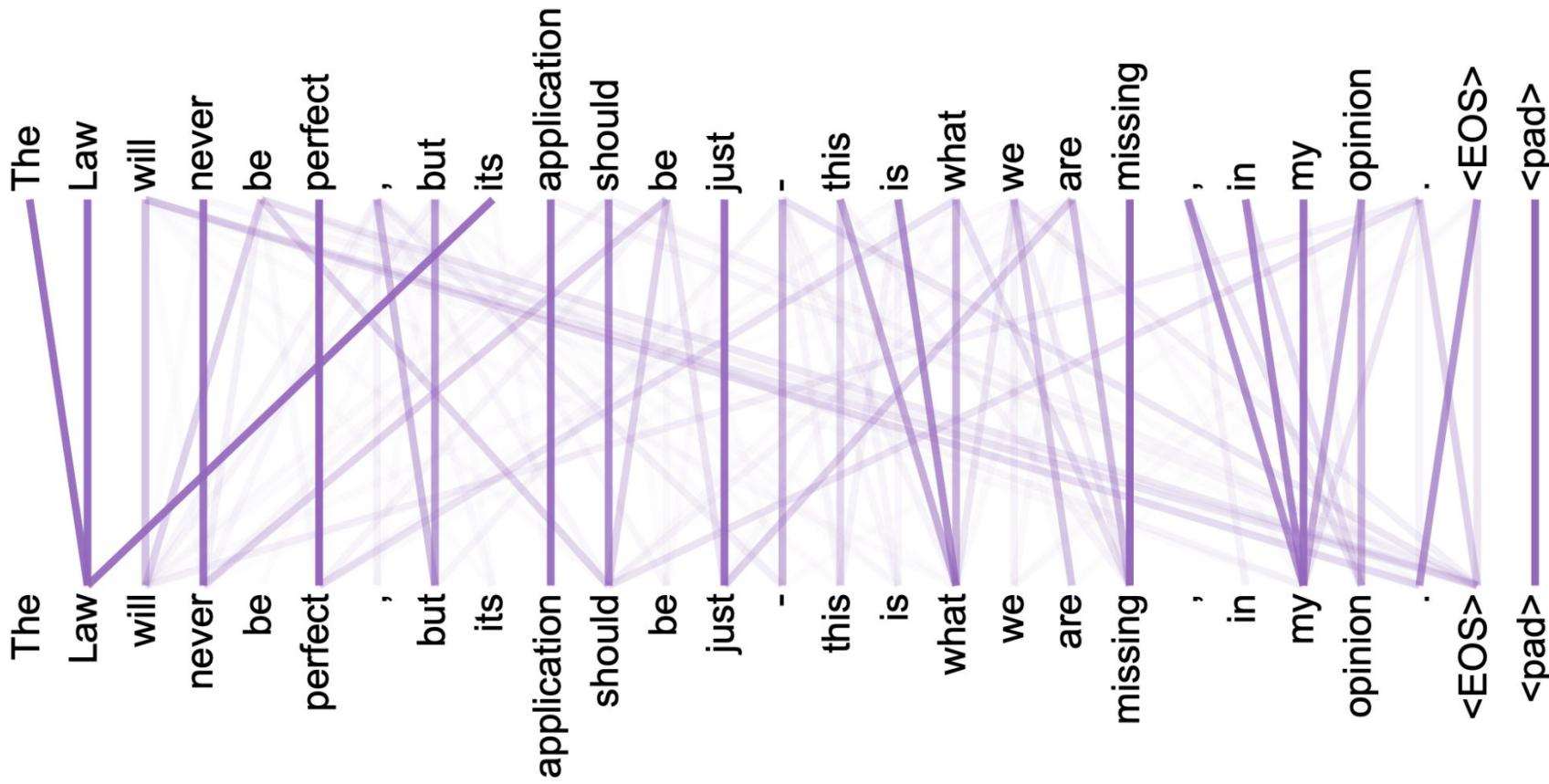
# Decoder (one layer)



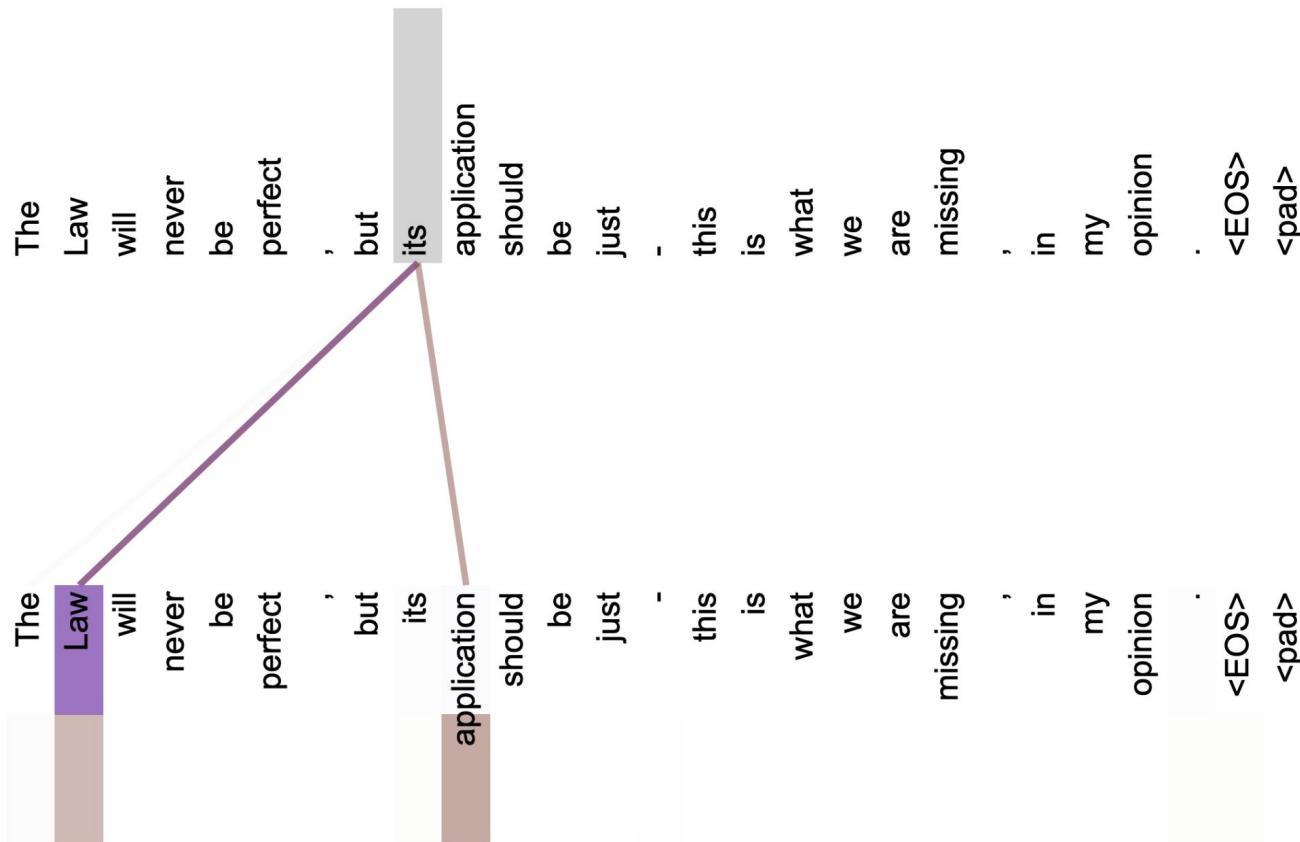
# Decoder (N layers)



# Attention visualizations



# Attention visualizations (cont'd)

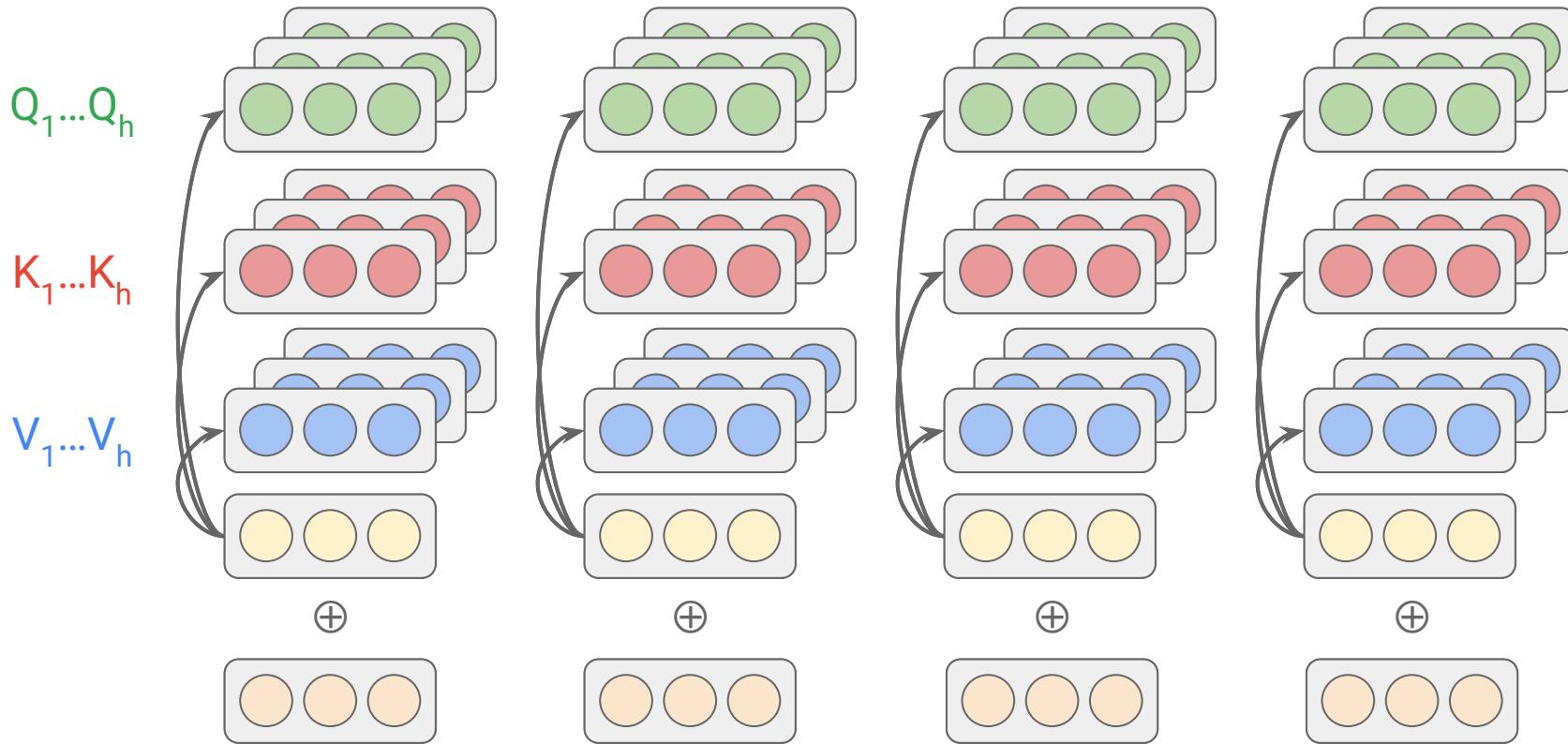


# Sinusoidal positional encoding

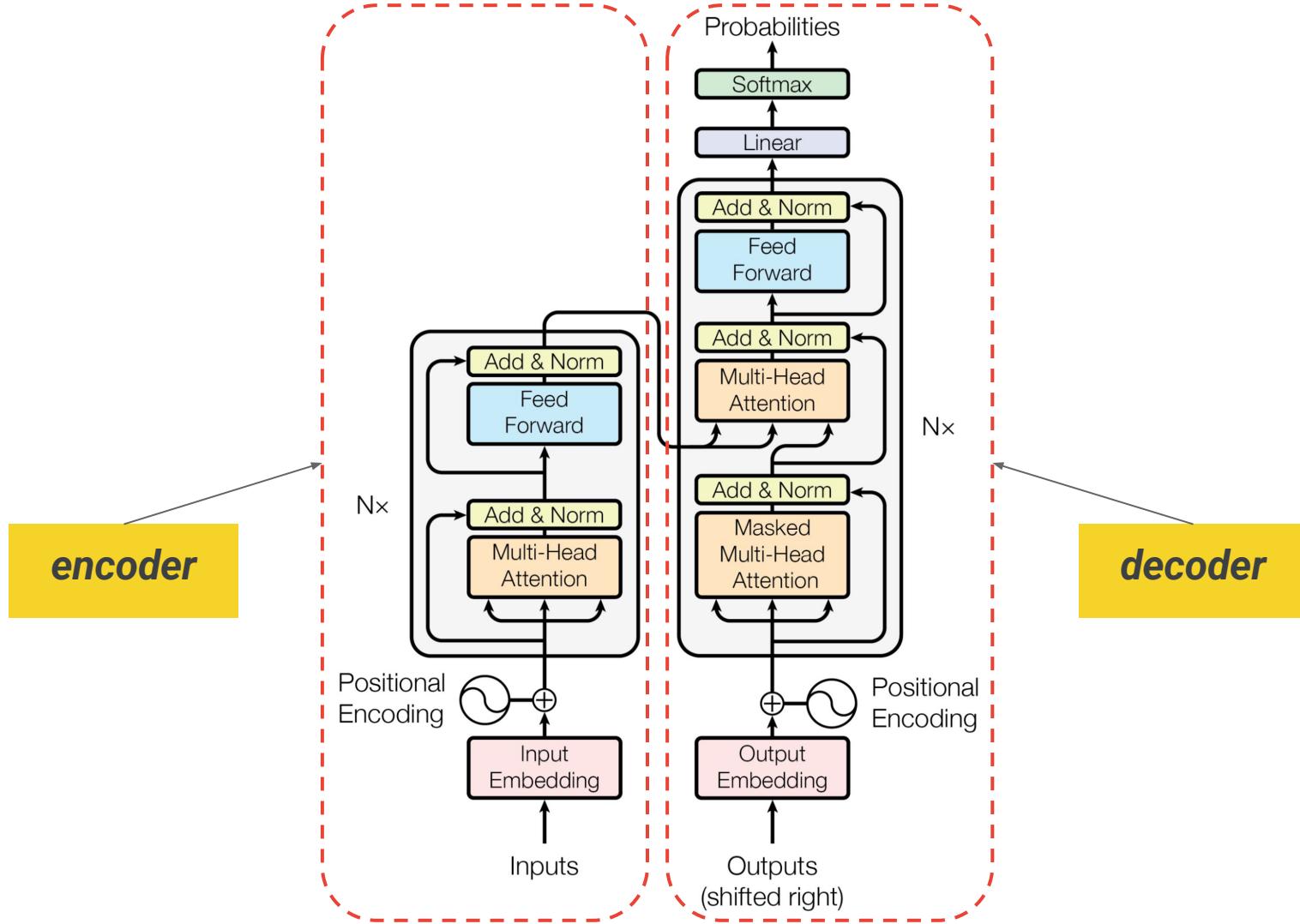
$$PE_{(pos,2i)} = \sin(pos/10000^{2i/d_{\text{model}}})$$

$$PE_{(pos,2i+1)} = \cos(pos/10000^{2i/d_{\text{model}}})$$

# Positional Encoding (cont'd)



# Transformer block (putting it together)



**Thank you!**