

# Language modeling

**CS 5624: Natural Language Processing**  
*Spring 2025*

<https://tuvllms.github.io/nlp-spring-2025>

**Tu Vu**



# Office hours

- **Instructor:** Tu Vu
  - **Office hours:** Thursday 3:00 - 4:00 PM, [D&DS](#) 374
- **Teaching Assistant:** Rishab Balasubramanian
  - **Office hours:** Monday 1:00 - 2:00 PM, [D&DS](#) 260E

Office hours (both in-person and via Zoom) will start next Monday, January 27<sup>th</sup>. Zoom links will be posted on Piazza.

# Final project

- The class size has exceeded 50 students and is still growing
- Groups of ~~2-3~~ 4-5; all groups should be formed by January 31<sup>st</sup>
- A Google form for submitting group information will be available next week
- Search for teammates on Piazza  
<https://piazza.com/class/m63qacreewc2fs/post/5>  
or reach out to us at [cs5624instructors@gmail.com](mailto:cs5624instructors@gmail.com)

## Final project (cont'd)

*“If I were given one hour to save the planet, I would spend **59 minutes** defining the problem and one minute resolving it.”*

— Albert Einstein?

# Homework

- Homework 0 will be released tomorrow (due February 7<sup>th</sup>)

## Reminder

- Conditional probability  $P(B|A) = \frac{P(A, B)}{P(A)}$

Rewriting

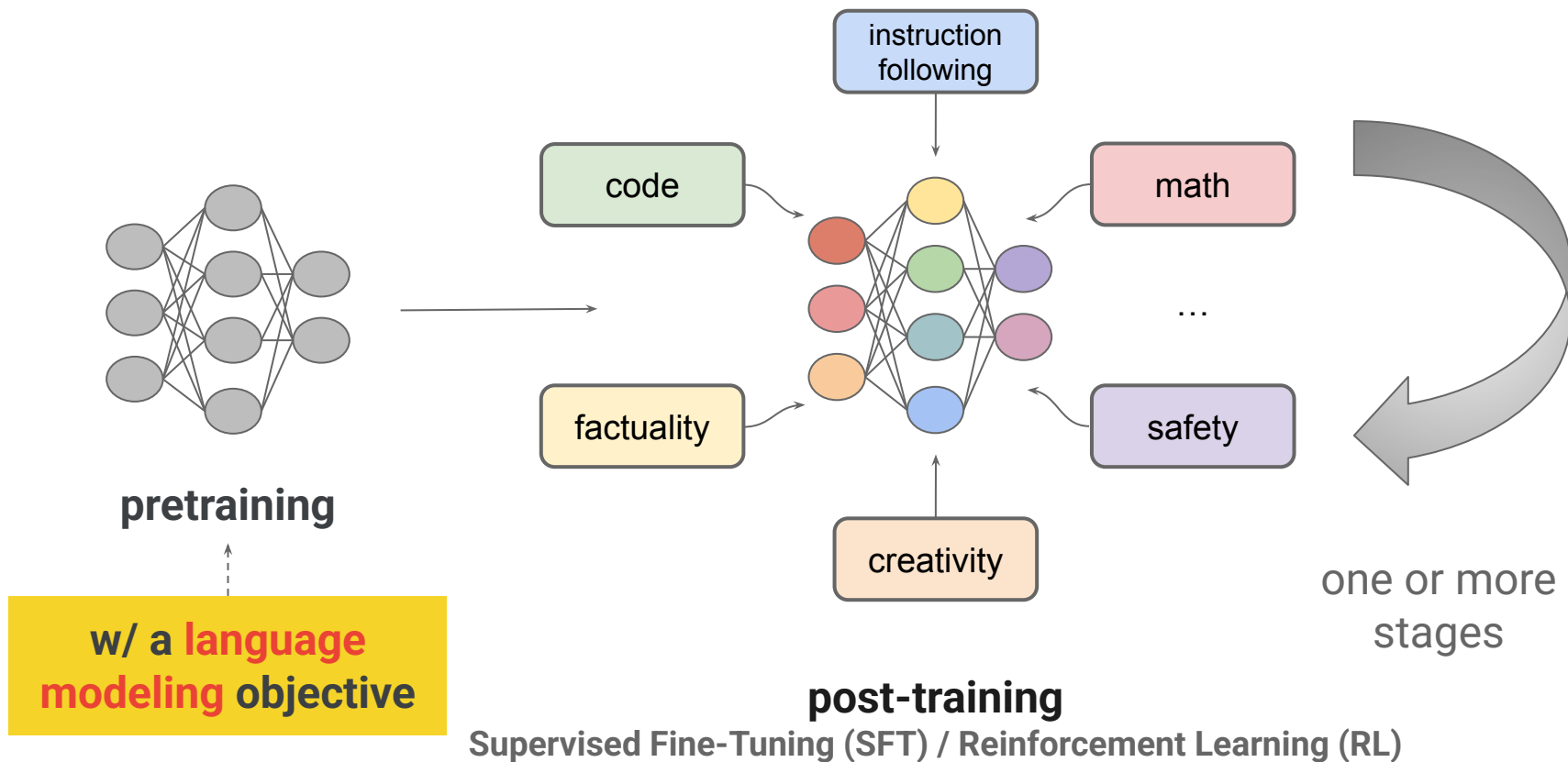
$$P(A, B) = P(A) \times P(B|A)$$

- Chain rule

$$\begin{aligned} P(X_1, X_2, \dots, X_n) &= P(X_1, X_2, \dots, X_{n-1}) \times P(X_n | X_1, X_2, \dots, X_{n-1}) \\ &= P(X_1, X_2, \dots, X_{n-2}) \times P(X_{n-1} | X_1, X_2, \dots, X_{n-2}) \times P(X_n | X_1, X_2, \dots, X_{n-1}) \\ &= P(X_1) \times P(X_2 | X_1) \times \dots \times P(X_n | X_1, X_2, \dots, X_{n-1}) \end{aligned}$$

$$P(w_1, w_2, \dots, w_n) = P(w_1) \times P(w_2 | w_1) \times \dots \times P(w_n | w_1, w_2, \dots, w_{n-1})$$

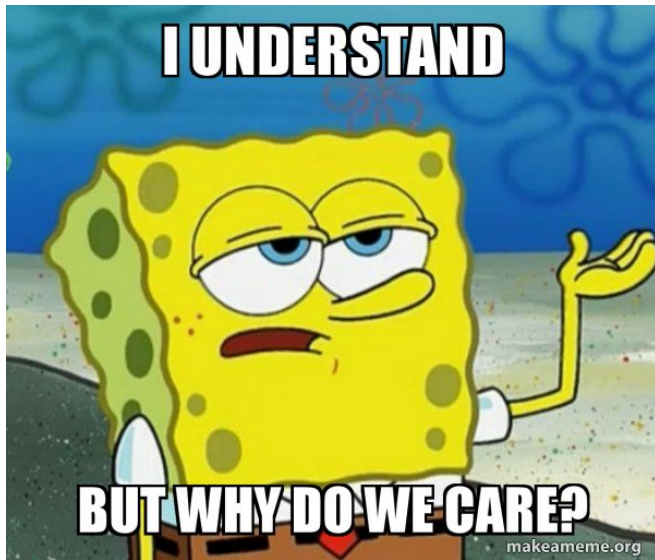
# The development of modern LLMs



# Language modeling

- Predicting the next/missing word

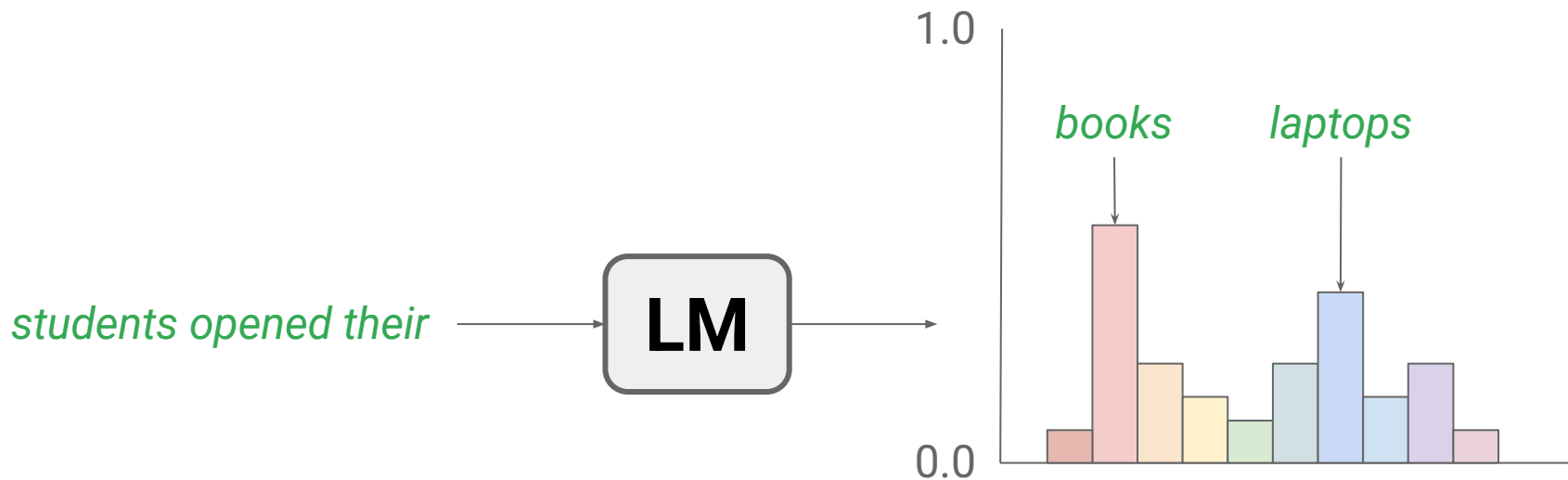
Example: “The cat is on the \_\_\_\_.” → Predicted: “mat”.





# What is a language model?

- A machine learning model that assigns a ***probability*** to each possible next word, or a ***probability distribution*** over possible next words



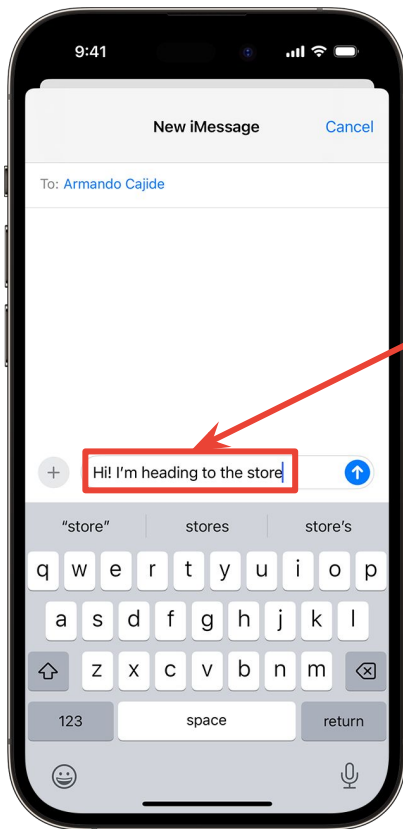
## What is a language model? (cont'd)

- A language model can also assign a probability to an entire sentence

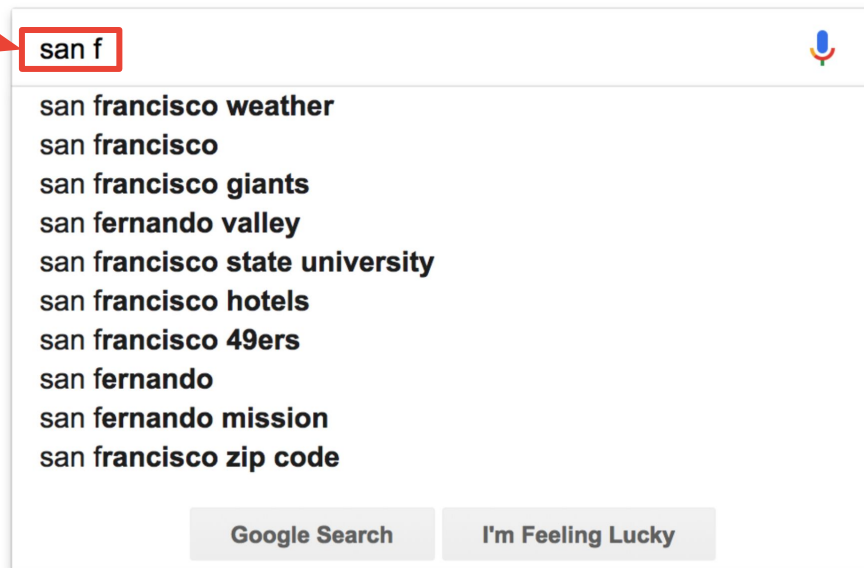
$$P(w_1, w_2, \dots, w_n) = P(w_1) \times P(w_2|w_1) \times \dots \times P(w_n|w_1, w_2, \dots, w_{n-1})$$

$$\begin{aligned} P(\text{"The cat is on the mat"}) &= P(\text{"The"}) \times P(\text{"cat"} | \text{"The"}) \times P(\text{"is"} | \\ &\text{"The cat"}) \times P(\text{"on"} | \text{"The cat is"}) \times P(\text{"the"} | \text{"The cat is on"}) \times \\ &P(\text{"mat"} | \text{"The cat is on the"}) \end{aligned}$$

# You use language models everyday!



prefix



source: [Apple Support](#)

source: [Google Blog](#)

# Two categories of language models

- **Statistical language models**
  - **N-gram / Count-based language models**
- **Neural language models (e.g., ChatGPT, Gemini)**

# N-grams

- An n-gram is a sequence of n words
- Unigram (n=1)
  - “The”, “water”, “of”, “Walden”, “Pond”
- Bigram (n=2)
  - “The water”, “water of”, “of Walden”, “Pond”
- Trigram (n=3)
  - “The water of”, “water of Walden”, “of Walden Pond”
- 4-gram
- ...

## N-grams (cont'd)

- Notation
  - **word type**: a unique word in our vocabulary
  - **token**: an individual occurrence of a word type

Example: “I am Sam. Sam am I. I do not like green eggs and ham.”

→ one word type of “I”, three tokens of “I”

## N-grams (cont'd)

- How to compute the probabilities?

$$P(w_1, w_2, \dots, w_n) = P(w_1) \times P(w_2|w_1) \times \dots \times P(w_n|w_1, w_2, \dots, w_{n-1})$$

$$P(\textit{"blue"} \mid \textit{"The water of Walden Pond is so beautifully"})$$

*What is the problem with this approach?*

# The Markov assumption

- n-gram model: Approximate the prefix by just the last *n-1* words
- bigram (n=2) model

$$\begin{aligned} P(\text{"blue"} \mid \text{"The water of Walden Pond is so beautifully"}) \\ = P(\text{"blue"} \mid \text{beautifully}) \end{aligned}$$

- trigram (n=3) model

$$\begin{aligned} P(\text{"blue"} \mid \text{"The water of Walden Pond is so beautifully"}) \\ = P(\text{"blue"} \mid \text{so beautifully}) \end{aligned}$$



# The Markov assumption (cont'd)

- unigram model

$$\begin{aligned}P(w_1, w_2, \dots, w_n) &= P(w_1) \times P(w_2|w_1) \times \dots \times P(w_n|w_1, w_2, \dots, w_{n-1}) \\&\approx P(w_1) \times P(w_2) \times \dots \times P(w_n) \\&= \prod_{k=1}^n P(w_k)\end{aligned}$$

- bigram model

$$\begin{aligned}P(w_1, w_2, \dots, w_n) &\approx P(w_1) \times P(w_2|w_1) \times \dots \times P(w_n|w_{n-1}) \\&= \prod_{k=1}^n P(w_k|w_{k-1})\end{aligned}$$

# Maximum likelihood estimation (MLE)

$$P(w_n|w_{n-1}) = \frac{\text{Count}(w_{n-1}w_n)}{\sum_w \text{Count}(w_{n-1}w)} = \frac{\text{Count}(w_{n-1}w_n)}{\text{Count}(w_{n-1})}$$

<s> I am Sam </s>

<s> Sam I am </s>

<s> I do not like green eggs and ham </s>

relative frequency

Here are the calculations for some of the bigram probabilities

$$P(\text{I} | \text{<s>}) = \frac{2}{3} = 0.67 \quad P(\text{Sam} | \text{<s>}) = \frac{1}{3} = 0.33 \quad P(\text{am} | \text{I}) = \frac{2}{3} = 0.67$$

$$P(\text{</s>} | \text{Sam}) = \frac{1}{2} = 0.5 \quad P(\text{Sam} | \text{am}) = \frac{1}{2} = 0.5 \quad P(\text{do} | \text{I}) = \frac{1}{3} = 0.33$$

## Example

- From a restaurant corpus

“can you tell me about any good cantonese restaurants close by”

“tell me about chez panisse”

“i’m looking for a good place to eat breakfast”

“when is caffe venezia open during the day”

# Example (cont'd)

unigram  
counts

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

target									
prefix		i	want	to	eat	chinese	food	lunch	spend
	i	5	827	0	9	0	0	0	2
	want	2	0	608	1	6	6	5	1
	to	2	0	4	686	2	0	6	211
	eat	0	0	2	0	16	2	42	0
	chinese	1	0	0	0	0	82	1	0
	food	15	0	15	0	1	4	0	0
	lunch	2	0	0	0	0	1	0	0
	spend	1	0	1	0	0	0	0	0

want  
followed  
i 827  
times

## Example (cont'd)

827/2533

	<b>i</b>	<b>want</b>	<b>to</b>	<b>eat</b>	<b>chinese</b>	<b>food</b>	<b>lunch</b>	<b>spend</b>
<b>i</b>	0.002	0.33	0	0.0036	0	0	0	0.00079
<b>want</b>	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
<b>to</b>	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
<b>eat</b>	0	0	0.0027	0	0.021	0.0027	0.056	0
<b>chinese</b>	0.0063	0	0	0	0	0.52	0.0063	0
<b>food</b>	0.014	0	0.014	0	0.00092	0.0037	0	0
<b>lunch</b>	0.0059	0	0	0	0	0.0029	0	0
<b>spend</b>	0.0036	0	0.0036	0	0	0	0	0

Here are a few other useful probabilities:

$$P(i | \langle s \rangle) = 0.25$$

$$P(\text{food} | \text{english}) = 0.5$$

$$P(\text{english} | \text{want}) = 0.0011$$

$$P(\langle s \rangle | \text{food}) = 0.68$$

$$P(\langle s \rangle \text{ i want english food } \langle s \rangle)$$

$$= P(i | \langle s \rangle) P(\text{want} | i) P(\text{english} | \text{want})$$

$$P(\text{food} | \text{english}) P(\langle s \rangle | \text{food})$$

$$= 0.25 \times 0.33 \times 0.0011 \times 0.5 \times 0.68$$

$$= 0.000031$$

## Example (cont'd)

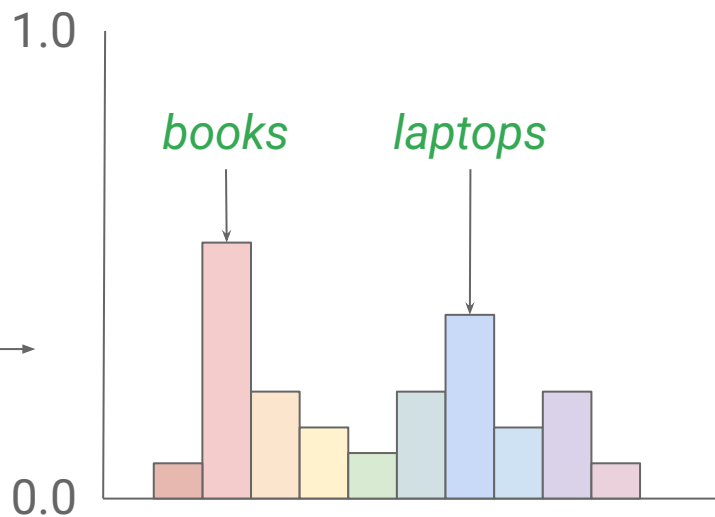
sparsity  
issue

	<b>i</b>	<b>want</b>	<b>to</b>	<b>eat</b>	<b>chinese</b>	<b>food</b>	<b>lunch</b>	<b>spend</b>
<b>i</b>	0.002	0.33	0	0.0036	0	0	0	0.00079
<b>want</b>	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
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<b>lunch</b>	0.0059	0	0	0	0	0.0029	0	0
<b>spend</b>	0.0036	0	0.0036	0	0	0	0	0

# How to sample sentences from a language model?

- Decoding strategies
  - Greedy decoding
  - Sampling
  - Others (future lecture)

*students opened their*



# Sample generations

1 gram	<p>–To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have</p> <p>–Hill he late speaks; or! a more to leg less first you enter</p>
2 gram	<p>–Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow.</p> <p>–What means, sir. I confess she? then all sorts, he is trim, captain.</p>
3 gram	<p>–Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done.</p> <p>–This shall forbid it should be branded, if renown made it empty.</p>
4 gram	<p>–King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv'd in;</p> <p>–It cannot be but so.</p>

from King John

**Figure 3.4** Eight sentences randomly generated from four n-grams computed from Shakespeare's works. All characters were mapped to lower-case and punctuation marks were treated as words. Output is hand-corrected for capitalization to improve readability.



## Is a 4-gram model insufficient for language modeling?

- In general, this is insufficient for language because it fails to account for **long-distance dependencies**.

Example: “The computer which I had just put into the machine room on the fifth floor crashed.”

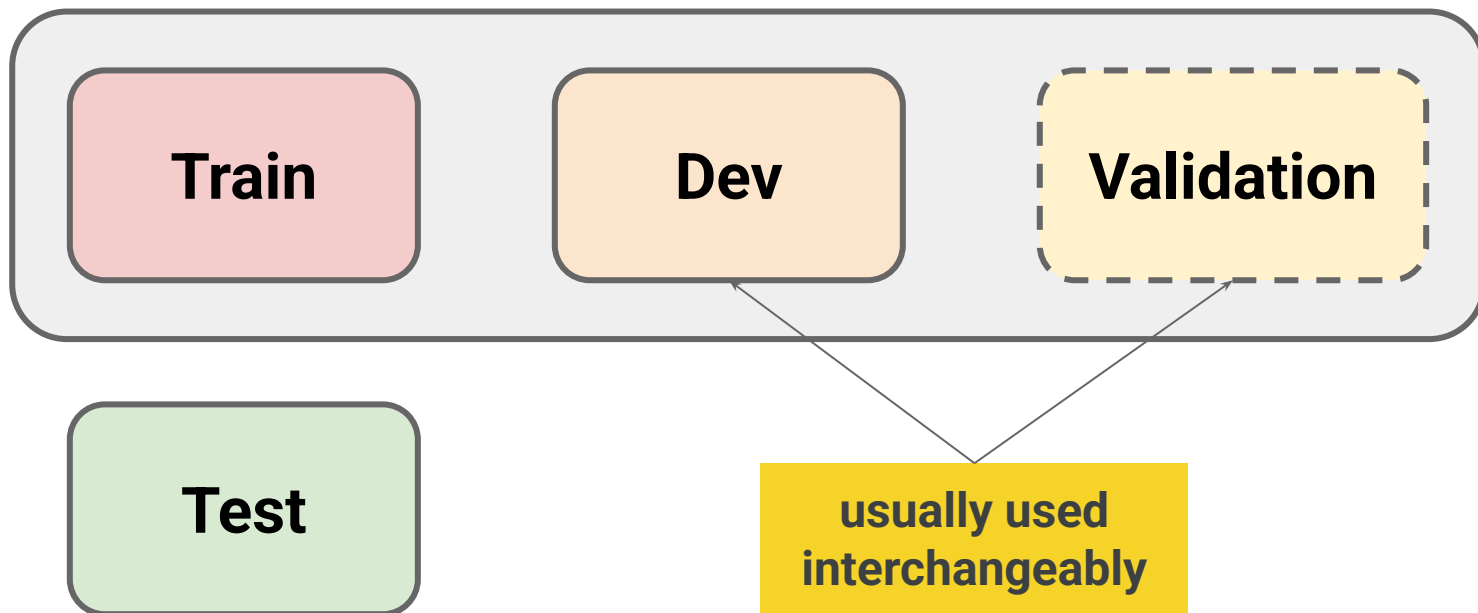
## *Should we increase the value of $n$ ?*

- As  $n$  increases, the number of possible  $n$ -grams grows exponentially (many  $n$ -grams have insufficient or no data)
- Storing and processing large  $n$ -grams requires more memory and computational power
- Beyond a certain point, increasing  $n$  may not yield significant performance improvements, especially if the dataset does not contain sufficient examples of longer  $n$ -grams

# Shakespeare as corpus

- T=884,647 tokens, V=29,066
- Shakespeare produced 300,000 bigram types out of  $V^2=844,000,000$  possible bigrams.
- **99.96%** of the possible bigrams have zero entries in the bigram table (were never seen)!

# Evaluating language models



# Never train on the test set!



Susan Zhang  
@suchenzang

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...

MBPP might've also been used somewhere in the Phi-1.5 dataset.

Just like we truncated one of the GSM8K problems, let's try truncating the MBPP prompts to see what Phi-1.5 will autocomplete with.

[h/t to @drjwrae for suggesting this too: [x.com/drjwrae/status...](https://x.com/drjwrae/status...)]



Part 2



Susan Zhang  
@suchenzang · Sep 12, 2023

I think Phi-1.5 trained on the benchmarks. Particularly, GSM8K.



[x.com/suchenzang/status...](https://x.com/suchenzang/status...)

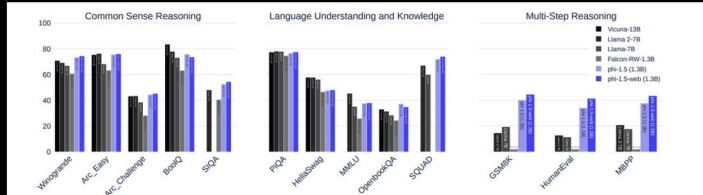


Figure 1: Benchmark results comparing **phi-1.5**, its version enhanced with filtered web data **phi-1.5-web**, and other state-of-the-art open-source LLMs. Sizes range from **phi-1.5**'s 1.3 billion parameters (Falcon-RW-1.3B [PMH\*23]) to 10x larger models like Vicuna-13B [ZCS\*23], a fine-tuned version of Llama-13B [TLI\*23]). Benchmarks are broadly classified into three categories: common sense reasoning, language skills, and multi-step reasoning. The classification is meant to be taken loosely, for example while HellaSwag requires common sense reasoning, it arguably relies more on "memorized knowledge". One can see that **phi-1.5** models perform comparable in common sense reasoning and language skills, and vastly exceeds other models in multi-step reasoning. Note that the numbers are from our own evaluation pipeline, to ensure consistency between models, and thus they might differ slightly from numbers reported elsewhere.



Susan Zhang  
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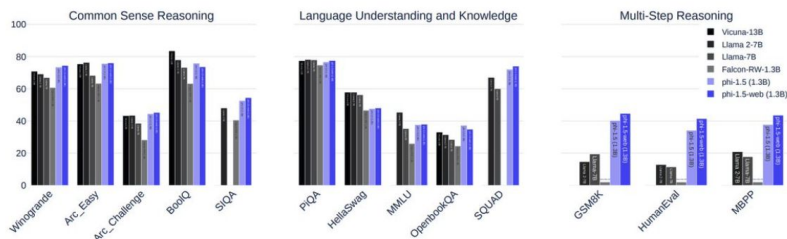


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Susan Zhang  
@suchenzang · Aug 2, 2023

Never trust a result in 2023 that doesn't mention the risk of dataset contamination. [x.com/mathemagic1an/...](https://x.com/mathemagic1an/...)

# Perplexity

$$\text{perplexity}(W) = P(w_1 w_2 \dots w_N)^{-\frac{1}{N}}$$

We normalize by the number of words N by taking the Nth root

$$= \sqrt[N]{\frac{1}{P(w_1 w_2 \dots w_N)}}$$

Or we can use the chain rule to expand the probability of  $W$ :

$$\text{perplexity}(W) = \sqrt[N]{\prod_{i=1}^N \frac{1}{P(w_i | w_1 \dots w_{i-1})}}$$

# Perplexity as Weighted Average Branching Factor

- Suppose a sentence consists of random digits.  
What is the perplexity of this sentence for a model that assigns a probability of  $1/10$  to each digit?

**Lower perplexity = Better language model**

	<b>Unigram</b>	<b>Bigram</b>	<b>Trigram</b>
<b>Perplexity</b>	962	170	109



In practice, we use log probs

$$\log \prod p(w_i | w_{i-1}) = \sum \log p(w_i | w_{i-1})$$

logs to avoid  
numerical underflow



**sentence:** I love love love love love the movie

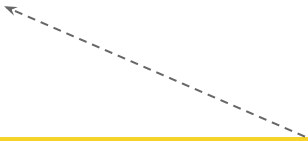
$$p(i) \cdot p(\text{love})^5 \cdot p(\text{the}) \cdot p(\text{movie}) = 5.95374181\text{e-}7$$

$$\log p(i) + 5 \log p(\text{love}) + \log p(\text{the}) + \log p(\text{movie})$$

$$= -14.3340757538$$

In practice, we use log probs (cont'd)

$$\textit{perplexity}(W) = \exp\left(-\frac{1}{N} \sum_i^N \textit{logp}(w_i | w_{<i})\right)$$



**perplexity is the  
exponentiated token-level  
negative log-likelihood**

# Infini-gram: Scaling Unbounded n-gram Language Models to a Trillion Tokens

<https://arxiv.org/pdf/2401.17377>

**Thank you!**