5.1 Baseline Analysis

The approach presented in Section 4.3 has some shortcomings especially in its space consumption. In this section we will look at some possible reasons for these shortcomings.

The biggest shortcoming of the *in-path* oracle is the space consumption. We found the oracle to be very large even on relatively small instances. Furthermore it was not possible to test instances of similar size to the instances used by [1]. This bakes the question for the cause of the large size of the oracle.

5.1.1 Theoretical

Definition 5

Radius

Let r be the average of $r_a^F, r_a^B, r_b^F, r_b^B$ such that $4r = r_a^F + r_a^B + r_b^F + r_b^B$.

We can use r to get an upper bound for the average over all the specific radii which should give us an idea how large the block pairs can be in relation to their distance.

Lemma 6

In-Path Radius Upper Bound

With d_D denoting the detour through p for any block pair (A, B) to be *in-path* the average radius is bound by:

$$r \leq \frac{d_N(s,t)\varepsilon - d_D}{4 + 2\varepsilon}$$

PROOF: Using Lemma 3 gives us:

$$\begin{aligned}
\frac{d_N(s,t) + d_D + 2r}{d_N(s,t) - 2r} &\leq 1 + \varepsilon \\
d_N(s,t) + d_D + 2r &\leq (1+\varepsilon)(d_N(s,t) - 2r) \\
4r &\leq d_N(s,t)\varepsilon - 2r\varepsilon - d_D \\
4r + 2r\varepsilon &\leq d_N(s,t)\varepsilon - d_D \\
r(4 + 2\varepsilon) &\leq d_N(s,t)\varepsilon - d_D \\
r &\leq \frac{d_N(s,t)\varepsilon - d_D}{4 + 2\varepsilon}
\end{aligned}$$

■

We can see r can be at most $\frac{1}{4}$ of $d_N(s,t)\varepsilon - d_D$ for a block pair to be *in-path*. This is especially bad for small ε because then $d_N(s,t)\varepsilon$ is small which in turn causes r to be a small fraction of $d_N(s,t)$. Moreover, d_D is subtracted from $d_N(s,t)\varepsilon$ causing r to have to be even smaller or even zero.

Lemma 7

Not In-Path Radius Upper Bound

With d_D denoting the detour through p for any block pair (A, B) to be not *in-path* the average radius is bound by:

$$r \leq \frac{d_D - d_N(s,t)\varepsilon}{4 + 2\varepsilon}$$

PROOF: Using Lemma 4 gives us:

$$\begin{aligned}
\frac{d_N(s,t) + d_D - 2r}{d_N(s,t) + 2r} &\geq 1 + \varepsilon \\
d_N(s,t) + d_D - 2r &\geq (1+\varepsilon)(d_N(s,t) + 2r) \\
4r + 2r\varepsilon &\leq d_D - d_N(s,t)\varepsilon \\
r &\leq \frac{d_D - d_N(s,t)\varepsilon}{4 + 2\varepsilon}
\end{aligned}$$

■

For a block pair to be not *in-path* r is primarily bound by d_D which makes sense because a large detour increases the difference to the detour limit and thus increases the size a block can have without containing a node which can have a detour within the limit.

5.1.2 Practical Worst Cases

In order to get a better understanding of the performance of Algorithm 2 we build a tool to visualize the results produced by the algorithm. The tool also allows us to have a look at intermediate results occurring during the execution of the algorithm.

- One-Way
- etc.