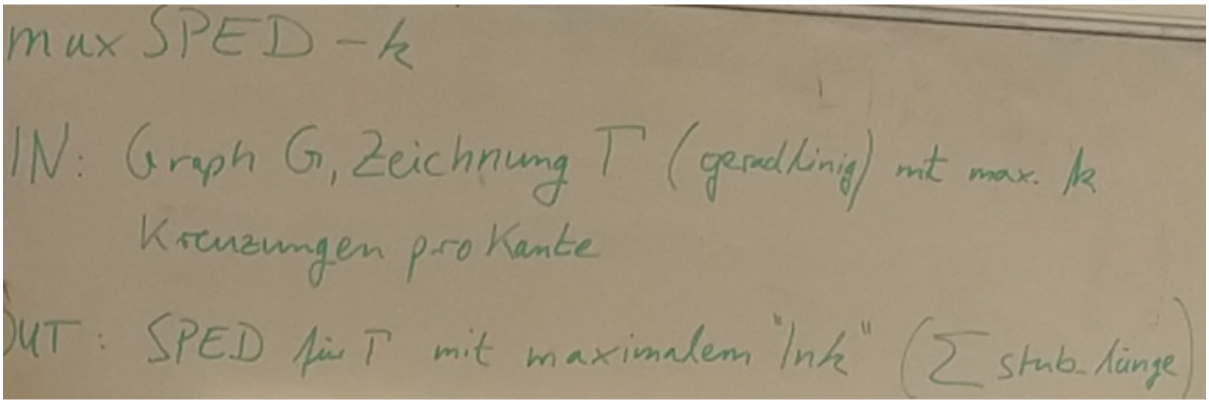
Research Session protocol on Graph Drawing, 4.12.2017 @ Favouritenstraße 9-11

Topic selection

maxSPED for k>2 in Partial Edge Drawings was selected, since it is the most interesting topic and most realistic one to get results in reasonable time. (parameter k means only k-planar graphs are considered)

Problem definition



We already learned about maxSPED with k=2, which could make use of cycle and path structures to prove their complexity. With k>2, this is not possible. Thus, other approaches need to be used.

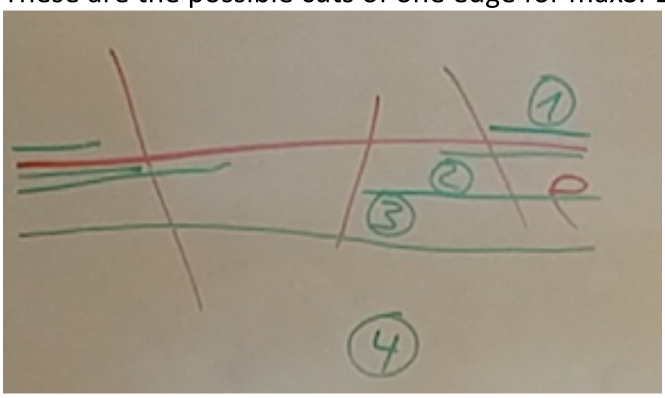
Strategy:

1. Try to prove complexity for k=3
2. If successful, then generalize this to k>2 (make use of XP complexity class = polyn. Runtime for fixed k)

www.graphclasses.org is a good source of information when working with graph classes and their properties.

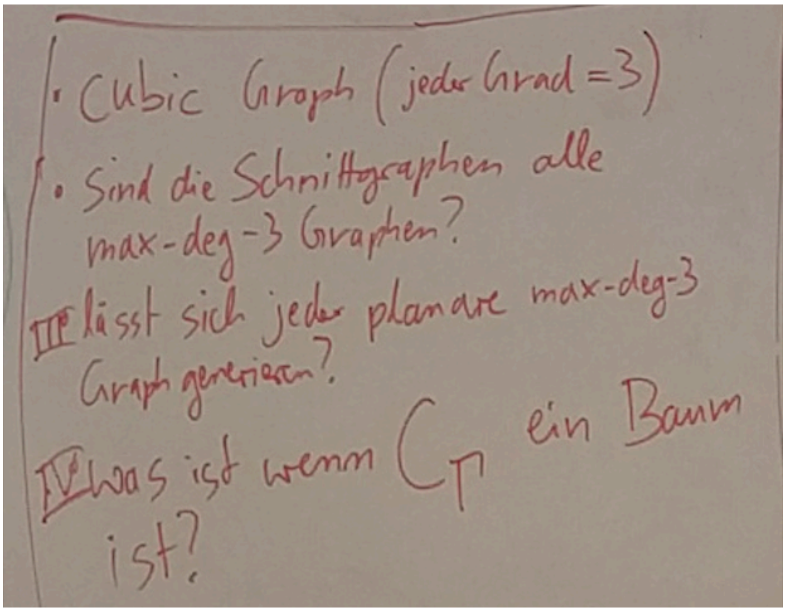
Prove ideas for maxSPED k=3

These are the possible cuts of one edge for maxSPED with k=3:



There are different classes of graphs that can be considered. Solving multiple subclasses could produce interesting minor results and give ideas for the general prove (k<2). Those are possible subclasses and questions:

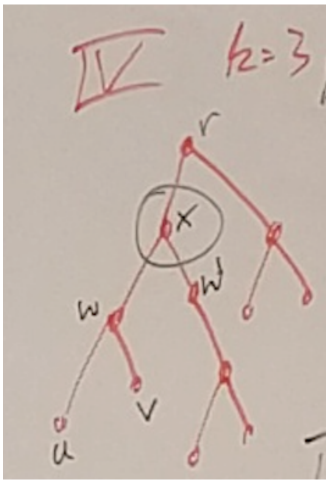
- Cubic
- 3-planar graphs with tree-structure
    - = Binary trees
- Bipartite graphs $K_{3,3}$

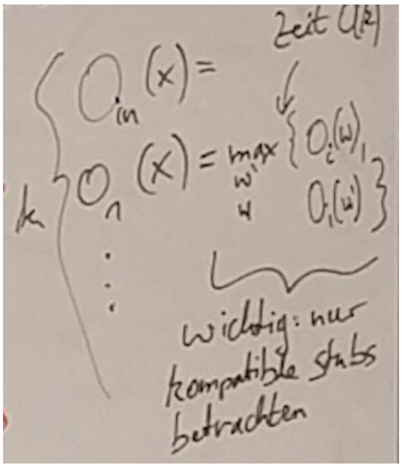


3-planar graphs with tree-structure

Can be drawn as binary trees.

Idea: Bottom-Up approach beginning from leafs, go up towards root





Differentiate the following possible cut points of an edge:
- O_{in} … whole edge
- O_1 … shortest Stub
- O_2 … medium Stub
- O_3 … longest Stub

Alternative (might be helpful):
- O_{in} … whole edge
- O_1 … parent
- O_2 … left child
- O_3 … right child

Can dynamic programming be applied for k=3 (as seen for k=2)?

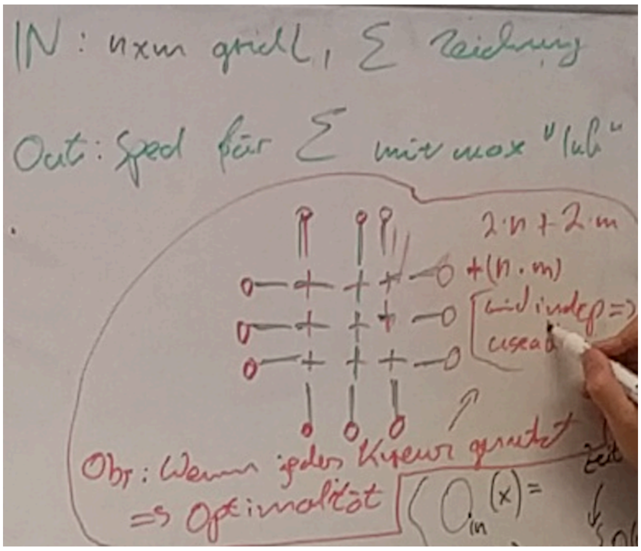
Runtime (Guess)
$O(k^2)$ for each vertex
Overall $O(n*k^2)$

Thus, for any k fixed and k>2, this would lead to polynomial runtime.
Complexity comes from building the "Schnittgraph" O(n log n)
+ Dynamic Programming still possible for k=3 and tree-structure

Bipartite graph (→ max. $K_{3,3}$)
Can be drawn as 3x3 grid.
Find optimal SPED for grid

Strategy:
1. Find optimal SPED for unilength grid
2. Find optimal SPED for weighted grid

Unilength grid
Given (m x n) grid, draw the grid consisting of components "line" and "crossing" for each part of the grid.
Observation: A solution with a line or crossing set for each possible crossing component is an optimal solution.



Since we can consider subgraphs of the real graph to be $K_{3,3}$ in isolation, this can be used for each such subgraph.