$$L \triangleright W = \bigcup_{u \in L} u \triangleright W$$

$$u \triangleright W := Min_{\sqsubseteq}\{w : w \in W \wedge u \sqsubseteq w\}$$

$$V = \{v : \forall w(w \in W \rightarrow w \not\sqsubseteq v)\} \qquad pref(V) = \{u : \exists v(u \sqsubseteq v \wedge v \in V)\}$$

**Eigenschaften:**

$$u \in L \rightarrow u \in (u \triangleright L)$$

$$u \in L \rightarrow (u \triangleright L = \{u\})$$

$$L \triangleright L = L$$

$$U \triangleright L \subseteq L$$

$$L \triangleright (U \cup V) \not\supseteq (L \triangleright U) \cup (L \triangleright V)$$

$$L \triangleright (U \cup V) \subseteq^{?} (L \triangleright U) \cup (L \triangleright V)$$

$$L \triangleright (U \cap V) \not\subseteq (L \triangleright U) \cap (L \triangleright V)$$

$$L \triangleright (U \cap V) \supseteq^{?} (L \triangleright U) \cap (L \triangleright V)$$

$$L \triangleright (U \cdot V) \not\subseteq (L \triangleright U) \cdot (L \triangleright V)$$

$$L \triangleright (U \cdot V) \not\supseteq (L \triangleright U) \cdot (L \triangleright V)$$

$$(L_1 \triangleright L_2) \triangleright L_3 \not\supseteq L_1 \triangleright (L_2 \triangleright L_3)$$

$$(L_1 \triangleright L_2) \triangleright L_3 \subseteq^{?} L_1 \triangleright (L_2 \triangleright L_3)$$

$$L_1 \text{ ist regulär}, L_2 \text{ ist regulär} \rightarrow L_1 \triangleright L_2 \text{ ist regulär}$$

$$W \triangleright V = \emptyset$$

$$W \triangleright pref(V) = \emptyset$$

$$W \triangleright W \cdot X^* = W$$

$$W \cdot X^* \triangleright V = \emptyset$$

$$W \cdot X^* \triangleright X^* = W \cdot X^*$$

$$W \cdot X^* \triangleright W = W$$

$$W \cdot X^* \triangleright pref(V) = \emptyset$$

$$pref(V) \triangleright W \cdot X^* =$$

$$L_1 \subseteq L_2 \rightarrow L_1 \triangleright L_2 = L_1$$

$$L_2 \subseteq L_1 \rightarrow L_1 \triangleright L_2 = L_2$$

$$L \triangleright (U \cup V) \not\supseteq (L \triangleright U) \cup (L \triangleright V),$$

$$\text{Gegenbeispiel: } L = \{a\} \quad A = \{abb, aaba\} \quad B = \{aab, aba\}$$

$$L \triangleright (A \cup B) = \{abb, aba, aab\} \text{ ,aber } L \triangleright A \cup L \triangleright B = \{abb, aaba\} \cup \{aab, aba\} = \{abb, aaba, aab, aba\}$$

$$L \triangleright (U \cap V) \not\subseteq (L \triangleright U) \cap (L \triangleright V),$$

$$\text{Gegenbeispiel: } L = \{a, b\} \quad A = \{a, aa\} \quad B = \{aa, b\}$$

$$L \triangleright (A \cap B) = \{aa\} \text{ ,aber } L \triangleright A \cap L \triangleright B = \{a\} \cap \{b\} = \emptyset$$

$$L \triangleright (U \cdot V) \not\supseteq (L \triangleright U) \cdot (L \triangleright V),$$

$$L \triangleright (U \cdot V) \not\subseteq (L \triangleright U) \cdot (L \triangleright V),$$

$$\text{Gegenbeispiel: } A = \{aa\} \quad B = \{b, bb\}$$

$$L \triangleright (A \cdot B) = \{aab\} \text{ ,aber } L \triangleright A \cdot L \triangleright B = \{aa\} \cdot \emptyset = \{aa\}$$

$$(L_1 \triangleright L_2) \triangleright L_3 \not\supseteq L_1 \triangleright (L_2 \triangleright L_3)$$

$$\text{Gegenbeispiel: } L_1 = \{ab, aa\} \quad L_2 = \{a, ab\} \quad L_3 = \{aa\}$$

$$(L_1 \triangleright L_2) \triangleright L_3 = \{ab\} \triangleright \{aa\} = \emptyset \text{ ,aber } \{ab, aa\} \triangleright \{aa\} = \{aa\}$$

$$
\begin{aligned}
L \triangleright (W \cup V) &= \bigcup_{l \in L} l \triangleright (W \cup V) \\
&= \bigcup_{l \in L} (\min_{\sqsubseteq} \{w : w \in (W \cup V) \wedge l \sqsubseteq w\}) \\
&= \bigcup_{l \in L} (\min_{\sqsubseteq} \{w : (w \in W \vee w \in V) \wedge l \sqsubseteq w\}) \\
&= \bigcup_{l \in L} (\min_{\sqsubseteq} \{w : w \in W \wedge l \sqsubseteq w \vee w \in V \wedge l \sqsubseteq w\}) \\
&= \bigcup_{l \in L} (\min(\{l\} \cdot X^* \cap \{w : w \in W \wedge l \sqsubseteq w \vee w \in V \wedge l \sqsubseteq w\})) \\
&= \bigcup_{l \in L} (\min(\{l\} \cdot X^* \cap \{w : w \in W \wedge l \sqsubseteq w\} \cup \{l\} \cdot X^* \cap \{w : w \in V \wedge l \sqsubseteq w\})) \\
&= \bigcup_{l \in L} (\min(l \triangleright W \cup l \triangleright V)) \\
&= \min((L \triangleright W) \cup (L \triangleright V))
\end{aligned}
$$