

$$L \triangleright W = \bigcup_{u \in L} u \triangleright W$$

$$u \triangleright W := \text{Min}_{\sqsubseteq} \{w : w \in W \wedge u \sqsubseteq w\}$$

$$V = \{v : \forall w (w \in W \rightarrow w \not\sqsubseteq v)\} \quad \text{pref}(V) = \{u : \exists v (u \sqsubseteq v \wedge v \in V)\}$$

**Eigenschaften:**

$$u \in L \rightarrow u \in (u \triangleright L)$$

$$u \in L \rightarrow (u \triangleright L = \{u\})$$

$$L \triangleright L = L$$

$$U \triangleright L \subseteq L$$

$$L \triangleright (U \cup V) \neq (L \triangleright U) \cup (L \triangleright V)$$

$$L \triangleright (U \cap V) \neq (L \triangleright U) \cap (L \triangleright V)$$

$$L \triangleright (U \cdot V) \neq (L \triangleright U) \cdot (L \triangleright V)$$

$$(L_1 \triangleright L_2) \triangleright L_3 \neq L_1 \triangleright (L_2 \triangleright L_3)$$

$$L_1 \text{ ist regulär, } L_2 \text{ ist regulär} \rightarrow L_1 \triangleright L_2 \text{ ist regulär}$$

$$W \triangleright V = \emptyset$$

$$W \triangleright \text{pref}(V) = \emptyset$$

$$W \triangleright W \cdot X^* = W$$

$$W \cdot X^* \triangleright V = \emptyset$$

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$$W \cdot X^* \triangleright W = W$$

$$W \cdot X^* \triangleright \text{pref}(V) = \emptyset$$

$$\text{pref}(V) \triangleright W \cdot X^* =$$

$$L_1 \subseteq L_2 \rightarrow L_1 \triangleright L_2 = L_1$$

$$L \triangleright (U \cup V) \neq (L \triangleright U) \cup (L \triangleright V),$$

$$\text{Gegenbeispiel: } A = \{abb, aaba\} \quad B = \{aab, aba\}$$

$$L \triangleright (A \cup B) = \{abb, aba, aab\}, \text{ aber } L \triangleright A \cup L \triangleright B = \{abb, aaba\} \cup \{aab, aba\} = \{abb, aaba, aab, aba\}$$

$$L \triangleright (U \cap V) \neq (L \triangleright U) \cap (L \triangleright V),$$

$$\text{Gegenbeispiel: } A = \{abb, ab\} \quad B = \{abb, aab\}$$

$$L \triangleright (A \cap B) = \{abb\}, \text{ aber } L \triangleright A \cap L \triangleright B = \{ab\} \cap \{abb, aab\} = \emptyset$$

$$L \triangleright (U \cdot V) \neq (L \triangleright U) \cdot (L \triangleright V),$$

$$\text{Gegenbeispiel: } A = \{aa\} \quad B = \{b, bb\}$$

$$L \triangleright (A \cdot B) = \{aab\}, \text{ aber } L \triangleright A \cdot L \triangleright B = \{aa\} \cdot \emptyset = \{aa\}$$

$$\begin{aligned}
L \triangleright (W \cup V) &= \bigcup_{l \in L} l \triangleright (W \cup V) \\
&= \bigcup_{l \in L} (\min_{\sqsubseteq} \{w : w \in (W \cup V) \wedge l \sqsubseteq w\}) \\
&= \bigcup_{l \in L} (\min_{\sqsubseteq} \{w : (w \in W \vee w \in V) \wedge l \sqsubseteq w\}) \\
&= \bigcup_{l \in L} (\min_{\sqsubseteq} \{w : w \in W \wedge l \sqsubseteq w \vee w \in V \wedge l \sqsubseteq w\}) \\
&= \bigcup_{l \in L} (\min(\{l\} \cdot X^* \cap \{w : w \in W \wedge l \sqsubseteq w \vee w \in V \wedge l \sqsubseteq w\})) \\
&= \bigcup_{l \in L} (\min(\{l\} \cdot X^* \cap \{w : w \in W \wedge l \sqsubseteq w\} \cup \{l\} \cdot X^* \cap \{w : w \in V \wedge l \sqsubseteq w\})) \\
&= \bigcup_{l \in L} (\min(l \triangleright W \cup l \triangleright V)) \\
&= \min((L \triangleright W) \cup (L \triangleright V))
\end{aligned}$$