Home Exam
FYS-2008

UiT - Norges Arktiske Universitet

December 5, 2023

Problem 1:

In this problem we are considering the Wheatstone bridge depicted in the figure down below, with one sensor arm represented by the resistance R_1.

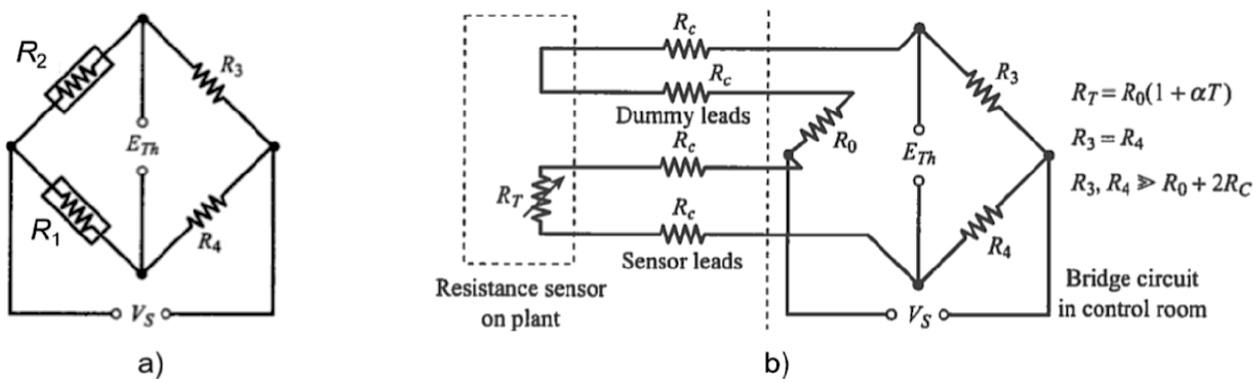

Figure 1: **a)** Wheatstone bridge with a sensing arm represented by R1 . **b)** Four-lead bridge circuit with restive temperature sensor. *Figure and text copied from the home exam paper.*

a)

In this task we are asked to derive the normalized output voltage E_{th}/V_s as function of the four bridge resistance, which would be the figure 1a. To make it easier for us to visualize, we can sketch the relevant nodes and how the current is behaving in the circuit. As shown in the figure below.

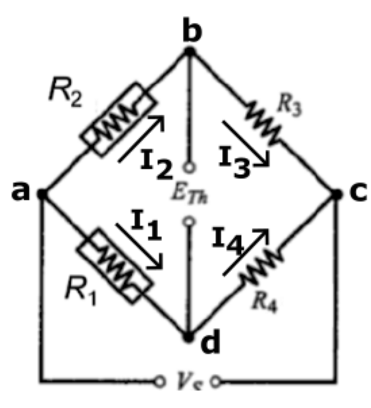

Figure 2: Circuit a) from figure 1, with labeled nodes and currents.

To start deriving the normalized output voltage, we need to take Ohm's law and Kirchhoff's law in consideration.
We start by looking at the current flowing through node b. Since R_2 and R_3 is in series, we have.

$$I_2 = I_3 = I_b \tag{1}$$

Than we use Ohm's law.

$$V_s = R_2 I_2 + R_3 I_3 \Rightarrow I_2 + I_3 = \frac{V_s}{R_2 + R_3} = I_b \tag{2}$$

Now we look at the opposite side, where we have the same that R_1 and R_4 is in series.

$$I_1 = I_4 = I_d \tag{3}$$

$$V_s = R_1 I_1 + R_4 I_4 \Rightarrow I_2 + I_4 = \frac{V_s}{R_1 + R_4} = I_d \tag{4}$$

E_{th} is measuring the voltage difference between node b and d, so we can start by expressing the voltage difference between node a and b than for node a and d. As follow.

$$V_b - V_a = (I_2 + I_3)R_2 = I_b R_2 = \frac{V_s R_2}{R_2 + R_3} \tag{5}$$

$$V_d - V_b = (I_1 + I_4)R_1 = I_d R_1 = \frac{V R_1}{R_1 + R_4} \tag{6}$$

Than we can start expressing E_{th}.

$$E_{th} = V_d - V_b = (V_d - V_a) - (V_b - V_a) = (\frac{V_s R_1}{R_1 + R_4}) - (\frac{V_s R_2}{R_2 + R_3}) \tag{7}$$

$$= V_s(\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3})$$

Noe we can formulate this for E_{th}/V_s, which gives.

$$\frac{E_{th}}{V_s} = \underline{\underline{\frac{R_1 R_3 - R_2 R_4}{(R_1 + R_4)(R_2 + R_3)}}} \tag{8}$$

b)

To find the unknown resistance of R_1, when the Wheatstone bridge is balanced. We know that the voltage difference of E_{th} is zero at balance. Hence:

$$E_{th} = (V_b - V_d) = 0 \tag{9}$$

So if we use equation (5) and (6) from the previous task, which represented V_b and V_d, we have.

$$V_b = V_d \Rightarrow \frac{R_2}{R_2 + R_3} = \frac{R_1}{R_1 + R_4} \tag{10}$$

Than we can solve for R_1:

$$
\begin{aligned}
R_2(R_1 + R_4) &= R_1(R_2 + R_3) && (11)\\
\cancel{R_2R_1} + R_2R_4 &= \cancel{R_1R_2} + R_1R_3 \\
R_2R_4 &= R_1R_3 \\
R_1 &= \underline{\underline{\frac{R_2R_4}{R_3}}} && (12)
\end{aligned}
$$

And we have obtained the unknown resistance of R_1.

c)

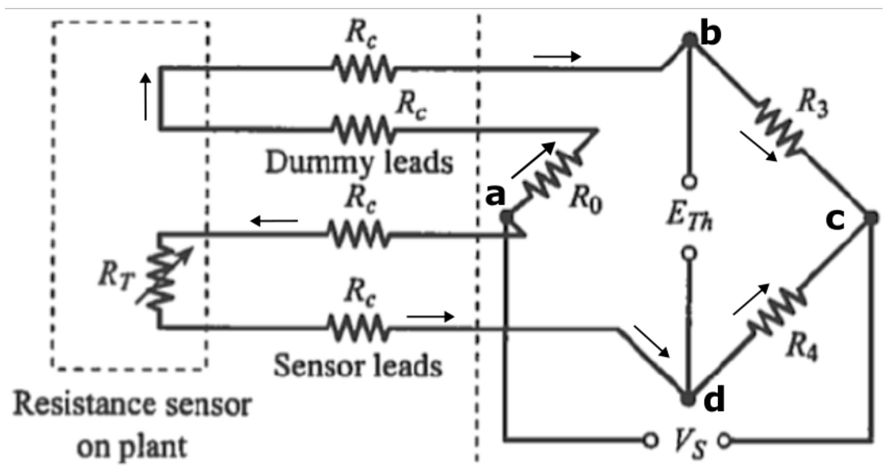

Figure 3: Circuit B in figure 1, with labeled nodes and currents.

Now we gone consider the circuit B in figure 1, also shown above with nodes labeled and where the current is moving. Where R_T is a resistance temperature sensor and R_c is the resistance of leads connecting the sensor to the bridge circuit. We are asked to derive a expression for E_{th}.
We start by repeating the similarly steps, as we did in task 1b).

$$
V_s = (R_0 + 2R_c)I_2 + R_3I_3 \Rightarrow I_2 + I_3 = \frac{V_s}{R_0 + 2R_c + R_3} \tag{13}
$$

$$
V_s = (R_T + 2R_c)I_1 + R_4I_4 \Rightarrow I_1 + I_4 = \frac{V_s}{R_T + 2R_c + R_4} \tag{14}
$$

Than we can start to calculate the differences in node b and d, for b we will have.

$$
V_b - V_a = (I_2 + I_3)R_3 = \frac{V_sR_3}{R_0 + 2R_c + R_3} \tag{15}
$$

Now we express the voltage difference in node d.

$$
V_d - V_a = (I_1 + I_4)R_4 = \frac{V_sR_4}{R_T + 2R_c + R_4} \tag{16}
$$

We can now compute this for E_{th}, which is the voltage difference between node b and d.

$$
\begin{aligned}
E_{th} &= (V_b - V_a) - (V_d - V_a) = V_b - V_d && (17) \\
&= \Big(\frac{V_s R_3}{R_0 + 2R_c + R_3}\Big) - \Big(\frac{V_s R_4}{R_T + 2R_c + R_4}\Big) && (18)
\end{aligned}
$$

Using that $R_3 = R_4$ and calculate the given expression above with reciprocal.

$$
E_{th} = V_s\Big(\frac{R_3(R_T - R_0)}{(R_0 + 2R_c + R_3)(R_T + 2R_c + R_4)}\Big) \tag{19}
$$

Since $R_3 \gg R_0 + 2R_c$, we can reformulate as.

$$
E_{th} = V_s\Big(\frac{\cancel{R_3}(R_T - R_0)}{\cancel{R_3}(R_T + 2R_c + R_4)}\Big) = V_s\Big(\frac{R_T - R_0}{R_T + 2R_c + R_4}\Big) \tag{20}
$$

By having that $R_T = R_0(1 + \alpha T)$ And $R_3 = R_3$.

$$
E_{th} = V_s\Big(\frac{R_0(1 + \alpha T) - R_0}{R_T + 2R_c + R_3}\Big) = V_s\Big(\frac{R_0 \alpha T}{R_T + 2R_c + R_3}\Big) \tag{21}
$$

Now since we have that $R_3 \gg R_0 + 2R_c$, we assume that R_3 is way bigger than $R_T + 2R_c$. Hence:

$$
E_{th} = V_s \frac{R_0 \alpha T}{R_T + 2R_c + R_3} \quad \underset{\approx R_3}{\Rightarrow} \quad E_{th} = V_s \frac{R_0 \alpha T}{R_3} \tag{22}
$$

Little bit of reorganization, and we obtain the correct expression.

$$
E_{th} = \underline{\underline{V_s \frac{R_0}{R_3} \alpha T}} \tag{23}
$$

d)

The bridge is said to be balanced, when E_{th} (output voltage) is equal to zero. This balanced condition occurs at a specific temperature (T). In task above, we found that E_{th} can be expressed as.

$$
E_{th} = V_s \frac{R_0}{R_3} \alpha T
$$

So we set $E_{th} = 0$ for the balanced condition, and we get.

$$
0 = V_s \frac{R_0}{R_3} \alpha T \quad \Longrightarrow \quad T = 0°C \tag{24}
$$

So at our bridge is in balance when $T = 0°C$. This also, applies that:

$$
\frac{R_T}{R_2} = \frac{R_4}{R_3} \tag{25}
$$

When the bridge is in balance.
To make the bridge balance at different temperature's (T_{bal}), we need to take in consideration the inner resistance of temperature sensor (R_T). Previously we had R_T defined as.

$$R_T = R_0(1 + \alpha T)$$

Where R_0 is the inner resistance of R_T. So, by taking the consideration of inner resistance would change with temperature, and still be able to gain a balanced bridge. We set R_0 to be R_x for the inner resistance of R_T. Now we can, find a expression for T_{bal} by using equation 25.

$$\frac{R_1}{R_2} = \frac{R_4}{R_3} \qquad \overset{R_3=R_4}{\Longrightarrow} \qquad R_1 = R_2 \tag{26}$$

Now we can set in $R_1 = R_T + 2R_c$ and $R_2 = R_0 + 2R_c$.

$$R_x(1 + \alpha T_{bal}) + 2R_c = R_0 + 2R_c \tag{27}$$

$$R_x(1 + \alpha T_{bal}) = R_0 \tag{28}$$

$$T_{bal} = \underline{\underline{\frac{R_0}{R_x \alpha} - \frac{1}{\alpha}}} \tag{29}$$

e)

The advantages of using the bridge configuration as the circuit b in figure 1, as the temperature sensor is used as one of the bridge resistors it allows for a automatic compensation of variations in the sensors resistance. Which means that, the bridge can obtain balance by it's self as we showed in task (c) with T_{bal}. Rather than in circuit a, where we would need to change R_2 to find the balance of the circuit. This would also lead to, that the measurement made from circuit b would be more precise.

Problem 2:

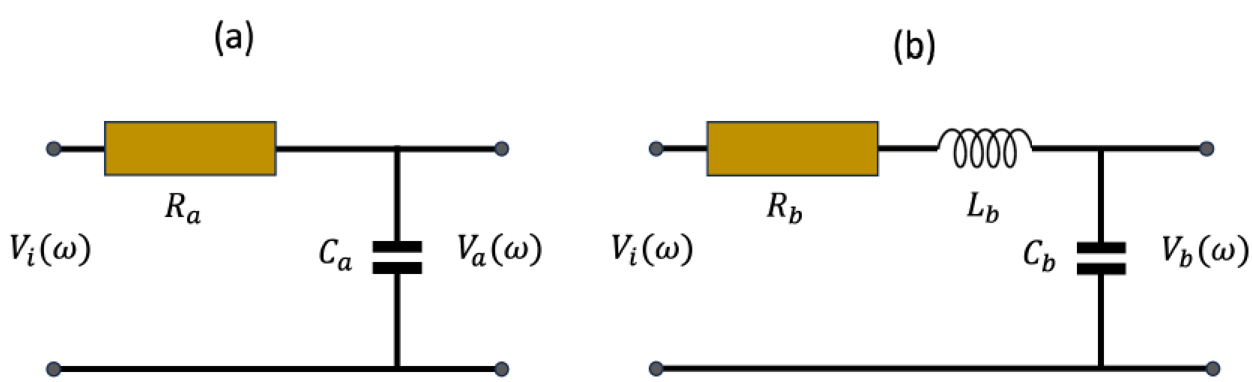

Figure 4: 1^{st} and 2^{nd} order low-pass filters. *Figure and text copied from the home exam paper.*

a)

Here we gone derive the frequency response, of circuit (b) shown in figure 4.
From the lecture notes, we are informed that Kirchhoff's voltage law over the loop is.

$$-RL - L\frac{dI}{dt} - \frac{1}{C}\int I dt + V(t) = 0 \tag{30}$$

In the lecture notes, the relation between V_C and current I is already shown, also the relation between V_C and other voltages as V_R and V_L is also already executed. Which makes it possible for us, to have the rearranged Kirchhoff's law over the loop defined as.

$$LC\frac{d^2V_C}{dt^2} + RC\frac{dV_C}{dt} + V_C = V(t) \tag{31}$$

Which would be the ODE for voltage V_C over the capacitor. By using this, we can take the Fourier transform of both sides directly without the need to express the current in terms of voltage and charge.
To take the Fourier transform of the expression 31, er use the following expression (transformation of derivative, *from lecture notes*).

$$\mathcal{F}\left\{\frac{d^nU}{dt^n}\right\} = (i\omega)^n\mathcal{F}(U(t)) \tag{32}$$

We can than applye this to equation 31.

$$LC(i\omega)^2\mathcal{F}(V_C) + RC(i\omega)\mathcal{F}(V_C) + \mathcal{F}(V_c) = \mathcal{F}(V(t)) \tag{33}$$

Rearrange for $\mathcal{F}(\mathcal{V}_C)$.

$$\begin{aligned}\mathcal{F}(V_C)(LC(i\omega)^2 + RC(i\omega) + 1) &= \mathcal{F}(V(t)) \\ \mathcal{F}(V_C) &= \frac{\mathcal{F}(V(t))}{LC(i\omega)^2 + RC(i\omega) + 1}\end{aligned} \tag{34}$$

We know that the natural frequency and the damping coefficient can be expressed as.

$$\omega_n = \frac{1}{\sqrt{LC}} \qquad \text{and} \qquad \xi = \frac{R}{2\sqrt{LC}}$$

And by looking at the lecture notes, specific at the standard form for the differential equation and see that we are dealing with step response. We will have that $\mathcal{F}(V(t)) = 1$. So by using the this, and the above. Our equation take's the following form.

$$\mathcal{F}(V_C) = \frac{1}{\frac{1}{\omega_n^2}(i\omega)^2 + 2i\xi(\frac{\omega}{\omega_n}) + 1} \tag{35}$$

$$H_b(\omega) = \frac{1}{-\frac{\omega^2}{\omega_n^2} + 2i\xi(\frac{\omega}{\omega_n}) + 1} \tag{36}$$

Which is our final expression, and compering with the expression given in the exam paper we see that we have obtained the correct expression.

b)

Now we gone find a general expression of the amplitude $|H_b(\omega)|$ of $H_b(\omega)$, and than we gone look what happens when $\xi = 1/\sqrt{2}$.
We start by finding the expression for the amplitude also called the magnitude response of $H_b(\omega)$ which is a complex function, which can be calculated by the following.

$$|H_b(\omega)| = \sqrt{Re\{H_b(\omega)\}^2 + Im\{H_b(\omega)\}^2} \tag{37}$$

Here our real- and imaginary part will be:

$$Re\{H_b(\omega)\} = 1 - \frac{\omega^2}{\omega_n^2} \qquad \text{and} \qquad Im\{H_b(\omega)\} = 2\xi\frac{\omega}{\omega_n}$$

Than we can apply this.

$$|H_b(\omega)| = \frac{1}{\sqrt{(1 - \frac{\omega^2}{\omega_n^2})^2 + (2\xi\frac{\omega}{\omega_n})^2}} \tag{38}$$

$$\tag{39}$$

Than we recall the following, math properties.

$$\frac{1}{(1-x)^2} = \frac{1}{1 - 2x + x^2}$$

Where x, is our real part ($Re\{H_b(\omega)\}$). Using this, gives us the following.

$$\underline{\underline{|H_b(\omega)| = \frac{1}{\sqrt{1 + \frac{\omega^4}{\omega_n^4} - 2\frac{\omega^2}{\omega_n^2} + 4\xi^2\frac{\omega^2}{\omega_n^2}}}}} \tag{40}$$

Which would be our general expression for the amplitude $|H_b(\omega)|$.
If we now set $\xi = 1/\sqrt{2}$.

$$|H_b(\omega)| = \frac{1}{\sqrt{1 + \frac{\omega^4}{\omega_n^4} - 2\frac{\omega^2}{\omega_n^2} + 4(\frac{1}{\sqrt{2}})^2\frac{\omega^2}{\omega_n^2}}} \tag{41}$$

$$= \frac{1}{\sqrt{1 + \frac{\omega^4}{\omega_n^4} - 2\frac{\omega^2}{\omega_n^2} + 2\frac{\omega^2}{\omega_n^2}}} \tag{42}$$

$$= \underline{\underline{\frac{1}{\sqrt{1 + \frac{\omega^4}{\omega_n^4}}}}} \tag{43}$$

And we have obtained the same expression, given in the exam paper.

c)

Here we gone simulate the circuits in LTspice, we have that both filters have the resistance $R_a = R_b = 100\Omega$ and we want to have a cut-off frequency at $f_c = 1MHz$.
We start by finding the capacitance (C_a) in circuit (a).

$$f_c = \frac{1}{2\pi R_a C_a} \qquad \Rightarrow \qquad C_a = \frac{1}{2\pi R_a f_c} = 1.59 \cdot 10^{-9} F \tag{44}$$

Now for the RLC-circuit, we need to find the inductions (L_b) and the capacitance (C_b). We start by finding the capacitance (C_b), which would be the same as the previous calculation for C_a. Hence, $C_b = 1.59 \cdot 10^{-9} F$. So, than we go forward and calculate the inductions (L_b).

$$f_c = \frac{1}{2\pi\sqrt{L_b C_b}} \qquad \Rightarrow \qquad L_b = \frac{1}{4\pi^2 C_b f_c^2} = 1.59 \cdot 10^{-5} H \tag{45}$$

Than by simulating this, in LTspice. We obtain the following plot.

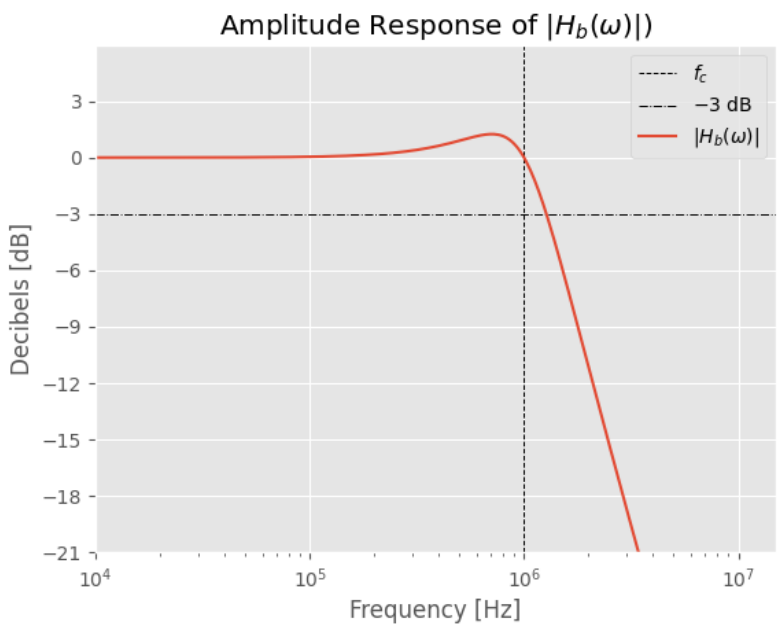

Figure 5: Plot of the frequency response of the RLC-circuit.

We can see from the figure above (fig 5), that the frequency response is off by some values for the filter to have a cut-off frequency of $1MHz$. So than we use the try and error method, to find the right value for C_b. This method gave score, when we have $C_b = 2.16 \cdot 10^{-5} F$. As we can see from the plot below (fig 6), where the frequency response of filter (a) is included.

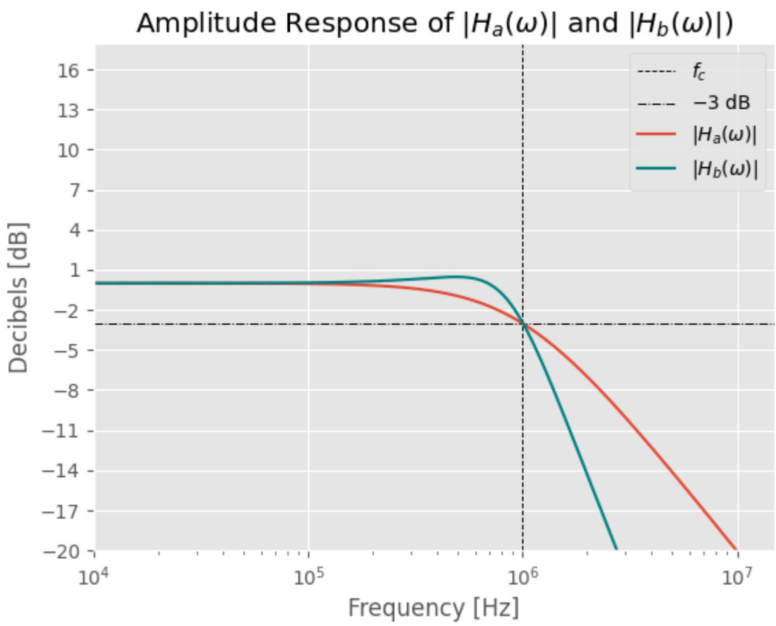

Figure 6: Frequency response of circuit a and b.

Here the frequency response of circuit b, represents a Butterworth response. The circuits used to simulate the two circuits in LTspice, have the following representation.

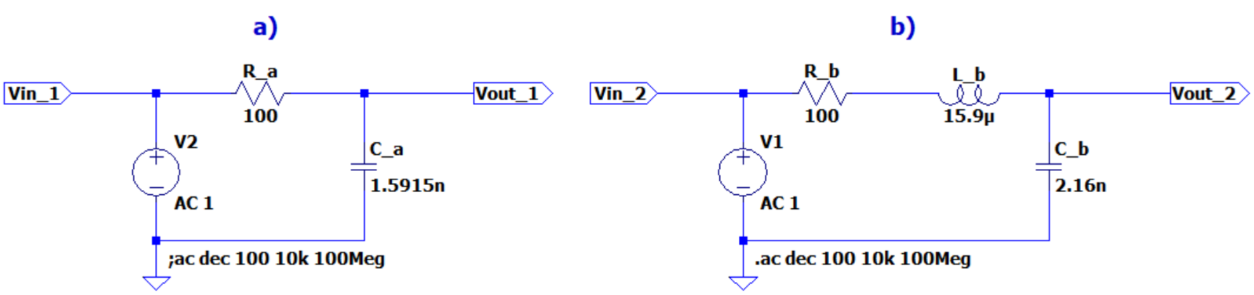

Figure 7: Circuit (a) and (b) from figure 4.

d)

By changing R_b by 20% up and down, we get the following plot.

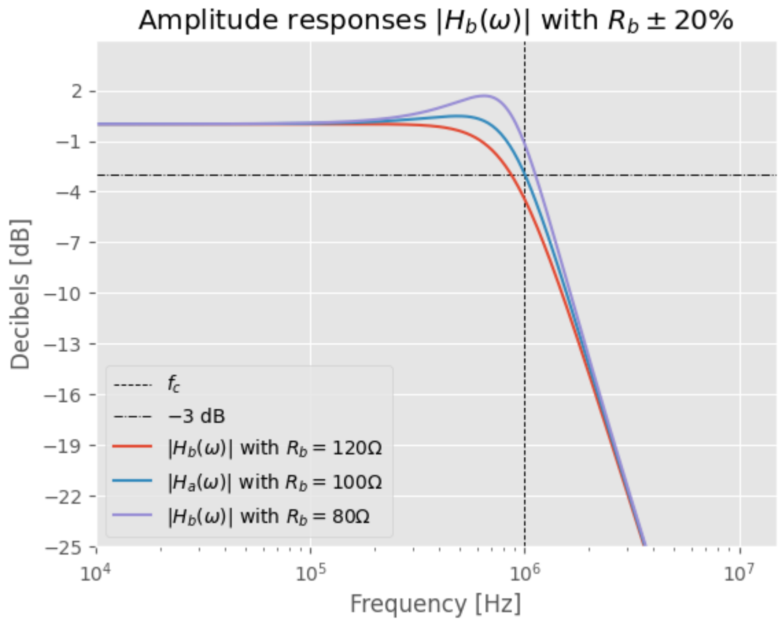

Figure 8: Frequency response of circuit b) in figure 7, with $R_b \pm 20\%$.

We see that, by changing the resistance of R_b the filter will not have the correct cut-off frequency. If we look beside that the cut-off frequency is off, and think about what would happen if we use the the filter with the different values of R_b and their associative cut-off frequency. For $R_b = 80\Omega$ the f_c would be smaller than originally, so the passed frequencies (passed band) would be smaller and since we have that the slope is not as steep it's a probability of a signal contains more noise. When $R_b = 120\Omega$ we would have the opposite, sharper curvature probability for noise is less and the pass band is bigger.
By using a 2^{nd} order filter ($H_b(\omega)$), we see from plot 6 that it it declining faster than the 1^{st} order filter ($H_a(\omega)$). This means several things. First, since the roll-off is sharper the rate of the filter attenuating frequencies is faster. Second, as mentioned above it decline way faster at the cut-off frequency meaning that it's reducing the presence of unwanted frequencies or noise. So the the 2^{nd} order filter, will be better for isolating specific range of frequencies than the 1^{st} order filter.

Problem 3:

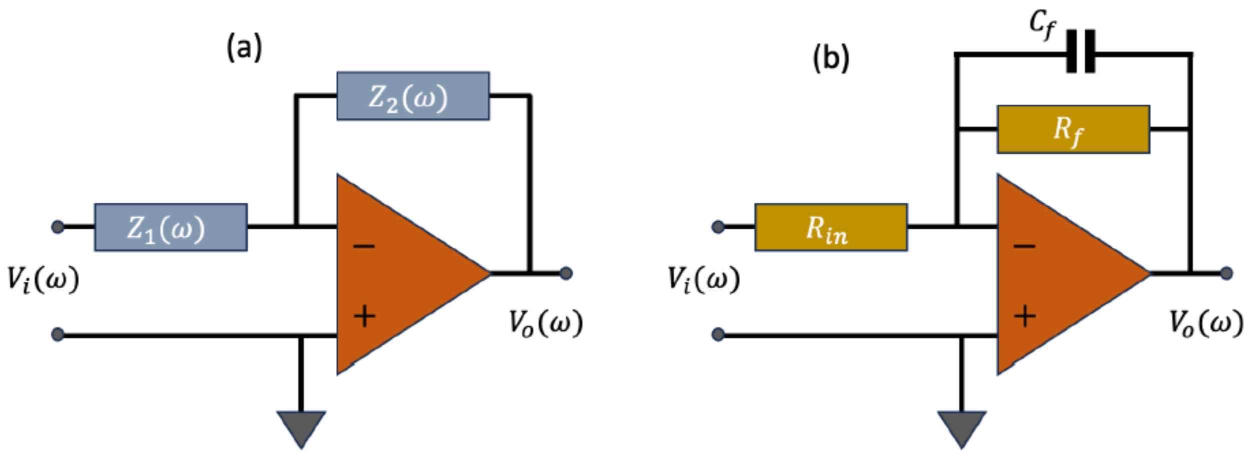

Figure 9: General filter [Fig (a)] and specific filter [Fig (b)] using an op-amp with negative feedback. *Figure and text copied from the home exam paper.*

a)

In this task we are asked to derive an expression for the output voltage of the amplifier, for circuit (a) in figure 9. Here we assume that we have a general inverted amplifier configuration, we also assume that we have a ideal amplifier.

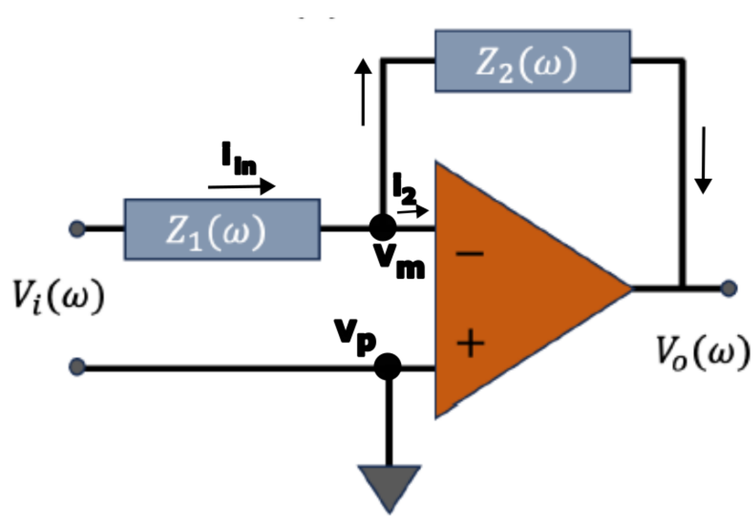

Figure 10: Circuit (a) from figure 9, with nodes and currents.

Since we assume that the amplifier is ideal, it will have infinity resistance so $I_2 = 0$ in the figure above and same yields for $V_m = 0$. We also assume that the current goes from the left to the right. Using Ohm's law, the current through Z_1, can be expressed as.

$$I = \frac{V_i(\omega) - V_m(\omega)}{Z_1(\omega)} = \frac{V_i(\omega)}{Z_1(\omega)} \tag{46}$$

Than for Z_2.

$$I = \frac{V_m(\omega) - V_o(\omega)}{Z_2(\omega)} \tag{47}$$

Since the current I is the same for both Z_1 and Z_2. We set them against each other.

$$\frac{V_i(\omega)}{Z_1(\omega)} = -\frac{V_o(\omega)}{Z_2(\omega)} \tag{48}$$

Now we can solve for the output voltage ($V_o(\omega)$), and we obtain.

$$V_o(\omega) = \underline{\underline{-\frac{Z_2(\Omega)}{Z_1(\omega)}V_i(\omega)}} \tag{49}$$

And we have obtained the same expression, given in the exam paper.

b)

Here we are asked to find an analytic expression for the frequency response, of circuit (b) in figure 9.

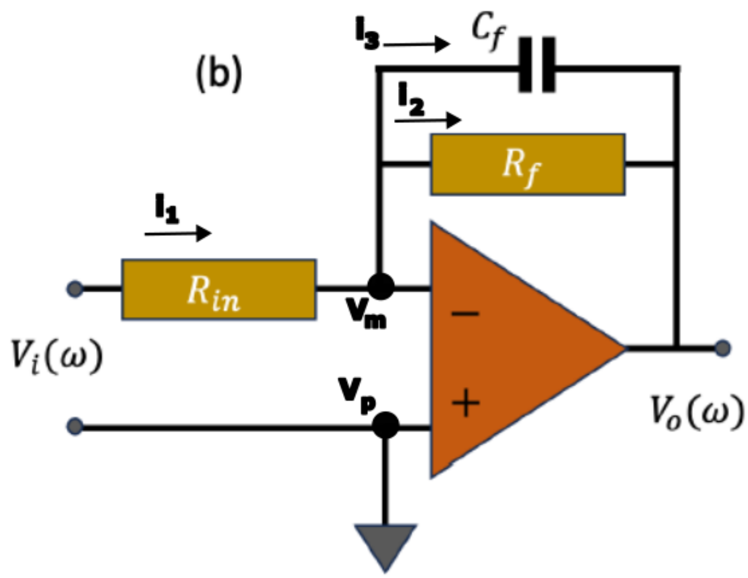

Figure 11: Circuit (b) from figure 9, with nodes and currents.

From the previous task, we managed to define.

$$V_o(\omega) = -\frac{Z_2(\Omega)}{Z_1(\omega)}V_i(\omega)$$

We know that the frequency response can be expressed as the following, with the differences of the input- and output voltage in frequency domain.

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} \tag{50}$$

So than, we can redefine equation 49.

$$\frac{V_o(\omega)}{V_i(\omega)} = -\frac{Z_2(\omega)}{Z_1(\omega)} \tag{51}$$

We can now calculate the impedance $Z_2(\omega)$ and $Z_1(\omega)$. First we calculate the impedance $Z_2(\omega)$, which in our system would be the parallel of C_f and R_f in frequency domain. The impedance of C_f and R_f is defined as.

$$Z_{C_f} = \frac{1}{j\omega C_f} \qquad\qquad and \qquad\qquad Z_{R_f} = R_f$$

The total impedance of the parallel combination, will be.

$$\frac{1}{Z_2(\omega)} = \frac{1}{Z_{R_f}(\omega)} + \frac{1}{Z_{R_f}} \tag{52}$$

$$\frac{1}{Z_2(\omega)} = \frac{1}{R_f} + \frac{1}{\frac{1}{j\omega C_f}} \tag{53}$$

$$\frac{1}{Z_2(\omega)} = \frac{1}{R_f} + j\omega C_f \tag{54}$$

$$Z_2(\omega) = \frac{R_f}{1 + j\omega C_f R_f} \tag{55}$$

For the one resistor in series with the parallel configuration, have the impedance as.

$$Z_{R_{in}} = R_{in}$$

Than we can compute the above, back into the equation 51.

$$\frac{V_o(\omega)}{V_i(\omega)} = -\frac{\frac{R_f}{1+j\omega C_f R_f}}{R_{in}} = -\frac{R_f}{1 + j\omega C_f R_f} \cdot \frac{1}{R_{in}} = -\frac{R_f}{R_{in}(1 + j\omega C_f R_f)} \tag{56}$$

Forward we set $s = j\omega$, and we obtain.

$$H(\omega) = -\frac{R_f}{R_{in}(1 + sC_f R_f)} = \underline{\underline{-\frac{R_f}{R_{in}}\left(\frac{1}{1 + sC_f R_f}\right)}} \tag{57}$$

c)

For this task we gone simulate circuit (b), in figure 9. Where $R_{in} = 100\Omega$ and $R_f = 1k\Omega$. To find the value of C_f, we can use the following definition and compute for C_f.

$$f_c = \frac{1}{2\pi R_f C_f} \qquad \Rightarrow \qquad C_f = \frac{1}{2\pi R_f f_c} = \underline{1.59 \cdot 10^{-10} F} \tag{58}$$

Now we can, add the values for the resistors and the capacitor to LT-spice as shown below.

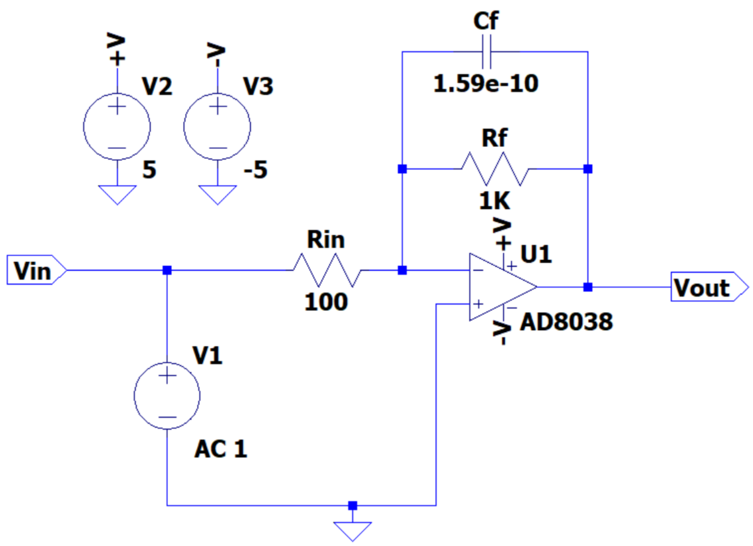

Figure 12: Circuit (b) in LT-spice

By using the decay function in LT-spice, to simulate the circuit and see if we have obtained the correct cut-off frequency. We get the following plot.

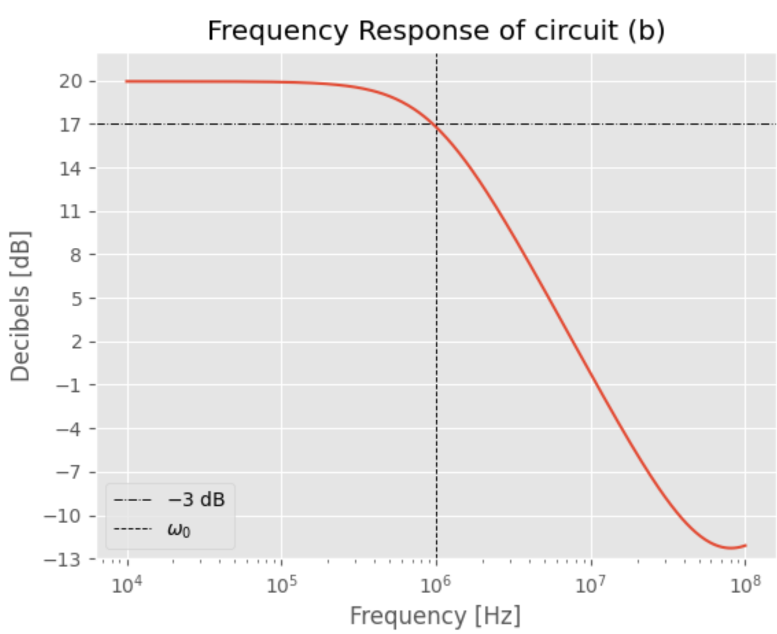

Figure 13: Frequency response of circuit (b) from figure 9, where $R_{in} = 100\Omega$, $R_f = 1k\Omega$ and $C_f = 15.9nF$

We can see that the circuit have a cut-off frequency at $1MHz$.

d)

Now we gone repeat what we did in (c), but now we gone simulate circuit by using `.tran` and have a step response as the input. `.tran` takes one argument which is the simulation time, which is given as $4\mu s$. The step response we apply the input, takes six arguments initial value (`Voff`), pulsed value (`Von`), delay (`Tdelay`), rise time (`Tr`), fall time (`Tf`), on time (`Ton`). Here we use that `Voff` $= 0V$ and `Von` $= 0.1V$, where `Von` is given in the task. I also gone set `Tr` and `Tf` equal to $0.01\mu s$, to give the pulse an abrupt change which can represent a square wave. If this value was equal to zero, change of the input pulse would be increase with time, than get flat at 0.1 and so decrease with time. Output pulse, would have the same behavior. `Von` is set to $4\mu s$, to make one single pulse. Now we have the following command in LTspice:

```
.tran 4u
PULSE(0 0.1 0 0.01u 0.01u 4u)
```

Now by running this in LTspice we get the following plot.

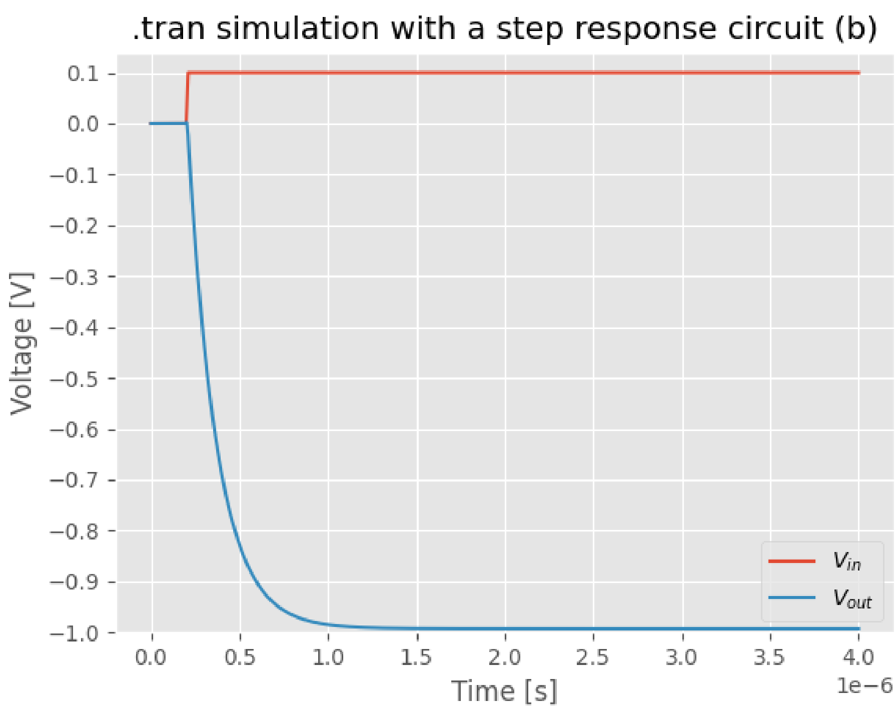

Figure 14: circuit (fig 15) simulated by using `.tran` and have a step response as the input.

e)

Now we gone make a band-pass filter based on the impedance model in figure 9a, where i gone suggest some components which we gone add to the circuit to get at band-pass between $10kHz$ and $1MHz$.

To make a band-pass filter, i will have a resistor (R_{in}) and a conductor (C_{in}) in series which

would replace $Z_1(\omega)$. For $Z_2(\omega)$ we will have a resistor R_f and a conductor C_f in a parallel configuration. Having the R_{in} and C_{in} in series will work as a high-pass filter, R_f and C_f in parallel works as a low-pass filter. By adding these filters together, we obtain a band-pass filter.
We will use the previous values, for R_{in} and R_f.

$$R_{in} = 100\Omega \qquad \text{and} \qquad R_f = 1k\Omega$$

Now we can calculate the value of C_{in}, in the high-pass filter. Which have a cut-off frequency at $10kHz$.

$$f_{c_1} = \frac{1}{2\pi R_{in} C_{in}} \qquad \Rightarrow \qquad C_{in} = \frac{1}{2\pi R_{in} f_{c_1}} = 1.59 \cdot 10^{-7} F \tag{59}$$

Than for the C_f in the low-pass filter configuration.

$$f_{c_2} = \frac{1}{2\pi R_f C_f} \qquad \Rightarrow \qquad C_f = \frac{1}{2\pi R_f f_{c_2}} = 1.59 \cdot 10^{-10} F \tag{60}$$

Now we can build our circuit in LTspice and add our values for the resistors and capacitors. Which would look like the following, in LTspice

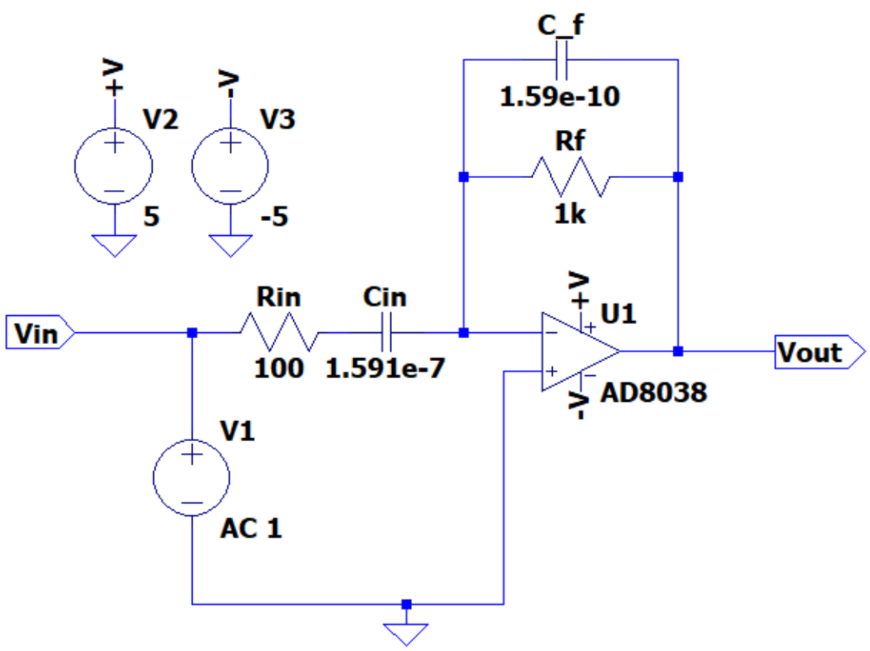

Figure 15: Caption

By using AC analysis with decay as type of sweep, we get the plot.

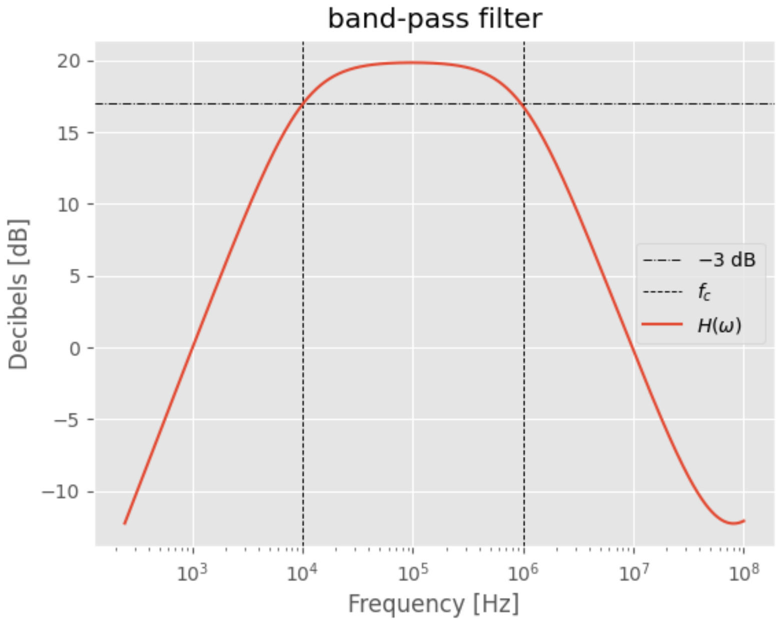

Figure 16: Frequency response of circuit 15

From the plot we see that we have obtained a band-pass filter, which have a band-pass between $10kHz$ and $1MHz$. It could look like if the graph do not have the cut-off frequency exactly at $1MHz$, but is really close.

f)

By doing some researcher from [1], we find the following relation.

$$db = 20log_{10}(\tau f) \tag{61}$$

So if we want a $-3db$ at 10Mhz we can implement it to the expression above, and solve for τ.

$$\tau = \frac{10^{15}}{10^7} \approx \underline{\underline{7.079 \cdot 10^{-8}s}} \tag{62}$$

Than we can add this to LTspice, by having the input as a square pulse and simulate with `.Trans` for the same circuit 15 we had in the previous task. This gives the following plot in time domain.

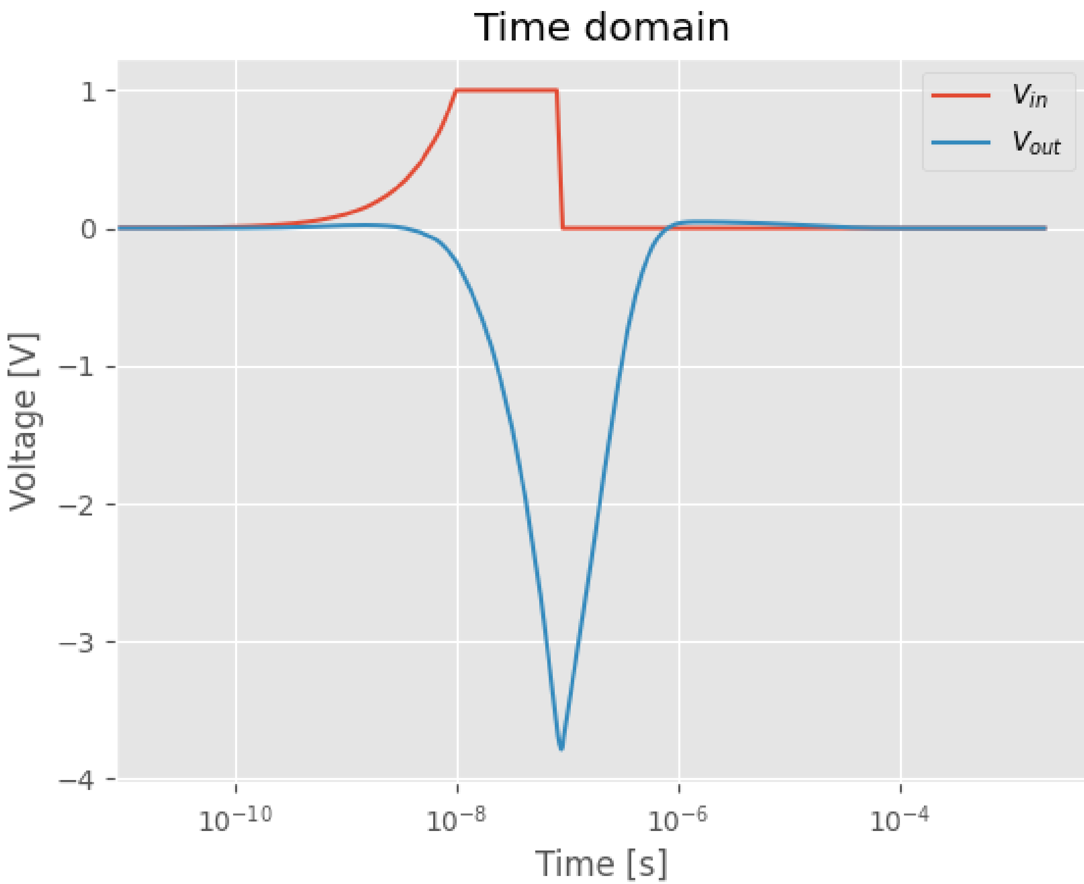

Figure 17: $\tau = 7.079 \cdot 10^{-8}s$ in LTspice's pulse generator, plot of the time domain

Now we can use the `FFT` tool in LTspice, for the plot above and we obtaine.

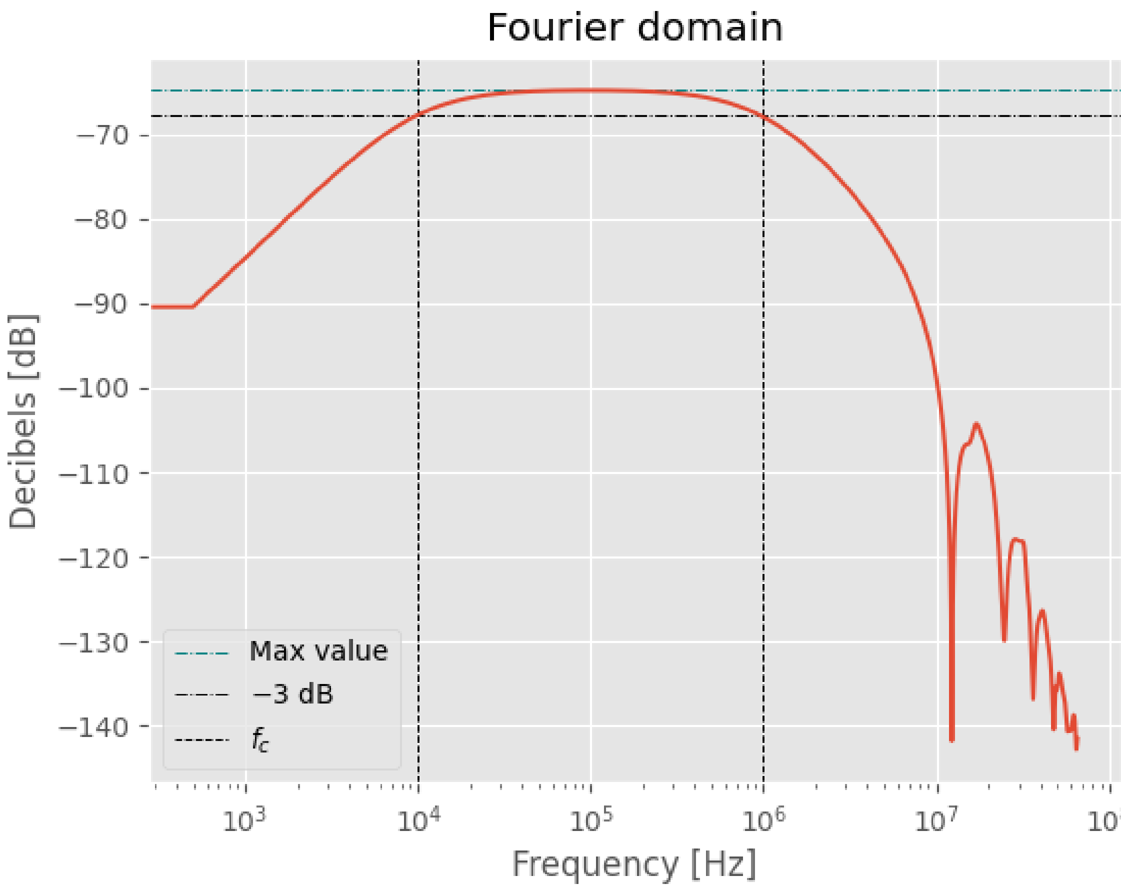

Figure 18: FFT of plot 17

As we can see from the plot, by simulating it this way. We get the same cut-off frequencies as we did from the previous task, which we can see in plot fig.16. Which makes sense since the band-pass filter we designed in the previous task, was designed to have a pass-band between 10kHz and 1MHz.

Appendix

References

[1] Amy Jones. The deciBel (dB) Logarithmic Unit to Express Ratio Between 2 Values. *HCT America*, June 2021. [Online; accessed 17. Oct. 2023].