To derive the rotation matrices **P** and **Q** that relate the position and velocity vectors in an elliptical orbit to the Earth-centered reference frame, we need to use the concepts of orbital elements and the rotation matrices for orbital frame transformation. The orbital elements describe the orientation and shape of the elliptical orbit, and these elements are:

1. Semi-Major Axis (a) 2. Eccentricity (e) 3. Inclination (i) 4. Argument of Perigee (ω) 5. Longitude of the Ascending Node (Ω) 6. Eccentric Anomaly (E)

Let's derive **P** and **Q** step by step:

First, define the position vector in the orbital frame:

$$\mathbf{r}_o = \begin{bmatrix} a(\cos E - e) \\ a\sqrt{1-e^2}\sin E \\ 0 \end{bmatrix}$$

And the velocity vector in the orbital frame:

$$\dot{\mathbf{r}}_o = \sqrt{\mu/a} \begin{bmatrix} -\sin E \\ \sqrt{1-e^2}\cos E \\ 0 \end{bmatrix}$$

Now, we need to perform three sequential rotations to transform these vectors from the orbital frame to the Earth-centered reference frame:

1. Rotate by Ω (Longitude of the Ascending Node):

$$R_1(\Omega) = \begin{bmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Rotate by i (Inclination):

$$R_2(i) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{bmatrix}$$

3. Rotate by ω (Argument of Perigee):

$$R_3(\omega) = \begin{bmatrix} \cos\omega & -\sin\omega & 0 \\ \sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, combine these rotation matrices to obtain **P** and **Q**. First, calculate **P**:

$$\mathbf{P} = R_3(\omega) \cdot R_2(i) \cdot R_1(\Omega)$$

Substituting the rotation matrices:

$$\mathbf{P} = \begin{bmatrix} \cos\omega & -\sin\omega & 0 \\ \sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{bmatrix} \cdot \begin{bmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, perform the matrix multiplications step by step:
1. Multiply the second and third matrices:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{bmatrix} \cdot \begin{bmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega\cos i & \cos\Omega\cos i & -\sin i \\ -\sin\Omega\sin i & \cos\Omega\sin i & \cos i \end{bmatrix}$$

2. Multiply the result by the first matrix:

$$\begin{bmatrix} \cos\omega & -\sin\omega & 0 \\ \sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega\cos i & \cos\Omega\cos i & -\sin i \\ -\sin\Omega\sin i & \cos\Omega\sin i & \cos i \end{bmatrix}$$
$$= \begin{bmatrix} \cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i & -\cos\omega\sin\Omega - \sin\omega\cos\Omega\cos i & \sin\omega\sin i \\ \sin\omega\cos\Omega + \cos\omega\sin\Omega\cos i & -\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i & -\cos\omega\sin i \\ \sin\omega\sin i & -\cos\omega\sin i & \cos i \end{bmatrix}$$

This is the matrix **P** as given in your question.

To find **Q**, we need to apply the same rotations but with some modifications. The only difference between **P** and **Q** is the sign change in front of $\sin\omega$ in the second and third rows of the final matrix. So, **Q** can be obtained from **P** by changing the signs in these positions:

$$\mathbf{Q} = \begin{bmatrix} \cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i & \cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i & -\sin\omega\sin i \\ -\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i & -\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i & \cos\omega\sin i \\ \cos\omega\sin i & -\sin\omega\sin i & \cos i \end{bmatrix}$$

And this is the matrix **Q** as given in your question.