Home Exam II: Radar

FYS-3001 Physics of remote sensing

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Tromsø - Norway
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Read the data

Task 1:

To start of by reading the SLC data and plot it, we do this by taking the absolute value of the SLC image. By doing this we obtain the image represented in figure 1 below.

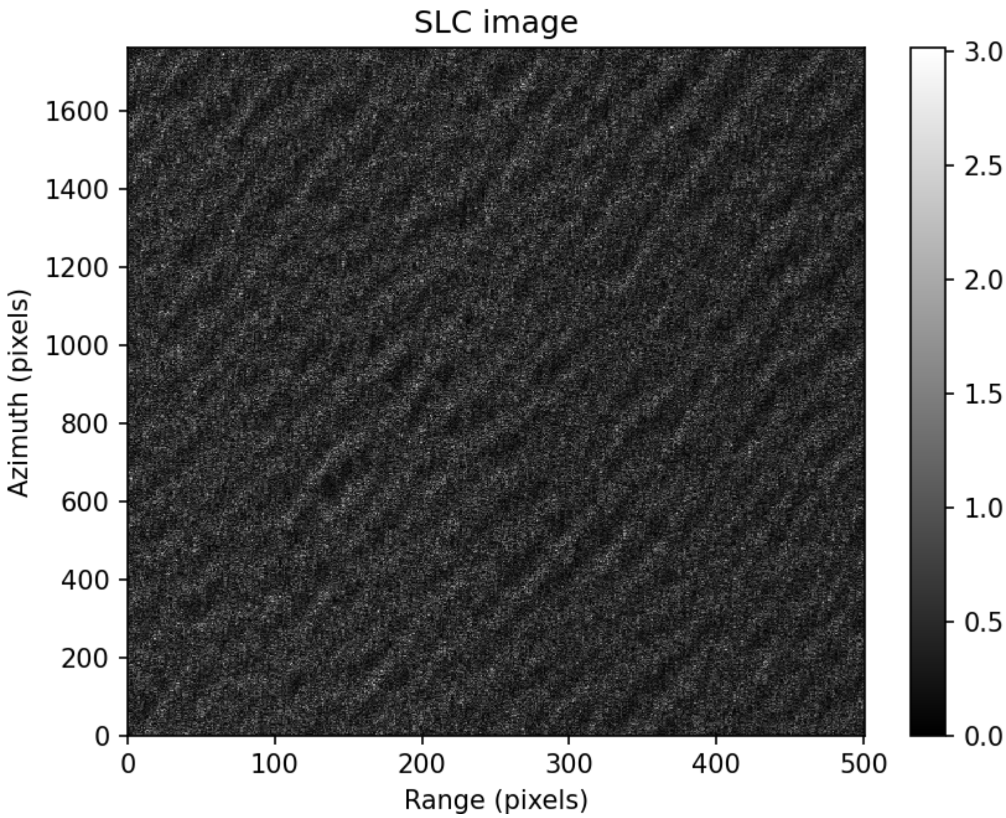


Figure 1: Absolute value of the complex image.

By comparing with figure 1 given in the exam paper, we can conclude with that we have obtained the correct complex image.

A. Image Statistics

Task 1:

Figure 2 shows the real and imaginary parts of the SLC image. By interpreting the histograms, I would say we have a *Laplace distribution* histogram. The plots show that we don't have a fully symmetric distribution on both sides of the mean, as some higher and lower peaks are evident and we have one instinct peak in both plots. However it is a possibility for the histogram to be Gaussian, as often histograms from SLC images is Gaussian.

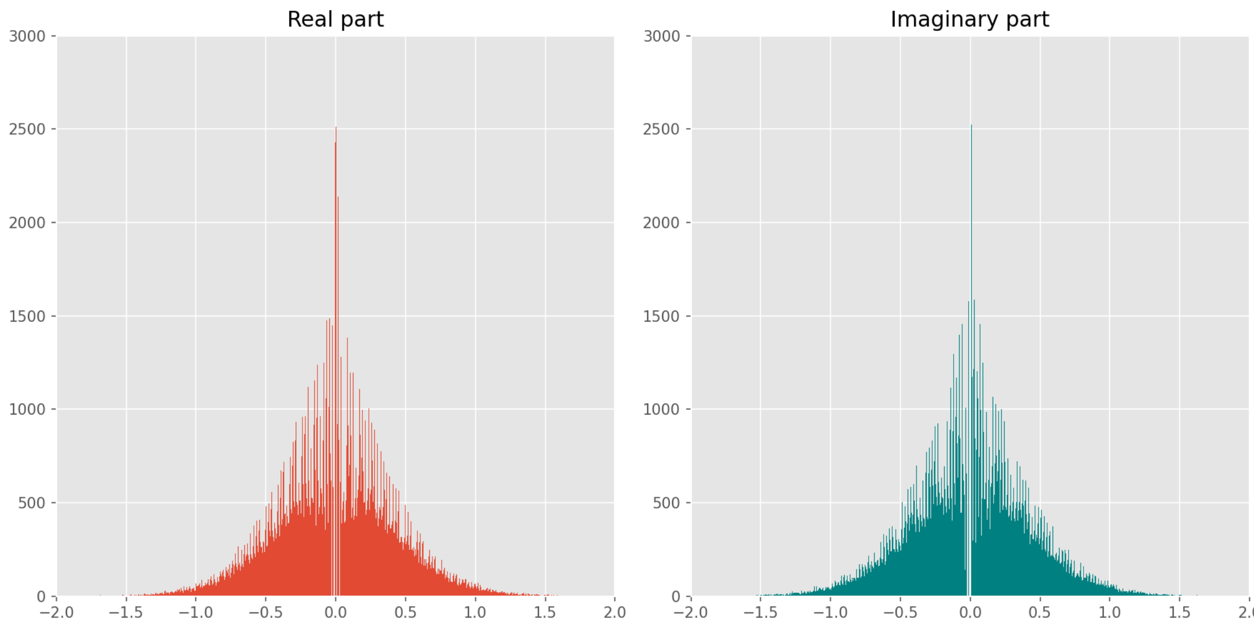


Figure 2: Histograms of the real and the imaginary part of the SLC image. (bins=6000)

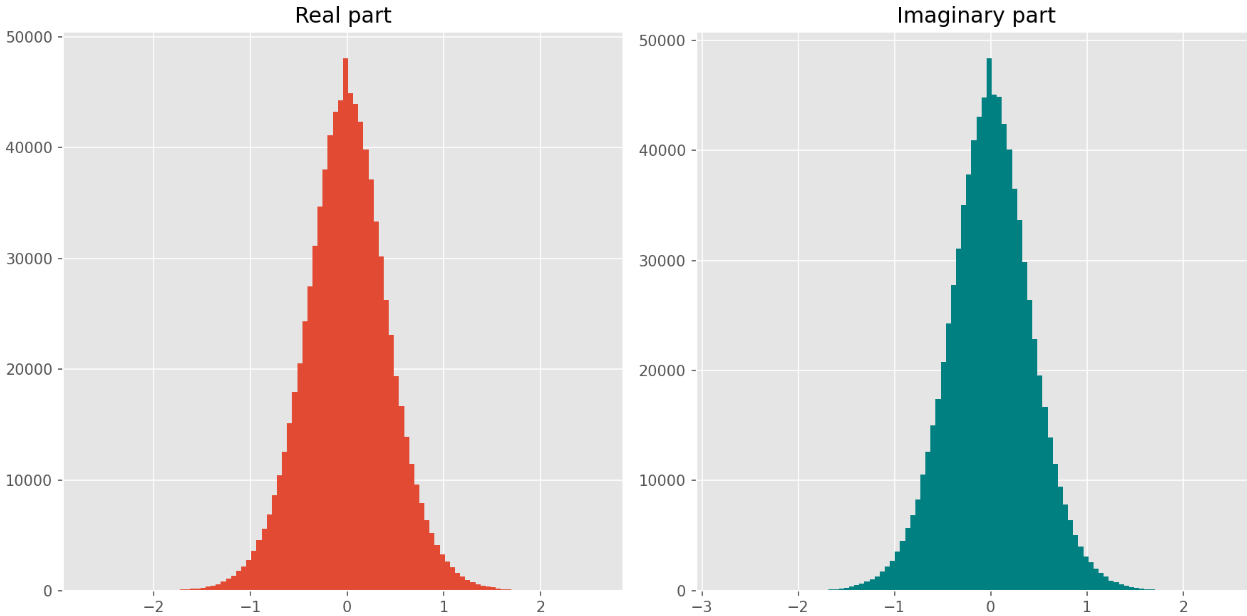


Figure 3: Histograms of the real and the imaginary part of the SLC image. (bins=100)

But to further investigate if we have a Laplace or Gaussian distribution, we can compute the histograms corresponding Laplace - and Gaussian distribution shown in figure 4. Here we can see that there is a bigger probability for this to be a Gaussian distribution regarding not being a perfect Gaussian distribution, also as the most common type of histograms for SLC images is Gaussian I will interpret that the type of histogram we have here is a Gaussian distributed histogram.

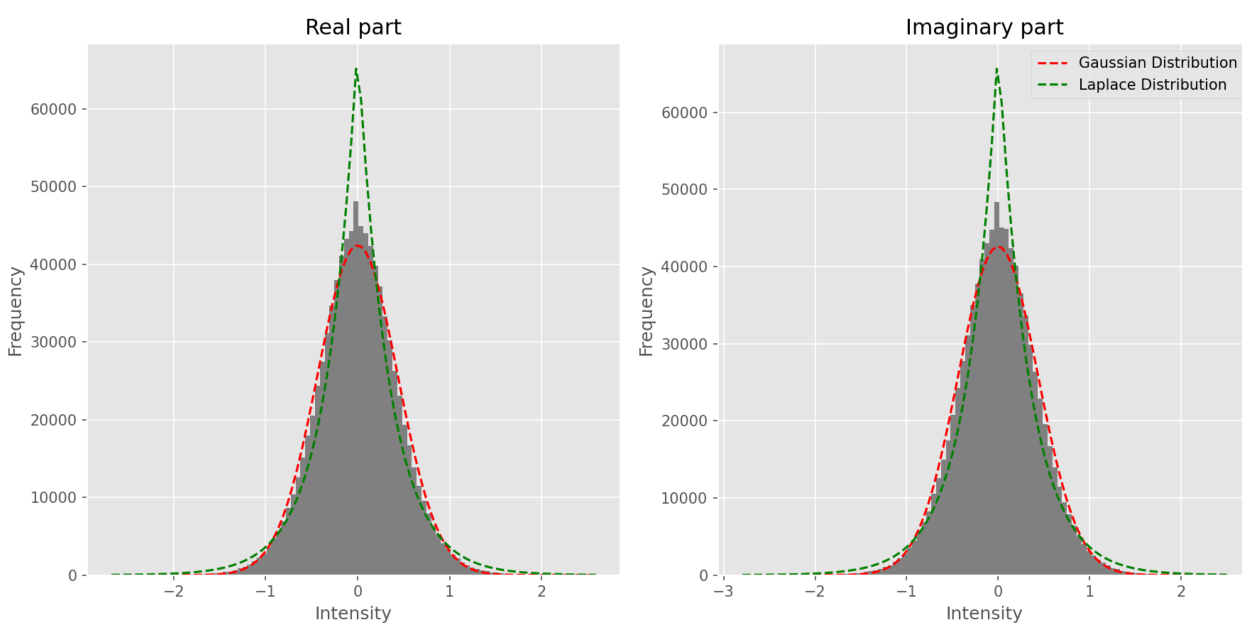


Figure 4: Histogram (bins=100) real and the imaginary part of the SLC image, with Gaussian distribution and Laplace distribution.

Task 2:

Figure 5 shows intensity from the complex SLC image, Where the x- and y-axis are normalized. The histogram represents a *Gamma distribution*, as we have a clear peak and a skew right.

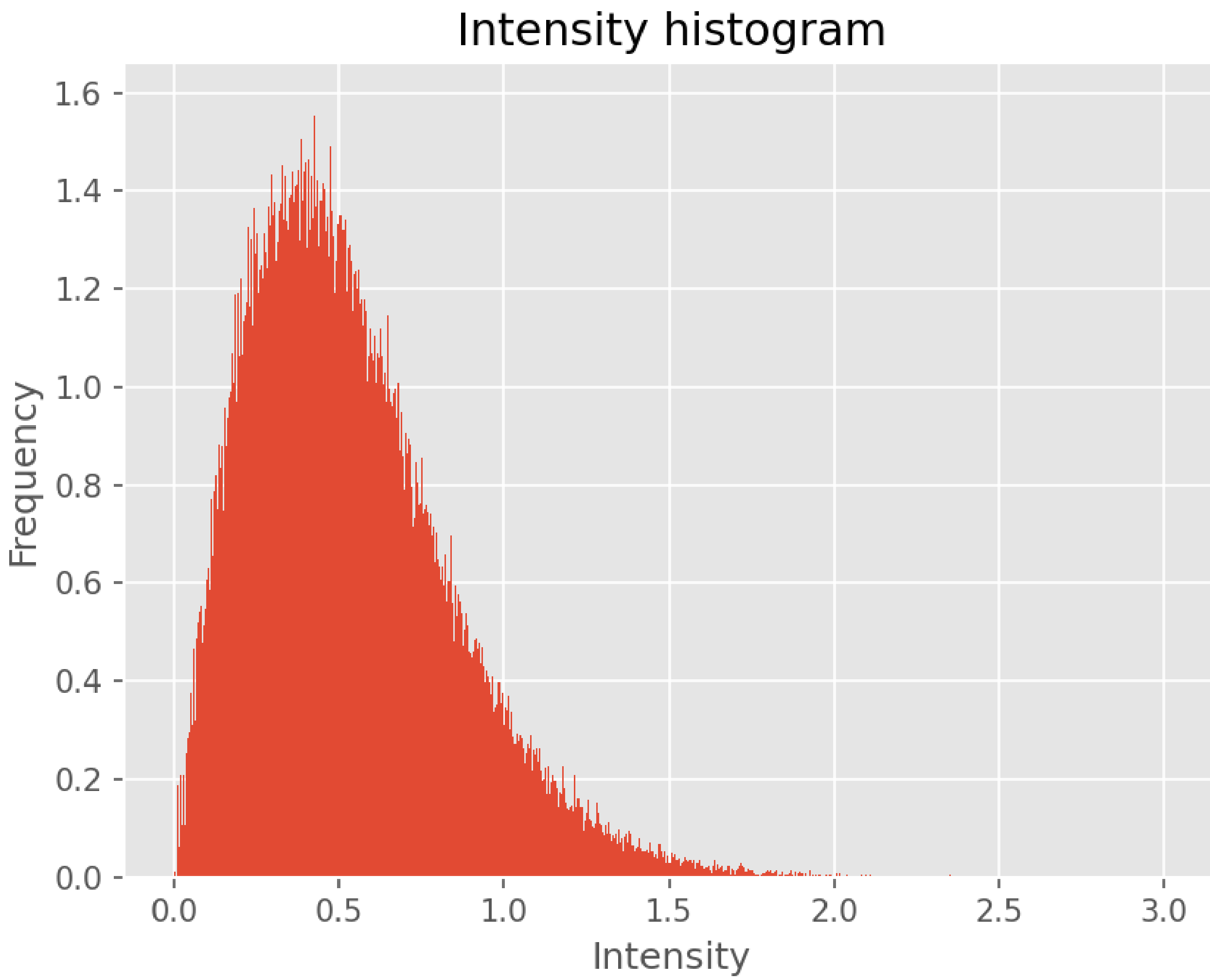


Figure 5: Histogram of the image intensity from the complex SLC image. (bins=6000)

We can further investigate if the Gamma distribution is correct, by plotting the related Gaussian distribution, Rayleigh distribution, and Gamma distribution shown in the figure 6. Which makes it clear that we have a Gamma-distributed histogram type.

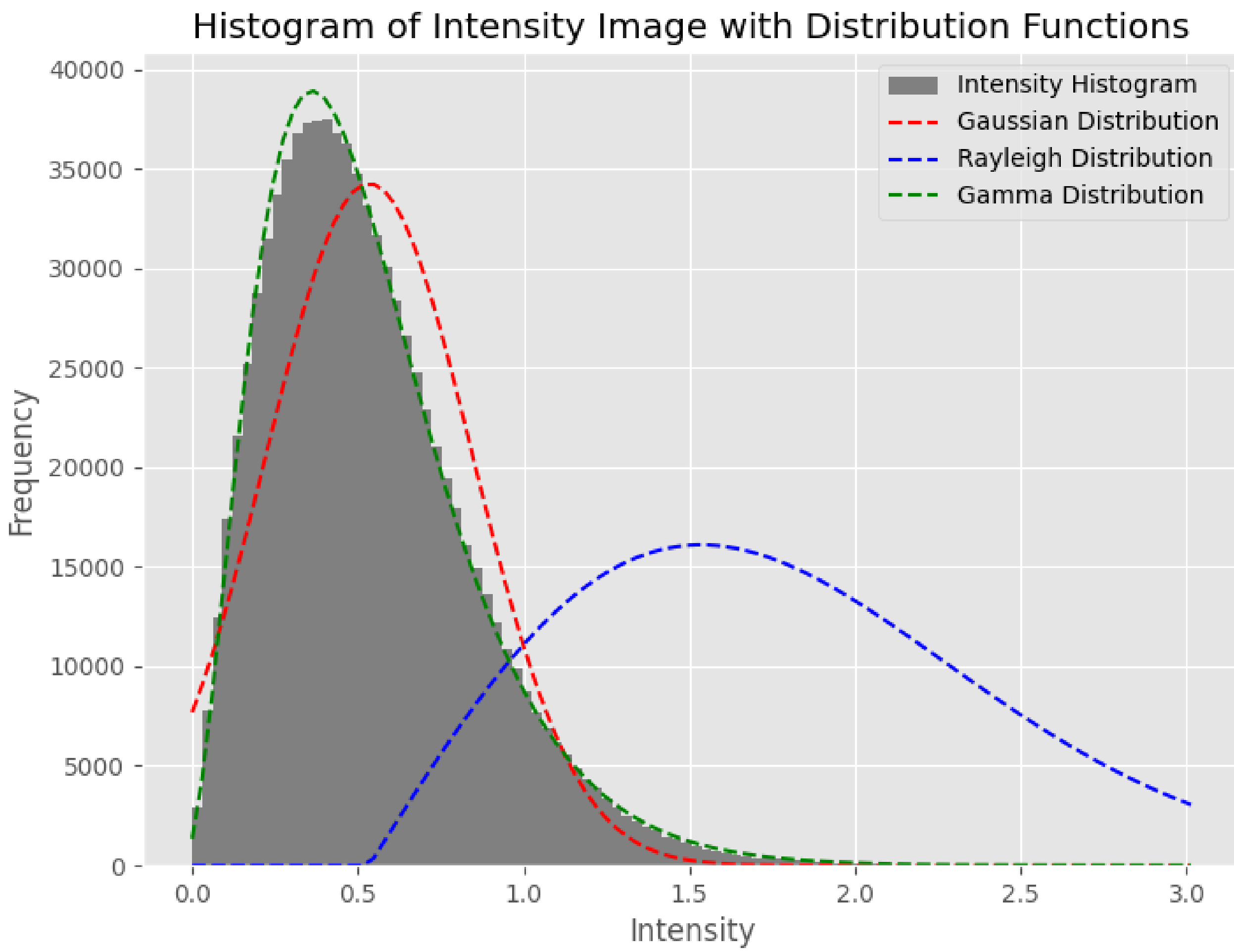


Figure 6: Intensity histogram (bins=100), with Gaussian distribution, Rayleigh distribution, and Gamma distribution.

By calculating the normalized variance of the intensity image, we obtain a normalized variance of **0.18**.

Task 3:

Figures 7 show a histogram of the smoothed intensity image, this is done by convolving the previous intensity image (figure 5) with a 5×5 sized kernel.

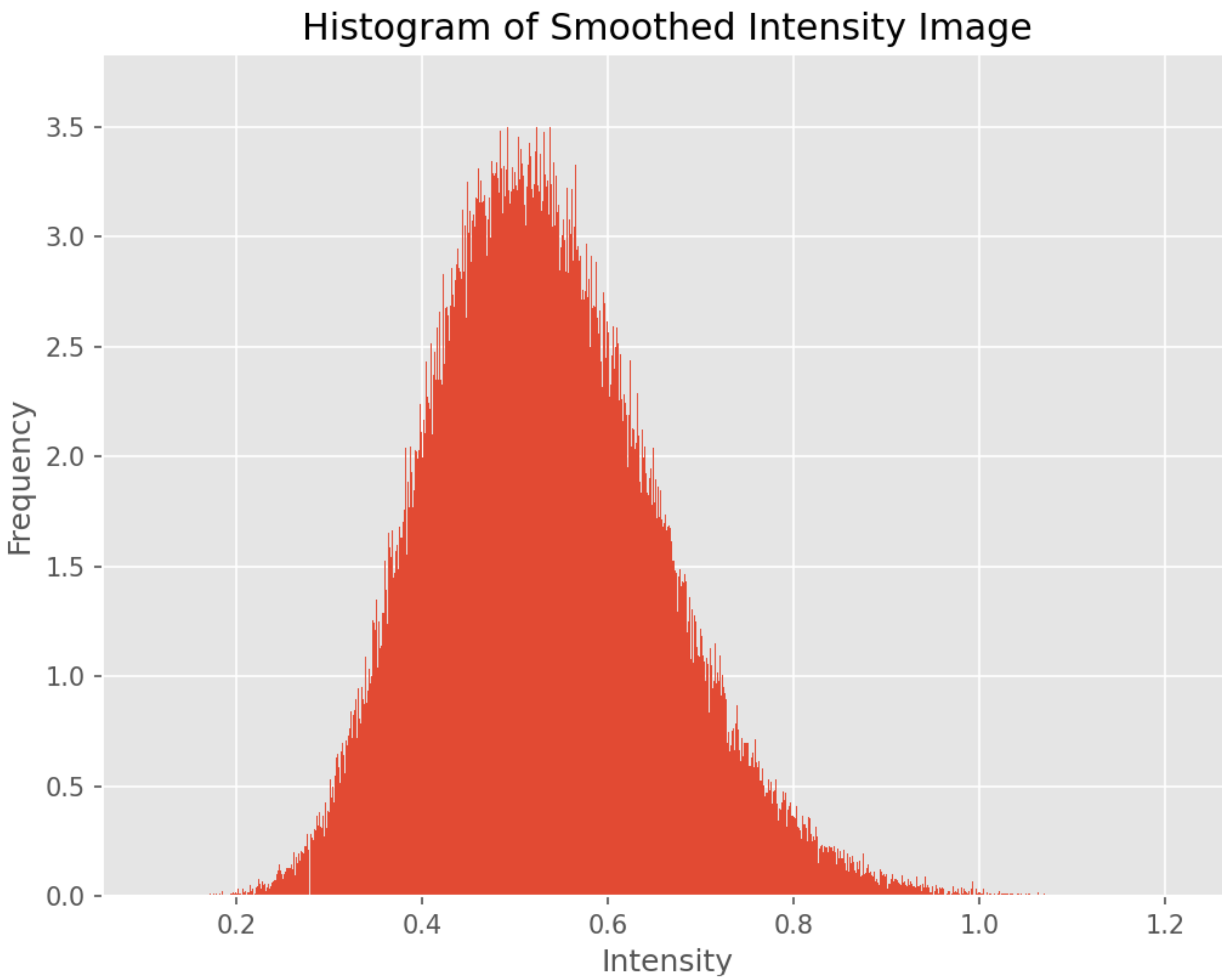


Figure 7: Histogram of smoothed intensity image, using a 5×5 kernel. (bins=6000)

For investigating which type of histogram we are working with, we have plotted the histogram with it's respectable Gaussian-, Rayleigh-, and Gamma distribution which is shown in figure 8. It becomes evident that we have a gamma-distributed histogram.

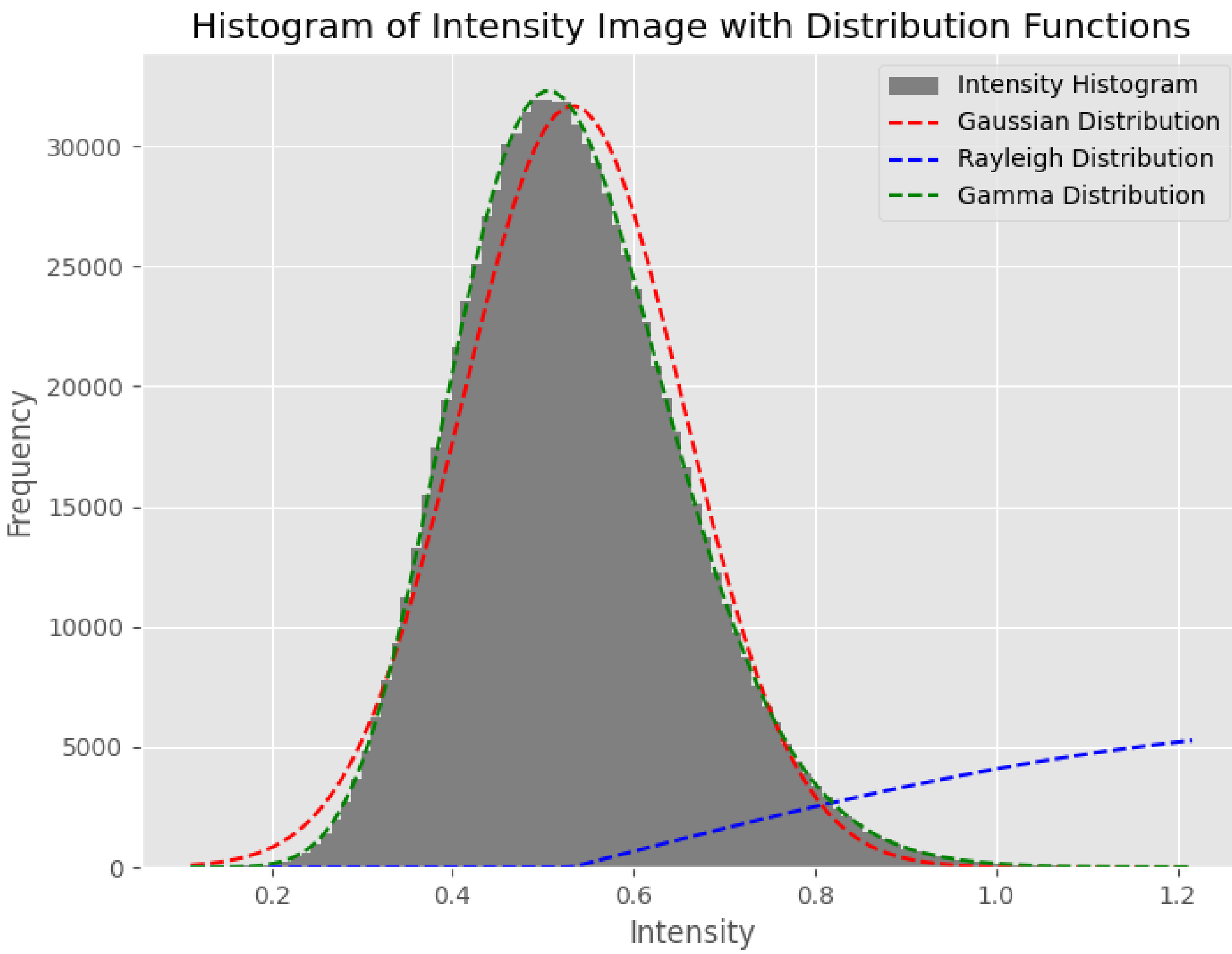


Figure 8: Smoothed Intensity histogram (bins=100), with Gaussian distribution, Rayleigh distribution, and Gamma distribution.

After the smoothing operation on the intensity image, we obtain a normalized variance of **0.028**.

Note: In tasks 1A to 3A we have included histogram plots containing 6000 bins and 100 bins, this is used to see if we have loss of information as we lower the number of bins.

Task 4:

In tasks 1a to 3a we analyzed the histogram of the SLC image, which is affected by a phenomenon called speckle. Speckle is a granular noise (i.e. salt-and-pepper noise) that arises in coherent imaging systems due to the interference of scattered electromagnetic waves.
The histogram of the real and imaginary parts (figure 2) of the SLC image typically exhibits a characteristic distribution, which often resembles Gaussian or Rayleigh. In speckle theory, the real and imaginary parts of the image are often modeled as independent, identical distributed complex Gaussian variables (V_x, V_y). As we obtained a histogram looking like

a distribution between Gaussian and Laplace distribution with one instinct peak, this may occur from non-Gaussian speckle statistics or sensor noise.
In the intensity histogram (figure 5), which was derived from the magnitude of the complex SLC image. The speckle theory predicts that the intensity of the image should follow a Gamma distribution rather than a Gaussian or Rayleigh distribution with a skewed distribution, Which comes true by looking on the plot 6 where the distribution functions are plotted. This comes from the multiplicative noise in speckles. We found that the normalized variance was **0.18**, which provides an insight into the amount of speckle noise/variability.
In the last histogram (figure 7), where we have plotted a smoothed intensity histogram using a 5×5 spatial filter. This is done to reduce the effects of speckle noise while preserving the important image features. By looking at figure 7, this becomes clear as the histogram now goes over a broader range of intensities. Here we will predict that the image will look, smoother. We are smoothing the image, as it can help mitigate speckle noise and make the image more interpretable. Since we just smoothed the intensity image, we will predict that the histogram is still gamma distributed which comes true in figure 8. To be certain that this reduced the amount of speckle, we computed again the normalized variance that ended up being **0.028** hence the speckle noise is reduced by a vast amount.

B. Look Extraction and Fourier Spectral Estimation

Task 1:

In figure 9, we have a complex-valued 2D spectra of the SLC image. This is obtained by performing a Fourier transform of the SLC image, then shifting it so that the zero wavenumber k=(0,0) becomes in the center of the array.

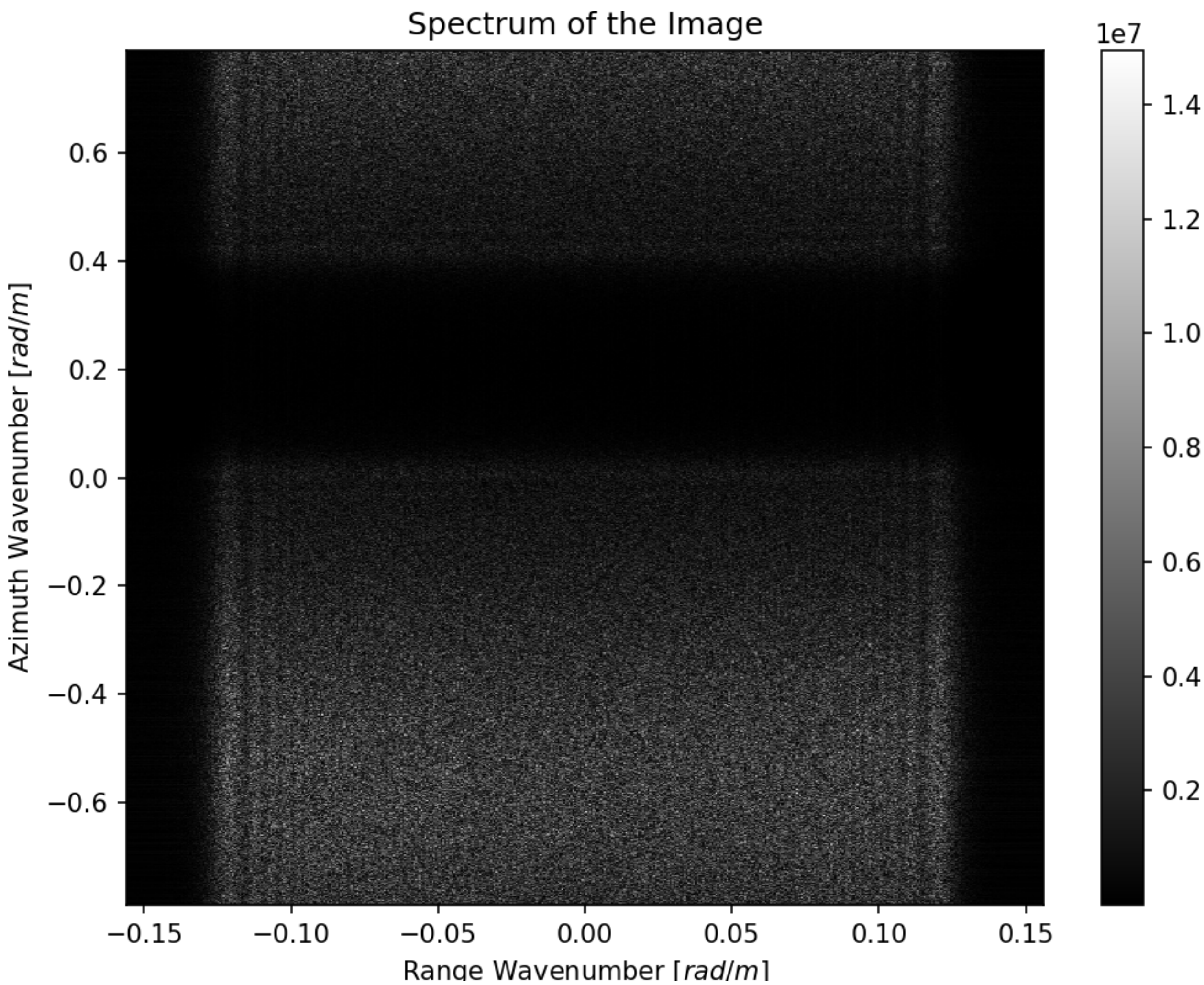


Figure 9: Complex valued 2D spectra

Here the axis is computed by first calculating the resolution of the pixels in range (x) and azimuth (y) of the input image using the equation.

$$\Delta x = \frac{c}{2 f_{sf} sin\theta} \quad \text{and} \quad \Delta y = \frac{V}{f_{prf}} \quad [m] \tag{1}$$

Where f_{sf} is the range, f_{prf} is the azimuth, V is the speed of radar ground velocity, c is the speed of light, and θ is the radar incidence angle. We can then use this to calculate the bin size in our 2D spectra, with the following formula.

$$\Delta k_x = \frac{2\pi}{N_x \Delta x} \qquad \text{and} \qquad \Delta k_y = \frac{2\pi}{N_y \Delta x} \qquad [rad/m] \tag{2}$$

Where N_x and N_y is the size of the Fourier transform in range and azimuth. The highest wavenumber in the spectra can be calculated using.

$$k_x^{max} = \frac{\pi}{\Delta x} \qquad \text{and} \qquad k_y^{max} = \frac{\pi}{\Delta y} \qquad [rad/m] \tag{3}$$

<u>Task 2:</u>

Now we generate a spectral profile in the azimuth direction by averaging the absolute value of the 2D spectra in the range direction, and we obtain the following plot shown in figure 10 of the azimuth Fourier domain with a multi-modal distribution of the frequencies as we have two small peaks and one high peak at zero.

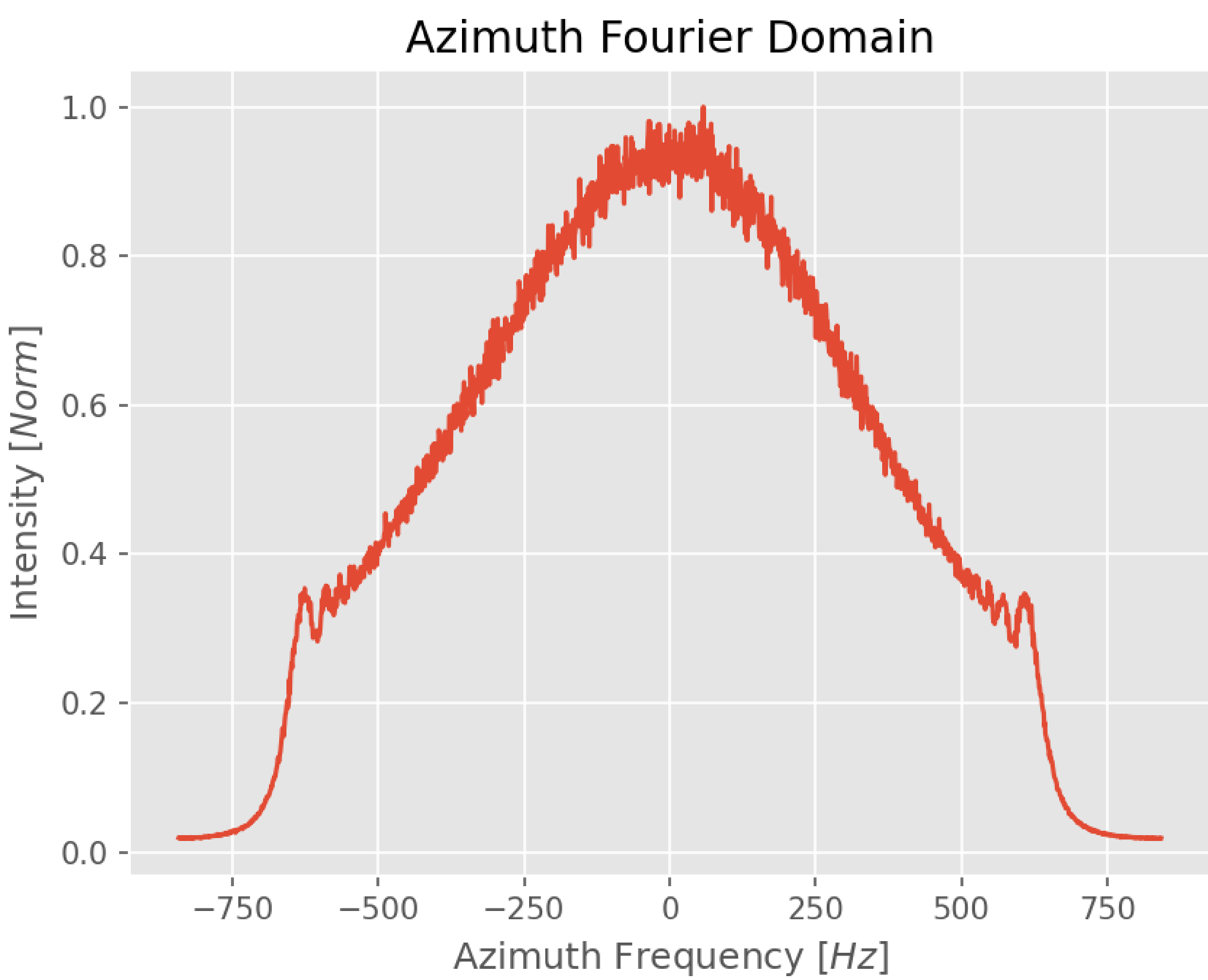
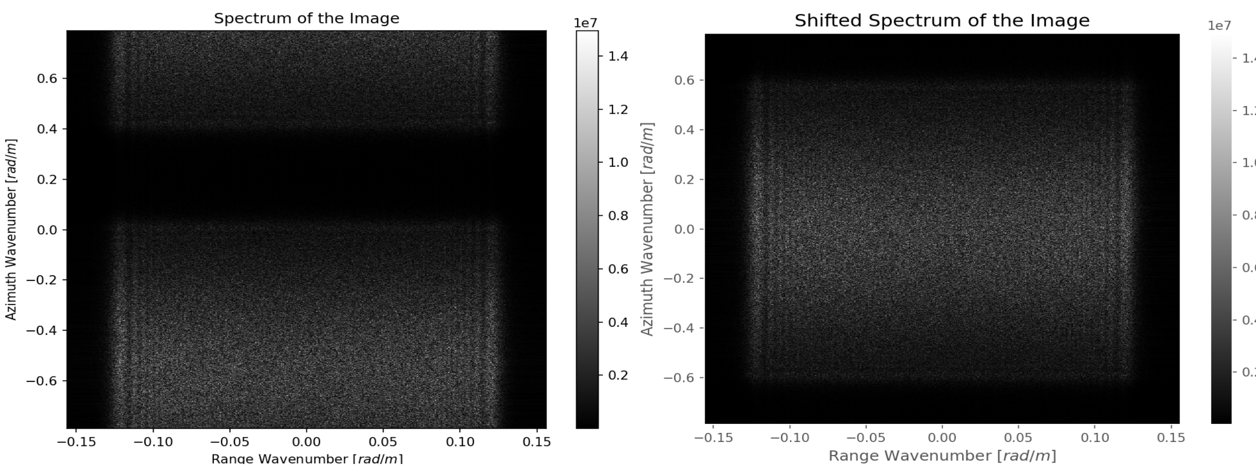


Figure 10: Azimuth Fourier domain

Where we have the minimum value of the azimuth frequency to be $-841.35Hz$, and the maximum value of $841.35Hz$. From the plot and computation, we find that the spectral profile is shifted around the zero frequency of about -194Hz. This also comes true by looking at figure 9. As we have two instinct peaks, we will have a pointer of where our 3 separate looks regarding their frequency bands. [1]

Task 3:

In the previous task, we found that the azimuth frequency was shifted by -194Hz, so we can shift the complex spectrum in the opposite azimuth direction to obtain a complex 2D spectrum with it's parts centered around k=(0,0). This is shown in figure 11, where plot 11a shows the original spectrum and plot 11b shows the azimuth shifted spectrum.



(a) (b)

Figure 11: Plot 11a shows the original complex 2D spectra obtained in task 1B, plot 11b shows the complex 2D spectra where the azimuth is shifted.

Task 4:

Now we bandpass filter the azimuth Fourier domain of the shifted 2D spectra from the previous task, into three equal looks. As shown in figure 12, the azimuth Fourier domain shown is obtained from task 2B hence not shifted. It is just for illustrating the bandpass ranges.

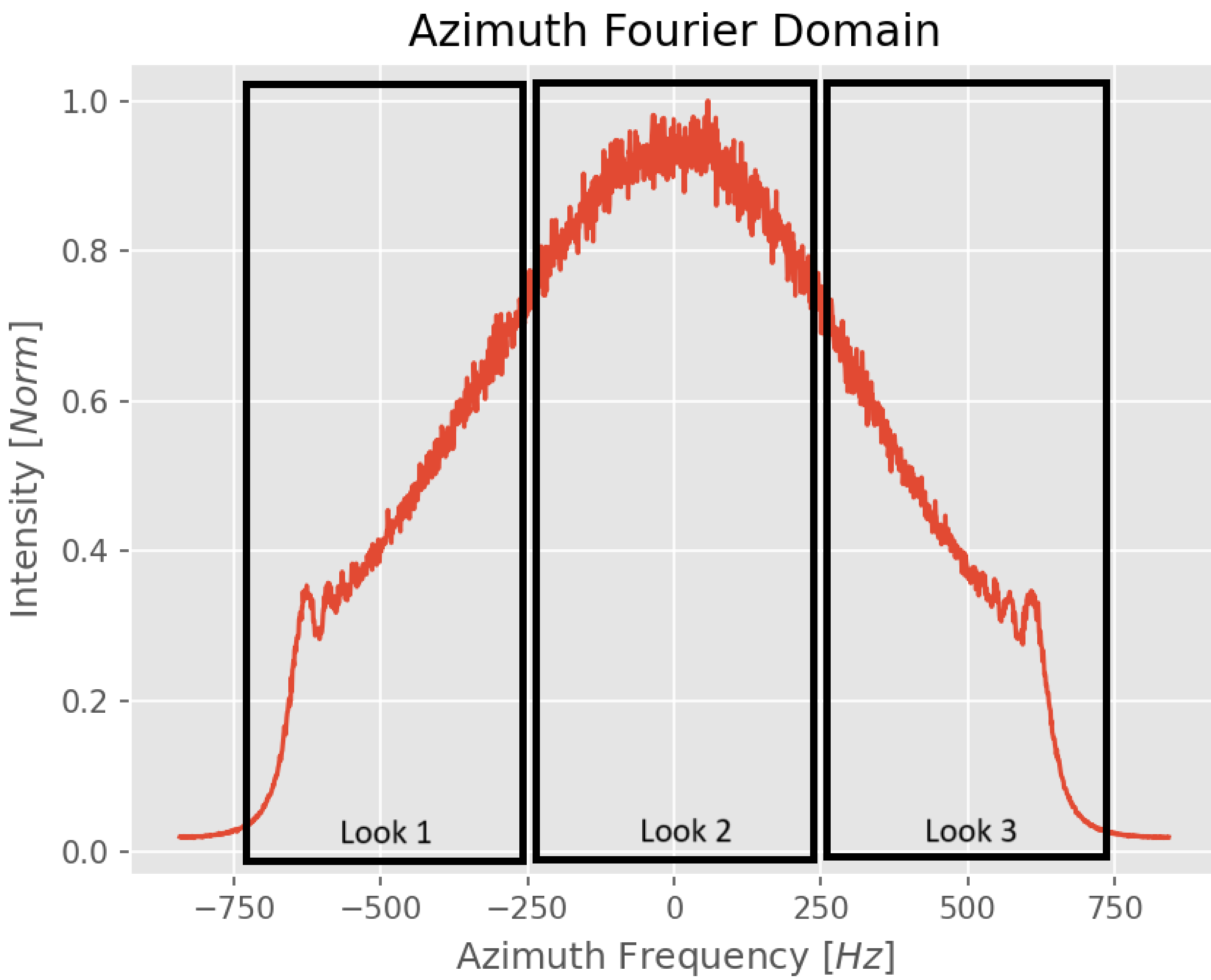


Figure 12: The look-extraction filters, the Azimuth Fourier domain is shifted but gives a good illustration. [1]

We can now extract each of the complex look and go back to the spatial domain by inverse Fourier transformation, and we end up with 3 intensity images shown in figure 13.

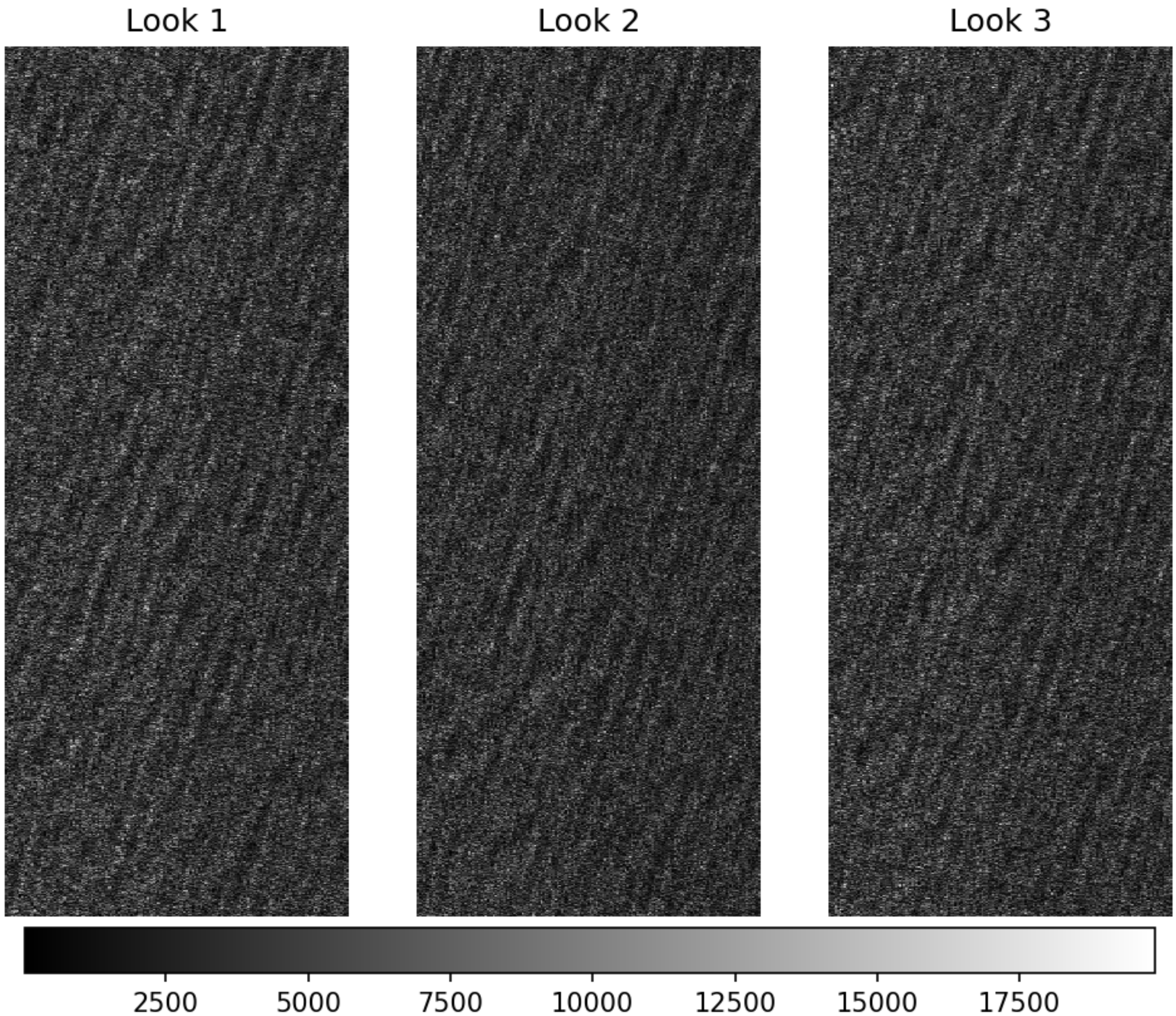


Figure 13: 3 intensity images, obtained from their respective filter ranger illustrated in figure 12.

Task 5:

As we have obtained the 3 intensity images, we can compute the co- and cross-image spectra between the images.
We compute the various spectra by computing the different products between the corresponding Fourier transforms, forward we remove the mean intensity and normalize the intensity images before the spectra computation ($I = (I - < I >)/ < I >$) and which gets followed up by taking the complex conjugate of the "2-look" (e.g. 1-look × conj(2-look)). This gives

the following co- and cross-spectra.

Co-Spectra:

$sub1 \times sub1$	$sub2 \times sub2$	$sub3 \times sub3$

Cross-spectra

$sub1 \times sub2$	$sub2 \times sub3$	$sub1 \times sub3$

Hence, we end up with 6 spectra. We average with the same look separation time, and we end up with the spectra shown in figure 14.

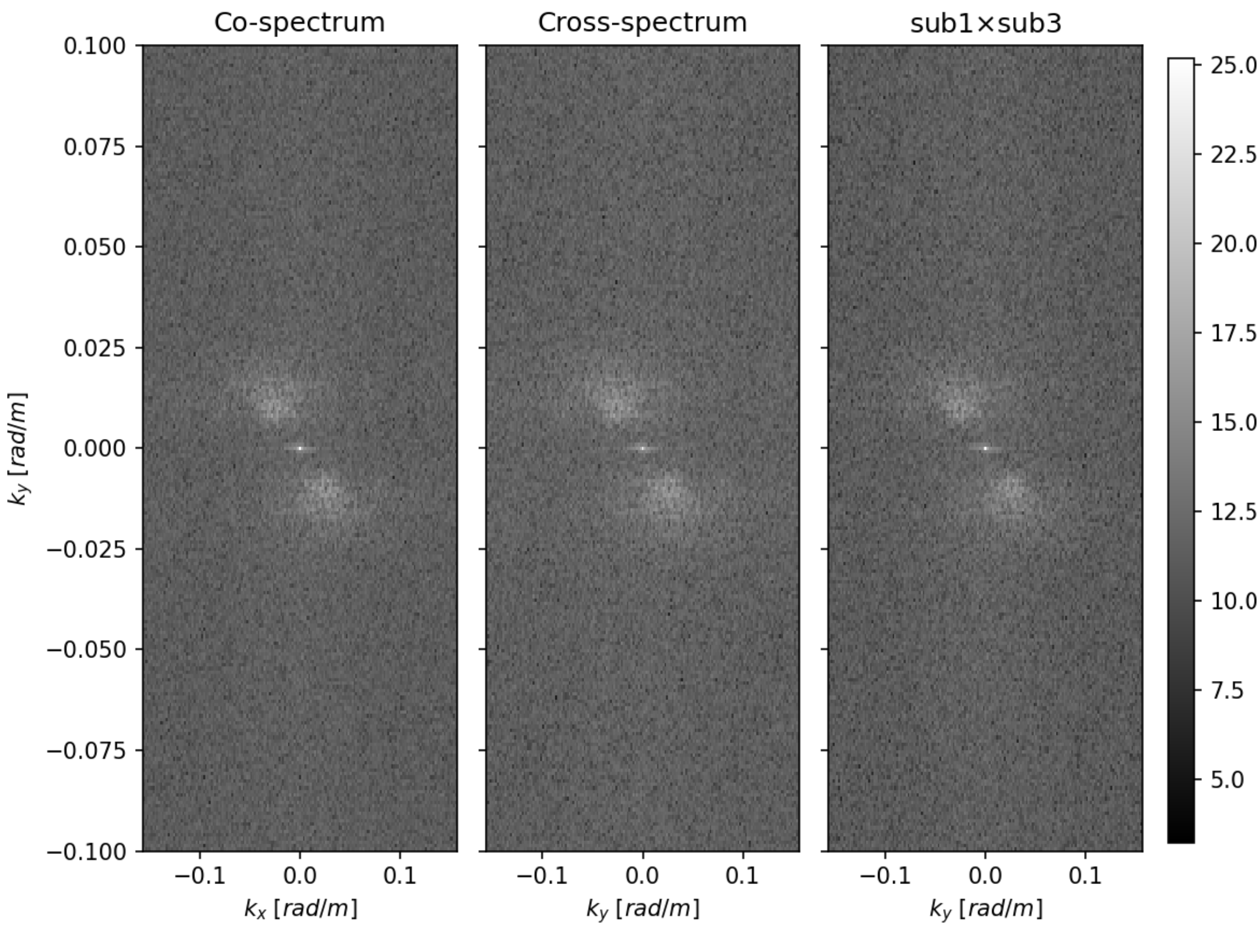


Figure 14: Co- and Cross-spectra between the 3 intensity images, in figure 13.

C. Analysis of 2D Spectra

Task 1:

Analyzing the co- and cross-spectra provides insights into wave propagation and the dominant spectral characteristics of the data.
Co-spectrum represents the spectral energy associated with the covariation of two signals at each frequency and wavenumber bin. The radar image (SLC), captures the correlation between the two components of the scattered signal: real and imaginary.
Cross-spectra represents the spectral energy associated with the cross-covariation between two signals at each frequency. In the SLC image, it captures the correlation between two different signals.
Figure 15 shows the Co-, cross- and cross(1×3)-spectra, with the related real and imaginary parts. As we take the complex conjugate, the imaginary part of the co-spectra will show zero. From the plots using the positive value for the spectral energy, we can calculate the wavelength of where the spectral energy maximizes. First, we convert the wavelength of the azimuth- and range-wavenumber to wavelength then use Pythagoras, this gives a wavelength of approx $611m$ where the spectral energy maximizes.

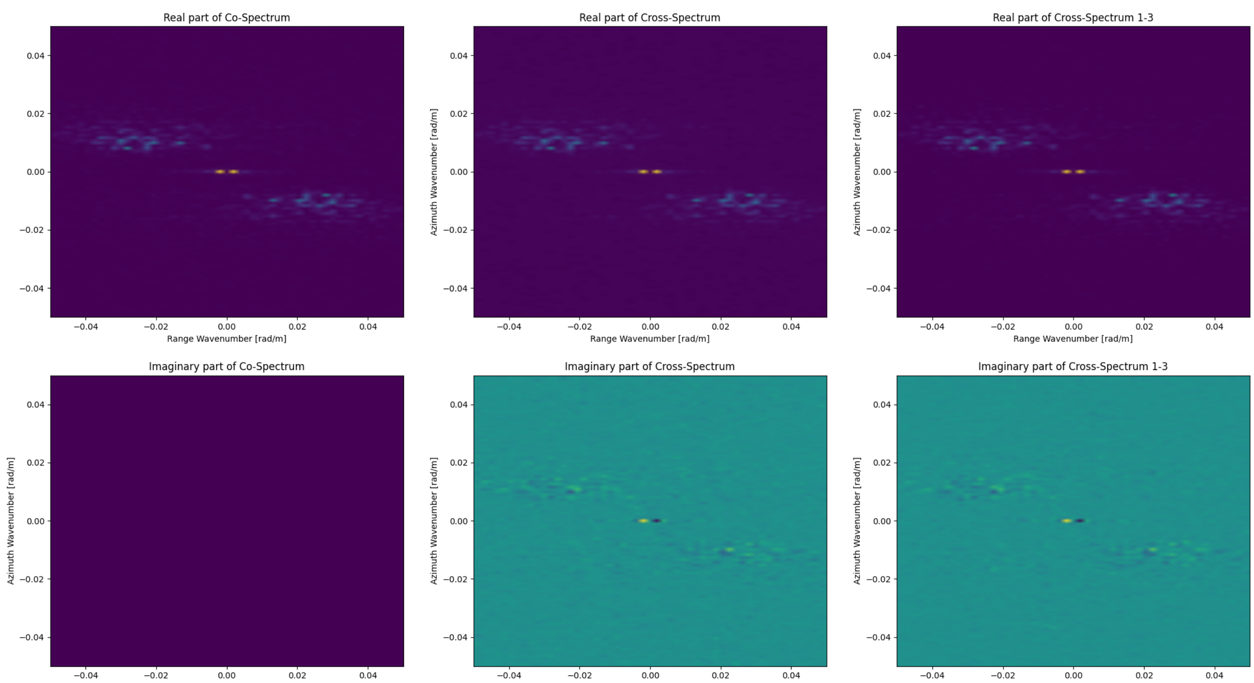


Figure 15: Plot of the real and imaginary parts of the 3 spectra, shown in figure 14.

Looking at figure 16, we see that we have one positive pulse on the left and one negative pulse on the right (where the arrow points). As we have a displacement from positive to negative in the right direction, it means that we have a unique wave propagation direction to the right, in the SLC image.

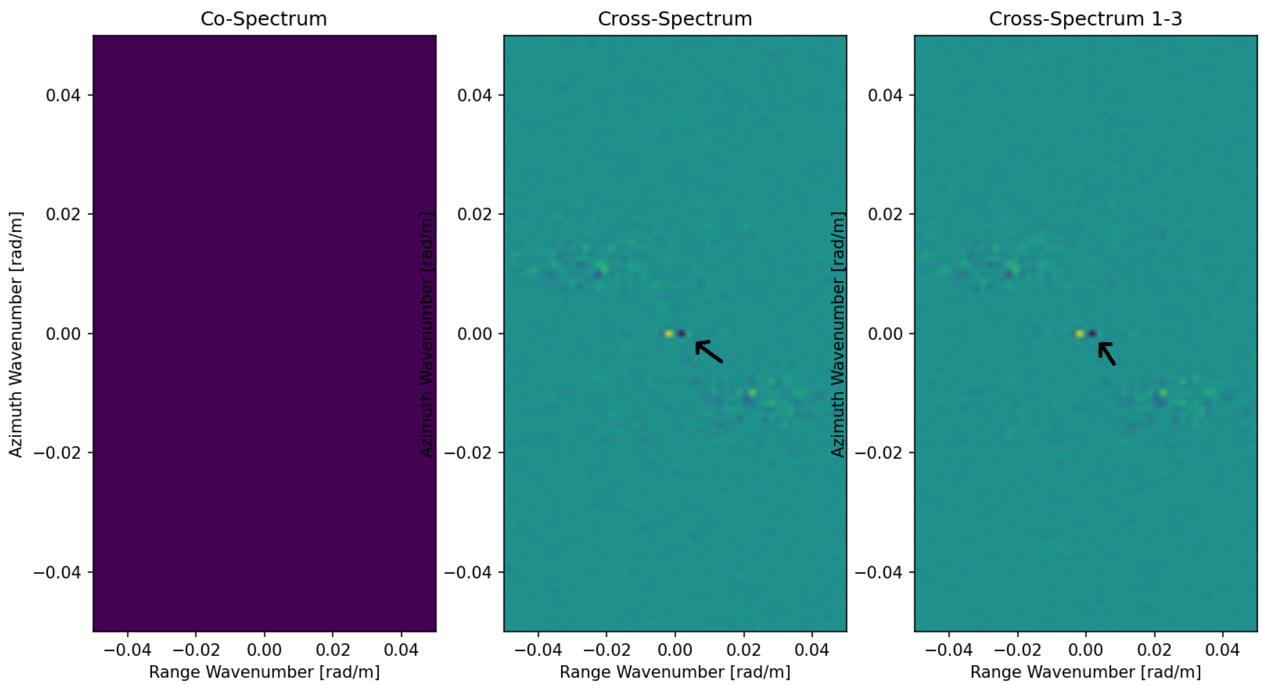


Figure 16: Imaginary parts of the Co- and Cross-spectra

Appendix

References

[1] R.Husson and P.Vincent. Sentinel-1 ocean swell wave spectra (osw) algorithm definition. *s-1 MPC*, 1.5(1.5):0–58, 2022,Oct.10.