Homework 3

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Question 1

a

From theorem 8, every resident weakly prefers the output of RADA to any other stable matching.Let μ be the output of RADA, and let μ' be another stable matching. Then, for every $r \in R$, $\mu(r) \succeq_r \mu'(r)$. In RADA, r_6 is matched to h_2 thus $\mu(r_6) = h_2$. If there exist an stable matching μ' such that $\mu'(r_6) = h_4$ then $\mu(r_6) \prec_{r_6} \mu'(r_6)$ which leads to contradiction with theorem 8. Thus, there is no stable matching that r_6 is matched to h_4

b

From Rural Hospitals Theorem, every under subscribed hospital have exactly the same residents matched to it in all stable matchings. h_3 is an under subscribed hospital as it has capacity 3 but has only 2 residents match to it under RADA. So all other stable matching, h_3 should have the same residents set (r_5, r_9), while r_3 is not in this set. Thus there is no stable matching that r_3 is matched to h_3

c

From Rural Hospitals Theorem, every hospital has exactly the same number of residents matched to it in all stable matchings. In RADA, h_4 has 3 residents thus h_4 will have 3 residents in another stable matching. Conclusion: there is no stable matching where h_4 has exactly 2 residents assigned,.

d

From theorem 11, For every $h \in H$ and stable matchings μ and μ', either h surely prefers μ and μ' or it surely prefers μ' and μ . Denote μ is stable matching where h_1 is assigned $r_9; r_{10}$. Denote μ' is stable matching where h_1 is assigned $r_6; r_1$. If h_1 surely prefer μ than μ' then $r_9 \succ_{h_1} r_6$, which is not true. In addition, if h_1 surely prefer μ' than μ' then $r_1 \succ_{h_1} r_9$, which is also not true. Conclusion: there is no stable matching where h_1 is assigned to $\{r_6, r_1\}$

Question 2

Denote μ is a Pareto efficient matching. A matching is Pareto efficient if it is not Pareto dominated by any other matching. Thus in μ, every $a \in A$ has its best choice among unassigned h, because if there exist an a' that does not get it best choice (denote it is h', then μ will be Pareto dominated by a matching that have $(a'$,$h')$ and other pairs keep the same. In addition, if every $a \in A$ has its best choice among unassigned h, then μ is Serial Dictatorship because we can make an ordering to satisty that.