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Electric vehicle scheduling and optimal charging problem: complexity, exact and heuristic approaches

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This paper deals with the Electric Vehicle (EV) Scheduling and Optimal Charging Problem. More precisely, given a fleet of EVs and Combustion Engine Vehicles (CVs), a set of tours to be processed by vehicles and a charging infrastructure, the problem aims to optimise the assignment of vehicles to tours and minimise the charging cost of EVs while considering several operational constraints mainly related to chargers, electricity grid and EVs driving range. We prove that the Electric Vehicle Scheduling and Charging Problem (EVSCP) is NP-hard in the ordinary sense. We provide a mixed-integer linear programming formulation to model the EVSCP and use CPLEX to solve small and medium instances. To solve large instances, we propose two heuristics: a Sequential Heuristic (SH) and a Global Heuristic (GH). The SH considers the EVs sequentially. To each EV, it assigns a set of tours and guarantees the feasibility of a charging schedule. Then, it generates an optimal charging schedule for this EV. However, the GH computes, in the first step, a feasible assignment of tours to all EVs. In the second step, it applies a global Min-Cost-Flow-based charging algorithm to minimise the charging cost of the EVs fleet. To evaluate the efficiency of our solving approaches, computational results on a large set of real and randomly generated test instances are reported and compared.

Keywords: electric vehicle; optimal charging; complexity; scheduling; mixed integer programming; greedy algorithms; maximum weight clique; minimum cost flow; experiments

1. Introduction

The transport sector is responsible for energy consumption and greenhouse gas emissions. To tackle environmental and energy challenges in this sector, Electric Vehicles (EVs) seem to be a potential alternative that offers a reduction in both petroleum consumption and greenhouse gas emissions.

However, the EV industry is still facing many weaknesses. The first is the limited EV driving range which varies between 80 and 130 km for light duty EVs. The second weakness is related to the long charging time of EVs (fully charging the battery pack can take up to 8 h with Level 1 chargers). The third weakness concerns the development of charging infrastructure networks and the limited capacity of the electricity grid.

In order to cope with some of the cited weaknesses, several initiatives are made to support a successful deployment of EVs. For example, the Transport and Tourism Committee of the European Commission voted a directive that ensures the deployment of 456,000 charging stations for public usage in Europe by 2020. Furthermore, some services related to charging infrastructures such as mapping services and roaming services are already proposed by several start-ups and companies. However, to ensure a successful deployment of EVs in the short-term, it is significant to target development towards (1) specific usage categories for which the EV is the most suitable (e.g. urban transport, urban logistics, business fleets) in terms of driving range, battery capacity, and operating cost, and (2) an optimal management of EVs ecosystem (vehicles – chargers – electricity grid – fleet management) by focusing on new optimisation challenges aiming to develop efficient models and decision tools to manage the ecosystem of EVs.

This study is a part of the French National Project Infini Drive, led by La Poste Group (Postal services company, France), ERDF (French company that manages the public electricity distribution network) and six other companies and research laboratories. This R&D project has been supported by ADEME (French Environment and Energy Management Agency) as part of the *Vehicle of the Future* program. It aims at designing, with a progressive approach, an intelligent system that manages the charging infrastructures and allows for an economical and ecological sustainable deployment of EVs fleet in the business context.

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The project Infini Drive focuses on the development of efficient decision tools that allow (i) an optimal assignment of already constructed routes to EVs and Combustion Engine Vehicles (CVs) while considering the energy requirement, the routes length, the EVs driving range and the technical constraints related to the power grid, with the objective of ensuring the profitability of EVs, (ii) producing optimal EVs charging planning that allows charging electric vehicles with sufficient energy in order to process all routes and minimise the total charging costs.

Note that the routes construction is not considered within the framework of this project. In fact, the routes construction in the business process of ERDF is a complex operation for which the availability, the skills of agents (drivers) and the qualification of agents asked by the demands of clients should be considered. The routes construction for both industrial users is provided by advanced optimisation software, which are specific to each industrial. Thus, the routes are considered as input data of the decision tools of the project. Furthermore, depending on the working day shift of agents, 80% of routes of both companies (La Poste and ERDF) have a total length of less than 75 km and can be made by EVs.

2. Related work

Optimisation problems related to EVs can be classified into four classes: (i) problems related to EV used in the context of Smart Grid or Vehicle to Grid (V2G), (ii) controlled EV charging problems, (iii) charging infrastructure network design problems and (iv) EV routing problems.

In the V2G context, EVs batteries are used to store energy. Those batteries can serve as an energy source by sending electricity back into the grid in order to either prevent or postpone load shedding. Research based on the V2G problems has mainly focused on how to connect the batteries of EVs to the power grid (Tomic and Kempton 2007), on proving the validity of the V2G concept (Kempton and Letendre 1997), identifying new markets (Kempton and Tomic 2005a; Kempton and Tomic 2005b) and controlling V2G systems (Hutson, Venayagamoorthy, and Corzine 2008; Shi and Wong 2011). The goal of the V2G control is to decide whether the EV should be charged, discharged or provide frequency regulation at each hour. Hutson, Venayagamoorthy, and Corzine (2008) consider the problem of maximising the EVs owners' profit by selling excessive energy to grid. A V2G control algorithm, based on the application of a binary particle swarm optimisation technique, is proposed. Shi and Wong (2011) considerthe real-time control problem of V2G and present a greedy algorithm. Lopez et al. (2013) address the problem of EV charging and V2G capabilities for congestion management. They propose an algorithm, based on power distribution factors, for optimal EV congestion management. Other studies on V2G problems include the works (Soares et al. 2013; Jian et al. 2014; Chen et al. 2015).

The problem of designing the charging infrastructure network is considered in many studies including the works (Wang 2007, 2008; He et al. 2013; Wang and Lin 2013). This problem is addressed by Wang (2007) who develops a model using an integer program in order to determine the locations of charging stations. Furthermore, Wang (2008) presents an integer programming model to optimise the locations and the number of battery exchange stations. Wang and Lin (2013) use the concepts of set covering problems to model the allocation of multiple types of charging stations. A mathematical program with complementarity constraints is developed by He et al. (2013) to determine an optimal allocation of a given number of public charging stations. Other studies related to infrastructure location problems include the works (Wang and Lin 2013; You and Hsieh 2014; Zheng et al. 2014; Hosseini and MirHassani 2015).

The controlled EV charging problems consist of a better management of the charging load and include delayed, timed and more advanced types of smart charging (Erol-Kantarci and Mouftah 2010; Deilami et al. 2011; Rotering and Ilic 2011). The objective of those problems is to minimise the charging cost. Sundstrom (2010) describe an approach based on a Mixed Integer Programming (MIP) formulation to optimise EV battery charging behaviour with the goal of minimising charging costs. The design of a simulation environment, which produces charging schedules using a multi-objective evolutionary optimisation algorithm is presented by Ramezani et al. (2011). Lee et al. (2011) expose an energy consumption scheduler that is able to reduce peak power load in smart places based on genetic algorithms. A concept of real-time scheduling techniques for EV charging that minimises the impact on the power grid and guarantees the satisfaction of consumer's charging requirements is suggested by Kang, Duncan, and Mavris (2013). Other studies related to the controlled EV charging problems can be found in Olivella-Rosell et al. (2015) and Chen et al. (2015).

The EV routing problem is less considered in the literature. The energy-optimal routing problem is addressed by Artmeier et al. (2010) and it is modelled as the shortest path problem with additional constraints concerning the energy consumption. The authors propose search algorithms that satisfy the energy constraints. Erdogan and Miller-Hooks (2012) formulate the Green Vehicle Routing Problem as a MIP. Two constructive heuristics are developed to solve this problem. Schneider, Stenger, and Goeke (1989) combine an EV routing problem with time windows and the possibility of charging EVs at stations along the route. The objective is to minimise the number of employed vehicles and the total travelled distance. A hybrid algorithm that combines a variable neighbourhood search algorithm with a tabu search is proposed. In Goeke and Schneider (2014), the EV Routing Problem with Time Windows and Mixed Fleet to optimise the routing of a mixed fleet of EVs and

Combustion Engine Vehicles (CVs) is addressed. To solve this problem, an Adaptive Large Neighborhood Search algorithm that is enhanced by a local search is proposed. In Hiermann, Puchinger, and Hartl (2014), the authors introduce the Electric Fleet Size and Mix Vehicle Routing Problem with Time Windows and Recharging Stations. The objective is to optimise the fleet and the vehicle routes including the choice of recharging times and recharging stations. To solve this problem, a hybrid heuristic, which combines an Adaptive Large Neighbourhood Search and an embedded local search and labelling procedure for intensification, is introduced. In Felipe et al. (2014), the authors present a variation of the EV routing problem in which different charging technologies are considered and partial EV charging is allowed. Constructive and local search heuristics are proposed. A recent overview of topics related to EVs use for goods distribution can be found in Pelletier, Jabali, and Laporte (2014), Afroditi et al. (2014) and Lin et al. (2014).

In this paper, we address the Electric Vehicle Scheduling and Charging Problem (EVSCP). The EVSCP is defined as follows: Given a set of tours, a fleet of EVs and CVs and a charging infrastructure, the objective is to seek an optimal way to assign tours to vehicles and to minimise the EVs charging cost while satisfying the constraints related to the electricity grid, chargers and EVs batteries capacities. An extended abstract of this paper is presented in Sassi and Oulamara (2014).

The remainder of the paper is organised as follows: Section 3 presents the problem and describes the notation used in the remaining sections. In Section 4, we prove the NP-Hardness of the problem. Section 5 provides a detailed description of the mathematical formulation of the problem. In Sections 6 and 7, our exact and heuristic approaches are presented. Section 9 summarises the computational results. Concluding remarks are given in Section 10.

3. Problem description

We consider a set $M_1 = \{1, \dots, m_1\}$ of EVs and a set $M_2 = \{1, \dots, m_2\}$ of CVs required to process n tours during the time horizon $[0, T]$. Each vehicle can process, at most, one tour at a time. Each EV j operates with a battery characterised by its capacity B_j (kWh) and its State of Charge $\left(\mathrm{SoC}_j\right)$ defined as the available capacity and expressed as a percentage of its nominal capacity B_j (0 = empty; 1 = full). Let SoC_j^0 be the initial state of charge of EV j at time $t = 0$. In order to improve batteries lifetime after repeated use (Bashash et al. 2011) and to respect the security issues, at each time, SoC_j should be in $\left[\mathrm{SoC}_j^{\min}, \mathrm{SoC}_j^{\max}\right]$, where $\mathrm{SoC}_j^{\min}$ and $\mathrm{SoC}_j^{\max}$ are the minimal and the maximal allowable values of SoC, respectively. We assume that there are m_1 chargers available to charge EVs during $[0, T]$. This time horizon is divided into T equidistant time periods, $t = 1, \dots, T$, each of length d, where t represents the time interval $[t-1, t]$. For our problem, we fix d to 15 min, this setting is justified by the fact that EV manufacturers recommend that a charging phase should last at least 15 min to avoid undesirable chemical reaction in the lithium-ion batteries. Furthermore, the data concerning the available energy for EV charging provided by the electricity grid provider are given by a step of 15 min. At each time period t, each charger can provide, to EV j, a charging power $p_{jt} \in \left[p^{\min}, p^{\max}\right]$, where $p^{\min}$ and $p^{\max}$ are, respectively, the minimal and maximal powers that can be delivered by the charger. Thus, an EV charged with a power p_{jt} during the time period t retrieves a total amount of energy equal to $d \times p_{jt}$ (kWh). We denote by g_t the electricity grid capacity available for EV charging at time t. Let c_t be the energy cost during time period t. There is a set of n tours to be processed by the vehicles. Each tour i is characterised by a start time s_i, a finish time f_i, a weight w_i (km) and energy E_i (kWh) required to perform the tour with an EV. Two tours i and j overlap if their intersection is nonempty, i.e. $[s_i, f_i] \cap \left[s_j, f_j\right] \neq \emptyset$, otherwise they are disjoint. Given a tour i, let $V(i)$ be the set of tours such that $\forall\, j \in V(i)$, i and j overlap. Note that all tours are already constructed. Thus the start and finish times of each tour are known in advance and, to satisfy operational constraints, pre-emption of tours is not allowed and the vehicles cannot charge while in tour. The objective consists in maximizing the total weight of tours processed by EVs and minimising the charging costs.

This problem can be seen as a fixed interval scheduling problem (Kolen et al. 2007; Kovalyov, Ng, and Cheng 2007) with additional constraints of energy. This problem has, so far, never been considered in the literature.

4. NP-hardness result

In this section, we prove the NP-hardness of the EVSCP. We use the reduction to the Partition problem, which is known to be NP-complete in the ordinary sense (Garey and Johnson 1979). This problem can be stated as follows: Given a set $P = \{a_1, a_2, \dots, a_p\}$ of p integers such that $\sum_{i=1}^{p} a_i = 2A$, is there a partition of P into two subsets P_1 and P_2 such that $\sum_{i \in P_1} a_i = \sum_{i \in P_2} a_i = A$?

THEOREM 1 *EVSCP is NP-hard in the weak sense.*

Table 1. Characteristics of the instance ($\mathcal{I}$).

Tour	Start time	Finish time	Energy consumption
$T_i;\ i = 1, \ldots, 2p$	$A + \sum_{k=0}^{i-1} a_k$	$A + \sum_{k=0}^{i} a_k$	a_i
T_{2p+1}	$3A$	$3A + 1$	A
T_{2p+2}	$3A$	$3A + 1$	A

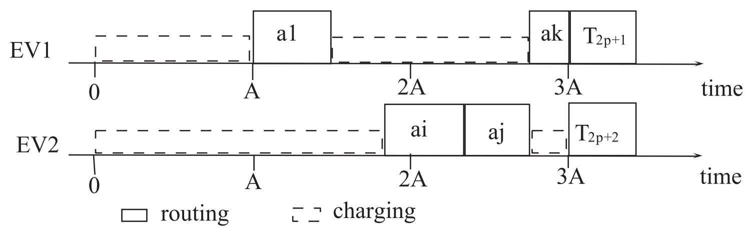

Figure 1. Solution to the scheduling problem on two EVs – EV1 and EV2.

Proof Given an arbitrary instance of the Partition problem, we build an instance ($\mathcal{I}$) of the EVSCP with a set of $2p + 2$ tours. The start times, the finish times and the energy consumptions of those tours are given in Table 1. We assume that $a_0 = 0$.

Without loss of generality, we restrict our analysis to the case of two available EVs with a battery capacity of $2A$ (kWh). We assume that there exist two chargers. Each charger can provide one discrete power of 1 (kW). The grid capacity available for EV charging is equal to 2 (kW) at each time interval. At $t = 0$, the batteries of both EVs are empty and we consider $\text{SoC}^{\min} = 0$ and $\text{SoC}^{\max} = 1$ for both vehicles. In what follows, we show that there exist a schedule of tours and a feasible charging planning of EVs if, and only if, the Partition problem admits a solution (Figure 1).

First, assume that the Partition problem has a solution and let P_1 and P_2 be the required subsets of P. The desired schedule of tours is constructed by building two sets L_1 and L_2 of tours $T_1, \ldots, T_{2p}$, where tours of L_1 and L_2 correspond to the sets P_1 and P_2, respectively, i.e. $T_i \in L_j$, if $a_i \in P_j$, $i = 1, \ldots, 2p$, $j = 1, 2$. Assign tours of L_1 to EV1 and tours of L_2 to EV2. The tours T_{2p+1} and T_{2p+2} are assigned to EV1 and EV2, respectively. In order to execute the assigned tours, the EVs are charged as follows: during the time interval $[0, A]$, each EV is charged with a power of 1 (kW); at $t = A$ each EV recovers A (kWh). During the time interval $[A, 3A+1]$, EV1 processes all tours of L_1, for A units of time, and is charged when it is not processing any tour i.e. EV1 is charged during the intervals $\left[A + \sum_{k=0}^{i-1} a_k,\ A + \sum_{k=0}^{i} a_k\right]$, $i \in L_2$, and, finally it processes the tour T_{2p+1}, whereas EV2 processes all tours of L_2 and gets charged during the intervals $\left[A + \sum_{k=0}^{i-1} a_k,\ A + \sum_{k=0}^{i} a_k\right]$, $i \in L_1$ before performing the tour T_{2p+2}. It is easy to see that, during the different charging phases, each EV recovers enough amount of energy to process all tours, and at each time the SoC of each EV is in the interval $\left[\text{SoC}^{\min}, \text{SoC}^{\max}\right]$. Therefore, the obtained schedule is feasible.

Conversely, assume that there exist a feasible schedule of tours and a charging planning of EVs. Since T_{2p+1} and T_{2p+2} have the same start and finish times, we assume that T_{2p+1} and T_{2p+2} are executed by EV1 and EV2, respectively. Let R_1 and R_2 be the two sets of tours assigned to EV1 and EV2, respectively. Let $\sum_{a_i \in R_1} a_i = A_1$ and $\sum_{a_i \in R_2} a_i = A_2$ be the amounts of energy required by the EVs to execute the sets R_1 and R_2 of tours, respectively. Then, EV1 and EV2 need a total amount of energy equal to $A + A_1$ and $A + A_2$, respectively. Assume that $A_1 > A$. Then, EV1 needs an amount of energy greater than $2A$. EV1 can be charged only during intervals $[0, A] \bigcup_{i \in R_2} \left[A + \sum_{k=0}^{i-1} a_k,\ A + \sum_{k=0}^{i} a_k\right]$ and it may recover at most a total amount of energy $E = 3A - A_1 < 2A$ (kWh). Since $E < 2A$, there is not enough energy to process the set of tours R_1 and T_{2p+1}, then $A_1 \leq A$. Following the same reasoning, we have $A_2 \leq A$. Since $A_1 + A_2 = 2A$, we conclude that $A_1 = A$ and $A_2 = A$. Therefore, R_1 and R_2 represent a solution to the Partition problem. □

5. Problem formulation

In this section, we propose a MIP for the EVSCP. Before presenting the MIP model, in the following we summarise the nomenclature of the problem.

$[0, T]$: optimisation time horizon.
$t = 1, \ldots, T$: discretisation of T.
$M_1 = \{1, \ldots, m_1\}$: set of m_1 EVs.
$M_2 = \{1, \ldots, m_2\}$: set of m_2 CVs.
B_j: EV j battery capacity (kWh).
SoC_j^0, SoC_j^t: state of Charge of EV j at times 0 and t, respectively.
$\mathrm{SoC}_j^{\min}$, $\mathrm{SoC}_j^{\max}$: minimal and maximal allowable values of SoC.
$p^{\min}$, $p^{\max}$: minimal and maximal power that could deliver a charger.
g_t: electricity grid capacity available at t.
c_t: energy cost during t.
n: number of tours.
s_i, f_i: start and finish times of tour i.
w_i: length of tour i.
E_i: amount of energy required to perform tour i.

We introduce the following decision variables:
x_{ij} : 0–1 variable equals 1 if the vehicle j, $j = 1, \ldots, m_1 + m_2$, is allocated to tour i, $i = 1, \ldots, n$, and 0 otherwise.
y_{jt} : 0–1 variable equals 1 if the EV j, $j = 1, \ldots, m_1$, is charged during the time interval $t = 1, \ldots, T$ and 0 otherwise.
p_{jt} : real variable denotes the charging power level applied to EV j, $j = 1, \ldots, m_1$ at time interval $t = 1, \ldots, T$.

In order to reduce the complexity of the proposed MIP and, without loss of generality, we assume that the energy E_i required to perform the tour i is consumed during the last period f_i of tour i; i.e. during the time interval $[f_i - 1, f_i]$ $\left(E_{i,f_i} = E_i \text{ and } E_{i,l} = 0, l = s_i, s_i + 1, \ldots, f_i - 1\right)$. The EVSCP is formulated as a MIP. Its mathematical formulation $(\mathcal{P})$ is the following:

$$\mathrm{Lex}\left(\max \sum_{i=1}^{n}\sum_{j=1}^{m_1} w_i \times x_{ij}\ ;\ \min \sum_{j=1}^{m_1}\sum_{t=1}^{T} d \times c_t \times p_{jt}\right) \tag{1}$$

$$\sum_{j=1}^{m_1+m_2} x_{ij} = 1, \quad \forall i \tag{2}$$

$$x_{ij} + \sum_{i' \in V(i)} x_{i'j} \leq 1, \quad \forall i, \forall j \tag{3}$$

$$\sum_{t=s_i}^{f_i} y_{jt} + (f_i - s_i + 1) \times x_{ij} \leq (f_i - s_i + 1), \quad \forall i, \forall j \in M_1 \tag{4}$$

$$\sum_{j=1}^{m_1} p_{jt} \leq g_t, \quad \forall t \tag{5}$$

$$p^{\min} \times y_{jt} \leq p_{jt}, \quad \forall t, \forall j \in M_1 \tag{6}$$

$$p_{jt} \leq p^{\max} \times y_{jt}, \quad \forall t, \forall j \in M_1 \tag{7}$$

$$\mathrm{SoC}_j^0 + \frac{\sum_{t \leq s_i - 1} d \times p_{jt} - \sum_{l=1/f_l \leq s_i - 1}^{n} E_{l,f_l} \times x_{lj}}{B_j} \leq \mathrm{SoC}_j^{\max}, \quad \forall i, \forall j \in M_1 \tag{8}$$

$$\mathrm{SoC}_j^0 + \frac{\sum_{t \leq f_i} d \times p_{jt} - \sum_{l=1/f_l \leq f_i}^{n} E_{l,f_l} \times x_{lj}}{B_j} \geq \mathrm{SoC}_j^{\min} \quad \forall i, \forall j \in M_1 \tag{9}$$

Constraint (2) ensure that each tour is assigned to exactly one vehicle. Constraint (3) guarantee that no vehicle can be assigned to overlapping tours. Constraint (4) prohibit charging the EV when it is in tour. Constraint (5) ensure that, at each time period t, the total power used to charge the EVs does not exceed the electricity grid's maximum capacity. Constraints (6) and (7) guarantee the respect of the minimum and the maximum powers of chargers when charging the EVs. Constraints (8) and (9) ensure that the SoC of each EV is in the interval $\left[\mathrm{SoC}^{\min}, \mathrm{SoC}^{\max}\right]$ during the whole time horizon $[0, T]$. In fact, the quantity $\sum_{t \le s_i - 1} d \times p_{jt} - \sum_{l=1/f_l \le s_i - 1}^{n} E_{l,f_l} \times x_{lj}$ represents the quantity of energy recovered by the EV j before starting route i minus the quantity of energy consumed by the EV to process routes finishing before i. This quantity should be less than $B_j \times \left(\mathrm{SoC}_j^{\max} - \mathrm{SoC}_j^0\right)$. The quantity $\sum_{t \le f_i} d \times p_{jt} - \sum_{l=1/f_l \le f_i}^{n} E_{l,f_l} \times x_{lj}$ is the quantity of electricity that remains in the EV j battery after processing all routes finishing before route i as well as route i. This quantity should be superior to $B_j \times \left(\mathrm{SoC}_j^{\min} - \mathrm{SoC}_j^0\right)$.

Our goal is to optimise two lexicographical objective functions (Ehrgott 2000; Marler and Arora 2004); i.e. the objective functions are arranged in order of their importance and the optimisation problems are solved one at a time. More precisely, if z^* is the optimal value of the primary objective function then the optimisation of the secondary objective function is subject to that the primary objective function is bounded by z^*.

Our first objective consists in maximising the EVs travelled distance $\left(\sum_{i=1}^{n} \sum_{j=1}^{m_1} w_i \times x_{ij}\right)$. The second objective is to minimise the EVs charging costs $\left(\sum_{j=1}^{m_1} \sum_{t=1}^{T} c_t \times p_{jt}\right)$. The lexicographical optimisation of our two objective functions is motivated by the real application. In fact, in our case the vehicles are leased and must be sold with a target of 64,000 km after 4 years, and since the range of EVs is limited, the aim of the fleet managers is to maximise the use of EVs.

We can also think that it may be easier to optimise the fleet's total cost of ownership (TCO) rather than maximising the EVs kilometres travelled and minimising the charging costs. However, since the introduction of EVs in the fleets of companies is recent, not enough data are available to correctly evaluate the EVs TCO.

6. Exact approach

The proposed MIP is solved in two steps using CPLEX. In the first step, the objective is to maximise the EVs kilometres travelled $\left(\sum_{i=1}^{n} \sum_{j=1}^{m_1} w_i \times x_{ij}\right)$. The optimal solution generated in the first step serves as the starting solution of the second step. A new constraint which ensures that the number of EVs kilometres travelled is greater than the objective function of the first step is added to the MIP in the second step. Then, the new MIP is solved with the second objective function $\left(\sum_{j=1}^{m_1} \sum_{t=1}^{T} c_t \times p_{jt}\right)$. These two steps are described in Algorithm 1.

Algorithm 1. Exact method

1: **Input:** Data of EVSCP
2: **Output:** A solution to the EVSCP
3: **Step 1**: Solve the MIP with the objective function $K = \sum_{i=1}^{n} \sum_{j=1}^{m_1} w_i \times x_{ij}$
4: Let K^* be the value of the objective function of the optimal solution S^*
5: **Step 2**: Add the constraint $\sum_{i=1}^{n} \sum_{j=1}^{m_1} w_i \times x_{ij} \ge K^*$ to the MIP formulation
6: Solve the new MIP with the objective function $\sum_{j=1}^{m_1} \sum_{t=1}^{T} c_t \times p_{jt}$ starting from the solution S^*

Algorithm 2. Overall heuristic's algorithm

1: **Input:** Data of the EVSCP
2: **Output:** A solution to the EVSCP
3: **for** Each EV **do**
4: Apply the *Tours Selection Algorithm* to get a set of tours achievable by that EV
5: Given the set of selected tours, apply the *Charging Schedule Algorithm* to optimise the charging cost of that EV
6: **end for**
7: **for** Each CV **do**
8: Apply the *Tours Selection Algorithm*
9: **end for**

7. Sequential heuristic approach

In this section, we describe a Sequential Heuristic (SH) to solve the EVSCP. This heuristic is mainly based on the following idea: For each EV, a set of tours is selected and assigned to that EV. Then, a charging schedule is proposed while satisfying all the constraints described in Section 5. This heuristic interleaves two steps: the tours selection step and the charging schedule step. The overall SH is given in Algorithm 2.

7.1 *Tours selection algorithm*

To assign a set of tours to an EV and maximise the kilometres travelled, the Maximum Weight Clique Problem (MWCP) (Brijnesh and Wysotski 2004; Yamaguchi and Masuda 2008) is deployed. Recall that the MWCP is defined as follows: Let $\mathcal{G} = (\mathcal{V}; \mathcal{E})$ be an arbitrary undirected and weighted graph, where $\mathcal{V}$ is the set of nodes and $\mathcal{E}$ is the set of edges. To each node, i is associated a weight w_i. Two distinct nodes are said to be adjacent if they are connected by an edge. Given a subset $\mathcal{S}$ of nodes, the weight of $\mathcal{S}$ will be denoted by $W(\mathcal{S}) = \sum_{i \in S} w_i$. A clique of graph $\mathcal{G}$ is a subset of $\mathcal{V}$ in which all nodes are pairwise adjacent. A clique S is called maximal if no strict superset of $\mathcal{S}$ is a clique. A maximal weight clique $\mathcal{S}$ is a clique that is not contained in any other clique having a weight larger than $W(\mathcal{S})$.

The problem of tours selection is represented through an undirected weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where each node $i \in \mathcal{V}$ represents a tour which is characterised by a weight (the length of tour in kilometres). We assume that the nodes are indexed in a nondecreasing order of their starting times s_i. Two nodes i and j are connected by an edge if the tours i and j are disjoint. The tours selection problem is then equivalent to the MWCP in graph $\mathcal{G}$ with additional constraints. The MWCP is a NP-hard problem (Garey and Johnson 1979). Balas and Yu (1989) provide some polynomial solvable cases of MWCP in specific graphs. The interval graph is one of the specific graphs for which the MWCP is a polynomial problem. However, even if graph $\mathcal{G}$ is an interval graph, the MWCP in graph $\mathcal{G}$ should ensure that a feasible charging schedule for EVs exists.

To solve the tours selection problem, we propose a heuristic that finds the maximum weight clique in graph $\mathcal{G}$ and makes sure that the charging schedule exists.

7.1.1 *Overview of the algorithm*

Our algorithm consists of two steps. In the first step, a clique $\mathcal{C}$ is constructed using a greedy heuristic and in the second step, the weight of $\mathcal{C}$ is improved by removing and/or adding one or several nodes. The two steps are repeated till stop conditions are fulfilled. The best weighted clique is updated at each iteration. The details of both steps are as follows:

- **Initial step.** Initially, the clique $\mathcal{C}$ is empty. The algorithm randomly selects a vertex i and adds it to $\mathcal{C}$. To expand this clique, a list of candidate nodes $L = \{v \in \mathcal{V} | v$ is connected to all nodes of $\mathcal{C}$ and $v \notin \mathcal{C}\}$ is created. The algorithm, then, selects a node v^* from L and adds it to $\mathcal{C}$. v^* is the node with the largest total sum of weights of its adjacent vertices ($W(v^*) = \max_{v \in L} \sum_{k \in V(v)} w_k$, where $V(v)$ is the set of nodes adjacent to v) that satisfies the *admission condition* (described later) related to the feasibility of a charging schedule. This process is repeated until L is empty or all the nodes of L do not satisfy the *admission condition*.
- **Improvement step.** When there are no more nodes that could increase the weight of clique $\mathcal{C}$, for each $i \in \mathcal{C}$, the algorithm computes the list of nodes $N_i = \{v \notin \mathcal{C} |$ *v is connected to all nodes of* $\mathcal{C} \setminus \{i\}\}$, and removes from $\mathcal{C}$ the node i with $W(N_i) \geq W(N_j), \forall j \in \mathcal{C}$, where $W(N_i) = \sum_{k \in N_i} w_k$. A new list L of candidate nodes is then selected, and the best node of L is added to $\mathcal{C}$ as described in *Initial step*. The process of adding and/or removing nodes is repeated until a specific timeout is reached. When the improvement step is stopped, the best clique is updated, the current clique is cleared and the algorithm restarts with the initial step until the stop condition is reached. Finally, the algorithm generates the best clique found among all constructed cliques and removes the best clique's nodes from $\mathcal{G}$.
- **Stop condition.** The algorithm continues until either a fixed limited processing time is reached or the best clique weight reaches the target clique weight; i.e. when a clique that has the same weight as the total weight of the graph, is found.

7.1.2 *Admission condition*

The admission condition ensures that the tours corresponding to the current clique's nodes can be processed by an EV; i.e. there is a feasible charging schedule that allows the EV to perform all tours of the current clique. The existence of a feasible schedule is guaranteed by the metric R. R is the minimal amount of energy that the EV should recover before starting the first tour of the clique, i.e. $R = \sum_{k=1}^{|\mathcal{C}|} (E_k - Y_k)$, where Y_k is the amount of energy that may be charged between successive tours $k-1$ and k. In other words, $Y_k = d \times \sum_{t=f_{k-1}+1}^{s_k-1} \min\left\{p^{\max}, p'_t\right\}$, where p'_t is the residual electricity grid power during time period t. We assume that $Y_1 = 0$. A node v is added to the current clique C, if, following the chronological order, for each node $j \in \mathcal{C}$, $R_j = \sum_{k=1}^{j}(E_k - Y_k)$ is lower than the EV's battery capacity and $R_j \le d \times \sum_{t=0}^{s_1-1} \min\left\{p^{\mathrm{Max}}, p'_t\right\}$.

7.2 *Charging schedule algorithm*

Given the best clique $\mathcal{C}$ found by the *Tours Selection Algorithm*, the objective here is to provide an optimal charging schedule that minimises the total charging cost and satisfies the constraints described in Section 5. Below, two optimal charging schedule algorithms are proposed. The first is based on a Minimum Cost Flow Formulation and runs in $O\left(\left((T+|\mathcal{C}|)\log(T+|\mathcal{C}|)\right)^2\right)$ time and the second runs in $O\left((T+|\mathcal{C}|)^2\right)$ time.

7.2.1 *Min-cost-flow-based charging algorithm*

In this section, we show that the charging problem can be reduced to the Minimum Cost Flow Problem (MCFP) (Vaidyanathan and Ahuja 2010) in a given network. For a given set V of vertices of the clique $\mathcal{C}$, a network $\mathcal{G} = (\mathcal{W}, \mathcal{A})$ is defined as follows:

- The set of nodes $\mathcal{W}$ consists of (i) a source s, (ii) the nodes t_l representing the time periods $[l-1, l]$, except the time periods when the EV is not available for charging, i.e. $l \in \{1, \ldots, T\} \setminus \cup_{v \in \{1,\ldots,|V|\}}\{s_v+1, \ldots, f_v\}$, (iii) the nodes v_k, $k = 1, \ldots, |V|$, representing the tours indexed in increasing order of their start times s_k, (iv) a sink p.
- The set $\mathcal{A}$ of directed arcs with restricted capacities consists of (i) arcs (s, t_l) with a maximum capacity $a_l = \min\left\{p^{\max}, p'_l\right\} \times d$, where p'_l is the residual electricity grid power during time period l, and a cost c_l corresponding to the energy cost during the time period l, (ii) arcs (t_l, v_k) if $l < s_k + 1$ and $l > f_{k-1}$ with a capacity $+\infty$ and a cost equal to zero, (iii) arcs (v_{k-1}, v_k), $k = 2, \ldots |V|$ with a capacity equal to $B_j - E_{k-1}$ and a cost equal to zero, (iv) arcs (v_k, p), $k = 1, \ldots, v_{|V|}$ with a lower bound and an upper bound of the capacity equal to E_k, and a cost equal to zero.

Let $f(i, j)$ be the flow on the arc $(i, j) \in \mathcal{A}$ in an optimal solution to the MCFP with a total cost $\sum_{l \in \{1,\ldots,T\} \setminus \cup_{v \in V}\{s_v+1,\ldots,f_v\}} c_l \times f(s, t_l)$. We define a feasible schedule to the corresponding charging problem as follows: at each time period l, where $l \in \{1, \ldots, T\} \setminus \cup_{v \in V}\{s_v+1, \ldots, f_v\}$, apply on EV j a charging power $p_{j,l} = \frac{f(s,t_l)}{d}$. Using this procedure, any feasible solution to the MCFP can be converted to a feasible solution S to the charging problem with a cost $C(S) = \sum_{l \in \{1,\ldots,T\} \setminus \cup_{v \in \mathcal{V}}\{s_v+1,\ldots,f_v\}} c_l \times f(s, t_l)$. Thus, when the flow cost is minimised, the total charging cost is also minimised.

LEMMA 1 *An optimal solution to the charging schedule problem can be found in* $O\left(\left((T+|V|)\log(T+|V|)\right)^2\right)$ *polynomial time.*

Proof The MCFP can be solved in $O\left(\left((T+|V|)\log(T+|V|)\right)^2\right)$ time (see for e.g. Orlin 1993). Obviously, the optimal solution to the resulting MCFP converts into an optimal solution to the charging schedule problem in $O(T)$ time. □

In the following paragraph, a greedy charging schedule algorithm, running in $O(T \times |V|)$ time, is proposed.

7.2.2 *Greedy charging schedule algorithm*

Let $C = (c_1, \ldots, c_T)$ and $G = (g_1, \ldots, g_T)$ be two vectors of T elements, where c_t and g_t are the electricity cost and the electricity grid capacity during time period $[t-1, t]$, respectively. Let $E = (e_1, \ldots, e_T)$ be the energy vector, where e_t, $t = 1, \ldots, T$, represents either the quantity of energy charged during the time interval $[t-1, t]$ or the energy consumed if the EV is in tour during the time interval $[t-1, t]$. e_t is initialised to $\frac{-E_k}{f_k - s_k + 1}$ if $t \in [s_k, f_k]$, $\forall k \in V$, and to zero otherwise.

The goal of the algorithm is to provide each EV with the maximum possible amount of energy during the cheapest time intervals. At each time interval $[t-1, t]$, e_t is equal to the maximum amount of energy that could be charged during the time interval $[t-1, t]$ if the EV is not in tour. To compute the value of e_t, the residual capacity of EV's battery and the residual available capacity of the electricity grid during the time interval $[t-1, t]$ are considered. More precisely, the algorithm starts by sorting the vector C in a nondecreasing order of c_t. For each value c_t such that $e_t = 0$; i.e. the time interval $[t-1, t]$ is not already scanned and the EV$_j$ is available for charging, let V_t be the set of tours such that $V_t \subseteq V$ and $v_k \in V_t$ if, and only if, $s_k > t$. For each $v_k \in V_t$ sorted in order of their indices, (i) compute the energy $E_k^{t,B}$ that could be charged without exceeding the capacity of the battery, i.e. $E_k^{t,B} = \mathrm{SoC}^{\max} \times B_j - \sum_{l=1}^{s_k} e_l$, (ii) calculate the energy $E_k^{t,V}$ that should be charged in order to process the tour v_k, i.e. $E_k^{t,V} = \mathrm{SoC}_j^{\min} \times B_j - \sum_{l=1}^{f_k} e_l$. Let $E_k^t = \min\{E_k^{t,B}, E_k^{t,V}\}$. After scanning all tours of V_t, the energy E^t that will be charged during the time interval $[t-1, t]$ is $E^t = \min\left\{p^{\max} \times d, g_t \times d, \min_{k \in V_t}\left\{E_k^t\right\}\right\}$. Note that we assume that $p^{\min} = 0$. Finally, vectors E and G are updated, i.e. $e_t := E^t$ and $g_t := g_t - \frac{E^t}{d}$. The overall greedy approach is as described by Algorithm 3.

THEOREM 2 *Algorithm 3 provides an optimal solution to the charging schedule problem and runs in $O(T \times |V|)$ polynomial time.*

Proof Let S be the charging solution generated by the *Greedy Charging Schedule Algorithm*, and let S^* be the optimal solution provided by the *Min-Cost-Flow-Based Charging Algorithm*. Let $C(S)$ and $C(S^*)$ be the costs of solutions S and S^*, respectively. Let t_1 be the first time period such that $p_{j,t_1}(S) \neq p_{j,t_1}(S^*)$ where c_{t_1} is the smallest electricity cost among all electricity costs. By construction, the *Greedy Charging Schedule Algorithm* applies, at each time period, the maximal available power to charge EV j. We have then $p_{j,t_1}(S) > p_{j,t_1}(S^*)$. Let t_2 be another time period such that $p_{j,t_2}(S) < p_{j,t_2}(S^*)$. Let S' be a new solution derived from S^* where $p_{j,t_i}(S^*) = p_{j,t_i}\left(S'\right)\ \forall i \neq 1, 2$, $p_{j,t_1}\left(S'\right) = p_{j,t_1}\left(S'\right) + \epsilon$ and $p_{j,t_2}\left(S'\right) = p_{j,t_2}\left(S'\right) - \epsilon$, where $0 < \epsilon \leq p_{j,t_1}(S) - p_{j,t_1}(S^*)$. It is easy to see that S' is a feasible solution to the charging problem, furthermore, $C\left(S'\right) = C(S^*) - c_{t_2} + c_{t_1} < C(S^*)$ since $c_{t_2} > c_{t_1}$. This contradicts the fact that S^* is an optimal solution. It then follows that, $\forall i$, $p_{j,t_i}(S) = p_{j,t_i}(S^*)$. Therefore, S is an optimal solution to the charging problem. The complexity of the greedy charging algorithm is dominated by the two loops 5 and 7 of Algorithm 3. The greedy algorithm produces an optimal solution in $O(T \times |V|)$-time. □

Algorithm 3. Greedy charging schedule algorithm

1: **Input:** A set of tours assigned to EV j
2: **Output:** A charging schedule of EV j
3: Let $C = (c_1, \ldots, c_T)$, $G = (g_1, \ldots, g_T)$ and $E = (e_1, \ldots, e_T)$ be three vectors of T elements, where c_t, $t = 1, \ldots, T$, is the electricity cost during the time interval $[t-1, t]$, g_t, $t = 1, \ldots, T$, is the residual capacity of the electricity grid during the time interval $[t-1, t]$ and e_t, $t = 1, \ldots, T$, is either the quantity of energy charged during the time interval $[t-1, t]$ or the energy consumed during the time interval $[t-1, t]$ if the EV is in routing. Parameter e_t is initialised to $\frac{-E_k}{f_k - s_k + 1}$ if $t \in [s_k, f_k]$, $\forall k \in V$ and to zero otherwise.
4: Sort the vector C in the nondecreasing order of c_t and let $C = \left(c_{\pi(1)}, \ldots, c_{\pi(T)}\right)$ be the sorted vector
5: **for** each time interval $[\pi(t) - 1, \pi(t)]$ such that $e_{\pi(t)} = 0$ **do**
6: Let $V_{\pi(t)}$ be a set of tours such that $V_{\pi(t)} = \{v_k \in V | s_k > \pi(t)\}$ sorted in the increasing order of the indices of tours.
7: **for** each v_k in $V_{\pi(t)}$ **do**
8: calculate $E_k^{\pi(t)} = \min\left\{\mathrm{SoC}^{\max} \times B_j - \sum_{l=1}^{s_k} e_l\ ;\ \mathrm{SoC}_j^{\min} \times B_j - \sum_{l=1}^{f_k} e_l\right\}$
9: **end for**
10: Calculate $E^{\pi(t)} = \min\left\{p^{\max} \times d, g_{\pi(t)} \times d\ ;\ \min_{k \in V_{\pi(t)}}\left\{E_k^{\pi(t)}\right\}\right\}$
11: Update $e_{\pi(t)} = E^{\pi(t)}$ and $g_{\pi(t)} = g_{\pi(t)} - \frac{E^{\pi(t)}}{d}$
12: **end for**
13: The charging schedule is given by the power vector $P = \left(\frac{\max\{0, e_1\}}{d}, \ldots, \frac{\max\{0, e_T\}}{d}\right)$ that should be applied to EV j.

8. Global heuristic approach

In this section, we propose a new Global Heuristic (GH) to solve the EVSCP problem. The GH algorithm consists of two steps. The first step aims at assigning tours to vehicles and guaranteeing the feasibility of a charging schedule. Then, the second step determines the optimal charging schedule for all EVs using the MCFP. Both steps are detailed in the next paragraph.

In the first step, the *Tours Selection Algorithm* presented in Section 7.1 is used to assign tours to vehicles, and a new *Estimation Procedure* is introduced. In fact, in *Sequential Algorithm*, for each vehicle, the charging schedule is calculated after the tours selection. This allows to update the electricity grid power available at each time. However, in GH algorithm, the charging schedule of all vehicles is calculated in the second step. In order to ensure the existence, in the second step, of a charging schedule of all vehicles, we need to estimate the remaining capacity of the electricity grid after the assignment of a set of tours to each vehicle. Thus, for each vehicle, after the tours selection algorithm, a new *Estimation Procedure* of the available electricity grid power is introduced. Indeed, the electricity grid power is updated by considering the fact that the current vehicle is charged at the last possible time. More precisely, the *Greedy Charging Schedule Algorithm* presented in Section 7.1 is applied to simulate the charging schedule of the EV, and update the electricity grid power available at each time. To ensure that the vehicle is charged at the last possible time, in Algorithm 3, instead of sorting the cost vector C (step 3 of Algorithm 3) in nondecreasing order of c_i, vector C is sorted in the decreasing order of time indices t.

The second step of GH determines an optimal charging schedule for all vehicles. More precisely, given the sets $L_1, \ldots, L_{m_1}$, of tours assigned to $\mathrm{EV}_1, \ldots, \mathrm{EV}_{m_1}$, respectively, the optimal charging schedule problem is modelled as a MCFP in a network $\mathcal{G} = (\mathcal{W}, \mathcal{A})$ that is defined as follows:

- The set of nodes $\mathcal{W}$ consists of (i) a source s, (ii) the nodes t_l representing the time periods $[l-1, l]$, (iii) the nodes $v_k^j \in L_j$, $j = 1, \ldots, m_1$ representing the tours indexed in the increasing order of their start times and (iv) a sink p.
- The set $\mathcal{A}$ of directed arcs with restricted capacities, consists of (i) arcs (s, t_l) with bounded capacity $a_l = \min\{p^{\max}, g_l\} \times d$ and a cost c_l corresponding to the energy cost during the time period l, (ii) arcs $\left(t_l, v_k^j\right)$ if $t_l < s_k^j + 1$ and $t_l > f_{k-1}^j$ with a capacity $+\infty$, and a cost equal to zero, (iii) arcs $\left(v_{k-1}^j, v_k^j\right)$, $k = 2, \ldots |L_j|$ with a maximum capacity equal to $B_j - E_{k-1}$ and a cost equal to zero and (iv) the arcs $\left(v_k^j, p\right)$, $k = 1, \ldots, |L_j|$, $j = 1, \ldots, m_1$, with a lower bound and an upper bound equal to E_k^l, and a cost equal to zero.

Let $f(i, j)$ be the flow on the arc $(i, j) \in \mathcal{A}$ in an optimal solution to the MCFP with a total cost $\sum_{l=1}^{T} c_l \times f(s, t_l)$. We define an optimal charging schedule as follows: at each time interval l, where $l \in \{1, \ldots, T\}$, apply to EV j a charging power $p_{j,l} = \frac{f\left(t_l, v_k^j\right)}{d}$, where $\left(t_l, v_k^j\right) \in \mathcal{A}$. It is easy to show that the obtained charging schedule is optimal. The proof is similar to that for Theorem 2. Using this procedure, an optimal solution to the MCFP can be converted into an optimal solution S to the charging problem. Thus, when the flow cost is minimised, the total charging cost is also minimised. An optimal solution to the charging schedule problem can be found in $O\left(\left(\left(T + |L|\right)\log\left(T + |L|\right)\right)^2\right)$-time, where $L = \cup_{j=1}^{m_1} L_j$.

9. Computational experiments and discussion

To evaluate the quality of solutions obtained using our exact and heuristic methods, extensive numerical experiments are conducted on real and random instances. We use real instances provided by one of the industrial partners of the project. All the experiments are carried out on an Intel Xeon E5620 2.4 GHz processor, with 8 GB RAM memory. The tested algorithms are coded in the C++ language. ILOG CPLEX 12.5 is used to solve the MIP. The results obtained are compared in terms of quality of the solutions and run times. The optimisation procedure is based on a 24 h time horizon. Prices of electricity are based on those proposed by EDF (French electric utility Company). Note that at most m_1 Level 1 chargers, with a range of 0.0(kW) − 3.7(kW), could be used.

The first real instance is composed of 18 CVs, 8 EVs and 45 tours. All EVs have 22(kWh) battery packs. The second instance is composed of 46 tours, 14 CVs, 8 EVs with a battery capacity of 22(kWh) and 4 other EVs with a battery capacity of 23.5(kWh). For these two instances, CPLEX and the heuristics succeeded to generate optimal solutions within short execution time (<3 s).

For confidentiality reasons, we are not allowed to present the details of those instances. However, we summarise our main computational results in Table 2. Column *Existing solution* refers to the implemented solution in the industrial site, and column *Exact approach* refers to solutions provided by CPLEX. Furthermore, the following indicators are reported: (i) the number of EVs and CVs used in the solution, (ii) the number of kilometres done by EVs and CVs, (iii) the amount of energy consumed by EVs, (iv) the cost of energy and (v) the peak of power load when charging EVs. From Table 2, we can make the following observations:

- All EVs and less CVs are used in the optimal solution compared to the existing solution.
- The number of EVs kilometres travelled is improved by 71.3% for instance 1 and by 21.8% for instance 2. The energy required by EVs increases by 60.2% for instance 1 and by 23.5% for instance 2, which is consistent with the percentage of additional kilometres done by EVs.
- The cost of energy consumed by EVs in instance 1 is close to the cost of energy in the existing solution, and it is even better for instance 2. This can be explained by the fact that, in the existing solution, EVs are charged when they return to the depot and no charging optimisation is performed. However, in the optimal solution, EVs are charged when the cost of energy is cheap and the charging curve is smoothed during the charging period, which explains that the power peak is lower in the optimal solution.

The second part of the experiments focuses on testing our solving methods on random instances which are generated as follows. The number of vehicles $nv = m_1 + m_2$ takes its values from {40, 80, 120, 160, 200}. The number of EVs is a ratio of nv; i.e. $m_1 = ev \times nv$. ev takes its values from {0.25, 0.5, 0.75, 1.0}. A parameter a is used to distinguish the capacity

Table 2. Computational results on real instances.

	Instance 1		Instance 2	
	Existing solution	Exact approach	Existing solution	Exact approach
Number of EV	8	8	12	12
Number of CV	18	17	14	12
Number of km done by EV	380	651	562	685
Number of km done by CV	1020	749	516	393
Amount of energy consumed by EV (kWh)	94.36	152.07	72.88	90.04
Cost of energy consumed by EV (euro)	4.28	4.90	5.72	4.93
Power peak (kVA)	24	23.5	36	27

of EVs batteries, namely $a = 1$ means that all EVs have a capacity of 22(kWh) and $a = 2$ means that 50% of EVs have a capacity equal to 22(kWh) and the rest of EVs have a capacity of 16(kWh). At most, m_1 chargers, providing a power ranging between 0(kW) and 3.7(kW), could be used to charge EVs. The parameter tt is related to the number of tours. If $tt = 1$, then $n = nv \times c$, where $c \in [1.2, 1.3]$. If $tt = 2$, then $n = nv \times c$, where $c \in [1.5, 1.6]$. All tours start no earlier than 6 am and finish no later than 8 pm, and their energy need is between 0.15(kWh) and 0.35(kWh) per kilometre. 50% of those tours last between 5 and 7 h, start between 6 am and 1 pm, and their lengths lie within [50(km), 80(km)]. The rest of tours last between 2 and 4 h and their lengths are in [20(km), 45(km)]. A fraction of $\frac{1}{6}$ of medium tours start between 6 am and 8 am, $\frac{1}{3}$ start between 8 am and 10 am, $\frac{1}{6}$ finish between 6 pm and 8 pm, and $\frac{1}{3}$ finish between 4 pm and 6 pm. Regarding the electricity grid capacity, we assumed that, if $t \in$[00 am, 6 am], $g_t = \frac{2}{3} \times ev \times nv \times p^{\mathrm{Max}}$, if $t \in$[6 am, 6 pm], $g_t = \frac{1}{5} \times ev \times nv \times p^{\mathrm{Max}}$ and if $t \in$[6 pm, 00 am], $g_t = \frac{1}{2} \times ev \times nv \times p^{\mathrm{Max}}$, where $p^{\mathrm{Max}} = 3.7$(kW).

For each value of nv, 16 classes of instances are then constructed; and for each class, 10 instances are randomly generated. Each class is denoted by $nv\_w\_ev\_x\_a\_y\_tt\_z$, where $w = nv$, x takes the values 1, 2, 3, 4 if ev is equal to : 0.25, 0.50, 0.75, 1, respectively, $y = a = \{1, 2\}$ and $z = tt = \{1, 2\}$. The generated instances are available on Sassi and Oulamara (2014). The computational results are summarised in Tables 3 and 4. A time limit of 3600 s is set for each instance. Table 3 provides results concerning the first objective Function, while Table 4 reports the results obtained using the second objective function. Note that the total charging cost depends on the EVs assignment to tours, on the number of EVs kilometres travelled and on the number of EVs used. Thus, the values of the second objective function, obtained by our methods, may be different even if the value of the first objective function is the same. To provide an efficient comparison between our three solving approaches on the basis of the second objective function, in Table 4, we report the results obtained on instances with a full EV fleet. In fact, for these instances, our solving methods generate almost the same assignment solutions.

In Table 3, the following indicators are reported: (i) *Success*: percentage of solved instances among the tested instances within the same class. Solved instances refer to instances for which at least one solution is generated, within the time limit, even if it is not optimal, (ii) $\mathrm{Gap}_{\mathrm{UB}}(M)$: gap between the best solution S_M produced by the method $M \in \{\mathrm{EM, SH, GH}\}$ (EM, SH, and GH are exact method [CPLEX], serial heuristic and global heuristic, respectively) in relation to the Upper Bound (UB) generated by CPLEX, i.e. $\mathrm{Gap}_{\mathrm{UB}}(S_M) = \frac{\mathrm{UB}-S_M}{S_M}$, (iii) $\mathrm{Gap}_{mx}(M)$: gap between the best solution generated by methods EM, SH and GH and the solution generated by the method $M \in \{\mathrm{EM, SH, GH}\}$, i.e. $\mathrm{Gap}_{mx}(M) = \frac{\max\left(S_{\mathrm{EM}}, S_{\mathrm{SH}}, S_{\mathrm{GH}}\right)-S_M}{S_M}$, and, (iv) $Time$: average run time needed to generate a solution.

From Table 3, we can make the following observations:

- The exact algorithm solves small and medium instances (fleets with 40 and 80 vehicles), but fails to solve large instances (fleets with more than 120 vehicles). For large instances with a small number of EVs, i.e. $ce = 0.25$, the exact algorithm succeeds to find feasible solutions. Furthermore, when the EVs fleet is heterogeneous, i.e. $a = 2$, the exact algorithm performs poorly in finding feasible solutions compared to the same classes of instances with homogenous fleets of EVs.
- GH succeeds in solving all instances, whereas SH fails to solve 44 instances out of 800. This particularly concerns large instances (for example, instances of classes $nv\_200\_ce\_3\_a\_2\_tt\_1$ and $nv\_200\_ce\_4\_a\_1\_tt\_2$). This is due to the fact that SH computes a charging schedule for each EV sequentially without anticipating the existence of a feasible charging schedule for the last EVs.
- Concerning the performance of the proposed heuristics, Table 3 shows that both heuristics exhibit an excellent performance. GH solves to optimality 180 instances out of 800 and provides the best performances with $\mathrm{GAP}_{\max}$

Table 3. Performance results of EM, SH and GH heuristics considering the first objective function.

	Exact method				SH algorithm				GH algorithm			
Instance	Success (%)	Gap_{UB} (EM) (%)	Gap_{mx} (EM) (%)	Time	Success (%)	Gap_{UB} (SH) (%)	Gap_{mx} (SH) (%)	Time	Success (%)	Gap_{UB} (GH) (%)	Gap_{mx} (GH) (%)	Time
nv_40_ce_1_a_1_tt_1	90.00	5.27	0.00	3600.00	100.00	6.72	1.36	0.07	100.00	5.94	0.62	0.29
nv_40_ce_1_a_1_tt_2	90.00	10.62	1.02	3602.00	100.00	10.88	1.29	0.09	100.00	9.46	0.00	0.26
nv_40_ce_1_a_2_tt_1	90.00	10.29	2.95	3601.98	100.00	7.36	0.29	0.07	100.00	7.14	0.00	0.23
nv_40_ce_1_a_2_tt_2	100.00	14.85	3.68	3608.23	100.00	11.38	0.61	0.09	100.00	10.71	0.00	0.27
nv_40_ce_2_a_1_tt_1	90.00	8.54	0.00	3602.18	100.00	13.70	4.76	0.09	100.00	9.84	1.21	6.39
nv_40_ce_2_a_1_tt_2	100.00	11.96	0.00	3601.00	100.00	16.57	4.17	0.12	100.00	12.61	0.63	6.57
nv_40_ce_2_a_2_tt_1	90.00	13.39	3.32	3601.00	100.00	12.75	2.76	0.09	100.00	9.73	0.00	5.86
nv_40_ce_2_a_2_tt_2	100.00	17.97	4.37	3601.00	100.00	16.43	3.13	0.14	100.00	12.89	0.00	7.36
nv_40_ce_3_a_1_tt_1	90.00	0.50	0.00	3601.00	100.00	0.50	0.00	0.11	100.00	0.50	0.00	0.59
nv_40_ce_3_a_1_tt_2	100.00	7.19	0.00	3602.00	100.00	8.46	1.18	0.16	100.00	8.23	0.96	0.93
nv_40_ce_3_a_2_tt_1	90.00	1.40	0.90	3606.47	100.00	0.46	0.46	0.12	100.00	0.00	0.00	0.61
nv_40_ce_3_a_2_tt_2	100.00	12.57	4.58	3601.00	100.00	8.12	0.49	0.17	100.00	7.59	0.00	1.26
nv_40_ce_4_a_1_tt_1	90.00	0.00	0.00	13.47	100.00	0.46	0.46	0.12	100.00	0.00	0.00	0.60
nv_40_ce_4_a_1_tt_2	100.00	0.00	0.00	29.96	100.00	0.00	0.00	0.22	100.00	0.00	0.00	1.19
nv_40_ce_4_a_2_tt_1	90.00	0.00	0.00	19.14	100.00	0.00	0.00	0.13	100.00	0.00	0.00	0.60
nv_40_ce_4_a_2_tt_2	100.00	0.00	0.00	52.64	100.00	0.00	0.00	0.19	100.00	0.00	0.00	1.32
nv_80_ce_1_a_1_tt_1	100.00	18.65	1.80	3601.00	100.00	19.75	2.77	0.33	100.00	16.53	0.00	14.19
nv_80_ce_1_a_1_tt_2	100.00	26.14	5.57	3601.00	100.00	23.19	3.15	0.51	100.00	19.43	0.00	12.37
nv_80_ce_1_a_2_tt_1	100.00	24.43	7.00	3601.00	100.00	20.05	3.25	0.29	100.00	16.27	0.00	10.84
nv_80_ce_1_a_2_tt_2	100.00	32.55	10.40	3601.00	100.00	22.83	2.57	0.41	100.00	19.75	0.00	12.33
nv_80_ce_2_a_1_tt_1	100.00	20.47	5.57	3601.20	100.00	14.12	0.31	0.39	100.00	13.77	0.00	3.44
nv_80_ce_2_a_1_tt_2	100.00	28.40	11.23	3601.48	100.00	14.58	0.14	0.55	100.00	14.42	0.00	4.18
nv_80_ce_2_a_2_tt_1	100.00	35.45	18.26	3601.85	100.00	14.30	0.47	0.35	100.00	13.76	0.00	3.15
nv_80_ce_2_a_2_tt_2	100.00	52.92	32.61	3602.36	100.00	14.76	0.32	0.49	100.00	14.40	0.00	3.92
nv_80_ce_3_a_1_tt_1	100.00	1.18	0.00	818.78	100.00	1.46	0.27	0.39	100.00	1.35	0.17	4.88
nv_80_ce_3_a_1_tt_2	100.00	15.49	4.89	3601.98	100.00	10.05	0.49	0.63	100.00	9.51	0.00	5.98
nv_80_ce_3_a_2_tt_1	100.00	7.00	5.47	3606.53	100.00	1.47	0.09	0.41	100.00	1.38	0.00	4.20
nv_80_ce_3_a_2_tt_2	60.00	25.87	15.54	3602.01	100.00	8.61	0.00	0.60	100.00	8.72	0.10	5.24
nv_80_ce_4_a_1_tt_1	100.00	0.00	0.00	117.31	90.00	0.12	0.12	0.47	100.00	0.00	0.00	4.93
nv_80_ce_4_a_1_tt_2	90.00	0.00	0.00	798.39	90.00	0.12	0.12	0.56	100.00	0.00	0.00	6.34
nv_80_ce_4_a_2_tt_1	100.00	0.00	0.00	217.83	100.00	0.00	0.00	0.42	100.00	0.00	0.00	5.01
nv_80_ce_4_a_2_tt_2	30.00	0.00	0.00	1062.29	90.00	0.00	0.00	0.55	100.00	0.00	0.00	4.29

(*Continued*)

Table 3. (*Continued*).

nv_120_ce_1_a_1_tt_1	100.00	24.62	6.93	3601.74	100.00	17.05	0.47	0.64	100.00	16.51	0.00	3.54
nv_120_ce_1_a_1_tt_2	100.00	48.84	22.46	3601.21	100.00	20.88	0.00	1.02	100.00	20.88	0.00	3.93
nv_120_ce_1_a_2_tt_1	100.00	37.80	17.91	3601.01	100.00	17.01	0.39	0.61	100.00	16.56	0.00	3.14
nv_120_ce_1_a_2_tt_2	100.00	73.91	40.38	3601.76	100.00	21.77	0.41	1.08	100.00	21.27	0.00	4.15
nv_120_ce_2_a_1_tt_1	100.00	103.17	63.42	3602.63	100.00	15.66	0.83	0.76	100.00	14.72	0.00	8.35
nv_120_ce_2_a_1_tt_2	50.00	111.66	78.48	3602.42	100.00	19.50	1.46	1.33	100.00	17.78	0.00	18.65
nv_120_ce_2_a_2_tt_1	100.00	115.33	80.87	3603.01	100.00	15.41	0.73	0.83	100.00	14.57	0.00	8.66
nv_120_ce_2_a_2_tt_2	20.00	95.76	66.32	3602.42	100.00	18.57	1.20	1.23	100.00	17.16	0.00	17.01
nv_120_ce_3_a_1_tt_1	100.00	31.21	29.70	3603.29	100.00	1.48	0.71	0.85	100.00	0.77	0.00	15.15
nv_120_ce_3_a_1_tt_2	0.00	*	*	*	100.00	10.45	1.88	1.48	100.00	8.41	0.00	15.86
nv_120_ce_3_a_2_tt_1	90.00	29.44	28.09	3603.97	100.00	1.19	0.58	0.89	100.00	0.61	0.00	15.10
nv_120_ce_3_a_2_tt_2	0.00	*	*	*	100.00	10.86	2.26	1.72	100.00	8.41	0.00	13.23
nv_120_ce_4_a_1_tt_1	90.00	0.00	0.00	504.42	100.00	0.00	0.00	0.79	100.00	0.00	0.00	12.05
nv_120_ce_4_a_1_tt_2	0.00	*	*	*	100.00	2.10	2.10	1.61	100.00	0.00	0.00	14.22
nv_120_ce_4_a_2_tt_1	30.00	0.00	0.00	730.19		0.33	0.33	0.80	100.00	0.00	0.00	8.13
nv_120_ce_4_a_2_tt_2	0.00	*	*	*	0.00	*	*	*	100.00	0.13	0.00	8.58
nv_160_ce_1_a_1_tt_1	100.00	374.04	146.35	3603.88	100.00	20.50	0.56	1.37	100.00	19.84	0.00	5.87
nv_160_ce_1_a_1_tt_2	50.00	223.20	155.19	3603.36	100.00	25.20	0.12	1.99	100.00	25.06	0.00	8.22
nv_160_ce_1_a_2_tt_1	90.00	882.71	450.83	3602.62	100.00	19.36	0.00	1.35	100.00	19.36	0.00	1.65
nv_160_ce_1_a_2_tt_2	50.00	150.93	100.99	3604.58	100.00	23.86	0.24	2.35	100.00	23.56	0.00	6.68
nv_160_ce_2_a_1_tt_1	10.00	294.76	240.23	3606.24	100.00	16.82	0.68	1.41	100.00	16.03	0.00	63.09
nv_160_ce_2_a_1_tt_2	0.00	*	*	*	100.00	19.70	0.89	2.49	100.00	18.90	0.00	132.13
nv_160_ce_2_a_2_tt_1	40.00	223.84	181.29	3603.72	100.00	13.97	0.67	1.51	100.00	13.22	0.00	40.83
nv_160_ce_2_a_2_tt_2	0.00	*	*	*	100.00	19.31	0.83	2.45	100.00	18.33	0.00	113.56
nv_160_ce_3_a_1_tt_1	0.00	*	*	*	100.00	6.16	4.39	1.57	100.00	1.69	0.00	168.93
nv_160_ce_3_a_1_tt_2	0.00	*	*	*	100.00	19.14	6.68	2.84	100.00	11.69	0.00	152.09
nv_160_ce_3_a_2_tt_1	0.00	*	*	*	100.00	6.27	4.33	1.57	100.00	1.86	0.00	159.00
nv_160_ce_3_a_2_tt_2	0.00	*	*	*	100.00	19.20	6.80	2.75	100.00	11.61	0.00	129.82
nv_160_ce_4_a_1_tt_1	0.00	*	*	*	90.00	0.06	0.95	1.60	100.00	0.00	0.00	26.24
nv_160_ce_4_a_1_tt_2	0.00	*	*	*	30.00	1.00	0.94	2.59	100.00	0.05	0.00	137.98
nv_160_ce_4_a_2_tt_1	0.00	*	*	*	100.00	0.00	0.00	1.51	100.00	0.00	0.00	29.09
nv_160_ce_4_a_2_tt_2	0.00	*	*	*	20.00	1.54	1.54	2.69	100.00	0.00	0.00	121.15

(*Continued*)

Table 3. (*Continued*).

*nv*_200_*ce*_1_*a*_1_*tt*_1	60.00	390.39	390.39	3607.04	100.00	21.75	0.43	2.11	100.00	21.23	0.00	16.83
*nv*_200_*ce*_1_*a*_1_*tt*_2	0.00	*	*	*	100.00	22.28	0.32	4.32	100.00	22.06	0.00	15.26
*nv*_200_*ce*_1_*a*_2_*tt*_1	20.00	558.04	447.35	3600.00	100.00	20.92	0.80	2.16	100.00	20.22	0.00	15.11
*nv*_200_*ce*_1_*a*_2_*tt*_2	0.00	*	*	*	100.00	22.56	0.80	4.79	100.00	21.58	0.00	16.73
*nv*_200_*ce*_2_*a*_1_*tt*_1	0.00	*	*	*	100.00	22.42	4.01	3.06	100.00	17.70	0.00	184.33
*nv*_200_*ce*_2_*a*_1_*tt*_2	0.00	*	*	*	100.00	26.08	3.79	6.40	100.00	21.48	0.00	109.74
*nv*_200_*ce*_2_*a*_2_*tt*_1	0.00	*	*	*	100.00	22.39	4.06	3.03	100.00	17.62	0.00	185.07
*nv*_200_*ce*_2_*a*_2_*tt*_2	0.00	*	*	*	100.00	29.07	4.07	6.17	100.00	24.03	0.00	127.91
*nv*_200_*ce*_3_*a*_1_*tt*_1	0.00	*	*	*	100.00	13.01	6.66	2.96	100.00	5.95	0.00	164.35
*nv*_200_*ce*_3_*a*_1_*tt*_2	0.00	*	*	*	30.00	27.48	7.13	6.79	100.00	18.99	0.00	173.80
*nv*_200_*ce*_3_*a*_2_*tt*_1	0.00	*	*	*	100.00	12.59	6.55	2.88	100.00	5.67	0.00	122.31
*nv*_200_*ce*_3_*a*_2_*tt*_2	0.00	*	*	*	30.00	28.17	7.24	7.09	100.00	19.52	0.00	188.96
*nv*_200_*ce*_4_*a*_1_*tt*_1	0.00	*	*	*	100.00	0.00	0.00	2.60	100.00	0.00	0.00	183.20
*nv*_200_*ce*_4_*a*_1_*tt*_2	0.00	*	*	*	70.00	0.29	0.29	5.22	100.00	0.00	0.00	109.60
*nv*_200_*ce*_4_*a*_2_*tt*_1	0.00	*	*	*	100.00	0.00	0.00	13.27	100.00	0.00	0.00	110.67
*nv*_200_*ce*_4_*a*_2_*tt*_2	0.00	*	*	*	70.00	0.25	0.25	6.05	100.00	0.00	0.00	156.08

Note: Entries ‘*’ mean that the run-time limit was reached without finding any feasible solution.

Table 4. Performance results of EM, SH and GH heuristics considering the second objective function.

	Exact method		SH		GH	
	$\text{Gap}_{\text{LB}}(\%)$	$\text{Gap}_{\min}(\%)$	$\text{Gap}_{\text{LB}}(\%)$	$\text{Gap}_{\min}(\%)$	$\text{Gap}_{\text{LB}}(\%)$	$\text{Gap}_{\min}(\%)$
$nv\_40\_ce\_4\_a\_1\_tt\_1$	3.87	1.64	8.88	6.75	2.28	0.00
$nv\_40\_ce\_4\_a\_1\_tt\_2$	8.88	5.87	10.68	7.62	3.31	0.00
$nv\_40\_ce\_4\_a\_2\_tt\_1$	3.74	1.70	8.71	6.77	2.08	0.00
$nv\_40\_ce\_4\_a\_2\_tt\_2$	8.80	6.00	10.54	7.71	3.06	0.00
$nv\_80\_ce\_4\_a\_1\_tt\_1$	13.18	8.74	14.17	10.97	4.88	0.00
$nv\_80\_ce\_4\_a\_1\_tt\_2$	13.22	8.98	12.47	9.41	4.66	0.00
$nv\_80\_ce\_4\_a\_2\_tt\_1$	13.69	8.70	15.60	12.19	5.48	0.00
$nv\_80\_ce\_4\_a\_2\_tt\_2$	13.13	7.28	17.78	12.26	6.30	0.00
$nv\_120\_ce\_4\_a\_1\_tt\_1$	13.47	9.16	15.16	11.12	4.73	0.00
$nv\_120\_ce\_4\_a\_2\_tt\_1$	13.67	6.45	14.57	9.28	7.70	0.00

less than 1% comparing to the best solutions found by CPLEX and SH and a gap GAP_{UB} of at most 20% compared to the UB found by CPLEX. SH succeeds in finding optimal solutions for 89 instances and provides good solutions with a $\text{GAP}_{\max}$ of at most 7% and a GAP_{UB} of at most 30% compared to the UB found by CPLEX for large instances.

- The running times of SH and GH are relatively short. About 190 s are needed to solve large instances using GH. A shorter running time, with at most 13 s, is needed to find good solutions for large instances using SH.

In Table 4, we consider the instances $nv\_w\_ev\_4\_a\_y\_tt\_z$ for which our solving methods have provided almost the same solutions. The following indicators are reported: (i) $\text{Gap}_{\text{LB}}(M)$: gap between the best solution S_M produced by the method $M \in \{\text{EM, SH, GH}\}$ in relation to the Lower Bound (LB) generated by CPLEX, i.e. $\text{Gap}_{\text{LB}}(S_M) = \frac{S_M - \text{LB}}{\text{LB}}$, (ii) $\text{Gap}_{\min}(M)$: gap between the best solution generated by methods EM, SH and GH and the solution generated by the method $M \in \{\text{EM, SH, GH}\}$, i.e. $\text{Gap}_{\min}(M) = \frac{S_M - \min\left(S_{\text{EM}}, S_{\text{SH}}, S_{\text{GH}}\right)}{\min\left(S_{\text{EM}}, S_{\text{SH}}, S_{\text{GH}}\right)}$.

From Table 4, we can see that GH provides the best performances with a lower gap of less than 8%. Moreover, it generates the best solutions for all instances comparing to the solutions provided by the other methods.

10. Conclusion

In this paper, we considered the problem of simultaneously optimising the allocation of tours to EVs and minimising the charging costs. This problem was shown to be NP-hard in the ordinary sense. To solve the EVSCP, a MIP was proposed. An exact method based on two phases was developed to solve the MIP with CPLEX. We also proposed two heuristics. The first heuristic is a sequential one and interleaves two subalgorithms to assign a set of tours to each EV using the MWCP, and plan its charging process using a Minimum Cost Flow formulation. The second is a global heuristic. It generates a solution to the assignment problem of tours to all EVs while guaranteeing the feasibility of a charging schedule. After that, it applies a global charging algorithm to minimise charging costs. The computational results showed the efficiency and the effectiveness of the proposed heuristics.

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