Integrated Vehicle and Crew Scheduling for Electric Buses with Realistic Charging Behavior

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1 Introduction

Electric vehicles are beginning to make up large portions of the fleet for public transport providers. In The Netherlands for instance, around 21% of all registered buses are already electric according to the RDW (2025). However, in order to meet European regulations (2018) on climate and sustainability, individual line operators such as the Dutch Qbuzz (2024) are slowly replacing even more of their old combustion based fleet with electric vehicles. It is therefore almost certain that this share will only grow in coming years across different countries.

This shift in energy source has introduced new challenges to the methods currently in use to keep our public transport moving. Of primary concern are charging infrastructure and vehicle ranges, both of which are much more limited than their traditional combustion based counterparts. The planning process which has traditionally been used, as outlined in Figure 1, therefore requires additional research into each of its steps in order to incorporate these newly relevant constraints.

In this work, we will focus on incorporating electric vehicle restrictions into two of these steps: vehicle scheduling and crew scheduling. Specifically, we will consider the scheduling of electric buses and their drivers, as these two steps make up a majority of day-to-day costs within the bus transit sector.

The goal of vehicle scheduling is generally to assign vehicles such that a set of trips is covered. In the case of buses, trips are given by the timetables for the individual routes as generated in the previous planning step. Our goal is therefore to assign sequences of compatible trips to buses such that each individual trip is driven.

In order to do this, a collection of vehicle tasks must be generated. A vehicle task can be seen as the individual schedule that a bus will follow throughout the day: it may start at a bus storage facility (more commonly called a depot), then perform one or more trips, before finally returning to the depot. The driving actions performed between between a depot and a trip, as well as those between trips themselves are called deadheads. The costs of driven deadheads along with the total number of buses used are the main focus for minimization, as the costs incurred for driving the trips themselves are fixed. This cost minimization problem is often referred to as the vehicle scheduling problem or VSP. The single-depot VSP with a single vehicle type is solvable in polynomial time, however extensions such as multiple vehicle types or limited vehicle ranges are shown to be NP-Hard by Bunte and Kliewer (2009) and Haghani and Banihashemi (2002) respectively.

Once vehicle tasks are known, we can move on to scheduling our crew. Here, our goal is to find an assignment of crew members to vehicles such that the previously made vehicle tasks can actually be performed. Continuing within the context of bus planning, we now need to match drivers with our previously planned buses such that there is always a driver present while the bus is moving. In
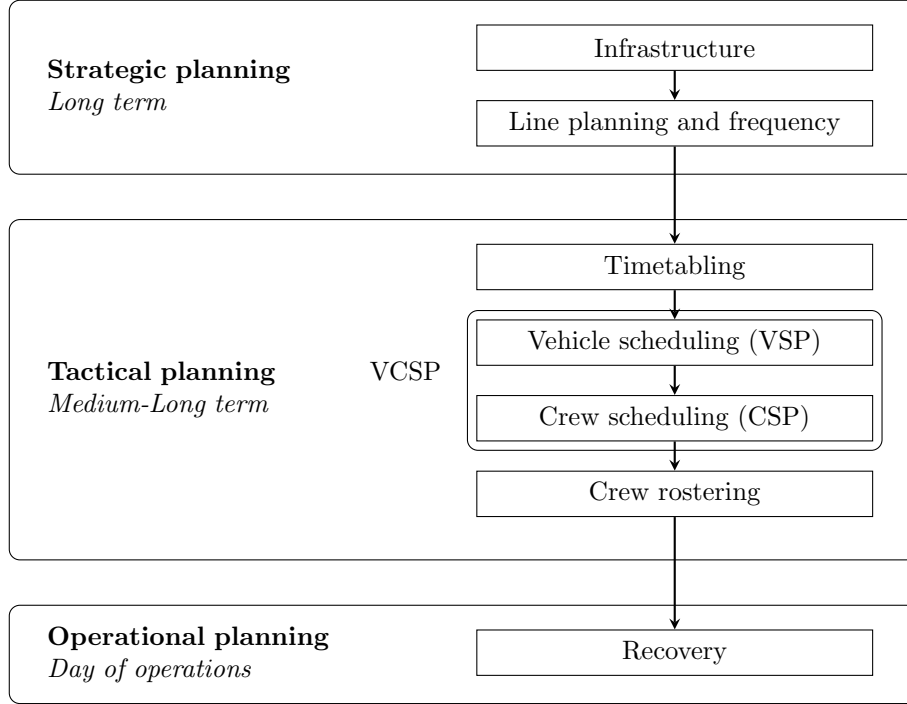
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Figure 1: A general overview of the public transport planning process, based on Ceder and Wilson (1986), Ibarra-Rojas et al. (2015), and Perumal et al. (2022).

order to do this, a set of crew tasks must be created that completely cover the given vehicle tasks. A one-to-one mapping between crew tasks and vehicle tasks is not always possible; overall driving time for a driver is limited in a single day due to labor regulations, and for longer shifts breaks are mandatory as well. As vehicles don't have these constraints, a vehicle task may not be feasible to perform in its entirety for a single driver. It is therefore necessary to split vehicle tasks up into multiple segments, whereafter we can create crew tasks that consist of a sequence of one or more compatible segments. The costs of vehicle tasks themselves are fixed as they must always be driven; in order to minimize costs, we must therefore find a covering set of crew tasks which minimizes the number of required crew members and paid hours in which the driver is not actively driving. This minimization problem is referred to as the crew scheduling problem or CSP, and has been shown to be NP-Hard by Fischetti et al. (1989).

Crew scheduling is the greatest contributor to day-to-day costs; a recent estimate by Perumal et al. (2019) puts it at around 60% of the overall operational costs for bus transit providers in Northern Europe. As can be seen however, the VSP and CSP are very closely related. The vehicle tasks that are selected in the VSP directly determine what crew tasks are feasible within the CSP. It is therefore not always optimal to fully minimize costs in the vehicle scheduling process, as this might incur higher overall costs due to crew scheduling. We can therefore roughly split the solving of vehicle and crew assignments into two separate approaches: sequential, in which the VSP and CSP are solved separately; or integrated, in which the VSP and CSP are solved such that overall costs are minimized simultaneously. The integrated approach is often referred to as the vehicle and crew scheduling problem, or VCSP.

A lot of work has already been done for the VSP, CSP and VCSP. Both the sequential and integrated approach have been extensively studied since the 1980s, as shown by surveys such as Bodin and Golden (1983). The introduction of electric vehicles has however introduced significant constraints on charging and vehicle ranges, invalidating a formerly often made assumption that a vehicle was able to drive an entire day without being refueled. This most directly effects the VSP, as charging periods now need to be added throughout the day in order to effectively use buses. The version of the vehicle scheduling problem which incorporates these constraints, referred to as the E-VSP, has been the focus of many studies going back to around 2014. We refer the reader to a survey by Perumal et al. (2022) for a detailed overview of recent progress.

Limited literature does exist on the integrated VCSP with electric vehicles (E-VCSP), however simplifying assumptions are made which might limit real world applicability or accurate modeling of costs. Most notably, assumptions are currently made about charging locations (such as only being able to charge at a bus depot) or charging behavior (such as modeling the process as being purely linear or only allowing full charges). Additionally, to the best of our knowledge battery degradation due to usage patterns has not been included in any integrated models at the time of writing. Our aim is to introduce an integrated E-VCSP model which incorporates more realistic behavior for battery charging and usage, by including the following:

- Nonlinear battery charging times.

- Cost of battery degradation due to usage patterns.

- Capacitated charging stations at both depots and the endpoints of a trip.

- Partial charging of the battery throughout the day.

Using this model, we compare the overall costs of the integrated approach to an equivalent sequential formulation using instances provided by Qbuzz. [*TODO: Alleen kosten vergelijken lijkt me met minimale doel; Het liefst zou ik ook nog iets van gevoeligheidsanalyse doen voor invloed van oplaadlocaties/batterijcapaciteit/etc, maar dat is afhankelijk van hoeveel tijd het model werkend krijgen kost*]. This work is organized as follows. In Section 2, we will discuss work related to the E-VCSP and give an overview of common ways of modeling and solving the problem. In Section 3, we give our formal problem definition. [*TODO: Meer secties*].

2   Related work

In this section, we will discuss work related to our research into the E-VCSP. An overview of the nomenclature used has been included in Table 1, and an summary of how batteries and charging behavior is modeled in the discussed works has been included in Table 2.

Abbreviation	Definition
ALNS	Adaptive Large Neighborhood Search
B&P	Branch-and-Price
CG	Column Generation
CP	Constraint Programming
CSP	Crew Scheduling Problem
E-...	Problem ... with electric vehicles
LNS	Large Neighborhood Search
LS	Local Search
MDVSP	Multi Depot Vehicle Scheduling Problem
MIP	Mixed Integer Program
SAA	Simulated Annealing Algorithm
SDVSP	Single Depot Vehicle Scheduling Problem
SoC	State of Charge
TCO	Total Cost of Ownership
ToU	Time of Usage
TVSP	Integrated Timetabling and Vehicle Scheduling Problem
VCSP	Integrated Vehicle and Crew Scheduling Problem
VSP	Vehicle Scheduling Problem

Table 1: Nomenclature used in this work

	Model	ToU	SoC	Nonlinear Ch.	Partial Ch.	Ch. Location	Degradation
Li (2014)	E-VSP	No	D	No	No	D	No
van Kooten Niekerk et al. (2017)	E-VSP	Yes	C/D	Yes	Yes	D/T	Yes
Olsen and Kliewer (2020)	E-VSP	No	C	Yes	Yes	D/T	No
Zhang et al. (2021)	E-VSP	No	C/D	Yes	Yes	D	Yes
Parmentier et al. (2023)	E-VSP	No	C	Yes	Yes	D/T	No
de Vos et al. (2024)	E-VSP	No	D	Yes	Yes	D/T	No
Perumal et al. (2021)	E-VCSP	No	C	No	No	D	No
Wang et al. (2022)	E-VCSP	Yes	C	No	Yes	D	No
Sistig and Sauer (2023)	E-VCSP	No	C	No	Yes	D/T	No
Shen and Li (2023)	E-VCSP	No	C	No	No	D/T	No
Cong et al. (2024)	E-VCSP	Yes	C	No	Yes	D	No
Ham and Park (2021)	E-VRPTW	Yes	C	No	Yes	D	No
Stadnichuk et al. (2024)	E-TVSP	No	C	No	Yes	D/T	No

Table 2: A brief overview of battery modeling in E-VCSP related literature. SoC modeled as (D)iscrete or (C)ontinuous variable, Charge locations at (D)epot or (T)erminal trip stops, Degradation of battery in cost function

2.1 (E-)VSP

Before considering previous work on the E-VSP, we will first cover the most basic form of vehicle scheduling: that which only considers a single depot, single vehicle type and unlimited vehicle ranges. This problem, referred to as the single depot vehicle scheduling problem (SDVSP), forms the underlying basis of both the multi-depot and electric vehicle extensions that we will consider later. We will therefore give a brief summary of one the most common models and solution methods used for the SDVSP, thereby having a baseline to which we can compare extensions. For a more comprehensive overview on different models used for the SDVSP and multi-depot VSP (MDVSP), we refer the reader to a review by Bunte and Kliewer (2009).

As mentioned before, our goal in the SDVSP is to create a set of vehicle tasks throughout the day in which the number of vehicles and the cost of driven deadheads is minimized while covering all trips. In order to solve this problem, a graph consisting of the trips and depot can be created. In this graph, we can then solve a min-cost flow problem in which all trips are covered; the optimal vehicle tasks then follow from the flow paths along the trips.

More formally, given a set of trips T and a single depot d, a graph $G = (V, A)$ can be constructed. Let $V = \{v_t^s \mid t \in T\} \cup \{v_t^e \mid t \in T\} \cup \{v_d^s, v_d^e\}$, in which each pair of nodes (v_t^s, v_t^e) represent the start and end of trip t respectively. Additionally, the pair of nodes (v_d^s, v_d^e) represent the depot at the start and end of the day. Next, let $A_T = \{(v_t^s, v_t^e) \mid t \in T\}$ represent the arcs connecting the start and end of the trip, each with cost equal to the cost of driving the trip. Let $A_{DH} = \{(v_t^e, v_{t'}^s) \mid (t, t') \in T \times T, t'$ can be driven after $t\}$ represent a set of arcs which model feasible deadhead trips; once again, each with cost equal to those incurred when driving the deadhead between t and t'. Let $A_D = \{(v_d^s, v_t^s) \mid t \in T\} \cup \{(v_t^e, v_d^e) \mid t \in T\}$ represent deadheads from the depot to a trip and vice versa, once again with costs equal to those when driving the deadheads. If fixed costs per vehicle are required, these can be added to arcs leaving the depot. Finally, let $A = A_T \cup A_{DH} \cup A_D$, and let all arcs have capacity equal to 1.

Setting v_d^s as our source and v_d^e as our sink, we can now find a min-cost max-flow across the graph G. Due to our construction, all trips must be covered by exactly 1 flow, resulting in flow paths which we can directly use as vehicle tasks due to the assumption that our vehicles have enough range. It is therefore also shown that the SDVSP can be solved in polynomial time, as polynomial time min-cost max-flow algorithms exist and the graph elements are of size $|V| = O(|T|)$ and $|A| = O(|T|^2)$.

Two common extensions to the problem make it NP-Hard: the inclusion of multiple vehicle types, as well the use of multiple depots under the assumption that vehicles must return to their depot of origin. Both of these extensions are also discussed in Bunte and Kliewer (2009). The modification to the SDVSP flow network is the same in either case: an additional source/sink pair can be added for each new depot or vehicle type, and connected to the trips in the same way as the original depot. The problem then turns into into an integral multi-commodity flow, which has been shown to be NP-Hard by Even et al. (1975).

The introduction of any resource constraints within the VSP has also been shown to be NP-Hard by Bodin and Golden (1983). The E-VSP specifically deals with constraints on the driving range of vehicles, thereby making it closely related to the vehicle scheduling problem with route time constraints (VSP-RTC) as described by Haghani and Banihashemi (2002). The key difference between these two problems is that the E-VSP allows for (partial) recharging of a vehicle throughout the operating period, whereas the VSP-RTC assumes a fixed maximum travel time for the vehicle within the given period. The E-VSP has been shown to be NP-Hard by Sassi and Oulamara (2014).

Li (2014) was one of the first to consider a solution method for the E-VSP. A single-depot case with a single vehicle type is considered, in which the assumption is made that recharging (or battery swaps) can be performed in a fixed 5-minute window. The model is based on an extension of the SDVSP network, with the inclusion of total driving time constraints. Additionally, extra time-discretized nodes are added to represent capacitated battery charging/swap stations. For smaller instances, the model can be solved to optimality using column generation and branch-and-price (B&P). For larger instances, an alternate approach using truncated column generation followed by a local search to find a local optimum is used instead. The proposed methods are tested on trips in the San Francisco Bay Area, with a maximum instance size of 242 trips. These tests resulted in optimality gaps of < 5% for buses able to drive 150km, and between 7-15% for a range of 120km depending on the instance.

van Kooten Niekerk et al. (2017) introduces two models which aim to solve the single depot E-VSP while taking into account time dependent energy prices (ToU pricing), nonlinear charging times and battery degradation due to depth of discharge. The first model only allows for linear charging and no consideration for degradation or ToU, but uses continuous state of charge (SoC) variables which are added to the SDVSP network. The second model does allow for the extra inclusions, achieving this by duplicating trip nodes in the SDVSP network for discrete SoC values. The second model is solved using column generation and lagrangian relaxation, resulting in a possibly non-optimal solution. Tests are performed using data provided by Belgian bus company De Lijn in the city Leuven, using a total of 543 trips. They show that the discretized model can be solved in a considerably shorter time frame for large instances with similar results to the continuous model.

Olsen and Kliewer (2020) consider a multi-depot E-VSP, in which they which model the nonlinear phase of charging as an exponential function. In order to solve, they propose a greedy heuristic to construct vehicle tasks. Their primary focus is comparing (piecewise) linear approximations for the second phase of charging with an exponential function based approximation. They conclude that SoC and required charging times are more comparable to real life behavior when using the exponential function.

Zhang et al. (2021) apply a similar method to the one found in van Kooten Niekerk et al. (2017). They consider a single depot with capacitated charging infrastructure, with multiple round trip lines originating from the depot. In addition to this, they also incorporate nonlinear charging behavior and battery depreciation using discrete SoC and time nodes in the SDVSP network. They solve using a combination of CG and B&P. Tests are done on both randomly generated instances as well as 6 not yet electrified lines with up to 160 and 197 trips respectively.

Parmentier et al. (2023) consider a scalable approach to the E-VSP with non-linear charging. They introduce the concept of nondominated charging arcs, which are represented as deadhead arcs within the SDVSP. Their use considerably reduces the amount of candidate charging arcs when multiple charging points are available, as an arc is only considered if there is not another arc available with higher resulting charge and lower cost. In order to solve, a combination of CG and B&P techniques are used. A more computationally efficient version of the pricing problem is also provided by the nondominated charging arcs when charging infrastructure is uniform. Testing is done on the *large* instances introduced by Wen et al. (2016) which included up to 8 depots, 16 charging stations and 500 trips. Here, they are able to find solutions that only have an 0.06% optimality gap.

de Vos et al. (2024) consider the E-VSP with partial recharges and capacitated charging stations. Their model includes discrete SoC trip nodes and discrete SoC and time charging location nodes, similar to the model found in Zhang et al. (2021). Using these, a primal network is created using pessimistic SoC rounding when connecting trips with feasible deadheads. In order to solve, they apply CG with two separate approaches: branch-and-price and a diving heuristic. To overcome the limitations of dual bounds resulting from a discretized model, they incorporate ideas from Boland

et al. (2017) resulting in a dual network with optimistic connections. This gives the same bounds as the ones found in the non-discretized model. Testing is performed on a bus concession south of Amsterdam with 816 trips, with subsets being used as smaller instances. Optimality gaps of 1.5-2.7% are achieved across instances. They additionally note that the framework as provided can easily be extended for nonlinear charging functions and depth-of-discharge battery degradation.

2.2   CSP

Given a solution to the (E-)VSP, the corresponding CSP is most often solved as a set partitioning (or set covering) problem. Here, the tasks described by the sequences of trips generated during vehicle scheduling must be covered by the individual schedules of crew members. This problem has been shown to be NP-Hard in general by Fischetti et al. (1989).

Research into this subject is primarily done in the context of airline crew planning; crew costs in this field are generally even higher than those found in the more general public transport sector, as shown in Barnhart et al. (2003). Additionally, strong labor unions and restrictive labor legislation due to safety concerns cause a large number of constraints to be applied to crew schedules, resulting in a non-trivial problem to solve.

Results achieved in the aviation space quite easily generalize to other sectors, and we therefore refer the reader to a recent review by Deveci and Demirel (2018) for an overview of the state of the art.

2.3   (E-)VCSP

The VCSP has been a widely studied problem. Following the call for integrated methods by Bodin and Golden (1983) and others in the 1980s, a large number of different methods has been applied to integrate the VSP and CSP. We refer the reader to a recent review by Ge et al. (2024) for a more general overview of work done in the field in the past years.

One work that we will individually highlight is that of Huisman et al. (2005), due to its use of Lagrangian relaxation to connect the VSP and CSP . For readers unfamiliar with the technique, we recommend an introduction by Beasley (1993). Huisman et al. consider the multi-depot variant, and use a combination of CG and Lagrangian relaxation to solve both the MDVSP as well as the connection with the CSP. Of note is their assumption that crew members from each individual depot are only allowed to work on trips connected to said depot, allowing for individual depot CSPs to be solved as a subproblem. They test on instances in the Randstad metro area in the Netherlands with a maximum of 653 trips and 4 depots.

As for the electric counterpart of the VCSP, at time of writing we are aware of only five other works that discuss the integrated variant.

Perumal et al. (2021) were the first to offer a solution to the E-VCSP. They consider an instance of the problem in which only full recharges at the depot with a fixed duration of 120 minutes are possible. In order to solve, an ALNS was introduced which incorporates a B&P heuristic which has been previously used to solve the MDVSP, E-VSP and VCSP. The authors tested using real life data from lines in Denmark and Sweden with a maximum instance size of 1109 trips and multiple depots, and report an improvement of $1.17 - 4.37\%$ across different instances when compared to a sequential approach.

Wang et al. (2022) introduce a two layered model using particle swarms and an ϵ-constraint based mechanism which allows for a mix of traditional combustion and electric buses. The model incorporates partial depot charging, as well as measures to ensure that crew is primarily assigned to the same vehicle throughout the day. A circular bus route with a single depot in Changchun, China

with 68 daily trips is used as a basis for testing, with a focus on electric versus diesel usage and driver satisfaction.

Sistig and Sauer (2023) also offered an ALNS based approach, which aimed to improve upon the approach presented by Perumal et al. (2021) by including partial recharges, opportunistic charging at terminal stops of trips and non-fixed ranges for the vehicles. In order to solve, they implement a selection of 3-step ALNS neighborhoods consisting of E-VSP modification, finding a solution to the corresponding CSP and consequently modifying the CSP solution. Tests were done using an instance of a city route in Germany, with a single depot and a total of 282 trips. Different scenarios based on possible crew break and relief locations were considered in order to compare diesel and electric TCO. Additionally, sensitivity analysis of the TCO was done for parameters such as costs for electricity and drivers.

Shen and Li (2023) provide a minimum-cost flow framework for the E-VSP which is integrated with a set partitioning based approach for the E-CSP. They only provide full recharge capabilities at the depot, however focus on the inclusion of a distinction between energy use when driving and standing still in order to more accurately model real life traffic. A city line in China with 270 daily trips and a single depot is used for testing, resulting in cost savings of up to 8.7% when compared to a sequential approach.

Cong et al. (2024) provide a hybrid MIP and SAA based approach to optimizing a mixed fleet of combustion and electric vehicles with ToU electricity pricing. In each SAA iteration, a collection of new E-VSP trip assignments are created using neighborhood operations, after which two MIP models are sequentially employed to solve for charging and crew schedules. The methods are tested on a collection of 3 bus routes originating from the same depot in Changchun City, China with a total of 520 trips across all routes. When compared to the sequential approach, the integrated vehicle schedule was able to reduce costs by 0.8%.

2.4 Other related fields

The VSP is closely related to the vehicle routing problem (VRP); in this problem, the aim is to find minimum cost routes for vehicles originating from a depot and needing to pass multiple stops, most commonly for pickup or delivery with capacity constraints. The extension of the E-VRP which includes arrival time windows (E-VRPTW) is most closely related to the E-VSP, as the use of 0-width windows allows us to define the same precedence constraints as those naturally defined by trips in the VSP.

An example of work done on the E-VRPTW is that of Ham and Park (2021). They consider a single depot case in which they model ToU pricing and partial recharges during delivery routes. In order to model costs, a lexicographical minimization is done over the number of vehicles used, total distance traveled and energy recharged. In order to solve, a hybrid MIP and CP algorithm is used in which CP is used to model ToU related variables, and MIP is used to model the rest of the constraints.

Research has also been done into integrating the E-VSP with the step before it in the planning sequence: timetable planning. This problem, the E-TVSP, has recently been studied in the work of Stadnichuk et al. (2024). They allowed results of the E-VSP to introduce optimality cuts into the MIP used for creating timetable plans, thereby reducing overall cost. This is achieved by transforming the E-VSP problem into one of bin packing with conflicts, after which three different heuristic methods are applied and compared. They additionally prove that the bounds of the used heuristics are tight for their given instances.

3 Problem definition

In this section, we will give a definition of the integrated electric vehicle and crew scheduling problem (E-VCSP). As mentioned in previous sections, our goal is to create minimum cost schedules for both vehicles and crew members; vehicle tasks must be constructed for individual vehicles such that each trip is covered, and crew tasks must be constructed such that the vehicle tasks can be driven. Before defining these tasks, we will first formally define the data which is given. A summary of the notation used has been provided in Table 3.

Let T be a set of trips that need to be driven, let D be a set of depots that can be used, and let L be a set of locations. A trip is defined as a travel between two locations in L; let $l_{t,start} \in L$ and $l_{t,end} \in L$ represent the starting and ending location of trip t respectively, and let $l_d \in L$ represent the location of depot d. Next, let $p_{t,start}$ and $p_{t,end}$ refer to the planned starting and ending time of trip t, and let $d_t = p_{t,end} - p_{t,start}$ be the duration of the trip. Lastly, let k_t be equal to the number of kilometers traveled during the trip.

As multiple vehicles types may be in use, let Φ represent the set of all vehicle types. For each trip t, a subset of these vehicle types $\phi_t \subseteq \Phi$ may be used in order to drive the trip. Each depot d also has a preallocated number of each vehicle type available; let this be represented by $g_d^\phi \in \mathbb{N}^+$. Different vehicles might have different battery capacities; let σ_{min}^ϕ and σ_{max}^ϕ represent the minimum and maximum allowable SoC of vehicle type ϕ, both expressed in KWh. Additionally, let σ_{start}^ϕ represent the capacity that a vehicle starts with at the beginning of a day, and let c_t^ϕ represent the amount of charge a vehicle of type ϕ would use when traveling the trip, expressed in KWh.

Now, let us define given metrics when traveling between different locations. As the exact travel between locations may differ depending on the time of day, we will define these relations on a trip basis instead of location basis. Travels to and from depots are of course also allowed; let us therefore define these relations on some pair (s, s') where s and s' must be either a trip or depot. Of these, at least one must be a trip, giving us a departure time.

Let $d_{s,s'}$ represent the duration of the drive between (the end of) s and (start of) s', adjusted for departure time. Similarly, let $k_{s,s'}$ represent the amount of kilometers traveled, and let $c_{s,s'}^\phi$ represent the amount of battery charge used. Note that this last metric is dependent on the vehicle type ϕ, as different vehicle types might have different usage patterns.

Lastly, let us define our recharging behavior. In charging behavior, we will make two key assumptions: firstly, we assume that it is only possible for a vehicle to charge between trips. Charging at a depot is therefore not directly modeled unless one of the trip locations is the depot itself. Secondly, we assume charging may only be done at the start of a deadhead; if charging is desired at the end of a deadhead instead, a dummy 0-duration trip must be introduced at the deadhead target to model this behavior. Using these two assumptions, we can determine a maximum amount of charge that can be added during a deadhead from t to t' while still allowing for travel between the two trips.

As charging is a nonlinear process, the starting SoC σ of a vehicle at the end of t is of importance. Additionally, charging speeds may differ depending on the vehicle type ϕ. Combining this, we define $u_{\sigma,t,t'}^\phi$ as being this maximum possible amount of charge gained in the deadhead from t to t' while still allowing for travel between the trips. Note that when using discrete values for σ, all values of $u_{\sigma,t,t'}^\phi$ can be predetermined. In this, charging infrastructure available at $l_{t,end}$ must be considered considered. If no such infrastructure is available, $u_{\sigma,t,t'}^\phi = 0$.

Using these given values, we will now start by formally defining a feasible vehicle task for a ve-

hicle of type ϕ. A vehicle task must start at a depot, then perform a sequence of compatible trips and charging actions, before finally returning to a depot. We will refer to this ordered sequence as $S = \{s_0, s_1, \ldots, s_n, s_{n+1}\}$, where each subtask $s \in S$ can represent one of two actions: driving a trip $t \in T$, or being at a depot $d \in D$. In this sequence, let $a_s \in \{depot, trip\}$ represent the specific action that is performed by s. In order for a sequence to be a feasible vehicle schedule, the following conditions must be met.

First, the vehicle must start at a depot, then perform a sequence of trips, before finally returning to the depot. Note that trips may have starting or ending locations that coincide with that of the depot, however their respective subtask is not considered to be a depot action.

$$a_{s_0} = \text{depot} \tag{1}$$
$$a_{s_{n+1}} = \text{depot} \tag{2}$$
$$a_{s_i} = \text{trip} \qquad\qquad \forall i = 1, \ldots, n \tag{3}$$

The type of the vehicle must match those allowed for driving a trip.

$$\phi \in \phi_{t_i} \qquad\qquad \forall i = 1, \ldots, n \tag{4}$$

Trips may only be driven sequentially if there is time to travel from one trip to the next. Note that we make the assumption that there is always time to drive to and from a depot at the start and end of the day.

$$p_{s_i,end} + d_{s_i,s_{i+1}} \le p_{s_{i+1},start} \qquad\qquad \forall i = 1, \ldots, n-1 \tag{5}$$

Lastly, we want to ensure that the battery capacity is always respected. As mentioned before, the assumption is made that charging can only take place at the end of a trip before traveling to a following trip.

$$\sigma'_0 = \sigma^\phi_{start} \tag{6}$$
$$\sigma_i = \sigma'_{i-1} - c^\phi_{s_{i-1},s_i} - c_{s_i} \qquad\qquad \forall i = 1, \ldots, n+1 \tag{7}$$
$$\sigma_i \ge \sigma^\phi_{min} \qquad\qquad \forall i = 1, \ldots, n+1 \tag{8}$$
$$\sigma'_i = \sigma_i + \delta_j \cdot u^\phi_{\sigma_i,s_j,s_{j+1}} \qquad\qquad \forall i = 1, \ldots, n \tag{9}$$
$$\sigma'_i \le \sigma^\phi_{max} \qquad\qquad \forall i = 1, \ldots, n \tag{10}$$

Where σ_i represents the SoC at the end of subtask i, and σ'_i represents the SoC after possible charging is done at the end of subtask i. Additionally, $0 \le \delta_i \le 1$ represents a fraction of the maximum charge received.

For a feasible vehicle schedule v, let us define the following the following:

$$c(v) = c(s_0, s_1) + \sum_{i=1}^{n}(c(s_i) + c(s_i, s_i + 1)) \tag{11}$$

$$d(v) = d(s_0, s_1) + \sum_{i=1}^{n}(d(s_i) + d(s_i, s_i + 1)) \tag{12}$$

$$k(v) = k(s_0, s_1) + \sum_{i=1}^{n}(k(s_i) + k(s_i, s_i + 1)) \tag{13}$$

$$cost(v) = \alpha_1 + \alpha_2 \cdot c(v) + \alpha_3 \cdot d(v) + \alpha_4 \cdot k(v) \tag{14}$$

Where $c(v)$, $d(v)$ and $k(v)$ represent the total charge used, active time and distance traveled, with some set of scalar values α. We will refer to the set of all feasible vehicle tasks as V.

Vehicle tasks can be split up into multiple segments, each of which must be performed continuously by a single crew member. In some cases, this can be a single trip; in other cases, it might not be possible for a driver to be replaced when arriving at the destination. Let $L_r \subseteq L$ be a subset of the locations at which the driver can be replaced; using these relief points as splitting points, for each vehicle task $v \in V$, let its set of segments be defined as $w(v)$; our overall set of of segments W can then be defined as $W = \bigcup_{v \in V} w(v)$. In this, $w \in W$ represents a single segment.

Now, let us consider crew tasks. A crew task will be defined as an ordered sequence of subtasks $E = \{e_1, \dots, e_n\}$. Each of these subtasks may be of one of 5 types: driving a segment, taking a break, entering a vehicle; leaving a vehicle, or waiting in a vehicle. In order to differentiate between subtasks types, let the action performed during subtask e be defined as $a_e \in \{$ drive, break, step-on, step-off, wait $\}$. For every subtask e, let us define q_e as an identifier of the vehicle associated with the action. If $a_e = break$, q_e is undefined. Additionally, let us define $l_{e,start}$ and $l_{e,end}$ as the starting and ending location of a subtask, and let $p_{e,start}$, $p_{e,end}$ be the starting and ending time. Lastly, let $d_e = p_{e,end} - p_{e,start}$ be the duration of the subtask. In order to keep notation concise, we also define $b(E) = \{e \mid e \in E, a_e = break\}$ or the set of all break subtasks within a sequence.

In order for a sequence of subtasks to be a feasible crew task, the following conditions need to be met. We base our conditions on Dutch labor regulations, however similar regulations are present in other countries as well.

First, overall time feasibility. A driver may have a duty length of at most 9 hours.

$$p_{e_n,end} - p_{e_0,start} \leq 9\text{hours} \tag{15}$$

Next, break time feasibility. For shifts shorter than 4 hours, no breaks are required. For shifts between 4 and 5.5 hours, one break of at least 15 minutes must be included. For shifts longer than 5.5 hours, at least 40 minutes of break time must be included, where each individual break must be at least 15 minutes and at least one of these breaks must be 20 minutes or more. Lastly, for shifts starting after 15:00, a break between 16:30 and 20:30 of at least 20 minutes must be included.

$$
\begin{align}
&\text{if } 4 \text{ hours} \leq p_{e_n,end} - p_{e_1,start} \leq 5.5 \text{ hours} \quad &\text{then} \quad &\exists e \in b(E) : d_e \geq 15 \text{ min} \tag{16} \\
&\text{if } 5.5 \text{ hours} \leq p_{e_n,end} - p_{e_1,start} \quad &\text{then} \quad &\forall e \in b(E) : d_e \geq 15 \text{ min} \wedge \tag{17} \\
& & &\exists e \in b(E) : d_e \geq 20 \text{ min} \wedge \tag{18} \\
& & &\sum_{e \in b(E)} d_e \geq 40 \text{ min} \tag{19} \\
&\text{if } p_{e_1,start} \geq 15{:}00 \quad &\text{then} \quad &\exists e \in b(E) : d_e \geq 20 \text{ min} \wedge \tag{20} \\
& & &16{:}30 \leq p_{e,start} \leq 20{:}30 - p_{e,end} \tag{21} \\
& & & \tag{22}
\end{align}
$$

Next, we must ensure feasibility of transfers between subtasks themselves. In general, for waiting, break and handover related actions, the physical position of the driver does not change. For trips on the other hand, the starting position may differ from the ending position. In order to be feasible, each sequential pair of subtasks within the schedule must be reachable; additionally, the starting and ending location of the driver must be the same in order for the driver to return home at the

end of the day.

$$l_{e_n,end} = l_{e_1,start} \tag{23}$$

$$l_{e_{i-1},start} = l_{e_i,end} \qquad \forall i = 2, \ldots, n \tag{24}$$

$$\text{if } a_{e_i} \neq \text{trip then} \qquad l_{e_i,start} = l_{e_i,end} \qquad \forall i = 1, \ldots, n \tag{25}$$

Handovers between drivers also need to be possible. A step-on or step-off subtask must be present in order for a driver to enter or exit a vehicle; without being in a vehicle, the corresponding segments may not be performed. Waiting may also not be performed without being in (or associated with) a vehicle. Note that the construction of our segments implicitly models the fact that drivers may only get on and off at designated locations; step-ons, step-offs and breaks are therefore always possible at segment ends.

$$\text{if } a_{e_i} = \text{drive then} \qquad a_{e_{i-1}} \in \{ \text{ wait, drive, step-on } \} \wedge q_{s_{i-1}} = q_{s_i} \qquad \forall i = 2, \ldots, n \tag{26}$$

$$\text{if } a_{s_{i-1}} = \text{drive then} \qquad a_{e_i} \in \{ \text{ wait, drive, step-off } \} \wedge q_{s_{i-1}} = q_{s_i} \qquad \forall i = 2, \ldots, n \tag{27}$$

$$\text{if } a_{s_i} = \text{wait then} \qquad a_{e_{i-1}} \in \{ \text{ drive, step-on } \} \wedge q_{s_{i-1}} = q_{s_i} \qquad \forall i = 2, \ldots, n \tag{28}$$

Lastly, the schedule must be continuous; the driver must always be performing one of the 5 specified actions, and must start with a step-on and end with a step-off.

$$a(s_1) = \text{step-on} \tag{29}$$

$$a(s_n) = \text{step-off} \tag{30}$$

$$e(s_i - 1) = s(s_i) \qquad \forall i = 2, \ldots, n \tag{31}$$

$$\tag{32}$$

For a feasible crew schedule c, let us define its cost as being $cost(c) = \beta_1 + \beta_2 \cdot (p_{e_n,end} - p_{e_n,start}$, where β is some set of scalar values representing the fixed and variable costs for a crew member respectively. Let the set of all feasible crew schedules be called C.

Now having our definitions of V, W and C, we can formulate our integrated problem. In this, let x_i and y_j be binary variables indicate the usage of $v_i \in V$ and $c_j \in C$ respectively. Additionally, let $n_{it} \in \{0, 1\}$ be a constant indicating that v_i covers trip $t \in T$, and let with $m_{ijw} \in \{0, 1\}$ indicate that trip $w \in w(v_i)$ is covered by crew task j. Now, our minimization is as follows:

$$\min \sum_{1 \leq i \leq |V|} x_i cost(v_i) + \sum_{1 \leq j \leq |C|} y_j cost(c_j) \tag{33}$$

Subject to:

$$\sum_{1 \leq i \leq |V|} x_i a_{it}^v >= 1 \qquad \forall t = 1, \ldots, |T| \tag{34}$$

$$\sum_{1 \leq j \leq |C|} y_j m_{ijw} >= x_i \qquad \forall i = 1, \ldots, |V|, w \in w(v_i) \tag{35}$$

$$x_i \in \{0, 1\} \qquad \forall i = 1, \ldots, |V| \tag{36}$$

$$y_j \in \{0, 1\} \qquad \forall j = 1, \ldots, |C| \tag{37}$$

Where (34) ensures that all trips are covered by at least one vehicle, and (35) ensures that the segments in used vehicle tasks are covered by corresponding crew tasks.

Notation	Definition
Given	
$v \in V$	Set of all vehicle tasks, v single vehicle task
$w \in W$	Set of all vehicle segments, w single vehicle segment
$v(w) \subseteq W$	Set of vehicle segments for task v
$c \in C$	Set of all crew tasks, c single crew task
$t \in T$	Set of all trips, t a single trip
$d \in D$	Set of all depots, d a single depot
$l \in L$	Set of all locations, l a single location
$L_r \subseteq L$	Set of relief locations
$\phi \in \Phi$	Set of vehicle types, ϕ a single type
$\phi_t \subseteq \Phi$	Set of compatible vehicle types for t
$g_d^\phi \in \mathbb{N}^+$	Number of vehicles of type ϕ at d
$s \in S$	Ordered sequence of vehicle steps, s a single step
$e \in E$	Ordered sequence of crew subtasks, e a single subtask
$l_{\chi,start}$	Starting location of χ
$l_{\chi,end}$	Ending location of χ
l_χ	Location of χ
$p_{\chi,start}$	Planned starting time of χ
$p_{\chi,end}$	Planned end time of χ
d_χ	Duration of χ
$d_{\chi,\chi'}$	Duration of drive between χ and χ'
k_χ	Kilometers traveled in χ
$k_{\chi,\chi'}$	Kilometers driven between χ and χ'
c_χ^ϕ	SoC used by χ on type ϕ
$c_{\chi,\chi'}^\phi$	SoC used by ϕ driving between χ and χ'
σ_{min}^ϕ	Minimum SoC of ϕ
σ_{max}^ϕ	Maximum SoC of ϕ
σ_{start}^ϕ	Starting SoC of ϕ
$u_{\sigma,t,t'}^\phi$	Max. SoC gained between t, t' for type ϕ, start SoC σ
a_χ	Action type of sequence element χ
q_e	Vehicle associated with subtask e
$n_{it} \in \{0,1\}$	Vehicle task v_i covers t
$m_{ijw} \in \{0,1\}$	Crew task c_j covers $w \in w(v_i)$
Decision variables	
$x_i \in \{0,1\}$	Usage of vehicle task v_i
$y_j \in \{0,1\}$	Usage of crew task c_j
Additional helper functions	
$d(v)$	Total duration of v
$k(v)$	Total kilometers traveled in v
$c(v)$	Total charge used by v
$b(E)$	Set of break actions in E
$cost(\chi)$	Cost of task χ

Table 3: Notation used for formal problem description, where χ is used as a placeholder when multiple argument types can be applied.

4 Global planning for thesis

The following constraints list of constraints seem to make the problem the most challenging (when compared to just SDVSP): Partial charges, Nonlinear charges, Battery degradation, Capacitated charging stations, Multiple vehicle types and Multiple depots. Other than that, percentage of used crew shifts must match x type constraints seem like they might make solving harder; I'm unsure of their effects on the results of CG within a set covering context as I haven't seen them mentioned before, however intuitively it seems like it seems like a larger part of the searchspace would need to be traversed in order to find a good solution.

Partial charges, nonlinear charges and battery degradation can all be solved by using a similar discrete model as the one used in van Kooten Niekerk et al. (2017), Zhang et al. (2021) or de Vos et al. (2024); all of these allow for handling of partial and nonlinear in the charging deadhead arcs, and battery degradation can be included during vehicle task column generation. As all of these basically come for free with the choice of modeling, it would seem wise to focus on getting this working first. Next, the addition of capacitated charging stations is an issue. In both Zhang et al. (2021) and de Vos et al. (2024), this is handled by adding even more nodes to the graph which model the individual charging stations, however this only models the number of individual charging spots, and not total charging power available (which is a required constraint in the Qbuzz context). I currently don't have a solution for this in mind yet which remains compatible with nonlinear charging, so some further thought needs to be put into this idea. Depending on results from the initially implemented solution without these shared capacity limits, it might be necessary to focus here next; if it turns out that solutions don't generally exceed charging capacity, it might be worth it to put this off until the end.

Next, the issue of multiple vehicle types and depots. Both of these can be approached in roughly the same manner, however from conversations with Qbuzz it seems wise to focus on doing vehicle types first as even the smallest instances use multiple vehicle types.

In conclusion, my personal guess is that the following list of priorities is the best going forward:

- Begin with extending one of the previously mentioned models (probably van Kooten Niekerk et al., 2017) in order to get solutions to the E-VSP which incorporate partial charging, nonlinear charging and battery degredation.

- Add extraction of task blocks for crew scheduling, link with a set cover approach to crew scheduling

- If charging capacity turns out to be an issue, start by modifying E-VSP to incorporate this.

- Otherwise, start with adding multiple vehicle types and depots.

- Include charging capacity if not done before.

Rough planning planning of this process is described in Table 4. Additionally, weekly meetings with Philip (and Han, however this can also be bi-weekly if this doesn't turn out to be necessary) seem like a good idea to keep a check on progress and motivation. Bi-weekly meetings with Qbuzz are also in the works, in order to validate model and possibly adjust planning if something interesting pops up.

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Date	Task description
Begin February - Mid February	Finalize proposal
Mid February - Mid March	Implementation of E-SDVSP
Begin March	Proposal Presentation for OR group (if slot available)
Mid March - Begin April	Link E-SDVSP with crew scheduling
Begin April - Mid April	Testing with E-SDVCSP, part of report w/ results, hand in draft
Mid April - Mid March	Extend to include multi-vehicle/cap constraints
Mid March - End March	Testing, write part of report, draft
End March - End June	Extend to include cap/multi-vehicle constraints
End June - Half July	Testing, write part of report, draft
Half July - End July	Finalize report, prepare for presentation
End July / Begin August	Hand in final report, final presentation

Table 4: Rough outline of planning

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