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# Evaluating and Optimizing Opportunity Fast-Charging Schedules in Transit Battery Electric Bus Networks

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**Abstract.** Public transport operators (PTOs) increasingly face a challenging problem in switching from conventional diesel to more sustainable battery electric buses (BEBs). In this study, we optimize the opportunity fast-charging schedule of transit BEB networks in order to minimize the charging costs and the impact on the grid. Two mixed-integer linear programming (MILP) formulations that use different discretization approaches are developed and compared. Discrete-Time Optimization (DTO) resembles a time-expanded network that discretizes the time and decisions to equal discrete slots. Discrete-Event Optimization (DEO) discretizes the time and decisions into nonuniform slots based on arrival and departure events in the network. In addition to the DEO's higher practicability, the comparative computational study carried out on the transit-bus network in the city of Rotterdam, Netherlands, shows that the DEO is superior to the DTO in terms of computational performance. To show the potential benefits of the optimal schedule, it is compared with two reference common-sense greedy strategies: First-in-First-Served and Lowest-Charge-Highest-Priority.

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**Keywords:** transit electric bus networks • charging schedule • mixed integer linear programming

## 1. Introduction

In recent years, global awareness of climate change and other sustainability issues has increased substantially. With global carbon emissions increasing from 6 billion to 10 billion metric tons per year between 1990 and 2014 (Boden, Andres, and Marland 2017), efforts to mitigate the severity and impact of global warming at an international level have been underway for decades, culminating in the 2015 Paris Agreement. Nevertheless, the transition to a sustainable, carbon-free economy continues to face critical challenges, particularly for industries that have fundamentally relied on fossil fuels for more than a century.

One such sector is transportation, which accounted for 14% of global greenhouse gas emissions in 2010 (IPCC 2015). Although public transport is already considered to be one of the more sustainable forms of mobility, transit-bus fleets, in particular, historically rely on diesel or natural gas. Seeking to replace these fuels with clean, renewable energy sources, public

transport operators (PTOs) across the globe are increasingly considering a switch to battery electric buses (BEBs). However, because of the low energy density of Li-ion batteries, such a transition poses a variety of strategic, tactical, and operational challenges. Although diesel buses (DBs) can often run for an entire day without having to refuel, BEBs may need several charging sessions a day. Even with fast-charging equipment, the time required for charging ranges up to more than an hour for a depleted battery. This also places additional stress on the power grid, as fast-charging represents a significant load. These additional restrictions have to be aligned with classical constraints, such as the PTO's network structure, schedule requirements, fleet size, and staffing considerations.

Given the complexity of the overall problem, this paper focuses on the insufficiently studied problem of the operational aspect of a battery electric bus fleet by analyzing and optimizing the charging schedule for a

given set of fast-charger locations. We contribute to the existing literature in optimizing and evaluating the effect of charging schedules on the electrified transit-bus networks operations. By optimizing the charging schedule, we decide where, when, and for how long each bus in a network should charge during a day. Using hourly electricity prices as proxies for the demand–supply balance of the power grid, we seek to minimize energy costs for the PTO—thereby minimizing the impact on the grid—subject to a given trip schedule and network structure. We formulate the PTO's charging scheduling problem as a mixed-integer linear programming (MILP) problem, introduce two discretization approaches, and compare their performance and practicability. We then present a computational case study for the city of Rotterdam, Netherlands, which is currently undergoing a partial electrification of its transit-bus fleet. In this setting, we also compare the optimal strategy derived from the MILP formulation to a selection of common-sense greedy charging strategies.

In the next section, we summarize the relevant literature related to our work. In Section 3, we describe the PTO's optimization problem, which is subsequently formalized according to our two competing discretization approaches in Section 4. Section 5 describes the case study of Rotterdam, and Section 6 concludes by providing key recommendations and suggestions for future research.

## 2. Related Work

Li (2013) carries out a comparative study after optimizing the trip schedule, with the objective of minimizing the operational costs, between an electrified network either using opportunity fast-charging or battery swapping and conventional transit networks with compressed natural gas, diesel, or hybrid–diesel buses. He assumes a single charging location in the network and no operational differences between fast-charging and battery swapping and uses additional constraints for the maximum route distance for conventional networks. The results show that the emissions and the operational costs, without considering capital costs, are consistently lower for the electrified networks without a significant need to increase the fleet size. His conclusion shows the importance and benefit of fully electrifying transit-bus networks.

As the electrification of a transit-bus network requires solutions to several interdependent problems at the strategic, tactical, and operational levels, previous studies have generally focused on one of these perspectives. The problem is further complicated, because it fundamentally depends on the charging technology in use, such as opportunity charging, opportunity fast-charging, or battery swapping. Strategic decision problems include deciding which of these

technologies to use, but also determining the locations, numbers, types, and power levels of the chargers and battery-swapping stations. Tactical planning takes these decisions as given and optimizes trip scheduling and assignment based on the resulting constraints. In this paper, we focus on the operational level, which seeks to develop charging strategies that determine where, when, and for how long a particular bus charges. Results from the operational level can, in turn, provide valuable information for tactical and strategic decision making.

At the strategic level, a critical decision is the selection of the charging technology, with each technology having both advantages and disadvantages. Opportunity charging and opportunity fast-charging are similar, as they both depend on charging the BEBs during layover between trips. Opportunity fast-charging uses high charging power, and, thus, the BEBs are charged much faster compared with opportunity charging, but it adds a greater load to the grid with substantially higher peaks. Battery swapping allows for the swift replacement of a depleted battery with a fully charged one. However, it comes with significantly higher capital cost, as swapping stations need to be installed, extra batteries need to be acquired, and bigger plots of land are needed compared with the other two technologies (Li 2016). Mohamed et al. (2017) compare the impact of three different charging methods (i.e., opportunity fast-charging, opportunity charging, and overnight slow-charging) on the grid. They simulate the bus network for each charging technology to calculate the required numbers of chargers and buses, so as not to violate the operation schedule, and estimate the resulting load profiles. Their results show that opportunity fast-charging and opportunity charging would require electrical transformers—used to change the voltage level in the grid—with ratings that are five to six times higher than those required for overnight charging. Moreover, they show that fast-charging is the least preferred technology when considering the impact on the grid. Another study by Chen, Yin, and Song (2018) investigates a newer charging technique by employing wireless inductive charging lanes. They carry out an empirical analysis to compare it to battery swapping and opportunity charging, with the objective of minimizing the total costs (i.e., infrastructure and fleet costs) by determining the optimal sizes of the fleet and the batteries. Their results show that using battery swapping generally results in lower total costs compared with the other two techniques. The charging-lane technique becomes more cost competitive if the network has low operating speed and high service frequency. Finally, they show that, as the charging power of the opportunity charging technique increases (i.e., faster charging), its total costs decrease and converge to those of the battery swapping. These

studies show the potential superiority of the fast-charging technique when compared with battery swapping and opportunity charging, and the importance of our study in solving its particular disadvantage of negatively impacting the grid by optimizing the charging schedule. Optimizing the charging strategy might lower the peak power demand at some locations, which could reduce the need for upgrading the electrical infrastructure and equipment, such as the cables and transformers.

Taking the technology choice as given, several studies focus on the strategic problem of allocating the opportunity (fast) chargers. Xylia et al. (2017) use mixed-integer linear programming to determine the optimal fast-charger locations within the transit-bus network in Stockholm, Sweden, minimizing either total costs or energy consumption. They find that the costs of the partially electrified network are only marginally higher than those of a comparable network of only conventional buses. This implies that the high capital costs of the BEB network can largely be compensated by lower fuel costs. A similar study by Kunitz, Mendelevitch, and Goehlich (2017) uses MILP to locate and estimate the required number of fast-chargers to achieve a feasible operation at minimum electrification costs. Their results also show how the infrastructure requirements are impacted by the buses' HVAC (heat, ventilation, and air-conditioning) systems, and the trade-off between battery size and infrastructure requirements. Sebastiani, Lüders, and Keiko (2016) use discrete-event simulation with a metaheuristic optimization algorithm to find the optimal locations for fast-chargers within the Curitiba public transit transportation network in Brazil. The algorithm minimizes the number of chargers and the overhead stopping time of the buses at the charging stations, conditional on no bus running out of energy. Although the optimal allocation of the chargers is a very important early stage decision in electrifying transit-bus networks, it is not sufficient to guarantee the feasibility and optimality of the eventual operation. Thus, further research is required to address bus-charging strategies within a transit BEB network.

Various studies evaluate the feasibility of electrifying existing conventional networks by using heuristic approaches subject to certain assumptions. For instance, Rogge, Wollny, and Sauer (2015) assess the feasibility of electrifying the bus network in Muenster, Germany, by using one fast-charger at each terminal station and calculating the required battery capacity for each route. Moreover, they evaluate the trade-off between battery capacity and passenger capacity and its effect on the feasibility of electrification. Paul and Yamada (2014) use a greedy algorithm assigning trips to BEBs according to their departure times to create the operation and charging

schedules that maximize the overall utilization of BEBs for a transit city bus network in Japan. They assign DBs to the trips that cannot be electrified. Although the results of these studies provide useful insights, the fact that they use (often simple) heuristics makes implications regarding global optimality difficult. In this work, we address this issue by applying mathematical programming.

At the operational level, several studies focus on operational decisions, such as the optimal charging strategy. De Filippo, Marano, and Sioshansi (2014) test different charging queuing policies and investigate their effect on the average queuing and charging durations with different numbers of chargers in The Ohio State University's bus-service network. They use a single charging location in the network and allow for delays in the operating schedule if the bus does not have sufficient energy and requires charging to perform the next trip. Ke, Chung, and Chen (2016) use a genetic algorithm to optimize charging time and determine the required number of buses with varying battery capacities and the number of chargers to minimize the total costs of the transit BEB network of Penghu, Taiwan. Ding, Hu, and Song (2015) introduce an energy-storage system to the fast-charging stations to shift the charging process to off-peak hours and minimize the charging costs. They use mixed-integer nonlinear programming to optimize the size of the energy-storage system and of the fast-chargers, which reduced total costs by 22.85%. For the bus network of the city of Tallahassee, FL, Qin et al. (2016) evaluate how the charging threshold, which is the state of charge (SoC) limit up to which BEBs charge, affects the demand charge—that is, the cost determined by the maximum power demand—for different fleet sizes. They show that optimizing this threshold can substantially reduce overall charging costs, without requiring any changes to the infrastructure. In a related application, Pelletier, Jabali, and Laporte (2018) optimize the charging schedule for electric freight vehicles in order to minimize the operational costs under varying electrical-energy prices during the day, while modeling the battery-degradation process. Their results show how various factors, such as grid restrictions, demand charge, energy price, battery size, and degradation costs, can affect the charging schedule and the total operational costs. Although there is some methodological overlap between our study and this work, the difference in the application results in several key differences. For the electric freight fleet, Pelletier, Jabali, and Laporte (2018) consider only central charging at the depot with a sufficient number of chargers during long layovers. In contrast, we consider a transit-bus network with opportunity fast-charging during short layovers at terminal stations with a limited number of chargers in between



trips. Thus, guaranteeing the reliability of the network is more critical in our application and needs to be carefully considered. Furthermore, the size of transit-bus networks is usually larger, resulting in larger optimization problems. The choice of the discretization technique is crucial in reducing the problem size and improving the computational performance. Hence, the trade-off between the quality of the solution and the computational performance becomes an important question that needs to be studied.

To summarize, previous research has analyzed both the challenges of electrifying transit-bus networks and their impact on the power grid and how different charging strategies affect the feasibility of BEB operations. Our work builds upon and complements this work by investigating how to optimally allocate the available charging slots to the BEBs during the course of a day in order to minimize the impact on the grid, while ensuring feasible operations under a predetermined trip schedule and network structure. By applying our methods to the transit-bus network in the city of Rotterdam, we show how the optimal charging schedule can substantially reduce the impact of introducing transit BEBs on the grid.

### 3. Problem Description and Research Outline

The structure of a traditional transit-bus network can be defined by its terminal stations, timetable, and trip-assignment schedule. We define a trip by its starting time, duration, and distance from one terminal station to another. Passenger demand for traveling between origin–destination pairs determines the bus-line structure and the timetable, which specifies the departure times of the buses from one terminal station to another. Thus, the timetable should ensure that bus frequency and passenger demand are aligned throughout the day. Afterward, the trip-assignment schedule is developed to assign the buses to the trips, which determines the required fleet size. Finally, it is important to mention that we study our problem under fully deterministic settings. Thus, no uncertainty is considered with respect to the duration or the energy consumption of the trips. However, we add some common delays to the trips' traveling durations, as suggested by the PTO.

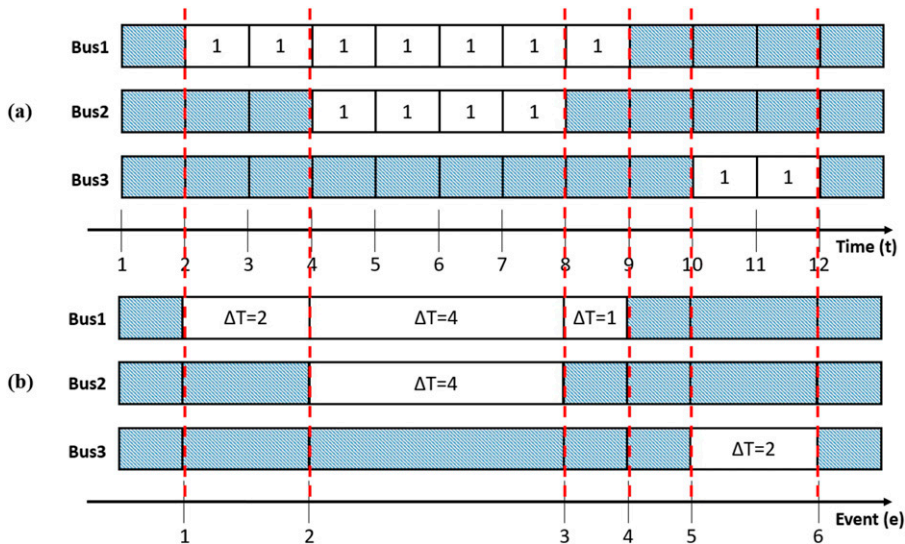
For the opportunity fast-charging transit BEB network, fast-chargers are installed at selected terminal stations. BEBs charge at these terminal stations during their layover time in between some trips. Thus, the trip-assignment schedule should grant sufficient layover time for recharging with fast-charging equipment. It is important to note that we only consider terminal stations in our settings, as there is no opportunity

for fast-charging at midway stations, making them irrelevant for our model. The number of chargers and their locations is an important strategic decision, which should be aligned with the schedule. However, the number of chargers at a terminal station could be limited by various factors, such as the available space and the maximum permissible electrical peak load at that location. Consequently, the number of the BEBs at a certain terminal station will often exceed the number of available chargers. Hence, developing a charging schedule is essential to decide which BEBs will be connected to the chargers in order to guarantee the feasibility of the operation. Finally, after fulfilling all their assigned trips, the BEBs go to the garage to fully recharge their batteries before the beginning of the next day's operations.

In this work, we focus on optimizing the transit BEB networks' charging schedule with opportunity fast-charging by deciding which bus should charge at which location and time during the day. Our main objective is to have a more sustainable public transportation network by minimizing the impact on the grid (i.e., by charging more during off-peak periods). We take the dynamic wholesale electricity prices as a proxy for the demand–supply balance of the power grid at each hour during the day. However, our methodology is applicable to any energy price function, such that balancing could also focus on the level of the distribution grid and local congestion. In either case, a higher electricity price implies a higher scarcity of electrical energy, whereas low prices reflect excess energy supply caused by, for instance, high renewable energy feed-ins. If the PTO's electricity tariff is proportional to these dynamic prices, our objective also implies a minimization of charging costs.

The optimization of the charging schedule is a time-dependent problem, which could be formulated in different ways. The most detailed one is to use time-expanded networks, in which the time and decisions are discretized to equally distributed time slots with the finest practical resolution. In our case, this is equivalent to discretizing the problem to one-minute time slots, corresponding to the timetable, which is communicated at a minute level. Thus, a decision should be made to determine which buses should use which chargers at the network for each minute. Although this formulation can correctly model all possible decisions and reach the best feasible solution with the minimum impact on the grid, the resulting schedule might lead to practical complications. In realistic settings, substituting the charging bus at a certain station would typically only occur at a moment a bus arrives or departs at this location.

**Figure 1.** (Color online) Schedule Data Preparation Example for the DTO Formulation (a) and the DEO Formulation (b)



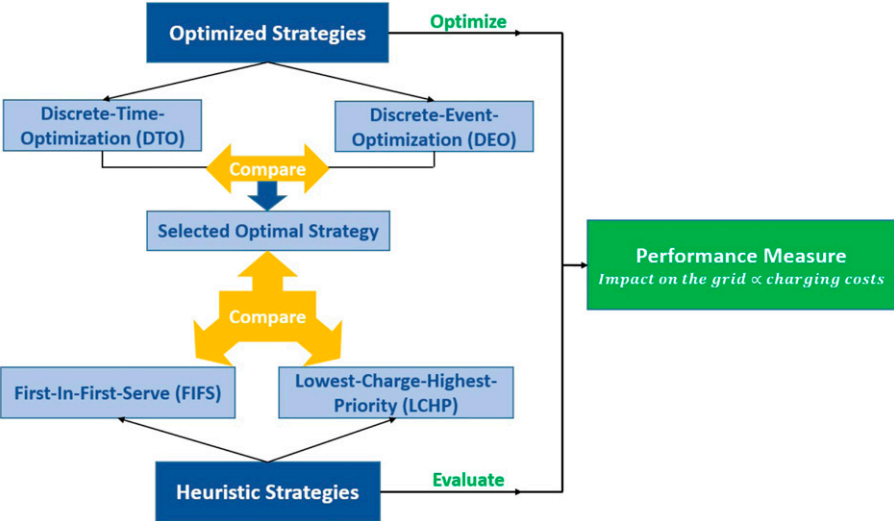
*Notes.* This table is for one station. There are similar tables for each station. The ones (1's) in the DTO tables mark that this bus is at this station during that minute (so it is allowed to charge at any of these one-minute slots). The dotted lines demonstrate the moments when an arrival or departure event occurs at this station. The shaded time slots show that the corresponding bus is not allowed to charge during that time slot as it is not at the station. The unshaded ones show the available time slots for charging for each bus.

This ensures that there is a driver who can plug/unplug the bus from the charger. Thus, another way is to discretize the network based on the occurrence of the relevant events only—arrivals and departures in our case—and not at each minute. Besides reaching a more practical charging strategy, this would also significantly reduce the number of variables and the size of the problem. Thus, we would expect a better computational performance from the event-based discretized formulation compared with the time-expanded one.

Previous research in various fields has explored different discretization approaches to deal with time-

expanded networks in efficient ways. Koné et al. (2011) formulate two different event-based discretized MILP formulations for the project-scheduling problem with limited resources and compare their results to two discrete time-based formulations and a continuous time one. Their results show that selecting the best-performing formulation depends on the instance, but, in general, the event-based and continuous time formulations solve the larger instances with longer planning periods more easily. In a closer research field to our study, Boland et al. (2017) develop a dynamic discretization algorithm to solve the time-expanded exact network to optimality in a faster way using

**Figure 2.** (Color online) Overview of the Study



an iterative algorithm. Their method depends on starting with a partially time-expanded network that does not contain all the time points and then continues updating the time points in the partial network through an iterative algorithm until it reaches optimality. However, this might not be needed in our problem, as the time-expanded network may yield a charging schedule with a better, but less practical, solution. This would be particularly relevant if the difference between the solutions of the two formulations is not substantially large. Finally, it is worth mentioning that the discretization in our problem could be changed from a time-expanded to an event-based network by constraining the charging events to start only upon the arrivals or departures of the BEBs.

Thus, we develop two optimization models with two different formulations and discretization techniques. First, the Discrete-Time Optimization (DTO) discretizes the whole day's operation into equal time slots of one minute, which is equivalent to the time-expanded network (see Figure 1(a)). The input to this model is the schedule that indicates which bus is at which station at each minute. Thus, the decision is to select which bus should charge at each minute and each station. As a result, the DTO formulation involves a discrete time variable for charging. Second, the Discrete-Event Optimization (DEO) only uses the time moments of bus arrival and departure events as the beginning of a new time slot (see dotted red lines in Figure 1(b)). This results in nonuniform and fewer time slots compared with those in the DTO formulation. It is obvious how the DEO's problem formulation could be extracted from the DTO's more general formulation by restricting the start of charging events to moments of bus arrivals and departures. Although the charging duration could be easily formulated as a continuous variable in the DEO formulation, we keep it as an integer variable, as the current practice is to communicate the schedule at the minute level. Thus, the charging duration is an integer number of minutes for both formulations.

The DTO model has higher flexibility than the DEO in changing the charging bus at the stations at more occasions. Thus, the DEO's feasible set is a subset of that of the DTO, and its optimal solution can never outperform the DTO's solution. As a result, the DTO yields an upper bound on the solution performance for the DEO. However, the DEO is expected to have a better computational performance due to the smaller number of variables and constraints. It is also better from a practical and operational point of view, as it is easier to change the charging bus at any station only upon arrival and departure events. This could decrease the required size of the crew, as no additional crew is needed to connect a bus to a charger a few minutes after its arrival.

Figure 2 gives an overview of our study. In Section 5, we conduct a comparative study between the two models to assess the additional cost of enforcing the practical requirements of the DEO and to compare the computational performance of the two formulations. Based on the results of this comparison, we select the preferred optimal schedule and compare it to some greedy strategies, such as First-in-First-Served (FIFS) and Lowest-Charge-Highest-Priority; (LCHP). Additionally, we evaluate how reducing the number of available fast-chargers in the network affects the relative performance of the three strategies.

## 4. Mathematical Formulations

In our models and study, all the buses start the operation with fully charged batteries, and a full-day operation is a 24-hour period. Additionally, we add constraints to guarantee certain levels of reliability and practicability. Buses are not allowed to charge during the day above a certain upper-limit threshold, which is assumed to be 90% in our study. This is because the charging behavior of the batteries becomes less efficient beyond 90% SoC. Buses go to the garage for overnight charging, and their batteries should be recharged to 100% SoC again before the start of the next day's operation (by the end of the 24 hours). For a reliable operation, the SoC is not allowed to drop below a certain lower-limit threshold, and we do not allow for delays in the trip schedule to recharge the batteries. Moreover, from practical perspectives, buses are not allowed to charge for less than a pre-specified minimum charging time. The setup time is excluded from this minimum charging time, which is the time needed to connect a bus to a charger. Additionally, buses are not allowed to be connected for overnight charging more than once because this would imply needing to hire someone to do that during the whole night. Finally, the hourly electricity prices from the daily day-ahead auction are taken as the cost function in our study.

### 4.1. Discrete-Time-Optimization Formulation

Table 1 shows the definitions of the sets, input parameters, and variables used in the DTO formulation. Equation (1) shows the objective function of minimizing the total charging costs ( $Q$ ), which also corresponds to minimizing the impact on the grid.  $P_t$  is the electrical energy price at minute  $t$ .  $X_{bt}$  is the binary decision variable, which indicates whether bus  $b$  is charging during minute  $t$  or not.  $I_{bt}$  is a binary variable that is one if bus  $b$  started the charging process at minute  $t$ . The setup time ( $\tau^s$ ) is removed from the charging time in Equation (1) and constraint (2), as no energy is transferred to the bus during setting it up to the charger.  $L_{bst}$  is the input parameter which represents the locations of the buses and takes the value

**Table 1.** Definitions of the Sets, Parameters, and Variables of the DTO Model

Sets	Definition
$B$	Set of buses in the network with index $b$
$S$	Set of stations in the network with index $s$
$T$	Set of one-minute time slots with index $t$
Parameters	
$L_{bst}$	Location of buses, 1 if bus $b$ is at station $s$ at minute $t$
$W_{bt}$	Energy consumed for performing a trip by bus $b$ starting at minute $t$ in kWh
$R_b$	Battery energy capacity of bus $b$ in kWh
$N_s$	Number of chargers at station $s$
$\beta_s$	Power of chargers at station $s$ in kW
$\eta_s$	% efficiency of chargers at station $s$
$P_t$	Electrical energy price at time $t$ in Euros/kWh
$\tau^s$	Setup time of buses to chargers in minutes
$\tau^m$	Minimum allowed charging time in minutes
$\tau_b^g$	Minute at which bus $b$ finishes all its trips and goes to the garage
$\tau^f$	Last minute of the planning period (24 hours)
$\tau_b^n$	Minute at which bus $b$ SoC drops below 90% for the first time
$\alpha_{bt}^u$	Upper limit SoC of each bus $b$ at time $t$ , equal to 0.9 for $\tau_b^n \leq t \leq \tau_b^g$ , and to 1 otherwise
$\alpha_b^l$	Lowest allowed SoC of each bus $b$ (ranging from 0 to 1)
Variables	
$X_{bt}$ (binary)	Decision to charge, 1 if bus $b$ is going to charge or is charging at minute $t$
$C_{bt}$ (continuous)	Effective SoC of bus $b$ at minute $t$ in kWh
$I_{bt}$ (binary)	Started charging, 1 if bus $b$ started charging at minute $t$
$U_{bt}$ (binary)	Binary product between $X_{b,t}$ and $X_{b,t-1}$
$Q$ (continuous)	Total charging costs in Euros

one if bus  $b$  is located at station  $s$  at minute  $t$ . Finally,  $\beta_s$  is the power of the chargers at station  $s$ .

$$Q(X_{bt}) = \min \left( \sum_{t \in T} \frac{P_t}{60} \sum_{b \in B} (X_{bt} - I_{bt} \times \tau^s) \sum_{s \in S} L_{bst} \times \beta_s \right). \quad (1)$$

The effective SoC ( $C_{bt}$ ) of bus  $b$  at minute  $t$ , after subtracting the energy required to perform a trip starting at  $t$ , is defined as shown in constraint (2). Thus, this value does not represent the actual SoC at each minute  $t$ . Here,  $R_b$  is the energy capacity of the battery of bus  $b$ .  $W_{bt}$  is the energy required for bus  $b$  to perform a trip starting at minute  $t$ .  $\eta_s$  is the efficiency of the chargers at station  $s$ .

$$C_{bt} = \begin{cases} R_b - W_{bt}, & \text{if } t = 0 \\ C_{b,t-1} - W_{bt} + (X_{bt} - I_{bt} \times \tau^s) \\ \times \left( \sum_{s \in S} L_{bst} \times \frac{\beta_s}{60} \times \frac{\eta_s}{100} \right), & \text{otherwise} \end{cases} \quad (2)$$

Constraints (3) and (4) are added to control the upper limit and lower limit of the SoC of each bus, respectively. Constraint (5) guarantees that all the buses retain their full charge before the beginning of the next day's operation. However, because energy is

added to the buses in a discretized minute-by-minute way, it cannot be formulated such that the final SoC is exactly equal to 100%. Thus, it is allowed to be within the range between 100% and less than 100% by the amount of energy added during one minute of charging at the garage.

$$C_{bt} \leq \alpha_{bt}^u \times R_b \quad \forall b \in B, t \in T, \quad (3)$$

$$C_{bt} \geq \alpha_b^l \times R_b \quad \forall b \in B, t \in T, \quad (4)$$

$$C_{bt} \geq R_b - \left( \sum_{s \in S} L_{bst} \times \frac{\beta_s}{60} \times \frac{\eta_s}{100} \right) \quad \forall b \in B, t = \tau^f. \quad (5)$$

The following constraint is added to prevent any charging to take place if the bus is not present at a station:

$$X_{bt} = 0 \quad \forall b, t \in A(b, t), \quad (6)$$

such that  $A(b, t) = \{b \in B, t \in T \mid \sum_s L_{bst} = 0\}$ . Furthermore, the number of buses charging at each station is restricted by the number of chargers at this station  $N_s$ , which is ensured by constraint (7).

$$\sum_b (X_{bt} \times L_{bst}) \leq N_s \quad \forall s \in S, t \in T. \quad (7)$$

The following set of constraints is used to calculate the binary variable  $I_{bt}$ , indicating that bus  $b$  started charging at minute  $t$ , which is necessary to account for the setup time. It is calculated through the variable  $U_{bt}$ , which is the binary product between  $X_{b,t}$  and  $X_{b,t-1}$ .



It is calculated in a linear way, as shown in constraints (9)–(11). Thus,  $I_{bt}$  takes the value one if and only if  $X_{b,t}$  is one and  $X_{b,t-1}$  is zero.

$$I_{b,t} = X_{b,t} - U_{b,t} \quad \forall b \in B, t \in T, \quad (8)$$

$$U_{b,t} \leq X_{b,t-1} \quad \forall b \in B, t \in T, \quad (9)$$

$$U_{b,t} \leq X_{b,t} \quad \forall b \in B, t \in T, \quad (10)$$

$$U_{b,t} \geq X_{b,t} + X_{b,t-1} - 1 \quad \forall b \in B, t \in T. \quad (11)$$

Moreover, because of practical concerns and as recommended by the PTO in the city of Rotterdam, buses are allowed to be connected to the chargers only once for overnight charging, which is represented in constraint (12). Finally, the minimum charging time is expressed by constraint (13).

$$\sum_{t \geq \tau_b^s} I_{bt} \leq 1 \quad \forall b \in B, \quad (12)$$

$$\sum_{t^* \geq t} X_{b,t^*} \geq (\tau^s + \tau^m) \times I_{b,t} \quad \forall b \in B, t \in T. \quad (13)$$

#### 4.2. Discrete-Event-Optimization Formulation

The definition of the sets and the parameters in the DEO formulation is very similar to that in the DTO formulation. However, the set  $T$  of the minutes is replaced by the set  $E$  of the events or time slots, with the index  $e$ . Table 2 shows the additional sets, parameters, and variables, or those that are defined differently in the DEO model compared with their

previous definitions in the DTO model. In the DEO formulation, there are as many time slots as there are number of events minus one. Each bus arrival or departure and the last minute of the 24-hour planning period are marked as events. Additionally, because the electricity prices are assumed to change each hour, the start of each hour is also counted as an event in order to differentiate between the charging costs at the different time slots at different hours.

The objective function shown in Equation (14) of minimizing charging costs is similar to that of the DTO model shown before in Equation (1).  $Y_{be}$  is the charging duration for bus  $b$  at time slot  $e$ .

$$Q(Y_{be}) = \min \left( \sum_{e \in E} \frac{P_e}{60} \sum_{b \in B} (Y_{be} - I_{be} \times \tau^s) \cdot \sum_{s \in S} L_{bse} \times \beta_s \right). \quad (14)$$

The SoC of each bus at each occurring event is defined as the SoC at the previous event plus any added charge during the last time slot and minus the energy required to start a trip at the occurrence of event  $e$  and is calculated as follows:

$$C_{be} = \begin{cases} R_b - W_{be}, & \text{if } e = 0 \\ C_{b,e-1} - W_{be} + (Y_{be} - I_{be} \times \tau^s) \\ \times \left( \sum_{s \in S} L_{bse} \times \frac{\beta_s}{60} \times \frac{\eta_s}{100} \right), & \text{otherwise} \end{cases} \quad (15)$$

Similar to constraints (3)–(5) in the DTO model, the following constraints are added to control the buses'

**Table 2.** Definitions of the Sets, Parameters, and Variables of the DEO Model

Sets	Definition
$E$	Set of the selected events in the network with index $e$ , each time slot $e$ lies in between the two successive events $e$ and $e + 1$
Parameters	
$L_{bse}$	Location of the buses, 1 if bus $b$ is at the station $s$ at the occurrence of event $e$
$W_{be}$	Energy consumed for performing a trip by bus $b$ starting at event $e$ in kWh
$H_{se}$	Event at station, 1 if there is an arrival or departure at station $s$ at the occurrence of event $e$
$T_e$	Minute at which event $e$ occurs and time slot $e$ starts
$\Delta T_e$	Length of time slot $e$ , defined as $T_{e+1} - T_e$
$V_e$	Number of next events required starting from (and including) event $e$ to cover setup time + minimum charging time
$P_e$	Electrical energy price during time slot $e$ in Euros/kWh
$\epsilon_b^g$	Event at which bus $b$ finishes all its assigned trips and goes to the garage
$e^f$	Last event of the day before the next day's operation starts
$\epsilon_b^n$	Event at which bus $b$ 's SoC drops below 90% for the first time
$\alpha_{be}^u$	Upper limit SoC of each bus $b$ at event $e$ , which is equal to 0.9 for $\epsilon_b^u \leq e \leq \epsilon_b^g$ , and is equal to 1 otherwise
Variables	
$Y_{be}$ (integer)	Charging duration of bus $b$ during time slot $e$ , between events $e$ and $e + 1$
$X_{be}$ (binary)	Indicates if bus $b$ is charging at time slot $e$ (1 if $Y_{be} > 0$ )
$C_{be}$ (continuous)	SoC of bus $b$ at occurrence of event $e$ in kWh
$I_{be}$ (binary)	Started charging, 1 if bus $b$ started charging at time slot $e$
$U_{be}$ (binary)	Binary product between $X_{b,e}$ and $X_{b,e-1}$

SoC upper limit, lower limit, and final value before the beginning of the next day's operation, respectively:

$$C_{be} \leq \alpha_{be}^u \times R_b \quad \forall b \in B, e \in E \quad (16)$$

$$C_{be} \geq \alpha_{be}^l \times R_b \quad \forall b \in B, e \in E \quad (17)$$

$$C_{be} \geq R_b - \left( \sum_{s \in S} L_{bse} \times \frac{\beta_s}{60} \times \frac{\eta_s}{100} \right) \quad \forall b \in B, e = e^f. \quad (18)$$

The following constraint is added to restrict the charging period of the buses between any two successive events to the length of the corresponding time slot  $\Delta T_e$ .  $L_{bse}$  is a binary variable that equals one whenever bus  $b$  is at station  $s$  at the occurrence of event  $e$ .

$$Y_{be} \leq (\Delta T_e) \left( \sum_{s \in S} L_{bse} \right) \quad \forall b \in B, e \in E. \quad (19)$$

The following two big-M constraints are used to calculate  $X_{be}$ , which indicates whether bus  $b$  is charging at time slot  $e$ . In practice, we set the  $M$  value equal to the length of each time slot plus one ( $\Delta T_e + 1$ ). This guarantees that  $M$  is larger than any feasible value for  $Y_{be}$ .

$$Y_{be} \geq 1 - M \times (1 - X_{be}) \quad \forall b \in B, e \in E, \quad (20)$$

$$Y_{be} \leq M \times X_{be} \quad \forall b \in B, e \in E. \quad (21)$$

The next constraint is added to ensure that the number of buses charging at a given station does not exceed the available number of chargers.

$$\sum_b (X_{be} \times L_{bse}) \leq N_s \quad \forall s \in S, e \in E. \quad (22)$$

The next set of constraints is similar to those in (8)–(11), which were used in the DTO model to calculate the binary variable that indicates the beginning of a charging process. Thus,  $I_{be}$  is one if bus  $b$  started charging when event  $e$  took place.  $U_{be}$  is the binary product between  $X_{b,e}$  and  $X_{b,e-1}$  and is calculated in a linear way by applying constraints (24)–(26).

$$I_{b,e} = X_{b,e} - U_{b,e} \quad \forall b \in B, e \in E, \quad (23)$$

$$U_{b,e} \leq X_{b,e-1} \quad \forall b \in B, e \in E, \quad (24)$$

$$U_{b,e} \leq X_{b,e} \quad \forall b \in B, e \in E, \quad (25)$$

$$U_{b,e} \geq X_{b,e} + X_{b,e-1} - 1 \quad \forall b \in B, e \in E. \quad (26)$$

The next constraint is added to restrict bus  $b$  to only start charging if there is an arrival or departure at its location at event  $e$ . Thus,  $H_{se}$  is one if there is a bus arrival or departure that is occurring at station  $s$  at event  $e$ .

$$I_{be} \leq \sum_s (L_{bse} \times H_{se}) \quad \forall b \in B, e \in E. \quad (27)$$

The following constraint is added to ensure that if a bus is going to charge at two consecutive time slots, it should charge for the whole time of the first time slot. This constraint is added to increase the practicability of the schedule.

$$Y_{be} \geq U_{b,e+1} \times (T_{e+1} - T_e) \quad \forall b \in B, e \in E. \quad (28)$$

Finally, constraint (29) is added to limit the number of connections of each bus for overnight charging to only once, which corresponds to constraint (12) in the DTO model. The last two constraints are added to control the minimum charging time, similar to constraint (13) in the DTO model.

$$\sum_{\substack{e \leq e^f \\ e \geq e_b^s}} I_{be} \leq 1 \quad \forall b \in B, \quad (29)$$

$$\sum_{\substack{e^* \leq e+V_e-1 \\ e^* \geq e}} Y_{b,e^*} \geq (\tau^s + \tau^m) \times I_{b,e} \quad \forall b \in B, e \in E, \quad (30)$$

$$\sum_{\substack{e^* \leq e+V_e-1 \\ e^* \geq e}} X_{b,e^*} \geq (V_e) \times I_{b,e} \quad \forall b \in B, e \in E. \quad (31)$$

## 5. Numerical Case Study and Results

### 5.1. Data Description

The city of Rotterdam has a very extensive transit-bus network that is currently operated by using conventional DBs. It consists of 61 different transit-bus lines to serve passengers, covering nearly the whole city and also reaching to some neighboring towns. In this study, we consider seven essential two-way lines, which are planned to be electrified within a year. Eleven terminal stations serve these seven lines. Six out of the seven lines are going to be fully electrified, and one will be partially electrified. Forty-seven BEBs are planned to operate on these seven lines, with batteries' energy capacity of 240 kWh.

Previous studies examined different factors that affect the energy consumption in BEBs. Zhou et al. (2016) calculated average energy consumption for the large 12m and medium 8m BEBs, which recorded 1.38–1.75 and 0.79 kWh/km, respectively. They also showed that the HVAC load is more important than the passengers load. The driving style, route type, number of stops, topologies, and elevation were also proven to have a significant effect on the BEBs' energy consumption (Kontou and Miles 2015). Thus, we assume the average energy consumption in our study to be constant at 1.5 kWh/km. As a result, the total consumed energy per day is estimated to be 15,230.1 kWh for the 47 BEBs. Thus, the average energy consumption per bus is 324 kWh, which is substantially greater than the 240-kWh battery capacity of the buses. It is also

worth mentioning that the Rotterdam PTO added some extra time (caused by common delays) to the trip durations as a safety margin for the planning. These delays have values that vary per trip according to its corresponding line and the time of day. Finally, the setup time is assumed to be one minute in the whole study.

The PTO installed 13 fast-chargers at only six of the 11 terminal stations. This is because of short layover times at some stations and the lack of available space for chargers and a maximum permissible electrical load at some terminal stations. Additionally, some buses stop at the garage during the day for a layover period between two successive trips. Two fast-chargers have also been installed in the garage. Thus, 15 fast-chargers with a charging power of 240 kW are distributed among six terminal stations and the garage. Moreover, 47 chargers, with a charging power of 50 kW, are installed at the garage for overnight charging of all buses. In Section 5.3, we examine different network structures and use heuristic criteria to reduce the number of chargers in the network. We then observe how this affects the performance of the operation while using different charging strategies.

## 5.2. Computational Comparison Between DTO and DEO Formulations

In this subsection, we carry out a computational comparison of the performance of the DTO and DEO formulations. We create large and small networks to test the computational performance for different problem sizes. The larger one includes the full network with 47 BEBs, as mentioned in the previous section. The smaller network consists of only one line with nine BEBs, which was selected because it is one of the most vital lines in the city. Within each network, we consider 16 different instances with two different levels for each of the following parameters: number of chargers, charging efficiency, minimum charging time, and lower-limit SoC.

The first level of the number of chargers is equal to the summation of the maximum number of simultaneously collocated BEBs at each of these terminal stations during the day. Thus, it is guaranteed that any arriving BEB finds a free charging slot. This gives a total of four fast-chargers in the nine-BEB network and 17 in the 47-BEB network. The distribution of the 17 chargers in the full network is similar to the distribution of the 15 chargers suggested by the PTO, but with four fast-chargers at the garage instead of two. The second level of the number of chargers is chosen by removing the least-used fast-chargers during the day, which results in two fast-chargers in the nine-BEB network and nine in the 47-BEB network. The criteria for selecting the removed chargers are discussed in detail in the next subsection. In both cases,

there are as many overnight chargers in the garage as there are buses. Additionally, the assumed two levels for each of the other parameters are 90% and 100% for the charging efficiency, one and three minutes for the minimum allowed charging time, and 25% and 40% for the lower limit of the SoC.

The results are obtained by the MILP solver of CPLEX 12.7 operating on a Windows personal computer with a 2.7-GHz i7 CPU and 16 GB of RAM and implemented in AIMMS. The solution-search stopping criteria are set to a 900-second time limit and 0.01% optimality gap. In the results shown next, instance names are labeled as follows: “B,” number of buses; “C,” number of chargers; “E,” charging efficiency; “M,” the minimum charging time; and “L,” the lower-limit SoC. The strictest instance would be the one with the lower levels of number of chargers and charging efficiency and the higher levels of the minimum charging time and the lower-limit SoC. Thus, 9B-2C-90E-3M-40L and 47B-9C-90E-3M-40L are the strictest instances for the small (i.e., nine BEBs) and the large (i.e., 47 BEBs) networks, respectively.

The optimization results are shown in Table 3. On the one hand, the results show that the DEO consistently performs better on execution time. Among the 24 feasible cases of the 32 instances, the DEO formulation was able to find a solution within 0.01% of optimality within the prespecified time limit and in less than one second for the smaller network (i.e., nine BEBs) and 302 seconds for the larger network (i.e., 47 BEBs). In contrast, the DTO formulation was able to find a solution with an optimality gap less than 0.01% within the 900-second time limit for six instances only within a duration ranging from 28 seconds to 42 seconds, all of them belonging to the smaller network and with the less restrictive operating conditions (higher number of chargers). Moreover, it was unable to find an integer solution within the 900-second time limit for two feasible instances in the larger network (instances 18 and 30). Nevertheless, the DTO was able to find a close-to-optimal solution (within 0.15%) for all of the other feasible instances within the specified 900-second time limit. The DTO’s average execution times amounted to 467.25 seconds for the smaller network and 903.47 seconds for the large network, whereas the DEO’s average execution times were 0.51 seconds and 53.92 seconds, respectively.

On the other hand, and as expected, the DTO can reach a better solution with lower charging costs and impact on the grid for all feasible instances. However, the results show that the difference in the optimal charging costs between the two formulations is only ranging from 0.17% to 0.32%. Thus, the cost of applying the DEO schedule instead of the DTO’s to improve the practicability is very small. Moreover, our results show that the DEO formulation performs consistently

**Table 3.** Optimization Results with the Objective of Minimizing the Charging Costs

Instance number	Instance label	DEO			DTO		
		Optimal costs (Euros)	Gap (%)	Execution time (s)	Optimal costs (Euros)	Gap (%)	Execution time (s)
1	9B-4C-100E-1M-25L	153.42	0.0099	0.95	153.08	0.0096	28.80
2	9B-4C-100E-1M-40L	160.00	0.0000	0.42	159.65	0.0603	900.36
3	9B-4C-100E-3M-25L	154.75	0.0045	0.36	154.41	0.0095	30.89
4	9B-4C-100E-3M-40L	163.58	0.0036	0.38	163.10	0	33.21
5	9B-4C-90E-1M-25L	171.51	0.0000	0.45	171.21	0.0099	36.29
6	9B-4C-90E-1M-40L	179.85	0.0041	0.36	179.54	0.0091	36.65
7	9B-4C-90E-3M-25L	174.05	0.0042	0.34	173.74	0.0042	41.37
8	9B-4C-90E-3M-40L	Infeasible			Infeasible		
9	9B-2C-100E-1M-25L	155.33	0.0098	0.99	154.90	0.0748	900.25
10	9B-2C-100E-1M-40L	165.40	0.0054	0.53	165.03	0.0730	901.33
11	9B-2C-100E-3M-25L	156.10	0.0100	0.70	155.60	0.0716	900.36
12	9B-2C-100E-3M-40L	Infeasible			Infeasible		
13	9B-2C-90E-1M-25L	174.93	0.0092	0.61	174.57	0.0143	900.24
14	9B-2C-90E-1M-40L	Infeasible			Infeasible		
15	9B-2C-90E-3M-25L	175.99	0.0000	0.53	175.66	0.0522	900.25
16	9B-2C-90E-3M-40L	Infeasible			Infeasible		
17	47B-17C-100E-1M-25L	758.95	0.0001	22.33	756.82	0.1103	903.61
18	47B-17C-100E-1M-40L	796.32	0.0076	22.58	—	—	904.40
19	47B-17C-100E-3M-25L	764.03	0.0011	20.14	761.82	0.1116	904.65
20	47B-17C-100E-1M-40L	Infeasible			Infeasible		
21	47B-17C-90E-1M-25L	849.07	0.0000	48.06	847.10	0.1241	904.81
22	47B-17C-90E-1M-40L	892.28	0.0000	23.91	890.32	0.1054	904.21
23	47B-17C-90E-3M-25L	855.23	0.0021	21.28	853.16	0.1217	901.39
24	47B-17C-90E-3M-40L	Infeasible			Infeasible		
25	47B-9C-100E-1M-25L	759.56	0.0064	74.31	757.38	0.1242	903.34
26	47B-9C-100E-1M-40L	797.94	0.0179	301.14	795.71	0.1447	901.58
27	47B-9C-100E-3M-25L	764.64	0.0026	32.86	762.47	0.1270	904.71
28	47B-9C-100E-3M-40L	Infeasible			Infeasible		
29	47B-9C-90E-1M-25L	849.88	0.0093	28.42	847.92	0.1131	905.28
30	47B-9C-90E-1M-40L	895.19	0.0100	21.88	—	—	901.20
31	47B-9C-90E-3M-25L	855.97	0.0092	30.11	853.88	0.1038	902.48
32	47B-9C-90E-3M-40L	Infeasible			Infeasible		

better than the DTO regarding execution time. The results also show that this difference in the optimal solution values of the two formulations becomes smaller as the network operating conditions become less strict. This occurs because the DTO is more flexible in changing the bus being charged at the terminal stations at every minute and can benefit from this under stricter instances. However, the execution time also increases as the network conditions become stricter or size increases for the two formulations, but the effect is more salient for the DTO.

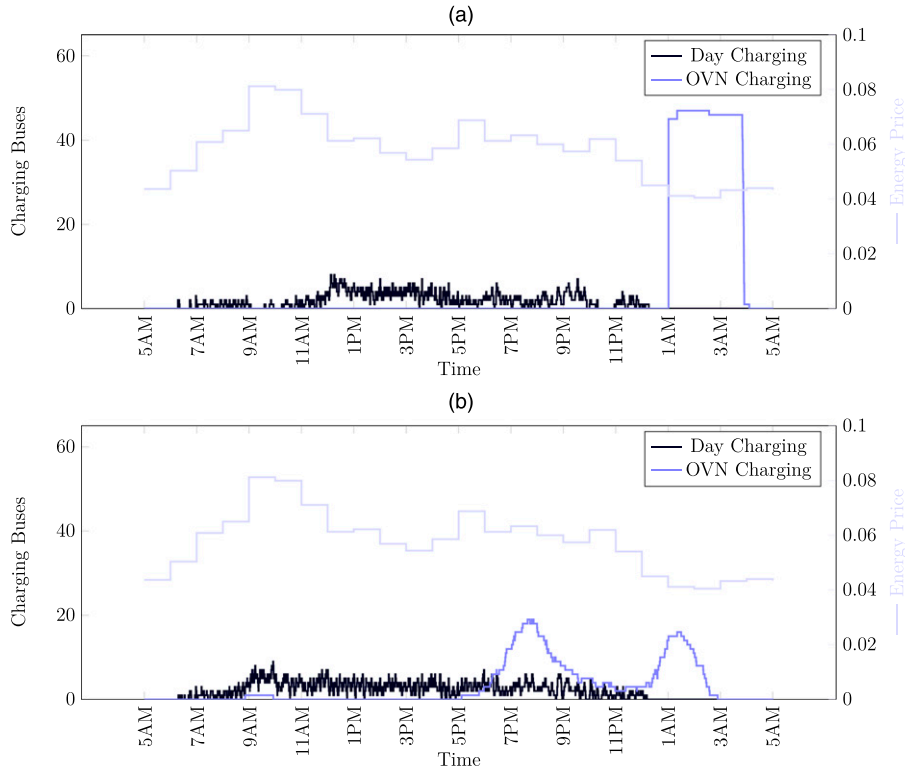
### 5.3. Comparison of Optimal Charging Schedule and Benchmark Greedy Charging Strategies

In this subsection, we compare the performance of the optimal charging schedule to some benchmark greedy charging strategies. We carry out this comparative study under different network conditions by reducing the number of fast-chargers. Then, we study the effect on the electrification feasibility and the main performance measures when using the optimal schedule and compare these to greedy strategies, which are

evaluated through a simulator. Based on the results of the comparative study from the previous subsection, we select the optimal schedule as the solution of the DEO formulation. It will be referred as the optimal schedule (OPT) for the rest of this section.

We implement two greedy strategies as a reference to compare them to the optimal schedule: First-in-First-Served and Lowest-Charge-Highest-Priority. The two strategies use common-sense algorithms to select which bus should get the charging slot. FIFOs arranges the buses in the queue according to their arrival time, whereas LCHP arranges them according to their SoC, so that the bus with the lowest SoC has the highest priority to take the charging slot. LCHP allows an arriving bus with a lower SOC to take a charging slot from a charging bus only if the SoC of the bus being charged is above a 50% threshold. Once its SoC reaches 50%, it leaves the queue and is replaced by a bus with a lower SOC. For the two simulated strategies, similar to the parameters of the OPT strategy, buses do not enter the charging queue during the day if their SoC is above a certain threshold (90%) or if the remaining time before



**Figure 3.** (Color online) Distribution of Charging Events and Energy Prices Throughout the Day for the OPT Strategy (a) and FIFS/LCHP Strategy (b)

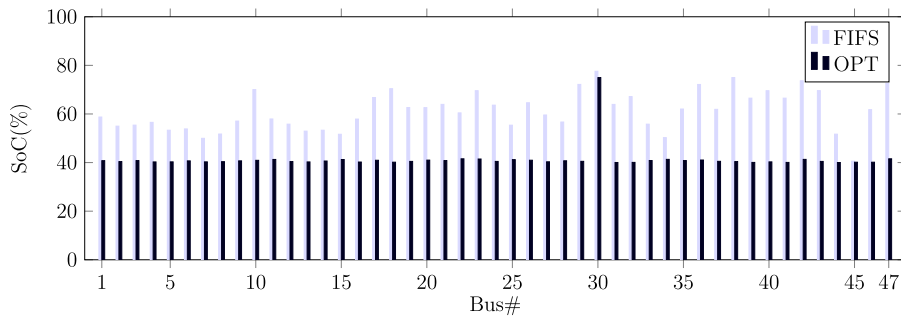
Note. OVN, overnight.

the next trip is less than a certain amount of minimum charging time (two minutes, including a setup time of one minute). Because the numbers of overnight chargers and buses are equal, each bus is connected to a charger once it arrives at the garage in both strategies.

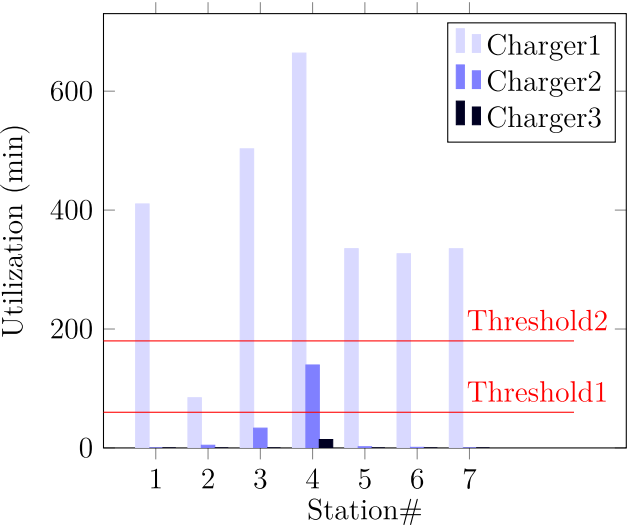
Similar to the analysis in the previous section, we consider the full 47-BEB network with the maximum number of 17 fast-chargers at the same locations with a charging power of 240 kW and 47 overnight chargers at the garage with a charging power of 50 kW. The setup and minimum charging durations are set to one minute in both the optimal and the simulated strategies, and the upper-limit SoC is set to 90% (equivalent to instance 18 in the previous section;

results are in Table 3). The lower-limit SoC in the optimal strategy is set to 40%. Using the maximum number of fast-chargers implies that any arriving bus at a terminal station with chargers can find a free charging slot, which also means that the FIFS and LCHP would have exactly the same results.

The total charging cost under the FIFS or LCHP strategy amounts to €953.33, compared with €796.3 for the optimal strategy. Thus, the optimal schedule can reduce the charging costs by 16.5%. This also implies decreasing the impact on the grid by 16.5% if the energy prices per hour are directly linearly proportional to the gap between the supply and demand at the grid. Figure 3 shows the distribution of the

**Figure 4.** (Color online) Lowest SoC of FIFS/LCHP Compared with OPT

**Figure 5.** (Color online) The Utilization per Charger at Each Station



charging events and energy prices throughout the whole day for the optimal strategy compared with FIFS and LCHP. It shows how the optimal strategy outperforms the other two strategies because it avoids charging during peak times and thereby reduces charging costs and minimizes the impact on the grid. However, this reduction in charging costs implies that SoC levels drop lower during the day. Figure 4 shows the minimum SoC per each bus for the FIFS/LCHP in comparison with the optimal strategy. It shows how the optimal schedule keeps the minimum SoC of all the buses at exactly the predefined permissible lower limit, avoids any unnecessary charging during the day, and charges more during the night at lower energy prices. As a result, the number of fast-charging events is smaller for the OPT strategy, which has a positive effect on the batteries' lifetime. In our case, the total number of charging events is 508 with the OPT schedule and 624 for the FIFS and LCHP strategy.

In the next analysis, we reduce the number of chargers by removing the least-used chargers in the full network. Figure 5 shows the actual utilized time of each charger for the full network with the maximum number of chargers under the FIFS and LCHP strategies. It also shows that four chargers are redundant, and 13 fast-chargers are sufficient to obtain the same results. This happens because some buses might stop at some terminal stations at the beginning of the schedule, but might not charge, as their SoC is above 90%. Thus, these four chargers will only be used if we allow buses to charge during the day, even if their SoCs are above 90%.

For the purpose of creating new, stricter instances, it is a reasonable approach to remove the fast-chargers in the network that are utilized for less than a certain duration. Thus, we restrict the network a couple of times by removing all the fast-chargers that are utilized for less than 60- and 180-minute thresholds (see Figure 5), which results in networks with eight and six chargers, respectively.

The results of the three strategies are shown in Table 4. With eight chargers, the LCHP has four more charging events than the FIFS, which are events when charging buses were forced out of their charging slot. The results show that the OPT strategy reduces the charging costs by 16% and 15.8%, compared with the FIFS and the LCHP, respectively. It also reduces the number of charging events to 490, compared with 616 under the FIFS strategy and 620 under the LCHP strategy.

With six chargers, the results show that the LCHP is now performing better than the FIFS charging strategy regarding the buses' lowest SoC. The SoC of six buses dropped below the 40% limit while adopting the FIFS strategy, whereas this happens to only four BEBs with LCHP. Additionally, the lowest minimum SoC among all buses was 5.16% in the FIFS, compared with 11.82% for the LCHP. It is now infeasible to have an OPT strategy that guarantees a lower-limit SoC of 40% for all the buses. Therefore, we relax the

**Table 4.** Summary of the Results of the Comparison Between the Optimal and Greedy Strategies with a Reduced Number of Chargers

Network configuration	Charging strategy	Charging costs	Number of charging events	Minimum SoC (%)	Number of buses with SoC < 40%
17 (or 13) chargers	FIFS	953.33	624	40.56	0
	LCHP	953.33	624	40.56	0
	OPT	796.30	508	40.00	0
8 chargers	FIFS	950.10	616	40.56	0
	LCHP	948.10	620	40.56	0
	OPT	798.04	490	40.00	0
6 chargers	FIFS	937.50	553	5.16	6
	LCHP	938.23	569	11.82	4
	OPT	801.82	484	16.00	1

minimum SoC constraint on a bus-by-bus level. As a starting point, we take the lowest SoC of each bus according to the results of the LCHP. Then, the lower-limit SoC of each bus that recorded a minimum SoC that is lower than 40% was increased gradually until it reached 40% or the feasible limit. This resulted in having only one bus with a minimum SoC of 16% that is lower than the 40% limit in the OPT strategy, which is also a better solution than that obtained by the LCHP and the FIFS strategies. Moreover, the results show that the OPT strategy is capable of reducing the charging costs now by around 14.5% compared with both the FIFS and LCHP charging strategies. Finally, the optimal strategy is again superior to the greedy strategies regarding the number of charging events and reduced it to 484, compared with 553 and 569 for the FIFS and LCHP, respectively.

As a further validation of our results, we also compare OPT to adapted greedy strategies, which take known intraday price variations into account. Specifically, LCHP and FIFS are augmented such that they seek to decrease charging costs by reducing the amount of charging during peak hours. Charging during peak hours is only allowed for those BEBs with an SoC value below a certain threshold. However, the results summarized in Table A.1 in the appendix show that following the optimal strategy is still clearly a better option for the PTO.

## 6. Conclusion and Future Research

In this study, we tackled the transit BEBs' charging-strategy problem, which is one of the most essential practical complications that PTOs are facing today, when replacing their current DB fleets with electric ones. More importantly, we focused on the problem from the perspective of the electrical-grid operators and the PTO. We presented different optimized charging strategies that guarantee and meet the operating requirements of the PTO, in addition to minimizing the impact on the grid, and increasing the sustainability of the public transportation network. The optimized charging schedule reduced the impact on the grid by up to 16.5% compared with greedy charging strategies. We also showed how better operational performance can be achieved under stricter operating conditions, such as reducing the number of chargers. Additionally, we adopted a reasonable heuristic approach to determine the number of required fast-chargers at the selected terminal stations.

We developed and compared two different formulations for optimizing the charging strategy based on different time-discretization techniques. The computational comparisons showed that, besides the

DEO's higher practicality, it is also superior in terms of computational performance. On the other hand, and as expected, the DTO can reach a better solution with lower charging costs and impact on the grid. Nevertheless, the difference in the optimal solutions between the two formulations was only marginal and ranged from 0.17% to 0.29%. Our results also show that this difference becomes larger as the network operating conditions become stricter due to the DTO's higher flexibility in changing the charging buses. However, the execution time also increases as the network gets larger or conditions become more restrictive.

Finally, there are still more aspects of the problem that need to be studied in more detail in the future. The fact that we consider the trip schedule and assignment, the fleet size, and the charging locations as input parameters could hinder us from reaching the global optimal solution in the larger problems. Thus, optimizing the trip and charging schedules should ideally be done simultaneously. However, this would result in a massive optimization problem with many interrelated decision variables. Moreover, although we added common delays suggested by the PTO to the schedule, we did not study how the proposed charging strategies would be affected by the stochasticity of the operating conditions, different types of contingencies, and extra delays. Additionally, introducing such a large fleet of BEBs could also offer a great opportunity of using them as a virtual power plant, by feeding electrical energy back into the grid, to help in balancing the demand and supply at periods of high scarcity of electrical energy (Kahlen, Ketter, and van Daen 2018).

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## Appendix

In this additional experiment, we assess the performance of the greedy strategies after they have been adapted to reduce the amount of charging during peak hours. We allow BEBs to start charging during peak hours only if their SoC is below 50%. We define peak hours as those hours that have the highest energy prices. The highest charging prices in our data occur from 9 a.m. to 12 p.m. We analyze two different settings in our experiment, with the first reducing charging from 9 a.m. to 11 a.m. and the second from 9 a.m. to 12 p.m. The network configurations and the numbers of chargers remain the same as in the main study. The results are shown in Table A.1.

**Table A.1.** Performance of Modified Greedy Strategies

Network configuration	Peak hours	Charging strategy	Charging costs	Number of charging events	Minimum SoC (%)	Number of buses with SoC < 40%
13 chargers	9 a.m.–11 a.m.	FIFS	913.7	534	22.37	11
		LCHP	913.7	534	22.37	11
	9 a.m.–12 p.m.	FIFS	897.91	499	22.37	18
		LCHP	897.91	499	22.37	18
8 chargers	9 a.m.–11 a.m.	FIFS	909.3	525	15.7	13
		LCHP	907.63	530	22.37	16
	9 a.m.–12 p.m.	FIFS	894.47	487	15.7	21
		LCHP	892.89	495	19.16	26
6 chargers	9 a.m.–11 a.m.	FIFS	900.72	474	5.16	16
		LCHP	900.02	491	11.82	19
	9 a.m.–12 p.m.	FIFS	888.25	443	5.15	27
		LCHP	887.17	456	11.82	30

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