Integrated Vehicle and Crew Scheduling for Electric Buses with Realistic Charging Behavior

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1 Introduction

Electric vehicles are beginning to make up large portions of the fleet for public transport providers. In The Netherlands for example, approximately 21% of all registered buses utilize an electric drivetrain, as reported by the RDW (2025). To comply with strict Dutch and the European European regulations (2018) on climate and sustainability, this proportion must increase substantially; from 2025 onward, the procurement of new buses powered by non-renewable energy sources will no longer be permitted, with the objective that by 2030, all public transport buses operate with zero emissions. Individual line operators such as Qbuzz (2024) are therefore rapidly replacing their remaining combustion based buses, resulting in primarily electric-powered fleets.

Effective use of these new electric vehicles is however more challenging than that of a combustion based fleet. One of the logistical steps in which this is most apparent is that of vehicle scheduling. The Vehicle Scheduling Problem (VSP) aims to assign vehicles such that a set of trips is covered; in the case of buses, these trips are defined by the timetables for individual lines. A typical solution to this problem comes in the form of a collection of vehicle tasks, each of which can be seen as a schedule that an individual bus will follow throughout the day: it may start at a bus storage facility (more commonly called a depot), then perform one or more trips, before finally returning to the depot. In order to string multiple trips together, a bus must make an empty travel between the ending location of one trip to the starting location of the next. These empty travels, often called deadheads, are the main focus when determining the overall costs of a collection of vehicle tasks. There is ample choice as to which deadheads can be driven, as in theory the deadhead between each pair of compatible trips can be included during the creation of a vehicle task. It is therefore necessary to determine which of these deadheads are required to be driven such that costs are minimized. In addition, the total number of used vehicles may also be minimized. Different variants of the VSP exist, such as the having a single/multiple vehicle types or including a single/multiple depots.

In each of these variants, a commonly made assumption is that a vehicle can travel an entire day without having to refuel. With electric vehicles, this assumption is often not valid: recharging throughout the day is required as the range of an electric vehicle is limited. Additionally, recharging takes a long time when compared to the refueling of a combustion vehicle; refueling can be done in a matter of minutes, whereas recharging may take multiple hours for a full charge. The electrification of fleets therefore requires the VSP to be adapted to include maximum range constraints, as well as charging possibilities within a vehicle task. This extended version of the problem, often referred to as the Electric Vehicle Scheduling Problem (E-VSP), is significantly harder than its non-electric counterpart; the VSP with a single depot can be solved in polynomial time, whereas the E-VSP is NP-Hard as shown by Bunte and Kliewer (2009) and Sassi and Oulamara (2014) respectively.

In addition to this added computational complexity in the E-VSP, other logistical planning steps are effected as well. The Crew Scheduling Problem (CSP) is commonly the step that follows vehicle scheduling. Here, schedules are made for individual crew members such that the planned vehicles always have a driver associated with them. As crew members are subject to labor regulations, additional constraints are now added when compared to vehicle scheduling such as maximum driving time and required breaks. The CSP is also NP-Hard, as shown by Fischetti et al. (1989).

Costs in this step often exceed those associated with the vehicles themselves, with a recent estimate by Perumal et al. (2019) putting crew member costs at around 60% of all operational costs. Optimal solutions for the CSP are however directly influenced by the selected vehicle tasks; they determine where and at what time handovers are possible between crew members. Sequentially minimizing costs in the VSP and CSP may therefore not result in a solution with overall lowest costs, as the solution to the CSP may be improved greatly by incurring higher costs in the VSP. This was already pointed out in the 1980s by Bodin and Golden (1983), who instead advocated for an integrated approach; here, the VSP and CSP are considered at the same time, resulting in the Vehicle and Crew Scheduling Problem (VCSP).

A lot of work has already been done on the VSP, CSP and VCSP. Both the sequential and integrated approach have been extensively studied since the 1980s, as shown by reviews such as Ibarra-Rojas et al. (2015) and Ge et al. (2024). As mentioned before however, the introduction of electric vehicles has introduced significant constraints on recharging times and vehicle ranges, invalidating the critical assumption in many of these works that a vehicle was able to drive an entire day without being refueled. As a response to this, the E-VSP has also been the focus of many studies going back to around 2014. We refer the reader to a survey by Perumal et al. (2022) for a detailed overview of recent progress in that regard.

The integrated VCSP with electric vehicles (E-VCSP) has seen less attention; to the best of our knowledge, only 5 works consider this problem. In these, simplifying assumptions are made which might limit real world applicability or accurate modeling of costs. Most notably, assumptions are currently made about charging locations (such as only being able to charge at a bus depot) or charging behavior (such as exclusively allowing full charges). In this work, our aim is therefore to introduce an integrated E-VCSP model which incorporates more realistic behavior for battery charging and usage.

The rest of this work is organized as follows. In Section 2, we will give more background information on the E-VCSP and provide a formal problem definition. In Section 3, we review literature related to the E-VCSP and identify gaps in the current research. . . .

2 Problem definition

In this section, we give a definition of the integrated Electric Vehicle and Crew Scheduling Problem (E-VCSP). In order to do so, we will first give a global overview of the problem at hand. Afterwards, we discuss the data which is given, before finally formalizing our the resulting problem.

2.1 Overview

The E-VCSP is part of the planning process used in bus public transit, as can be seen in Figure 1. It comes directly after the timetabling step, in which trips are defined for each line according to passenger demand. Each individual timetabled trip represents a bus traveling along a line at a specified time. Oftentimes, these trips are scheduled regularly throughout the day from the start to the end of a line, however irregular schedules and partial travels are also possible.

With these trips being laid out, we now need to determine how they will be driven. Individual

trips often have lengths of around 0.5-2 hours, therefore making it possible to perform multiple trips with a single vehicle and driver in a day. In order to perform a trip, a vehicle must travel to its starting location before driving it; this travel is called a deadhead. Deadheads occur between trips and between a trip and a depot. During a deadhead, the vehicle itself is empty except for the driver. It is therefore beneficial to minimize the total amount of time that vehicles and crew members spend driving deadheads, as costs incurred during this time do not have any direct benefit to passengers. When considering electric vehicles, the time between trips is not only used for deadheads. As electric vehicles generally have small ranges and long recharging times, this downtime can also be used in order to recharge the vehicle. Often, recharging stations will be available at the depot, however additional recharging stations may also be present at the starting or ending locations of trips (also called the terminal trip stops). Charging locations in the middle of a trip are generally not present. Charging a vehicle at one of these locations can take a long time: even with fast charging infrastructure, time for a full recharge can be on the order of hours for a single vehicle. Additionally, recharging is often a non-linear process that can be seen as two phases: from 0 to 80%, where the rate is linear, and from 80 to 100%, where it slows significantly. These phases often take similar amounts of time.

The additional vehicle downtime resulting from charging necessitates careful planning of crew schedules. Crew members must follow local labor regulations, which often consist of maximum driving times throughout the day and required breaks. In order to minimize crew costs, it is beneficial to maximize the time spent actually driving instead of waiting for a vehicle to be ready. Handovers of vehicles between crew members throughout the day are therefore common in order to minimize paid crew downtime. Additionally, long recharging periods may be combined with crew breaks in order to minimize lost productivity.

Handovers between crew members are often only possible at a predetermined set of locations; these relief points have waiting areas or break rooms connected to them, providing a place for a crew member to stay before entering their next vehicle. This means that handovers cannot always occur on a trip by trip basis; the schedules of vehicles and drivers can be broken up into multiple blocks, each of which starts and ends at a relief location. Each of these blocks must be driven continuously by a single driver.

Overall, the goal is to find a set of feasible vehicle and crew schedules that minimize costs. In this, vehicle schedules must ensure that each individual trip is driven while minimizing deadhead and charging costs. Crew schedules, on the other hand, must ensure that each vehicle schedule segment is covered while having feasible handovers.

2.2 Formal definition

We will now formally define the E-VCSP. In this, we will consider the variant with a single depot, single vehicle type and charging stations without capacity limitations. A summary of the notation used has been provided in Table 2.1

Given timetables for each of the considered lines, let $\mathcal{T}$ be the set of trips that they define. Additionally, let $\mathcal{DH}$ be the set of deadheads that represent either a travel to or from the depot $\mathcal{D}$, or a travel between trips in $\mathcal{T}$. In this, only deadheads are present which can feasibly be driven; that is, the travel time of the deadhead must be less than the time between its departure and the start of a target trip. Note that deadheads including a depot as either the origin or target are always considered to be time-feasible.

Next, let us define our battery behavior. For each trip and deadhead, let d indicate the amount of charge that is used during its travel. Additionally, let u_σ indicate the maximum amount of charge

Notation	Definition
Given	
$\mathcal{D}$	The depot
$t \in \mathcal{T}$	Set of all trips, t a single trip
$dh \in \mathcal{DH}$	Set of all feasible deadheads, dh a single deadhead
$b \in B$	Set of all blocks, b a single block
$v \in V$	Set of all vehicle tasks, v single vehicle task
$c \in C$	Set of all crew tasks, c single crew task
d	Amount of charge used driving a trip/deadhead
u_σ	Max. charge gained in deadhead with starting SoC σ
σ_{start}	SoC at beginning of block/task
σ_{min}	Minimum SoC of vehicle
σ_{max}	Maximum SoC of vehicle
$k_{i,t}$	Binary constant, vehicle task v_i covers trip t
$l_{i,b}$	Binary constant, vehicle task v_i uses block b
$m_{j,b}$	Binary constant, crew task c_j uses block b
Decision variables	
$x_i \in \{0,1\}$	Usage of vehicle task v_i
$y_j \in \{0,1\}$	Usage of crew task c_j
Helper functions	
$b(v) \subseteq B$	Set of blocks covered by v
$cost(\chi) \in \mathbb{R}^+$	Cost of task χ

Table 1: Notation used for formal problem description, where χ is used as a placeholder when multiple argument types can be applied.

that can be gained during a deadhead if a vehicle starts at a State of Charge (SoC) of σ; in this, let charging at any possible given charging location be considered. Depending on available charging infrastructure, this may imply that a deadhead is not driven directly, instead making a time-feasible detour via a charging location which provides the fastest charging capabilities. SoC gained at a location depends on given (possibly non-linear) charging curves for the vehicles, allowing for different amounts of charge to be achieved in the same time span depending on the initial SoC.

Lastly, let us define relief points. A subset of the terminal locations of trips may serve as relief points; these locations are the only ones at which a handover or break for a crew member can take place. Using these relief points, we can now start formally defining our schedules.

Let a *block* be a sequence of trips and/or deadheads that can be driven sequentially, in which only the first and last location in the sequence are relief points. This implies that a block must be driven by a single driver, as there is no opportunity to hand over the vehicle to another driver during the block itself. In order to be feasible, the block must match the following requirements:

- A block consists of at least one trip or deadhead.

- Only the first and last location visited in the block are relief points.

- If there is an element in the block directly following a trip, this must be a deadhead originating from that trip.

- If there is an element in the block directly following a deadhead, this must be the deadheads target trip.

- For each deadhead driven, the vehicle must charge with some value $\alpha \cdot u_\sigma$, where $0 \leq \alpha \leq 1$ represents a fraction of the maximal charge gained.

- Given an SoC σ_{start} at the beginning of the block, the SoC of the vehicle should remain within the range $[\sigma_{min}, \sigma_{max}]$ throughout the block.

Let the set of all feasible blocks be B. A schedule for a vehicle throughout the day can now be constructed as a sequence of blocks. Let us refer to such a schedule as a *vehicle task*. In order for a vehicle task to be feasible, the following conditions need to be met:

- A vehicle task consists of one or more blocks.

- Sequential pairs of blocks must be compatible; that is, for each sequential pair of blocks b and b', b must either end in a deadhead whose target is the starting trip of b', or b must end in a trip which is the origin of the starting deadhead of b'.

- The task must start and end at the depot.

- Given a SoC σ_{start} at the beginning of the task, the SoC of the vehicle should always remain within $[\sigma_{min}, \sigma_{max}]$ during all blocks.

Let the set of all feasible vehicle tasks be V. For a vehicle task v, let us define its cost as being $cost(v)$; let this be some combination of fixed and variable costs based on the trips and deadheads driven within the task.

Next, let us consider crew tasks. Crew members are bound by labor regulations; in most cases, this means that we will need to include break time in a crew schedule. Additionally, idle time at a handover location may be required in order to wait for an arriving vehicle. A crew task will therefore consist of a sequence of blocks and idle/break times. Exact feasibility constraints for crew tasks will differ per country and region, however we base the following constraints on a simplified version of the Dutch labor regulations:

- The crew task consists of one or more blocks and break/idle times.

- Sequential pairs of blocks must be compatible. That is, for each sequential pair of blocks b and b', the location at the end of b must be the same as b'.

- The crew task must start and end at the depot.

- The crew task may have a duty length of at most 9 hours

- For shifts between 4 and 5.5 hours, at least one break of 15 minutes must be included.

- For shifts longer than 5.5 hours, at least 40 minutes of break time must be included. All breaks must be longer than 15 minutes, and one break must be at least 20 minutes.

- For shifts starting after 15:00, at least one break between 16:30 and 20:30 of at least 20 minutes must be included.

Of these, the first three constraints will be applicable regardless of local labor regulations. Let the set of all feasible crew tasks be C. For a crew task c, let us define its cost as being $cost(c)$; let this be some combination of fixed and variable costs based based on duty and break length.

Using the definitions of the blocks B, vehicle tasks V and crew tasks C, we can formulate a linear program to describe our integrated problem. In this, let x_i and y_j be binary variables indicate the usage of $v_i \in V$ and $c_j \in C$ respectively. Additionally, let $k_{i,t} \in \{0,1\}$ indicate that v_i covers trip $t \in \mathcal{T}$, let $l_{i,b}$ indicate that v_i contains block $b \in B$, and let with $m_{j,b} \in \{0,1\}$ indicate that c_j includes block $b \in B$.

$$\min \quad \sum_{1 \le i \le |V|} x_i \cdot cost(v_i) + \sum_{1 \le j \le |C|} y_j \cdot cost(c_j) \tag{1}$$

Subject to:

$$\sum_{1 \le i \le |V|} x_i a_{it} = 1 \qquad\qquad \forall t = 1, \dots, |T| \tag{2}$$

$$\sum_{1 \le i \le |V|} x_i l_{i,b} = \sum_{1 \le j \le |C|} y_j m_{j,b} \qquad\qquad \forall b \in B \tag{3}$$

$$x_i \in \{0,1\} \qquad\qquad \forall i = 1, \dots, |V| \tag{4}$$

$$y_j \in \{0,1\} \qquad\qquad \forall j = 1, \dots, |C| \tag{5}$$

Here, constraint (2) ensures that all trips are covered by a vehicle, and constraint (3) ensures that for each block, the number of assigned drivers is equal to the number of assigned vehicles. This integrated formulation minimizes the costs for both vehicle and crew tasks simultaneously. For comparison, when the E-VSP and CSP are solved sequentially, the formulation becomes the following. First, vehicle tasks are selected such that their costs are minimized:

$$\min \quad \sum_{1 \le i \le |V|} x_i \cdot cost(v_i) \tag{6}$$

Subject to:

$$\sum_{1 \le i \le |V|} x_i a_{it} \ge 1 \qquad\qquad \forall t = 1, \dots, |\mathcal{T}| \tag{7}$$

$$x_i \in \{0,1\} \qquad\qquad \forall i = 1, \dots, |V| \tag{8}$$

Here, constraint (7) once again ensures that all trips are covered by a vehicle task. Once the vehicle tasks are selected, we can use them to derive the set of blocks that are driven. Let $b(v) \subseteq B$ represent the set of blocks driven by a vehicle task v. The overall set of blocks driven is then $B' = \bigcup_{v_i \in V, x_i = 1} b(v_i)$. Note that each block is present at most once due to the fact that all trips must be covered exactly once.

Using this, our formulation for crew tasks becomes the following:

$$\min \quad \sum_{1 \le j \le |C|} y_j \cdot cost(c_j) \tag{9}$$

Subject to:

$$\sum_{1 \le j \le |C|} y_j m_{j,b} = 1 \qquad\qquad \forall b \in B' \tag{10}$$

$$y_j \in \{0,1\} \qquad\qquad \forall j = 1, \dots, |C| \tag{11}$$

Abbreviation	Definition
ALNS	Adaptive Large Neighborhood Search
B&P	Branch-and-Price
CG	Column Generation
CP	Constraint Programming
CSP	Crew Scheduling Problem
E-...	Problem ... with electric vehicles
LNS	Large Neighborhood Search
LS	Local Search
MDVSP	Multi Depot Vehicle Scheduling Problem
MIP	Mixed Integer Program
SAA	Simulated Annealing Algorithm
SDVSP	Single Depot Vehicle Scheduling Problem
SoC	State of Charge
TCO	Total Cost of Ownership
ToU	Time of Usage
TVSP	Integrated Timetabling and Vehicle Scheduling Problem
VCSP	Integrated Vehicle and Crew Scheduling Problem
VSP	Vehicle Scheduling Problem

Table 2: Nomenclature used in this work [*TODO: formatting*]

Here, we now consider a greatly reduced number of available blocks to cover in constraint (10), resulting in a formulation with significantly fewer constraints overall.

3 Related work

In this section, we discuss work related to our research into the E-VCSP. An overview of the nomenclature used has been included in Table 3, and an summary of how batteries and charging behavior is modeled in the discussed works has been included in Table 3.

	Model	ToU	SoC	Nonlinear Ch.	Partial Ch.	Ch. Location	Degradation
Li (2014)	E-VSP	No	D	No	No	D	No
Van Kooten Niekerk et al. (2017)	E-VSP	Yes	C/D	Yes	Yes	D/T	Yes
Olsen and Kliewer (2020)	E-VSP	No	C	Yes	Yes	D/T	No
Zhang et al. (2021)	E-VSP	No	C/D	Yes	Yes	D	Yes
Parmentier et al. (2023)	E-VSP	No	C	Yes	Yes	D/T	No
De Vos et al. (2024)	E-VSP	No	D	Yes	Yes	D/T	No
Perumal et al. (2021)	E-VCSP	No	C	No	No	D	No
Wang et al. (2022)	E-VCSP	Yes	C	No	Yes	D	No
Sistig and Sauer (2023)	E-VCSP	No	C	No	Yes	D/T	No
Shen and Li (2023)	E-VCSP	No	C	No	No	D/T	No
Cong et al. (2024)	E-VCSP	Yes	C	No	Yes	D	No
Ham and Park (2021)	E-VRPTW	Yes	C	No	Yes	D	No
Stadnichuk et al. (2024)	E-TVSP	No	C	No	Yes	D/T	No

Table 3: A brief overview of battery modeling in E-VCSP related literature. SoC modeled as (D)iscrete or (C)ontinuous variable, Charge locations at (D)epot or (T)erminal trip stops, Degradation of battery in cost function

3.1 (E-)VSP

Before considering previous work on the E-VSP, we first cover the most basic form of vehicle scheduling: that which only considers a single depot, single vehicle type and unlimited vehicle ranges. This problem, often referred to as the single depot vehicle scheduling problem (SDVSP), forms the underlying basis of both the multi-depot and electric vehicle extensions that we consider later. We therefore give a brief summary of one the most common models and solution methods used for the SDVSP, thereby having a baseline to which we can compare extensions. For a more comprehensive overview on different models used for the SDVSP and multi-depot VSP (MDVSP), we refer the reader to a review by Bunte and Kliewer (2009).

In order to find a solution the SDVSP, the problem can be transformed into one of finding a min-cost max-flow in a graph. The graph can be constructed as follows: For all trips T, add a pair of nodes representing the start and end of the trip respectively. Add an arc from the start of each trip to its end with capacity 1 and cost equal to that of driving the trip. Next, connect the end of each trip t to the start of each trip t' for which the deadhead between the two is feasible; that is, there is enough time to drive the deadhead from t to t' before the scheduled starting time of t'. Let each of these arcs have a capacity 1 and a cost equal to that of driving the deadhead from t to t'. Lastly, let us introduce a pair of nodes representing the depot at the start and end of the day respectively. Connect the depot start node to the start of each trip, and do the same for the depot end node and the end of each trip. For each of these arcs, let the capacity once more be 1 and let the cost be equal to that of driving of the deadhead between the depot and trip. Fixed vehicle costs can be represented by adding costs to the arcs leaving the depot.

We can now find the min-cost max-flow on this graph; in this, let the depot start node be the source, and let de depot end node be the sink. Due to our construction, all trips must be covered by exactly 1 flow, resulting in flow paths which we can directly use as vehicle tasks due to the assumption that our vehicles have infinite range. It is therefore also shown that the SDVSP can be solved in polynomial time, as polynomial time min-cost max-flow algorithms exist and the graph elements are of size $|V| = O(|T|)$ and $|A| = O(|T|^2)$.

Two common extensions to the problem make it NP-Hard: the inclusion of multiple vehicle types, as well the use of multiple depots under the assumption that vehicles must return to their depot of origin. Both of these extensions are also discussed in Bunte and Kliewer (2009). The modification to the SDVSP flow network is the same in either case: an additional source/sink pair can be added for each new depot or vehicle type, and connected to the trips in the same way as the original depot. The problem then turns into into an integral multi-commodity flow, which has been shown to be NP-Hard by Even et al. (1975).

The introduction of any resource constraints within the VSP has also been shown to be NP-Hard by Bodin and Golden (1983). The E-VSP specifically deals with constraints on the driving range of vehicles, thereby making it closely related to the vehicle scheduling problem with route time constraints (VSP-RTC) as described by Haghani and Banihashemi (2002). The key difference between these two problems is that the E-VSP allows for (partial) recharging of a vehicle throughout the operating period, whereas the VSP-RTC assumes a fixed maximum travel time for the vehicle within the given period. The E-VSP has been shown to be NP-Hard by Sassi and Oulamara (2014).

Li (2014) was one of the first to consider a solution method for the E-VSP. They consider a single-depot case with a single vehicle type, in which the assumption is made that full recharging (or battery swaps) can be performed in a fixed 5-minute time window. The model is based on an extension of the SDVSP network, with the inclusion of total driving time constraints. Additionally, time-discretized nodes are added to represent capacitated battery charging/swap stations. Connec-

tions are made from the trip nodes to the charging nodes when a travel between the two is feasible, allowing for a vehicle to perform one or more charging action during its task. By discretizing the nodes, charging station capacity is enforced by setting arc capacity to 1. For smaller instances, the model can be solved to optimality using column generation and branch-and-price (B&P). For larger instances, an alternate approach using truncated column generation followed by a local search to find a local optimum is used instead. The proposed methods are tested on trips in the San Francisco Bay Area, with a maximum instance size of 242 trips. These tests resulted in optimality gaps of < 5% for buses able to drive 150km, and between 7-15% for a range of 120km depending on the instance.

Van Kooten Niekerk et al. (2017) introduce two models which aim to solve the single depot E-VSP while taking into account time dependent energy prices (ToU pricing), nonlinear charging times and battery degradation due to depth of discharge. The first model only allows for linear charging and no consideration for degradation or ToU, but uses continuous state of charge (SoC) variables which are added to the SDVSP network. The second model does allow for the extra inclusions, achieving this by duplicating trip nodes in the SDVSP network for discrete SoC values. The second model is solved using column generation and lagrangian relaxation, resulting in a possibly non-optimal solution. Tests are performed using data provided by Belgian bus company De Lijn in the city Leuven, using a total of 543 trips. They show that the discretized model can be solved in a considerably shorter time frame for large instances with similar results to the continuous model.

Olsen and Kliewer (2020) consider a multi-depot E-VSP, in which they model the nonlinear phase of charging as an exponential function. In order to solve, they propose a greedy heuristic to construct vehicle tasks. Their primary focus is comparing (piecewise) linear approximations for the second phase of charging with an exponential function based approximation. They conclude that SoC and required charging times are more comparable to real life behavior when using the exponential function.

Zhang et al. (2021) apply a similar method to the one found in Van Kooten Niekerk et al. (2017). They consider a single depot with capacitated charging infrastructure, with multiple round trip lines originating from the depot. In addition to this, they also incorporate nonlinear charging behavior and battery depreciation using discrete SoC and time nodes in the SDVSP network. They solve using a combination of CG and B&P. Tests are done on both randomly generated instances as well as 6 not yet electrified lines with up to 160 and 197 trips respectively.

Parmentier et al. (2023) consider a scalable approach to the E-VSP with non-linear charging. They introduce the concept of nondominated charging arcs, which are represented as deadhead arcs within the SDVSP. Their use considerably reduces the amount of candidate charging arcs when multiple charging points are available, as an arc is only considered if there is not another arc available with higher resulting charge and lower cost. In order to solve, a combination of CG and B&P techniques are used. A more computationally efficient version of the pricing problem is also provided by the nondominated charging arcs when charging infrastructure is uniform. Testing is done on the *large* instances introduced by Wen et al. (2016) which included up to 8 depots, 16 charging stations and 500 trips. Here, they are able to find solutions that only have an 0.06% optimality gap.

De Vos et al. (2024) consider the E-VSP with partial recharges and capacitated charging stations. Their model includes discrete SoC trip nodes and discrete SoC and time charging location nodes, similar to the models found in van Kooten Niekerk et al. (2017) and Zhang et al. (2021). As with those models, power used during deadhead arcs is rounded up to the nearest discrete value; this results in an underestimation of the actual SoC of the vehicle during its task, however ensures solutions that can be feasibly driven. This pessimistically rounded graph, which De Vos et al. refer to as the primal network, is accompanied by a dual network; this . They solve the problem by applying CG with two separate approaches: branch-and-price and a diving heuristic. To overcome the

limitations of dual bounds resulting from a discretized model, they incorporate ideas from Boland et al. (2017) resulting in a dual network with optimistic connections. This gives the same bounds as the ones found in the non-discretized model. Testing is performed on a bus concession south of Amsterdam with 816 trips, with subsets being used as smaller instances. Optimality gaps of 1.5-2.7% are achieved across instances. They additionally note that the framework as provided can easily be extended for nonlinear charging functions and depth-of-discharge battery degradation.

3.2 CSP

Given a solution to the (E-)VSP, the corresponding CSP is most often solved as a set partitioning (or set covering) problem. Here, the tasks described by the sequences of trips generated during vehicle scheduling must be covered by the individual schedules of crew members. This problem has been shown to be NP-Hard in general by Fischetti et al. (1989).

Research into this subject is primarily done in the context of airline crew planning; crew costs in this field are generally even higher than those found in the more general public transport sector, as shown in Barnhart et al. (2003). Additionally, strong labor unions and restrictive labor legislation due to safety concerns cause a large number of constraints to be applied to crew schedules, resulting in a non-trivial problem to solve.

Results achieved in the aviation space quite easily generalize to other sectors, and we therefore refer the reader to a recent review by Deveci and Demirel (2018) for an overview of the state of the art.

3.3 (E-)VCSP

The VCSP has been a widely studied problem. Following the call for integrated methods by Bodin and Golden (1983) and others in the 1980s, a large number of different methods has been applied to integrate the VSP and CSP. We refer the reader to a recent review by Ge et al. (2024) for a more general overview of work done in the field in the past years.

One work that we will individually highlight is that of Huisman et al. (2005), due to its use of Lagrangian relaxation to connect the VSP and CSP . For readers unfamiliar with the technique, we recommend an introduction by Beasley (1993). Huisman et al. consider the multi-depot variant, and use a combination of CG and Lagrangian relaxation to solve both the MDVSP as well as the connection with the CSP. Of note is their assumption that crew members from each individual depot are only allowed to work on trips connected to said depot, allowing for individual depot CSPs to be solved as a subproblem. They test on instances in the Randstad metro area in the Netherlands with a maximum of 653 trips and 4 depots.

As for the electric counterpart of the VCSP, at time of writing we are aware of only five other works that discuss the integrated variant.

Perumal et al. (2021) were the first to offer a solution to the E-VCSP. They consider an instance of the problem in which only full recharges at the depot with a fixed duration of 120 minutes are possible. In order to solve, an ALNS was introduced which incorporates a B&P heuristic which has been previously used to solve the MDVSP, E-VSP and VCSP. The authors tested using real life data from lines in Denmark and Sweden with a maximum instance size of 1109 trips and multiple depots, and report an improvement of $1.17 - 4.37\%$ across different instances when compared to a sequential approach.

Wang et al. (2022) introduce a two layered model using Particle Swarm Optimization and an ϵ-constraint based mechanism which allows for a mix of traditional combustion and electric buses. The model incorporates partial depot charging, as well as measures to ensure that crew is primar-

ily assigned to the same vehicle throughout the day. A circular bus route with a single depot in Changchun, China with 68 daily trips is used as a basis for testing, with a focus on electric versus diesel usage and driver satisfaction.

Sistig and Sauer (2023) also offered an ALNS based approach, which aimed to improve upon the approach presented by Perumal et al. (2021) by including partial recharges, opportunistic charging at terminal stops of trips and non-fixed ranges for the vehicles. In order to solve, they implement a selection of 3-step ALNS neighborhoods consisting of E-VSP modification, finding a solution to the corresponding CSP and consequently modifying the CSP solution. Tests were done using an instance of a city route in Germany, with a single depot and a total of 282 trips. Different scenarios based on possible crew break and relief locations were considered in order to compare diesel and electric TCO. Additionally, sensitivity analysis of the TCO was done for parameters such as costs for electricity and drivers.

Shen and Li (2023) provide a minimum-cost flow framework for the E-VSP which is integrated with a set partitioning based approach for the E-CSP. They only provide full recharge capabilities at the depot, however focus on the inclusion of a distinction between energy use when driving and standing still in order to more accurately model real life traffic. A city line in China with 270 daily trips and a single depot is used for testing, resulting in cost savings of up to 8.7% when compared to a sequential approach.

Cong et al. (2024) provide a hybrid MIP and SAA based approach to optimizing a mixed fleet of combustion and electric vehicles with ToU electricity pricing. In each SAA iteration, a collection of new E-VSP trip assignments are created using neighborhood operations, after which two MIP models are sequentially employed to solve for charging and crew schedules. The methods are tested on a collection of 3 bus routes originating from the same depot in Changchun City, China with a total of 520 trips across all routes. When compared to the sequential approach, the integrated vehicle schedule was able to reduce costs by 0.8%.

3.4   Other related fields

The VSP is closely related to the vehicle routing problem (VRP); in this problem, the aim is to find minimum cost routes for vehicles originating from a depot and needing to pass multiple stops, most commonly for pickup or delivery with capacity constraints. The extension of the E-VRP which includes arrival time windows (E-VRPTW) is most closely related to the E-VSP, as the use of 0-width windows allows us to define the same precedence constraints as those naturally defined by trips in the VSP.

An example of work done on the E-VRPTW is that of Ham and Park (2021). They consider a single depot case in which they model ToU pricing and partial recharges during delivery routes. In order to model costs, a lexicographical minimization is done over the number of vehicles used, total distance traveled and energy recharged. In order to solve, a hybrid MIP and CP algorithm is used in which CP is used to model ToU related variables, and MIP is used to model the rest of the constraints.

Research has also been done into integrating the E-VSP with the step before it in the planning sequence: timetable planning. This problem, the E-TVSP, has recently been studied in the work of Stadnichuk et al. (2024). They allowed results of the E-VSP to introduce optimality cuts into the MIP used for creating timetable plans, thereby reducing overall cost. This is achieved by transforming the E-VSP problem into one of bin packing with conflicts, after which three different heuristic methods are applied and compared. They additionally prove that the bounds of the used heuristics are tight for their given instances.
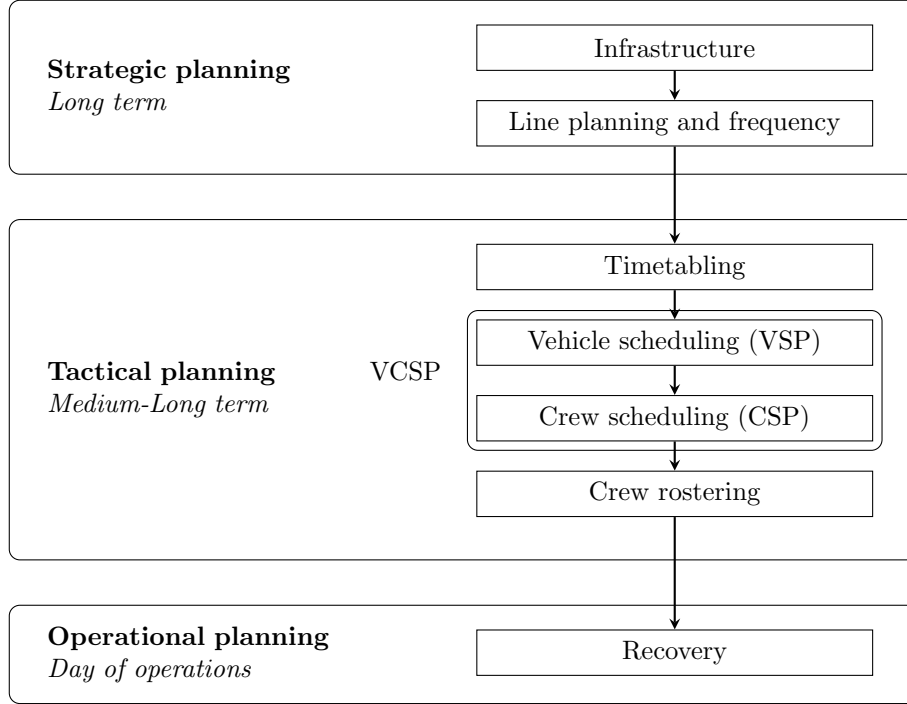
Figure 1: A general overview of the public transport planning process, based on Ceder and Wilson (1986), Ibarra-Rojas et al. (2015), and Perumal et al. (2022).

4 Rough planning

The following constraints list of constraints seem to make the problem the most challenging (when compared to a single-depot single-vehicle type combustion based model): Partial charges, Nonlinear charges, Capacitated charging stations, Multiple vehicle types and Multiple depots.

Partial and nonlinear charges can both be solved by using a similar discrete model as the one used in Van Kooten Niekerk et al. (2017), Zhang et al. (2021) or De Vos et al. (2024); all of these allow for handling of partial and nonlinear in the charging deadhead arcs, and battery degradation costs/ToU pricing could be included during vehicle task column generation. As all of these basically come for free with the choice of modeling, it would seem wise to focus on getting this working with a single depot / single vehicle type instance first.

Next, the issue of multiple vehicle types and depots. Both of these can be approached in roughly the same manner, however from conversations with Qbuzz it would seem that vehicle types should be done first as even the smallest instance provided to me now uses multiple vehicle types. Here, an approach similar to that found in Huisman et al. (2005) might be adaptable.

Lastly, the addition of capacitated charging stations is an issue. In both Zhang et al. (2021) and De Vos et al. (2024), this is handled however only the number of individual charging spots is constrained; total charging power available is also constrained in the Qbuzz context. I'm unsure of how to easily integrate this into a discretized model at time of writing without underestimating total power available, and doing so would probably require some pretty drastic changes to the model as a whole.

In conclusion, my personal guess is that the following list of priorities is the best going forward:

- Begin with implementing an adaptation of one of the previously mentioned models (probably a combination of Van Kooten Niekerk et al. (2017) and De Vos et al. (2024)) in order to get solutions to the E-VSP which incorporate partial charging, nonlinear charging and battery degredation.

- Add extraction of task blocks for crew scheduling, link with a set cover approach to crew scheduling.

- Expand model to include multiple vehicle types/depots.

- Expand model to include charging capacity (if time allows).

A rough planning has been outlined in Table 4. At time of writing I'm very unsure of the exact time required for each implementation step as this is my first time implementing most of the relevant techniques, so I've tried to leave some buffer room.

Date	Task description
Mid February	Finalize project proposal
Mid February - End March	Implementation of E-SDVSP
Begin April - End April	Link E-SDVSP with crew scheduling
Begin May - Mid May	Testing with E-SDVCSP, report findings, draft
Mid May - End May	Buffer
Start June - End June	Extend to include multi-vehicle/multi-depot constraints
Start July - Mid July	Testing with E-MDVCSP, report findings, draft
Mid July - End July	Finalize report, prepare final presentation
Start August	Finalize project
If time available	Charging station capacities

Table 4: Rough outline of planning

References

Barnhart, C. et al. (2003). "Airline Crew Scheduling". In: *Handbook of Transportation Science*. Ed. by R. W. Hall. Boston, MA: Springer US, pp. 517–560. ISBN: 978-0-306-48058-4. DOI: `10.1007/0-306-48058-1_14`. URL: `https://doi.org/10.1007/0-306-48058-1_14`.

Beasley, J. E. (1993). "Lagrangian relaxation". In: *Modern Heuristic Techniques for Combinatorial Problems*. USA: John Wiley & Sons, Inc., 243–303. ISBN: 0470220791.

Bodin, L. and B. Golden (1983). "Routing and scheduling of vehicles and crews: The state of the art". In: *Computers & Operations Research* 10.2. Routing and Scheduling of Vehicles and Crews. The State of the Art, pp. 63–211. ISSN: 0305-0548. DOI: `https://doi.org/10.1016/0305-0548(83)90030-8`. URL: `https://www.sciencedirect.com/science/article/pii/0305054883900308`.

Boland, N. et al. (2017). "The Continuous-Time Service Network Design Problem". In: *Operations Research* 65.5, pp. 1303–1321. DOI: `10.1287/opre.2017.1624`. eprint: `https://doi.org/10.1287/opre.2017.1624`. URL: `https://doi.org/10.1287/opre.2017.1624`.

Bunte, S. and N. Kliewer (2009). "An overview on vehicle scheduling models". In: *Public Transport* 1.4, pp. 299–317. ISSN: 1613-7159. DOI: `10.1007/s12469-010-0018-5`. URL: `https://doi.org/10.1007/s12469-010-0018-5`.

Ceder, A. and N. H. Wilson (1986). "Bus network design". In: *Transportation Research Part B: Methodological* 20.4, pp. 331–344. ISSN: 0191-2615. DOI: `https://doi.org/10.1016/0191-2615(86)90047-0`. URL: `https://www.sciencedirect.com/science/article/pii/0191261586900470`.

Cong, Y. et al. (2024). "Collaborative vehicle-crew scheduling for multiple routes with a mixed fleet of electric and fuel buses". In: *Energy* 298, p. 131400. ISSN: 0360-5442. DOI: `https://doi.org/10.1016/j.energy.2024.131400`. URL: `https://www.sciencedirect.com/science/article/pii/S0360544224011733`.

De Vos, M. H., R. N. van Lieshout, and T. Dollevoet (2024). "Electric Vehicle Scheduling in Public Transit with Capacitated Charging Stations". In: *Transportation Science* 58.2, pp. 279–294. DOI: `10.1287/trsc.2022.0253`. eprint: `https://doi.org/10.1287/trsc.2022.0253`. URL: `https://doi.org/10.1287/trsc.2022.0253`.

Deveci, M. and N. C. Demirel (2018). "A survey of the literature on airline crew scheduling". In: *Engineering Applications of Artificial Intelligence* 74, pp. 54–69. ISSN: 0952-1976. DOI: `https://doi.org/10.1016/j.engappai.2018.05.008`. URL: `https://www.sciencedirect.com/science/article/pii/S0952197618301234`.

European regulations (2018). *Regulation - 2018/1999 - EN - EUR-Lex — eur-lex.europa.eu*. `https://eur-lex.europa.eu/eli/reg/2018/1999/oj`. [Accessed 08-12-2024].

Even, S., A. Itai, and A. Shamir (1975). "On the complexity of time table and multi-commodity flow problems". In: *16th Annual Symposium on Foundations of Computer Science (sfcs 1975)*, pp. 184–193. DOI: `10.1109/SFCS.1975.21`.

Fischetti, M., S. Martello, and P. Toth (1989). "The Fixed Job Schedule Problem with Working-Time Constraints". In: *Operations Research* 37.3, pp. 395–403. URL: `http://www.jstor.org/stable/171059`.

Ge, L. et al. (2024). "Revisiting the richness of integrated vehicle and crew scheduling". In: *Public Transport* 16.3, pp. 775–801. ISSN: 1613-7159. DOI: `10.1007/s12469-022-00292-6`. URL: `https://doi.org/10.1007/s12469-022-00292-6`.

Haghani, A. and M. Banihashemi (2002). "Heuristic approaches for solving large-scale bus transit vehicle scheduling problem with route time constraints". In: *Transportation Research Part A: Policy and Practice* 36.4, pp. 309–333. ISSN: 0965-8564. DOI: `https://doi.org/10.1016/S0965-8564(01)00004-0`. URL: `https://www.sciencedirect.com/science/article/pii/S0965856401000040`.

Ham, A. and M.-J. Park (2021). "Electric Vehicle Route Optimization Under Time-of-Use Electricity Pricing". In: *IEEE Access* 9, pp. 37220–37228. DOI: `10.1109/ACCESS.2021.3063316`.

Huisman, D., R. Freling, and A. P. M. Wagelmans (2005). "Multiple-Depot Integrated Vehicle and Crew Scheduling". In: *Transportation Science* 39.4, pp. 491–502. DOI: `10.1287/trsc.1040.0104`. eprint: `https://doi.org/10.1287/trsc.1040.0104`. URL: `https://doi.org/10.1287/trsc.1040.0104`.

Ibarra-Rojas, O. et al. (2015). "Planning, operation, and control of bus transport systems: A literature review". In: *Transportation Research Part B: Methodological* 77, pp. 38–75. ISSN: 0191-2615. DOI: `https://doi.org/10.1016/j.trb.2015.03.002`. URL: `https://www.sciencedirect.com/science/article/pii/S0191261515000454`.

Li, J.-Q. (2014). "Transit Bus Scheduling with Limited Energy". In: *Transportation Science* 48.4, pp. 521–539. ISSN: 00411655, 15265447. URL: `http://www.jstor.org/stable/43666940` (visited on 12/20/2024).

Olsen, N. and N. Kliewer (2020). "Scheduling electric buses in public transport: Modeling of the charging process and analysis of assumptions". In: *Logistics Research* 13.1, pp. 1–17.

Parmentier, A., R. Martinelli, and T. Vidal (2023). "Electric Vehicle Fleets: Scalable Route and Recharge Scheduling Through Column Generation". In: *Transportation Science* 57.3, pp. 631–

646. DOI: 10.1287/trsc.2023.1199. eprint: https://doi.org/10.1287/trsc.2023.1199. URL: https://doi.org/10.1287/trsc.2023.1199.

Perumal, S. S., R. M. Lusby, and J. Larsen (2022). "Electric bus planning & scheduling: A review of related problems and methodologies". In: *European Journal of Operational Research* 301.2, pp. 395–413. ISSN: 0377-2217. DOI: https://doi.org/10.1016/j.ejor.2021.10.058. URL: https://www.sciencedirect.com/science/article/pii/S0377221721009140.

Perumal, S. S. et al. (2019). "A matheuristic for the driver scheduling problem with staff cars". In: *European Journal of Operational Research* 275.1, pp. 280–294. ISSN: 0377-2217. DOI: https://doi.org/10.1016/j.ejor.2018.11.011. URL: https://www.sciencedirect.com/science/article/pii/S0377221718309366.

Perumal, S. S. et al. (2021). "Solution approaches for integrated vehicle and crew scheduling with electric buses". In: *Computers & Operations Research* 132, p. 105268. ISSN: 0305-0548. DOI: https://doi.org/10.1016/j.cor.2021.105268. URL: https://www.sciencedirect.com/science/article/pii/S0305054821000605.

Qbuzz (2024). *Qbuzz — qbuzz.nl*. https://www.qbuzz.nl/gd/actueel/laatste-nieuws/ZBnQDREAACEAlo3q/busvervoer-90-uitstootvrij-vanaf-2024. [Accessed 08-12-2024].

RDW (2025). *Opendata*. URL: https://opendata.rdw.nl/.

Sassi, O. and A. Oulamara (Mar. 2014). "Electric Vehicle Scheduling and Optimal Charging Problem: Complexity, Exact and Heuristic Approaches". In: *International Journal of Production Research* 55. DOI: 10.1080/00207543.2016.1192695.

Shen, Y. and Y. Li (Nov. 2023). "Minimum Cost Flow-Based Integrated Model for Electric Vehicle and Crew Scheduling". In: *Journal of Advanced Transportation* 2023, pp. 1–23. DOI: 10.1155/2023/6658030.

Sistig, H. M. and D. U. Sauer (2023). "Metaheuristic for the integrated electric vehicle and crew scheduling problem". In: *Applied Energy* 339, p. 120915. ISSN: 0306-2619. DOI: https://doi.org/10.1016/j.apenergy.2023.120915. URL: https://www.sciencedirect.com/science/article/pii/S0306261923002799.

Stadnichuk, V. et al. (2024). *Integrated Optimization of Timetabling and Electric Vehicle Scheduling: A Case Study of Aachen, Germany*. https://optimization-online.org/2024/09/integrated-optimization-of-timetabling-and-electric-vehicle-scheduling-a-case-study-of-aachen-germany/. [Accessed 13-01-2025].

Van Kooten Niekerk, M. E., J. M. van den Akker, and J. A. Hoogeveen (2017). "Scheduling electric vehicles". In: *Public Transport* 9.1, pp. 155–176. ISSN: 1613-7159. DOI: 10.1007/s12469-017-0164-0. URL: https://doi.org/10.1007/s12469-017-0164-0.

Wang, J. et al. (2022). "Collaborative Optimization of Vehicle and Crew Scheduling for a Mixed Fleet with Electric and Conventional Buses". In: *Sustainability* 14.6. ISSN: 2071-1050. DOI: 10.3390/su14063627. URL: https://www.mdpi.com/2071-1050/14/6/3627.

Wen, M. et al. (2016). "An adaptive large neighborhood search heuristic for the Electric Vehicle Scheduling Problem". In: *Computers & Operations Research* 76, pp. 73–83. ISSN: 0305-0548. DOI: https://doi.org/10.1016/j.cor.2016.06.013. URL: https://www.sciencedirect.com/science/article/pii/S0305054816301460.

Zhang, L., S. Wang, and X. Qu (2021). "Optimal electric bus fleet scheduling considering battery degradation and non-linear charging profile". In: *Transportation Research Part E: Logistics and Transportation Review* 154, p. 102445. ISSN: 1366-5545. DOI: https://doi.org/10.1016/j.tre.2021.102445. URL: https://www.sciencedirect.com/science/article/pii/S136655452100209X.