CSC175 Practice Assignment 1 Spring 2019 Name Solutions

1) Let the universal set be $U = \{0,1,2,\ldots,16,17,18\}$, $A = \{x \in U | x = 2n, n \in \mathbb{Z}\}$, and $B = \{x \in U | x = 3n, n \in \mathbb{Z}\}$.

a) List the elements of A. $A = \{0,2,4,6,8,10,12,14,16,18\}$ (even integers)

b) List the elements of B. $B = \{0,3,6,9,12,15,18\}$ (multiples of three)

c) List the elements of A^c. $A^c = \{1,3,5,7,9,11,13,15,17\}$ (not in A)

d) List the elements of $A \cap B$. $A \cap B = \{0,6,12,18\}$ (intersection means overlap)

e) Write $A \cap B$ in set-builder notation. $A \cap B = \{x \in U | x = 6n, n \in \mathbb{Z}\}$ (multiples of 6)

f) List the elements of $(A \cup B)^c$. $(A \cup B)^c = \{1,5,7,11,13,17\}$ (not in the union)

g) List the elements of $A^c \cap B^c$. $A^c \cap B^c = \{1,5,7,11,13,17\}$ (notice equal to part f)

h) List the elements of $A - B$. $A - B = \{2,4,8,10,14,16\}$ (in A but not in B)

i) List the elements of $A \oplus B$. $A \oplus B = \{2,3,4,8,9,10,14,15,16\}$ (In one or the other, not both)

j) True or False: $B \subseteq A$. Why? False. For example, $3 \in B$ but $3 \notin A$.

k) True or False: $\{9,15\} \subseteq B$. Why? True, since 9 and 15 are elements of B.

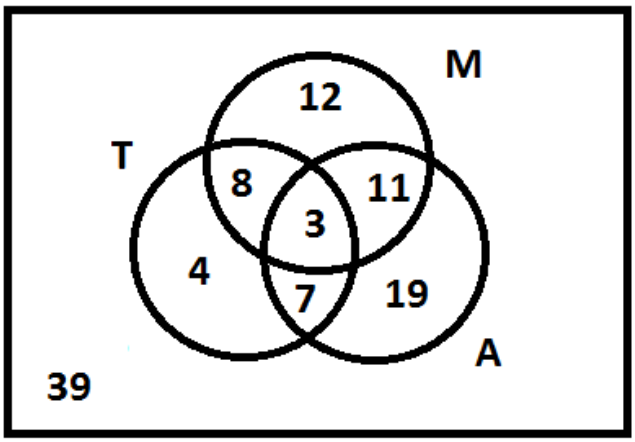
l) True or False: $\emptyset \in A$. Why? False since $\emptyset \subseteq A$, not an element of A.

m) True or False: $A \cap B \subseteq B$. Why? True, all elements of $A \cap B$ must be in B.

n) Find $|B|$. Why? Seven which is the number of elements in B.

o) List the elements of $D = \{x \in A \mid x < 8\}$ $\{0,2,4,6\}$ (in A and less than 8)

2) At Sunshine Academy, there are 103 seniors. In an activities survey, they were asked what activities they participated in during their high school experience. Here are the results: 40 in athletics, 34 in music, 22 in theatre, 11 in music and theatre, 14 in music and athletics, 10 in theatre and athletics, and 3 were in all three activities. Create a Venn Diagram (create your own or edit the attachment in Blackboard using Paint) and answer the following questions.



a) How many seniors participated only in theatre?
4 (no overlap)

b) How many seniors participated in athletics but not music?
19+7=26 (In A but not in M)

c) How many seniors participated in theatre and music but not athletics?
8 (In both T and M but not in A)

d) How many seniors participated in none of the activities?
39 (Total seniors minus sum of those in activities=103-64)

3) Let $V = \{a, i\}$ and $C = \{s, t, n\}$.

a) List $V \times C$ $\{(a,s),(a,t),(a,n),(i,s),(i,t),(i,n)\}$

b) Find $|V \times C|$ 6 ordered pairs

c) Is $(s,a) \in V \times C$? Why or why not? No, order is important! $s \notin V$ and $a \notin C$

d) Find the power set of V. (i.e. $\mathcal{P}(\{a,i\})$) $\{\emptyset,\{a\},\{i\},\{a,i\}\}$ (Power set always includes the empty set)

e) Find $|C^2|$ $|C^2| = |C \times C| = |C| \cdot |C| = 3 * 3 = 9$

4) Make the following conversions. Show work for credit.

a) Convert the hexadecimal number 1A7D to an integer. 1*4096+10*256+7*16+13*1 = 6781

b) Convert the binary number 10110110 to an integer. 128+32+16+4+2 = 182

c) Convert 587 to hexadecimal. 2*256+4*16+11 = 587 so we obtain the hexadecimal number $24B$

d) Use the algorithm (Ex 1.4.1) on page 24 of the textbook to find the binary representation of 587. (Show each step of the algorithm)

587 = 2*293 + 1 List = 1
293 = 2*146 + 1 List = 11
146 = 2*73 + 0 List = 011
73 = 2*36 + 1 List = 1011
36 = 2*18 + 0 List = 01011
18 = 2*9 + 0 List = 001011
9 = 2*4 + 1 List = 1001011
4 = 2*2 + 0 List = 01001011
2 = 2*1 + 0 List = 001001011
1 = 2*0 + 1 List = 1001001011 Answer = 1001001011

Check: 1001001011 means 512+64+8+2+1 = 587

5) Calculate each expression (show work)

a) $\sum_{i=0}^{3}(i+2)$ $(0+2)+(1+2)+(2+2)+(3+2)=14$

b) $\sum_{k=0}^{n}(3^k-2)$ for $n=0,1,2$

$n=0$: $3^0-2=1-2=-1$
$n=1$: $3^0-2+3^1-2=0$ Answer: $-1, 0, 7$
$n=2$: $3^0-2+3^1-2+3^2-2=7$

c) $\bigcup_{i=1}^{4}\{x \in \mathbb{Q} \mid i-1 < x \leq i\}$

$\{x \in \mathbb{Q} \mid 0 < x \leq 1\} \cup \{x \in \mathbb{Q} \mid 1 < x \leq 2\} \cup \{x \in \mathbb{Q} \mid 2 < x \leq 3\} \cup \{x \in \mathbb{Q} \mid 3 < x \leq 4\}$

Answer: $\{x \in \mathbb{Q} \mid 0 < x \leq 4\}$

d) $\bigcap_{i=1}^{4}\{x \in \mathbb{Q} \mid i-1 < x \leq i\}$

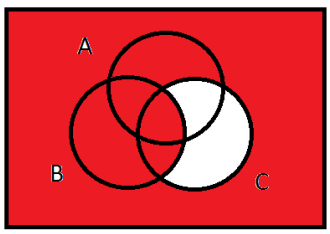
$\{x \in \mathbb{Q} \mid 0 < x \leq 1\} \cap \{x \in \mathbb{Q} \mid 1 < x \leq 2\} \cap \{x \in \mathbb{Q} \mid 2 < x \leq 3\} \cap \{x \in \mathbb{Q} \mid 3 < x \leq 4\}$

Answer: $\emptyset$ (There is no overlap)

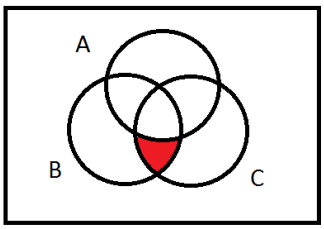
e) $\prod_{k=2}^{4}(3k-1)$ $(3*2-1)*(3*3-1)*(3*4-1)=5*8*11=440$

6) Edit the attachment in Blackboard using Paint to shade each region given in set notation:

a) $B \cup C^c$



b) $(B-A) \cap C$



7) Given the figure to the right, express the area in set notation.

Answer: $(B \cup C)^c$ or $B^c \cap C^c$