CSC175 Practice Assignment 7 Spring 2019 Name Solution

1) Use the following algorithms to sort the following list. Be sure to show and highlight each swap.

12 9 17 23 13 10

a) Bubble sort

12 9 17 23 13 10
9 12 17 **23 13** 10
9 12 17 13 **23 10**
9 12 **17 13** 10 23
9 12 13 **17 10** 23
9 12 **13 10** 17 23
9 **12 10** 13 17 23
9 10 12 13 17 23

b) Selection sort

12 9 17 23 13 10
9 | **12** 17 23 13 **10**
9 10 | **17** 23 13 **12**
9 10 12 | **23 13** 17
9 10 12 13 | **23 17**
9 10 12 13 17 | 23
9 10 12 13 17 23

c) Insertion sort

12 | **9** 17 23 13 10
9 12 | **17** 23 13 10
9 12 17 | **23** 13 10
9 12 17 23 | **13** 10
9 12 13 17 23 | **10**
9 10 12 13 17 23

d) Merge sort

12 9 17 23 13 10
12 9 17 23 13 10
12 9 17 23 13 10
9 12 17 13 23 10
9 12 17 10 13 23
9 10 12 13 17 23

2) Use the algorithm to find the binary number for the decimal number 1453.

1453 = 2*726 + 1	**List: 1**
726 = 2*363 + 0	**List: 01**
363 = 2*181 + 1	**List: 101**
181 = 2*90 + 1	**List: 1101**
90 = 2*45 + 0	**List: 01101**
45 = 2*22 + 1	**List: 101101**
22 = 2*11 + 0	**List: 0101101**
11 = 2*5 + 1	**List: 10101101**
5 = 2*2 + 1	**List: 110101101**
2 = 2*1 + 0	**List: 0110101101**
1 = 2*0 + 1	**List: 10110101101**

3) Given the sets $U = \{0,1,2,3,4,5,6,7,8,9\}, A = \{0,3,6,9\}, and\ B = \{2,3,4,5,6\}$, find:

a) $A \oplus B$ $\{\mathbf{0,2,4,5,9}\}$

b) $(A \cup B)^c$ $\{\mathbf{1,7,8}\}$

c) $B - A$ $\{\mathbf{2,4,5}\}$

d) Set builder notation for A. $\{\boldsymbol{x \in U | \ x = 3n, n \in \mathbb{N}}\}$

e) $|A \times B|$ $\mathbf{4 \cdot 5 = 20}$

4) Use the binomial theorem to expand $(5x-3y)^6$. Show all steps as shown on the class PowerPoint.

$$\binom{6}{0}(5x)^6(-3y)^0+\binom{6}{1}(5x)^5(-3y)^1+\binom{6}{2}(5x)^4(-3y)^2+\binom{6}{3}(5x)^3(-3y)^3+\binom{6}{4}(5x)^2(-3y)^4$$
$$+\binom{6}{5}(5x)^1(-3y)^5+\binom{6}{6}(5x)^0(-3y)^6$$

$$1\cdot 5^6x^6(-3)^0y^0+6\cdot 5^5x^5(-3)^1y^1+15\cdot 5^4x^4(-3)^2y^2+20\cdot 5^3x^3(-3)^3y^3+15\cdot 5^2x^2(-3)^4y^4$$
$$+6\cdot 5^1x^1(-3)^5y^5+1\cdot 5^0x^0(-3)^6y^6$$
$$15625x^6-56250x^5y+84375x^4y^2-67500x^3y^3+30375x^2y^4-7290xy^5+729y^6$$

5) Create a truth table to show $\sim(p\vee\sim q)\wedge\sim q$ is a contradiction.

p	q	$\sim q$	$p\vee\sim q$	$\sim(p\vee\sim q)$	$\sim(p\vee\sim q)\wedge\sim q$
T	T	F	T	F	F
T	F	T	T	F	F
F	T	F	F	T	F
F	F	T	T	F	F

Since the entire column of $\sim(p\vee\sim q)\wedge\sim q$ is false, the compound proposition is a contradiction.

6) Name the law used in each step.

a) $\sim(p\vee\sim q)\wedge\sim q\Leftrightarrow(\sim p\wedge\sim\sim q)\wedge\sim q$ **DeMorgan's Law**

b) $(\sim p\wedge\sim\sim q)\wedge\sim q\Leftrightarrow(\sim p\wedge q)\wedge\sim q$ **Double Negation**

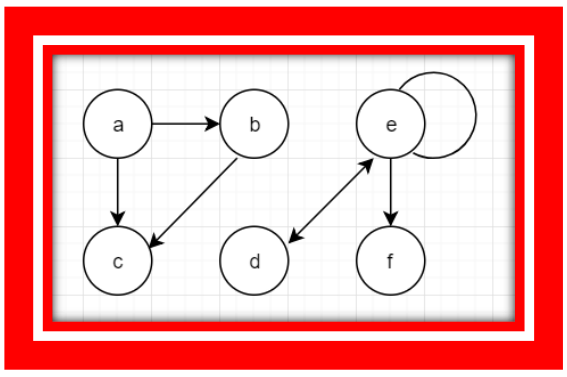
c) $(\sim p\wedge q)\wedge\sim q\Leftrightarrow\sim p\wedge(q\wedge\sim q)$ **Associative**

d) $\sim p\wedge(q\wedge\sim q)\Leftrightarrow\sim p\wedge c$ **Negation Law**

e) $\sim p\wedge c\Leftrightarrow c$ **Null Law**

Note: This is a proof for the truth table in #5. You will be writing these proofs yourself in MAT220. ☺

7) a) Use technology to create a digraph for the relation $S=\{(a,b),(d,e),(a,c),(e,e),(e,f),(b,c),(e,d)$.



b) Is S reflexive, symmetric, antisymmetric, and/or transitive? Why or why not?

Not reflexive, only loop is at e. Not symmetric because only one double arrow. Not antisymmetric because not all double arrows. Not transitive because (d,e) and (e,f) would become (d,f) which is not an element of S.

c) Create a table for the indegree and outdegree of each vertex.

Vertex	Indegree	Outdegree
a	0	2
b	1	1
c	2	0
d	1	1
e	2	3
f	1	0

8) Use the Euclidean Algorithm to find GCD(810, 205) (show each step of the algorithm)

810 = 3(205) + 195
205 = 1(195) + 10
195 = 19(10) + 5
10 = 2(5) + 0
GCD(810, 205) = 5

9) Give the Big O runtime for the JustDoIt algorithm. Explain your answer.

```
Begin JustDoIt (List of numbers A)

        Enter N (length of A)
        For i = 1 to N
        If A[i]<100 and A[i]>9
           Print  A[i]
        End If
        End For
        For j = 1 to N-1
            For k=2 to N
            If  A[j]<A[k]
            Print A[j]
            End If
            End For
        End For

    End JustDoIt
```

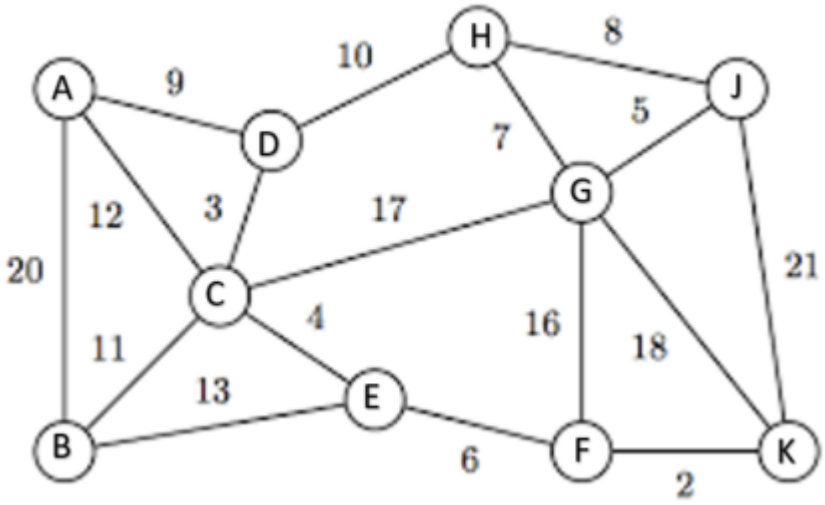
For the first for loop, we are make two comparisons for each element, which gives us 2N comparisons. The next two for loops are nested. We have one comparison but there are N-1 elements for j and N-1 elements for k. Thus, since they are nested, we multiply giving us: (N-1)*(N-1) = N^2-2N+1 which is asymptotic to N^2. Thus we have N^2+2N. Dropping the constants and taking the dominate term, we obtain a runtime O(N^2).

10) Explain what is happening in the JustDoIt algorithm in #9.

The first time we go through the list and print any numbers greater than 9 and less than 100 (two digit numbers if the list is all integers). The second part checks if each number in the list is less than any of the numbers that

follow **it in the list. If it is, it is printed. Thus, if a number is printed four times, that means that four numbers that follow it are bigger than the printed number.**

11) Use Prim's algorithm to find the minimum spanning tree starting at vertex B. List the edges in the correct order as found by the algorithm and give the sum.



$$\{BC, CD, CE, EF, FK, AD, DH, HG, GJ\}$$
$$SUM = 11 + 3 + 4 + 6 + 2 + 9 + 10 + 7 + 5 = 57$$