

1) Let  $A = \{2,3,4,5\}$ . Let  $R$  be a relation on  $A$  defined by  $(a, b) \in R$  if and only if  $a < b$ . Are the following elements of  $R$ ? Why or why not?

- a)  $(3,3)$  No, 3 is not less than 3
- b)  $(2,5)$  Yes,  $2 < 5$  and 2 and 5 are elements of  $A$
- c)  $(4,6)$  No, 6 is not an element of  $A$  and  $R$  is defined on  $A$
- d)  $(4,3)$  No, 4 is not less than 3 ( $R$  is antisymmetric)

2) Let  $A = \{2,3,4,5,6\}$  and  $B = \{14, 15, 16, 17\}$ . Let  $R$  be a relation from  $A$  into  $B$  defined by  $(a, b) \in R$  if and only if  $a|b$ . Find all elements of  $R$ .

$$R = \{(2,14), (2,16), (3,15), (4,16), (5,15)\}$$

3) Let  $B = \{14, 15, 16, 17\}$  and  $A = \{2,3,4,5,6\}$ . Let  $S$  be a relation from  $B$  into  $A$  defined by  $(b, a) \in S$  if and only if  $b$  is a multiple of  $a$ . Find all elements of  $S$ .

$$S = \{(14,2), (15,3), (15,5), (16,2), (16,4)\}$$

4) Given the relations  $R$  and  $S$  in numbers 2 and 3, find:

- a)  $RS$ .  $RS = \{(2,2), (2,4), (3,3), (3,5), (4,4), (4,2), (5,5), (5,3)\}$
- b)  $SR$ .  $SR = \{(14,14), (14,16), (15,15), (16,16), (16,14)\}$

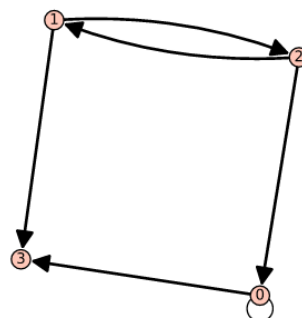
5) Let  $T$  be the relation on a set  $C = \{1,2,3,4\}$  whose digraph is given to the right.

- a) List the elements of  $T$ .  $T = \{(0,0), (0,3), (1,2), (1,3), (2,1), (2,0)\}$

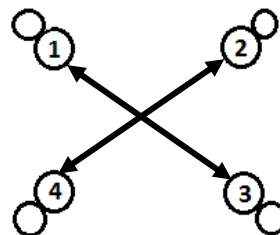
Note: parentheses since directed edges (order matters)

- b) List the outdegree and indegree of each vertex of the digraph in a table.

Vertex	Outdegree	Indegree
0	2	2
1	2	1
2	2	1
3	0	2



6) Let  $A = \{1,2,3,4\}$ . Define the relation  $R$  on  $A$  by  $aRb$  if and only if  $b - a$  is even. (Remember, 0 and -2 are also even numbers!) Use technology to create a digraph.



7) Consider the relation  $R$  on  $A$  in number 6.

a) Is  $R$  reflexive? Why or why not? **Yes,  $aRa, a \in A$**

b) Is  $R$  antisymmetric? Why or why not? **No, for example  $2R4$  and  $4R2$  but  $2 \neq 4$**

c) Is  $R$  symmetric? Why or why not? **Yes, if  $aRb$ , then  $bRa$ , for  $a, b \in A$**

d) Is  $R$  transitive? Why or why not? **Yes,  $aRb$  &  $bRc \rightarrow aRc$  is true for all ordered pairs in  $R$**

e) Is  $R$  an equivalence relation? Why or why not? **Yes,  $R$  is reflexive, symmetric, and transitive**

f) Is  $R$  a partial ordering? Why or why not? **No,  $R$  is not antisymmetric**

8) Let  $A = \{2,3,4,5,6,7,8\}$ . List the elements of the relation  $a \equiv_3 a$  (congruence mod 3).

**$\{(2,5), (5,2), (5,8), (8,5), (2,8), (8,2), (2,2), (5,5), (8,8), (3,6), (6,3), (3,3), (6,6), (4,7), (7,4), (4,4), (7,7)\}$**

9) Consider the undirected graph to the right.

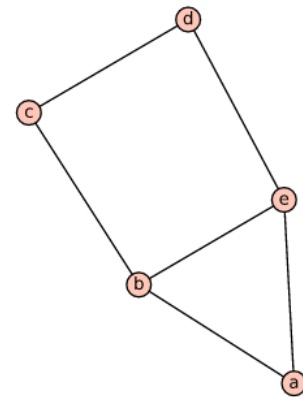
a) List the vertices.  **$V = \{a, b, c, d, e\}$**

b) List the edges.  **$E = \{\{a, b\}, \{a, e\}, \{b, e\}, \{b, c\}, \{c, d\}, \{d, e\}\}$**

**Note: curly braces since undirected edges (order doesn't matter)**

c) List the degree of each vertex in a table.

Vertex	Degree
a	2
b	3
c	2
d	2
e	3



d) Is it a simple graph? Why or why not? **Yes, no loops or multiedges**