## CSC 591/791 and ECE 591: Quantum Communications and Network
## Course Project - Bell Inequality Test
## Due Date: 11:59pm, April 18, 2025

Please consider the following quantum communication setting. The entanglement source (ES) generates entangled photons, which may not be perfect EPR pairs, from a Poisson statistics with average rate of 15,000 entanglements per second. ES can be precisely controlled to generate ONE of the few states below:

- Maximally entangled EPR pairs:

$$\left|\Phi^+\right\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)$$

- Maximally entangled EPR pairs with mixture:

$$\rho = 0.7\left|\Phi^+\right\rangle\left\langle\Phi^+\right| + 0.15\,|HV\rangle\langle HV| + 0.15\,|VH\rangle\langle VH|$$

- Non-maximally entangled pair:

$$|\phi\rangle = \frac{1}{\sqrt{5}}|HH\rangle + \frac{2}{\sqrt{5}}|VV\rangle$$

- Non-maximally entangled pair with mixture:

$$\rho' = 0.7\,|\phi\rangle\langle\phi| + 0.15\,|HV\rangle\langle HV| + 0.15\,|VH\rangle\langle VH|$$

where the $H$ and $V$ represent horizontal and vertical polarization, respectively.

The generated photon pairs are sent to Alice and Bob, each of which holds one part of a pair. Alice and Bob employ a single-photon SPAD detector as we learned from the paper Nature'18. The detector's specifications are: $10\%$ efficiency, $1,000$ Hz dark count rate, and $4$ microseconds dead time. Alice and Bob can analyze photons in any basis such as HH, VV, HV, DA (diagonal $45°$ and anti-diagonal $225°$), etc.

The system runs for $30$ seconds. That is to say, over $30$ seconds, ES keeps generating entanglement of a specific type while Alice and Bob keep detecting photons. Afterwards, Alice and Bob send their results to a coincidence monitor (CM) which adopts a coincidence window of 1 nanosecond. By analyzing the coincident events, CM calculates the total count rate, coincidence rate, fidelity, and Bell inequality (BI) — the $S$ term in our slides (recall that $S \leq 2$ for locality while $2 < S \leq 2\sqrt{2}$ for non-locality).

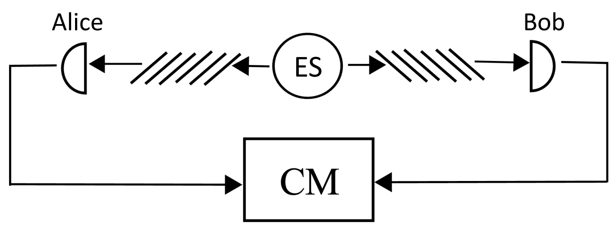


Fig. 1. Setup for the Bell inequality test

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The following questions will be graded for all students.

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Please write a program using MATLAB or Python (or whatever language you are comfortable with) to simulate the above well-known Bell Inequality Test. A very important hint is to select the appropriate analyzing basis for Alice and Bob. (Please refer to the paper Nature'18) In your simulation, please calculate the above four metrics for each one of the four states — $|\Phi^+\rangle$, $\rho$, $|\phi\rangle$, $\rho'$. Then, you will make a comparison about how different entanglement states affects these metrics.

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The following questions will be graded only for CSC 791 students.

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Can you find a state, which violates the Bell inequality with the minimum possible $S$ value that you can find? You don't need to prove your state is the minimum, but you need to show how you try to find such a state. As a hint, you may either use your simulator for "trial-and-error" starting from the above states, or resort to some theory in the literature to more quickly identify good "tunable" states. A Werner state may be a good point to get started if you take the second path.