

# Better Tensor Core Support in TVM with CUTLASS

Leyuan Wang, Dec. 16, 2021

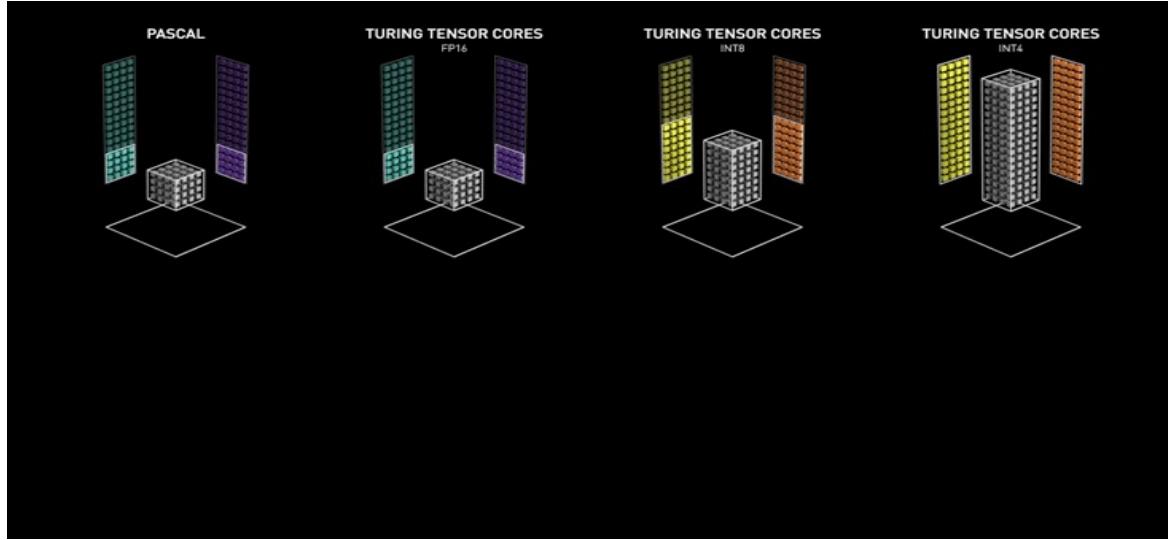
# Outline

- I. Motivation
- II. Why CUTLASS
- III. Integration: BYOC CUTLASS
- IV. Some performance numbers



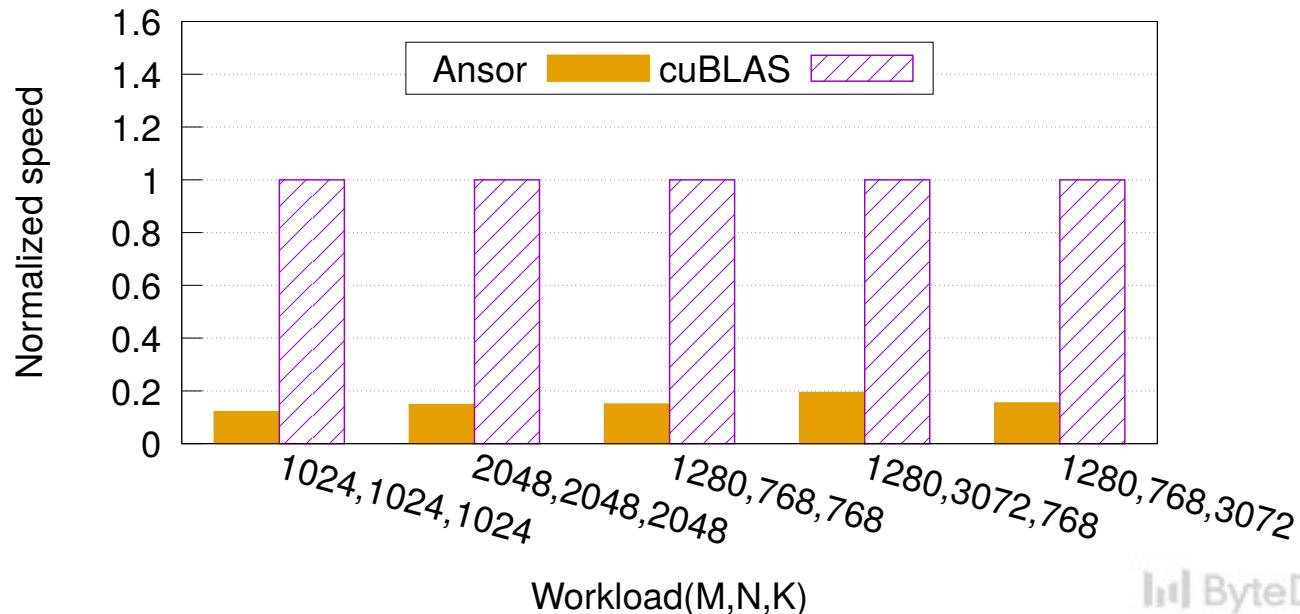
# Motivation

- Nvidia GPUs are mostly used in machine learning cloud computing
- Tensor Core acceleration for mixed precision (e.g. fp16) deep learning model inference is widely used.



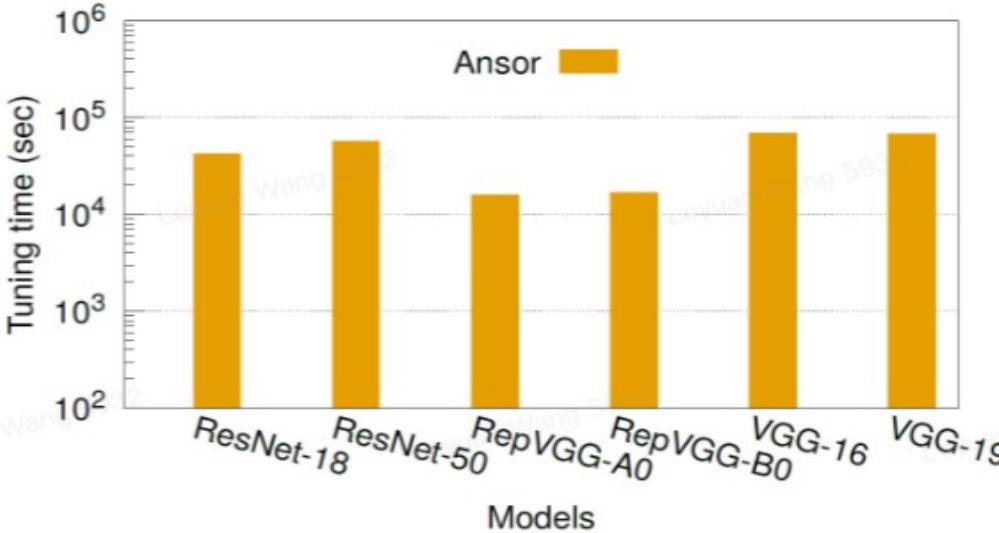
# Motivation

- Currently there's performance gap between TVM and vendor libraries on compute intensive operators (e.g. gemm, conv, etc.) in deep learning workloads using tensor cores on NVIDIA GPUs



# Motivation

- Long tuning time for searching best-performance compute intensive operators in current autotvm.
- A tophub data base can't solve the whole problem because of dynamic workloads and cost to maintain.



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# Why choosing CUTLASS

- An open-source CUDA C++ templated header library, containing many modularized and customizable components.
- For single compute intensive kernels, CUTLASS can compete with SOTA close-source vendor libraries on NVIDIA GPUs.
- Support epilogue fusion, persistent kernel fusion, etc. which is a beneficial feature compared with other vendor libraries.
- Support different functionalities such as GEMM and conv, etc..
- Support different data type tensor core instructions (b1, int4, int8, fp16, bf16, fp32, tf32, fp64, complex, quaternion).
- Support different generations of hardware (Volta, Turing, Ampere, etc.).
- Support different versions of CUDA compilers.

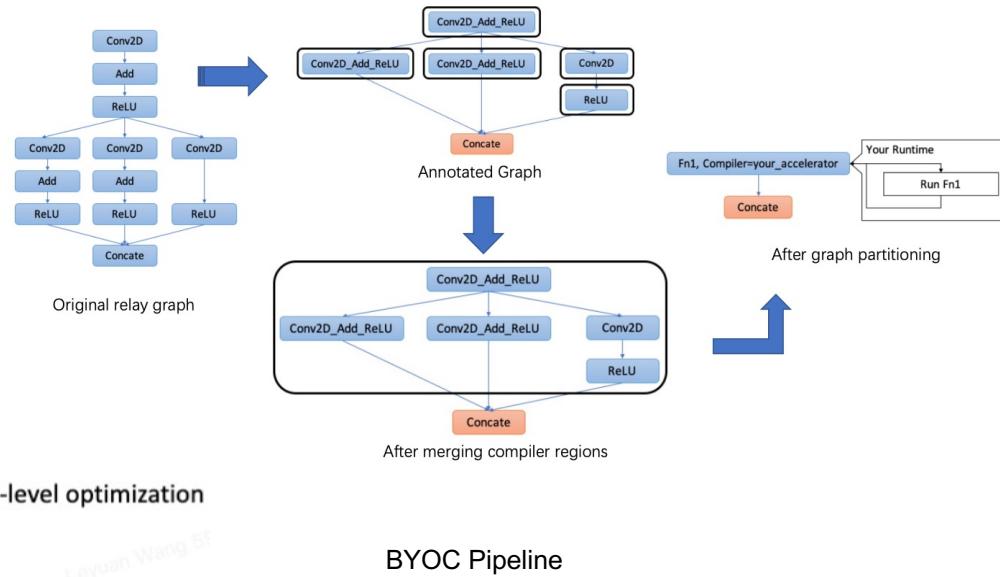
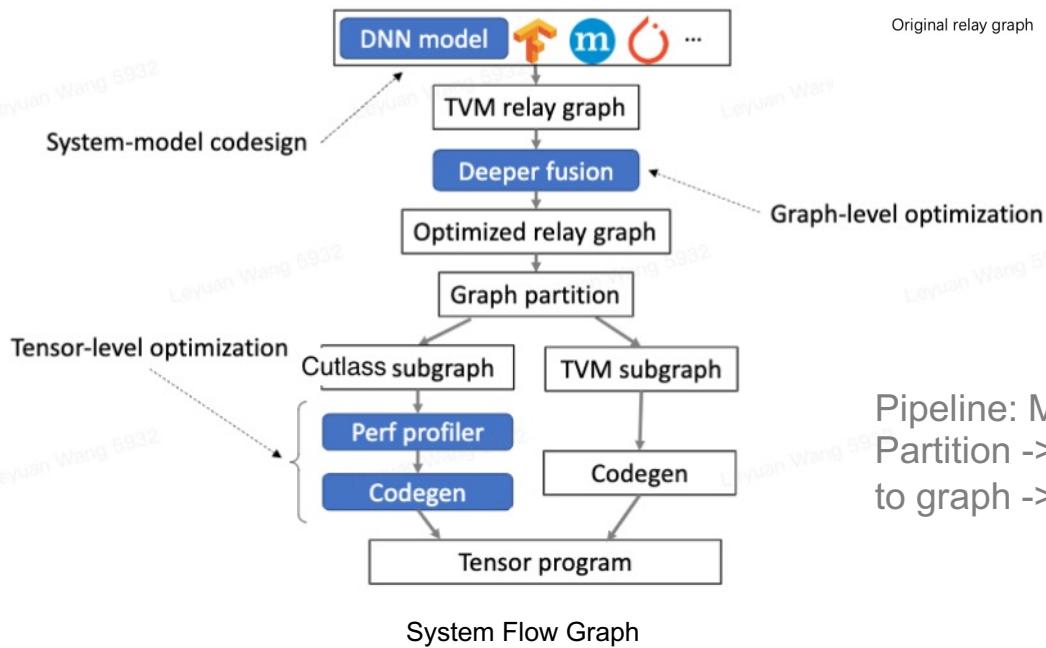
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# Integration method

## BYOC (Bring Your Own Codegen)



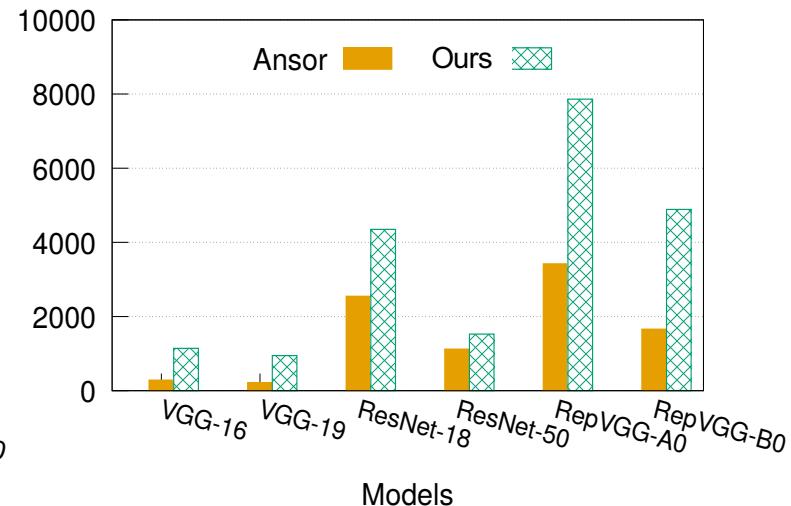
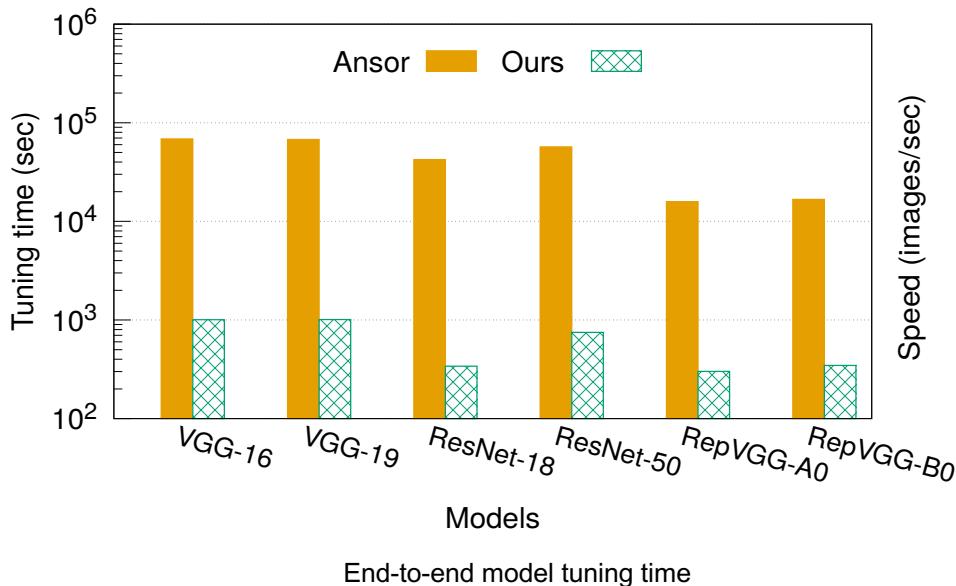
Pipeline: MergeComposite -> Annotate Target -> Graph Partition -> Tuner -> Codegen -> Compile -> Write back to graph -> Runtime

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# Less Tuning Time, Faster Speed

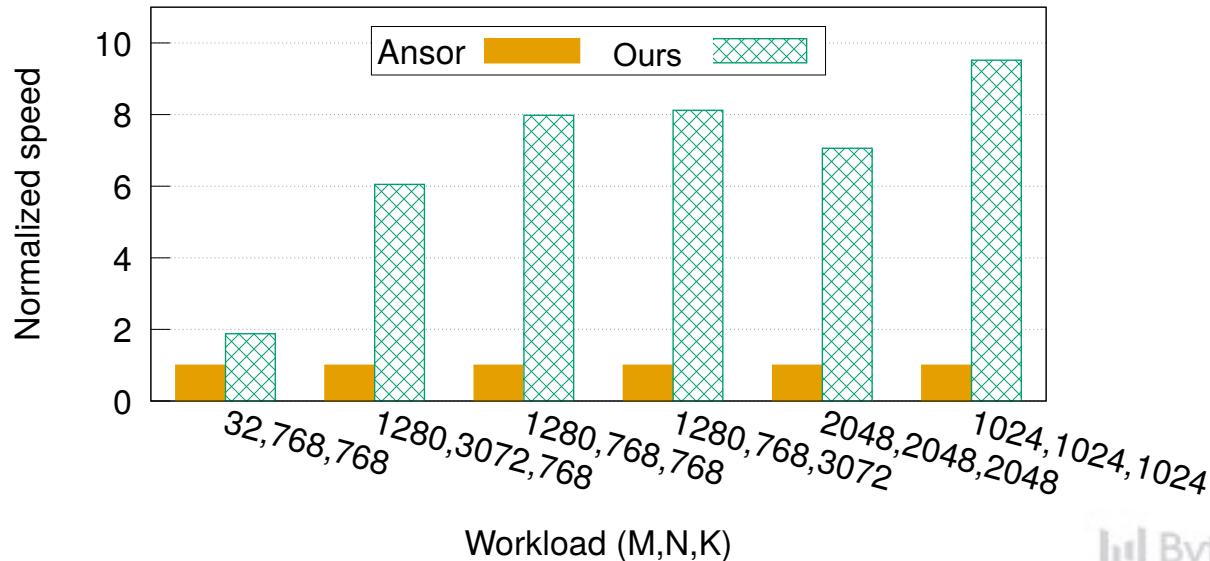
- Baseline: Ansor tune w/ trials =  $900 * \# \text{ of tasks}$
- We finish tuning within 20 minutes for all models while Ansor takes 12 hours on average



End-to-end model inference speed

# GEMM Performance

- Data type: FP16
- Baseline: Tune each operator for 2000 trials using Ansor
- GEMM workloads: square matrix + Bert GEMMS with bs 32, seq\_length 40
- Hardware: NVIDIA T4 GPU



# CUTLASS integration is merged into TVM

apache/tvm

## [BYOC] CUTLASS integration (#9261)

2k lines changed +1864 -6



Laurawly committed October 29, 2021 541f9f2



[BYOC] CUTLASS integration (#9261) · apache/tvm@541f9f2



THANKS.

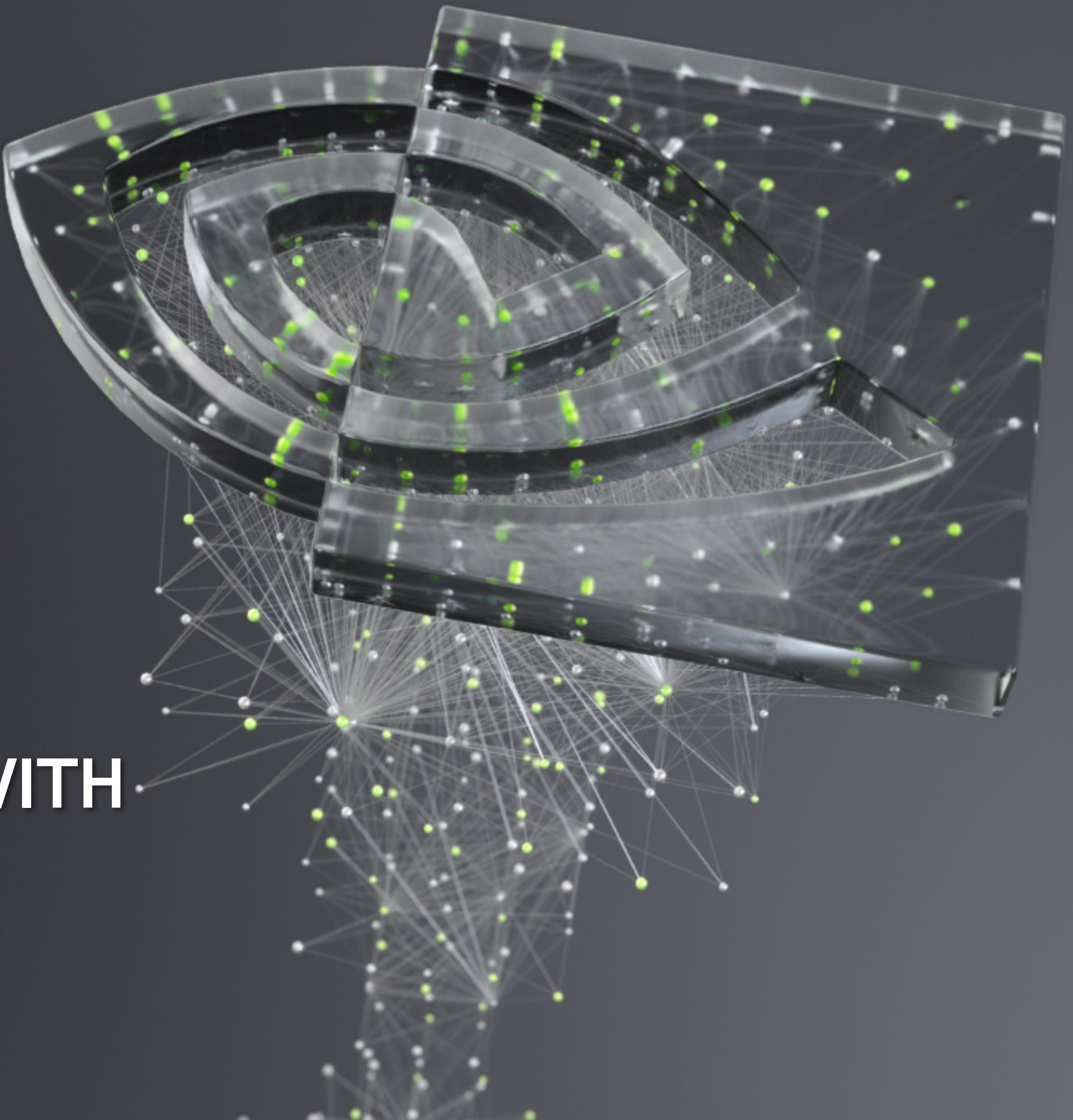




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# CUTLASS + TVM: FUSION WITH GEMM AND CONVOLUTION

Andrew Kerr, December 16, 2021



# ACKNOWLEDGEMENTS

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## CUTLASS GitHub Community

19.5K clones, 1.4K stars, and many active users

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## CUTLASS Team

Andrew Kerr, Haicheng Wu, Manish Gupta, Dustyn Blasig, Duane Merrill, Pradeep Ramani, Cris Cecka, Vijay Thakkar

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## Contributors

Timothy Costa, Naila Farooqui, Markus Hohnerbach, Alan Kaatz, Wei Liu, Piotr Majcher, Dhiraj Reddy Nallapa, Mathew Nicely, Kyrylo Perelygin, Aniket Shivam, Paul Springer, Pawel Tabaszewski, Chinmay Talegaonkar, John Tran, Jin Wang, Yang Xu, Scott Yokim

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## Acknowledgements

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## AGENDA

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### Overview

NVIDIA Ampere Architecture and CUTLASS

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### Abstractions for Tensor Cores

Accelerated matrix operations

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### Efficient Epilogues

Data exchange for efficient access to Global Memory

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### Epilogue Fusion Design Patterns

Opportunities for fusing custom operations with GEMM and CONV

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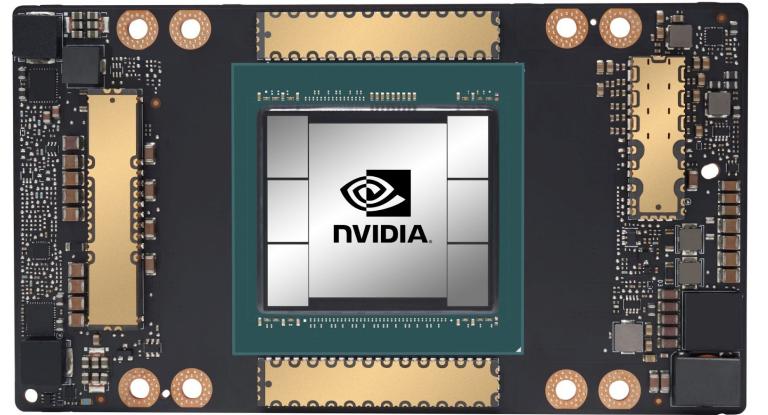
# OVERVIEW

# NVIDIA AMPERE ARCHITECTURE

## NVIDIA A100

### New and Faster Tensor Core Operations

- Floating-point Tensor Core operations **8x** and **16x** faster than F32 CUDA Cores
- Integer Tensor Core operations **32x** and **64x** faster than F32 CUDA Cores
- New IEEE double-precision Tensor Cores **2x** faster than F64 CUDA Cores



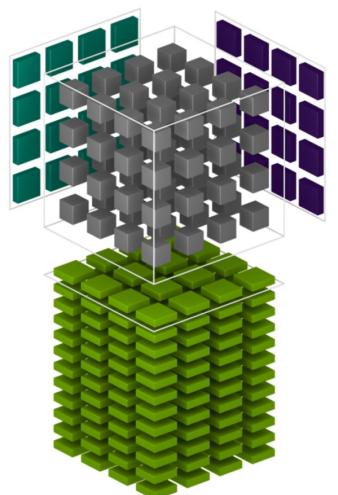
### Additional Data Types and Mode

- Bfloat16, double, Tensor Float 32

### Asynchronous copy

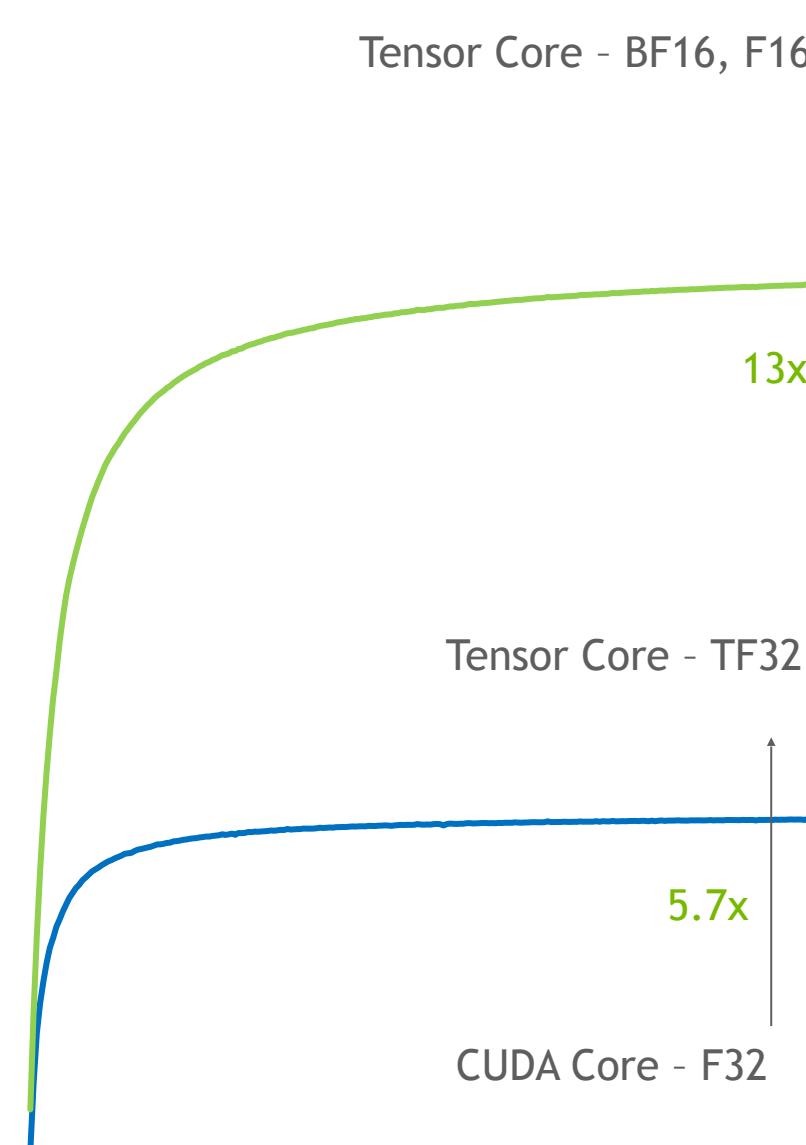
- Copy directly into shared memory - deep software pipelines

Many additional new features - see “Inside NVIDIA Ampere Architecture”

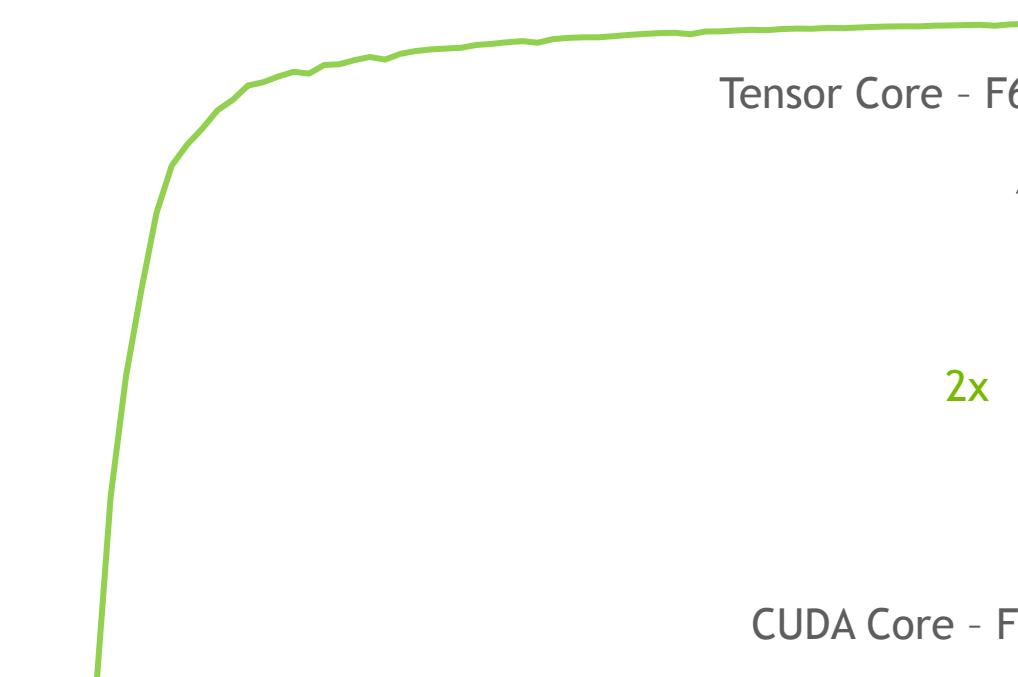


# CUTLASS PERFORMANCE ON NVIDIA AMPERE ARCHITECTURE

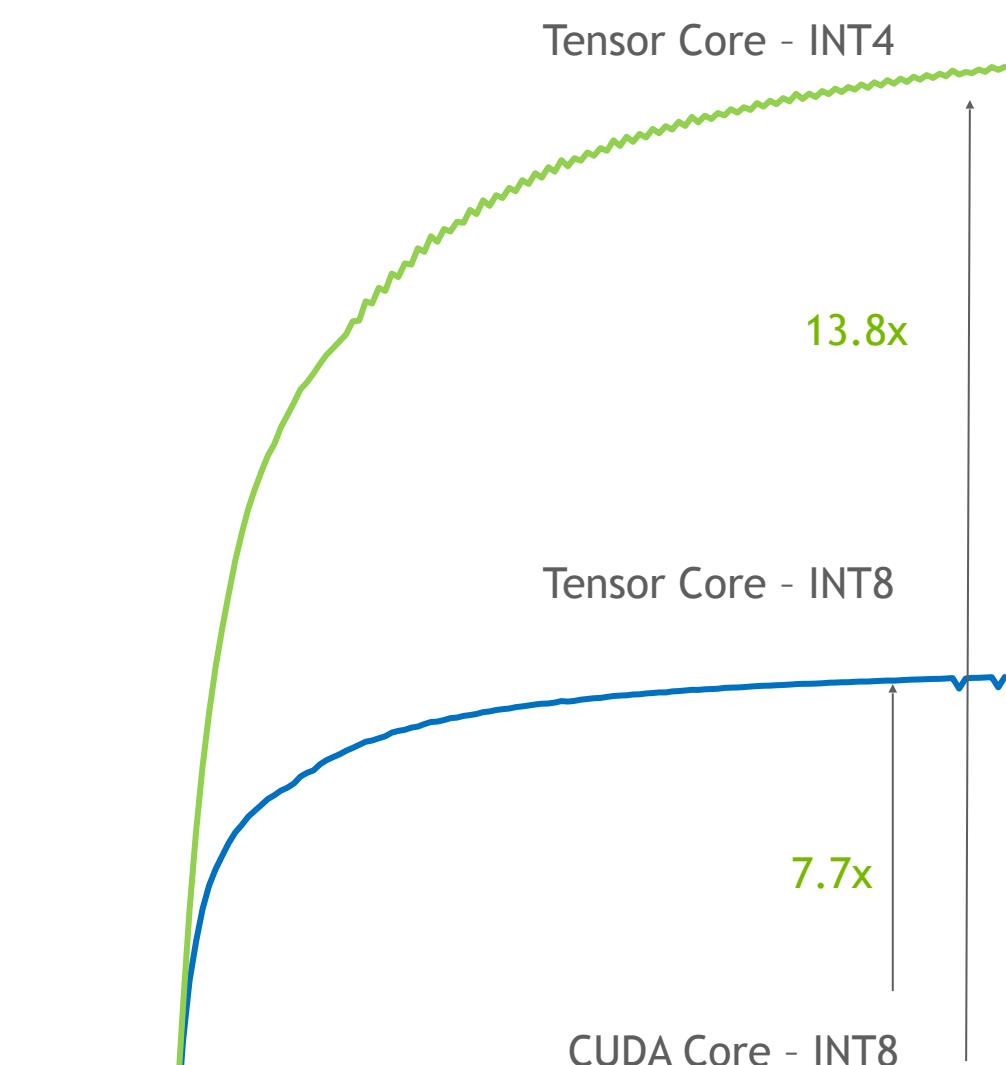
Mixed Precision Floating Point



Double Precision Floating Point



Mixed Precision Integer



m=3456, n=4096

# CUTLASS

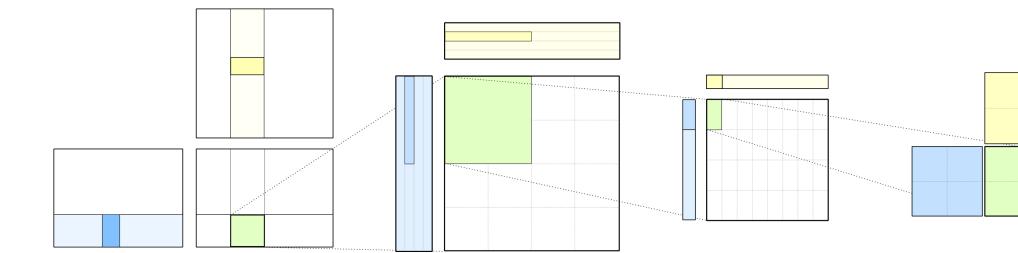
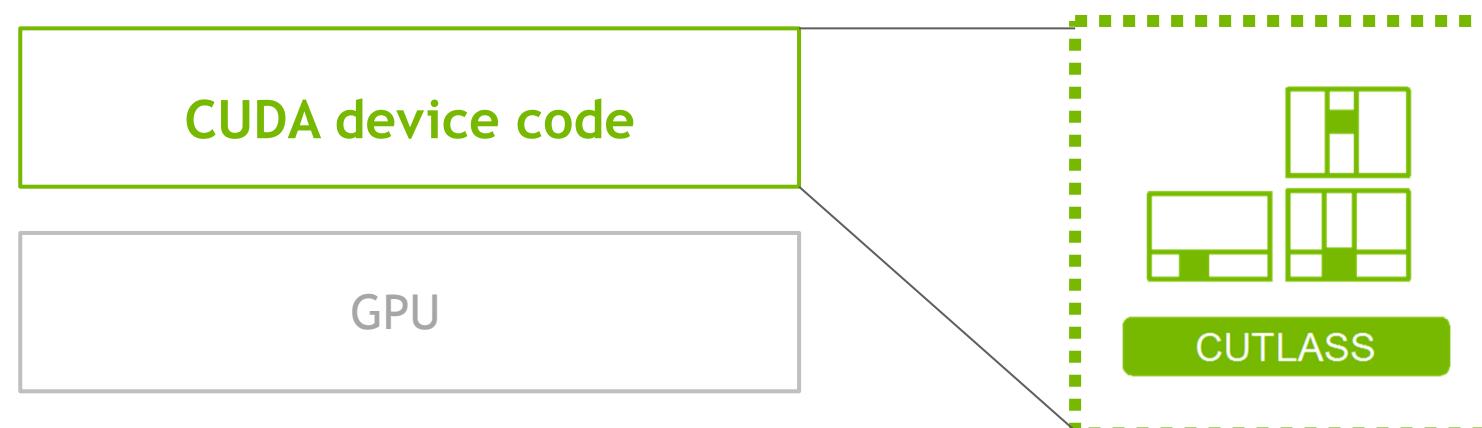
CUDA C++ Template Library for Deep Learning and High Performance Computing

**CUTLASS:** optimal CUDA C++ templates at all scopes and scales

Device	{ GEMM, Convolution, Reductions } x { all data types } x { SIMT, Tensor Cores } x { all architectures }
Kernel	GEMM, Batched GEMM, Convolution, Reduction, Fused output operations, Fused input operations
Threadblock	Pipelined Matrix Multiply, Epilogue, Collective access to tensors, Convolution matrix access
Warp	Tensor Core Multiply-Add operations, Efficient access to permuted tensor layouts
Thread	Numeric conversion, <functional> operators on arrays, complex<T>, fast math algorithms

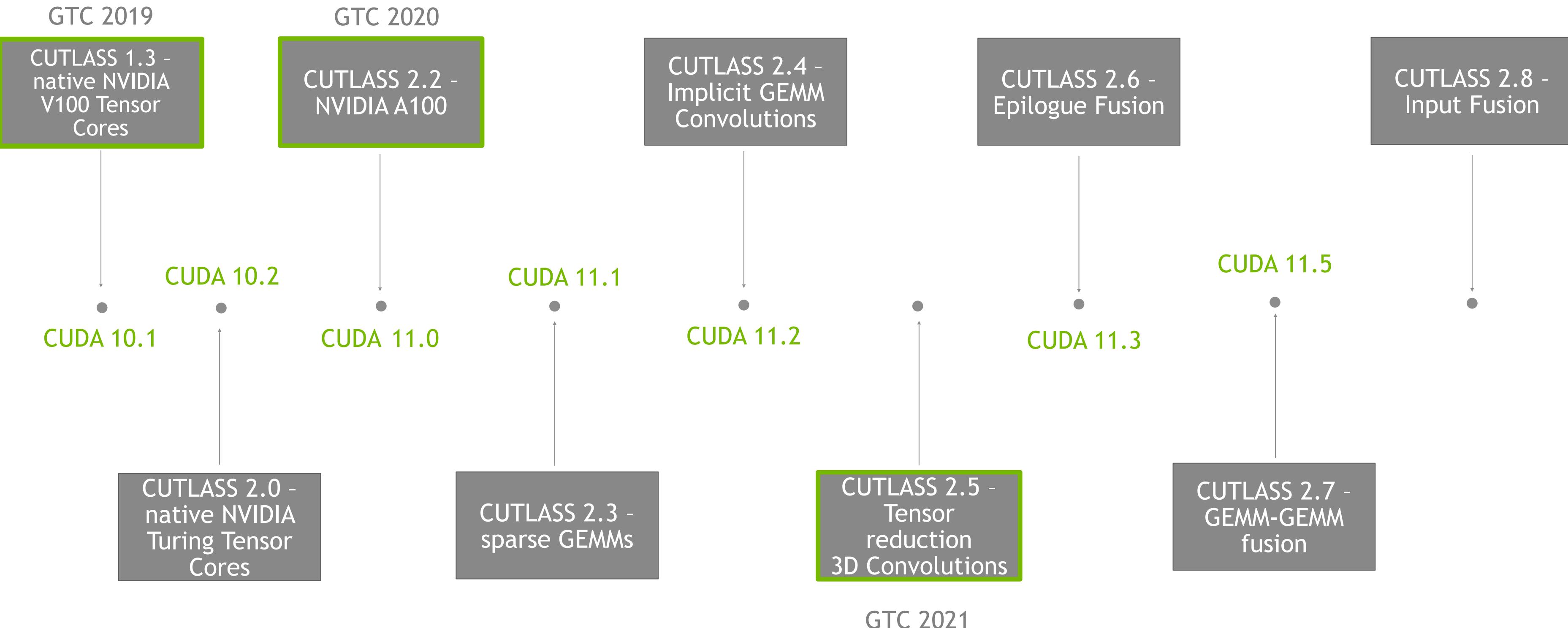
Open source: <https://github.com/NVIDIA/cutlass> Architecture intrinsic Templates wrapping architecture-specific PTX instructions (e.g. mma, cp.async, ldmatrix, cvt)

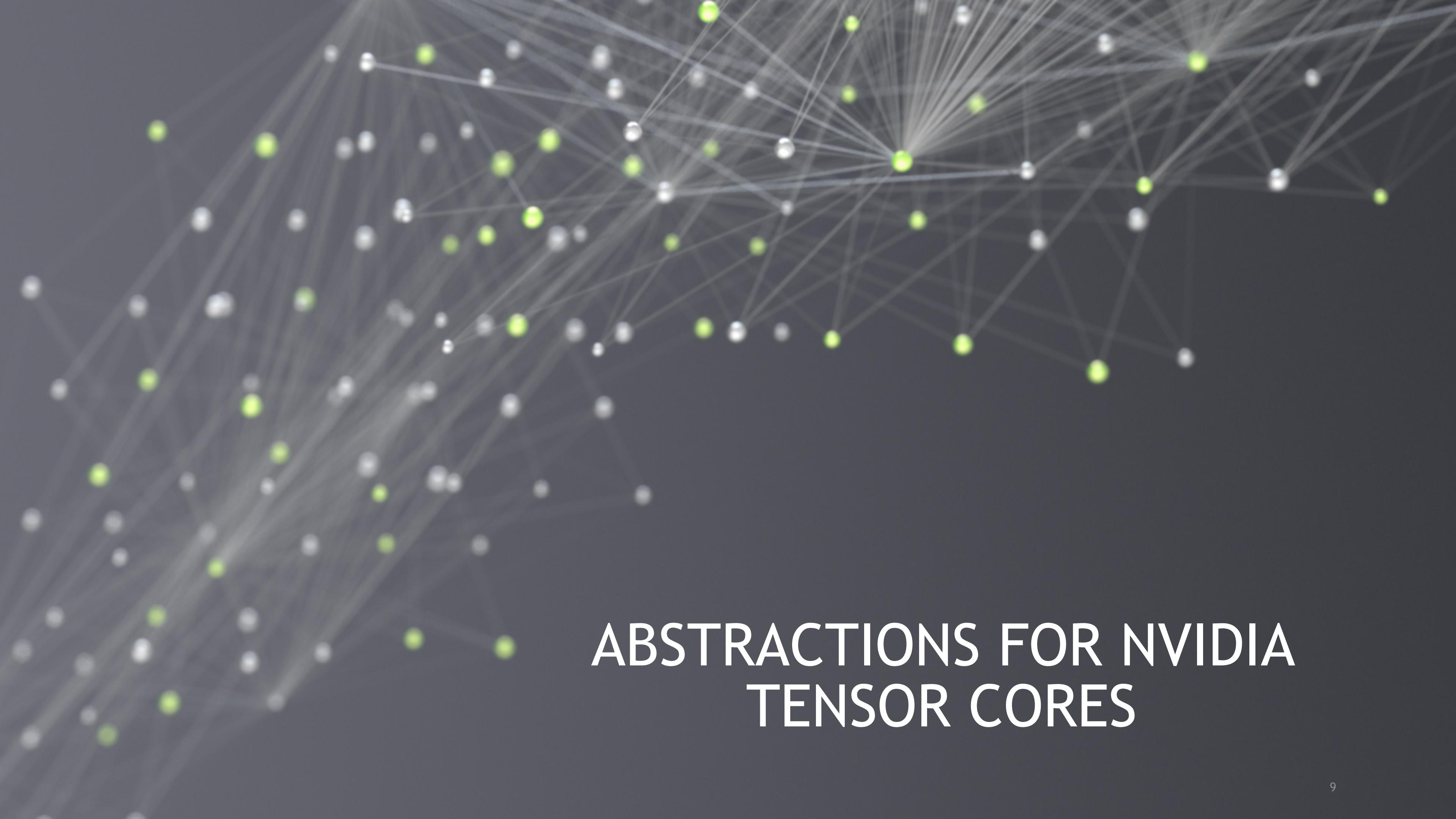
- Latest revision: CUTLASS 2.8
- Documentation: <https://github.com/NVIDIA/cutlass#documentation>
- Functionality: <https://github.com/NVIDIA/cutlass/blob/master/media/docs/functionality.md>



# CUTLASS

## CUDA C++ Templates for Deep Learning and Linear Algebra





# ABSTRACTIONS FOR NVIDIA TENSOR CORES

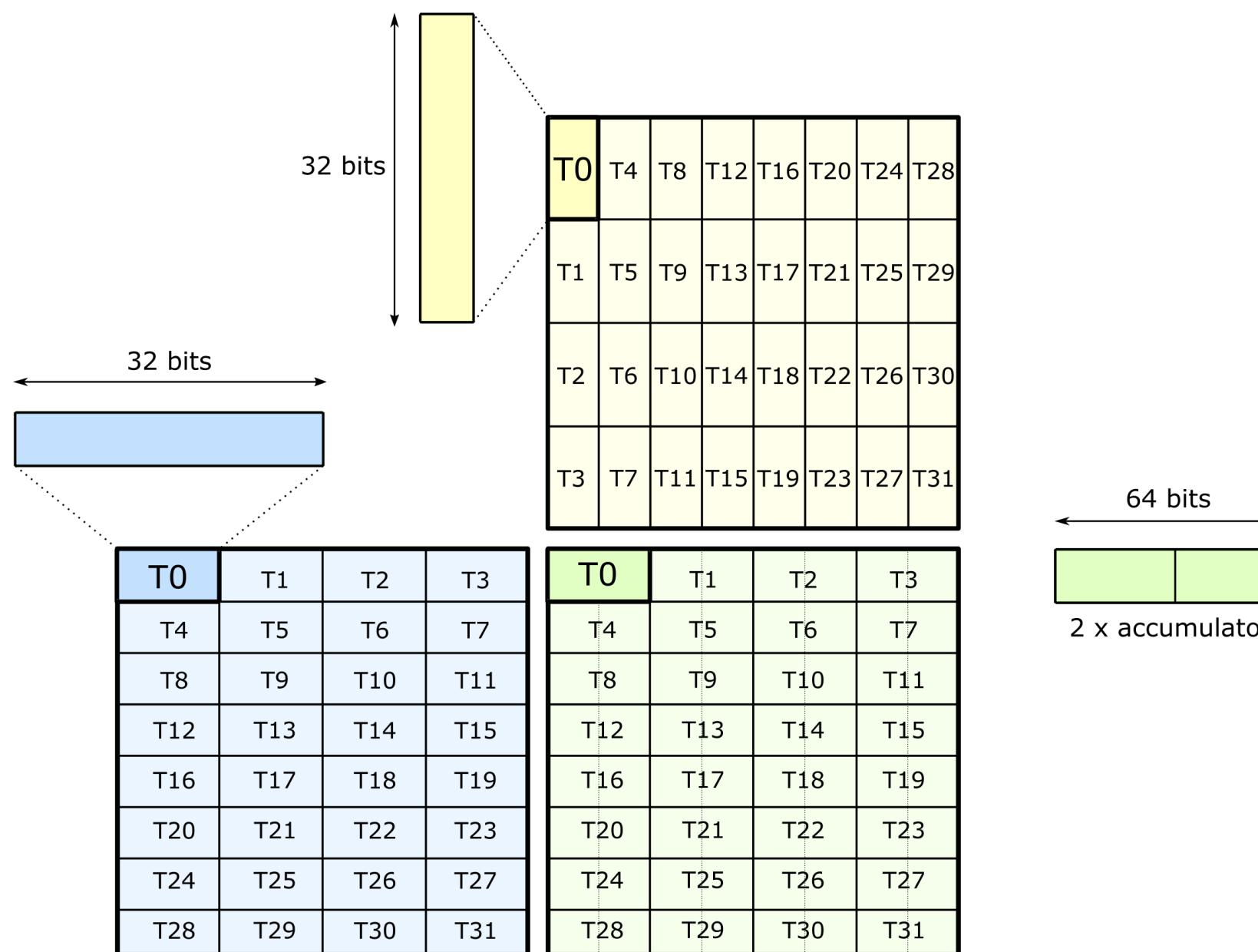
# HELLO WORLD: TENSOR CORES

Map each thread to coordinates of the matrix operation

Load inputs from memory

Perform the matrix operation

Store the result to memory



## CUDA example

```
__global__ void tensor_core_example_8x8x16(
    int32_t          *D,
    uint32_t const   *A,
    uint32_t const   *B,
    int32_t const    *C) {

    // Compute the coordinates of accesses to A and B matrices
    int outer = threadIdx.x / 4;           // m or n dimension
    int inner = threadIdx.x % 4;           // k dimension

    // Compute the coordinates for the accumulator matrices
    int c_row = threadIdx.x / 4;
    int c_col = 2 * (threadIdx.x % 4);

    // Compute linear offsets into each matrix
    int ab_idx = outer * 4 + inner;
    int cd_idx = c_row * 8 + c_col;

    // Issue Tensor Core operation
    asm(
        "mma.sync.aligned.m8n8k16.row.col.s32.s8.s8.s32 "
        " { %0, %1 }, "
        " %2,
        " %3,
        " { %4, %5 }; "
        :
        "=r"(D[cd_idx]), "=r"(D[cd_idx + 1])
        :
        "r"(A[ab_idx]),
        "r"(B[ab_idx]),
        "r"(C[cd_idx]), "r"(C[cd_idx + 1])
    );
}
```

# PERFORMANCE IMPLICATIONS

Load A and B inputs from memory: 2 x 4B per thread  
Perform one Tensor Core operation: 2048 flops per warp

2048 flops require 256 B of loaded data  
→ 8 flops/byte

## NVIDIA A100 Specifications:

- 624 TFLOP/s (INT8)
  - 1.6 TB/s (HBM2)
- 400 flops/byte

8 flops/byte \* 1.6 TB/s → 12 TFLOP/s

This kernel is global memory bandwidth limited.

## CUDA example

```
__global__ void tensor_core_example_8x8x16(
    int32_t          *D,
    uint32_t const   *A,
    uint32_t const   *B,
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    int outer = threadIdx.x / 4;      // m or n dimension
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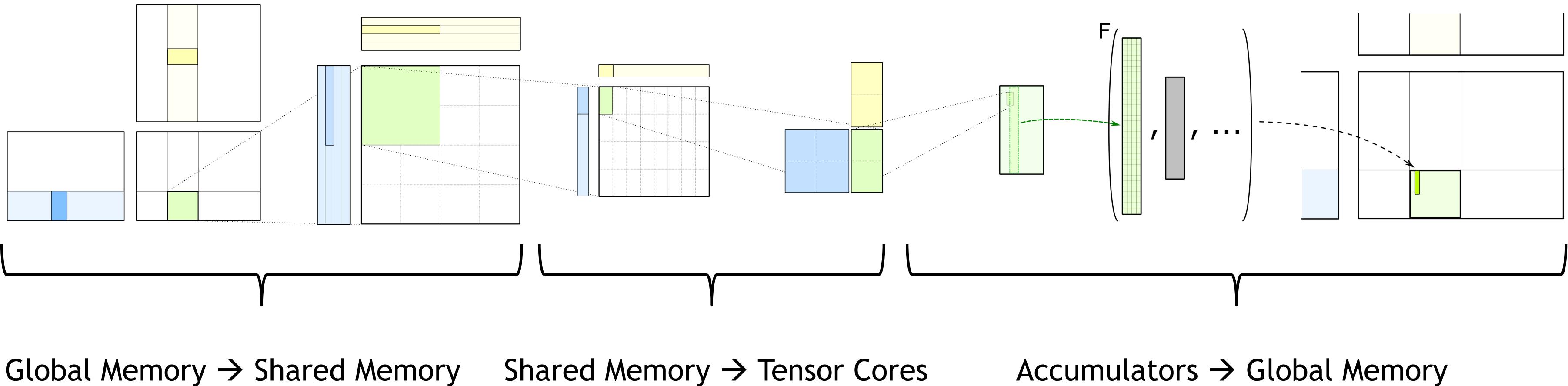
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    asm(
        "mma.sync.aligned.m8n8k16.row.col.s32.s8.s8.s32 "
        " { %0, %1 }, "
        " %2,
        " %3,
        " { %4, %5 }; "
        :
        "=r"(D[cd_idx]), "=r"(D[cd_idx + 1])

        :
        "r"(A[ab_idx]),
        "r"(B[ab_idx]),
        "r"(C[cd_idx]), "r"(C[cd_idx + 1])
    );
}
```

# CUTLASS GEMM MODEL

Decoupling application-specific behavior, Tensor Core operations, and Fused output operations

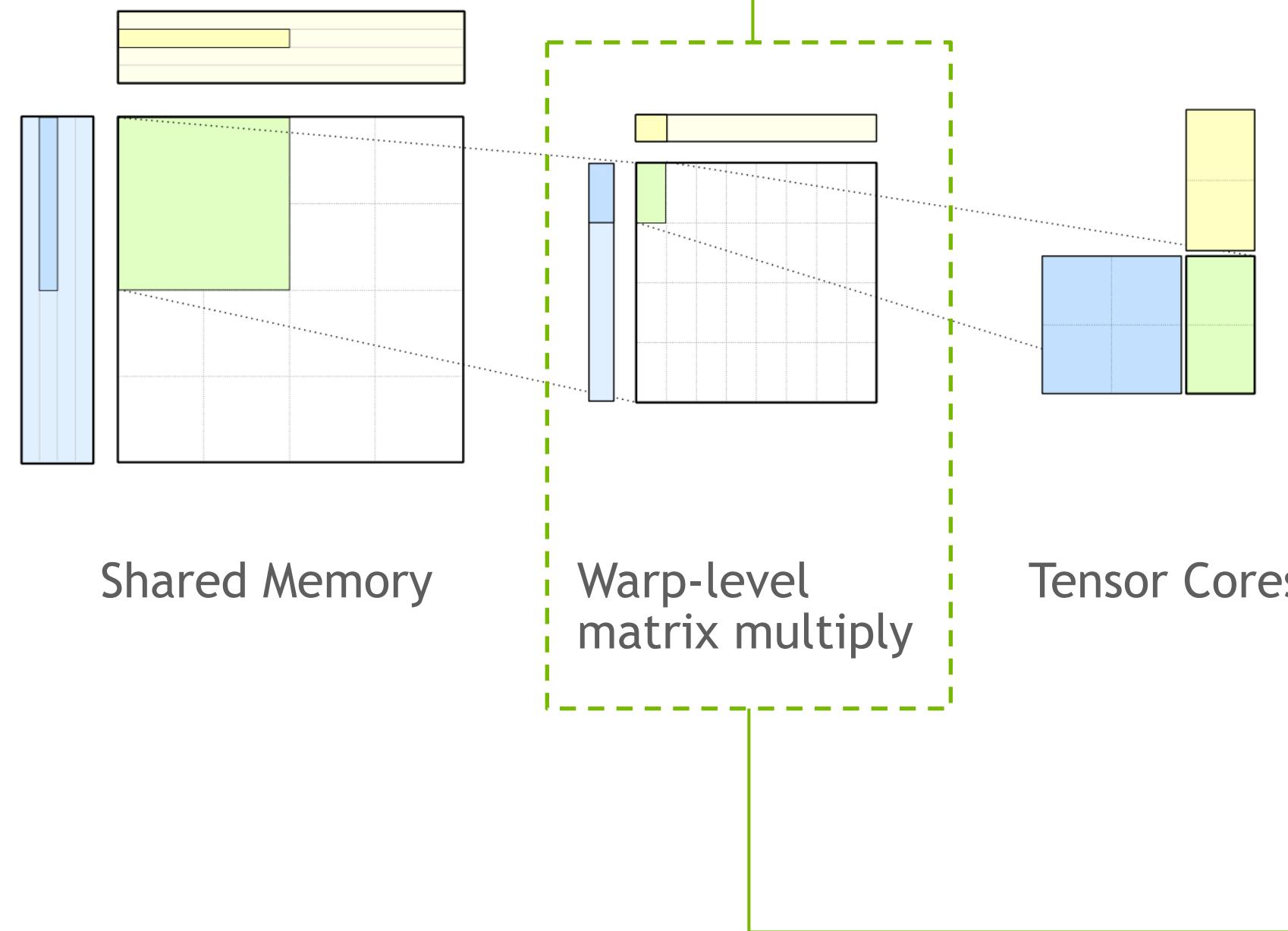
Matrix Multiply-Accumulate Phase



Tiled, hierarchical model: reuse data in Shared Memory and in Registers

See [CUTLASS GTC 2018](#) talk for more details about this model.

# CUTLASS: OPTIMAL ABSTRACTION FOR TENSOR CORES



```
using MmaWarp = cutlass::gemm::warp::DefaultMmaTensorOp<
    GemmShape<64, 64, 16>,
    half_t, LayoutA,
    half_t, LayoutB,
    float, RowMajor
>;
```

```
__shared__ ElementA smem_buffer_A[MmaWarp::Shape::kM * GemmK];
__shared__ ElementB smem_buffer_B[MmaWarp::Shape::kN * GemmK];
```

```
// Construct iterators into SMEM tiles
MmaWarp::IteratorA iter_A({smem_buffer_A, lda}, thread_id);
MmaWarp::IteratorB iter_B({smem_buffer_B, ldb}, thread_id);
```

```
MmaWarp::FragmentA frag_A;
MmaWarp::FragmentB frag_B;
MmaWarp::FragmentC accum;
```

```
MmaWarp mma_warp;
```

```
accum.clear();
```

```
#pragma unroll 1
for (int k = 0; k < GemmK; k += MmaWarp::Shape::kK) {
```

```
    iter_A.load(frag_A); // Load fragments from A and B matrices
    iter_B.load(frag_B);
```

```
    ++iter_A; ++iter_B; // Advance along GEMM K to next tile in
                         // and B matrices
```

```
                         // Compute matrix product
    mma_warp(accum, frag_A, frag_B, accum);
}
```

# CUTLASS: OPTIMAL ABSTRACTION FOR TENSOR CORES

## Tile Iterator Constructors:

Initialize pointers into permuted Shared Memory buffers

## Fragments:

Register-backed arrays holding each thread's data

## Tile Iterator:

load() - Fetches data from permuted Shared Memory buffers

operator++() - advances to the next logical matrix in SMEM

## Warp-level matrix multiply:

Decomposes a large matrix multiply into Tensor Core operations

```
using MmaWarp = cutlass::gemm::warp::DefaultMmaTensorOp<
    GemmShape<64, 64, 16>,
    half_t, LayoutA,                                     // GEMM A operand
    half_t, LayoutB,                                     // GEMM B operand
    float, RowMajor                                    // GEMM C operand
>;
```

```
__shared__ ElementA smem_buffer_A[MmaWarp::Shape::kM * GemmK];
__shared__ ElementB smem_buffer_B[MmaWarp::Shape::kN * GemmK];
```

```
// Construct iterators into SMEM tiles
MmaWarp::IteratorA iter_A({smem_buffer_A, lda}, thread_id);
MmaWarp::IteratorB iter_B({smem_buffer_B, ldb}, thread_id);
```

```
MmaWarp::FragmentA frag_A;
MmaWarp::FragmentB frag_B;
MmaWarp::FragmentC accum;
```

```
MmaWarp mma_warp;
```

```
accum.clear();
```

```
#pragma unroll 1
for (int k = 0; k < GemmK; k += MmaWarp::Shape::kK) {
```

```
    iter_A.load(frag_A); // Load fragments from A and B matrices
    iter_B.load(frag_B);
```

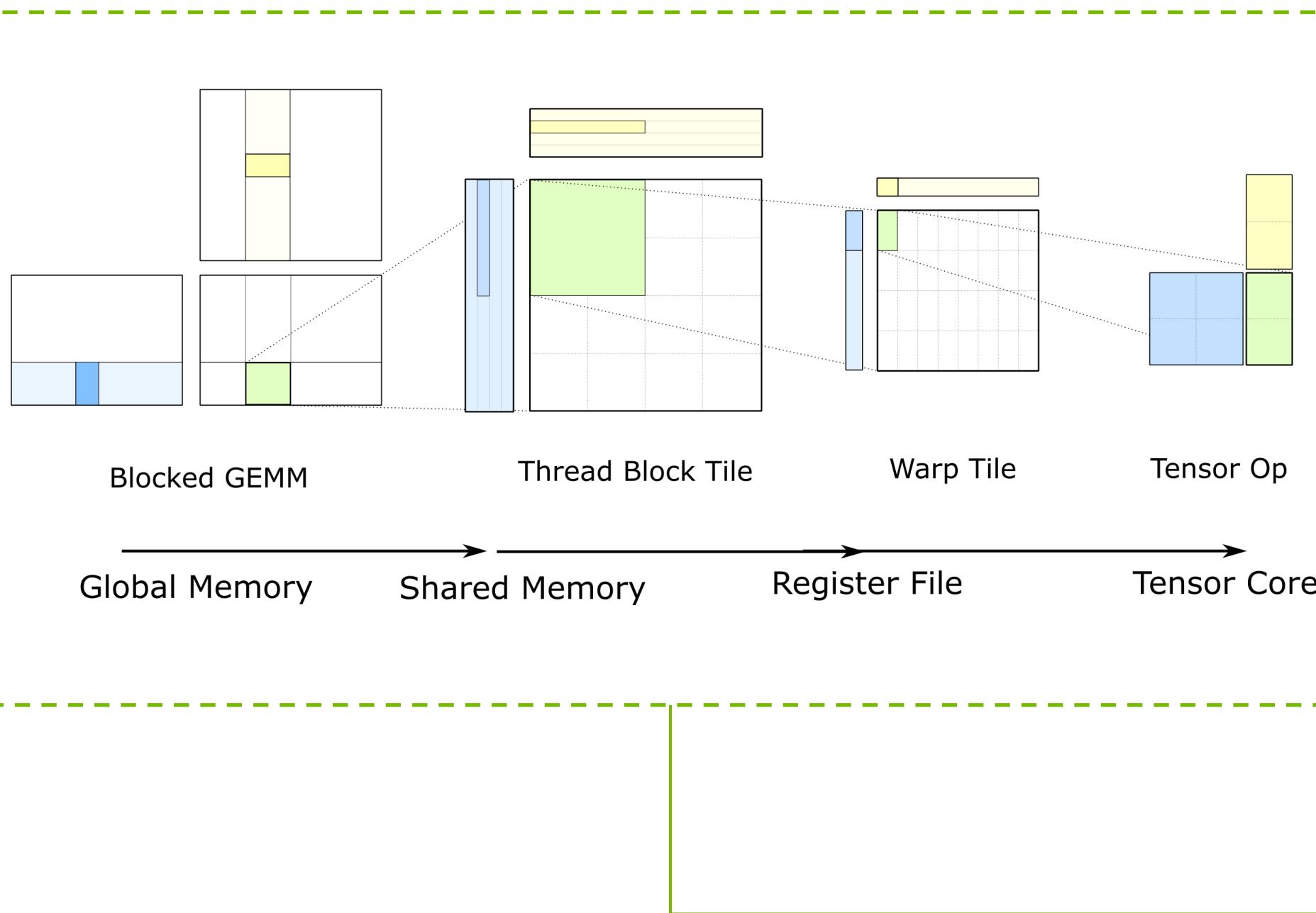
```
    ++iter_A; ++iter_B; // Advance along GEMM K to next tile in
                        // and B matrices
```

```
                        // Compute matrix product
    mma_warp(accum, frag_A, frag_B, accum);
}
```

# CUTLASS: ABSTRACTION FOR PIPELINED MATRIX MULTIPLY

## Threadblock-scoped operator for Matrix Multiply-Accumulate

`cutlass::gemm::threadblock::MmaMultistage`



“Kernel scoped” device code

```
// Compute position within threadblock
int thread_idx = threadIdx.x;

// Construct iterators to A and B operands
typename MmaMultistage::IteratorA iterator_A(
    params.params_A,
    ptr_A,
    {params.problem_size.m(), problem_size_k},
    thread_idx,
    tb_offset_A);

typename MmaMultistage::IteratorB iterator_B(
    params.params_B,
    ptr_B,
    {problem_size_k, params.problem_size.n()},
    thread_idx,
    tb_offset_B);

// Broadcast the warp_id computed by lane 0 to ensure dependent code
// is compiled as warp-uniform.
int warp_idx = __shfl_sync(0xffffffff, threadIdx.x / 32, 0);

int lane_idx = threadIdx.x % 32;

//
// Construct multistage, pipelined thread-scoped matrix multiply
//
MmaMultistage mma_multistage(
    shared_storage.main_loop, thread_idx, warp_idx, lane_idx);

typename MmaMultistage::FragmentC accumulators;

accumulators.clear();

// Compute matrix multiply
mma_multistage(
    gemm_k_iterations,
    accumulators,
    iterator_A,
    iterator_B,
    accumulators);
```

# CUTLASS: ABSTRACTION FOR PIPELINED MATRIX MULTIPLY

## Tile Iterator Constructors:

Initialize pointers and predicates into Global Memory

## CUDA optimization technique:

Compute warp index as a provably warp-uniform value for architecture-specific optimization by the compiler

## Construct matrix multiply-accumulate operator:

Constructs warp-level tile iterators, Shared Memory store iterators, and stage counters

## Accumulator Fragment:

Register-backed arrays holding Accumulators

## Threadblock-level matrix multiply:

Global to Shared Memory, Shared Memory to Registers, Tensor or CUDA Core multiply-add operations

```
// Compute position within threadblock
int thread_idx = threadIdx.x;

// Construct iterators to A and B operands
typename MmaMultistage::IteratorA iterator_A(
    params.params_A,
    ptr_A,
    {params.problem_size.m(), problem_size_k},
    thread_idx,
    tb_offset_A);

typename MmaMultistage::IteratorB iterator_B(
    params.params_B,
    ptr_B,
    {problem_size_k, params.problem_size.n()},
    thread_idx,
    tb_offset_B);

// Broadcast the warp_id computed by lane 0 to ensure dependent code
// is compiled as warp-uniform.
int warp_idx = __shfl_sync(0xffffffff, threadIdx.x / 32, 0);

int lane_idx = threadIdx.x % 32;

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accumulators.clear();

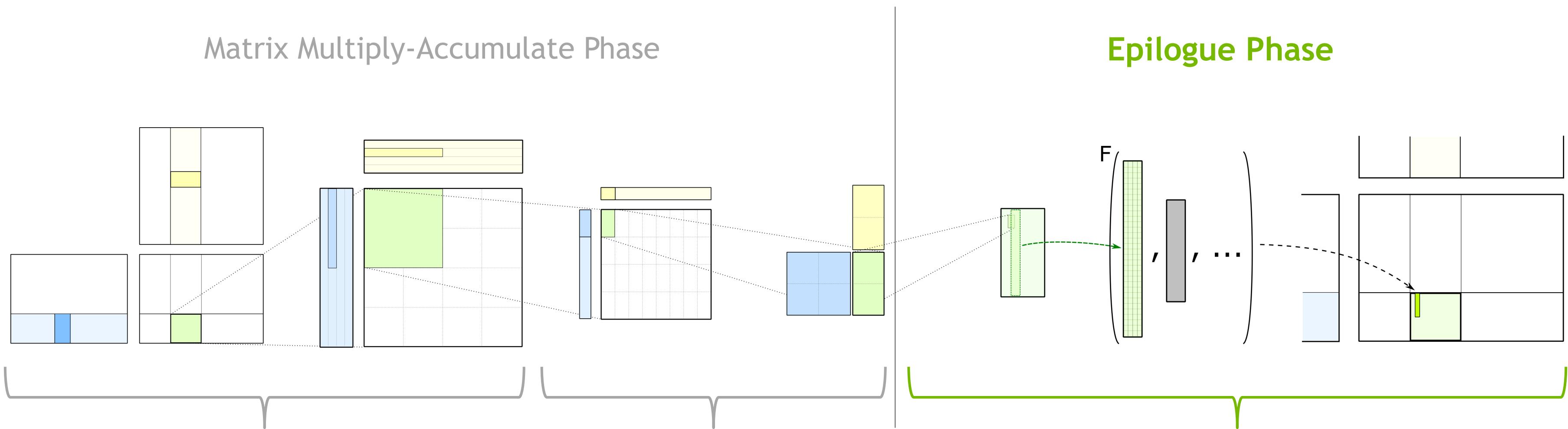
// Compute matrix multiply
mma_multistage(
    gemm_k_iterations,
    accumulators,
    iterator_A,
    iterator_B,
    accumulators);
```



# EFFICIENT EPILOGUES

# EPILOGUE PHASE

Efficiently storing GEMM output to Global Memory



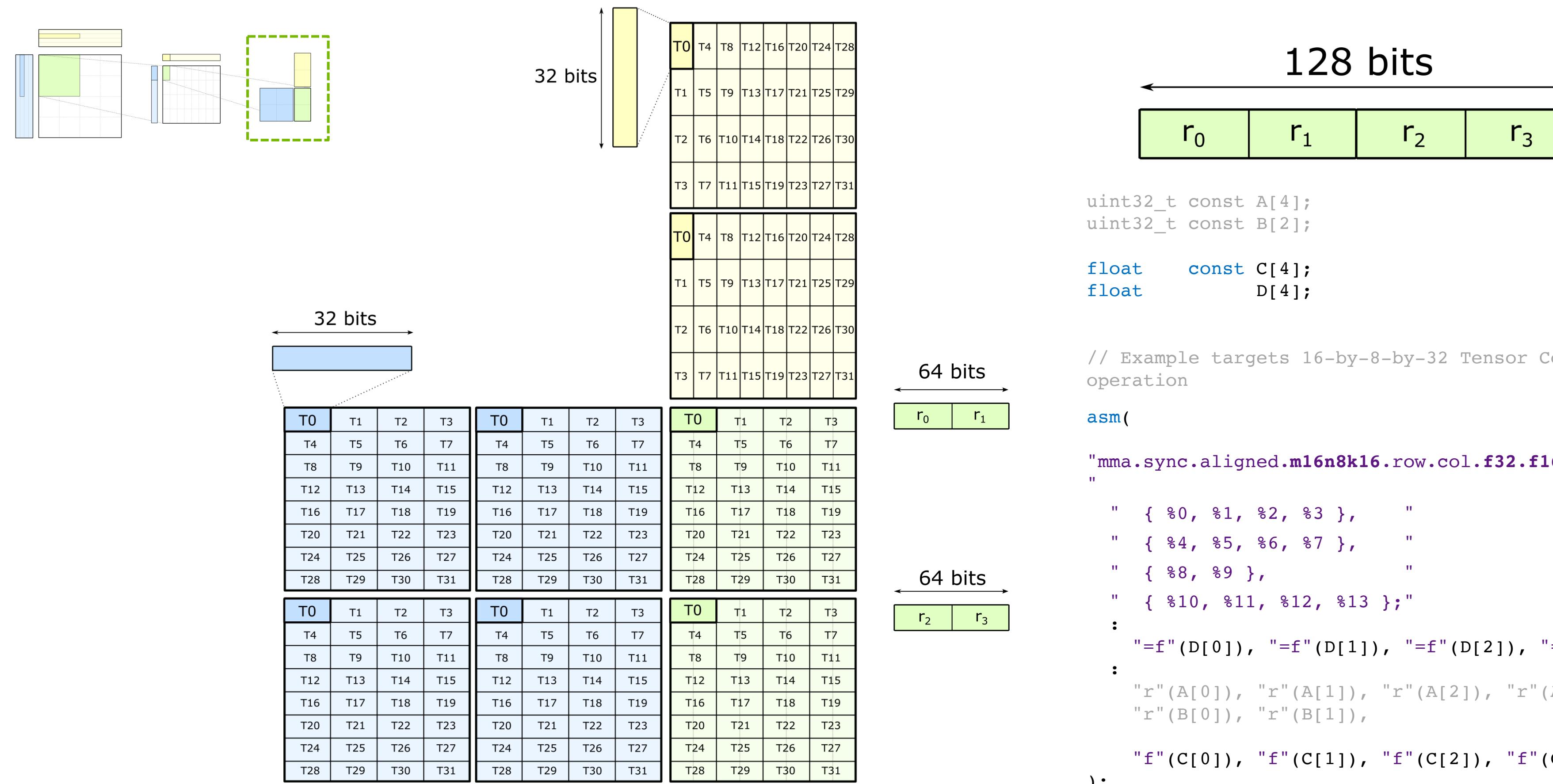
Global Memory → Shared Memory

Shared Memory → Tensor Cores

Accumulators → Global Memory

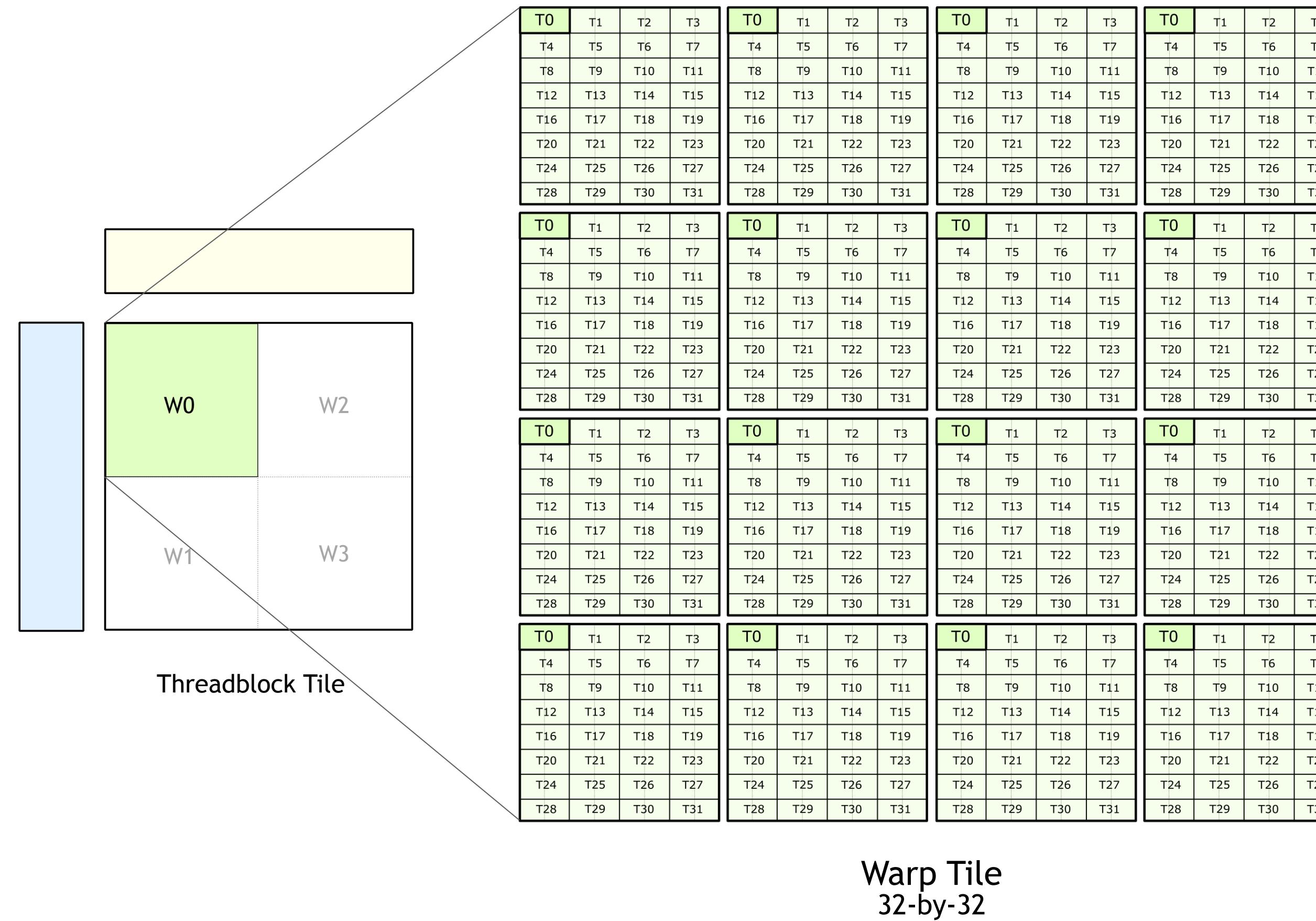
- Rearrangement of accumulators
- Fused math operations
- Type conversions and packing
- Coalesced access to Global Memory

# TENSOR CORE REGISTER ARRANGEMENT



Warp-wide Tensor Core operation: 16-by-8-by-256b

# WARP-WIDE ARRANGEMENT OF ACCUMULATORS

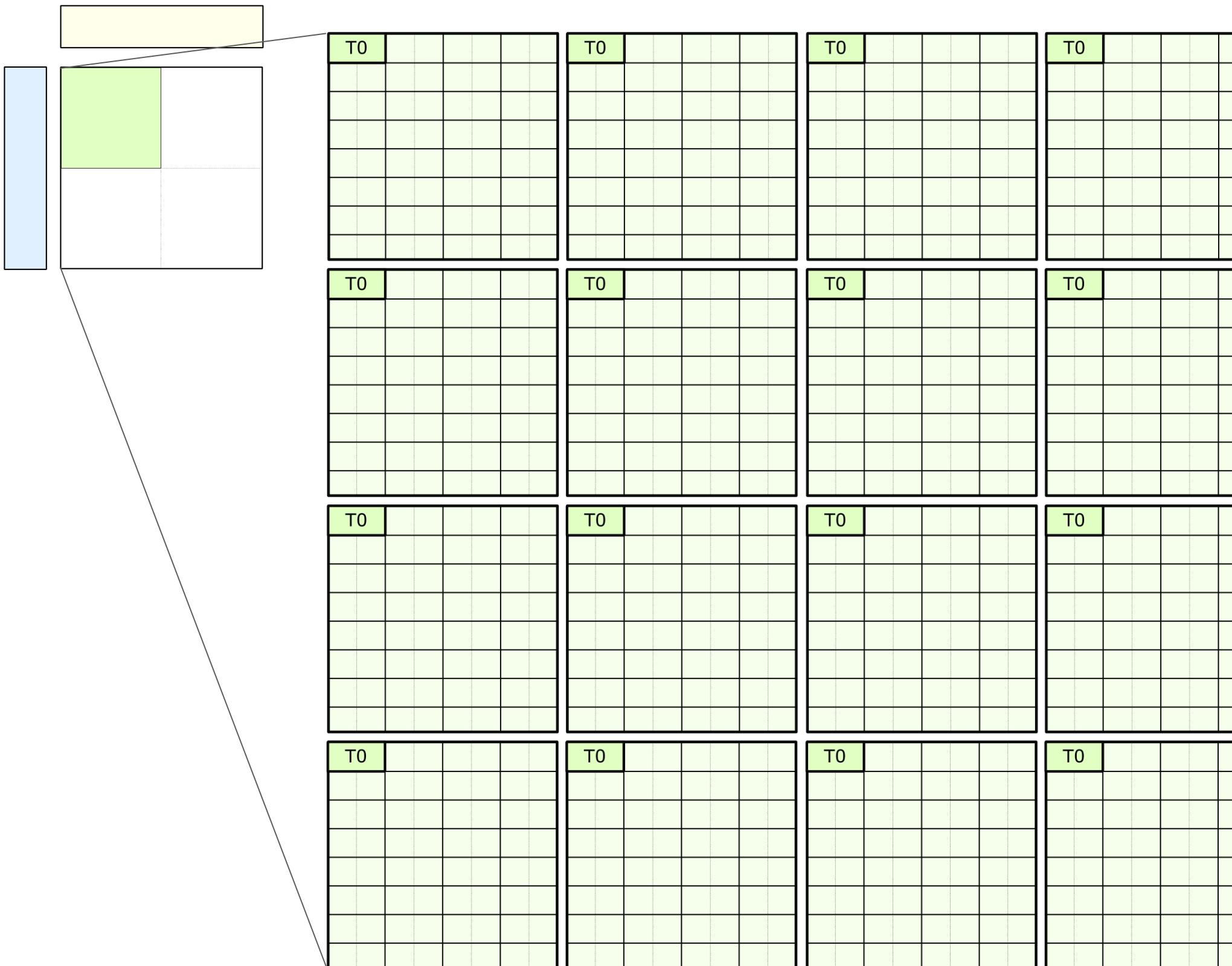


Typical threadblock configurations.

Threadblock Shape	Warp Shape	Accumulator Count
64-by-64	32-by-32	32
128-by-64	64-by-32	64
128-by-128	64-by-64	128
256-by-128	64-by-64	128

Four warps per threadblock to utilize SM

# Directly Store Accumulators



```
“Warp scoped” device code

int const kWarpCountM = 2;
int const kWarpCountN = 2;

int const kMmaCountM = 2;
int const kMmaCountN = 4;

int const kMmaShapeM = 16;
int const kMmaShapeN = 8;

// Accumulator array in Registers
float accumulators[kMmaCountM * kMmaCountN * 4];

// Output matrix in Global Memory
float *D_gmem = ...;
int ldm = ...;

// Indexing logic – map threads to coordinates
int warp_idx = (threadIdx.x / 32);
int thread_idx = (threadIdx.x % 32);

int warp_m = (warp_idx % kWarpCountM) *
(kWarpCountM * kMmaCountM * kMmaShapeM);

int warp_n = (warp_idx / kWarpCountN) *
(kWarpCountN * kMmaCountN * kMmaShapeN);

int thread_m = (thread_idx / 4);
int thread_n = (thread_idx % 4) * 2;

// Loop over MMA instructions in the M dimension
#pragma unroll
for (int mma_m = 0; mma_m < kMmaCountM; ++mma_m) {

    // Loop over MMA instructions in the N dimension
    #pragma unroll
    for (int mma_n = 0; mma_n < kMmaCountN; ++mma_n) {

        // Loop over accumulators within one MMA instruction
        #pragma unroll
        for (int i = 0; i < 4; ++i) {

            // Compute index in accumulator array
            int r_idx = i + mma_m * 4 + mma_n * 4 * kMmaCountM;

            // Compute output location within threadblock
            int m = warp_m + thread_m + mma_m * kMmaShapeM + (i / 2) * 8;
            int n = warp_n + thread_n + mma_n * kMmaShapeN + (i % 2);

            // Store to Global Memory
            D_gmem[m + n * ldm] = accumulators[r_idx];
        }
    }
}
```

# EFFICIENT EPILOGUES

Directly storing accumulators is not efficient for all data types and configurations

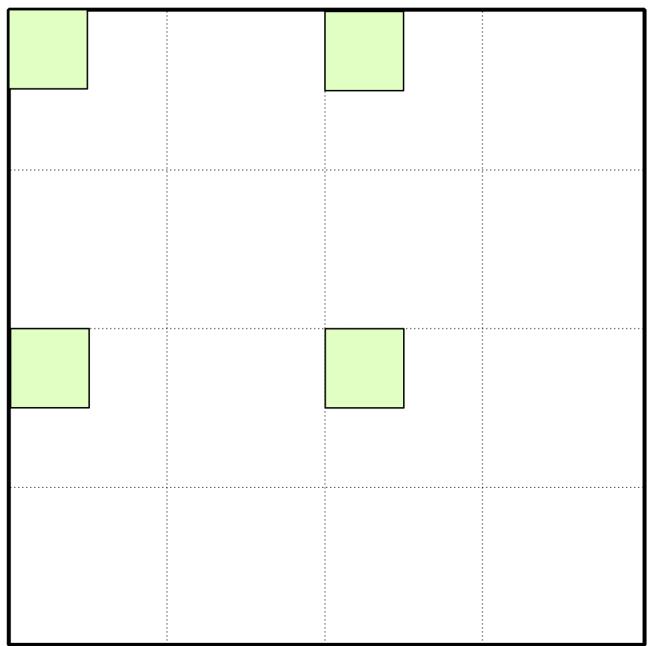
- Each warp writes to a disjoint 8-by-8 tile at a time
- Difficult for the hardware coalesce into accesses spanning large cache lines
- Mixed precision narrows the memory access size, resulting in small store operations

CUTLASS epilogues exchange accumulators through Shared Memory first

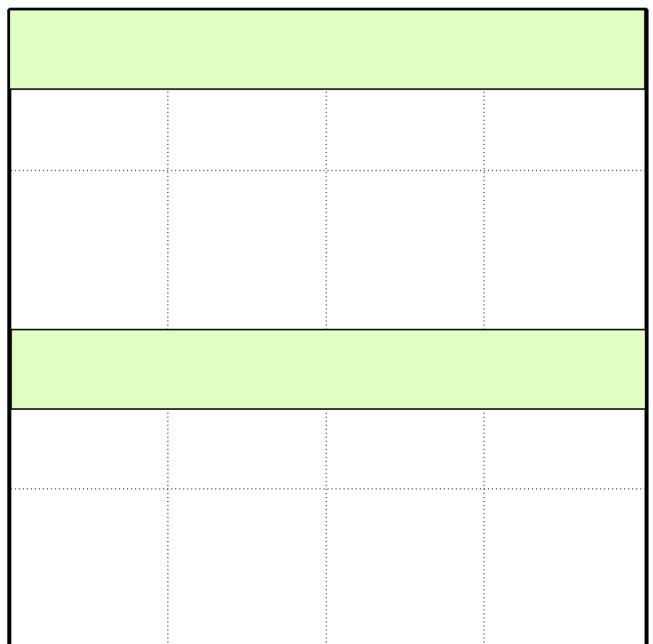
Objectives for efficiently accessing Global Memory

- Threads striped over (a few) large, contiguous tiles
- Store to whole cache lines, limited only by the width of Threadblock tiles
- Support larger vector access sizes
- Take advantage of data exchange by fusing other operations

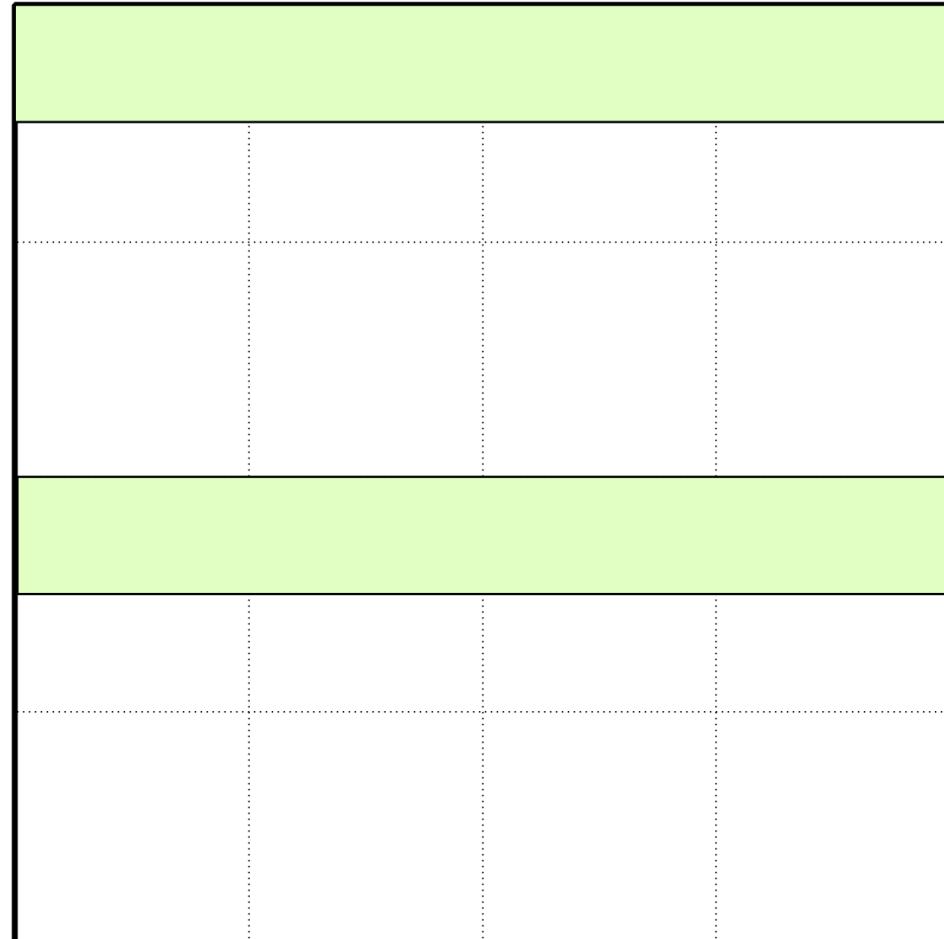
Direct Store access pattern



Coalesced accesses over contiguous tiles



# EFFICIENT EPILOGUES



for each “slice”

    \_\_syncthreads();

        Store a slice of accumulators to Shared Memory

    \_\_syncthreads();

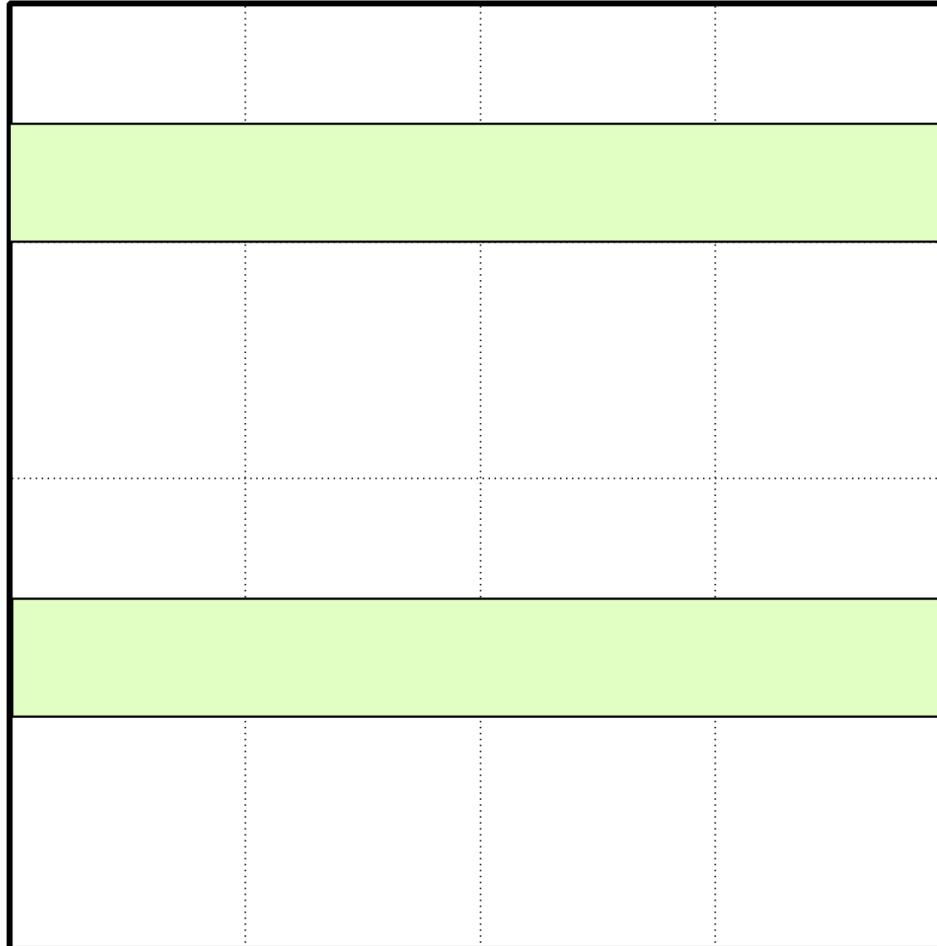
        Load the slice of accumulators from Shared Memory into registers

        Compute fused operations on accumulator slice and other matrices

        Store to Global Memory

        Advance Global Memory pointers

# EFFICIENT EPILOGUES



for each “slice”

`__syncthreads();`

        Store a slice of accumulators to Shared Memory

`__syncthreads();`

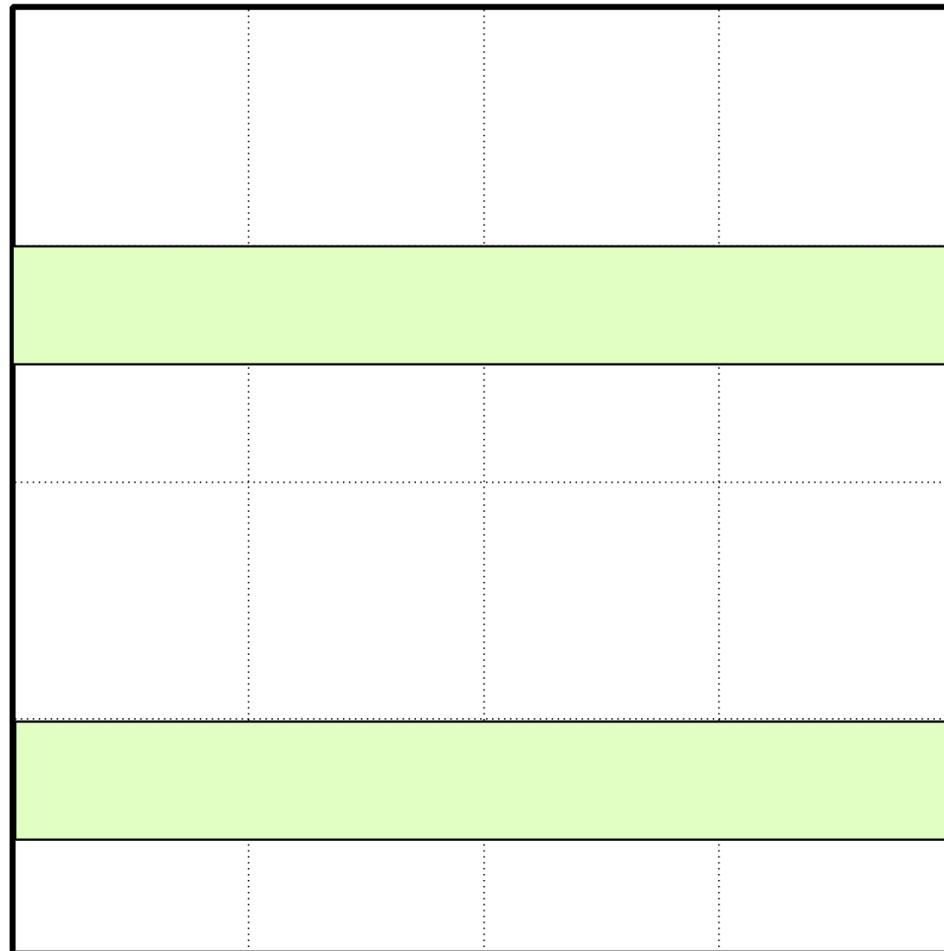
        Load the slice of accumulators from Shared Memory into registers

        Compute fused operations on accumulator slice and other matrices

    Store to Global Memory

    Advance Global Memory pointers

# EFFICIENT EPILOGUES



for each “slice”

`__syncthreads();`

        Store a slice of accumulators to Shared Memory

`__syncthreads();`

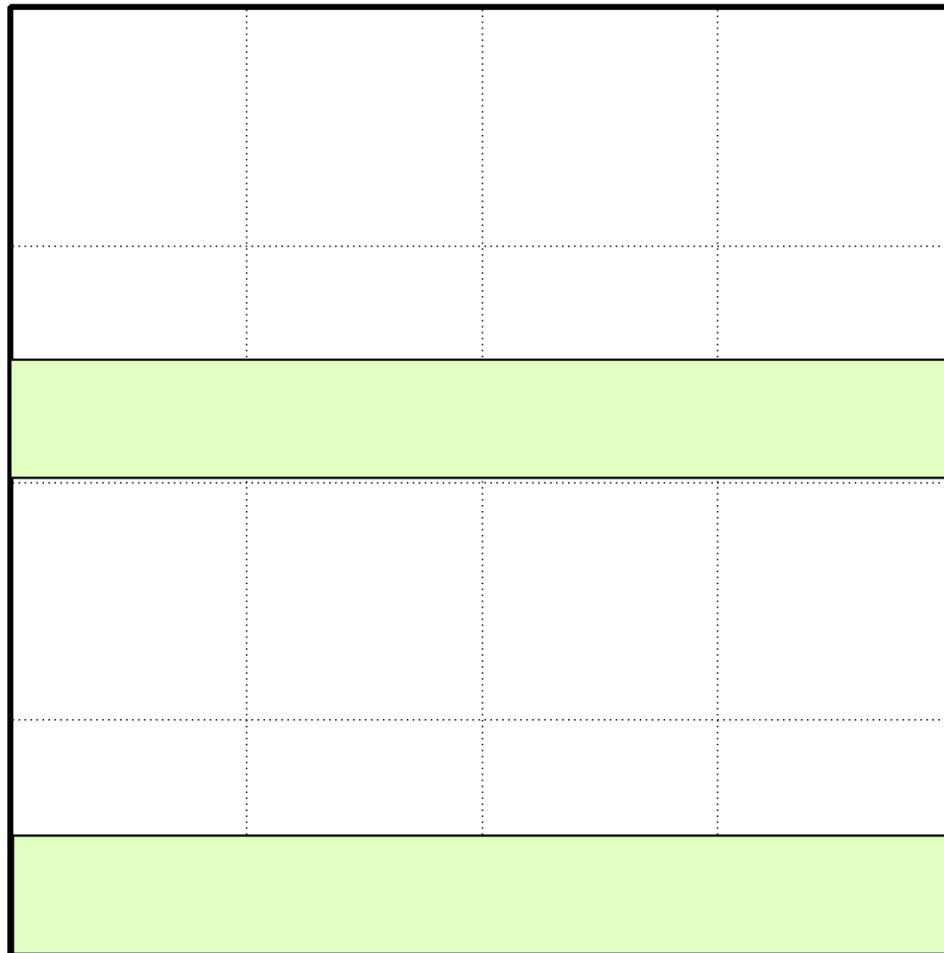
        Load the slice of accumulators from Shared Memory into registers

        Compute fused operations on accumulator slice and other matrices

    Store to Global Memory

    Advance Global Memory pointers

# EFFICIENT EPILOGUES



```
for each "slice"  
    __syncthreads();  
    Store a slice of accumulators to Shared Memory  
    __syncthreads();  
    Load the slice of accumulators from Shared Memory into  
    registers  
    Compute fused operations on accumulator slice and other  
    matrices  
    Store to Global Memory  
    Advance Global Memory pointers
```

# CUTLASS Epilogue

## Source fragment

Stores tiles from “C” matrix loaded from GMEM

## Accumulator “fragment iterator”

Iterates over “slices” of the overall accumulator fragment

## Load from Global Memory

Fetches a tile from the “C” matrix

## Store Accumulators to Shared Memory

Copies from accumulator fragment to Shared Memory

## Loads Accumulators from Shared Memory

Resulting ownership of data ‘aligned’ to vectors in the output matrix

## Elementwise functor

## Store to Global Memory

“Threadblock scoped” device code

```
template < ... >
class Epilogue {
public:

    // Executes the steps of the CUTLASS Epilogue
    CUTLASS_DEVICE
    void operator()
        OutputOp const &output_op,
        OutputTileIterator destination_iterator,
        AccumulatorTile const &accumulators,
        OutputTileIterator source_iterator
    ) {

        typename OutputTileIterator::Fragment source_fragment;
        source_fragment.clear();

        // Iterator over warp-level accumulator fragment
        AccumulatorFragmentIterator accum_fragment_iterator(accumulators);

        // Iterate over slices of the accumulator
        #pragma unroll
        for (int iter = 0; iter < OutputTileIterator::kIterations; ++iter) {

            // Load the source tensor from Global Memory
            source_iterator.load(source_fragment);
            ++source_iterator;

            __syncthreads();

            // Store slice of accumulators to Shared Memory
            typename AccumulatorFragmentIterator::Fragment accum_fragment;
            accum_fragment_iterator.load(accum_fragment);
            ++accum_fragment_iterator;

            this->warp_tile_iterator_.store(accum_fragment);

            __syncthreads();

            // Load fragments from Shared Memory
            typename SharedLoadIterator::Fragment aligned_accum_fragment;
            shared_load_iterator_.load(aligned_accum_fragment);

            // Compute an elementwise operation on the accumulated result
            typename OutputTileIterator::Fragment output_fragment;
            apply_output_operator_(output_fragment, output_op, aligned_accum_fragment, source_fragment);

            // Store the resulting fragment to Global Memory
            destination_iterator.store(output_fragment);
            ++destination_iterator;
        }
    };
}
```

# Combining Matrix Multiply and Epilogue Phases

Accumulator fragment

Contains result of matrix multiply operation

Execution of matrix multiply

Issues Tensor Core operations

Epilogue functor

Performs elementwise operations on result of matrix multiply

Epilogue output tile iterator

Loads and stores to output matrix in Global Memory

Epilogue constructor

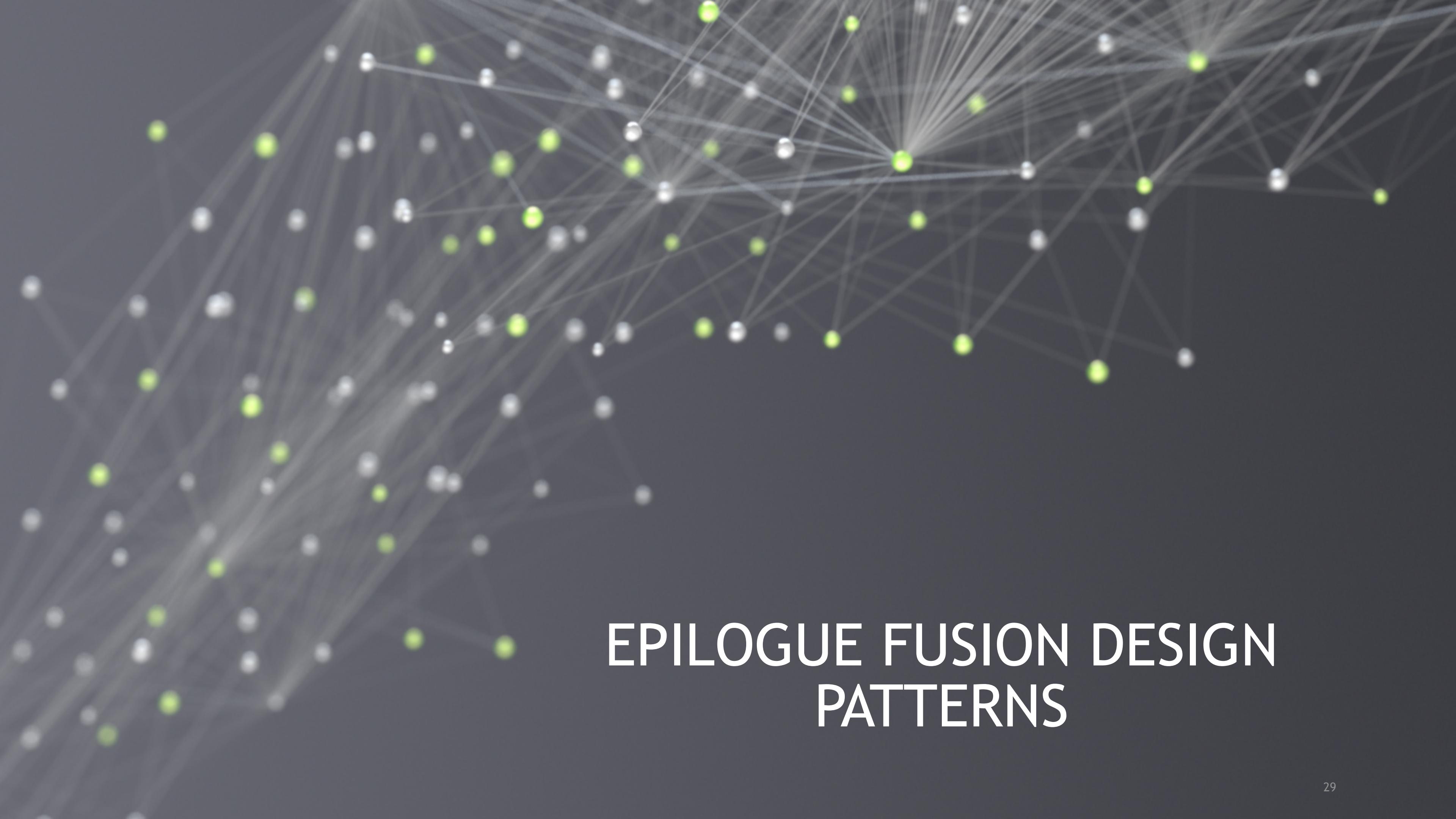
Initializes internal pointers to store and load from Shared Memory

Execution of Epilogue

Transforms accumulators and stores to Global Memory

```
//  
// Matrix multiply phase  
  
//  
  
MmaMultistage mma_multistage(  
    shared_storage.main_loop, thread_idx, warp_idx, lane_idx);  
  
typename MmaMultistage::FragmentC accumulators;  
  
// Execute matrix multiply  
mma_multistage(  
    gemm_k_iterations,  
    accumulators,  
    iterator_A,  
    iterator_B,  
    accumulators);  
  
//  
// Epilogue phase  
//  
  
EpilogueOutputOp output_op(params.output_op);  
  
// Compute threadblock's location within Global Memory  
MatrixCoord threadblock_offset(  
    threadblock_tile_offset.m() * MmaMultistage::Shape::kM,  
    threadblock_tile_offset.n() * MmaMultistage::Shape::kN  
);  
  
// Tile iterator loading from source tensor.  
typename Epilogue::OutputTileIterator iterator_C(  
    params.params_C,  
    ptr_C,  
    params.problem_size.mn(),  
    thread_idx,  
    threadblock_offset  
);  
  
Epilogue epilogue(  
    shared_storage.epilogue, thread_idx, warp_idx, lane_idx);  
  
// Execute the epilogue phase  
epilogue(  
    output_op, iterator_C, accumulators, iterator_C);
```

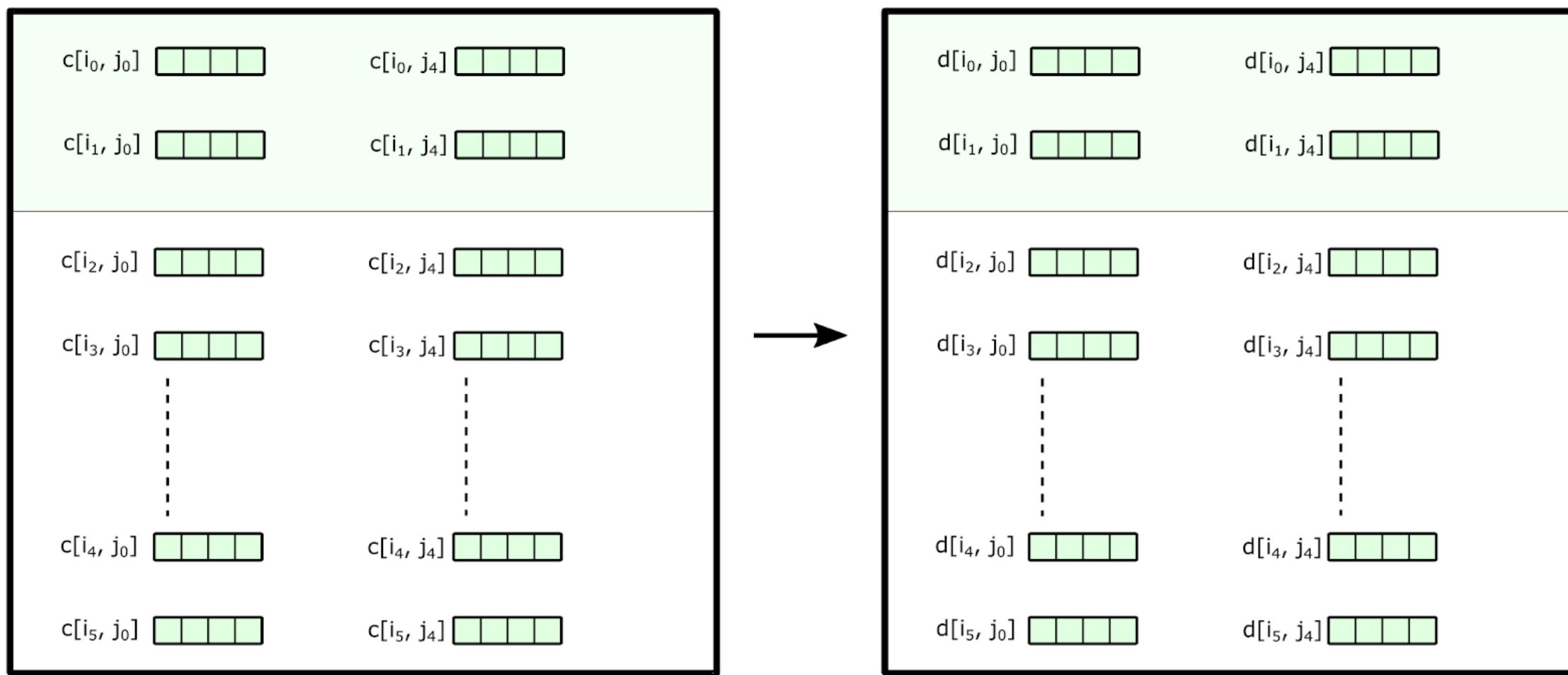
“Kernel scoped” device code



# EPILOGUE FUSION DESIGN PATTERNS

# ELEMENTWISE OPERATIONS

Fusing operations in the Epilogue



Elementwise operations may be performed on register-backed arrays before storing

Additional tensors may be loaded, subject to register liveness constraints

Vector sizes may be arbitrarily large; this is useful for storing bit-vectors for ReLU calculations (8 x 1b)

# Composing GEMM with Epilogue Functor

“Device wide” operator definition  
and launch in host code

Define the input, output, and compute types

Define the epilogue functor

Define the device-wide GEMM operator

E.g. GEMM “NN” using Tensor Cores

Construct “Arguments” structure

Problem size, pointers and strides, epilogue functor arguments

Launch the GEMM kernel grid

```
using ElementInput = cutlass::half_t;
using ElementOutput = float;
using ElementAccumulator = float;
using ElementEpilogueCompute = float;

using EpilogueFunctor = cutlass::epilogue::thread::LinearCombinationGELU<
    ElementOutput,                                     // Output tensor data type
    8,                                                 // Epilogue “vector size” – 16 bytes
    ElementAccumulator,                                // Accumulator data type – must match mainloop
    ElementEpilogueCompute                           // Epilogue “compute type” – type of alpha/beta
>;

using Gemm = cutlass::gemm::device::Gemm<
    ElementInput,   cutlass::layout::ColumnMajor,
    ElementInput,   cutlass::layout::ColumnMajor,
    ElementOutput, cutlass::layout::ColumnMajor,
    ElementAccumulator,
    cutlass::arch::OpClassTensorOp,
    cutlass::arch::Sm80,
    cutlass::gemm::GemmShape<128, 256, 64>,
    cutlass::gemm::GemmShape<64, 64, 64>,
    cutlass::gemm::GemmShape<16, 8, 16>,
    EpilogueFunctor,
    cutlass::gemm::threadblock::GemmIdentityThreadblockSwizzle<>,
    3
>;

// Collect arguments
typename Gemm::Arguments arguments{
    problem_size,
    tensor_A.device_ref(),
    tensor_B.device_ref(),
    tensor_C.device_ref(),
    tensor_D.device_ref(),
    {alpha, beta}           // Arguments passed to EpilogueFunctor
};

Gemm gemm_op;
gemm_op(arguments);          // Run the GEMM
```

# GEMM + Elementwise example functor: LinearCombinationGELU

“Params” nested structure contains arguments passed from caller

Constructor optionally fetches scalars via device pointers

Function call operator performs elementwise arithmetic and conversion

Returned value stored to Global Memory by Epilogue

```
class LinearCombinationGELU {
public:

    /// Host-constructable parameters structure
    struct Params {

        ElementCompute alpha;
        ElementCompute beta;
        ElementCompute const *alpha_ptr;
        ElementCompute const *beta_ptr;
        ...
    };

    ElementCompute alpha_, _beta;

    /// Constructs the function object, possibly loading from pointers in host memory
    CUTLASS_HOST_DEVICE
    LinearCombinationGELU(Params const &params) {

        alpha_ = (params.alpha_ptr ? *params.alpha_ptr : params.alpha);
        beta_ = (params.beta_ptr ? *params.beta_ptr : params.beta);
    }

    /// Computes: D = gelu( alpha * accumulator )
    CUTLASS_HOST_DEVICE
    FragmentOutput operator()(FragmentAccumulator const &accumulator) const {

        // Convert source to integral compute numeric type
        NumericArrayConverter<ElementCompute, ElementAccumulator, kCount, Round>
        accumulator_converter;

        ComputeFragment converted_accumulator = accumulator_converter(accumulator);

        // Perform elementwise operations
        ComputeFragment intermediate;

        multiplies<ComputeFragment> mul_add_accumulator;
        GELU_Taylor<ComputeFragment> gelu;

        intermediate = mul_add_accumulator(alpha_, converted_accumulator);      // D = alpha *
        Accum
        intermediate = gelu(intermediate);

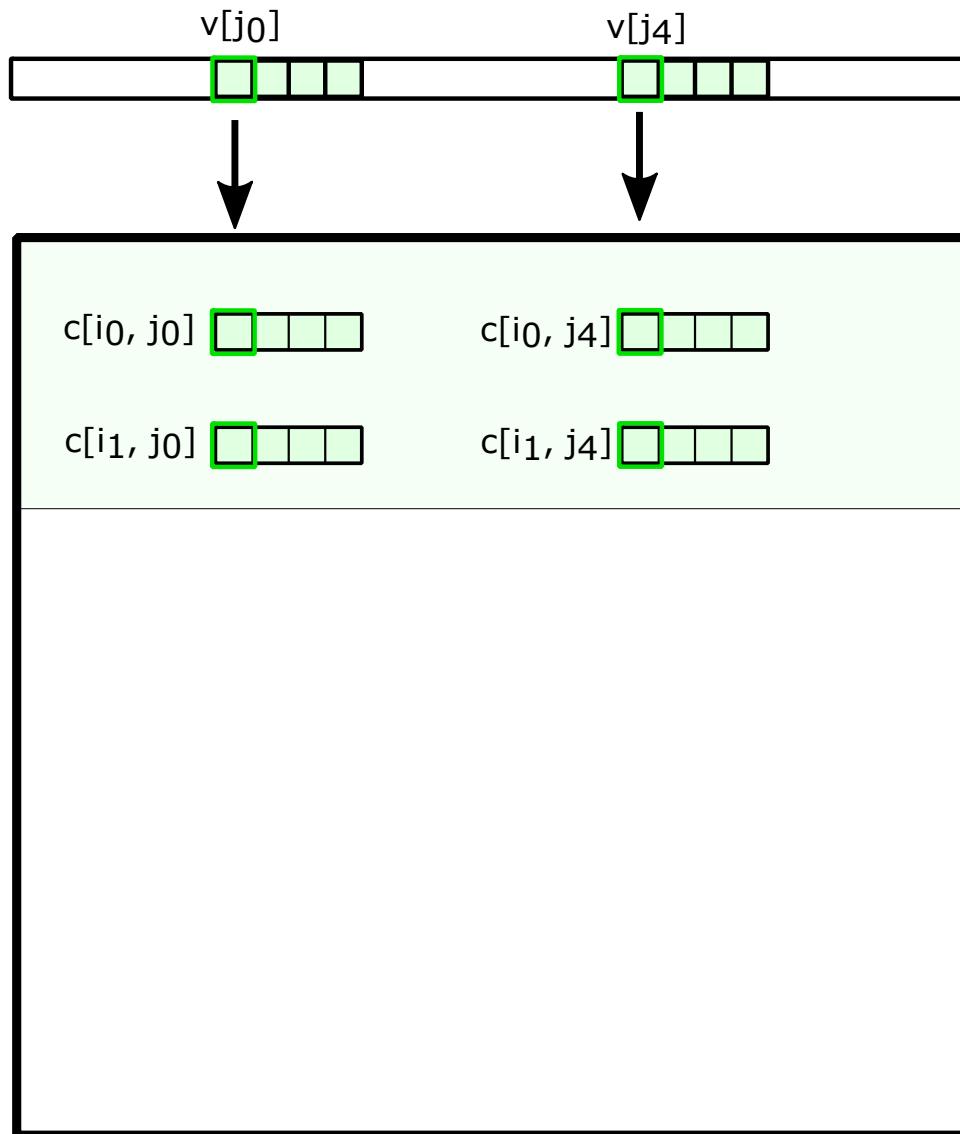
        // Convert to destination numeric type
        NumericArrayConverter<ElementOutput, ElementCompute, kCount, Round>
        destination_converter;

        return destination_converter(intermediate);
    }
}
```

“Thread-wide” epilogue functor

# BROADCASTING VECTORS

## Fusing operations in the Epilogue



Vector is broadcast over columns

```
# Matrix product computation
AB[m, n] = sum_k( A[m, k] * B[k, n] )

# Elementwise operations with broadcast
Z[m, n] = OutputOp0( AB[m, n], C[m, n], V[n] )
T[m, n] = OutputOp1( AB[m, n], C[m, n], V[n] )
```

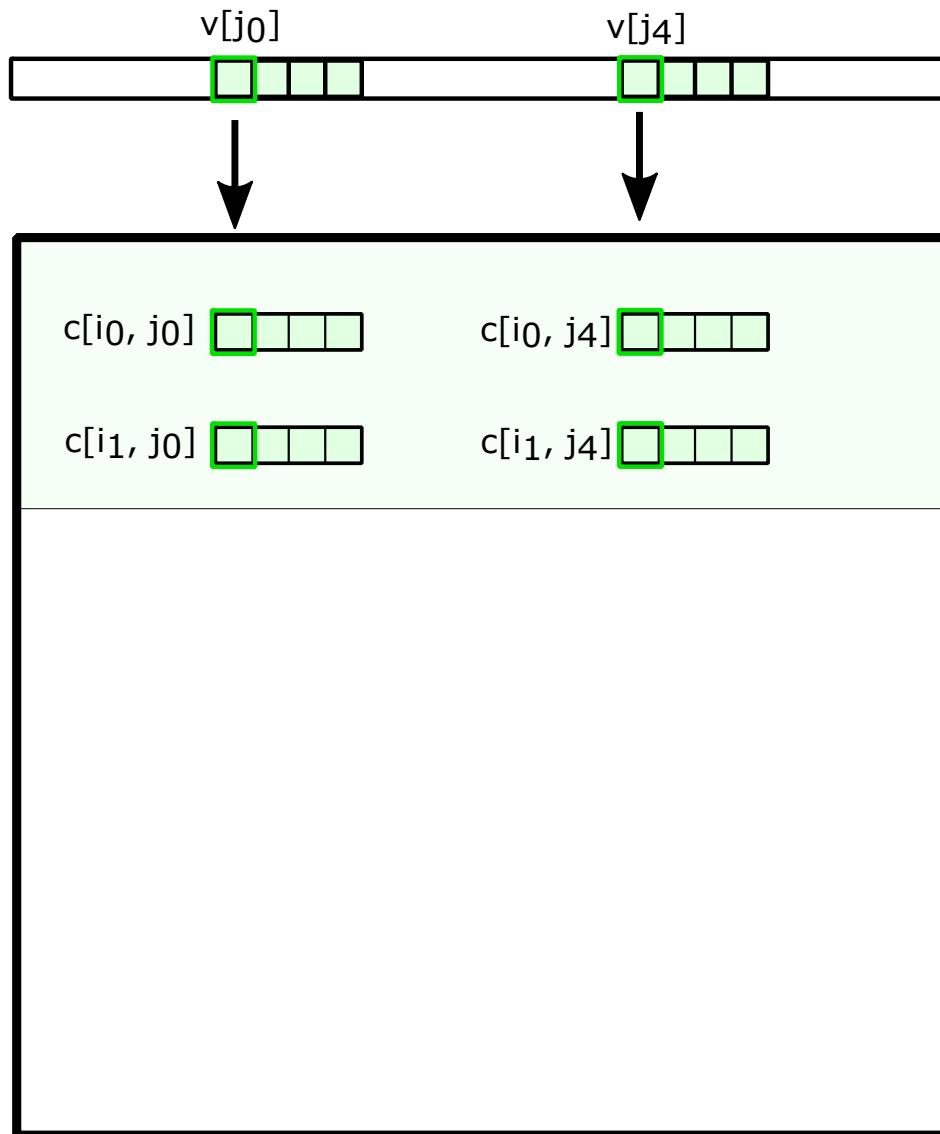
Elementwise operation may output two tensors Z and T

Vectors may be loaded prior to or during the Epilogue steps and broadcast to elementwise operations

See [include/cutlass/epilogue/threadblock/epilogue\\_with\\_broadcast.h](#)

# BROADCASTING VECTORS

## Fusing operations in the Epilogue



```
// Epilogue computes: Z, T = OutputOp(AB, C, V)
struct EpilogueWithBroadcast {

    CUTLASS_DEVICE void operator__()
        OutputOp const &output_op,
        ElementVector const * broadcast_ptr,
        ...
    }

    // Load the broadcast vector at the start of the epilogue
    BroadcastFragment broadcast_fragment;

    load_broadcast_fragment_()
        broadcast_fragment, broadcast_ptr, problem_size, threadblock_offset);

    ...

    // Iterate over accumulator tile
    #pragma unroll
    for (int iter = 0; iter < OutputTileIterator::kIterations; ++iter) {
        ...

        // Perform elementwise operation with broadcast vector
        #pragma unroll
        for (int i = 0; i < kOutputOpIterations; ++i) {

            output_op(
                frag_Z_ptr[i],
                frag_T_ptr[i],
                frag_AB_ptr[i],
                broadcast_fragment[i % ThreadMap::Iterations::kColumn]);
        }
        ...
    }
}
```

Vectors may be loaded prior to or during the Epilogue steps and broadcast to elementwise operations

See [include/cutlass/epilogue/threadblock/epilogue\\_with\\_broadcast.h](#)

# GEMM + Broadcast + Elementwise

“Device wide” operator definition  
and launch in host code

Define the epilogue functor for fused  
broadcast + elementwise

Define the device-wide GEMM operator  
E.g. GEMM “NN” using Tensor Cores

Construct “Arguments” structure

Additional arguments: pointers, leading  
dimensions, and batch strides

```
// GEMM with fused broadcast operation
//
// Computes the following:
//
//   Z[m, n] = OutputOp0( AB[m, n], C[m, n], V[n] )
//   T[m, n] = OutputOp1( AB[m, n], C[m, n], V[n] )

using EpilogueOutputOp = cutlass::epilogue::thread::LinearCombinationBiasElementwise<
    ElementOutput,
    ElementAccumulator,
    ElementCompute,
    ElementZ,
    ElementT,
    8,
    cutlass::epilogue::thread::GELU_taylor<float>
>;

using GemmKernel =
    typename cutlass::gemm::kernel::DefaultGemmWithBroadcast<
        cutlass::half_t, cutlass::layout::ColumnMajor, cutlass::ComplexTransform::kNone, 8,
        cutlass::half_t, cutlass::layout::ColumnMajor, cutlass::ComplexTransform::kNone, 8,
        cutlass::half_t, cutlass::layout::ColumnMajor,
        float,
        cutlass::arch::OpClassTensorOp,
        cutlass::arch::Sm80,
        cutlass::gemm::GemmShape<128, 128, 32>,
        cutlass::gemm::GemmShape<64, 64, 32>,
        cutlass::gemm::GemmShape<16, 8, 16>,
        EpilogueOutputOp,
        cutlass::gemm::threadblock::GemmIdentityThreadblockSwizzle<8>,
        3,
        cutlass::arch::OpMultiplyAdd
    >::GemmKernel;

using Gemm = cutlass::gemm::device::GemmUniversalAdapter<GemmKernel>;

typename Gemm::Arguments arguments{
    ...
    tensor_Broadcast.device_data(),
    tensor_T.device_data(),
    ...
    tensor_Z.layout().stride(0),
    tensor_T.layout().stride(0),
};
```

# GEMM + Broadcast + Elementwise example functor: LinearCombinationBiasElementwise

Functor outputs `frag\_Z` and `frag\_T`

`LinearCombinationBiasElementwise`  
is specialized for linear scaling with  
broadcast

Function call operator performs  
elementwise arithmetic and conversion

Output arguments are stored to Global  
Memory by EpilogueWithBroadcast

“Thread-wide” epilogue functor

```
template <
...
typename ElementwiseOp_ = Identity<ElementCompute_>,
typename BinaryOp_ = plus<ElementCompute_>
>
class LinearCombinationBiasElementwise {
public:

    /// Applies the operation when is_source_needed() is false
    CUTLASS_HOST_DEVICE
    void operator__()
        FragmentZ &frag_Z,
        FragmentT &frag_T,
        FragmentAccumulator const &AB,
        FragmentCompute const &V) const {

        ElementwiseOp elementwise_op;
        BinaryOp binary_op;

        NumericArrayConverter<
            ElementCompute, ElementAccumulator, kElementsPerAccess> accumulator_converter;

        FragmentCompute tmp_Accum = accumulator_converter(AB);
        FragmentCompute result_Z;
        FragmentCompute result_T;

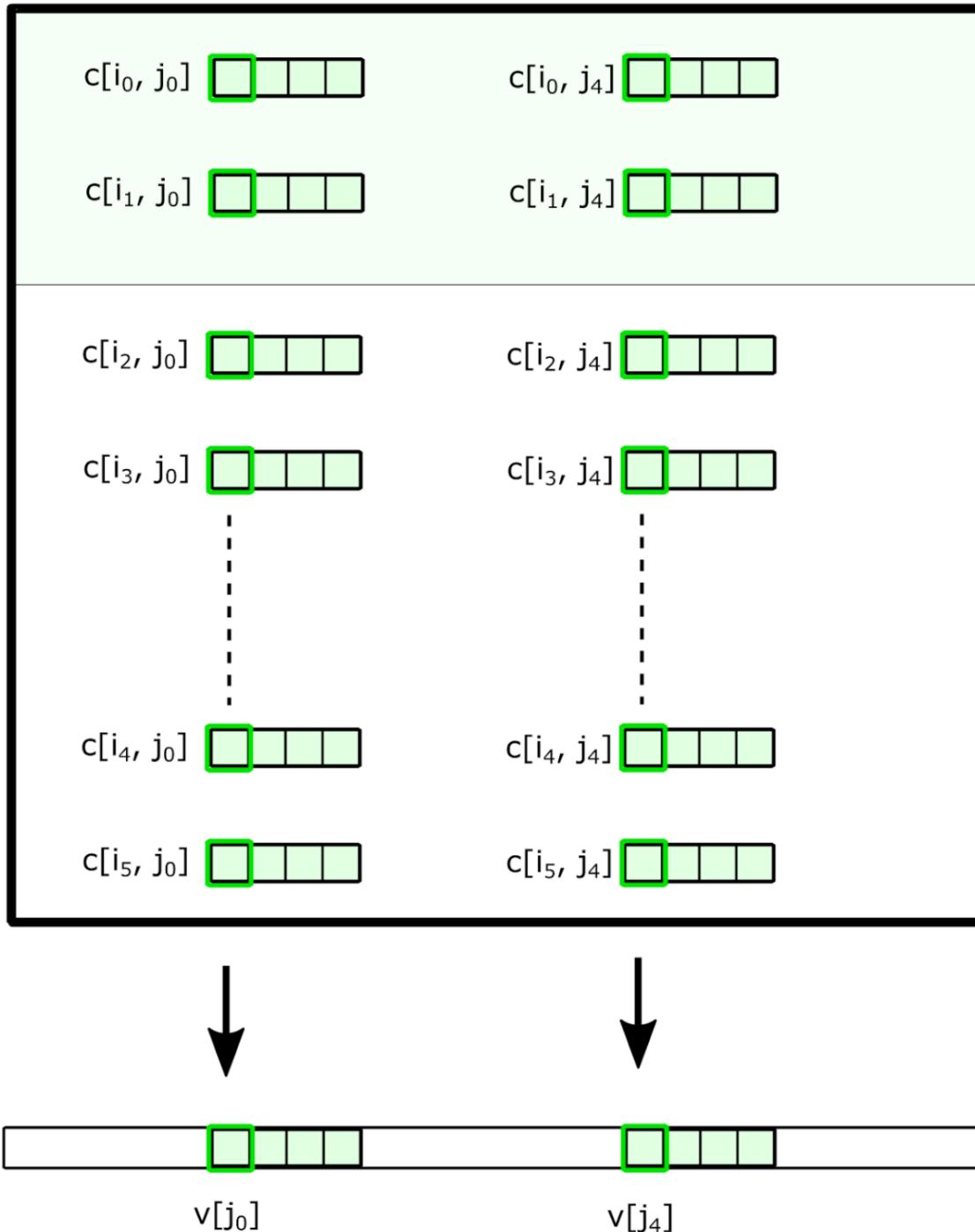
        CUTLASS_PRAGMA_UNROLL
        for (int i = 0; i < kElementsPerAccess; ++i) {
            ElementCompute z = binary_op(alpha_ * tmp_Accum[i], V[i]);
            result_Z[i] = z;
            result_T[i] = elementwise_op(z);
        }

        NumericArrayConverter<ElementZ, ElementCompute, kElementsPerAccess> convert_z;
        frag_Z = convert_z(result_Z);

        NumericArrayConverter<ElementT, ElementCompute, kElementsPerAccess> convert_t;
        frag_T = convert_t(result_T);
    }
};
```

# REDUCTIONS OVER GEMM OUTPUT

Fusing operations in the Epilogue



Columns reduced by custom reduction operator

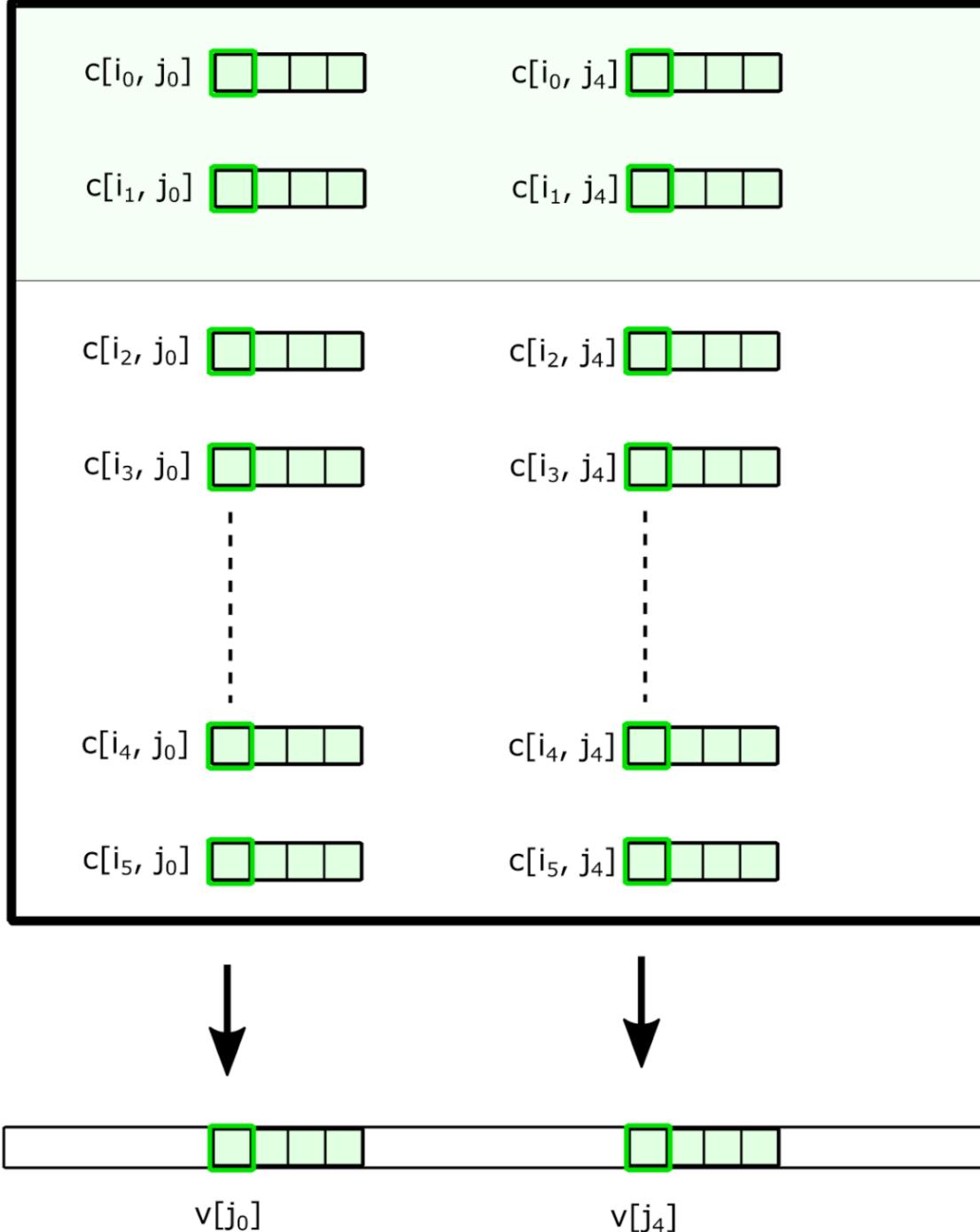
```
# Matrix product computation  
AB[m, n] = sum_k( A[m, k] * B[k, n] )  
  
# Elementwise operation  
Z[m, n] = OutputOp( AB[m, n], C[m, n], T[m, n] )  
  
# Reduction operation  
V[n]      = Reduce_m( Z[m, n], identity_element )
```

Reductions over Threadblock tile may be fused into Epilogue

See [include/cutlass/epilogue/threadblock/epilogue\\_with\\_reduction.h](#)

# REDUCTIONS OVER GEMM OUTPUT

## Fusing operations in the Epilogue



Reductions over Threadblock tile may be fused into Epilogue

See [include/cutlass/epilogue/threadblock/epilogue\\_with\\_reduction.h](#)

```
// Epilogue with reduction
struct EpilogueWithReduction {

    CUTLASS_DEVICE void operator()(

        OutputOp const &output_op,
        ElementVector * reduction_output_ptr,
        ...
    ) {

        ReductionFragment reduction_fragment;
        reduction_fragment.clear();

        // Iterate over accumulator tile
        for (int iter = 0; iter < OutputTileIterator::kIterations; ++iter) {
            ...

            // Elementwise operation
            for (int i = 0; i < kOutputOpIterations; ++i) {
                compute_fragment[i] = output_op(accumulator_fragment[i]);
            }

            // Partial reduction within a thread
            for (int column = 0; column < ReductionDetail::kColumnsPerThread; ++column) {
                for (int row = 0; row < ReductionDetail::kRowsPerThread; ++row) {

                    reduction_fragment[column] = reduction_op(
                        reduction_fragment[column],
                        compute_fragment[row * ReductionDetail::kColumnsPerThread + column]);
                }
            }
        }

        ...
    }

    __syncthreads();

    // Data exchange through Shared Memory, serial reduction, and store to Global
    Memory
    reduction_(problem_size, threadblock_offset, reduction_output_ptr,
    reduction_fragment);
}
};
```

# GEMM + Elementwise + Reduction

“Device wide” operator definition  
and launch in host code

```
// GEMM with fused partial reduction
//
// Computes the following:
//
//   Z[m, n] = OutputOp( AB[m, n], C[m, n], T[m, n] )
//
//   V[n]     = Reduce_m( Z[m, n], identity_element )

using EpilogueOutputOp = cutlass::epilogue::thread::LinearCombinationDReluConditionalBits<
    ElementCompute,
    ElementAccumulator,
    ElementOutput,
    8
>;

using ReductionOp = cutlass::plus<float>

using GemmKernel =
    typename cutlass::gemm::kernel::DefaultGemmWithReduction<
        cutlass::half_t, cutlass::layout::ColumnMajor, cutlass::ComplexTransform::kNone, 8,
        cutlass::half_t, cutlass::layout::ColumnMajor, cutlass::ComplexTransform::kNone, 8,
        cutlass::half_t, cutlass::layout::ColumnMajor,
        float,
        cutlass::arch::OpClassTensorOp,
        cutlass::arch::Sm80,
        cutlass::gemm::GemmShape<128, 128, 32>,
        cutlass::gemm::GemmShape<64, 64, 32>,
        cutlass::gemm::GemmShape<16, 8, 16>,

        EpilogueOutputOp,                                // Elementwise functor
        ReductionOp,                                    // Reduction functor

        cutlass::gemm::threadblock::GemmIdentityThreadblockSwizzle<8>,
        5,
        cutlass::arch::OpMultiplyAdd
    >::GemmKernel;

using Gemm = cutlass::gemm::device::GemmUniversalAdapter<GemmKernel>;
```

Define the elementwise functor

Define the reduction functor

Define the device-wide GEMM operator

E.g. GEMM “NN” using Tensor Cores

Specify elementwise and reduction functors

Reduction is performed over one threadblock  
Dependent kernel launch needed for final  
reduction

# GEMM + Elementwise + Reduction

example functor:

LinearCombinationDReluConditionalBits

Functor outputs one fragment prior to reduction

‘LinearCombinationDReluConditionalBits’ accepts an optional tensor of 1b condition bits produced by the forward pass

Condition bits are unpacked into ‘`bool`’ elements and used to conditionally output  $dy=0$ .

Returned value ‘`dy`’ stored to Global Memory by EpilogueWithReduction

“Thread-wide” epilogue functor

```
class LinearCombinationDReluConditionalBits {
public:

    // Computes linear scaling: D = alpha * accumulator
    #ifdef CUTLASS_HOST_DEVICE
    FragmentCompute operator__()
    #endif
    FragmentAccumulator const &accumulator,
    FragmentTensor const &tensor) const {

        // Convert source to internal compute numeric type
        NumericArrayConverter<
            ElementCompute, ElementAccumulator, kCount, Round> accumulator_converter;

        FragmentCompute converted_accumulator = accumulator_converter(accumulator);

        // Perform binary operations
        multiplies<FragmentCompute> mul_accumulator;

        // dy = alpha * Accum
        FragmentCompute dy = mul_accumulator(alpha_, converted_accumulator);

        // Obtain from packed bits
        bool conditions[kCount];

        UnpackPredicates<kCount> unpack_predicates;

        unpack_predicates(conditions, tensor);

        // dReLU = (cond ? dy : 0)
        #ifdef CUTLASS_PRAGMA_UNROLL
        for (int i = 0; i < kCount; ++i) {
            if (!conditions[i]) {
                dy[i] = ElementCompute();
            }
        }
        #endif
        return dy;
    };
}
```



# CONCLUSION

# CUTLASS PROVIDES REUSABLE TEMPLATES FOR FUSING WITH GEMM AND CONVOLUTION

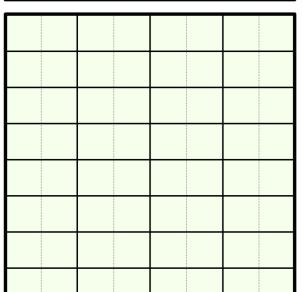
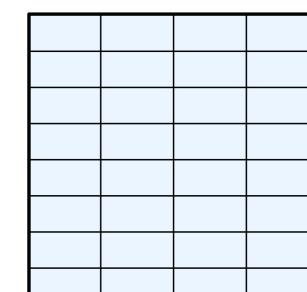
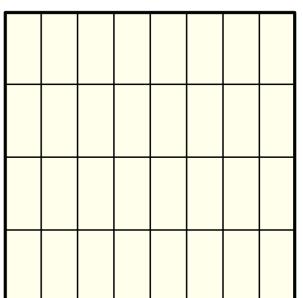
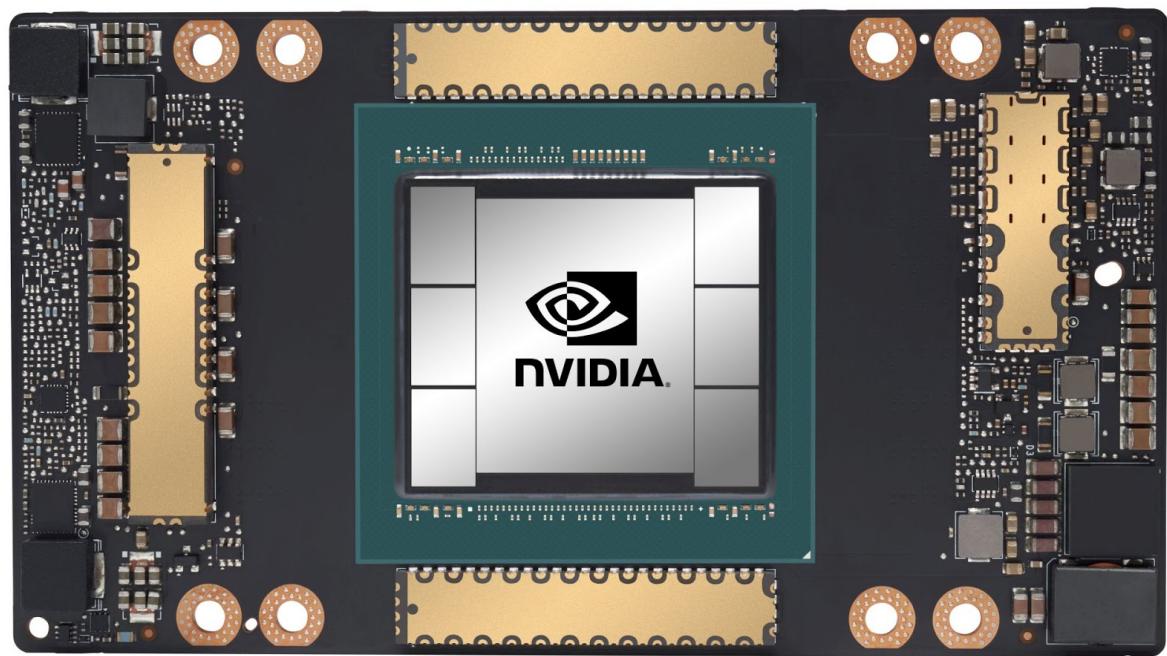
## CUTLASS

- CUTLASS provides tuned implementations of GEMM and CONV
- Extensible via custom functors on input and output
- Client defines functors for: elementwise, broadcast, reduction
- Capable of near-peak performance with CUDA 11.5 Toolkit

## CUTLASS 2.8: November 2021

- Block-resident CONV-CONV fused kernels
- Emulated single-precision with TensorFloat32
- Implicit GEMM convolution fusion on input
- Grouped GEMM implementation

Try it out! <https://github.com/NVIDIA/cutlass>



# REFERENCES

## NVIDIA Ampere Architecture:

- “Inside the NVIDIA Ampere Architecture” (GTC 2020 - S21730)
- “NVIDIA Ampere Architecture In-Depth” ([blog post](#))
- “CUDA New Features and Beyond” (GTC 2020 - S21760)
- “Tensor Core Performance on NVIDIA GPUs” (GTC 2020 - S21929)
- “Inside the Compilers, Libraries and Tools for Accelerated Computing” (GTC 2020 - S21766)

## CUTLASS

<https://github.com/NVIDIA/cutlass> (open source software, New BSD license)

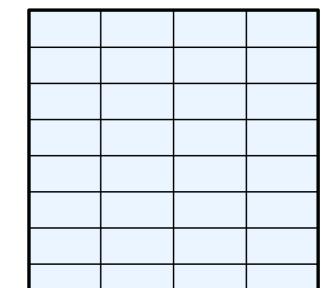
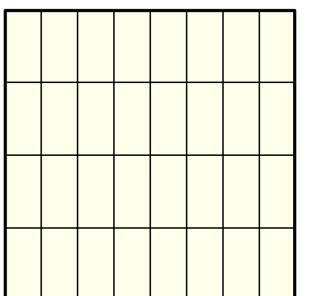
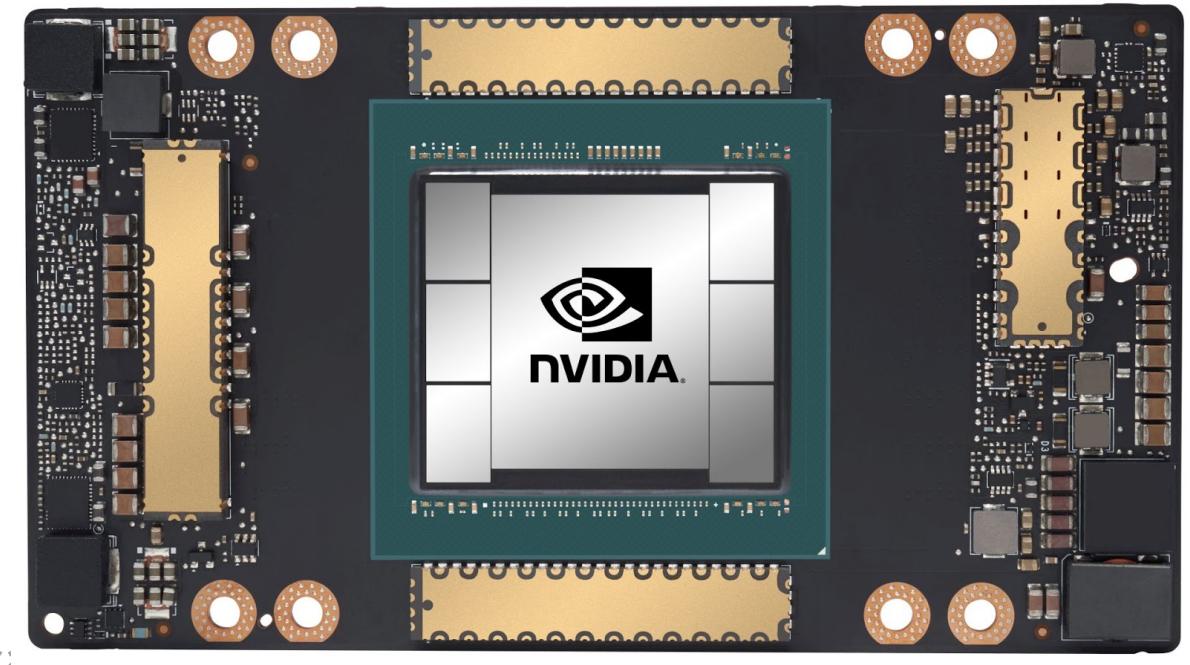
[CUTLASS Parallel For All blog post](#)

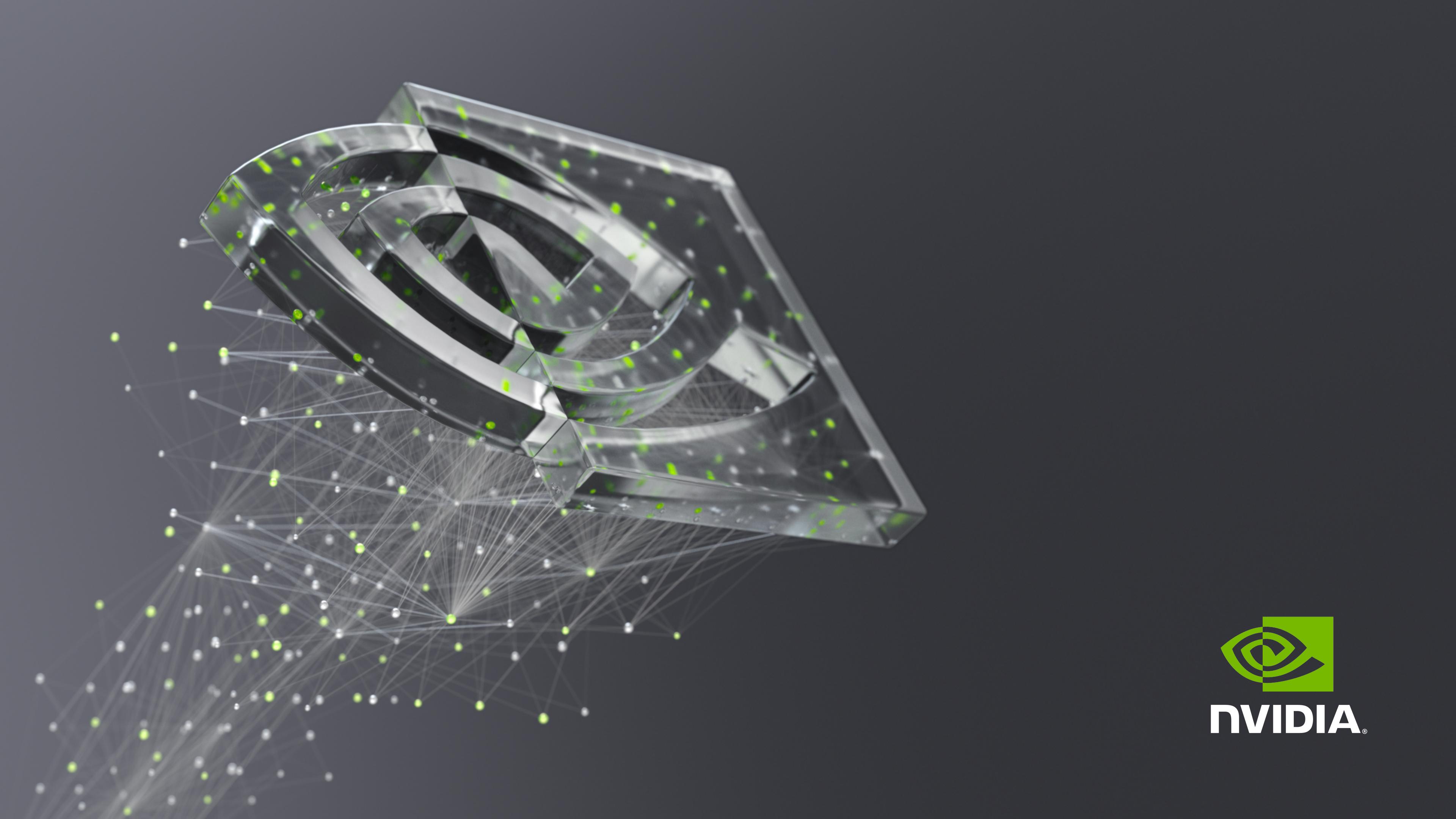
[GTC 2018 talk \(S8854\)](#) : CUTLASS: Software primitives for dense linear algebra at all levels and scales within CUDA

[GTC 2019 talk \(S9593\)](#) : cuTENSOR:High-performance Tensor Operations in CUDA (joint talk with cuTENSOR)

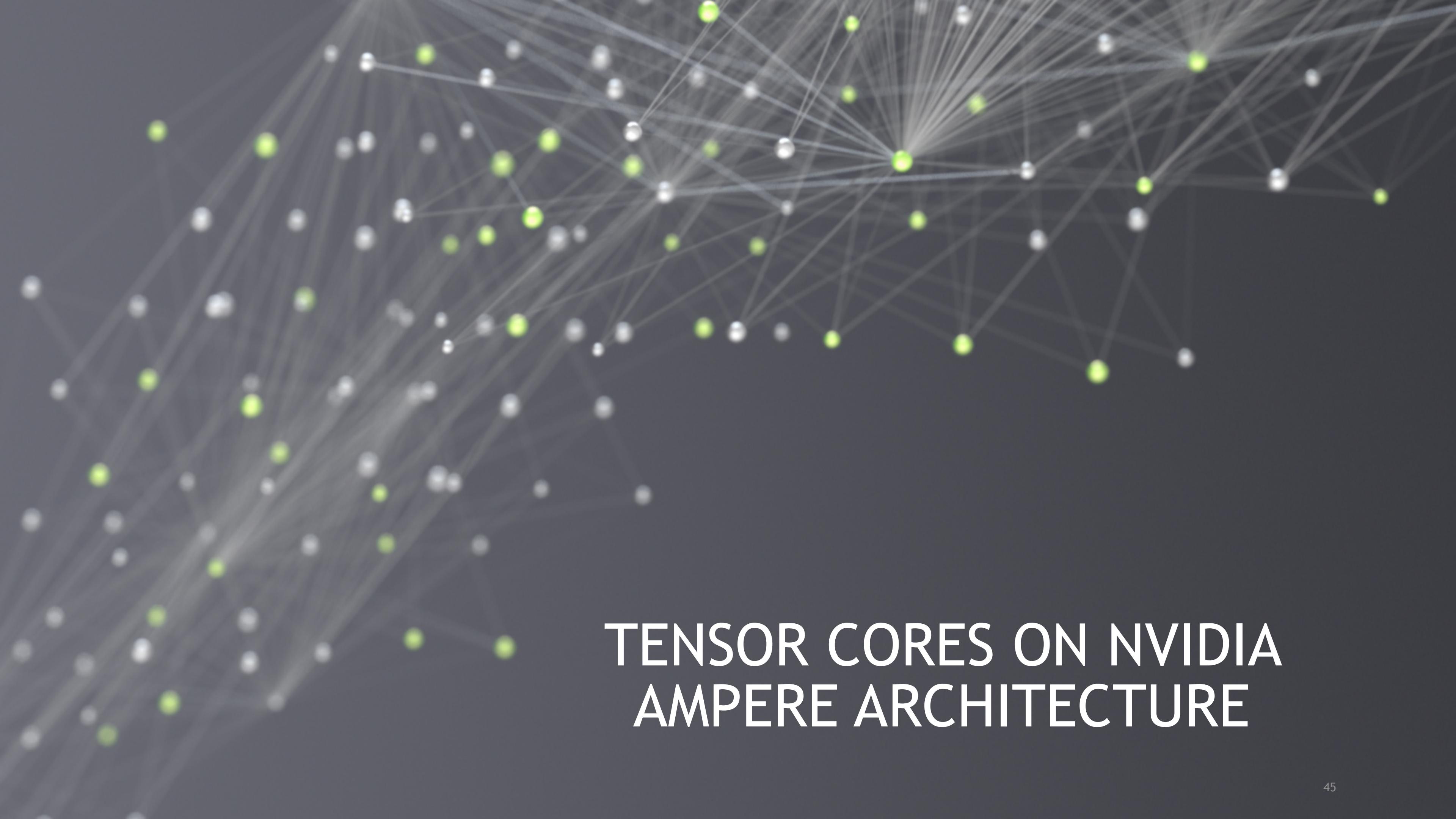
[GTC 2020 talk \(S21745\)](#) : Developing CUDA kernels to push Tensor Cores to the Absolute Limit on NVIDIA A100

[GTC 2021 talk \(S31883\)](#) : Accelerating Convolution with Tensor Cores in CUTLASS





NVIDIA®



# TENSOR CORES ON NVIDIA AMPERE ARCHITECTURE

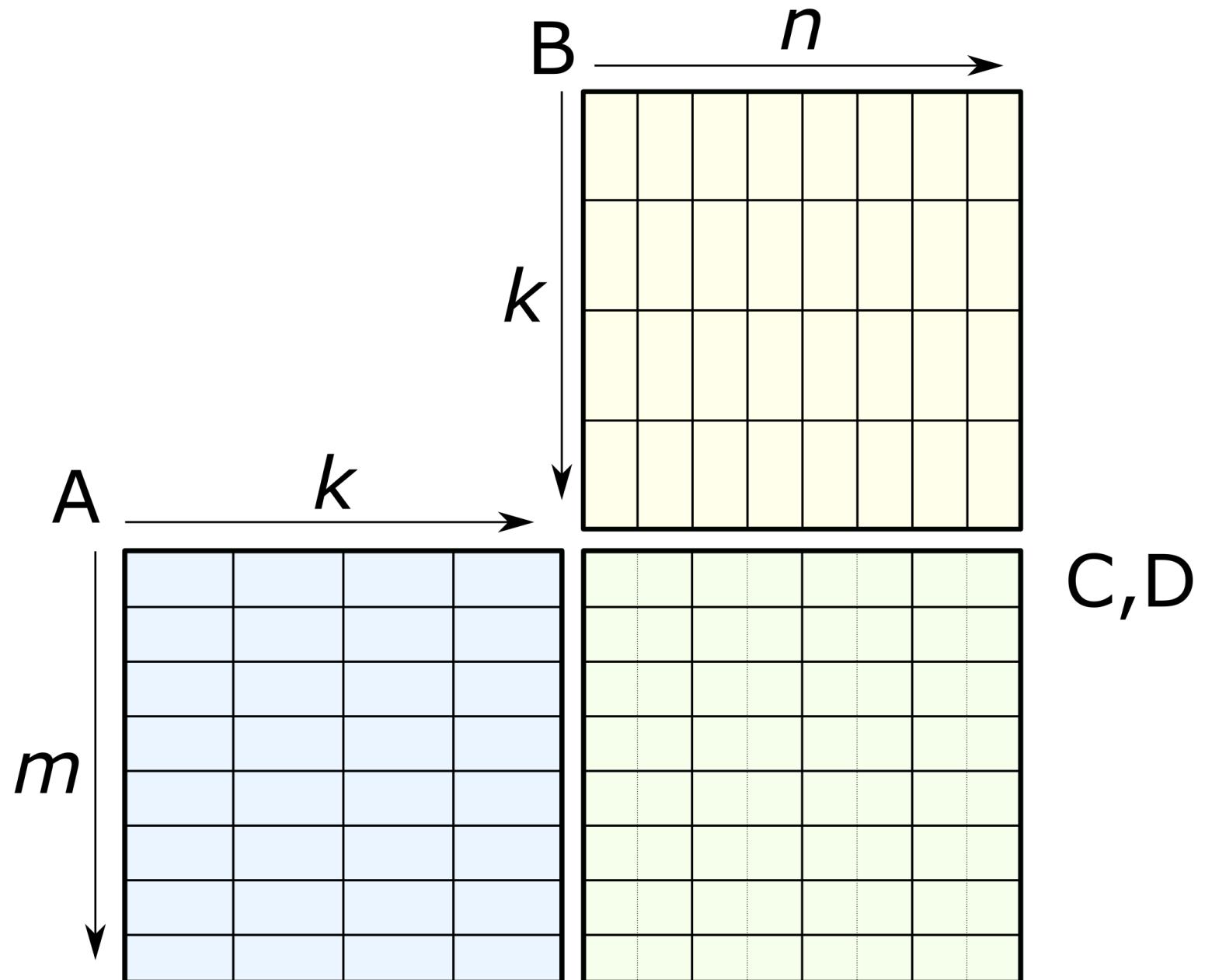
# WHAT ARE TENSOR CORES?

Matrix operations:  $D = \text{op}(A, B) + C$

- Matrix multiply-add
- XOR-POPC

$M$ -by- $N$ -by- $K$  matrix operation

- Warp-synchronous, collective operation
- 32 threads within warp collectively hold A, B, C, and D operands



# NVIDIA AMPERE ARCHITECTURE - TENSOR CORE OPERATIONS

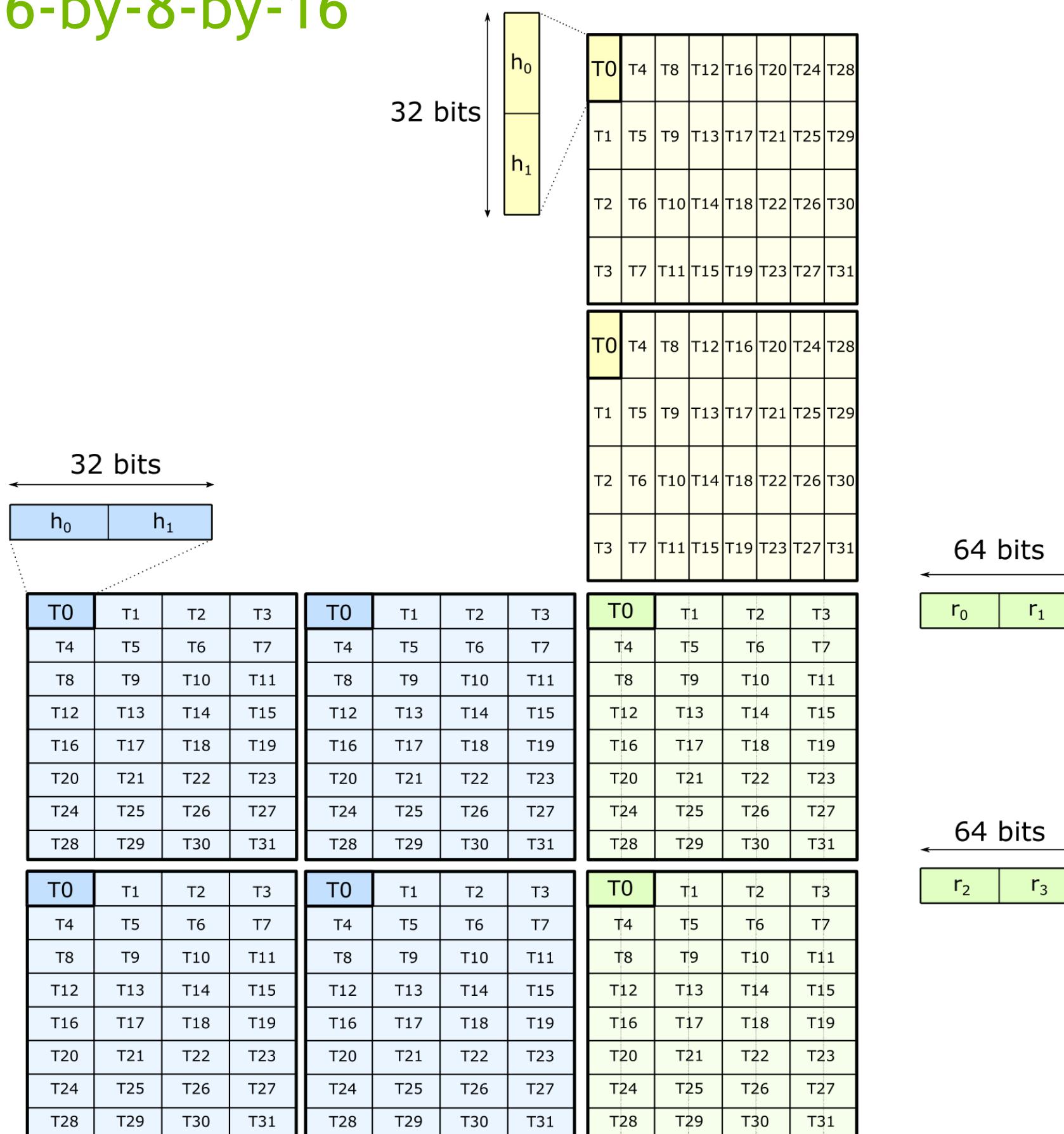
PTX	Data Types (A * B + C)	Shape	Speedup on NVIDIA A100 (vs F32 CUDA cores)	Speedup on Turing* (vs F32 Cores)	Speedup on Volta* (vs F32 Cores)
mma.sync.m16n8k16 mma.sync.m16n8k8	F16 * F16 + F16	16-by-8-by-16 16-by-8-by-8	16x	8x	8x
	F16 * F16 + F32				
	BF16 * BF16 + F32				
mma.sync.m16n8k8	TF32 * TF32 + F32	16-by-8-by-8	8x	N/A	N/A
mma.sync.m8n8k4	F64 * F64 + F64	8-by-8-by-4	2x	N/A	N/A
mma.sync.m16n8k32 mma.sync.m8n8k16	S8 * S8 + S32	16-by-8-by-32 8-by-8-by-16	32x	16x	N/A
mma.sync.m16n8k64	S4 * S4 + S32	16-by-8-by-64	64x	32x	N/A
mma.sync.m16n8k256	B1 ^ B1 + S32	16-by-8-by-256	256x	128x	N/A

<https://docs.nvidia.com/cuda/parallel-thread-execution/index.html#warp-level-matrix-instructions-mma-and-friends>

\* Instructions with equivalent functionality for Turing and Volta differ in shape from the NVIDIA Ampere Architecture in several cases.

# F16 \* F16 + F32

## 16-by-8-by-16



mma.sync.aligned  
(via inline PTX)

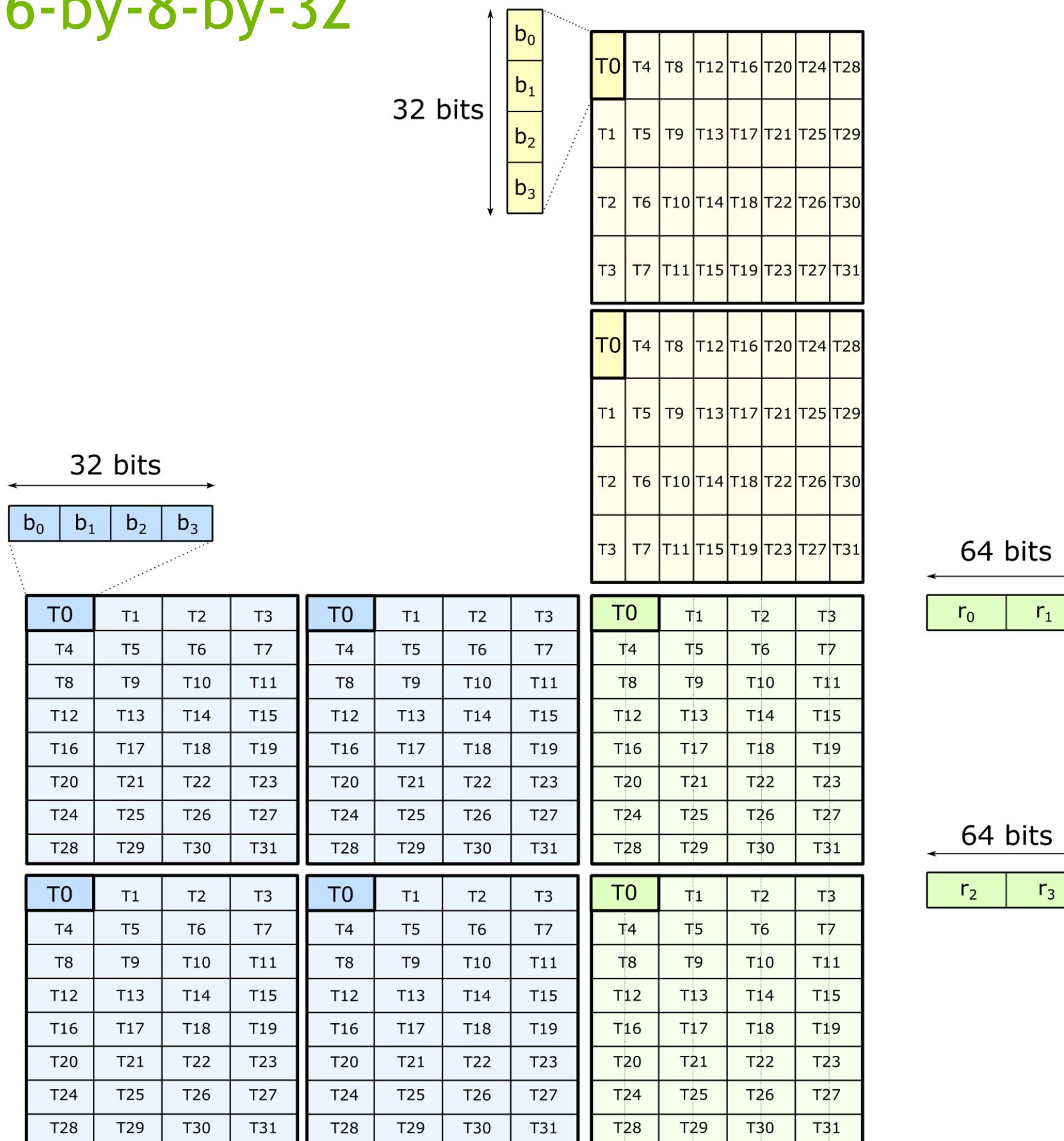
```
float          D[ 4 ];
uint32_t const A[ 4 ];
uint32_t const B[ 2 ];
float        const C[ 4 ];
```

```
// Example targets 16-by-8-by-32 Tensor Core operation

asm(
    "mma.sync.aligned.m16n8k16.row.col.f32.f16.f16.f32 "
    " { %0, %1, %2, %3 }, "
    " { %4, %5, %6, %7 }, "
    " { %8, %9 },
    " { %10, %11, %12, %13 }; "
    :
    "=f"(D[ 0 ]), "=f"(D[ 1 ]), "=f"(D[ 2 ]), "=f"(D[ 3 ])
    :
    "r"(A[ 0 ]), "r"(A[ 1 ]), "r"(A[ 2 ]), "r"(A[ 3 ]),
    "r"(B[ 0 ]), "r"(B[ 1 ]),
    "f"(C[ 0 ]), "f"(C[ 1 ]), "f"(C[ 2 ]), "f"(C[ 3 ])
);
```

# $S8 * S8 + S32$

## 16-by-8-by-32



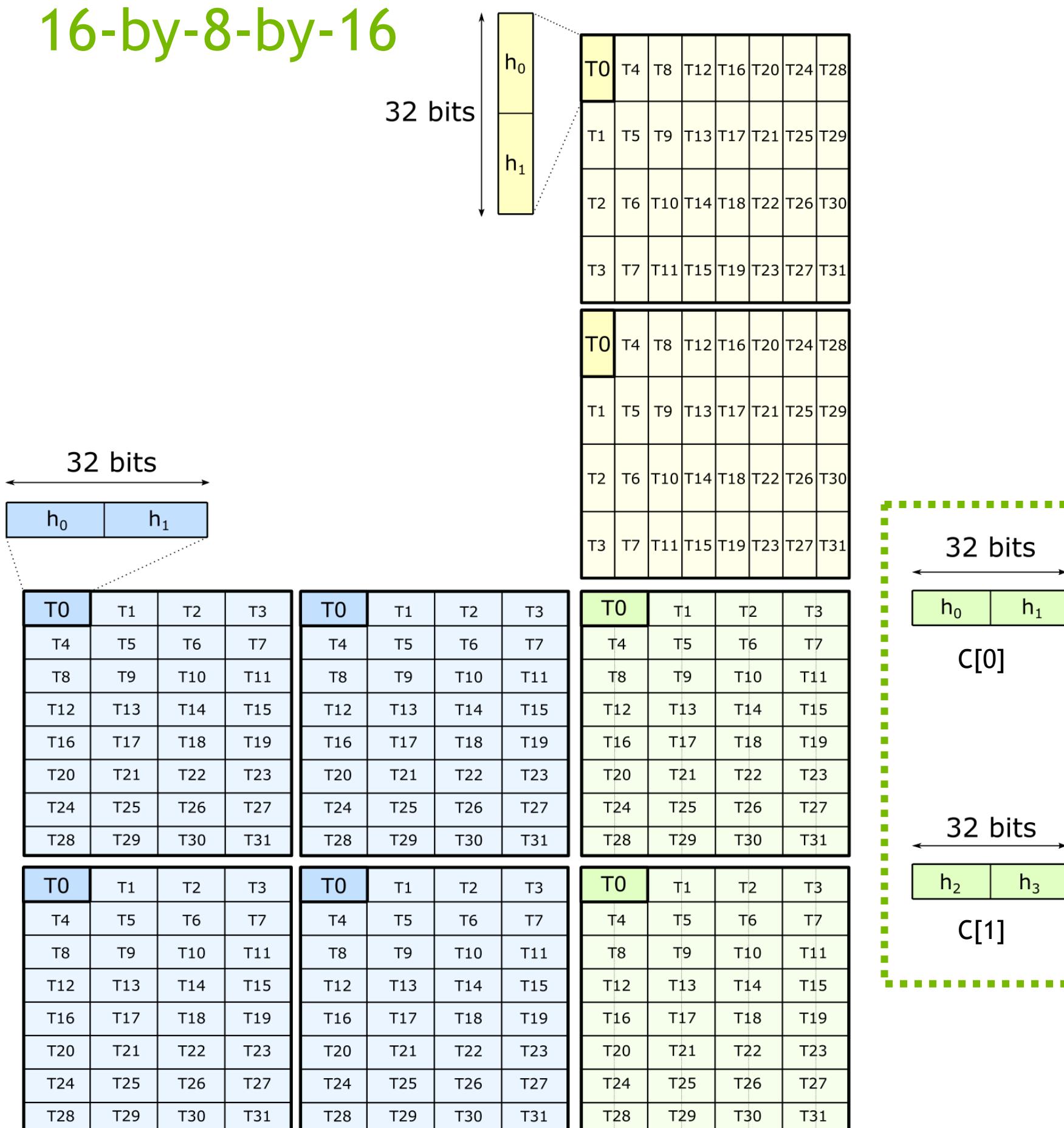
## mma.sync.aligned (via inline PTX)

```
int32_t          D[ 4 ];
uint32_t const  A[ 4 ];
uint32_t const  B[ 2 ];
int32_t const   C[ 4 ];
```

```
// Example targets 16-by-8-by-32 Tensor Core operation
asm(
    "mma.sync.aligned.m16n8k32.row.col.s32.s8.s8.s32 "
    " { %0, %1, %2, %3 },      "
    " { %4, %5, %6, %7 },      "
    " { %8, %9 },                "
    " { %10, %11, %12, %13 }; "
    " =r"(D[ 0 ]), =r"(D[ 1 ]), =r"(D[ 2 ]), =r"(D[ 3 ])
    " r"(A[ 0 ]), r"(A[ 1 ]), r"(A[ 2 ]), r"(A[ 3 ]),
    " r"(B[ 0 ]), r"(B[ 1 ]),
    " r"(C[ 0 ]), r"(C[ 1 ]), r"(C[ 2 ]), r"(C[ 3 ])
);
```

# HALF-PRECISION : F16 \* F16 + F16

16-by-8-by-16



mma.sync.aligned  
(via inline PTX)

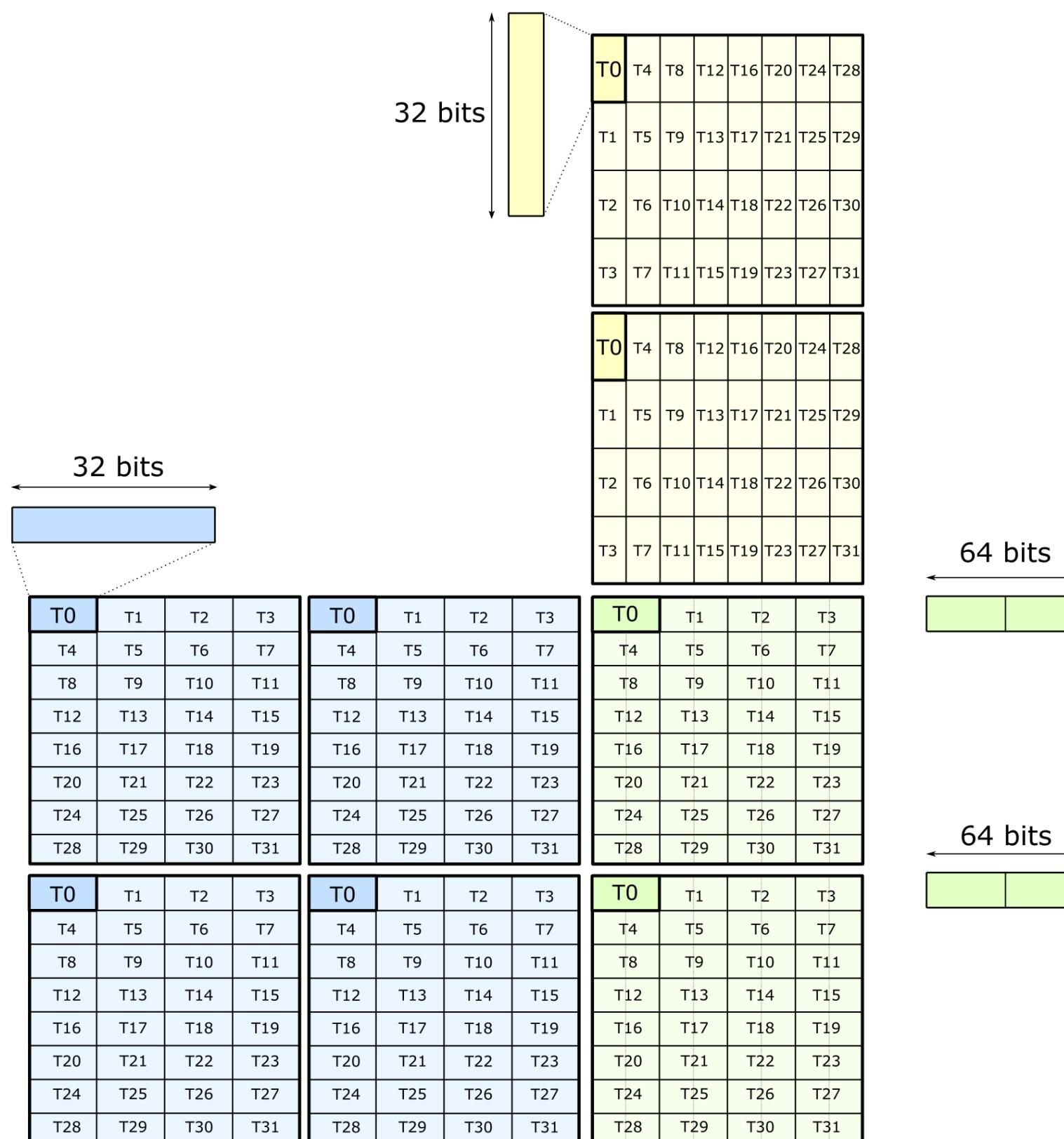
```
uint32_t          D[2]; // two registers needed (vs.
four)
uint32_t const A[4];
uint32_t const B[2];
uint32_t const C[2]; // two registers needed (vs.
four)
```

// Example targets 16-by-8-by-16 Tensor Core operation

```
asm(
    "mma.sync.aligned.m16n8k16.row.col.f16.f16.f16 "
    " { %0, %1}, "
    " { %2, %3, %4, %5 }, "
    " { %6, %7 }, "
    " { %8, %9 }; "
    :
    "=r"(D[0]), "=r"(D[1])
    :
    "r"(A[0]), "r"(A[1]), "r"(A[2]), "r"(A[3]),
    "r"(B[0]), "r"(B[1]),
    "r"(C[0]), "r"(C[1])
);
```

# CUTLASS: wraps PTX in template

*m-by-n-by-k*



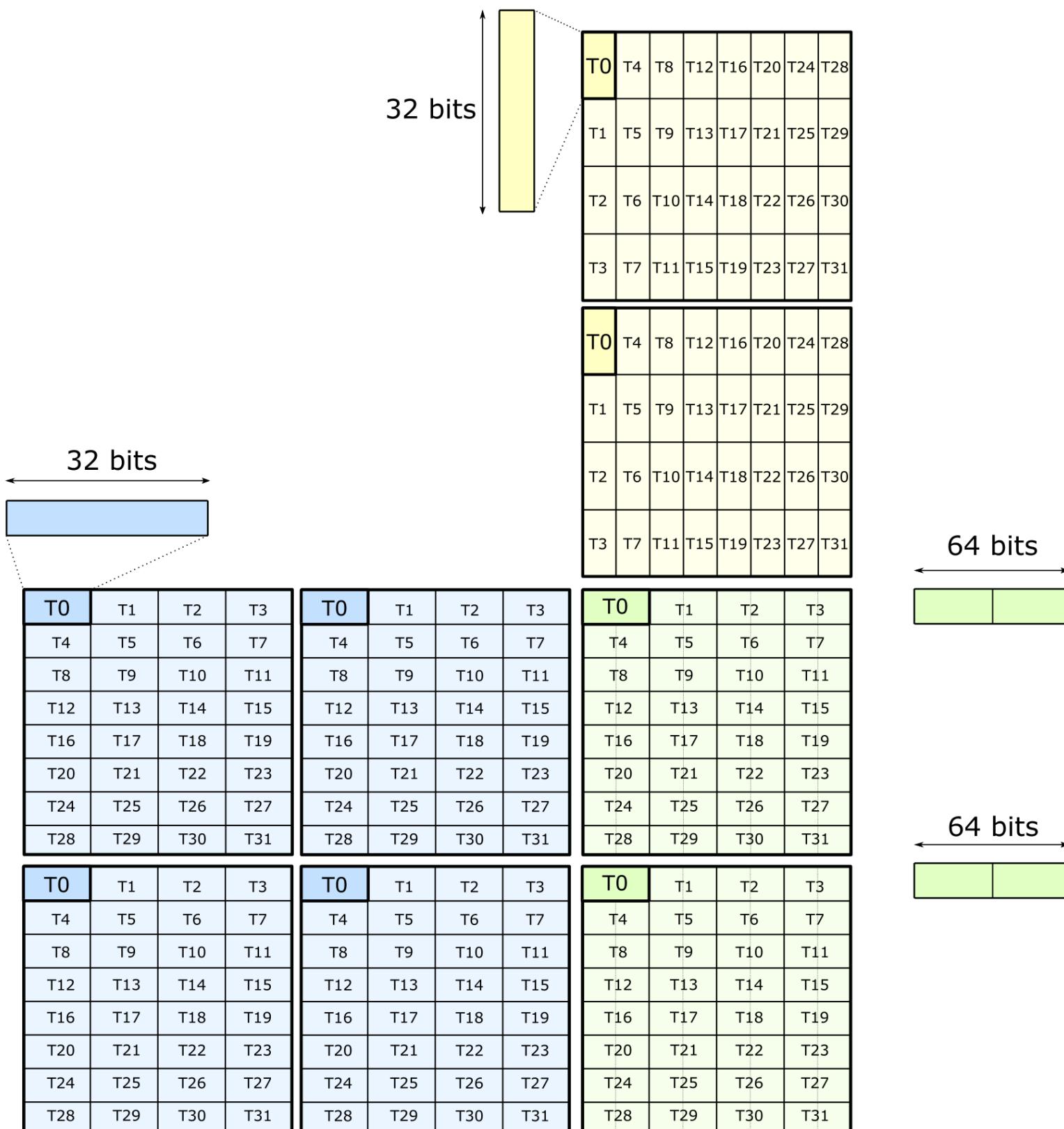
`cutlass::arch::Mma`

```
/// Matrix multiply-add operation
template <
    /// Size of the matrix product (concept: GemmShape)
    typename Shape,
    /// Number of threads participating
    int kThreads,
    /// Data type of A elements
    typename ElementA,
    /// Layout of A matrix (concept: MatrixLayout)
    typename LayoutA,
    /// Data type of B elements
    typename ElementB,
    /// Layout of B matrix (concept: MatrixLayout)
    typename LayoutB,
    /// Element type of C matrix
    typename ElementC,
    /// Layout of C matrix (concept: MatrixLayout)
    typename LayoutC,
    /// Inner product operator
    typename Operator
>

struct Mma;
```

# CUTLASS: wraps PTX in template

16-by-8-by-16



`cutlass::arch::Mma`

```
__global__ void kernel() {  
  
    // arrays containing logical elements  
    Array<half_t, 8> A;  
    Array<half_t, 4> B;  
    Array< float, 4> C;  
  
    // define the appropriate matrix operation  
    arch::Mma< GemmShape<16, 8, 16>, 32, ... > mma;  
  
    // in-place matrix multiply-accumulate  
    mma(C, A, B, C);  
  
    ...  
}
```