Autonomous Robot Motion

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1 | Introduction

This part will have the following setup:

- Context
- Definitions & Notations
- Outline
- Summary of our results
- My contribution

Definitions

We investigate a blind robot that navigates in a scene S from the starting point s to a target t which are a distance n apart. S contains a finite number of convex inpenetrable obstacles. The robot is assumed to be a mobile point automaton that is only aware of its absolute position and the position of t. It is blind, and only receives sensory feedback when it hits an obstacle. It can then follow the edge of the obstacle without knowing about its shape.

The robot will always try to move in a straight line to t. When the robot hits an obstacle, it will follow the shape of the object in the direction that initially minimizes the distance to the target until the path to t is clear again. It will repeat its behavior. We assume without loss of generality that if the robot hits the obstacle perpendicularly, the robot moves counterclockwise. This robotic behavior can be categorized as memoryless and dynamic, since it does not plan its path in advance, but decides on its direction every time it touches an obstacle, solely based on its current position, the position of the target and the gradient of the object it hits.

Obstacles may touch each other and the robot is a point, so the robot can "squeeze" through touching obstacles, avoiding the creation of concave objects out of convex ones.

We use a similar notation as (Blum, A., 198x). Let $R(S)$ be the total distance walked by the robot going from s to t in its heuristic path. $d(S)$ denotes the length of the shortest possible path. We are interested in $R(S)$ and will compare it to $d(s)$ and n using two ratios:

$$\rho = \max_S \frac{R(S)}{d(S)}, \quad \gamma = \max_S \frac{R(S)}{n}. \tag{1.1}$$

2 | Previous research: see papers & abstracts

- What do we do different
- Compare with our results

3 | Equal Size Squares

This section goes through the earlier work by Henk and Marijke on equal size squares. It basically gives the proof as presented in `paper.henk.pdf`.

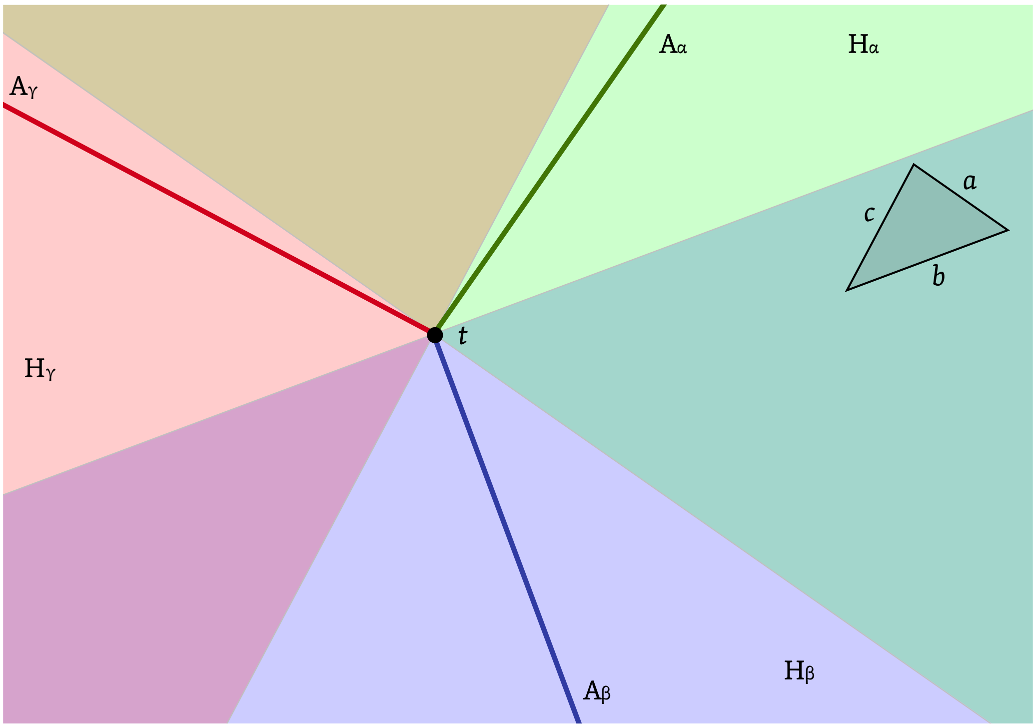
4 | Circles

This section will only be included if we do not manage to generalize our results to arbitrary convex objects. If we include it, it shows the numerical bounds for the worst-case heuristic paths over the optimal path or distance.

5 | Similar Same-Orientation *Sharp* Triangles

Asume all obstacles in S to be similar triangles of the same orientation. We use a coordinate system with t in the origin. The sides of triangle k are called a_k, b_k, c_k such that all sides a_i have the same direction α, all sides b_i have direction β and all sides c_i have direction γ. For each side-direction $x \in \{\alpha, \beta, \gamma\}$, we define a half plane H_x that is delimited by a line in the direction x through t. It indicates the region in which the robot could hit a triangle at a side in direction x. The half planes are well defined because the robot always moves towards t.

Next, define 'axes'. For a direction $x \in \{\alpha, \beta, \gamma\}$, define A_x as a ray from the origin into H_x that has a direction perpendicular to x. By definition, if a the robot hits an obstacle at the x-side, it will always be in H_x and decide to move towards A_x.



Illustration

Theorem 1.1

If the robot crosses an axis A_x at a Eudlidian distance λ from t, it will never cross the same axis at a distance $\geq \lambda$ later in time.

Proof

In order to prove theorem 1.1, we start by introducing some concepts that play a role in the argument. First, let's define a custom measurement M for a point's distance to t. Let Γ_1 be a triangle that is similar to the obstacles but rotated by $180°$ compared to the obstacles. Let Γ_1 have its orthocenter in t and a circumference of 1. Note that the vertices of Γ_1 are by definition on the axes A_α, A_β and A_γ. The distance $M(p)$ of a point p to t is defined as the factor $f \geq 0$ with which Γ_1 should be scaled with respect to t such that $p \in \Gamma_1$.

The heuristic path $HP(s, t)$ crosses the three axes 0 or more times. Let $\mathbf{c} = \{c_1, c_2, \ldots\}$ be a vector containing those crossing points and let $A(c_i)$ be the corresponding axes and $H(c_i)$ the corresponding half-plane.

Furthermore, for a point $p \in H_x$, define $D_x(p)$ as the M-distance between t and the projection of p on A_x. In the course of the proof, we will speak loosely about 'the D_x' of the robot in time.

Lemma 1.1.1

For any direction $x \in \{\alpha, \beta, \gamma\}$, looking at the D_x over time in the heuristic path of the robot, D_x can only increase while the robot follows the edge of an obstacle and it has started following that edge at a point not in H_x.

Proof
Suppose the robot starts following an edge in the half plane H_x and suppose that D_x increases while following the edge. If the edge is in the x-direction, D_x will stay constant. If the edge is not, D_x decreases, since the robot always aims towards t. This contradicts the assumptions and proves lemma 1.1.1.

Lemma 1.1.2

For any $x \neq y$, the intersection $A_x \cap H_y = \{t\}$.

Proof
This follows directly from the fact that the obstacles are sharp triangles. For a direction $x \in \{\alpha, \beta, \gamma\}$, the axis A_x is perpendicular to the direction x, whereas the angles between x and the other two directions are smaller than $90°$.

Lemma 1.1.3

An obstacle can only lie in 2 out of the 3 half planes.

Proof
This follows from geometric observations. If the object would be in all three half-planes, it would enclose t, which is not allowed.

Lemma 1.1.4

Let Γ be a triangle with sides a, b, c that intersects with the two axes A_a and A_b. The intersections of side c with these axes have the same M-distance.

Proof
This follows from the observation that along the side with direction c, the M-measure is constant by definition.

Lemma 1.1.5

If $M(c_i) = \lambda$ for some i, then $M(c_{i+1}) < \lambda$ or $M(c_{i+2}) < \lambda$.

Proof
Consider the edge of the obstacle O_i that followed after c_i. We distinguish two scenarios: (1) after following the edge, the robot is only in one half plane $H_a = H(c_i)$, and (2) the robot is both in H_a and another half plane H_b.

1. If the robot ends up only in the half plane H_a, it can only hit edges in the a-direction, and will move towards the axis A_a. Since, according to lemma 1.1.1, D_a will decrease, the path will cross A_a again, closer to t than before. $M(c_{i+1}) < \lambda$.
2. If the robot ends up in two half planes H_a and H_b, it will folow the same obstacle's b-side before leaving the edge of the O_i. After following this second side, it can either end up in (1) $H_a \cap H_b$ or (2) in H_b only:

    1. If the robot is still in $H_a \cap H_b$, it did not cross A_b (using lemma 1.1.2). Note that, if D_b would be $\geq \lambda$, the obstacle O_i, O_i would intersect with both A_a and A_b. This is in contradiction with the observation that the robot did not cross A_b. We conclude that at this moment, $D_a < \lambda$ and $D_b < \lambda$. Since those measures monotomely decrease while the robot is in H_a and H_b and the robot now moves towards A_a and A_b, it will intersect one of them at $M(c_{i+1}) < \lambda$.
    2. If the robot is now only in H_b, it will inevitably move towards A_b. Following the same argument as directly above, if the robot did not cross the axis A_b, $D_b < \lambda$ and $M(c_{i+1}) < \lambda$. Now we consider the case that the robot did cross A_b. Due to lemma 1.1.3, the robot is now only in half-plane H_b and will move towards A_b. Although its current D_b could be $\geq \lambda$, it will strictly move towards A_b. Since by lemma 1.1.4, the part of the axis with $D_b \geq \lambda$ is covered by O_i, the robot will now intersect the axis at a distance $M(c_{i+2}) < \lambda$.

Proof for theorem 1.1
If for some i, $A(c_i) = A(c_{i+1}) = A_x$, then $M(c_{i+1}) < M(c_i)$. This follows from the fact that D_x decreases while the robot is in H_x, and that the robot must cross another axis if it would leave H_x.

Furthermore, from lemma 1.1.5, we have that if the robot crosses two different axes after each other, it will eventually cross that axis at a smaller M-distance. Suppose the robot comes back to the axis A_x at the k'th crossing c_k after crossing a series of other axes. Let O_k be the object along which the robot crosses at c_k and let p be the point at which the robot starts following O_k. Assume now that (contrary to the theorem) $M(c_k) \geq \lambda$. Repeatedly using lemma 1.1.4 we know that $M(p) < \lambda$, therefore, from lemma 1.1.4, the obstacle O_k will cover the axis A_x all the way between c_k and the point on A_x at M-distance λ. This is not possibe, since the object O_i crosses the axis in this region too and objects are not allowed to intersect.

We conclude that if the robot crosses an axis A_x at a Eudlidian distance λ from t, it will never cross the same axis at a distance $\geq \lambda$ later in time.

6 | Proof Outline (old version)

Proof that for these triangles, the robot will eventually always get there in finite time.

- **Theorem** If the robot crosses an axis A_x at a Eudlidian distance p from t, it will never cross the same axis at a distance $\geq p$ later in time.
- Define a metric M with shape of triangle with orthocenter t.
- **Lemma** At some point after you crossed an axis at distance p in the M metric, you will cross another axis at a distance $\leq p$ in the M metric, or get to the target.
- **Lemma** After crossing an axis, the projection on axis on both sides decrease monotonely until the robot crosses an axis or reaches t.
- **Lemma** If, after crossing an axis at M-distance p, another axis is crossed at M-distance $\geq p$, it will go back to the axis.
- And then something smart for "and the next round is a big difference". Maybe use space-limitations, but which?