Mathematics for computer generated spoken documents.

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Abstract

This document contains the math examples used to test the audio formatting rules when developing Aster.

1 simple fractions and expressions.

$$a + b + c + d$$

$$a + \frac{b}{c} + d$$

$$\frac{a+b}{c+d}$$

$$\frac{a}{b} + c + d$$

$$\frac{a}{b+c+d}$$

$$a + \frac{b+c}{d+e} + x$$

$$a + bc + d$$

$$(a+b)(c+d)$$

2 superscripts and subscripts.

$$x_1^k + x_2^k + x_3^k + \cdots + x_n^k = 0$$

$$x^{k_1} + x^{k_2} + x^{k_3} + \cdots + x^{k_n} = 0$$

$$x_{k^1} + x_{k^2} + x_{k^3} + \cdots + x_{k^n} = 0$$

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$$x_{k_1} + x_{k_2} + x_{k_3} + \cdots + x_{k_n} = 0$$

$$x +_n y +_n z$$

3 Knuth's examples of fractions and exponents.

$$\frac{x+y^2}{k+1}$$

$$\frac{x+y^2}{k} + 1$$

$$x + \frac{y^2}{k+1}$$

$$x + \frac{y^2}{k} + 1$$

$$x + y^{\frac{2}{k+1}}$$

$$x^{2^y} \neq x^{2^y}$$

$$x^{2^y} = x^{2y}$$

4 A continued fraction.

$$1 + \cfrac{x}{1+\frac{x}{1+\frac{x}{1+\frac{x}{1+\frac{x}{1+\ddots}}}}}$$

5 Simple School algebra.

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

Given $ax^2 + bx + c = 0$, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

6 square roots.

$$\frac{1+\sqrt{5}}{2} = \phi$$

$$\frac{\sqrt{\pi}}{2} \neq \sqrt{\frac{\pi}{2}}$$

$$\sqrt{1 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots}}}}$$

7 Trigonometric identities.

$$\sin^2 x + \cos^2 x = 1$$

$$\sin x^2 + \cos x^2 \neq 1$$

$$\sin^{-1} x \neq \sin x^{-1}$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

8 Logarithms.

$$\log^2 x \neq 2 \log x$$

$$\log x^2 = 2 \log x$$

$$\frac{\log x}{\log a} = \log_a x$$

$$\log_{a^2} x = \frac{1}{\log_x a^2} = \frac{1}{2 \log_x a} = \frac{\log_a x}{2}$$

9 Series.

$$1 + x + x^2 + x^3 + x^4 + \cdots + x^{n-1} + \cdots = \frac{1}{1-x}$$

$$1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots + \frac{x^n}{n} + \cdots$$

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \pm \cdots = \log(1+x)$$

$$\gamma = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} - \log n$$

$$\log(1+x) - \log(1-x) = \log\frac{1+x}{1-x} = \sum_{i=1}^{\infty}\frac{x^{2i-1}}{2i-1}$$

10 Integrals.

$$\int \frac{dx}{x} = \log x$$

$$\int_1^a \int_1^b \int_1^c e^{x+y+z}\, dx\, dy\, dz$$

$$\int_1^\infty e^{x^2-x-1}\, dx$$

$$\int_1^\infty e^{x^{2-x}-1}\, dx$$

$$\int_0^1 \int_0^{\sqrt{1-y^2}} 1\, dx\, dy = \int_0^{\pi/2} \int_0^1 r\, dr\, d\theta$$

$$s = \int_a \int_b f\, dx\, dy + 1$$

11 Summations.

$$\sum_{i=1}^{n} a_i = 1$$

$$\sum_{1 \leq i \leq n} a_i = 1$$

$$\sum_{i=1}^{n} a_i + b_i = 1$$

12 Limits.

$$\lim_{x \to \infty} \int_0^x e^{-y^2} \, dy = \frac{\sqrt{\pi}}{2}$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

13 Cross referenced equations.

$$\cosh x = \frac{e^x + e^{-x}}{2} \tag{1}$$

$$\sinh x = \frac{e^x - e^{-x}}{2} \tag{2}$$

Squaring 1 and 2 and computing their difference gives

$$\cosh^2 x - \sinh^2 x = 1$$

14 Distance formula.

Given $x = (x_1, x_2), y = (y_1, y_2)$ the distance between the two points is given by:

$$d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$$

This is the distance formula.

15 Quantified expression.

$$\forall x \in X : \exists y \in Y : x = y$$

16 Exponentiation

Consider the expression:

$$e^{e^{e^x}}$$

Differentiating with respect to x gives:

$$e^{e^{e^x}} e^{e^x} e^x$$

Simplifying this expression gives:

$$e^{(e^{e^x} + e^x + x)}$$

17 A generic matrix

Notice the use of vertical and diagonal dots in the generic matrix shown below.

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

18 Faa de Bruno's formula

Let $D_x^k u$ represent the kth derivative of a function u with respect to x. The chain rule states that $D_x^1 w = D_u^1 w D_x^1 u$. If we apply this to second derivatives, we find $D_x^2 w = D_u^2 w (D_x^1 u)^2 + D_u^1 w D_x^2 u$. Show that the *general formula* is

$$D_x^n w = \sum_{0 \leq j \leq n} \sum_{\substack{k_1 + k_2 + \cdots + k_n = j \\ k_1 + 2k_2 + \cdots + nk_n = n \\ k_1, k_2, \ldots, k_n \geq 0}} D_u^j w \frac{n! (D_x^1 u)^{k_1} \cdots (D_x^n u)^{k_n}}{k_1! (1!)^{k_1} \cdots k_n! (n!)^{k_n}} \tag{3}$$