PrepInsta Handbook for Quantitative Ability

For Placement Preparation

PrepInsta

PrepInsta Technologies Pvt Ltd

Preface:

This mathematics book is a great guide for those who want to take the placement tests. For the above exams, the market is filled with various forms of guide books. You will see different books containing a small section of mathematics problems with answers, but no instructions about how to obtain these answers are given.

The variations of questions posed in different exams have been found on a memory basis in this book, which includes an accumulation of objective style questions with Example solutions by shortcut form.

It is hoped that the subject matter will instill trust in the applicants, and that the book will assist them in finding an ideal teacher.

Disclaimer: This book is made and published under the complete knowledge and expertise of the Author, however if there will be any error then it will be taken care of in the next Revised edition. Constructive suggestions and feedback are most welcome by our esteemed readers.

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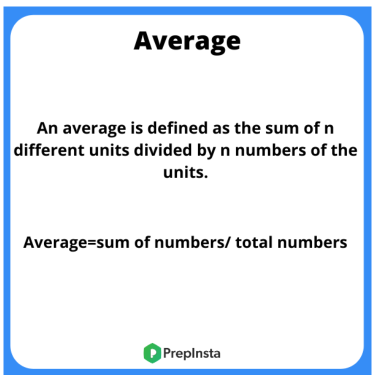
Chapter 1- Averages

The word "average" appears regularly in a number of ways. In statistics, the average is often used to minimize the amount of measurements needed to find the average when the data is high. We'll see how to use the expected mean to answer certain aptitude problems that are dependent on averages and weighted averages. The average is determined by dividing the total number of observations by the sum of the observations. The average is often referred to as the baseline. In our daily lives, we use the definition of average.

"Definition: An average is defined as the sum of n different units divided by n numbers of the units."



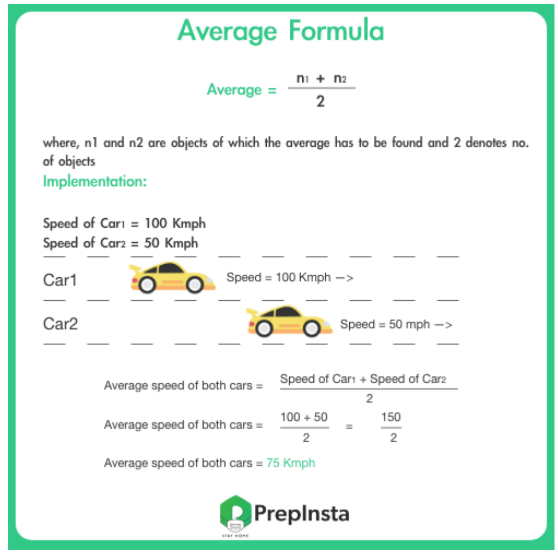
Basic Average Formulae:

It is defined mathematically as the ratio of the sum of all the numbers to the number of units in the sequence.

$$\frac{(X_1 + X_2 + X_3 + X_4 + ...X_n)}{n}$$

General formulae:

$$\text{Average} = \frac{\text{Sum of numbers}}{\text{Number of units}}$$



SOLVED EXAMPLE:

Example:- What will be the average weight of three bikes, whose respective weights are 46,54,52 ?

Solution: (sum of numbers/ total numbers)

= (46+54+53)/3

= 153/3

= 51

Example: Three years ago, the average age of the wife, husband, and their child was 28 years. When the average age of the mother and child is calculated 5 years ago, it was 22 years. Find out the present ager of the husband.

Sol: Total of the present ages of the family = (28 x 3+ 3 x 3)
= 93 years.
Total of the present ages of the mother and the child = (22 x 2 + 5 x 2)
= 54 years
Therefore, the present age of the husband = (93 - 54) = 39 years.

Example: The average age of children in a play is 15 years. The group is soon accompanied with another group of 20 children with an average age of 15 years.

The new average of the collective group now becomes 15.5 years. What is the number of children who initially went for the picnic?

Sol: Let the initial number of people going to the picnic be P
Also, 16P + 20 x 15 = 15.5 (P + 20)
Thus, P= 20

Example: 15 students in a class have an average of 15 years. Among these, 14 years is the average age for 5 students. The other nine students had an average age of 16 years. How old is the fifteenth student?

Sol: The age of the 15 th student can be calculated as
= [15 * 15 - (14 * 5 + 16 * 9)]
= 11 years.

Example: For 10 innings, the average runs of a cricketer were 32. Find the number of runs that he must score in the next innings to raise his average by 4.

Sol: Average = 32
Total runs = average of the runs x no. of innings
= 32 x 10 = 320
Increasing the average by 4 = 32 + 4 = 36
Total runs = new average x no of innings
= 36 x 11
= 396
Therefore, the runs scored at the end of the 11th inning
= 396 – 320 = 76.

Example: Find out the average of three tenths and four thousand.
Sol: Three tenths = 0.3
Four thousandths=0.004
The average is (0.3 + 0.004)/2
= 0.152

Example: If the average of 25 numbers is 0. Among these numbers, how many Maximum possible Numbers are non-zero?

Sol: Average = 0
Sum = (0 x 25) = 0.
It can be possible that 24 among them can hold positive values.
Also, if a is their sum, then their 25th number is (-a).
Therefore, none of them are greater than zero.

Example: The average of 11 results is 50, if the average of the first six results is 49 and that of the last six is 52. Find the sixth result?

Sol: 1 to 11 = 11 * 50 = 550
1 to 6 = 6 * 49 = 294
6 to 11 = 6 * 52 = 312
6th number = 294 + 312 – 550 = 56

Example: The average of 10 numbers is 23. If each number is increased by 4, what will the new average be?

Sol: Sum of the 10 numbers = 230
If each number is increased by 4, the total increase = 4 * 10 = 40
The new sum = 230 + 40 = 270 The new average = 270/10 = 27.

"AVERAGES FORMULA"

Average formula is broadly divided into two category i.e "**Average Speed and Velocity formula** and **Formula of Averages Related to Numbers"**

1.1 Average Speed

Average Speed = Total Distance / Total Time

CASE 1 : While driving at pace "a" for half of the time and "b" for the other half. The mathematical mean of the two speeds is then used to measure average speed.

Average speed = $\left(\frac{a+b}{2}\right)$

CASE II. When one travels at speed "a" for half the distance and "b" for the remaining distance. The harmonic mean of the two speeds is then used to calculate average speed.

Average speed = $\left(\frac{2ab}{a+b}\right)$

Case III: When traveling at speed a for one-third of the distance, speed b for the remaining one-third of the distance, and speed c for the remaining one-third of the distance

Average speed = $\left(\frac{3abc}{ab+bc+ca}\right)$

Case IV: Absolute displacement separated by total time can be used to quantify it.

Average Velocity = Displacement/ Total Time

1.2 Formula of Averages Related to Numbers

Case I : Average of 'n' consecutive Natural Numbers:
$$\left(\frac{n+1}{2}\right)$$

Case II: Average of the square of consecutive n natural numbers :
$$\left(\frac{(n+1)(2n+1)}{6}\right)$$

Case III: Average of cubes of consecutive n natural numbers :
$$\left(\frac{n(n+1)^2}{4}\right)$$

Case IV: Average of n consecutive even numbers **= (n+1)**

Case V: Average of consecutive even numbers till n=
$$\left(\frac{n}{2}+1\right)$$

Case VI: Average of n consecutive odd numbers **= n**

Case VII: Average of consecutive odd numbers till n =
$$\left(\frac{n+1}{2}\right)$$

Case VIII: Sum of 1st n even consecutive natural numbers is **n(n + 1)**

Case IX: Sum of 1st n odd consecutive natural numbers is n^2

How to solve Quickly:

1.1 Average Speed

Average Speed: It is the total distance traveled by a body over a given period of time. Formula to calculate speed is mentioned below:

Average Speed = Total Distance / Total Time

Example: A cyclist covered a distance of 28km in 2 hrs and 35km in 3 hrs . Find the average speed of the cyclist.

Answer :
Here , Total Distance = (28 + 35)km
= 63 km
Total Time = (2 + 3)hrs
= 5 hrs
Therefore, Average Speed = (63 / 5) km/hrs
= 12.6 km/hrs

Example: A travelled initial distance of 50km at a rate of 40kmph, then it travelled 20 km at uniform speed of 60 kmph and finally covered the remaining 40 km at a speed of 30kmph. Find the average time it will take to complete the journey.

Sol: Average Time= T1+T2+T3/3
T1=50/40=4/5
T2=20/60=1/3
T3=40/30=4/3
Average Time=37/15=2.5 hours (approx.)

Example: A vehicle travelling from Kolkata to Bangalore averages 50 kilometres per hour. It takes 20 hours to travel. It travels from Bangalore to Kolkata at a speed of 10 km/hr on the same route. What was the car's average speed on the round trip?

Sol: Distance = $50 \times 20 = 1000km$
$$Time = Distance/Speed = \frac{1000}{10} = 100hours$$
Average speed throughout the journey =
$$\frac{(1000+1000)}{(100+20)} = 16.66km/hr$$

CASE 1 : While driving at pace "a" for half of the time and "b" for the other half. The mathematical mean of the two speeds is then used to measure average speed.

Average speed = $\left(\frac{a+b}{2}\right)$

Example: An express train takes 4 hours to complete its journey. Find the average speed of the train if it

travelled with a speed of 60km/hr during the first half of the duration and covered the second half with 76 km/hr.

Answer :
According to the above formula ,
a = 60 km/hr
b = 76 km/hr
Therefore,

Average speed = $(\frac{60+76}{2})$
= 68 km/hr

Example: A boat runs at a speed of 90 km/h on the first half of its running time and at 50 km/h during the other half , then the average speed of the boat during the whole journey.
Sol: a= 90 km/hr , b= 50 km/hr
Thus, average speed =

$$\frac{(90+50)}{2} = 140/2 = 70km/hr.$$

CASE II. When one travels at speed "a" for half the distance and "b" for the remaining distance. The harmonic mean of the two speeds is then used to calculate average speed.

Average speed = $\left(\frac{2ab}{a+b}\right)$

Example: A gym instructor travels at a speed of 6km/hr from his home to the gym and returns back home at a speed of 4km/hr . Find the average speed for the whole journey .

Answer :
Here, according to the above formula ,
a = 6 km/hr
b = 4 km/hr
Therefore,

Average Speed = $(\frac{2*6*4}{6+4})$
= 4.8 km/hr

Example: Ajay travels from Delhi to Jaipur by a bus with an average speed of 45 km per hour. He travels from Jaipur to Delhi with an average speed of 30 km per hour. Find out the average speed in the complete journey.

Sol: The formula to calculate average speed
= 2pq/p+q km per hour
=2*45*30 / 45+30
= 30 km per hour

Case III: When traveling at speed a for one-third of the distance, speed b for the remaining one-third of the distance, and speed c for the remaining one-third of the distance

Average speed = $\left(\frac{3abc}{ab+bc+ca}\right)$

Example: An express train takes 3 hours to complete its journey. Find the average speed of the train if it travelled with a speed of 30km/hr during the first hour of the duration and covered the second hour with 45 km/hr and the remaining with 75 km/hr.

Answer :
Here, according to the above problem,
a = 30 km/hr
b = 45 km/hr
c = 75 km/hr
Therefore,
Average Speed =

$$(\frac{3*30*45*75}{30*45+45*75+75*30})$$

= 43.55 km/hr

Example: A car travels 3 hours to complete its journey. Find the average speed of the car if it travelled with a speed of 60km/hr during the first hour of the duration and covered the second hour with 30 km/hr and the remaining with 55 km/hr.

Sol: Here, according to the above problem,
a = 60 km/hr
b = 30 km/hr
c = 55 km/hr
Therefore,

Average Speed =
$$\frac{3*60*30*55}{60*30+30*55+55*60}$$ = 44 km/hr

Case IV: Absolute displacement separated by total time can be used to quantify it.

Average Velocity = Displacement/ Total Time

Example: A driver for 10km towards east in 2hrs and then 2 km towards west in 1hr . Find the average velocity of the driver .

Answer :
Here, Displacement = (10 - 2) km
Total Time = (2 + 1) hrs
Therefore,
Average Velocity = $(\frac{10-2}{2+1})$
= 2.67 km/hr

Example: In the same direction, a woman travels 13 kilometres in 8 hours and 5 kilometres in 2 hours. What was the man's average velocity during the journey?

Sol: Average velocity =
$$\frac{(13+5)}{(8+2)}=\frac{18}{10}=1.8km/hr$$

Example: A girl walks 6 km East in 3 hours and then 2 km West in 1 hour. Find the average velocity.

Sol: Average velocity =
$$\frac{(6-2)}{(3+1)}=\frac{4}{2}=2km/hr.$$

1.2 Formula of Averages Related to Numbers

Case I : Average of 'n' consecutive Natural Numbers:
$$\left(\frac{n+1}{2}\right)$$

Example: Find the average of the first 35 consecutive natural numbers.

Answer :
Here, n = 35
Therefore, applying the formula,
Average = $(\frac{35+1}{2})$
= 18

Example: When the average of 5 consecutive numbers is calculated, we get 15. Find out the largest among the five numbers.

Ans: Let the numbers be x,x+1,x+2,x+3,x+4
(x+x+1+x+2+x+3+x+4)/5 = 45
5x+10 = 75
x=13
If x=13 then largest number is x+4 = 13 +4
The answer is 17.

Example: The average of six numbers in a row is 25. Find the smallest among them.
Sol: Let the numbers be x,x+1,x+2,x+3,x+4,x+5
(x+x+1+x+2+x+3+x+4+x+5)/6 = 25
6x+15 = 150
x= 22.5
The smallest among them = 22.5

Example: What will be the average of 9 consecutive natural numbers.

Sol: n=9
Average = $\frac{9+1}{2}=\frac{10}{2}=5$

Case II: Average of the square of consecutive n natural numbers :
$$\left(\frac{(n+1)(2n+1)}{6}\right)$$

Example: Find the average of the square of consecutive 35 natural numbers.

Answer :
Here, n = 35
Therefore, applying the formula ,
Average = $[\frac{(35+1)(2*35+1)}{6}]$
= 426

Example: Find out the number which is non zero and its average with its square is 10 times the number.
Answer: Let us assume the number to be 'P'

$$\frac{(P+P^2)}{2} = 10P$$

$$P^2 = 20P - P$$

P= 19

Example:Which number is non zero whose average along with its square is 8 times from the number itself?

Sol: Let the number be 'n'
n*n/2 = 8n
n2 = 16n
n = 16

Case III: Average of cubes of consecutive n natural numbers :

$$\left(\frac{n(n+1)^2}{4}\right)$$

Example: Find the average of the cubes of consecutive 35 natural numbers.

Answer :
Here, n = 35
Therefore, applying the formula ,

Average = $[\frac{(35)(35+1)^2}{4}]$
= 11340

Example: What will be the average of the cubes of 29 natural numbers in a row.

Sol:
Here, n = 29
Therefore, applying the formula ,

Average = $[\frac{(29)(29+1)^2}{4}]$

= $\frac{(29)(30)^2}{4} = 6525$

Case IV: Average of n consecutive even numbers = (n+1)

Example: Find the average of 34 consecutive even numbers.

Answer :
Here, n = 34
Therefore, applying the formula,
Average = 34 + 1
= 35

Example: Find the average of 46 consecutive even numbers.

Sol: Here, n = 46
Therefore, applying the formula,
Average = 46 + 1
= 47

Case V: Average of consecutive even numbers till n=

$$\left(\frac{n}{2}+1\right)$$

Example: Find the average of consecutive even numbers till 34 .

Answer :
Here, n = 34
Therefore, applying the formula,

Average = $(\frac{34}{2}+1)$
= 18

Example: Find the average of consecutive even numbers till 90.

Sol: Here, n = 90
Therefore, applying the formula,

$$Average = (\frac{90}{2}+1)$$

= 45+1 = 46

Case VI: Average of n consecutive odd numbers = n

Example: Find the average of 35 consecutive odd numbers.

Answer :
Here, n = 35
Therefore, applying the formula,
Average = 35

Example: 61 is the average of five successive odd numbers. What is the result of the subtraction of the highest and the lowest number?

Answer: Let us assume the numbers are x, x+2, x+4, x+6, and x+8.
[x + (x + 2) + (x + 4) + (x + 6) + (x + 8)] / 5 = 61
Or, 5x + 20 = 305
x= 57
Subtracting the two numbers,
= (57 + 8) – 57
= 8

Case VII: Average of consecutive odd numbers till n = $\left(\frac{n+1}{2}\right)$

Example: Find the average of consecutive odd numbers till 35.

Answer :
Here, n = 35
Therefore, applying the formula ,
Average = $(\frac{35+1}{2}$
= 18

Example: Find the average of consecutive odd numbers till 17.
Sol: Here, n = 17
Therefore, applying the formula ,
Average = $\frac{17+1}{2}$
= $\frac{18}{2} = 9$

Case VIII: Sum of 1st n even consecutive natural numbers is n(n + 1)

Example: Find the sum of the first 34 even consecutive natural numbers .

Answer :
Here, n = 34
Therefore, applying the formula ,
Average = $34(34+1)$
= 1190

Example: Find the sum of the first 100 even consecutive natural numbers .
Sol: Here, n = 100
Therefore, applying the formula ,
Average = 100(100 + 1)
= $100 \times 101 = 10100$

Case IX: Sum of 1st n odd consecutive natural numbers is n^2

Example: Find the sum of the first 35 odd consecutive natural numbers.

Answer :
Here, n = 35
Therefore, applying the formula,
Average = $(35)^2$
= 1125

Example: Find the sum of the first 15 odd consecutive natural numbers.
Sol: Here, n = 15
Therefore, applying the formula,
Average = (15)^{2}
= 225

1.3 Tips and shortcuts to solve Averages:

Tip 1: If the value of each unit in a class is increased by some value x, then the average of the class also increases by x.

For example, if the marks obtained by Raj and Rohit increases by 20 marks each, the average of the total marks of both also increases by 20.
Tip 2: If the value of each unit in a class decreases by some value x, then the average of the class also decreases by x.

For example, if the score of Raj and Rohit in a match is decreased by 20 individually, the average score of both also decreases by 20.

__Tip 3:__ **The average of any number series or group is always between its smallest and the largest value.**

For example- If the average test score of four children is 6,9,10,11 then the average of all four names respectively is 9.

__Tip 4:__ **When a person leaves the group, and replacement is made of that person then:**

If the average age increases,

Age of new person = Age of separated person + (increase in the average × total number of persons).

If the average age decreases,

Age of new person = Age of separated person – (decrease in the average × total number of persons).

__Tip 5:__ **When a person joins the group,**

When the average age is increased
Age of new person = Previous average + (increase in average × total members including new member).

When the average age is decreased
Age of new person = Previous average – (decrease in average × total members including new member).

Exercise:
1.When a player weighing 45 kg is replaced by another player in a team of 20 players, the average increases by 2.5 kg, find the weight of the new player?

2. A batsman makes a score of 64 runs in the 16th innings and thus increases his average by 3. Find his average after the 16th inning?

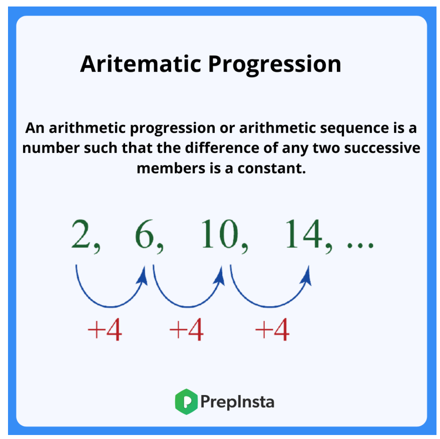
3.Five numbers averaged to 31. If one number is discarded, the average becomes 29. The discarded number is

4.The average salary of a person for the months of January, February, March and April is Rs.8000 and that for the months February, March, April and May is Rs.8500. If his salary for the month of May is Rs.6500, find his salary for the month of January?

5.The average age of a group of 10 persons was decreased by 3 years when one person, whose age was 42 years, was replaced by a new person. Find the age of the new person?

Chapter 2. Arithmetic Progression

An arithmetic progression or arithmetic sequence is a number such that the difference of any two successive members is a constant.Where common difference is denoted by d.n-th term of an arithmetic progression denoted by a_n. Sum of the first n elements denoted by S_n



An AP is represented in the form a, (a + d), (a + 2d), (a + 3d), …

where a = the first term, and d = the common difference.

General form, $T_n = a + (n-1)d$

__Basic concept of Arithmetic Progression:__

General form of an AP : a, a + d, a + 2d, a + 3d ………

First Term : a

Common Difference : d

Example: The given sequence or series is 2, 4, 6, 8, 10,…

Here, a = 2 and d = 2

__Finite or Infinite Arithmetic Progressions__

- **Finite Arithmetic Progression**

When there are a limited number of terms in the sequence then it is known as Finite Arithmetic Progression.

For example: 10, 20, 30, 40, 50

- **Infinite Arithmetic Progression**

When there are an unlimited number of terms in the sequence then it is known as Infinite Arithmetic Progression.

For example: 3, 5, 7, 9, 11, 13, ………

2.1 Properties of Arithmetic Progression

- If a fixed number is added or subtracted from each term of an AP, then the resulting sequence is also an AP and it has the same common difference as that of the original AP.
- If each term in an AP is divided or multiplied with a constant non-zero number, then the resulting sequence is also in an AP.
- If n^{th} is in linear expression then the sequence is in AP.
- If $a_1, a_2, a_3, \ldots, a_n$ and $b_1, b_2, b_3, \ldots, b_n$, are in AP. then $a_1+b_1, a_2+b_2, a_3+b_3, \ldots\ldots, a_n+b_n$ and $a_1–b_1, a_2–b_2, a_3–b_3, \ldots\ldots, a_n–b_n$ will also be in AP.
- If n^{th} term of a series is $T_n = An + B$, then the series is in AP
- Three terms of the A.P whose sum or product is given should be assumed as a-d, a, a+d.
- Four terms of the A.P. whose sum or product is given should be assumed as a-3d, a-d, a+d, a+3d.

There are main four types of Arithmetic Progression:

- Find nth term of series
- Find number of terms in the series
- Find sum of first ‘n’ terms of the series
- Find the arithmetic mean of the series.

2.2 Formulae of Arithmetic Progression

Case I: nth term of an AP

Formula to find the nth term of an AP is

$T_n = a + (n – 1) d$

Case II: Number of terms in an AP

- **Formula to find the numbers of term of an AP is**

$$n = \left[\frac{(l-a)}{d}\right] + 1$$

Case III: Sum of first n terms in an AP

- **Formula to find the sum of first n terms of an AP is**

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

OR

$$S_n = \frac{n}{2}(a + l)$$

Case IV: Arithmetic Mean

If a, b, c are in AP, then the Arithmetic mean a and c is b i.e.

$$b = \frac{1}{2}(a + c)$$

Case V: Sum of first n natural numbers

We derive the formula to find the sum of first n natural numbers

$$S = \frac{n(n+1)}{2}$$

Case VI: Sum of squares of first n natural numbers

- **Formula to find the sum of squares of first n natural numbers of an AP is**

$$S = \frac{n(n+1)(2n+1)}{6}$$

Case VII: Sum of first n odd numbers

- **Formula to find the nth term of an AP is the square of the number of terms**

$S = n^2$

Case VIII: Sum of first n even numbers

- **Formula to find the sum of of an AP is**

S = n(n+1)

2.2How to solve Quickly:

Case I: nth term of an AP

Formula to find the nth term of an AP is

T_n = a + (n – 1) d

where t_n = nth term,

a= the first term ,

d= common difference,

n = number of terms in the sequence.

Example: Find the 13th term of an arithmetic progression whose first term is 4 and the common difference is 5.

Answer: a = 4, n = 13, d = 5

nth term of A.P = a + (n-1)*d

= 4 + (13 - 1)*5

= 4 + 60 = 64

Question 1:

Find the 13th term of an arithmetic progression whose first term is 4 and the common difference is 5.

Answer:

a = 4, n = 13, d = 5

nth term of A.P = a + (n-1)*d

= 4 + (13 - 1)*5

= 4 + 60 = 64

Question 2:

Find the last number of the series where, a = 4, d = 7, and number of terms are 95 .

Answer :
According to the question,
a = 4, d = 7, n = 95
Therefore ,
last term = a+(n-1)d
or , last term= 4+(95-1)*7
or,last term=662

Question 3:

If $t_n = 36, d = -3 and n = 18$**, then the first term is ?**

Answer :
Here ,
$t_n = 36, d = -3 and n = 18$
Therefore,
$t_n = a + (n1)d$
or, $36 = a + (18 - 1) * (-3)$
or,36 = a + (17) * (-3) $36 = a + (17) * (-3)$
= a + (17) * (-3) $a + (17) * (-3)$ + (17) * (-3)
$17) * (-3)$) * (-3) (-3) -3)
or,36 = a – 51
or,a = 51+36
or,a = 87

Question 4:

If $t_n = 54, d = -7 and n = 44$, then the first term is?

Answer:
We know that,
$t_n = a + (n1)d$
or, $54 = a + (44 - 1) * (-7)$
or, 54= a + (43) * (-7)
or, 54= a – 301
or,a = 54+301
or,a = 355

Case II: Number of terms in an AP

- **Formula to find the numbers of term of an AP is**

$$n = \left[\frac{(l-a)}{d}\right] + 1$$

$$n = \left[\frac{(l-a)}{d}\right] + 1$$

where

n = number of terms,

a= the first term,

l = last term,

d= common difference.

Example: How many terms are there in A.P 3, 6, 9, 12, … , 384?

Answer: a = 3, l = 384, d = 3

n = $\frac{(l-a)}{d}+1\frac{(l-a)}{d}+1$

=129

Question 1:

How many terms are there in A.P 3, 6, 9, 12, … , 384?

Answer: a = 3, l = 384, d = 3

n = $\frac{(l-a)}{d}+1\frac{(l-a)}{d}+1$

=(384-3)/3 + 1

=128 + 1

=129

Question 2:

Find the 17th term of the series 2,5,8…

Answer :
In the given series a = 2, d = 3 n =14
Therefore,

$t_n = a + (n1)d\, t_n = a + (n1)d$

or, $t_{17} = 2 + (17-1)*3$

$t_{17} = 2 + (17-1)*3$

or, $t_{17} = 2 + 16*3\, t_{17} = 2 + 16*3$

or, $t_{17} = 2 + 48\, t_{17} = 2 + 48$

or, $]t_{17} = 50]\, t_{17} = 50$

Question 3: Find the last number of the series where, a = 6, d = 9, and number of terms are 72 .

Answer:
According to the question,

a = 6, d = 9, n = 72
Therefore,

$\frac{l-a}{d}+1\frac{l-a}{d}+1$

or, $72 = \frac{l-6}{9}+172 = \frac{l-6}{9}+1$

or,l=653

Question 4: Find the 11th term of the arithmetic progression 2, 4.5, 7, 9.5,11

Answer:
Here, d = 4.5 - 2
= 2.5

and, n = 11 , a is the first term 11th term
Therefore ,

$t_n = a + (n-1)d$

$t_n = a + (n-1)d$

Or, $t_{11} = 1 + (11-1)2.5$

$t_{11} = 1 + (11-1)2.5$

= 1 + 10 × 2.5
= 1 + 25
= 26

Question 5 : Find the 41st term of the arithmetic progression 3, 11, 19……….180

Answer:
Here, d = 11 - 3
= 8

and, n = 41 , a =3
Therefore ,

$t_n = a + (n-1)d$

$t_n = a + (n-1)d$

or, $t_{41} = 3 + (41-1)8$

$t_{41} = 3 + (41-1)8$

= 3 + 40 × 8
= 3+320
= 323

Case III: Sum of first n terms in an AP

- **Formula to find the sum of first n terms of an AP is**

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_n = \frac{n}{2}[2a + (n-1)d]$$

OR

$$S_n = \frac{n}{2}(a+l)$$ $$S_n = \frac{n}{2}(a+l)$$

where,

a = the first term,

d= common difference,

l = t_n = n^{th} term = a + (n-1)d

Example: Find the sum of the following series

4 + 7 + 10 + 13 …… to 26 terms

Answer: a = 4, d = 3, n = 26

So, Sn = $\frac{26}{2}[2 \times 4 + (26-1)3]$

$$\frac{26}{2}[2 \times 4 + (26-1)3]$$

=13[8+75]

=1079

Question 2: Find the sum of the series 100 + 94 + 88 …….. + 24

Answer :
In the given series ,

$$a = 100, d = -6, t_n = 24$$
$$a = 100, d = -6, t_n = 24$$
$$t_n = a + (n1)d$$ $$t_n = a + (n1)d$$

or, $24 = 100 + (n-1) * (-6)$
$$24 = 100 + (n-1) * (-6)$$
or, $24 = 100 - 6n + 6$
$$24 = 100 - 6n + 6$$
or, $n = 13.6 = 14(approx.)$
$$n = 13.6 = 14(approx.)$$

Now , sum of the series ,
$$S_n = (n/2) * (a+l)$$
$$S_n = (n/2) * (a+l)$$
or, $S_n = (14/2)(100+24)$
$$S_n = (14/2)(100+24)$$
or, $S_n = 7 * (124)$ $S_n = 7 * (124)$
or, $S_n = 868$ $S_n = 868$

Question 3: John begins to start saving from 1st January. He saves 1$ on 1st of January, 2$ on 2nd January and 3$ on 3rd of January. In a similar consecutive fashion, find his savings by the end of an ordinary year.

Answer :
According to the question, the series formed is
1,2,3,4,5,……365

Thus, a = 1 and d = 1
Therefore,
$$S_n = [2a + (n-1)d]$$
$$S_n = [2a + (n-1)d]$$
or, $n = 365$ $n = 365$

Now,
$$S_{365} = [2 * 1 + (365-1) * 1]$$
$$S_{365} = [2 * 1 + (365-1) * 1]$$
$$S_{365} = 66795_{\$}$$ $$S_{365} = 66795_{\$}$$

Question 4: Suppose the sum of 6 successive digits is 100 , then find the first number.

Answer :
According to the question,

d = 1 because there are 6 consecutive numbers
and, n = 6 , $S_n = 100\,S_n = 100$

Let the first digit in the progression be a
Therefore,

$$S_n = \frac{n}{2}[2a + (n-1) * d]$$
$$S_n = \frac{n}{2}[2a + (n-1) * d]$$

or, $100 = \frac{6}{2}[2a + (6-1) * 1]$

$$100 = \frac{6}{2}[2a + (6-1) * 1]$$

or, $100 = 3 * [2a + (5) * 1]$

$$100 = 3 * [2a + (5) * 1]$$

or, $100 = 3 * [2a + 5]$

$$100 = 3 * [2a + 5]$$

or, $\frac{100}{3} = 2a + 5\,\frac{100}{3} = 2a + 5$

or, $a = 14.17\,a = 14.17$

Question 5: Suppose the sum of 8 successive digits is 140 then find the first number.

Answer:
According to the question,
d = 1 because there are 8 consecutive numbers
and, n = 8 , $S_n = 140\,S_n = 140$

Let the first digit in the progression be a
Therefore,

$$S_n = \frac{n}{2}[2a + (n-1) * d]$$
$$S_n = \frac{n}{2}[2a + (n-1) * d]$$

or, $140 = \frac{8}{2}[2a + (8-1) * 1]$

$$140 = \frac{8}{2}[2a + (8-1) * 1]$$

or, $140 = 4 * [2a + (7) * 1]$

$$140 = 4 * [2a + (7) * 1]$$

or, $140 = 4 * [2a + 7]$

$$140 = 4 * [2a + 7]$$

or, $a = 14\,a = 14$

Case IV: Arithmetic Mean

If a, b, c are in AP, then the Arithmetic mean a and c is b i.e.

$$b = \frac{1}{2}(a + c)b = \frac{1}{2}(a + c)$$

Example: Find the arithmetic mean of the first 7 prime numbers

Answer: First 7 prime numbers : 2,3,5,7,11,13,17

Sum = 58

Mean = 58/7 = 8.28

Question 2: Calculate the arithmetic mean of series 4, 8, 12….100 .

Answer :
We know that ,

Arithmetic mean = $\frac{(a + c)}{2}\,\frac{(a + c)}{2}$

$$= \frac{(4 + 100)}{2} = \frac{(4 + 100)}{2}$$
$$= 52 = 52$$

Question 3: Calculate the arithmetic mean of series 6, 9, 12….240

Answer :
We know that ,

Arithmetic mean = $\frac{(a + c)}{2}\,\frac{(a + c)}{2}$

$$= \frac{(6+240)}{2} = \frac{(6+240)}{2}$$
$$= 123 = 123$$

Question 4: Find the arithmetic mean of series 8, 10, 12 , 14….250.

Answer :
We know that ,

Arithmetic mean = $\frac{(a+c)}{2}\frac{(a+c)}{2}$

$$= \frac{(8+250)}{2} = \frac{(8+250)}{2}$$

=129

Question 5: Find the arithmetic mean of the first five odd numbers.

Answer :
We know that,
First five odd natural number : 1 , 3, 5, 7, 9
Therefore,

Arithmetic Mean= $\frac{(1+9)}{2}\frac{(1+9)}{2}$
= 5

Case V: Sum of first n natural numbers

We derive the formula to find the sum of first n natural numbers

$$S = \frac{n(n+1)}{2} S = \frac{n(n+1)}{2}$$

where

S = Sum of first n natural numbers

n = number of natural numbers

Example: Find the sum of the first 21 natural numbers

Answer: $S = \frac{n(n+1)}{2} S = \frac{n(n+1)}{2}$

n = 21

S = 210

Question: Find the sum of first 200 terms,

Answer :
We know that the sum of first n terms ,

$$S = \frac{n(n+1)}{2} S = \frac{n(n+1)}{2}$$

Here , n = 200
Therefore ,

The sum of first 200 terms: $S = \frac{n(n+1)}{2}$

$$S = \frac{n(n+1)}{2}$$
$$= \frac{200(200+1)}{2} = \frac{200(200+1)}{2}$$

=20100

Question: Find the sum of first 65 terms,

Answer:
We know that the sum of first n terms ,

$$S = \frac{n(n+1)}{2} S = \frac{n(n+1)}{2}$$

Here , n = 65
Therefore ,

The sum of first 65 terms: $S = \frac{n(n+1)}{2}$

$$S = \frac{n(n+1)}{2}$$
$$= \frac{65(65+1)}{2} = \frac{65(65+1)}{2}$$

=2145

Question : Find the average of the first 35 consecutive natural numbers.

Answer :
Here, n = 35

Therefore, applying the formula,

Average = $(\frac{35+1}{2})(\frac{35+1}{2})$

= 18

Question: Find the sum of first 250 terms,

Answer:
We know that the sum of first n terms ,

$$S=\frac{n(n+1)}{2}$$ $$S=\frac{n(n+1)}{2}$$

Here , n = 250
Therefore ,

The sum of first 200 terms: $$S=\frac{n(n+1)}{2}$$

$$S=\frac{n(n+1)}{2}$$

$$=\frac{250(250+1)}{2}$$ $$=\frac{250(250+1)}{2}$$

=31375

Case VI: Sum of squares of first n natural numbers

- **Formula to find the sum of squares of first n natural numbers of an AP is**

$$S=\frac{n(n+1)(2n+1)}{6}$$

$$S=\frac{n(n+1)(2n+1)}{6}$$

where

S = Sum of first n natural numbers

n = number of natural numbers.

Example: Find the sum of square of first 15 natural numbers

Answer: $$S=\frac{n(n+1)(2n+1}{6}$$

$$S=\frac{n(n+1)(2n+1}{6}$$

n=15

S= 1240

Question : Find the average of the square of consecutive 35 natural numbers.

Answer :
Here, n = 35

We know that , $$S=\frac{n(n+1)(2n+1}{6}$$

$$S=\frac{n(n+1)(2n+1}{6}$$

Therefore, applying the formula ,

Average = $[\frac{(35+1)(2*35+1)}{6}]$

$[\frac{(35+1)(2*35+1)}{6}]$

= 426

Question : Find the sum of the squares of the first 100 natural numbers.

Answer:
We know that ,

$$S=\frac{n(n+1)(2n+1}{6}$$

$$S=\frac{n(n+1)(2n+1}{6}$$

Here, n=100
Therefore,

$$S=\frac{100(100+1)(2*100+1}{6}$$

$$S=\frac{100(100+1)(2*100+1}{6}$$

or, $S = 338350$ $S = 338350$

Question : Find the sum of the squares of the first 50 natural numbers.

Answer:
We know that ,

$$S = \frac{n(n+1)(2n+1}{6}$$

$$S = \frac{n(n+1)(2n+1}{6}$$

Here, n=50
Therefore,

$$S = \frac{50(50+1)(2*50+1}{6}$$

$$S = \frac{50(50+1)(2*50+1}{6}$$

or, $S = 42925$ $S = 42925$

Question :Find the sum of the squares of the first 20 natural numbers.

Answer:
We know that ,

$$S = \frac{n(n+1)(2n+1}{6}$$

$$S = \frac{n(n+1)(2n+1}{6}$$

Here, n=20
Therefore,

$$S = \frac{20(20+1)(2*20+1}{6}$$

$$S = \frac{20(20+1)(2*20+1}{6}$$

or,S=2870

Case VII: Sum of first n odd numbers

- **Formula to find the nth term of an AP is the square of the number of terms**

S =n^2

where

S = Sum of first n natural numbers

n = number of natural numbers

Example: Find the sum of first 12 odd numbers

Answer: $S = n^2$ $S = n^2$

n=12

S=144

Question: Find the sum of first 50 odd numbers

Answer:

Here, n=50

We know that ,

$$S = n^2 \, S = n^2$$

Therefore ,

$$S = (50)^2 \, S = (50)^2$$

or,S=2500

Question :Find out the sum of the odd numbers between 11 to 60 .

Answer:

$$S_{11-60} = S_{1-60} - S_{1-10}$$

$$S_{11-60} = S_{1-60} - S_{1-10}$$

or, $S_{11-60} = (30)^2 - (5)^2$

$$S_{11-60} = (30)^2 - (5)^2$$

or, $S_{11-60} = 900 - 25$

$$S_{11-60} = 900 - 25$$

or, $S_{11-60} = 875$ $S_{11-60} = 875$

Question :Find out the sum of the odd numbers between 21 to 50 .

Answer:

$$S_{21-50} = S_{1-50} - S_{1-20}$$
$$S_{21-50} = S_{1-50} - S_{1-20}$$

or, $S_{21-50} = (25)^2 - (10)^2$
$$S_{21-50} = (25)^2 - (10)^2$$

or, $S_{21-50} = 625 - 100$
$$S_{21-50} = 625 - 100$$

or, $S_{21-50} = 525\, S_{21-50} = 525$

Question :Find the sum of 35 consecutive odd numbers

.

Answer :
Here, n=35
We know that ,

$$S = n^2\, S = n^2$$

Therefore ,

$$S = (35)^2\, S = (35)^2$$

or, $S = 1225\, S = 1225$

Case VIII: Sum of first n even numbers

- **Formula to find the sum of of an AP is**

S = n(n+1)

where

S = Sum of first n natural numbers

n = number of natural numbers

Example: Find the sum of first 10 even numbers

Answer: S=n(n+1)

n=10

S=110

Question : Find the sum of the first 35 even numbers.

Answer:
Here , n=35
We know ,

$$S = n(n+1)\, S = n(n+1)$$

or,S=35(35+1)
or,S=1260

Question: Find the sum of the first 100 even numbers.

Answer:
Here , n=100
We know ,
S=n(n+1)
or,S=100(100+1)
or,S=10100

Question :Find the sum of the first 50 even numbers.

Answer:
Here , n=50
We know ,
S=n(n+1)
or,S=50(50+1)
or,S=2550

Question: The sum of the even numbers between 1 and n is 79*80, where n is an odd number, then find the value of n .

Answer:
We know that sum of first n consecutive even numbers ,

$$S = n(n+1)\, S = n(n+1)$$

According to the question ,
S=n(n+1)
or,79*80=n(n+1)
or,79(79+1)=n(n+1)

Comparing both sides, we get ,
n=79.

Exercise:

Question 1. A figure 24 is distributed into three parts which are in Arithmetic Progression and the total of their squares is 208. Find the largest number.

Question 2. In the given arithmetic progression, '33' would be a term in it. 7, 10, 13, 16, 19, 22………52.

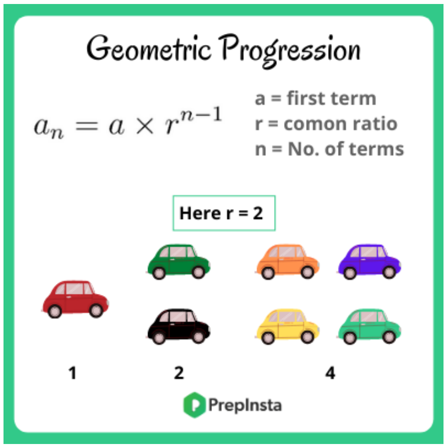
Question 3. An elastic toy bounces (3/4)th of its height after touching to the base from which it has fallen over. Calculate the full distance that it travels before coming to rest, if it is mildly fallen from a top of 360 metres.

Question 4. Anupam joined a company Pidilite industries in January 2018 and he got his first pay of Rs 2000. After that he got an increment every month of Rs 1500. Calculate his total pay after the end of 5 years of his job.

Question 5. If the 8th term of an Arithmetic Progression is zero, then the ratio of its 28th and 18th term will be?

Chapter 3: Geometric Progression

In this chapter, we will go into the in-depth definition of Geometric Progression, as well as some main formulas for answering questions about it.
We will also go through some pointers that will help to simplify the solution and clarify them with the help of some sample instances.



Geometric Progression

"Definition - A geometric progression (G.P) is a series of terms in which each succeeding term is formed by multiplying each preceding term by a constant value, whereas the constant value is referred to as the common ratio. For instance, 2, 4, 8, 16, 32, 64,... is a GP with a common ratio of 2."

3.1 - Formulae for Geometric Progression (G.P)

- **Common Ratio:**

The amount which we multiply each time in a geometric sequence is known as the common ratio of the sequence. The formula to find common ratio in a G.P is as follows:

$$r = \frac{a_2}{a_1} r = \frac{a_2}{a_1}$$

Where a_2 and a_1 are two consecutive terms on the G.P

- **Nth term of the series:**

The nth term of the sequence is basically the last term of the series. The formula to find the same is given as:

$$a_n = ar^{n-1} a_n = ar^{n-1}$$

Where a_n is the last term or the the term which has been asked to find **for example:** 3rd term or 10th term of the given sequence.
a is the first term
r is the common ratio
n is the number of terms

- **Sum of the first n terms in a G.P**

This formula is used to find the total sum of the definite number of terms given of a particular G.P. The formula is as follows.

$$S_n = \frac{a(r^n - 1)}{r - 1} S_n = \frac{a(r^n - 1)}{r - 1}$$

Where r ≠ 1
n = No. of terms present in G.P
r = Common Ratio
a = First term of the sequence

- **Sum of the infinite G.P**

If the number of terms in a GP is not finite, the GP is referred to as infinite GP. The formula for computing the sum to infinity of a given GP is:

$$S_\infty = \sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$$

$$S_\infty = \sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$$

Here,
S_∞ = Sum of infinite geometric progression
a = First term
r = Common ratio
n = Number of terms

- **Geometric Mean of G.P**

The geometric mean is a mean or average that uses the product of two numbers to determine the central trend or normal value of a series of numbers. The formula for finding out geometric mean is as follows:
If two non-zero numbers a and b are in GP, then there GM is
GM = $(ab)^{1/2}$

If three non-zero numbers a,b and c are in GP, then there GM is
GM = $(abc)^{1/3}$

3.2 - Properties of Geometric Progression

- If 'a' is the first term, r is the common ratio of a finite G.P. consisting of m terms, then the nth term from the end will be = $\mathbf{ar^{m-n}}$.
- The n^{th} term from the end of the G.P. with the last term 'l' and common ratio r is **$l / (r^{(n-1)})$.**
- Reciprocal of all the terms in G.P are also considered in the form of G.P.
- When all terms are GP raised to the same power, the new series of geometric progression is formed.

3.3 - How to Solve Geometric Progression Problems and its related Tips and Tricks.

Type 1: nth Term of G.P

Questions below are example of how to find the nth term of a Geometric Progression using the formula

$$a_n = ar^{n-1} a_n = ar^{n-1}$$

Type 2: Number of terms in a G.P

In this type you are showed how to find the total number of terms in a geometric progression using the formula

$$a_n = ar^{n-1} a_n = ar^{n-1}$$

Type 3: Sum of the "n" number of terms in a G.P

In this particular type you will be able to learn how to find the sum of the total number of terms present in the form of geometric progression by using the formula

$$S_n = \frac{a(r^n - 1)}{r - 1} S_n = \frac{a(r^n - 1)}{r - 1}$$

Where r ≠ 1

Type 4: Geometric Mean of the given series

In this type you will be able to find the geometric mean of the given geometric progression which will be in the form of

GM = $(ab)^{1/2}$

When two non-zero numbers a and b are in GP, and

GM = $(abc)^{1/3}$

When two non-zero numbers a, b and c are in GP

Exercise:

Question 1: The eighth term of geometric progression is 128 and the common ratio is 2. The product of the first five terms is?

Question 2: A ball is thrown from a height of 5000 meter on a ground the ball is bounced 4/5 times of its every last bounce then calculate the total distance covered by the ball till it stopped?

Question 3: The 7thterm of G.P is 8 times the 4thterm. What will be the 1stterm if its 5thterm is 48?

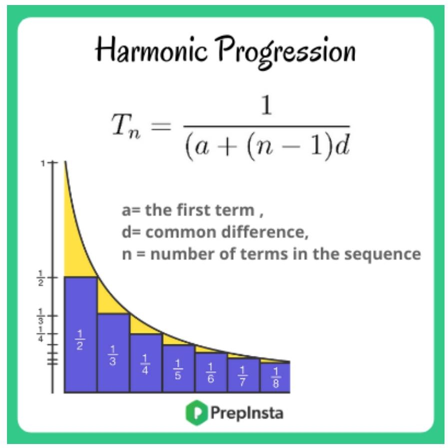
Question 4: The sum of n terms of series is 0.8 + 0.88 + 0.888 + 0.8888 + …… is?

Question 5: In a geometric progression the common ratio is half of the first term. If the 4th term of G.P is 32, then the value of the 15th term is?

Chapter 4: Harmonic Progression

In this chapter, we will go into the in-depth definition of Harmonic Progression, as well as some main formulas for answering questions about it.

We will also go through some pointers that will help to simplify the solution and clarify them with the help of some sample instances.



Harmonic Progression:

"Definition - A Harmonic Progression (HP) is a real-number series calculated by taking the reciprocals of the arithmetic progression that does not contain 0. Any concept in the series is known to be the harmonic mean of its two neighbors in harmonic progression. The series a, b, c, d,..., for example, is called an arithmetic progression; the harmonic progression can be written as 1/a, 1/b, 1/c, 1/d,..."

4.1 - Formulae to solve Harmonic Progression related Problems

- **Nth term of H.P**

The nth term as you all know discussed in the chapter of Geometric progression is known as the last term of a progression or series. The formula to find the nth term of H.P is given as:

$$T_n = \frac{1}{(a + (n1)d} T_n = \frac{1}{(a + (n1)d}$$

where tn = nth term,

a= the first term ,

d= common difference,

n = number of terms in the sequence

- **Harmonic Mean (H.M) of H.P**

A kind of numerical average is the harmonic mean. It is computed by dividing the total number of observations by the inverse of each number in the sequence. As a result, the harmonic mean is the inverse of the arithmetic mean of reciprocals. If a, b are in HP, then there HM is

$$HM = \frac{n}{\left(\frac{1}{a_1} + \frac{1}{a_2} + ... + \frac{1}{a_n}\right)}$$

$$HM = \frac{n}{\left(\frac{1}{a_1} + \frac{1}{a_2} + ... + \frac{1}{a_n}\right)}$$

Where, n = Total number of numbers or terms, a1, a2,…..an = Individual terms or individual values.

4.2 - Properties related to Harmonic Progression

- For two numbers, if A, G and H are respectively the arithmetic, geometric and harmonic means, then
  A≥ G ≥ H
- Relationship between arithmetic, geometric, and harmonic means **AM * HM = GM^2**
- Unless a = 1 and n = 1, the sum of a harmonic series will never be an integer. This is because at least one denominator of the progression is divisible by a prime number that does not divide any other denominator.
- Three consecutive numbers of a HP are: 1/(a–d), 1/a, 1/(a+d)
- Four consecutive numbers of a HP are: 1/(a–3d), 1/(a–d), 1/(a+d), 1/(a+3d).

4.3 - How to solve problems involving Harmonic Progression

Type 1: Find the nth term of the series

Questions below are example of how to find the nth term of a Geometric Progression using the formula

$$T_n = \frac{1}{(a + (n1)d} T_n = \frac{1}{(a + (n1)d}$$

Example: If the sum of reciprocals of the first 11 terms of an HP series is 110, find the 6th term.

Sol: Reciprocals of first 11 terms will be AP

Therefore, S_n = n /2 [2a + (n − 1) d]

S_{11}=110

n =11

110 = 11/2[2a + (11 − 1) d]

110 = 11/2 * [2a + 10d]

220 = 22a + 110d

22a + 110d = 220

a + 5d = 10

which is the 6th term of the AP series.

Therefore, the 6th term in HP = 1/10

Type 2: Harmonic mean of the series.

In this type you will be able to find the harmonic mean of the given harmonic progression which will be in the form of

$$HM = \frac{n}{\left(\frac{1}{a_1} + \frac{1}{a_2} + ... + \frac{1}{a_n}\right)}$$

$$HM = \frac{n}{\left(\frac{1}{a_1} + \frac{1}{a_2} + ... + \frac{1}{a_n}\right)}$$

Example: Arti walked first one-fourth of the distance at a speed of 2 km/hr. The next one-fourth of the distance was covered by running at the speed of 3km/hr. The last one-fourth of the distance was covered by cycling at the speed of 6 km/hr. Find the average speed for the whole journey covered by Arti?

Sol: According to the question, the distance covered is the same in all the three cases.

Therefore the average speed = HM of 2, 3, and 6

Average speed = 4 / (1/2 + 1/3 + 1/6)

Average speed = 4 / 1 = 4 km/hr

Exercise:

Question - 1: In HP the sum of the first two terms is 17/70 the sum of next two terms is 5/4 the sum of the following two terms is - 7/10 the sum of the following two terms will be?

Question - 2: In HP 2nd term of an HP is 40 / 9 and the 5th term is 20/3. Find the maximum possible number of terms in HP?

Question - 3: If a b c are in HP then a / (b + c), b/ (c + a), c / (a + b) are in?

Question - 4: If a_1, a_2, a_3, ….., a_n are in H.P then, the sum of $a_1a_2 + a_2a_3 + …. + a_{n-1}a_n$ will be?

Question - 5: The sum of the reciprocals of the three numbers in AP is 12 and the product of the number is 1/48 find the second number?

Chapter 5- Profit and Loss

What is Profit :- Ex.If a shopkeeper brings a cloth for Rs.100 and sells it for Rs.120, then he has made a profit of Rs.20/-.

What is Loss:- If a salesperson has bought a textile material for Rs.300 and he has to sell it for Rs.250/-, then he has gone through a loss of Rs.50/-.

There are two main factors use to solve Profit and Loss:

Cost Price – The price at which a commodity or object is bought at known as Cost Price or CP

Selling Price – The price at which the commodity is sold is known as Selling Price or SP.

Or in simple language

1. Cost Price – It is basically the price at which a commodity or object is bought at. **e.g.** Shopkeeper buying Sugar from Farmer to sell in his grocery store. In its short form it is denoted as **C.P.**

2. Selling Price – The price at which the commodity is sold at. e.g. Shopkeeper selling sugar to his customers. In its short form it is denoted as S.P.

3. **Gain or Profit –** If the Cost Price is lesser than Selling Price, gain is made.

4. Loss – If Cost price is greater than the Selling price, Loss is incurred.

5.1 Formulas for Profit and Loss

Mentioned below all the formulas used in Profit and Loss to solve the questions:

1. **Profit or Gain:** SP – CP
2. **Loss** = CP – SP
3. **Percentage Profit or Gain Percentage –**
$$\left(\frac{Gain \times 100}{C.P}\right)$$
4. **Loss Percentage –**
$$\left(\frac{Loss \times 100}{C.P}\right)$$
5. **C.P in Case of Gain =**
$$\left(\frac{(100)}{(100 + Gain)} \times S.P\right)$$
6. **C.P in Case of Loss =**
$$\left(\frac{(100)}{(100 - Loss)} \times S.P\right)$$
7. **S.P in Case of Gain =**
$$\left(\frac{(100 + Gain)}{(100)} \times C.P\right)$$
8. **S.P in Case of Loss =**
$$\left(\frac{(100 - Loss)}{(100)} \times C.P\right)$$
9. **If you sell two same items, first at x% profit and 2nd one at x% loss. Then a loss is incurred always, which is given by** = $\left(\frac{x^2}{10}\right)$
10. **Marked price and discount**

- **Selling price** = marked price – discount
- **Discount** = marked price – selling price
- **Marked price** = selling price + discount

11. **Discount Percentage** = (Discount/Marked price) x 100

12. **Successive discounts:-** If d1% , d2% d3%are successive discounts on marked price, selling price=marked price
$$\left(\frac{100-d_1}{100}\right)+\left(\frac{100-d_2}{100}\right)+\left(\frac{100-d_3}{100}\right)$$

Profit or Gain: SP – CP

5.2 How to solve Quickly:

1. **Profit or Gain:** SP – CP

Question- Raj spends Rs. 6700 on an old electronic item and Rs. 800 on its repairs. If he sells it for Rs. 7800, his profit is:

Answer-

Cost Price (C.P.) = Rs. (6700 + 800) = Rs. 7500.

Selling Price (S.P.) = Rs. 7800.

Gain = (S.P.) - (C.P.) = Rs.(7800 - 7500) = Rs. 300

Question : Mohan purchased a bluetooth speaker from his friend. He then sold it for Rs.180. Had it been sold for Rs.205 , the gain would have been 1/4 of the former loss. What is the cost price of the bluetooth speaker?

Answer :
We know that ,
Gain = SP - CP
Loss = CP - SP

So According to the question ,

$$(205-CP)=\frac{1}{4}(CP-180)$$

or,CP = Rs. 102

2. **Loss = CP – SP**

Question-Krithik goes to a store and finds a radio which costs him at a price of Rs. 9500. After a few days he finds the radio useless and plans to sell it. But for selling he has to bear some repair cost which was Rs. 200 and sells it for Rs. 6000, his loss is:

Answer-

Cost Price (C.P.) = Rs. (9500 + 200) = Rs. 9700.

Selling Price (S.P.) = Rs. 6000.

Loss = (C.P.)-(S.P.) = Rs.(9700 - 6000) = Rs. 3700

Question : Find out how much did Richa gain or lose in the entire transaction , if she purchased two fridge , but found that she needed to raise money urgently, so she sold them for Rs. 18000 each. On one she made 25% and on the other she lost 25%.

Answer :
Price of 1st fridge be Rs. X, then ,
18000 - 0.25X = X

or, $X=\frac{18000}{1.25}$

or, $X=Rs.14400$

Price for 2nd fridge be Rs.Y, then ,
8000+0.25Y = Y

or, $Y=\frac{18000}{0.75}$

or,Y = Rs. 24000

Total money spend on purchasing both the fridge = X+Y
= 14400+24000
= Rs.38400

Total money earned by selling the fridge= Rs.18000+18000
= Rs. 36000
Therefore,
Loss = Buying Price - Selling Price
= Rs. 38400-36000
= Rs. 2400

3. Percentage Profit or Gain Percentage –

$$\left(\frac{Gain\times 100}{C.P}\right)$$

Question : A store owner spent Rs.400 for a watch and sold it for Rs.500. What is the percentage of profit he makes?

Answer-

(C.P)=400, (S.P)=500

profit% = (SP-CP)/CP * 100 = (500-400)/400 * 100 = 100/400 * 100 = 25%

Question : A vendor bought 6 pens for a rupee. How many for a rupee must he sell to gain 20%?

Answer:

C.P. of 6 pen = Re. 1

S.P. of 6 pen = 120% of Re. 1 = Rs. $\frac{6}{5}$
Now,

For Rs. $\frac{6}{5}$, pens sold = 6.

For Re. 1, pens sold = $6 * \frac{5}{6} = 5$

Question : I make a profit of 20% by selling a piece of gold. What would be the profit percent if it were calculated on the selling price instead of the cost price?

Answer :
Let CP be Rs. 100
then SP = Rs. 120.
therefore,
Profit percentage if it was calculated on SP will be

$$\frac{20}{120} * 100$$
$$= 16.67\%$$

Question: If the selling price is doubled, the profit triples. Find the profit percent ?

Answer :
Let the C.P be Rs.100 and S.P be Rs.x, then,
Profit = (x-100)
Now the S.P is doubled, then the new S.P is 2x
New profit is (2x-100)
According to the question,

$$3(x - 100) = 2x - 100$$

By solving, we get
x = 200

Profit percent = $\frac{(200 - 100)}{100} * 100$ = 100%

Question: If the cost price is 30% of the selling price. Then what is the profit percent.
Answer:
Let the S.P = 100
then C.P. = 30

Therefore , Profit = 70
Now ,

Profit percentage = $\frac{70}{30} * 100 = 233.3\%$

4. Loss Percentage – $\left(\frac{Loss \times 100}{C.P}\right)$

Question A shopkeeper spent Rs.1200 for an item and sold it for Rs.500. What is the percentage of loss he makes?
Answer-
(C.P)=1200, (S.P)=500
loss% = (CP-SP)/CP * 100 = (1200-500)/400 * 100 = 700/400 * 100 = 175%

5. C.P in Case of Gain =

$$\left(\frac{(100)}{(100 + Gain)} \times S.P\right)$$

Question: Varun sells a dozen of products at Rs 5600 and the overall profit he earned on this order was 12%. For how much did he purchase 1 quantity of that product?
Answer-

C.P in Case of Gain = $\left(\frac{(100) \times S.P}{(100 + Gain)}\right)$

Cost Price of 12 Units = (100/100+12)*5600=5000
And CP for one unit = 5000/12 = Rs 417(approx.)

Question: Find the cost price of a pen if at 25% of profit, the selling price of two dozen pens is Rs.130.80.
Answer :
We know that ,

$$CP = \frac{100}{100 + Gain} * SP$$
$$CP = \frac{100}{125} * 130.80$$

or, $CP = 104.64$

The above cost price is for 2 Dozen
So, Price of a pen = Rs. 4.36

Question :Find the cost price of the pen if Jeena made a profit of 25% while selling a book for Rs.1250.

Answer :
We know that ,

$$CP = \frac{100}{100 + Gain} * SP$$

$$CP = \frac{100}{125} * 1250$$

or,CP=1000

Question : An electronic machine was sold for Rs. 2500. Had it been sold for Rs. 4000 there would have been an additional profit of 15%. What will be the cost price of the electronic machine?

Answer :
Let the cost price be Rs. x
and the Original Selling Price = Rs. 2500
Original Profit = (2500 - x) Rs.
Therefore,

Profit Percentage = $\frac{(2500 - x)}{CP} * 100$

Now, new Selling Price = Rs. 4000
Original CP = Rs. x
New profit = (4000 - x) Rs.

New profit percentage = $\frac{(4000 - x)}{CP} * 100$

Therefore,

$$\frac{(4000 - x)}{CP} * 100 - \frac{(2500 - x)}{CP} * 100 = 15$$

or, $15CP = 150000$

or, $CP = \frac{150000}{15}$ or, $CP = 10000$

So, the original cost price is Rs. 10000 .

Question : Arun sells a bike to Bharat at a profit of 10%. Bharat sells it to Chandra at a profit of 15%. If Chandra pays Rs. 250 for it, then what is the cost price of the bike for Arun ?

Answer :
Let the cost price for Arun = Rs. 100.
SP for Arun = CP for Bharat = 100 + 10% of 100= 110.
SP for Bharat = CP for Chandra = 110 + 15% of 110 = 126.5.
Given,
CP for Chandra = 250
So,
126.5 = 250
1 = 250/126.5
100 = (250 *100)/126.5 = Rs. 197.62.
Therefore , Cost Price for Arun = Rs. 197.62.

6. C.P in Case of Loss =

$$\left(\frac{(100)}{(100 - Loss)} \times S.P \right)$$

Question- Hrtithik purchased vegetables from a wholesale market and sold them in his locality. He sells these vegetables for Rs. 5000 but suffers a loss of 5%. At price did he buy these vegetables from the wholesale market.

Answer-

C.P in Case of loss= $\left(\frac{(100) \times S.P}{(100 - Loss)} \right)$

Cost Price of 12 Units = (100/100-5)*5000=5263(approx.)

Question : On selling 11 balls at Rs. 440, there is a loss equal to the cost price of 5 balls. Find the cost price of a ball .

Answer :
(C.P. of 11 balls) - (S.P. of 11 balls) = (C.P. of 5 balls)

C.P. of 11 balls = S.P. of 11 balls = Rs.440.

C.P. of 1 ball = $\frac{440}{11}$ = Rs. 40 .

7. S.P in Case of Gain =

$$\left(\frac{(100 + Gain)}{(100)} \times C.P \right)$$

Question- At what price must Roshan sell his typewriter so that he earns a profit of 7%. It is known that he bought the typewriter for Rs 12,000.

Answer-

S.P in Case of Gain =

$$\left(\frac{(100+Gain)\times C.P}{(100)}\right)$$

S.P = (100+7/100)*12,000 = 12,840 Rs.

8. S.P in Case of Loss =

$$\left(\frac{(100-Loss)}{(100)}\times C.P\right)$$

Question- Aarthy buys 12 units of books for 4800 Rs. and sells 6 units for 1200 how much loss did he bear on one book.

Answer-

S.P in Case of Loss =

$$\left(\frac{(100-Loss)\times C.P}{(100)}\right)$$

CP of 1 book = 4800/12=400Rs

SP of 1 book = 1200/6=200Rs

200=(100-loss)/100 * 400 => 200/4=100-loss => loss=50%

Question: If you sell two same items, first at x% profit and 2nd one at x% loss. Then a loss is incurred always, which is given by = $\left(\frac{x^2}{10}\right)$

Question-Two goods were purchased at different prices and then sold for the same price. The first item was sold at a 10% profit, while the second was sold at a 10% loss. What is the benefit or loss figure, if any?

Answer-

By formula - If you sell two same items, first at x% profit and 2nd one at x% loss. Then a loss is incurred always, which is given by = $(x/10)^2$

Thus, loss = (10/10)^2 = 1^2 = 1%

10. Marked price and discount

- Selling price = marked price – discount
- Discount = marked price – selling price
- Marked price = selling price + discount

Note:

(a) If there is no discount, the marked price is equal to the selling price

(b) Discount is always calculated on marked price unless otherwise stated.

11.**Discount Percentage** = (Discount/Marked price) x 100

Question - Anna gives her customers a ten percent discount on her beauty products and also makes a 20 percent profit. What is the true cost of the Rs. 400 beauty product to her?

Answer -

Marked price = Rs. 400

Discount = 10%

Profit = 20%

Therefore, the Selling Price = 90% of 400

Therefore 400 x 90/100 = Rs. 360

Selling price = Rs. 360

Profit = 20%

Cost price = 100/120 x 360 = Rs. 300

12. Successive discounts:- If d1% , d2% d3%are successive discounts on marked price,

selling price=marked price

$$\left(\frac{100-d_1}{100}\right)+\left(\frac{100-d_2}{100}\right)+\left(\frac{100-d_3}{100}\right)$$

Question - Rahul provides a 10% discount on cricket bats and a 5% discount on the reduced price to consumers who pay cash. For a cricket bat worth Rs. 200, how much does a customer have to pay in cash?

Answer - Price of the cricket bat = Rs. 200

After Discount of 10% Marked price would be Rs. 180

Since he is purchasing the bat by cash, a discount of 5% is applicable again on the reduced marked price. Thus, the final selling price of the cricket bat would be Rs. 171.

5.3 Tips and Tricks to solve Profit and Loss Question Quickly

Trick 1: Seller has two Articles for the same price, but the first article is sold at x% profit and other at x% loss. Total Profit/Loss incurred by him is not 0%

Way to solve this question is –

Apply direct formula Loss = $\left(\frac{x}{10}\right)^2 \%$

Trick 2: Where no CP or SP is given. But the whole concept is about Percentages.

Way to solve this type of questions

Assume the CP to be 100 and then solve the whole problem.

Trick 3: There are two Articles and you have to calculate total loss or profit.

Way to solve this type of questions

Now these problems are generally easy. But the whole point of solving is not to even use a pen and solve in 20 seconds.

Trick 4: CP of y items is the same as SP of x items and Profit or Loss of some percentage is made.

Way to solve this type of questions

$$\left(\frac{loss}{C.P}\right) \times 100$$

Trick 5: If the price of an item increases by r% , then the reduction in consumption so that expenditure remains the same is

or

If the price of a commodity decreases by r% then increase in consumption , so as not to decrease expenditure on this item is

Way to solve this type of questions

Just apply the following two formulas

Case I. $\frac{r}{100+r} \times 100\%$

Case II. $\frac{r}{100-r} \times 100\%$

Trick 6: A dishonest dealer claims to sell his goods at cost price ,but he uses a weight of lesser weight .Find his gain%.

Way to solve this type of questions

Apply the following formula directly

$$Gain\% = \frac{\text{true weight- false weight}}{\text{false weight}} \times 100$$

Trick 7: These questions will not be there for exams like AMCAT and Cocubes etc but for eLitmus.

A shopkeeper sells an item at a profit of x % and uses a weight which is y % less .find his total profit

$$gain\% = \frac{\%profit + \%\text{less in weight}}{100 - \%\text{less in weight}}$$

When a dealer sells goods at loss on cost price but uses less weight .

$$\text{Profit\% or loss\%} = \frac{\text{\%less weight- \% loss}}{100 - \text{\%less weight}} \times 100$$

A dishonest dealer sells goods at x % loss on cost price but uses a gms instead of b gms to measure as standard, his profit or loss percent :-

$$[\text{Profit \% or Loss\%}] = \frac{\text{original weight}}{\text{altered weight}} - 100$$

Note :- profit or loss will be decided according to sign .if +ive it is profit ,if –ve it is loss .

Exercise

Question 1. Man bought 10 cycles for Rs.500 each. He sold four cycles for Rs. 400, three for Rs. 600. At what price should he sell remaining cycles so as to earn an average profit of Rs. 300 per cycle approximately?

Question 2. A trader allows a discount of 12% in order to maintain the price line. He still makes a profit of 32% on the cost price. Calculate the profit percent he would have made on the selling price if he sold on the marked price.

Question 3. If bags bought at prices ranging from Rs. 200 to Rs. 350 are sold at prices ranging from Rs. 300 to Rs. 425, what is the greatest possible profit that might be made in selling eight bags ?

Question 4. Rohit sells a Shirt for Rs.800 and loses something, if he had sold it for Rs.980, his gain would have been double the former loss. Find the cost price of the shirt ?

Question 5. Daniel purchases an old bike for Rs. 4700. He spends Rs. 800 on its maintenance and sells it back for Rs. 5800. Calculate his profit percent.

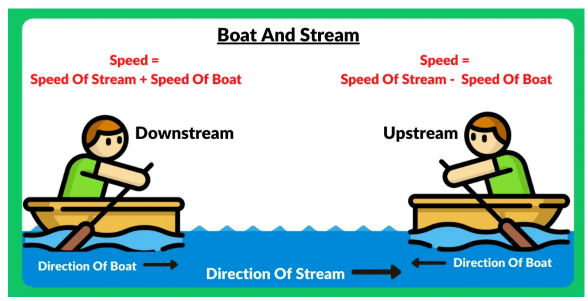
Chapter 6- Boats and streams

Introduction:

One of the most popular subjects in the quantitative portion of all career entrance exams is boats and streams. Many career tests, such as those offered by TCS and Infosys, include it in the Aptitude column. This is a very straightforward subject that requires the use of Formulas in a specific way to answer the query.

What is downstream: It refers to the direction of water flow in relation to an object; where an object or body flows downstream of a stream, it is referred to as Downstream.

What is Upstream : It also refers to the direction of water flow in reference to the object; as the object or body flows in the opposite direction, the stream is referred to as upstream. The time, direction, and distance principles are used in the boats and streams issues. In the event of certain issues, though, a few changes must be made.



Important Tips

- **In water, the direction along the stream is called Downstream**
- **The direction against the stream is called Upstream.**
- **The boats and streams problems are based on the concepts of time, speed, and distance. However, a few adjustments need to be made in case of such problems. There are some variations of these problems .**

6.1 Basic Formulas for Upstream and Downstream

Formula I : If the speed of a boat in still water is u km/hr and the speed of the stream is v km/hr, then

Speed downstream = (u + v) km/hr

Formula II : If the speed downstream is a km/hr and the speed upstream is b km/hr, then

Speed in still water: $\frac{1}{2}(a+b)km/hr$

Formula III: Let's Assume that a man can row at the speed of x km/hr in still water and he rows the same distance up and down in a stream which flows at a rate of y km/hr. Then his average speed throughout the journey is :

$$\frac{(SpeedDownstream) \times \text{ (Speed Upstream)}}{\text{Speed in still water}}$$

$$\frac{(x+y) \times \text{ (x-y)}}{\text{x}}$$

Formula IV: Let the speed of a man in still water be x km/hr and the speed of a stream be y km/hr. If he takes t hours more in upstream than to go downstream for the same distance, the distance travelled is

$$\frac{(x^2-y^2)\times t}{2y}$$

Formula V: A man rows a certain distance downstream in t_1 hours and returns the same distance upstream in t_2 hours. If the speed of the stream is y km/hr, then the speed of the man in still water

$$\frac{y(t_2+t_1)}{t_2-t_1}$$

Formula VI: A man can row a boat in still water at x km/hr in a stream flowing at y km/hr. If it takes him t hours to row a place and come back, then the distance between the two places is

$$\frac{t(x^2-y^2)}{2x}$$

6.2 How To solve Boats and Streams Questions:

Formula I : If the speed of a boat in still water is u km/hr and the speed of the stream is v km/hr, then

Speed downstream = (u + v) km/hr

Question: A boat can travel with a speed of 18 km/hr in still water. If the speed of the stream is 7 km/hr, find the time taken by the boat to go 75 km downstream.

Answer: Speed downstream = (18+7) = 25 km/hr

Time taken to travel 75 km downstream = 75/25 = 3 hours

Speed upstream = (u – v) km/hr.

Question: A man can row with a speed of 23 km/hr in still water. If the speed of the stream is 3 km/hr, find the time taken by the man to go 70 km upstream

Answer: Speed upstream = (23-3) = 20 km/hr

Time taken to travel 70 km upstream = 70/20 = 3.5 hours

More Question

Question. Calculate the speed of a motorboat in still water if current of the river flows at 4 kmph, and the boat takes 40 mins. To cover 10 km upstream and back again at the starting point.

Sol: Let the speed of the boat be ‘a’ (in still water)

Therefore speed downstream= a+2

And speed upstream= a-2

Time taken to cover 10 km back and forth= [10/(a+2) + 10/(a-2)] = 40/60

$$=> a^2 - 60a - 4 = 0$$

Or (a-10)(a+6)= 0

Since a cannot be negative therefore a = 10 kmph

Question. .Sam rows a boat at a speed of 6 kmph in still water. If the speed of the stream is 2 kmph, how much time will Sam take to cover 34 km downstream?

Sol: Speed of the boat with the stream= 6+2= 8 kmph

Therefore time taken by Sam to Cover 36 km= 36/8= 4.5 hours

Question. A man can row with a speed of 23 km/hr in still water. If the speed of the stream is 3 km/hr, find the time taken by the man to go 70 km upstream

Answer: Speed upstream = (23-3) = 20 km/hr

Time taken to travel 70 km upstream = 70/20 = 3.5 hours

Question. The speed of a boat in still water is 4.5 kmph, and the rate of the river flow is 3 kmph. Calculate the total time taken by Agatha to cover a distance of 75 km to and fro in the same river?

Sol: Speed of the boat upstream= 4.5-3= 1.5 kmph

Speed of the boat downstream= 4.5+3= 7.5 kmph

Therefore total time taken=(7.5/ 1.5 + 75/1.5) = 40 hrs

Question. A boat takes 20 hours to travel downstream from point P to Point Q. and coming back to a point R midway between P and Q. If the velocity of the stream is 5 kmph, and speed of the boat in still water is 13 kmph, then what is the distance between P and Q?

Sol: Velocity of boat= 13 kmph

Velocity of stream= 5 kmph

Time= 20 hours

Velocity of boat downstream= Velocity of boat+ Velocity of stream

= 13+5= 18 kmph

Velocity of boat upstream= Velocity of boat- Velocity of stream

= 13-5= 8 kmph

The distance between P and Q be S

S/8+0.5S/13= 20

= (13S+4S)/104= 20

= 17S= 2080

Therefore S= 2080/17= 122.4 kmph

Question. Calculate the distance covered by a boat downstream in 24 minutes if the speed of the boat in still water is 30 kmph and the speed of the current is 6 kmph?

Sol: Speed of the boat downstream= 30+6= 36 kmph

Therefore distance covered by the boat= 36* 24/60 = 14.4 km

Question. Calculate the speed of the boat in still water if it takes 2 hours to cover a certain distance downstream and takes 3 hours to return back to the starting point. And the speed of the stream is 6 kmph

Sol: Let the speed of the boat in stable water be a

Then speed of the boat downstream= a+6

Speed of the boat upstream= a-6

Therefore (a+6)*2= (a-6)* 3 or a= 30 kmph

Formula II : If the speed downstream is a km/hr and the speed upstream is b km/hr, then

Speed in still water: $\frac{1}{2}(a+b) km/hr$

Rate of stream = $\frac{1}{2}(a-b)$

Question: In one hour, a rower goes 12 km/hr along the stream and 6 km/hr against the stream. The speed of the rower in still water (in km/hr) is:

Answer: Speed in still water = $\frac{1}{2}(12+6)$

= 9 kmph

Rate of stream: $\frac{1}{2}(a-b) km/hr$

Question: A man can row at a speed of 15 kmph in still water and he rows the same distance up and down in a stream. A flow of the stream is 5 kmph. If the total distance is 160 km, find the total time taken by the man to cover the distance

Answer: Avg speed = $\frac{(15+5)(15-5)}{15}$

= 40/3

Time taken = 160/(40/3) = 12 hours

More Examples:

Question: Sam's Rowing speed is 10 kmph, but he takes double time in rowing the boat upstream in comparison to downstream. Calculate the rate of the stream?

Sol: Since it is mentioned that his speed downstream is twice that of the upstream.

So let Sam's speed upstream be 'a' then his speed downstream= 2a

Therefore (2a+a)= 10 or a= 6.66 kmph

Hence his speed upstream= 6.66 kmph and

His speed downstream= 6.66*2= 13.33 kmph

Therefore the rate of the current=

$$\frac{1}{2}(13.33 - 6.66) = 3.33km/ph$$

Question: Tom takes 2 hours to sail a boat for 4 km against the stream and can cover 2 km distance in 20 minutes if he rows the boat along with the current of the river. Calculate the time taken to cover 10 km in stagnant water?

Sol: Speed Downstream= 2/20 * 60 = 6 kmph

Speed Upstream= 2*4= 8 kmph

Speed in stable water= 1/2 * (6+8)= 7 kmph

Hence time taken to cover 10 km= 10/7 = 1 3/7 hrs

Question: Sam takes 12.5 minutes to cover 900 meters distance rowing the boat against the stream of the river and takes 7.5 minutes to row the same boat downstream. Calculate Sam's speed of rowing the boat in still water?

Sol: Let the speed of the boat in still water be

Sam takes 12.5 minutes 750 seconds to cover 900 m upstream

Therefore speed upstream= 900/750 = 1.2 mps

As the time taken downstream 7.5 min or 450 sec

Therefore speed downstream = 900/450= 2 mps

Therefore speed in stable water= 1/2 * (1.2+2)= 1.6 mps

Or 1.6* (3600/1000) = 1.6 * 18/5= 5.76 kmph

Question: Sam takes triple time to row a boat against the stream of the river than the time he takes to row with the stream to cover the same distance. What will be the ratio between the speed of the boat in still water to that of the flow?

Sol: Let Sam's speed upstream be a kmph

Speed downstream= 3a kmph

Therefore the speed of the boat in still water: Speed of river stream

= (3a+a)/2:(3a-a)/2

= 4a/2: 2a/2 Or 2: 1

Question: Tom rows for 7 hours to cover a distance of 24 km of the lake. Later he analyses that he takes the same time to cover 8 km distance with the stream and 6 km distance against the stream. Calculate the rate of the stream?

Sol: Let time taken to cover 8 km be a

Then speed downstream= 8/a

And speed upstream= 6/a kmph

Therefore 24/ (8/a) + 24/(6/a) = 7

Or a= 1 kmph

Therefore speed downstream= 8/7 and speed upstream= 6/7

Hence rate of stream= 1/2 (8/7 - 6/7) = 1/7

Question: Joe takes 4 hours to row a boat downstream to cover a distance of 32 km, whereas it takes him 8 hours to cover the same distance upstream. What will be the speed of the boat in stagnant water?

Sol: Speed of the boat downstream= 32/4 = 8 kmph

Speed of the boat upstream= 32/8 = 4 kmph

Therefore speed in still water= 1/2 * (8+4)= 6 kmph

Formula III: Let's Assume that a man can row at the speed of x km/hr in still water and he rows the same distance up and down in a stream which flows at a rate of y km/hr. Then his average speed throughout the journey is :

$$\frac{\text{(Speed Downstream)} \times \text{ (Speed Upstream)}}{\text{Speed in still water}}$$

$$\frac{(x+y) \times \text{ (x-y)}}{\text{x}}$$

Question: A man can row at a speed of 15 kmph in still water and he rows the same distance up and down in a stream. A flow of the stream is 5 kmph. If the total

distance is 160 km, find the total time taken by the man to cover the distance

Answer: Avg speed = $\frac{(15+5)(15-5)}{15}$

= 40/3

Time taken = 160/(40/3) = 12 hours

Formula IV: Let the speed of a man in still water be x km/hr and the speed of a stream be y km/hr. If he takes t hours more in upstream than to go downstream for the same distance, the distance travelled is

$$\frac{(x^2-y^2)\times t}{2y}$$

Question: Speed of a man in still water is 20 km/hr and the speed of stream is 5 km/hr. He takes 2 hours more in upstream than to go downstream. Find the total distance travelled

Answer:

$$\frac{(\text{speed in still water})^2-(\text{speed os stream})^2\times(\text{extra ti}}{2(\text{speed of stream})}$$

= $\frac{(20)^2-(5)^2\times 2}{2\times 5}$ =75 km

Formula V: A man rows a certain distance downstream in t_1 hours and returns the same distance upstream in t_2 hours. If the speed of the stream is y km/hr, then the speed of the man in still water

$$\frac{y(t_2+t_1)}{t_2-t_1}$$

Question: A man rows a distance downstream in 2.5 hours and returns the same distance in 5.5 hours. If the speed of the stream is 6 km/hr, find the speed of the man in still water

Answer: Speed of man in still water =

$$\frac{6(2.5+5.5)}{5.5-2.5}$$

=16 km/hr

Formula VI: A man can row a boat in still water at x km/hr in a stream flowing at y km/hr. If it takes him t hours to row a place and come back, then the distance between the two places is

$$\frac{t(x^2-y^2)}{2x}$$

Question: A man can row in still water at 18 km/hr in a stream flowing at 8 km/hr. If it takes him 9 hours to row to a place and come back, find the distance between two places

Answer: Distance between two places

= $\frac{9(18^2-8^2)}{2*18}$ =65 km

6.3 Tips and Tricks:

Type 1: Tips and Tricks for Finding Speed of Boat

Speed of the boat in still water = 1/2 (Downstream speed + Upstream speed)

Downstream speed = Speed of boat in still water + Speed of stream

Upstream speed = Speed of boat in still water – Speed of stream

Type 2. Tips and Tricks to Find the Speed of Stream

Speed of stream = 1/2 *(Downstream speed – upstream speed)

Speed downstream = (u + v) km/hr.

Speed upstream = (u – v) km/hr.

Exercise:

Question 1. A person can swim in still water at 4 km/h. If the speed of water is 2 km/h, how many hours will the man take to swim back against the current for 6km?

Question 2. A boat covers a certain distance in one hour downstream with the speed of 10 kmph in still water and the speed of current is 4 kmph. Then find out the distance travelled.

Question 3. A boat sails 15 km of a river towards upstream in 5 hours. How long will it take to cover the

same distance downstream, if the speed of current is one-fourth the speed of the boat in still water:

Question 4. A man goes down stream with a boat to some destination and returns upstream to his original place in 5 hours. If the speed of the boat in still water and the stream are 10km/hr and 4km/hr respectively, the distance of the destination from the string place is

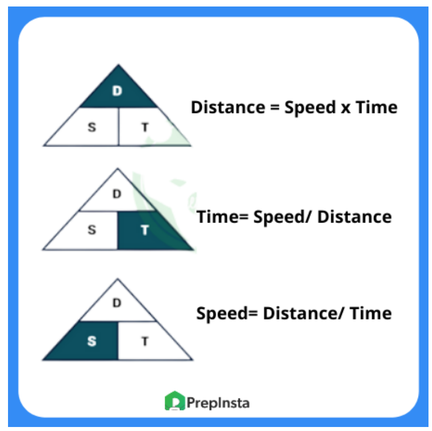
Question 5. A boat takes 19 hours for travelling downstream from point A to point B and coming back to a point C which is at midway between A and B. If the velocity of the stream is 4 kmph and the speed of the boat in still water is 14 kmph, what is the distance between A and B ?

Chapter 7- Speed Time and Distance

Introduction:

The rate at which an object travels from one location to another is called speed. The distance traveled divided by the time taken equals speed.

We can easily find the three values by using rpm, time, and distance formulas.



7.1 Formulas of Speed, Time and Distance

- Distance = Speed x Time
- $$Speed = \left(\frac{Distance}{Time}\right)$$
- $$Time = \left(\frac{Distance}{Speed}\right)$$

7.2 How to solve Quickly:

- **Speed:**

Speed can be defined as how quickly an object moves from one place to another.

Formula for speed = distance / time

Or it can be written as, speed = d/t

- **Time:**

Time is defined as distance divided by speed.

Formula for time = distance / speed

Or it can be written as, speed = d/s

- **Distance:**

Area covered by an object from moving one place to another in a uniform speed and time is known as Distance.

Formula for distance = speed x time

Or it can be written as , s × t

Question-A bus's maximum speed is 72 kmph when there are no delays, and 63 kmph when there are stops. How many minutes every hour does the bus come to a complete stop?

Answer-

Due to stoppages, the train covers 72-63=9 km less.

Thus, Time taken to cover 9 km=(9/total distance in one hour without delays)*60=(9/72)*60 min = 7.5 minutes

I. **Formula for Conversion of Km/hr to m/sec where x is in Km/hr**

1 km = 1000m

1 h = 3600 s

So 1km /h = $\frac{1000}{3600}$

$$\frac{5}{18}$$

Y *m/sec* = $\left(x \times \frac{5}{18}\right)$

Question- In 25 seconds, a train of 250 metres crosses a telegraph post. What is the train's speed in kmph?

Answer-

The speed of the train will be distance / time

Thus Speed = 250/25 = 10m/s

But we need the answer in kmph, thus conversion formula

10 * (18/5) = 36kmph

Equal Distance at two speeds

An object covers equal distance at speed S1 and other equal distance at speed S2 then his average speed for the distance is (2S1S2)/(S1 + S2)

Train Based Formula

- **ST = Speed of Train**
- **SO = Speed of Object**
- **LT = Length of Train**
- **LO = Length of Object**

Case 1 –

When Train Crosses a Stationary Object with no Length(e.g. Pole) in time t

$$S_T = \frac{L_T}{t}$$

Case 2 – When Train Crosses a Stationary Object with Length L_0 (e.g. Train Platform) in time t

$$\frac{S_T = (L_T + L_0)}{t}$$

Case 3- When Train Crosses a Moving Object with no Length (e.g. Car has negligible length) in time t

Objects moving in Opposite directions

$$(S_T + S_0) = L_T/t$$

Objects moving in Same directions

$$(S_T - S_0) = L_T/t$$

Case 4. When Train Crosses a Moving Object with Length L_0 (e.g. Another Train treated as an object) in time t

Objects(Trains) moving in Opposite directions

$$(S_T + S_0) = (L_T + L_0)/t$$

Objects(Trains) moving in Same directions

$$(S_T - S_0) = (L_T + L_0)/t$$

Note – In Case for Train 2 is treated as an object

7.2 Tips and Tricks to solve Questions

Trick 1: When Distance is constant in the given scenario

Way to solve this problem

We all know that Distance = Speed x Time

So, if Distance is constant then this equation will be

S_1 x T_1 = S_2 x T_2

Now, most of you will disregard this in the exam and it will never occur to you to use it. And if you already know what I'm talking about.

However, if you make it a habit to read questions carefully, not only for pace, time, and distance, but for any subject as well, you will be able to answer the question in a quarter of the time.

Similarly, in a study of eight engineering students solving a speed-time-distance problem, only two used this formula, and both correctly answered the question in 1/4th of the time it took the other six.

Trick 2: Object Travelling in Opposite Direction of the Train

Way to solve this problem

If the object has negligible length then use the formula –

Objects moving in Opposite directions

$$(S_T + S_0) = L_T/t$$

Objects moving in Same directions

$$(S_T - S_0) = L_T/t$$

If the Object has significant Length then use the formula

Objects(Trains) moving in Opposite directions

$$(S_T + S_0) = (L_T + L_0)/t$$

Objects(Trains) moving in Same directions

$$(S_T - S_0) = (L_T + L_0)/t$$

Trick 3: Train A leaves at X am from Position 1 and Reaches Position 2 at Y am, other train leaves from Position 2 at A Am and Reaches at B AM. Distance is given. When do two Trains Cross one another.

When they meet, b would have to voyage 120km more than Diff.

Is 120 thus, by l.c.m. it would be 360 and 240 So, 360 and 240 = 600.

Question: A and B start walking towards one another at the same time at given speeds and separated by a given distance. What is the time they cross one another at?

Trick 4: Had Object been Faster by a Km/hr then time taken and had it been slower then b Km/hr. What is the Distance?

Way 2 Solve Type 4 Problem

A will travel some distance d1 and B will travel d2. Sum of d1 and d2 will be D(given distance between the two) solved for this equation and time will be the same for both.

- d1 = s1 x t
- d2 = s2 x t

Solved Questions:

Question: A person travels from Chennai to Pondicherry in a cycle at 7.5 Kmph. Another person travels the same

distance in train at a speed of 30 Kmph and reached 30 mins earlier. Find the distance.

Solution:
t=d/s
(d/7.5)-(d/30)=(30/60)
(20d-5d)/150=1/2
d=5.
Question: A travels at 40kmph. B travels at 60kmph. They are travelling towards each other. BY the time they meet , B would have travelled 120 km more than A. Find the total distance.
Solution: A movement at 40kmph and b goes at 60kmph. It will be more useful.

They are going towards each other and begin in the meantime.

Question: A vehicle running at a velocity of 180 kmph. How much distance will it cover in 20 minutes?

Solution:

In 1 hour (60 mins) distance covered =180

Distance covered in 20 mins=180/60* 20=60 kms

Question: Rahul drives at a speed of 45 Km/hr and crosses the flyover in 15 minutes. Calculate the total length of the flyover.

Solution:

Distance = Time*Speed= 11.25 km

Question: A man travels to a destination at a rate of 30kmph and returns back at the rate of 10kmph. If the whole journey took 29/5 hours. Find the distance of the entire journey.

Solution:

Average speed = 2(30) (10)/ 30+10

= 2x 300/ 40 = 15

We know speed = distance/ time

15= d/ 29/ 5

15x29/5 = D

D= 87km

Question: Two friends start walking together. One walks at a speed of 3kmph, while the other walks at 4kmph. If the latter arrives 30 minutes earlier than the former. Calculate the distance.

Solution:

Let 30 minutes be 1/ 2

assume the distance travelled by friend walking at a distance of 3 kmph be X

then, distance travelled by friend walking at a distance of 4 kmph be (x-1/2)

3x = 4(x-1/2)

2 = 4x- 3x

x=2

putting the value of x we get x= 6

Question: A and B drive at the speeds of 18 Kmph and 27 kmph respectively.If B takes 9 hours less than what A takes for the same distance. Then distance is:

Solution:

Let A takes t hours then B takes t-9 hours

Because distance is same in both cases

So 18 * t = 27 (t-9)

=>18t=27t-243

=>t=243/9

=>t=27

=>18 * 27 = 486 km

Question: Two men set out on a bicycle towards each other. After passing each other they completed their journey in (5/6) hours and (8/3) hours respectively. At what rate does the second man cycle if the first cycle at 4 kmph?

Solution:
since both the men have the same distance,

d=s*t

let distance be d

d=5/6*4..........eq(1)

d=8/3*x.........eq(2)

equating both the equations

5/6*4=8/3*x

x=1.28

Question: Amit and Ajay are moving from two different ends of the road. They both take 2 hrs to cross each other. How long is the road if they move with the speed of 30 and 40 km/h respectively.

Solution:

Distance = speed* time

Distance= (30+40)× 2

= 140km

Question: A train starts from A towards B with some velocity. Due to an engine problem, after travelling 3/8 of its journey, it slows to 3/5 of its actual velocity. The train reaches B 1 hour later than the actual planned time. If the engine had failed after travelling 80km and if it would have slowed down to 4/5th of its initial velocity for another 80km and covered the remaining distance with 1/2 of its initial velocity, the train would have reached the destination one and half hours late. What is the distance between A and B in meters?

Solution:
Let the distance between A and B be X and the speed initially be V.

The train travels 3X/8km with speed V and the remaining distance(X - 3X/8)km with speed 3/5 of V. Ultimately the train was late by 1 hour.

According to the above condition with the formula distance/speed = time , we can have

[(3X/8)/V]+[(X-(3X/8))/(3V/5)] = [X/V]+1

[3X/8V] + 5(8X-3X)/24V = [X/V]+1

9X+25X-24X / 24V = 1

10X-24V = 0 ....eqn1

According to the question, if the train travelled 80km with speed V, another 80km with 4/5 th of V and the remaining distance(X-160)km with speed 1/2 of V then

[80/V]+[80/(4V/5)]+[(X-160)/(1V/2)] = [X/V]+3/2

80/V + 100/V + (2X-360)/V = X/V + 3/2

X-180 / V = 3/2

2X-3V = 360 ....eqn2

solving eqn1 and eqn2

we have, X=480 and V=200

Thus the distance between A and B is 480km and the speed of the train is 200km/hour.

Hence 480km = 480000meters is the answer.

Question: Joe started from Mumbai towards Pune and her friend julie in the opposite direction.they met at a point .Distance travelled by joe was 1.8 miles more than that of julie.after spending some both started there way. joe reaches in 2 hours while julie in 3.5 hours. Assuming both were travelling with constant speed.What is the distance between the two cities.

Solution:
The distance between 2 cities is 6.6 miles.Consider Both meet after X Hrs after they started .Consider Y Miles is the distance travelled by Julie in X Hrs=> Distance covered by Joe in X Hrs = Y + 1.8 Miles.
Hence Julie speed is Y/X Miles/Hr And Joe speed is (Y + 1.8) /X Miles/Hr

Distance travelled by Joe (Bombay - Pune) in 2 Hrs = Distance travelled

by Joe (Pune - Bombay) in

3.5 Hrs((Y + 1.8) /X) * 2 = (Y/X) * 3.52Y + 3.6 = 3.5Y 1.5 Y =3.6Y=

2.4Hence Distance between Pune -Bombay = Y + (Y +1.8) = 2Y + 1.8 =

(2*2.4) +1.8 = 4.8 + 1.8 = 6.6 Miles

Question: A & B two places. C & D are two people. C started from A and D started from B. When they met each other in the way C traveled 18 m more than D. Then C took 13 and half a minute and D took 24 minutes to

reach the other end. What was the distance between A & B?

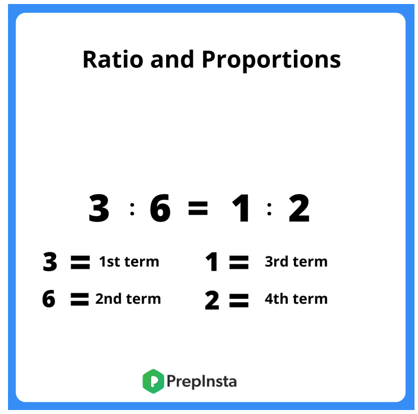
Solution:
let d travelled x km, then c travelled 18+x km. then c takes 13.5 min to travel x distance and d takes 24 min to travel 18+x so their speeds are x/13.5 & (x+18)/24
Initially time taken by c & d to travel distances 18+x and x are the same.

$t=d/s \Rightarrow (18+x)/(x/13.5) = x/((x+18)/24)$

$\Rightarrow (18+x)^2 = 1.777x^2$

On solving the quadratic equation x=54 m
Distance between them is 2x+18= 2*54+18 = 126 m.

Chapter 8: Ratio and Proportion



Ratio:

The fraction that one quantity is of the other is the ratio of two quantities in the same units. It's a relationship formed by the division of two numbers of the same kind. As a result, the ratio of 2 to 3 is 2/3, or 2:3. The antecedent and consequent terms of a ratio are the first and second terms, respectively. It's also worth noting that 10: 15 equals 10/15, and 2/3 equals 2: 3. As a result, multiplying each word of a ratio by the same number has no bearing on the ratio.

Or in other words
When compared to executing their difference, the comparison by division process makes sense in certain ways. As a result, the ratio between two numbers is defined as the contrast of two quantities using the division method. Only when the two numbers in a ratio have the same unit can they be compared. Ratios are used to compare two concepts. **A ratio is denoted by the symbol ':'.**

Proportion:
Proportion is defined as the equality of two ratios. It's used to calculate the proportion of one class to the total. To put it another way, the proportion is a component that defines the relative relationship between the overall part and the proportion.

As a result, since 2: 3 equals 4: 6, we can write 2: 3 :: 4: 6 and conclude that 2, 3, 4, and 6 are proportional. As a result, the numbers 2, 3, 4, and 6 are referred to as 1st, 2nd, 3rd, and 4th proportional, respectively. The extreme terms are the first and fourth proportional terms, and the mean terms are the second and third proportional terms.

The product of the means and the product of the extrapolation are equal.

Or in other words

Proportions and ratios are said to be two sides of the same coin. When the values of two ratios are equal, they are said to be in proportion. To put it another way, it contrasts two ratios. **The symbol '::' or '=' is used to represent proportions.**

Few Important Points:

- Fourth proportional

If x : y = z: a, then a is called the fourth proportional to x, y, z.

- Third proportional

If x: y = y: z, then z is called the third proportional to x and y.

- Mean proportional

Mean proportional between x and y is $\sqrt{xy}$

- Comparison of ratios

We say that (x: y) > (z: a), then (x/y) > (z /a)

- Compounded ratio

The compounded ratio of the ratios (x: y), (z: a), (b : c) is (xzb: yac)

8.1 Formulas & Properties of Ratio

- The ratio of two people a and b is denoted as a : b.
- a : b = ma : mb, where m is a constant.
- x : y : z = X : Y : Z is equivalent to x/X=y/Y=z/Z
- If x/y = z/a then, x+y/x-y = z+a/z-a

Formulae Property of Proportion

- x/y=z/a , this means x : y ::z:a

Ratio and Proportion Questions are widely classified in 4 categories:

- Type 1: Ratio and Proportions Tricks- Compound Ratio Based On Individual Ratios
- Type 2: Tricks and Shortcuts- Distributing Any Quantity Based On Ratios
- Type 3: Ratio and Proportions Tips and Tricks- Coins Based Ratio Problems
- Type 4: Tips and Tricks- Mixtures & Addition Based Ratio Problems

8.2How to solve Ratio and Proportion Question:

Type 1: Ratio and Proportions Tricks- Compound Ratio Based On Individual Ratios

Question: Between the two bus stops, the ratio of AC and second-class non AC fares is 12:5, and the number of passengers travelling by first AC and non AC is 1:20. What is the amount received from AC passengers if the fare is Rs. 1120?

Answer:

Ratio of the amounts collected from AC and non AC class = (12 × 1): (5 × 20) = 12 : 100.

Amount collected from AC class passengers = (12/112)*1120 = 120

Question: Seats left in two classes for new admissions are in the ratio 4:6. There is a thought to increase the seats by 20% and 50% respectively. Find out the ratio of increased seats?

Sol: Let the number of seats left in the classes be 4y and 6y

Increased seats = (120/100* 4y) (150/100*6y)

= 48y/10 , 9y

= 48:90

The ratio will be 16: 30,

= 8: 15

Question: What is the third proportional to 6 and 9?

Sol: Let the third proportion to 6 and 9 be z

6:9 = 9: z

z = 9*9/6= 13.5

Question: If 74 is divided into three parts proportional to 6, 3, 2. Then calculate the smallest part.

Sol: Given ratio = 6:3:2, sum of the ratio = 11

Then the smallest part will be = 74* 2/11= 13.5

Question: If the amount of rs.780 is divided into three subsequent parts, which are proportional to 2/3 : 4/5 : 5/6, then calculate the first part.

Sol: Given the ratio = 2/3 : 4/5 : 5/6

Taking the L.C.M we get 20+ 24+ 25

30

Calculating the ratio of first part 780* 20/69 = 226

Question: Calculate the 4th proportion in 3, 39 and 78

Sol: Using the formula, if x : y and z : a, then it can be solved as (x*z)/(y*a)

Here, 3:39 : : 78:x

So, x = 39*78/ 3 = 1014

Question: if c: d = 6:7 and d: e = 8:7. What will be c: d: e?

Sol: d is common in both the ratio,

So, the value of d = 7 and 8, as they are not the same.

Multiplying 6:7 up and down by 8

6*8/7*8= 48/56 = c/d

Similarly multiplying 8:7 up and down by 7

8*7/7*7 = 56/49 = d/e

Value of d is 56

Therefore, c: d: e = 48: 56: 49

Question: The present ages of three brothers are in the proportion 12:14 :17. The difference between the ages of elder and the eldest is 6 years. What will be the proportion of their ages after 4 years?

Sol: 14 - 17 = 3 parts = 6 years

= 1 part is 2 year

24, 28, 34 = present

28,32,38 = After 4 years

14:16:19

Question: What is the sum (in Rs) Which when divided among A,B,C,D in the proportion 2:3:5:8 provides Rs 8420 less to D than what it provides to him when the proportion is

$\frac{1}{2} : \frac{1}{3} : \frac{1}{5} : \frac{1}{8}$?

Sol: Case I - A:B:C:D = 2:3:5:8--(i)

Case II - A:B:C:D = $\frac{1}{2} : \frac{1}{3} : \frac{1}{5} : \frac{1}{8}$

= 60:40:24:15 --(ii)

=> difference of proportion of D is

$$(i) - (ii) => \left(\frac{8}{18} - \frac{15}{139}\right) x = 8420$$

$$=> x = 8420 \left(\frac{18 \times 139}{8 \times 139 - 18 \times 15}\right)$$

$$=> x = 8420 \left(\frac{18 \times 139}{842}\right)$$

=> x = 25020

Type 2: Tricks and Shortcuts- Distributing Any Quantity Based On Ratios

Question: Naveen, Kenny, and Tanay each invested Rs. 12000, Rs. 15000, and Rs. 18000 in their respective businesses. After a period of eight months since the company's inception. Additional funds were invested by Kenny and Tanay in a 3:5 ratio, respectively. What was the additional sum invested by Kenny after 8 months if the share ratio of Naveen and Kenny was 3: 4 at the end of the year?

Answer:

For easy understanding lets take - > Naveen as P, Kenny as Q and Tanay as R

Let the ratio additional amount of Q and R be 3x and 5x,

The ratio of profit of P: Q: R =[12000*12]: [15000*8 + (15000 + 3x) *4]: [18000*8 +(18000 + 5x) *4]

= 144000: (120000 + 60000 +12x): (144000+ 72000 + 20x)

The ratio of share of P and Q = 3:4

144000/(180000+ 12x) -(3/4)

192000 = 180000 + 12x

12000=12x

X = 1000

Q's additional investment after 8 months = 3x

- Rs. 3000

Question: Ritesh and Jitesh together have Rs. 1500. If 6/10 is Ritesh's amount is equal to 2/5 of Jitesh's amount. How much amount does Jitesh have?

Sol: $\frac{6}{10}R = \frac{2}{5}J$

$$R = \frac{2}{5} \times \frac{10}{6}J$$

$$R = \frac{2}{3}J$$

R/J = 2/3

R: J = 2:3

Therefore, Jitesh's share = 2/5 *1500 = 600

Question: Two boxes are filled with stones 40% and 80% which is more than the third box. Find the ratio of two boxes?

Sol: Let the third box be Z

Then, first box = 140% of z = 140z/100 = 7/5

Then, second box = 180% of z = 180z/100 = 9/5

Therefore, ratios of the two boxes will be 7z/5 : 9z/5 = 35z : 45z = 7:9

Question: Total profit earned by a company was distributed in Anil, Bharat, Chetan and Dinesh in the proportion of 6:3:3:2. If Chetan gets 1000 more than Dinesh, what is Bharat's share?

Sol: Let the shares be = 6x : 3x : 3x : 2x respectively

Then 3x- 2x = 1000

x = 1000

Therefore, Bharat's share = 3x

= 3 * 1000 = 3000

Question: Wages of Ronit and Sahil are in the ratio 4: 6. If the wages are increased by Rs. 8000 the new ratio becomes 40:50. What is Sahil's wages?

Sol: Let the original salaries of Ronit and Sahil be Rs. 4x and Rs. 6x respectively

Then,4x+8000/6x+8000=40/50

⇒ 50(4x + 8000) = 40(6x + 8000)

⇒ 40x =80,000

⇒ x= 2000

Sumit's present salary = (6x + 8000) = Rs. (12000 + 8000) = Rs. 20,000

Question: Rubbers, Rulers and colour pens in a shop are in the ratio of 6: 4: 2. If there are 150 rubbers, the number of colour pens in the shop is ---

Sol: Let the rubbers be 6x, rulers be 4x and colours be 2 x

Now, 6x = 150

So, x = 150 /6 = 25

The number of colour pens = 2*25 = 50

Question; Raju saves rs. 4488/- from his salary. He paid rent, phone bill and water bill in the ratio 20:22:26. Find the money paid for each bill?

Sol: Let the total money paid for bill be 20x, 22x, and 26x

20x+ 22x+ 26x = 4488

x = 4488/68 = 66

So, 20*66= 1320, 22*66 = 1452, 26*66 = 1716

Question: Salaries of Ravi and Sumit are in the ratio 2:3. If the salary of each is increased by Rs. 4000, the new ratio becomes 40:57. What is Sumit's salary?

Sol: Let the original salaries of Ravi and Sumit be Rs. 2x and Rs. 3x respectively.

Then,

(2x+4000) / (3x+4000) = 40 / 57

⇒ 57 × (2x + 4000) = 40 × (3x+4000)

⇒ 6x = 68,000

⇒ 3x = 34,000

Sumit's present salary = (3x + 4000) = Rs.(34000 + 4000) = Rs. 38,000

Question: Three years ago, the ratio of ages of A and B was 7:1. After three years from now, the ratio of their ages will be: 4:1. What is the difference between their ages (in years) after seven years from now?

Sol: Three year ago age Ratio => A:B

7:1 (3 years ago)

After three year age Ratio A:B

4:1

Duplicate ratio = 8:2

7:1 +1

8:2

1 unit difference = 6 years

Age difference always same for any two members

=> difference between them is 6 units = 6*6 = 36

Type 3: Ratio and Proportions Tips and Tricks- Coins Based Ratio Problems

Question: A box contains 25 paise and 50 paise coins in the ratio of 4: 6 amounting to Rs. 320 Find the number of coins of each type respectively.

Sol: Let the number of coins be 4x and 6x.

Total amount given = Rs.320

=> (.25)(4x) + (.50)(6x) = 320

Hence we get x = 1x + 3x = 320

x = 80

= 25 paise coins = 320

= 50 paisa coins = 480

Question: Geeta has 1800 rupees in the denomination of 5 paisa, 25 paisa, and 75 paisa in ratio 6 : 3 : 1. Calculate how many 25 paise coins he has.

Sol: Let the number of 5 paisa coins be 6x

Let the number of 25 paise coins be 3x

Let the number of 75 paisa coins be x

Then, 5*6x/100 + 25*3x/100 + 75x/100 = 180x/100

Given, 180x/100 = 1800

Therefore, 1800*100/180 = x

x = 1000

Hence, 25 paise coins = 3*1000 = 3000

Question: A bag contains 50 P, 25 P and 10 P coins in the ratio 5: 9: 4, amounting to Rs. 206. Find the number of coins of each type respectively.

Sol: Let the common term be x.

Hence no. of coins be 5x ,9x , 4x respectively

Now given total amount = Rs.206

=> (.50)(5x) + (.25)(9x) + (.10)(4x) = 206

we get x = 40

=> No. of 50p coins = 200

=> No. of 25p coins = 360

=> No. of 10p coins = 160

Question: Jo's collection contains US, Indian and British stamps. If the ratio of US to Indian stamps is 5 to 2 and the ratio of Indian to British stamps is 5 to 1, what is the ratio of US to British stamps?

Sol: Indian stamps are common to both ratios. Multiply both ratios by factors such that the Indian stamps are represented by the same number.

US : Indian = 5 : 2, and Indian : British = 5 : 1. Multiply the first by 5, and the second by 2. Now US : Indian = 25 : 10, and Indian : British = 10 : 2 Hence the two ratios can be combined and US : British = 25 : 2

Question: The savings of an employee equals income-expenditure. If the income of A, B, C is in the ratio 1:2:3, expenses 3:2:1. What is the order of employees A, B. C in the increasing order of the size of their savings?

Sol: income ratio should be=1:2:3

expense ratio should be=3:2:1

savings of a=1/6-(1/6*3/6)=1/12

savings of b=2/6-(2/6*2/6)=2/9

savings of c=3/6-(3/6*1/6)=5/12

so,savings ratio=1/12:2/9:5/12=3/36:8/36:15/36

so,answer is c>b>a

Question: There are 25p, 10p, and 5p coins in a sack in the ratio of 3: 2: 1. How many 25 p coins are there in total if there are Rs. 50?

Sol: Let the number of 25 p, 10 p and 5 p coins be 3x, 2x, x respectively.

$$Total amount = \frac{(25 \times 3x)}{100} + \frac{(10 \times 2x)}{100} + \frac{(5 \times x)}{100} = \frac{100x}{100}$$

Acc to the problem,

$$\frac{100x}{100} = 50$$

x=50

Number of 25p coins =$3 \times 25 = 75$

Question: A jug is filled with a mixture of three parts water and five parts syrup. To achieve a 1:1 syrup-to-water ratio, how much of the mixture must be drained and replaced with water?

Answer:

Ratio of syrup and water 5:3

Let syrup = 5x

Water 3x and total mixture -> 8x

Let 8y units of mixture is drawn off it means in 8y units of mixture 5y units will be syrup and 3 y units will be water (Because ratio of syrup and water is 5 : 3)

New quantity of syrup = 5x - 5y

New quantity of water = 3x - 3y + 8y = 3x + 5y (8y units of water is added)

They are equal to each other.

-> 5x - 5y - 3x + 5y

2x - 10

y/x 2/10-1/5

> Ky/8x - 1/S

So, 1/5th of the mixture must be taken out and replaced with water

Question: In a bucket of 30 litres, the ratio of Orange juice and its pulp is 4:2, if the ratio is to be 2:4, then the quantity of pulp to be further added will be ---

Sol: Quantity of juice = (30 *4/6) liters = 20 litres

Quantity of water = (30- 20) = 10 litres

Another ratio = 2:4

Let the quantity of pulp to be added further be x times

Then, juice: pulp (20/10+x)

Now, (20/10+x) = 2/4

80 = 20 + 2x

x = 60/2 = 30

Question: In what ratio should water be mixed with soda costing Rs 24 per litre, so as to make a profit of 25% by selling the diluted liquid at Rs 27. 50 per litre?

Sol: S.P = 27.50

P = 25 %

S.P = C.P x 125/100

27.50 = C.P x 125/100

C.P = Rs 11

W : S = 11:1

Question: A container contains a mixture of two liquids P and Q in the ratio 7 : 5. When 9 litres of mixture are drawn off and the container is filled with Q, the ratio of P and Q becomes 7 : 9. How many litres of liquid P was contained in the container initially?

Sol: Suppose the can initially contains 7x and 5x of mixtures A and B respectively.

Quantity of A in mixture left =7x - 7/12 x 9 litres = 7x - 21/4 litres.

Quantity of B in mixture left = 5x - 5/12 x 9 litres = 5x - 15/4 litres.

=> 7x - 21/4 / 5x - 15/4 = 7/9

=> 252x - 189 = 140x + 147

=> x = 3.

i.e 7x = 7 x 3

So, the can contained 21 litres of A.

Question: There are two kinds of alloys of tin and copper. The first alloy contains tin and copper such that 93.33% of it is tin. In the second alloy there is 86.66% tin. What weight of the first alloy should be mixed with some weight of the second alloy so as to make a 50 kg mass containing 90% of tin?

Sol: Consider x and y,

x - Weight of the first alloy

y - Weight of the second alloy

Here,

x + y = 50

Framing an equation,

14x / 15 + 13y / 15 = 90 / 100 * 50

14x + 13y = 675

Solving,

x + y =50

14x + 13y = 675

Solving for x,

We get,

x = 25 kg

Hence, 25 kg of the first alloy should be mixed with some weight of the second alloy so as to make a 50 kg mass containing 90% of tin.

Question: A container has a capacity of 20 gallons and is full of spirit. 4 gallons of spirit is drawn out and the container is again filled with water. This process is repeated 5 times. Find out how much spirit is left in the resulting mixture finally?

Sol: The amount of spirit left = 20 × 4/5 × 4/5 × 4/5 × 4/5 × 4/5 = 4096/625 = 6 (346/625).

Question: A jar was filled with 80 litres of buttermilk, the ratio of the curd and water was 6:4. If the ratio is to be 4:6, then calculate the quantity of water added in the end?

Sol: Quantity of curd = 80*6/10 = 48

Quantity of water = (80-48) = 32

New ratio = 4:6

Let the quantity of water be,

Curd: water = (80/ 32- x) = 4/6

480 = 128+4x

4x= 352, x = 352/4 = 88

Question: In what ratio should two varieties of tea be priced at Rs. 240 and Rs 280 respectively. Be mixed so that the cost of the combination is Rs 254per kg?

Sol: 280-254 = 26

254-240 = 14

Ratio of mixture = 26:14 = 13:7

Question: A mixture contains alcohol and water in the ratio 4 : 3. If 5 liters of water is added to the mixture, the ratio becomes 4: 5. Find the quantity of alcohol in the given mixture.

Sol: Let the quantity of alcohol and water be 4x litres and 3x litres respectively

4x/(3x+5) = 4/5

20x = 4(3x+5)

8x = 20

x = 2.5

Quantity of alcohol = (4 x 2.5) litres== 10 litres.

Question: The volumes of three containers are in the ratio 2:3:4. They are full of mixtures of milk and water in the ratios 3:1 , 3:2 , 4:3 respectively. When the contents of all these three containers are poured into a fourth container, the ratio of milk and water in the fourth container is

Sol: Volume Three container = 2:3:4

milk and water ratio

A= 3:1

B= 3:2

C= 4:3

So consider volume as common multiply ratios then

A:B:C=2:3:4

1400:21000:2800

A volume 1400

m= 1050

w= 350

B volume 2100

m= 1260

w= 840

C volume = 1600

m=1600

w=1200

Total milk : water = 3910 : 2390

= 391:239 (co primes)

<u>8.3 Tips and Tricks to solve Ratio and Proportion:</u>

With a few basic tips and tricks, you can easily solve problems involving ratio and proportion. Here are some easy tips and tricks on Ratio.

1. If x : y and z : a, then it can be solved as (x*z)/(y*a).
2. If x/y=z/a=b/c, then each of these ratios is equal to (x+z+e)⁄(y+a+f)
3. If x/y=z/a, then y/x=a/z (Invertenao)
4. If x/y=z/a, then x/z=y/a (Alterenao)
5. If x/y=z/a, then (x+y)/y=(z+a)/a (Componendo)
6. If x/y=z/a, then (x-y)/y=(z-a)/a (Dividenao)
7. If x/y=z/a, then (x+y)/(x-y)=(z+a)/(z-a) (Componendo and Dividendo)
8. Four numbers x, y, z ana a are said to be in proportion if x : y = z : a. If on the other hand, x : y = y : z = z : a, then the four numbers are said to be in continued proportion.
9. Let us consider the ratios, x : y = y : z. Here y is called the mean proportional and is equal to the

square root of the product of x and z i.e. y2 = x *z
⇒ y = √xz

10. If the three ratios, x : y, y : z, z : a is known, we can find x : a by the multiplying these three ratios x/a = x/y * y/z * z/a
11. If x, y, z, and a are four terms and the ratios x : y, y : z, z : a are known, then one can find the ratio x : y : z : a.

Solved Examples:

Question: Between the two bus stops, the ratio of AC and second-class non AC fares is 12:5, and the number of passengers travelling by first AC and non AC is 1:20. What is the amount received from AC passengers if the fare is Rs. 1120?

Answer:

Ratio of the amounts collected from AC and non AC class = (12 × 1): (5 × 20) = 12 : 100.

Amount collected from AC class passengers = (12/112)*1120 = 120

Question: Seats left in two classes for new admissions are in the ratio 4:6. There is a thought to increase the seats by 20% and 50% respectively. Find out the ratio of increased seats?

Sol: Let the number of seats left in the classes be 4y and 6y

Increased seats = (120/100* 4y) (150/100*6y)

= 48y/10 , 9y

= 48:90

The ratio will be 16: 30,

= 8: 15

Question: What is the third proportional to 6 and 9?

Sol: Let the third proportion to 6 and 9 be z

6:9 = 9: z

z = 9*9/6= 13.5

Question: If 74 is divided into three parts proportional to 6, 3, 2. Then calculate the smallest part.

Sol: Given ratio = 6:3:2, sum of the ratio = 11

Then the smallest part will be = 74* 2/11 = 13.5

Question: If the amount of rs.780 is divided into three subsequent parts, which are proportional to 2/3 : 4/5 : 5/6, then calculate the first part.

Sol: Given the ratio = 2/3 : 4/5 : 5/6

Taking the L.C.M we get 20+ 24+ 25

30

Calculating the ratio of first part 780* 20/69 = 226

Question: Calculate the 4th proportion in 3, 39 and 78

Sol: Using the formula, if x : y and z : a, then it can be solved as (x*z)/(y*a)

Here, 3:39 : : 78:x

So, x = 39*78/ 3 = 1014

Question: If c: d = 6:7 and d: e = 8:7. What will be c: d: e?

Sol: d is common in both the ratio,

So, the value of d = 7 and 8, as they are not the same.

Multiplying 6:7 up and down by 8

6*8/7*8= 48/56 = c/d

Similarly multiplying 8:7 up and down by 7

8*7/7*7 = 56/49 = d/e

Value of d is 56

Therefore, c: d: e = 48: 56: 49

Question: The present ages of three brothers are in the proportion 12:14 :17. The difference between the ages of elder and the eldest is 6 years. What will be the proportion of their ages after 4 years?

Sol: 14 - 17 = 3 parts = 6 years

= 1 part is 2 year

24, 28, 34 = present

28,32,38 = After 4 years

14:16:19

Question: What is the sum (in Rs) Which when divided among A,B,C,D in the proportion 2:3:5:8 provides Rs 8420 less to D than what it provides to him when the proportion is $\frac{1}{2} : \frac{1}{3} : \frac{1}{5} : \frac{1}{8}$

Sol: Case I - A:B:C:D = 2:3:5:8--(i)

Case II - A:B:C:D = $\frac{1}{2}:\frac{1}{3}:\frac{1}{5}:\frac{1}{8}$

= 60:40:24:15 --(ii)

=> difference of proportion of D is

(i)-(ii)=> $\left(\frac{8}{18}-\frac{15}{139}\right)x = 8420$

=> $x = 8420\left(\frac{18\times 139}{8\times 139-18\times 15}\right)$

=> $x = 8420\left(\frac{18\times 139}{842}\right)$

=> $x = 25020$

Question: Naveen, Kenny, and Tanay each invested Rs. 12000, Rs. 15000, and Rs. 18000 in their respective businesses. After a period of eight months since the company's inception. Additional funds were invested by Kenny and Tanay in a 3:5 ratio, respectively. What was the additional sum invested by Kenny after 8 months if the share ratio of Naveen and Kenny was 3: 4 at the end of the year?

Answer:

For easy understanding lets take - > Naveen as P, Kenny as Q and Tanay as R

Let the ratio additional amount of Q and R be 3x and 5x,

The ratio of profit of P: Q: R =[12000*12]: [15000*8 + (15000 + 3x) *4]: [18000*8 +(18000 + 5x) *4]

= 144000: (120000 + 60000 +12x): (144000+ 72000 + 20x)

The ratio of share of P and Q = 3:4

144000/(180000+ 12x) -(3/4)

192000 = 180000 + 12x

12000=12x

X = 1000

Q's additional investment after 8 months = 3x

- Rs. 3000

<u>Exercise:</u>

1.Ritesh and Jitesh together have Rs. 1500. If 6/10 is Ritesh's amount is equal to 2/5 of Jitesh's amount. How much amount does Jitesh have?

2. Two boxes are filled with stones 40% and 80% which is more than the third box. Find the ratio of two boxes?

3. Total profit earned by a company was distributed in Anil, Bharat, Chetan and Dinesh in the proportion of 6:3:3:2. If Chetan gets 1000 more than Dinesh, what is Bharat's share?

4. Wages of Ronit and Sahil are in the ratio 4: 6. If the wages are increased by Rs. 8000 the new ratio becomes 40:50. What is Sahil's wages?

5. Rubbers, Rulers and colour pens in a shop are in the ratio of 6: 4: 2. If there are 150 rubbers, the number of colour pens in the shop is ---

6. Raju saves rs. 4488/- from his salary. He paid rent, phone bill and water bill in the ratio 20:22:26. Find the money paid for each bill?

7. Salaries of Ravi and Sumit are in the ratio 2:3. If the salary of each is increased by Rs. 4000, the new ratio becomes 40:57. What is Sumit's salary?

8. Three years ago, the ratio of ages of A and B was 7:1, After three years from now, the ratio of their ages will be: 4:1. What is the difference between their ages (in years) after seven years from now?

9. A box contains 25 paise and 50 paise coins in the ratio of 4: 6 amounting to Rs. 320 Find the number of coins of each type respectively.

10. Geeta has 1800 rupees in the denomination of 5 paisa, 25 paisa, and 75 paisa in ratio 6 : 3 : 1. Calculate how many 25 paise coins he has.

11.A bag contains 50 P, 25 P and 10 P coins in the ratio 5: 9: 4, amounting to Rs. 206. Find the number of coins of each type respectively.

Chapter 9: Linear Equations

A linear equation is an algebraic equation in which each term has a single exponent. The equation is represented graphically as a straight line.

The standard form of a linear equation. y = m x + b.

The variables are x, y, m, and b, and the constants are x, y, m, and b.

A linear equation is an equation where variable quantities are in the first power only and whose graph is a straight line.

Linear Equations Definitions

- A linear equation is an algebraic equation in which each term has an exponent of one and the graphing of the equation results in a straight line.
- Standard form of the linear equation is y = mx + b. Where, x is the variable and y, m, and b are the constants.

9.1 Formulas of Linear equations in one variable

- A Linear Equation in one variable is defined as **ax + b = 0**
- Where, a and b are constant, **a ≠ 0**, and x is an unknown variable
- The solution of the equation ax + b = 0 is x $-\frac{b}{a}$

  We can also say that $-\frac{b}{a}$ is the root of the linear equation ax + b = 0.

Formulas of Linear equations in two variable

- A Linear Equation in two variables is defined as **ax + by + c = 0**
- Where a, b, and c are constants and also, both a and b ≠ 0

Formulas for Linear equations in three variable

- A Linear Equation in three variables is defined as **ax + by + cz = d**
- Where a, b, c, and d are constants and also, a, b and c ≠ 0

9.2 How to solve Linear Equations carefully:

1. **Substitution Method**

Step 1: Solve one of the equations either for x or y.
Step 2: Substitute the solution from step 1 into the other equation.
Step 3: Now solve this equation for the second variable.

Question : Solve for x and y using the Substitution Method.

y-2x=5
3x+y=10
Answer :
The above equations can be rewritten as ,
y=2x+5......(i)
3x+y=10.......(ii)
Substituting eq(i) in eq(ii) , we get ,
3x+(2x+5)=10
or,5x+5=10
or, x=1
Now, substituting the value of x in eq(i), we get ,
or, y=2(1)+5
or, y=7
Therefore , x=1 and y=7

Question : Solve the linear system using the substitution method .

y=-x+4
2y-4x=2

Answer :
The above equations can be rewritten as ,
y=-x+4....eq(i)
2y-4x=2.....eq(ii)
Substituting eq(i) in eq(ii) , we get ,
(-x+4)-2x=1
or,-3x=-3
or, x=1
Now, substituting the value of x in eq(i), we get ,
y=-1+4
or, y=3, Therefore, x=1 and y=3

Question : Solve the linear system using the substitution method .

x=36-9y

$$3y + \frac{x}{3} = 12$$

Answer :
The above equations can be rewritten as,

$$\frac{x}{3} = 12 - 3y$$ ......eq(i)

$$3y + \frac{x}{3} = 12$$ .....eq(ii)

Substituting eq(i) in eq(ii) , we get ,
3y+12-3y=12
or, 12=12
Therefore, the system is dependent and the solution is
$x = 36 - 9y$..

Question : Solve the linear system using the substitution method .
7y+2x=16
-21y-6x=24

Answer :
The above equations can be rewritten as,
7y+2x=16.....eq(i)
-7y-2x=8......eq(ii)
Now, in eq(i) ,
2x=-7y+16
or, $x = -\frac{7y}{2} + 8$ ....eq(iii)

Substituting the value of eq(iii) in eq(i) , we get ,

$$-7y - 2(-\frac{7y}{2} + 8) = 8$$

or,-7y+7y-16=8
or,-2=1
Therefore, the system is inconsistent and no solution exists.

Question : Solve the linear system using the substitution method .
2x+3y=16
x+7y=19
Answer :
The above equations can be rewritten as ,
2x+3y=16....eq(i)
x=-7y+19.....eq(ii)
Substituting eq(ii) in eq(i) , we get ,
2(-7y+19)+3y=16
or,-14y+38+3y=16
or,-11y=-22
or,y=2
Substituting the value of y in eq(ii) , we get ,
x=-7(2)+19
or,x=5

Therefore , x=5 and y=2

2. **Elimination Method**

Step 1: Multiply both the equations with such numbers to make the coefficients of one of the two unknowns numerically same.
Step 2: Subtract the second equation from the first equation.
Step 3: In either of the two equations, substitute the value of the unknown variable. So, by solving the equation, the value of the other unknown variable is obtained.

Question : Solve the linear system using the elimination method .

-y+2x=5
3x+y=10

Answer :
The above equations can be rewritten as ,
2x-y=5....eq(i)
3x+y=10....eq(ii),
Multiplying eq(i) by 1 and eq (ii) by 1 and subtracting eq(ii) from eq(i) , we get ,
(3x-2x)+(y-y)=(10-5)
or,x=5
Substituting the value of x in eq(i) , we get ,
2(5)-y=5
or,y=5
Therefore, x=5 and y=5

Question : Solve the linear system using the elimination method .
3x+y=9
5x+4y=22

Answer :
The given equations are ,
3x+y=9....eq(i)
5x+4y=22....eq(ii)
Multiplying eq(i) by 4 and eq(ii) by 1 , we get
12x+4y=36...eq(iii)
5x+4y=22...eq(iv)
Now, Subtracting equation (iv) from eq(iii) , we get ,
(12x-5x)+(4y-4y)=(36-22)
or,7x=14
or,x=2
Now, substituting the value of x in eq(i) , we get ,
3(2)+y=9
or,6+y=9
or,y=3
Therefore, x=2 and y=3

Question : Solve the linear system using the elimination method .

2y-4x=2
y=-x+4

Answer :
The above equations can be rewritten as ,
-4x+2y=2...eq(i)
x+y=4...eq(ii)
Now, Multiplying eq(i) by 1 and eq(ii) by 4 , we get ,
-4x+2y=2...eq(iii)
4x+4y=16...eq(iv)
Adding both the above equations , we get ,
(-4x+4x)+(2y+4y)=(2+16)
or, 6y=18
or, y=3
Substituting the value of y in eq(ii), we get ,
x+3=4
or,x=1
Therefore , x=1 and y=3

Question: Solve the linear system using the elimination method .

-4x-3y=-1
-2x-2y=4

Answer :
The given equations are ,
-4x-3y=-1...eq(i)
-2x-2y=4.....eq(ii)
Multiplying eq(i) by 2 and eq(ii) by (-3) , we get
-8x-6y=-2....eq(iii)
6x+6y=-12.....eq(iv)
Now, Adding eq(i) and eq(ii) , we get ,
(-8x+6x)+(-6y+6y)=(-2-12)
or,-2x=-14
or,x=7
Substituting the value of x in eq(ii) , we get ,
-2(7)-2y=4
or,-14-2y=4
or,y=9
Therefore , x=7 and y=9

Question: Solve the linear system using the elimination method .

5x-2y=-3
2x+3y=3

Answer :
The given equations are ,
5x-2y=-3...eq(i)
2x+3y=3...eq(ii)
Multiplying eq(i) by 3 and eq(ii) by 2 , we get ,
15x-6y=-9...eq(iii)
4x+6y=6....eq(iv)
Adding eq(iii) and eq(iv) , we get ,
(15x+4x)+(-6y+6y)=(-9+6)
or,19x=-3

Or, $x = \frac{-3}{19}$

Substituting the value of x in eq(i) , we get

$$y = \frac{21}{19}$$

Therefore , $x = \frac{-3}{19} andy = \frac{21}{19}$

Question : Solve the linear system using the elimination method .

$$2x - \frac{3}{y} - 12 = 0$$

$$5x + \frac{7}{y} - 10 + 9 = 0$$

Answer :
The above equations can be rewritten as ,

$$2x - \frac{3}{y} = 12$$ .....eq(i)

$$5x + \frac{7}{y} = 1$$ .....eq(ii)

Put $\frac{1}{y} = a$, then we get ,
2x-3a=12.....eq(iii)
5x+7a=1.....eq(iv)

Multiplying equation (iii) by 5 and equation (iv) by 2 , we get
10x-15a=60...eq(v)
10x+14a=2..eq(vi)
Subtracting eq(v) and eq(vi) , we get ,
(10x-10x)+(-15a-14a)=(60-2)
or-29a=58
or,a=-2

or, $\frac{1}{y} = -2[Since, \frac{1}{y} = a]$

or, $y = -\frac{1}{2}$

Substituting the value of y in eq(i) , we get ,

$$2x - 3(-2) = 12$$

or,x=3

Therefore, $x = 3andy = -\frac{1}{2}$

Question: Solve the linear system using the elimination method .

$$\frac{y}{2} + \frac{2}{3}x + 1 = 0$$

$$y - \frac{1}{3}x - 10 + 7 = 0$$

Answer :
The given above equations can be rewritten as ,

$$\frac{2}{3}x + \frac{y}{2} = -1$$ ....eq(i)

$$-\frac{1}{3}x + y = 3$$ .....eq(ii)

Multiplying eq(i) by 6 and eq(ii) by 3 , we get ,
4x+3y=-6...eq(iii)
-x+3y=9....eq(iv)
Subtracting eq(iv) from eq(iii) , we get ,
(4x-(-x))+(3y-3y)=-6-9
or,5x=-15
or,x=-3

Substituting the value of x in eq(ii) , we get ,

$$-\frac{1}{3}(-3) + y = 3$$

or,1+y=3
or,y=2

Question: Ajay bought two copies and one pencil cost Rs. 35 and three copies and four pencils cost Rs. 65 from a stationery store.. Find the cost of copy and pencil separately bought by him.

Answer :
Let the cost of a copy be Rs. x
and the cost of a pencil be Rs. y

According to the question ,
2x+y=35...eq(i)
3x+4y=65..eq(ii)
Multiplying eq(i) by 4 and eq(ii) by 1 , we get ,
8x+4y=140...eq(iii)
3x+4y=65...eq(iv)
Subtracting eq(iv) by eq(iii) , we get ,
(8x-3x)+(4y-4y)=(140-65)
or,5x=75
or,x=15
Substituting the value of x in eq(i) , we get ,
2(15)+y=35
or,y=5
Therefore , Cost of a copy = Rs. 15
Cost of a pencil =Rs. 5

3. <u>Cross-Multiplication Method</u>

Suppose there are two equation,

$$p_1x + q_1y = r_1 \, p_1x + q_1y = r_1$$

$$p_2x + q_2y = r_2 \, p_2x + q_2y = r_2$$

Multiply Equation (1) with p_2
Multiply Equation (2) with p1

$$p_1p_2x + q_1q_2y = r_1p_2$$

$$p_1p_2x + p_1q_2y = p_1r_2$$

Subtracting,

q1p2y–p1q2y=r1p2–p1r2

or, y ($q_1 p_2 – q_2p_1$) = $r_2p_1 – r_1p_2$

$$\text{therefore, y} = \frac{r_2p_1 - r_1p_2}{q_1p_2 - q_2p_1}$$

$$\text{therefore, y} = \frac{r_1p_2 - r_2p_1}{q_2p_1 - q_1p_2}$$

where ($p_1q_2 – p_2q_1$) ≠ 0

Therefore,

$$\text{therefore, y} = \frac{y}{r_1p_2 - r_2p_1} = \frac{1}{q_2p_1 - q_1p_2}$$

Multiply Equation (1) with q_2

Multiply Equation (2) with q1

$$p_1q_2x + q_1q_2y = r_1q_2$$

$$q_1p_2x + q_1q_2y = q_1r_2$$

Subtracting,

$$p_1q_2x - p_2q_1x = q_1r_2 = r_1q_2 - q_1r_2$$

or , $x(p_1q_2 - p_2q_1) = (q_1r_2 - q_2r_1)$

$$or, x = \frac{q_1r_2 - r_1q_2}{p_1q_2 - p_2q_1}$$

$$\text{therefore, } = \frac{x}{q_1r_2 - r_1q_2} = \frac{1}{p_1q_2 - p_2q_1}$$

where $(p_1q_2 - p_2q_1) \neq 0$

From equations (3) and (4), we get,

$$\frac{x}{q_1r_2 - r_1q_2} = \frac{y}{r_1p_2 - r_2p_1} = \frac{1}{q_2p_1 - q_1p_2}$$

where $(p_1q_2 - p_2q_1) \neq 0$

Note: Shortcut to solve this equation will be written as

$$\frac{x}{q_1r_2 - r_1q_2} = \frac{y}{r_1p_2 - r_2p_1} = \frac{1}{q_2p_1 - q_1p_2}$$

Which means,

$$x = \frac{q_1r_2 - r_1q_2}{q_2p_1 - q_1p_2}$$

$$y = \frac{r_1p_2 - r_2p_1}{q_2p_1 - q_1p_2}$$

Question 1:

Solve the linear system using the cross multiplication method

.

$$x + y = -10$$
$$3x + 7y = -2$$

Answer :
The above equations can be rewritten as ,
$x + y + 10 = 0$ ....eq(i)
$3x + 7y + 2 = 0$ .....eq(ii)

Here,
The coefficients of x are, i.e. $p1 = 1andp2 = 3.$
The coefficients of y are, i.e. $q1 = 1andq2 = 7.$
The constant terms are, i.e. $r1 = 10andr2 = 2.$
Now,

$$\frac{x}{q1r2 - r1q2} = \frac{y}{r1p2 - r2p1} = \frac{1}{q2p1 - q1p2}$$

or, $x = \dfrac{q1r2 - r1q2}{q2p1 - q1p2}$

or, $y = \dfrac{r1p2 - r2p1}{q2p1 - q1p2}$

Hence , $x = -17andy = 7$

Question : Solve the linear system using the cross multiplication method .

$$\frac{x}{a} + \frac{y}{b} = a + b$$
$$\frac{x}{a^2} + \frac{y}{b^2} = 2$$

Answer:
Here,
The coefficients of x are, i.e.

$$p1 = \frac{1}{a}andp2 = \frac{1}{a^2}.$$

The coefficients of y are, i.e.
The constant terms are, i.e.

$$r1 = -(a+b) and r2 = -2.$$

$$p1 = \frac{1}{a} and p2 = \frac{1}{a^2}.$$

Now,

$$\frac{x}{q1r2 - r1q2} = \frac{y}{r1p2 - r2p1} = \frac{1}{q2p1 - q1p2}$$

or, $x = \dfrac{q1r2 - r1q2}{q2p1 - q1p2}$

or, $y = \dfrac{r1p2 - r2p1}{q2p1 - q1p2}$

Hence ,

$$x = a^2 and y = b^2$$

Question: Solve the linear system using the cross multiplication method .

$$3x + 4y - 24 = 0$$
$$20x - 11y - 47 = 0$$
$$20x - 11y - 47 = 0$$

Answer:
Here,
The coefficients of x are, i.e.

$$p1 = 3 and p2 = 20.$$

The coefficients of y are, i.e.

$$q1 = 4 and q2 = -11.$$

The constant terms are, i.e.

$$r1 = -24 and r2 = -47.$$

Now,

$$\frac{x}{q1r2 - r1q2} = \frac{y}{r1p2 - r2p1} = \frac{1}{q2p1 - q1p2}$$

or, $x = \dfrac{q1r2 - r1q2}{q2p1 - q1p2}$

or, $y = \dfrac{r1p2 - r2p1}{q2p1 - q1p2}$

Hence ,

$$x = 4 and y = 3$$

Question : Solve the linear system using the cross multiplication method .

$$a(x+y) + b(x-y) = a^2 - ab + b^2$$
$$a(x+y) + b(x-y) = a^2 - ab + b^2$$

Answer:
The above equations can be rewritten as ,

$$(a+b)x + (a-b)y = a^2 - ab + b^2$$
$$(a+b)x + (a-b)y = a^2 - ab + b^2$$

Here,
The coefficients of x are, i.e.

$$p1 = (a+b) and p2 = (a-b).$$

The coefficients of y are, i.e.

$$q1 = (a-b) and q2 = (a+b).$$

The constant terms are, i.e.

$$r1 = -(a^2 - ab + b^2) and r2 = -(a^2 + ab + b^2).$$

Now,

$$\frac{x}{q1r2 - r1q2} = \frac{y}{r1p2 - r2p1} = \frac{1}{q2p1 - q1p2}$$

or, $x = \dfrac{q1r2 - r1q2}{q2p1 - q1p2}$

or, $y = \dfrac{r1p2 - r2p1}{q2p1 - q1p2}$

Hence ,

$$x = \frac{b^2}{2a} and y = \frac{2a^2 + b^2}{2a}$$

Question: Solve the linear system using the cross multiplication method .

$$ax + by - c^2 = 0$$
$$a^2x + b^2y - c^2 = 0$$

Answer :
Here,
The coefficients of x are, i.e.

$$p1 = a and p2 = a^2.$$

The coefficients of y are, i.e. $q1 = b and q2 = b^2.$

The constant terms are, i.e.

$$r1 = -c^2 and r2 = -c^2.$$

Now,

$$\frac{x}{q1r2 - r1q2} = \frac{y}{r1p2 - r2p1} = \frac{1}{q2p1 - q1p2}$$

or, $x = \dfrac{q1r2 - r1q2}{q2p1 - q1p2}$

or, $y = \dfrac{r1p2 - r2p1}{q2p1 - q1p2}$

Hence ,

$$x = \frac{c^2(1-b)}{a(a-b)}$$

$$y = \frac{c^2a(a-1)}{b(a-b)}$$

Question : Solve the linear system using the cross multiplication method .

$$ax + by = 1$$

$$bx + ay = \frac{(a+b)^2}{a^2+b^2} - 1$$

Answer :

The above equations can be rewritten as ,

$$ax + by - 1 = 0$$

$$bx + ay - \frac{(2ab)}{a^2+b^2} = 0$$

Here,

The coefficients of x are, i.e. $p1 = aandp2 = b.$

The coefficients of y are, i.e. $q1 = bandq2 = a.$

The constant terms are, i.e.

$$r1 = -1andr2 = -\frac{(2ab)}{a^2+b^2}.$$

Now,

$$\frac{x}{q1r2 - r1q2} = \frac{y}{r1p2 - r2p1} = \frac{1}{q2p1 - q1p2}$$

or, $x = \dfrac{q1r2 - r1q2}{q2p1 - q1p2}$

or, $y = \dfrac{r1p2 - r2p1}{q2p1 - q1p2}$

Hence ,

$$x = \frac{(a)}{a^2+b^2}$$

$$y = \frac{(b)}{a^2+b^2}$$

Question: Solve the linear system using the cross multiplication method .

$$(a-b)x + (a+b)y = a^2 - 2ab - b^2$$

$$(a+b)(x+y) = a^2 + b^2$$

Answer :

The above equations can be rewritten as ,

$$(a-b)x + (a+b)y - (a^2 - 2ab - b^2) = 0$$

$$(a+b)x + (a+b)y - (a^2 + b^2) = 0$$

Here,

The coefficients of x are, i.e.

$$p1 = (a-b)andp2 = (a+b).$$

The coefficients of y are, i.e.

$$q1 = (a+b)andq2 = (a+b).$$

The constant terms are, i.e.

$$r1 = -(a^2 - 2ab - b^2)andr2 = -(a^2 + b^2).$$

Now,

$$\frac{x}{q1r2 - r1q2} = \frac{y}{r1p2 - r2p1} = \frac{1}{q2p1 - q1p2}$$

or, $x = \dfrac{q1r2 - r1q2}{q2p1 - q1p2}$

or, $y = \dfrac{r1p2 - r2p1}{q2p1 - q1p2}$

Hence ,

$$x = a + b$$

$$y = \frac{(2ab)}{a+b}$$

4. <u>**Number of solutions**</u>

Suppose, there are two linear equations: $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$

Now,

A. If $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$, then there will be numerous solutions, and the graphs will have coincident lines.

B. If $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$ then there will be no solution, and the graphs will have parallel lines.

Question : State whether the following systems will have one , none or infinite solutions .

$$3x + y = 7$$
$$4x + y = 7$$

Answer :
Here ,
$a1 = 3, a2 = 4; b1 = 1, b2 = 1; c1 = 7, c2 = 7$
Therefore ,

$$\frac{3}{4} \neq \frac{1}{1}$$

Hence , the above equations will have one solution.

Question: State whether the following systems will have one , none or infinite solutions .

$$6x + 2y = 10$$
$$3x + y = 5$$

Answer :
Here ,
$a1 = 6, a2 = 3; b1 = 2, b2 = 1; c1 = 10, c2 = 5$
Therefore ,

$$\frac{6}{3} = \frac{2}{1} = \frac{10}{5}$$

Hence , the above equations will have infinite solutions.

Question: State whether the following systems will have one , none or infinite solutions .

$$x - y = 3$$
$$3x - 3y = 6$$

Answer :

Here ,
$a1 = 1, a2 = 3; b1 = -1, b2 = -3; c1 = 3, c2 = 6$
Therefore ,

$$\frac{1}{3} = \frac{-1}{-3} \neq \frac{3}{6}$$

Hence , the above equations will have no solutions.

Question: Find the number of solutions in the following system of linear equations.

$$14x + 7y = 28$$
$$2x + y = 4$$

Answer :
Here ,
$a1 = 14, a2 = 7; b1 = 7, b2 = 1; c1 = 28, c2 = 4$
Therefore ,

$$\frac{14}{7} = \frac{7}{1} = \frac{28}{4}$$

Hence , the above equations will have multiple solutions.

Question 6: Find the number of solutions in the following system of linear equations.

$$-3x + y = 3$$
$$2x + y = -4$$

Answer :
Here ,
$a1 = -3, a2 = 2; b1 = 1, b2 = 1; c1 = 3, c2 = -4$
Therefore ,

$$\frac{-3}{2} \neq \frac{1}{1}$$

Hence , the above equations will have one solution.

Question : Find the number of solutions in the following system of linear equations.

$$22x = -11y + 44$$
$$7y = -14x + 28$$

Answer :
The above equations can be rewritten as ,

$$22x + 11y = 44$$
$$14x + 7y = 28$$

Here ,
$a1 = 22, a2 = 14; b1 = 11, b2 = 7; c1 = 44, c2 = 28$
Therefore ,
$$\frac{22}{14} = \frac{11}{7} = \frac{44}{28}$$

Hence , the above equations will have multiple solutions.

Question : Find the number of solutions in the following system of linear equations.

$$y = -3x + 9$$
$$3y = -9x + 9$$

Answer :
The above equations can be rewritten as ,
$$3x + y = 9$$
$$9x + 3y = 9$$

Here ,
$a1 = 3, a2 = 9; b1 = 1, b2 = 3; c1 = 9, c2 = 9$
Therefore ,
$$\frac{3}{9} = \frac{1}{3} \neq \frac{9}{9}$$

Hence , the above equations will have no solution.

Question: Find the value of x for which $ax + b = 3$ and $a + bx = 2$ are inconsistent ?

Answer :
The given equations are ,
$ax + b = 2$....eq(i)
$a + bx = 1$....eq(ii)

The above two equations will be inconsistent if
$$\frac{k}{1} = \frac{1}{k} \neq \frac{3}{2}$$
i.e. , $k^2 = 1$
or, $k \pm 1$

Therefore , the two given equations will be inconsistent if $k \pm 1$.

Question: Solve the following pairs of equations .

$$2a - 3b = 2$$
$$4a - 6b = 6$$

Answer :
The given equations are ,
$2a - 3b = 2$...eq(i)
$4a - 6b = 6$...eq(ii)

Comparing the coefficients of the above two equations , we get
$$\frac{2}{4} = \frac{-3}{-6} \neq \frac{2}{6}.$$

Therefore the equations have no solutions .

Question: Determine whether the following pair of equations represents simultaneous equations or not.

$$-3a + 7b = 5$$
$$5a - 3b = 10$$

Answer :
The given equations are ,
$-3a + 7b = 5$...eq(i)
$5a - 3b = 10$...eq(ii)
Comparing the coefficients of the above two equations , we get
$$\frac{-3}{5} \neq \frac{7}{-3}$$

Therefore the above pair of equations has one solution and thus , it represents simultaneous equations.

<u>Tips and Tricks to solve Linear Equation:</u>

Here, we have provided quick and easy tips and tricks for you on Linear Equation questions which are efficient in competitive exams as well as other recruitment exams that must help to find a better place.

- It can be easily solved by eliminating the wrong options. It means put the given values in the equation and check which one is satisfying the equation.
- Standard form of linear equations is y= mx+b
- There are 2 types of questions asked in exams explained below.

Exercise:

Question: Solve the linear system using the elimination method .

$$-y + 2x = 5$$
$$3x + y = 10$$

Question: Solve the linear system using the elimination method .

$$3x + y = 9$$
$$5x + 4y = 22$$

Question: Solve the linear system using the elimination method .

$$2y - 4x = 2$$
$$y = -x + 4$$

Question: Solve the linear system using the elimination method .

$$-4x - 3y = -1$$
$$-2x - 2y = 4$$

Question: Solve the linear system using the elimination method .

$$5x - 2y = -3$$
$$2x + 3y = 3$$

Question: Solve the linear system using the elimination method .

$$2x - \frac{3}{y} - 12 = 0$$
$$5x + \frac{7}{y} - 10 + 9 = 0$$

Question: Solve the linear system using the elimination method .

$$\frac{y}{2} + \frac{2}{3}x + 1 = 0$$
$$y - \frac{1}{3}x - 10 + 7 = 0$$

Chapter 10: Quadratic Equation

A quadratic equation is one that can be written in standard form as ax2+ bx + c = 0, where x is a variable and a, b, and c are constants, and an is less than 0. Because there is no term when a = 0, the equation is linear, not quadratic.

Formulas for Quadratic Equations & Definitions

- An equation where the highest exponent of the variable is a square. Standard form of quadratic equation is $ax^2+bx+c = 0$
- Where, x is the unknown variable and a, b, c are the numerical coefficients.

10.1 Basic Formulae for Quadratic Equations:

- If $ax^2+bx+c = 0$ is a quadratic equation, then the value of x is given by the following formulae
- Generic Formulae:

$$X = \frac{-b \pm \sqrt{b^2 4ac}}{2a}$$

10.2 Tips and Tricks to solve Quadratic Equation:

- On solving the quadratic equations generally we get two linear equations. Here are some quick shortcuts and easy tips and tricks for you to solve Quadratic Equation questions quickly, easily, and efficiently in various exams of recruitment drive and competitive exams and recruitment exams.
- There are generally three ways in Quadratic Equations.

1. Factorization
2. Completing the square method.
3. Discriminant and Quadratic Formula

More Tips:

- In order to find the sign of roots we use the sign given in the equation.

Sign of coefficien t 'x'	Sign of coefficien t 'y'	Sign of roots
+	+	– –

+	–	– +
–	+	+ +
–	–	+ –

For example: $x^2 + 3x – 4 = 0$

The factors are 4 and 1
Now will find the sign of roots. According to the table, if the sign given in the equation is + and – then their sign of roots is – and +.

Therefore, the roots of the equation are -4 and 1

- When one equation has positive roots and the other has negative, or equations have the same sign roots, then compare the roots and find the relation between them.
- When the roots of the equation are positive and negative, we cannot find the relation between them.
- A quadratic equation is an equation having the form $\mathbf{ax^2 + bx + c = 0}$**.** Where, x is the unknown variable and a, b, c are the numerical coefficients.
- In each of these questions, two equations are given. You have to solve these equations and find out the values and relation between x and y.

<u>10.3 More Formulae to solve Quadratic Equations:</u>

- <u>Factorization</u>

It is a very simple method to solve quadratic equations. Factorization give 2 linear equations

For example: $x^2 + 3x – 4 = 0$

Here, a = 1, b = 3 and c = -4

Now, find two numbers whose product is – 4 and sum is 3.

So, the numbers are 4 and -1.

Therefore, two factors will be 4 and -1

- <u>Completing the Square Method</u>

Every Quadratic equation always has a square term. If we could get two square terms of quality sign we can get linear equations. Middle term is called as ‘b’ and splitted by

$$\left(\frac{b}{2}\right)^2$$

Other basic concepts to remember while solving quadratic equations are:

1.Nature of roots

1. Nature of roots determines whether the given roots of the equation are real, imaginary, rational or irrational. The basic formula is $b^2 – 4ac$.

2. This formula is also called **discriminant or D**. The nature of the roots depends on the value of D. Conditions to determine the nature of the roots are:

3. If D < 0, then the given roots are imaginary.

4. If D = 0, then roots given are real and equal.

5. If D > 0, then roots are real and unequal.

6. Also, in case of D > 0, if the equation is a perfect square then the given roots are rational, or else they are irrational.

2. Sum and product of the roots

For any given equation the sum of the roots will always be $-\frac{b}{a}$ and the product of the roots will be $-\frac{c}{a}$ Thus, the standard quadratic equation can also be written as $x^2 – (A + B)x + A*B = 0$

3. Forming a quadratic equation

The equation can be formed when the roots of the equation are given or the product and sum of the roots are given.

<u>10.4 How to solve Quadratic Equation Questions:</u>

- <u>Factorization and completing square method:</u>

Question : Solve

$$2x^2 + 2x - 1 = x^2 + 6x - 5$$.

Answer :

The above equation can be rewritten as ,

$2x^2 + 2x - 1 = x^2 + 6x - 5$

or, $x^2 - 4x + 4 = 0$

Now ,

$x^2 - (2+2)x + 4 = 0$

Or, (x-2)(x-2)=0

or,x=2,2

Question :Solve $25a = 12a^2$.

Answer :

The above equation can be rewritten as ,

$12a^2 - 25a = 0$

or, $a(12a - 25) = 0$

Now ,

a(12a-25)=0

Or, a=0 and (12a-25)=0

or,a=0, $\frac{25}{12}$

Question: Solve $a^4 - 2a^3 = 3a^2$.

Answer :

The above equation can be rewritten as ,

$a^4 - 2a^3 - 3a^2 = 0$

or, $a^2(a^2 - 2a - 3) = 0$

Now ,

$a^2(a-3)(a+1) = 0$

or, $a = 0, (a-3) = 0 and (a+1) = 0$

or, $a = 0, 3, -1$

Question : Solve $y^{2) - 6y + 5 = 0}$

Answer :

The above equations can be rewritten as ,

$y^{2) - 6y = -5}$ ....eq(i)

In the above equations , coefficient of a=1 , therefore,

We will add , $(\frac{-6}{2})^2 = 9$ to both the sides of the equations to complete the square.

$y^{2) - 6y + 9 = -5 + 9}$

or, $y^{2) - 6y + 9 = 4}$

or, $(y-3)^{2) = 4}$

Therefore,

$(y-3) \pm 2$

Or, $y = 5 and y = 1$

Question : Solve $(a-2)^2 - 36 = 0$.

Answer :

The above equation can be rewritten as ,

$(a-2)^2 = 36$

Now ,

$(a-2)^2 = 36$

or, $(a-2) \pm 6$

or, $a = -4, 8$

Question 6 : Solve $(6a+1)^2 + 3 = 0$.

Answer :

The above equation can be rewritten as ,

$(6a+1)^2 = -3$

Now ,

$(6a+1) \pm \sqrt{-3}$

or, $(6a+1) \pm \sqrt{3}i$

or, $6a = -1 \pm \sqrt{3}i$

or,

$a = -(\frac{1}{6}) - (\frac{\sqrt{3}}{6})i - (\frac{1}{6}) + (\frac{\sqrt{3}}{6})i$

Question : Solve $a^{2) + 2a - 4 = 0}$

Answer :

The above equations can be rewritten as ,

$a^{2) + 2a = 4}$ ....eq(i)

In the above equations , coefficient of a=1 , therefore, We will add , $(\frac{2}{2})^2 = 1$ to both the sides of the equations to complete the square.

$a^{2)+2a+1=4+1}$

or, $a^{2)+2a+1=5}$

or, $(a+1)^{2)=5}$

Therefore,

$(a+1) \pm \sqrt{5}$

or, $a = -1 + \sqrt{5} and a = -1 - \sqrt{5}$

Question :

Solve $\frac{(a^2-10)}{(a+2)} + a - 4 = a - 3]$.

Answer :

The above equation can be rewritten as ,

$(a^2-10) + (a-4)(a+2) = (a-3)(a+2)$

Now ,

$a^2 - 10 + a^2 - 2a - 8 = a^2 - a - 6$

or, $a^2 - a - 12 = 0$

or, $(a-4)(a+3) = 0$

or, $a = 4, -3$

Question :

Solve $\frac{4a}{a+1} + \frac{5}{a} = \frac{6a+5}{a^2+a}$.

Answer :

The above equation can be rewritten as ,

$a(a+1)(\frac{4a}{a+1} + \frac{5}{a}) = (\frac{6a+5}{a^2+a})a(a+1)$

Now ,

$(a)(4a) + 5(a+1) = 6a + 5$

or, $4a^2 + 5a + 5 = 6a + 5$

or, $4a^2 - a = 0$

or, $a(4a-1) = 0$

Or, $4a - 1 = 0$

or, $a = \frac{1}{4}$

We ignored a=0 since that would give us division by zero.

Question : Solve $2a^{2)+3a+2=0}$

Answer :

The above equations can be rewritten as,

$2a^{2)+3a=-2}$ ...eq(i)

In the above equations , coefficient of $a \neq 1$, therefore,

We will multiply , $(\frac{1}{2})$ to both the sides of the equations.

Thus , $a^2 + \frac{3}{2}a = -1$

And now we will add $[(\frac{1}{2})(\frac{3}{2})]^2 = (\frac{9}{16})$ to both sides of the equations to complete the square.

$a^2 + (\frac{3}{2})a + (\frac{9}{16}) = -1 + (\frac{9}{16})$

Or, $a^2 + (\frac{3}{2})a + (\frac{9}{16}) = -(\frac{7}{16})$

or, $(a + (\frac{3}{4}))^2 = -(\frac{7}{16})$

Or, $(a + (\frac{3}{4})) = \pm\sqrt{(\frac{-7}{16})}$

Therefore,

or,

$a = a = -(\frac{3}{2})) + \sqrt{(\frac{-7}{16})} and a = a = -(\frac{3}{2})) - \sqrt{(\frac{-7}{16})}$

- <u>Discriminant and Quadratic Formula</u>

Question 1 :

Solve for y, $y^{2)=-2y+2}$

Answer :

The above equation can be rewritten as ,

$$y^2 + 2y - 2 = 0$$

Here , $a = 2, b = 2, c = -1$

Now , Discriminant , $D = b^2 - 4ac$

$= 2^2 - 4(2)(-1)$

=12

Since , D > 0 , the equation has real roots.

Substituting a,b and c in the quadratic formula , we get ,

$$y = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$y = \frac{-2 \pm \sqrt{(2^2 - 4(1)(-2))}}{2(1)}$$

or,

or,

$$y = y = -1 + \sqrt{3} and y = -1 - \sqrt{3}$$

Question 2 :

Find the roots of $5y^2 + 6y + 1 = 0$

Answer :

Here, $a = 5, b = 6, c = 1$

Now , Discriminant , $D = b^2 - 4ac$

$= 6^2 - 4(5)(1)$

$= 16$

Since , D > 0 , the equation has real roots.

Substituting a,b and c in the quadratic formula , we get ,

$$y = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$y = \frac{-6 \pm \sqrt{(6^2 - 4(5)(1))}}{2(5)}$$

Or,

or, $y = -1 and y = -0.2$

Question 3 :

Solve $x^2 + 2x = -1$

Answer :

The above equation can be rewritten as ,

$$x^2 + 2x + 1 = 0$$

Here, $a = 1, b = 2, c = 1$

Now , Discriminant , $D = b^2 - 4ac$

$= 2^2 - 4(1)(1)$

=0

Since , D = 0 , the equation has one root.

Substituting a,b and c in the quadratic formula , we get ,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2^2 - 4(1)(1))}}{2(1)}$$

or,

or, $x = -1$

Question 4 :

Solve $y^2 - 4y + 6.25 = 0$

Answer :

Here, a=1,b=-4,c=6.25

Now , Discriminant , $D = b^2 - 4ac$

$$= (-4)^2 - 4(1)(6.25)$$

=-9

Since , D < 0 , the equation has no real roots.

Substituting a,b and c in the quadratic formula , we get ,

$$y = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Or

$$y = \frac{-(-4) \pm \sqrt{((-4)^2 - 4(1)(6.25))}}{2(1)}$$

Or, $y = 2 + 1.5i and y = 2 - 1.5i$

Question 5 :

Solve y(y+2)=-2

Answer :

The above equations can be rewritten as,

$$y^2 + 2y + 2 = 0$$

Here, $a = 1, b = 2, c = 2$

Now , Discriminant , $D = b^2 - 4ac$

$$= 2^2 - 4(1)(2)$$
=-4

Since , D < 0 , the equation has no real roots.

Substituting a,b and c in the quadratic formula , we get ,

$$y = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

or, $y = \frac{-2 \pm \sqrt{(2^2 - 4(1)(2))}}{2(1)}$

or, $y = -1 + iandy = -1 - i$

Question 6 :

Solve $5y^2 + 2y + 1 = 0$

Answer :

The given equation is,

$$5y^2 + 2y + 1 = 0$$

Here, $a = 5, b = 2, c = 1$

Now , Discriminant , $D = b^2 - 4ac$
$$= 2^2 - 4(5)(1)$$
$$= -16$$

Since , D < 0 , the equation has no real roots.

Substituting a,b and c in the quadratic formula , we get ,

$$y = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

or, $y = \frac{-2 \pm \sqrt{(2^2 - 4(5)(1))}}{2(5)}$

or,

$y = -0.2 + 0.4iandy = -0.2 - 0.4i$

Question 7 :

Solve $2y^2 + 6y + 29 = 0$

Answer :

The given equation is,

$$2y^2 + 6y + 29 = 0$$

Here, $a = 2, b = 6, c = 29$

Now , Discriminant , $D = b^2 - 4ac$
$$= 6^2 - 4(2)(29)$$
=-196

Since , D < 0 , the equation has no real roots.

Substituting a,b and c in the quadratic formula , we get ,

$$y = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

or, $y = \frac{-6 \pm \sqrt{(6^2 - 4(2)(29))}}{2(2)}$

or,

$y = -1.5 + 3.5iandy = -1.5 - 3.5i$

Question 8 :

If 54 is the product of two consecutive positive multiples of three, then, find the sum of the two positive numbers ?

Answer :

Let us consider the first number be x

and the second number be (x+3)

According to the question ,

x(x+3)=54

or, $x^2 + 3x - 54 = 0$

or, $x^2 + 9x - 6x - 54 = 0$

or,(x+9)(x-6)=0

or,x=-9 , 6

But positive number is considered in the question , so ,

The two consecutive numbers are 6 and 9 .

Therefore,

Required Sum = 6 + 9

= 15

Question 9 :

If

$a(a^2 - 5a + 3) = -9, a > 0andb^2 + b = 2, b < 0$

, then find the value of $ab^2 + b$.

Answer :

Considering ,

$$a(a^2 - 5a + 3) = -9, a > 0$$

or, $a^3 - 5a^2 + 3a + 9 = 0$

or, $(a^2 - 6a + 9)(a + 1) = 0$

or, $(a - 3)^2(a + 1) = 0$

or, $a = 3, -1$
Since , a>0 , so a=3

Now,
$$b^2 + b - 2 = 0$$
or, $(b+2)(b-1) = 0$
or,b=-2,1
Since, b<0 , so , b=-2

Therefore ,
$$= 3(-2)^2 - 2$$
=10

Question 10 :
Let
$$f(a) = a^2 - 4a + 2, andg(a) = 6 - x$$
. Find the sum of the possible values of b such that f(b)=g(b).

Answer :
According to the question ,
f(b)=g(b)
Or, $b^2 - 4b + 2 = 6 - b$
or, $b^2 - 3b - 4 = 0$
Or, (b-4)(b+1)=0
or,b=4 , -1

Therefore ,
Required sum = 4 + (-1)
= 3

<u>Exercise:</u>

Question 1 :
Solve $2x^2 + 2x - 1 = x^2 + 6x - 5$.

Question 2 :
Solve $25a = 12a^2$.

Question 3 :
Solve $a^4 - 2a^3 = 3a^2$.

Question 4 :
Solve $y^{2)-6y+5=0}$

Question 5 :
Solve $(a-2)^2 - 36 = 0$.

Question 6 :
Solve $(6a+1)^2 + 3 = 0$.

Question 8 :
Solve $a^2 + 2a - 4 = 0$

Question 9 :
Solve $2a^2 + 3a + 2 = 0$

Chapter 11: Coordinate Geometry

The study of geometry using a coordinate system is known as coordinate geometry.

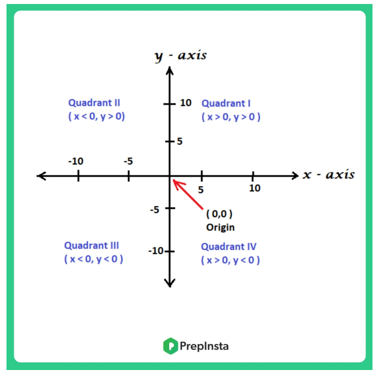
Or,

Coordinate geometry is a branch of geometry in which the position of points on a plane is identified using an ordered pair of numbers called coordinates.

It is also known as Analytical Geometry or Cartesian Geometry. It is a method of describing the location of points in a cartesian plane that was proposed by the French mathematician René Descartes (1596 – 1650).
Points are positioned on the coordinate plane of coordinate geometry. The x-axis, which runs across the plane, and the y-axis, which runs at right angles to it, are the two scales.

Coordinate Geometry Basic Concept

- Coordinate geometry is a branch of geometry in which the positions of points on a plane are determined using an ordered pair of numbers called coordinates.
- The origin is defined as the point where the x and y axes intersect. Both x and y are zero at this point.
- The positive values on the right-hand side of the x-axis are greater than the negative values on the left-hand side.



11.1 Formulas for Coordinate Geometry:

- Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$
  AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Slope of line when two points are given (x_1, y_1) and (x_2, y_2)
  m = $\dfrac{(y_1 - y_2)}{(x_1 - x_2)}$
- Slope of line when linear equation is given ax + by = c => $-\dfrac{a}{b}$
- Midpoint = $\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}$
- The coordinates of a point R(x,y) that divides a line segment joining two points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio m:n is given by
  x = $\dfrac{mx_2 + nx_1}{m + n}$

y = $\dfrac{my_2 + ny_1}{m + n}$

- The coordinates of a point R(x,y) that divides a line segment joining two points $A(x_1, y_1)$ and $B(x_2, y_2)$ externally in the ratio m:n is given by
  x = $\dfrac{mx_2 - nx_1}{m - n}$

y = $\dfrac{my_2 - ny_1}{m - n}$

- Centroid of a triangle with its vertices (x_1,y_1), (x_2,y_2), (x_3,y_3)
  C = $\frac{x_1+x_2+x_3}{3}$, $\frac{y_1+y_2+y_3}{3}$
- Area of a Triangle with its vertices A(x_1,y_1), B(x_2,y_2), C(x_3,y_3)
  A = $\frac{1}{2}[(x_1(y_2-y_3)+(x_2(y_3-y_1))+(x_3(y_1-$
- Division of a line segment by a point
  If a point p(x, y) divides the join of A(x_1, y_1) and B(x_2, y_2), in the ratio m: n, then
  x= $\frac{mx_2+nx_1}{m+n}$ and y= $\frac{my_2+ny_1}{m+n}$
- The equation of a line in slope intercept form is Y= mx+ c, where m is its slope.
  The equation of a line which has gradient m and which passes through the point (x_1, y_1) is =
  $y - y_1 = m(x - x_1)$.

11.2 How To solve Coordinate Geometry:

Formula : Distance between two points

Question 1 :

A delivery boy picked up a package from the store located at point A (7,3) and delivered it to the location located at point B(2,-6). Find the distance traveled by the delivery boy .

Answer :
Here , (x1,y1)=(7,3)
(x2,y2)=(2,-6)
Therefore , required Distance =

$$\sqrt{(x2-x1)^2+(y2-y1)^2}$$

$$=\sqrt{(2-7)^2+(3-(-6))^2}$$

$$=\sqrt{(5)^2+(9)^2}$$

$$=\sqrt{25+81}$$

$$=\sqrt{106}$$

Question 2 :

Find a point on the y-axis that is equidistant from the points (2,3) and (-1,2) .

Answer :
We know that the x-coordinate of any point on the y-axis is 0
Hence, we assume the point that is equidistant from the given points to be (0,a)
Therefore ,

$$\sqrt{(2-0)^2+(3-a)^2}=\sqrt{(-1-0)^2+(2-a)^2}$$

or,

$$\sqrt{(2)^2+(3-a)^2}=\sqrt{(-1)^2+(2-a)^2}$$

Squaring both sides , we get ,

$$4+9-6a+a^2=1+4-4a+a^2$$

or, $a=4$

Therefore , the required point is (0,4) .

Question 3 :

A,B and C are the points such that AB=BC whose coordinates are (6,-1) , (1,3) and (k,8) respectively . Find the value of k .

Answer :
Coordinate of A (6,-1) .
Coordinate of B (1,3)
Coordinate of C (k,8)

Distance between A and B (AB)

$$=\sqrt{(x2-x1)^2+(y2-y1)^2}$$

$$=\sqrt{(6-1)^2+(-1-3)^2}$$

$$=\sqrt{(5)^2+(-4)^2}$$

$$=\sqrt{25+16}$$

$$=\sqrt{41}$$

Distance between B and C (BC)

$$=\sqrt{(x2-x1)^2+(y2-y1)^2}$$

$$=\sqrt{(1-k)^2+(3-8)^2}$$

$$=\sqrt{(1-k)^2+(-5)^2}$$

$$= \sqrt{(1-k)^2 + 25}$$

According to the question ,
AB=BC

or, $\sqrt{41} = \sqrt{(1-k)^2 + 25}$

Squaring both sides , we get ,

$$41 = (1-k)^2 + 25$$

or, $(1-k)^2 = 41 - 25$

or, $(1-k)^2 = (4)^2$

or, $1 - k \pm 4$

Or, k=5,-3

Question 4 :

Find the area of the circle if the endpoints of the diameter of the circle is located at points (4,6) and (0,0).

Answer :
Length of the diameter is the distance between the two endpoints (4,6) and (0,0) .
So , Distance

$$= \sqrt{(x2-x1)^2 + (y2-y1)^2}$$
$$= \sqrt{(0-4)^2 + (0-6)^2}$$
$$= \sqrt{(-4)^2 + (-6)^2}$$
$$= \sqrt{24 + 36}$$
$$= \sqrt{60}$$
$$= 2\sqrt{15}$$

Therefore , $radius = \dfrac{diameter}{2}$

$$= \sqrt{15}$$

Now , Required area of the circle $= \Pi(radius)^2$

$$= \Pi(\sqrt{15})^2$$
$$= 15\Pi$$

Question 5 :

A rectangular box with vertices A,B,C and D are represented by points (7,7),(7,1),(3,1),(1,7) respectively . Calculate the diagonal length of the rectangular box .

Answer :
To determine the length of the diagonal , we will consider the vertices which are lying diagonally to each other .
We consider , (7,7) and (3,1)
Therefore , Required Distance

$$= \sqrt{(x2-x1)^2 + (y2-y1)^2}$$
$$= \sqrt{(3-7)^2 + (1-7)^2}$$
$$= \sqrt{(-4)^2 + (-6)^2}$$
$$= \sqrt{24 + 36}$$
$$= \sqrt{60}$$
$$= 2\sqrt{15}$$

<u>Formula : Slope of Line</u>

Question 1 :

Find the equation of a straight line passing through a point (2, –3) and having a slope of 1 unit.

Answer:
Here,
Given point : (2, –3)
and, slope = 1
According to the formula ,
(y – y1) = m (x – x1)
or, y – (–3) = 1 (x – 2)
or,y + 3 = x – 2
or,y – x + 5 = 0

Therefore , Required Equation : y – x + 5 = 0

Question 2 :

Find the slope of a line passing through a point (12,4) and is defined by y=mx+7.

Answer :
The given equation is y=mx+7.

Thus , the y-intercept is 7
and the point is (0,7)

Therefore , Required Slope , $m = \dfrac{y1 - y2}{x1 - x2}$

$$m = \frac{4-7}{12-0}$$

$$m = \frac{-3}{12}$$
$$m = \frac{-1}{4}$$

Question 3 :

Find the slope of the line $\frac{14}{3}x = \frac{1}{6}y - 7$.

Answer :

The given equation is $\frac{14}{3}x = \frac{1}{6}y - 7$

Or, y=28x-7

Therefore, the required slope is 28 .

Question 4 :

A straight line passes through the points (8,-4) and (7,3) . Find the slope of the straight line .

Answer :

The given points are (8,-4) and (7,3) .

Therefore , Required Slope $m = \frac{y1 - y2}{x1 - x2}$

$$m = \frac{-4 - 3}{8 - 7}$$
$$m = \frac{-7}{1}$$

m=-7

Question 5 :

A straight line has an equation 2x+3y-6=0. Find the slope of a line perpendicular to the given straight line .

Answer :

The given equation of the straight line can be rewritten as ,

2x+3y-6=0

or,2x+3y=6

or, $y = \frac{-2}{3}x + 2$

We know that the slope of the perpendicular line is reciprocal of the given line .

Therefore, Required Slope = Reciprocal of $\frac{-2}{3}$

$= \frac{-3}{2}$

Question 6 :

Find the slope of a line parallel to a straight line whose equation is given by 2x-y=4?

Answer :

The given equation of the straight line can be rewritten as ,

y=2x-4

Therefore , slope of the line = 2

We know that two parallel lines have the same slope .

Therefore, Required slope of the parallel line = 2 .

Question 7 :The vertices of a triangle are represented by the points A(0 , -1) , B(2 , 1) and C(-4 , 3) . Determine whether the triangle is the right triangle or not.

Answer :

The given vertices are A(0 , -1) , B(2 , 1) and C(-4 , 3)

Now, Slope of point A and point B ,

$$m1 = \frac{y1 - y2}{x1 - x2}$$
$$m1 = \frac{1 - (-1)}{2 - 0}$$
$$m1 = \frac{2}{2}$$
$$m1 = 1$$

Again , Slope of point A and point C ,

$$m2 = \frac{y1 - y2}{x1 - x2}$$
$$m2 = \frac{3 - (-1)}{-4 - 0}$$
$$m2 = \frac{4}{-4}$$
$$m2 = -1$$

Now , multiplying both the slopes m1 and m2 , we get ,

$$m1xm2 = -1$$

Since the product of the two slopes is -1 , therefore , the line representing the points AB and AC are perpendicular to each other .

Thus , the given triangle is a right triangle .

Question 8 :

Find the slope of the straight line x+3=0.

Answer :

The given above equation can be rewritten as , x=-3

So, the equation of the straight line is perpendicular to the x-axis .

Therefore , the slope is undefined.

Question 9 :

Determine whether the three points P(2 , 3) , Q(5 , 6) and R(0 , -2) are collinear or not.

Answer :

The given points are P(2 , 3) , Q(5 , 6) and R(0 , -2)

Now, Slope of point P and point Q ,

$$m1 = \frac{y1 - y2}{x1 - x2}$$

$$m1 = \frac{6 - 3}{5 - 2}$$

$$m1 = \frac{3}{3}$$

$$m1 = 1$$

Again , Slope of point Q and point R ,

$$m2 = \frac{y1 - y2}{x1 - x2}$$

$$m2 = \frac{-2 - 6}{0 - 5}$$

$$m2 = \frac{-8}{-5}$$

$$m2 = \frac{8}{5}$$

Therefore , $m1 \neq m2$

Thus , the given points are not collinear .

Question 10 :

Find the slope of the straight line y-9=2

Answer :

The given above equation can be rewritten as , y=11

So, the equation of the straight line is parallel to the x-axis .

Therefore , the slope is equal to 0 .

Formula : Midpoint

Question 1 :

If the origin is the mid-point of the line segment joined by the points (a,b) and (-3,2). Find the value of (a, b).

Answer :

The two points given are (a,b) and (-3,2) .

Origin , O = (0,0)

Here , x1=a , x2=-3

and y1=b , y2=2

We know that,

$$x = \frac{x1 + x2}{2}; y = \frac{y1 + y2}{2}$$

Applying the above formula , we get ,

$$0 = \frac{a + (-3)}{2}; 0 = \frac{b + 2}{2}$$

or,a=3 ; b=-2

Formula : Dividing Line Segment

Question 1 :

Calculate the coordinates of the point which will divide the line which joins the two points (3,5) and (11, 8) externally with a ratio of 5:2.

Answer :

We know that the formula for external division case is ,

$$X=\frac{mx2-nx1}{m-n},Y=\frac{my2-ny1}{m-n}$$

Here , $x1=3,x2=11$

and $y1=5,y2=8$

According to the given question, the ratio given is 5:2,

I.e. $m=5;n=2$

Putting the values in the formula , we get ,

$$X=\frac{49}{3};Y=10$$

Formula : Equation of Line

Question 1 :

What will be the equation of the line whose slope is 3 and its y-intercept is -4?

Answer :
Here, Slope m = 3, and c= -4
When we put these values into the equation, we get ,
y = mx +c
or,y = 3x – 4

Exercise:

Question 1 :A delivery boy picked up a package from the store located at point A (7,3) and delivered it to the location located at point B(2,-6). Find the distance traveled by the delivery boy .

Question 2 :Find a point on the y-axis that is equidistant from the points (2,3) and (-1,2) .

Question 3 : A,B and C are the points such that AB=BC whose coordinates are (6,-1) , (1,3) and (k,8) respectively . Find the value of k .

Question 4 : Find the area of the circle if the endpoints of the diameter of the circle are located at points (4,6) and (0,0).

Question 5 : A rectangular box with vertices A,B,C and D are represented by points (7,7),(7,1),(3,1),(1,7) respectively . Calculate the diagonal length of the rectangular box .

Question 6: Find the equation of a straight line passing through a point (2, –3) and having a slope of 1 unit.

Question 7 : Find the slope of a line passing through a point (12,4) and is defined by $y=mx+7$

Question 8 : Find the slope of the line

$$\frac{14}{3}x=\frac{1}{6}y-7$$.

Question 9 : A straight line passes through the points (8,-4) and (7,3) . Find the slope of the straight line .

Question 10 : A straight line has an equation $2x+3y-6=0$ **. Find the slope of a line perpendicular to the given straight line .**

Chapter 12: LCM and HCF

What is LCM Lowest Common Multiple –

As the name implies, LCM is the lowest common multiple of two or more Natural Numbers. For example, the LCM for 15 and 20 is 60 (don't worry, we'll explain how we did it).

Or,

LCF – Least Common Factor is a least number that divides any number and leaves remainder zero.

What is HCF Highest Common Factor or Greatest Common Divisor(GCD)-

As the name implies, LCM is the lowest common multiple of two or more Natural Numbers. For example, the LCM for 15 and 20 is 60 (don't worry, we'll explain how we did it).

Or,HCF – Highest Common Factor (HCF) of two or more numbers is the greatest number which divides each of them exactly.

Prime Factors –

These are a list of prime numbers that divide a larger number, for example, 20 – 2, 5 are prime Factors. (Don't worry, we'll show you how to get them below.)

12.1 HCF and LCM Tips and Tricks and Shortcuts

- The H.C.F of two or more numbers is smaller than or equal to the smallest number of given numbers
- The smallest number which is exactly divisible by a, b and c is L.C.M of a, b, c.
- The L.C.M of two or more numbers is greater than or equal to the greatest number of given numbers.
- The smallest number which when divided by a, b and c leaves a remainder R in each case. Required number = (L.C.M of a, b, c) + R
- The greatest number which divides a, b and c to leave the remainder R is H.C.F of (a – R), (b – R) and (c – R)
- The greatest number which divide x, y, z to leave remainders a, b, c is H.C.F of (x – a), (y – b) and (z – c)
- The smallest number which when divided by x, y and z leaves remainder of a, b, c (x – a), (y – b), (z – c) are multiples of M
  Required number = (L.C.M of x, y and z) – M

12.2 Properties of LCM and HCF:

Property 1 - LCM x HCF = Product of two numbers

H.C.F. and L.C.M. of Fractions:

$$H.C.F. = \frac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$$

$$L.C.M = \frac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$$

Property 2 - LCM ≥ Numbers ≥ HCF

Property 3- LCM is a multiple of HCF

Property 4 - If the HCF of two numbers is 1 then they are Co-Primes.

12.3 How to solve LCM and HCF:

Calculating Prime Factors -

In this method, you take the lowest prime number and see if the greater number is divisible by it. If it's not divisible then you move to the higher prime number.

Let us show you how to calculate the prime factors for numbers step by step.

Prime Factors of 12 -

- Lowest Prime number 2, 12 ÷ 2 = 6
- Again 6 ÷ 2 = 3
- Again 3 ÷ 2 is not possible so next prime number is 3
- Again 3 ÷ 3 = 1

We can write this as 12 = 2x2x3 = $2^2 \times 3$

Finding LCM of two Numbers -

Let us take an example HCF of 15 and 20, first we list out all the prime factors of each

15 = 3 x 5

20 = 2 x 2 x 5 = $2^2 \times 5$

Then multiply each factor by the greatest number of times it occurs in either number.

- 2 - Occurs 2 times
- 3 - Occurs 1 time

- 5 - Occurs 1 time

So LCM is (2x2)x(3x1)x(5x1) = 60

Finding HCF of two Numbers

Let us take an example HCF of 18 and 24, we already have listed out all the prime factors of each number

18 = 2 x 3 x 3

24 = 2 x 2 x 2 x 3 = $2^3 \times 3$

Now we find the factor that exists at least once in both of them.

There is 2 and 3 one in common factor

The GCD is 2 x 3= 6

Solve the Questions:

1. **What is the greatest number dividing 41, 90, and 180 which generates the same remainder for each number?**
2. **What will be the HCF of 513, 1134, and 1215?**
3. **What is the HCF of 0.63 and 1.05?**
4. **Find out the HCF of 2/5,11/5,8/9**
5. **22 is the H.C.F of two numbers. The LCM of these two numbers has factors 12 and 13. Which is the larger of the two numbers?**
6. **Five bells start to toll together at intervals 3, 5, 7, 9, and 11 seconds respectively. In the next 60 minutes, in how many instances do they ring together?**
7. **Find out the least number which should be added to 25 to generate a sum which leaves no remainder when divided by 3 and 9.**
8. **What will be the LCM of 1.05 and 2.1?**
9. **Find out the least number which when divided by 12, 20, and 15, leaves no remainder?**
10. **What will be the LCM of 2, 6, and 8?**
11. **Calculate the Lowest Common Multiple of 20, 30, and 40.**
12. **The LCM and HCF of two numbers are 50 and 5. If one of the numbers is 25, calculate the second number.**
13. **The ratio of two numbers is 3: 2. If the HCF of them is 10. What are the numbers?**
14. **The ratio of two numbers is 1 : 2. If their LCM is 20, then what is their sum?**
15. **23 is the HCF of two numbers. The other two factors of their LCM are 14 and 13. What is the larger number among them?**
16. **Find the 4-digit smallest number which when divided by 12, 15, 25, 30 leaves no remainder?**
17. **Find the least number which when divided by 2, 3, 4 and 5 leaves a remainder 3. But when divided by 9 leaves no remainder ?**
18. **The HCF of 2472, 1284 and a third number 'N' is 12. If their LCM is 2^3 × 3^2 × 5 × 103 × 107, then the number 'N' is -----.**
19. **2 gears. one with 12 teeth and the other one with 14 teeth are engaged with each other. One tooth in smaller gear and one tooth in bigger gear are marked and initially those 2 marked teeth are in contact with each other. After how many rotations of the smaller gear with the marked teeth in the other gear will again come into contact for the first time?**
20. **The numbers 272738 and 232342, when divided by n, a two digit number, leave a remainder of 13 and 17 respectively. Find the sum of the digits of n?**
21. **How many different integers can be expressed as the sum of three distinct numbers from the set {3, 10, 17, 24, 31, 38, 45, 52}?**
22. **Four Iron metal rods of lengths 78 cm, 104 cm, 117 cm and 169 cm are to be cut into parts of equal length. Each part must be as long as possible. What is the maximum number of pieces that can be cut?**
23. **How many prime numbers are there which are less than 100 and greater than 3 such that they are of the following forms 4x + 1 , 5y - 1?**

24. **All even numbers from 2 to 98 inclusive, except those ending 0, are multiplied together. What is the rightmost digit (the units digit) of the product?**

25. **What is the greatest power of 143 which can divide 125! exactly.**

26. **If the HCF of 180 and 432 is expressed as (180m + 432n) , where m and n are integers , then what is the difference between m and n?**

27. **The least number which when divided by 48, 60, 72, 108 and 140 leaves 38, 50, 62, 98 and 130 as remainders respectively, is ----**

28. **The sum of two numbers is 2604 and their hcf is 124. Which is smaller between them if their difference is the least possible?**

29. **What will be the least number which when doubled will be exactly divisible by 12, 18, 21 and 30 ?.**

30. **The least number which when divided by 5, 6 , 7 and 8 leaves a remainder 3, but when divided by 9 leaves no remainder, is ---.**

31. **A, B and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 seconds, B in 308 seconds and c in 198 seconds, all starting at the same point. After what time will they again at the starting point again ?.**

32. **The product of LCM and HCF of two numbers is 48. The difference of two numbers is 8. Find the numbers?**

33. **Four bells begin to toll together respectively at the intervals of 8, 10, 12 and 16 seconds. After how many seconds will they toll together again?**

34. **If the ratio of two numbers is 3:4 and LCM of the number is 180 then what is the number?**

35. **Which is the largest integer that divides all three numbers 23400,272304,205248 without leaving a remainder?**

Chapter 13. Percentages

Percentage is a statistic expressed as a percentage of 100 in Aptitude. It's done with the assistance of a ratio. Percentage is denoted by the notation " % ".

- In mathematics, a percentage is a number or ratio expressed as a fraction whose denominator (bottom) is 100.
  Thus, x percent means x hundredths, written as x%.
  We express x% as a fraction as $\frac{x}{100}$
- For example 10% = $\frac{10}{100} = \frac{1}{10}$

13.1 Percentages Formulas & Basic Concept

- To calculate a % of b = $\frac{\mathrm{a}}{100}\mathrm{b}$
- To find what percentage of a is b = $\frac{\mathrm{b}}{\mathrm{a}}100$
- To calculate percentage change in value
  Percentage change =
  $$\frac{\mathrm{Change}}{\mathrm{InitialValue}}100$$
- Percentage Increase or Decrease
  - Percentage increase =
    $\frac{\mathrm{R}}{100+\mathrm{R}}100$ %
  - Percentage decrease =
    $\frac{\mathrm{R}}{100-\mathrm{R}}100$ %
- Successive Percentage Change
  If there are successive percentage increases of a % and b%, the effective percentage increase is:
  $\mathrm{a}+\mathrm{b}+\frac{\mathrm{ab}}{100}$ %
- Successive Discount :

If there are successive discount of a % and b%, the effective discount is: $\mathrm{a}+\mathrm{b}-\frac{\mathrm{ab}}{100}$%

- Results on population
  Let the population of a town be P now and suppose it increases at the rate of R% per annum, then:
    - Population after n years = $$\mathbf{P(1+\frac{R}{100})^n}$$
    - Population n years ago = $$\mathbf{\frac{P}{(1+\frac{R}{100})^n}}$$
- Results on Depreciation:
  Let the present value of a machine be P. Suppose it depreciates at the rate of R% per annum. Then:
    - Value of the machine after n years = $$\mathbf{P(1-\frac{R}{100})^n}$$
    - Value of the machine n years ago = $$\mathbf{\frac{P}{(1-\frac{R}{100})^n}}$$
    - If A is R% more than B, then B is less than A by $\mathbf{\frac{R}{100+R}100}$ %
    - If A is R% less than B, then B is more than A by $\mathbf{\frac{R}{100-R}100}$ %
- Increase N by S% : $N(1+\frac{S}{100})$.
- Decrease N by S% : $N(1-\frac{S}{100})$

13.2 How to Solve Quickly Percentage Questions & Definition

- A percentage is a ratio expressed as a fraction whose denominator (bottom) is 100.Thus, x percent means x hundredths, written as x%.
- A percentage is a fraction of an amount expressed as a particular number of hundredths of that amount.

What is P percent of X?

- Written as an equation: Y = P% × X.
- The 'what' is Y that we want to solve for
- Remember to first convert percentage to decimal, dividing by 100
- Solution: Solve for Y using the percentage formula **Y = P% × X.**

13.3 Tips and tricks and shortcuts on Percentage

- Here are quick and easy tips and tricks on for Percentage problems swiftly, easily, and efficiently in competitive exams and other recruitment exams.
- If the value of an item goes up or down by x%, the percentage reduction or increment to be now made to bring it back to the original point is $\mathbf{\frac{x}{100+x}100}$ %
- If A is x% more or less than B, then B is $\mathbf{\frac{x}{100+x}100}$ % less or more than A.
- If the price of an item goes up/down by x %, then the quantity consumed should be reduced by $\mathbf{\frac{x}{100+x}100}$ % so that the total expenditure remains the same.
- Percentage – Ratio Equivalence table

Fraction Percentage

$\frac{1}{3}$ × 100 = 33.33% $\frac{1}{10}$ × 100 = 10%

$\frac{1}{4}$ × 100 = 25% $\frac{1}{11}$ × 100 = 9.09%

$\frac{1}{5}$ × 100 = 20% $\frac{1}{12}$ × 100 = 8.33%

$$\frac{1}{9}100 = 11.11\%$$

$\frac{1}{16}$ × 100 = 6.25%

Exercise:

1. **Gilchrist scored a total of 180 runs with 8 boundaries and 5 sixes. What % of runs did he make by running between the wickets?**

2. **Sam owned some Microsoft shares. He sold 35% of them and still left with 780 shares. How many shares he originally had?**

3. **In an office, 20% of the employees are below 30 years of age. 1/3 of employees are above 30 years of age which is equal to the number of employees with 50 years of age which is 24. What is the total no. of employees in the company?**

4. **Tom gave some money to son Harry for shopping. Harry spent 70% of the money and now is left with Rs 1200 only. How much money had he got from his father tom?**

5. **Tom failed in chemistry. He scored 300. And 40% were the passing marks. He fell short by 40 marks. What were the maximum marks he could have got?**

6. **Ben scored 8 marks more from what he did in the previous exam in which he scored 80 marks. And Jerry scored 10 marks more from what he did in his last exam where he got 50 marks. Who showed more improvement?**

7. **What % of Rs. 30 is 45 paise?**

8. **If 10% of X = Y, then Y% of 20 is equal to ---**

9. **What % of 8.9 Km is 2800meters?**

10. **The total strength of a class is 175. From which only 110 students had full attendance. Find out what % of students who did not get full attendance.?**

11. **1200 boys and 1500 girls appeared in a CA exam, out of which 45% boys and 65% girls passed the exam. How much % of total students (including boys and girls) were not able to clear the exam?**

12. **How much % is fraction 8/5 ?**

13. **What is 90% of a number whose 300% is 60?**

14. **What % of 9 km is 450 meters?**

Chapter 14. Probability

In Aptitude , Probability is the ratio of wanted outcomes to the total number of possible outcomes i.e.

P(A) =

$$\frac{\text{The Number Of wanted outcomes}}{\text{The total number Of Possible Outcomes}}$$

14.1 Formula & Definition for Probability

- **Probability** is a number that expresses the likelihood of a specific event occurring.
- The term "**probability**" refers to the likelihood of certain events occurring.
- When an event happens, such as throwing a ball or choosing a card from a deck, there must be some element of probability involved.
- In terms of mathematics, probability refers to the ratio of wanted outcomes to the total number of possible outcomes. There are three approaches to the theory of probability, namely:

Basic Definition

- **Random Event :-** When an experiment is repeated several times under comparable conditions, it does not always produce the same result, because the result of a trial is one of many possible outcomes, the experiment is referred to as a random event or a probabilistic event.

- **Elementary Event –** The result of each random event is referred to as the elementary event. Each related outcome is known as an elementary event whenever the random event is done.

- **Sample Space –** The set of all possible outcomes of a random event is referred to as Sample Space. When a coin is thrown, for example, the possible results are head or tail.

- **Event –** A subset of the sample space associated with a random event is referred to as an event.

- **Occurrence of an Event –** If any of the elementary events associated with a random event is an outcome, the event is said to occur.

Basic Probability Formulas

- **Probability Range -** $0 \leq P(A) \leq 1$

- **Rule of Complementary Events -** $P(A^C) + P(A) = 1$
- **Rule of Addition -** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- **Disjoint Events -** Events A and B are disjoint if $P(A \cap B) = 0$
- **Conditional Probability -** $P(A \mid B) = \dfrac{P(AB)}{2}$
- **Bayes Formula -** $P(A \mid B) = P(B \mid A) \cdot \dfrac{P(A)}{P(B)}$
- **Independent Events -** Events A and B are independent if. $P(A \cap B) = P(A) \cdot P(B)$.

14.2 How to Solve Quickly Probability questions

- You can solve many simple probability problems just by knowing two simple rules:
- The probability of any sample point can range from 0 to 1.
- The sum of probabilities of all sample points in a sample space is equal to 1.
- The probability of event A is denoted by P(A).

Exercise:

1. **Both Sruthi and Pooja Randomly Choose Balloons Of Color Red, Orange And Yellow. What Is The Probability That Both Choose a Red Balloon?**

2. **A box contains 4 Red erasers and 3 Green erasers. If Amit picks 3 erasers at a time, what is the probability that the erasers are of the same colour?**

3. **In a box there are 15 envelopes, some 5 out of them have Rs. 100 in it, while others are empty envelopes. One envelope is selected randomly, what is the probability that the selected envelope is empty?**

4. **A box contains 5 red marbles, 6 blue marbles, and 4 yellow marbles. Two marbles are drawn randomly. What is the probability that no yellow marble is drawn?**

5. **A box contains 5 red, 2 blue, and 3 black balls. 3 balls are selected at random. What is the probability that one ball is red and other is blue?**

6. **A container contains 25 electric lights, out of which 5 are defective. Three lights are chosen at random from this container. The probability that at least one of these is defective, is:**

7. **A packet contains 4 yellow, 6 green and 8 blue balls. Four balls are drawn at random from the packet. The probability that all of them are yellow, is:**

8. **6 Red, 4 Blue And 5 White Balls Are Kept In A Bag And Total 2 Balls Are Drawn At Random. What Is The Probability That The Ball Drawn Is A Red?**

9. **In A Game Show There Is 30; 500 Prize Cards And 20; 200 Prize Cards. A Contestant Is Asked To Choose A Card At Random. What Is The Probability That He Won A 500 Cards Prize?**

10. **A Bag Contains 10 Orange And 20 Pink Flavored Candies. One Candy Is Taken Out At Random. What Is The Probability That An Orange Flavored Candy Is Taken Out?**

Chapter 15. Compound Interest

The return on interest to the principal sum of an amount is known as compound interest. Instead of paying out interest, it is computed as a result of reinvesting it. Interest is received on the principal sum plus previously accrued interest in the following period as a result of this.

Compound Interest Formulas

- Compound interest is the interest calculated on the original principal and on the accumulated past interest of a deposit or loan. Compound interest is calculated based on the principal, interest rate (APR or annual percentage rate), and the time involved.

Formula of Compound Interest (CI) P - P

Formula of Amount =CI +P

$= P(1+\frac{r}{100})^n - P + P$

Here, P = Principal

r = rate of interest

t = the number of years the amount is deposited or borrowed for.

n= the number of times that interest is compounded per unit 't'.

15.1 Formulas for Compound Interest

- **Formulas for Compound Interest (When Interest is Compounded Annually)**
  - Amount = P$(1+\frac{r}{100})^n$
  - Compound Interest =Total amount – Principal
  - Rate of interest (R) = $[(\frac{A}{P})^{\frac{1}{t}} - 1]\%$
- **Formulas of Compound Interest (When Interest is Compound Half Yearly)**
  - Amount = **P** $(1+\frac{\frac{r}{2}}{100})^{2n}$
  - Compound Interest = Total amount – Principal
- **Compound Interest Formulas (When Interest is Compounded Quarterly)**
  - Amount = **P** $(1+\frac{\frac{r}{4}}{100})^{4n}$
  - Compound Interest = Total amount – Principal
- **Formulas of Compound Interest (When Interest is Compounded Monthly)**
  - Amount = **P** $(1+\frac{\frac{\mathbf{r}}{\mathbf{12}}}{\mathbf{100}})^{\mathbf{12n}}$
- **Compound Interest Formulas (When Interest is Compounded Annually but time is in fraction, say 2($\frac{3}{2}$)years)**
  - Amount = **P**

$$(1+\frac{\mathbf{r}}{\mathbf{100}})^2(1+\frac{\frac{3}{2}r}{100})$$

- **Formulas of Compound Interest (When rates are different for different years)**
  - Amount = P (1+$\frac{r_1}{100}$) (1+$\frac{r_2}{100}$)
  - (1+$\frac{r_3}{100}$)
- **Formulas for Compound Interest (Present worth of Rs. x due n years)**
  - Present worth = $\frac{x}{(1+\frac{r}{100})}$

15.2 How to solve Quickly:

- Compound interest is the interest calculated on the original principal and on the accumulated past interest of a deposit or loan.
- Compound interest calculated by multiplying the original principal amount one plus the annual interest rate raised to the number of compound periods minus one.

15.3 Tips and Tricks and Shortcuts on Compound Interest

- Here are quick and easy tips and tricks on Compound Interest. Learn the tricks and concept of compound interest.
- There are mainly 5 types of questions asked in exams. However, these questions can be twisted to check the students understanding level of the topic but it can be solved using tips and tricks.

MoreTricks and Tips and Shortcuts:

- A sum of money placed at compound interest becomes x time in ‘a’ years and y times in ‘b’ years. These two sums can be related by the following formula:
  $$x^{\frac{1}{a}} = y^{\frac{1}{b}}$$
- If an amount of money grows up to Rs x in t years and up to Rs y in (t+1) years on compound interest, then
  r% = $\frac{\mathbf{y} - \mathbf{x}}{\mathbf{x}} * 100$
- A sum at a rate of interest compounded yearly becomes Rs. A_1 in t years and Rs. A_2 in (t + 1) years, then
  P = A_1 $\left(\frac{A_1}{A_2}\right)$

<u>Exercise:</u>

1. **The compound interest at an amount 4% p.a. in 2 years is Rs. 51. Find the principal amount.**

2. **If the principle amount of Rs. 1000 needs to be compounded annually for the duration of 4 years, what will be the amount?**

3. **Calculate the compound interest for the sum of Rs. 12,000 at the rate of 5% per annum and for the duration of 1 year.**

4. **The cash price of a radio is Rs. 3000. A buyer paid Rs. 1200 in cash and promised to pay the remaining money in 2 monthly equal installments at the rate of 5% per annum compound interest. What is the value of each installment?**

5. **Calculate the compound interest for the amount of INR 20,000 at the rate of 10% annually for the duration of three years.**

6. **On a half-yearly basis, a bank offers 4% compound interest. If Anita deposits Rs. 1100 each on 1st Jan and 1st July of the year. What will be the amount of interest gained at the end of the year?**
7. **Calculate the compound interest for Rs. 20,000 after 2 years at the rate of 4% per**

8. **Aditi invested Rs. 10,000 in a scheme for the duration of two years at the rate of 2% compound interest. How much will she get at the maturity of the deposit?**

9. **If after a certain number of years, Rs. 10,000 amounts to Rs. 1,60,000 at compound interest, what will be Rs. 10,000 will amount in half of that time?**

10. **What will be the compound interest on Rs. 8000 at the rate of 15% per annum for the duration of 2 years and 4 months?**

11. **If compound interest is being calculated half-yearly at a rate of 5% for 1 year 6 months for ₹6400. Then find the amount.**

12. **Ankita made an initial investment of ₹1200 in Abhishek's future project. Abhishek promised that he will provide a 5 percent compound interest on the initial investment and after a time period of "x", she will end up receiving 123 rs extra. Find the value of x.**

13. **Axis Bank gives a difference of ₹122 on a particular sum of amount for the period of three years at the rate of 5 percent per annum. And the difference here is in between the interest of simple and compound interest, then what is the amount on which Axis Bank is giving difference?**

14. **The difference between the simple interest and compound interest is ₹57 which is given by the following banks like Punjab National Bank and Punjab and Sind Bank. Punjab National Bank is giving the simple interest on the sum for 4 years at 4 percent per annum and Punjab and Sind Bank is giving the compound Interest on the sum for 3 years at 5 percent per annum. Then what is the sum on which Punjab National Bank and Punjab and Sind Bank is giving the interest**

Chapter 16. Perimeter, Area and Volume

The path that surrounds or covers a two-dimensional shape is called a perimeter. Volume, on the other hand, is the amount of three-dimensional space surrounded by a closed surface. And the area of a two-dimensional figure or shape is the quantity that expresses its size.

Basic Points to keep in mind:

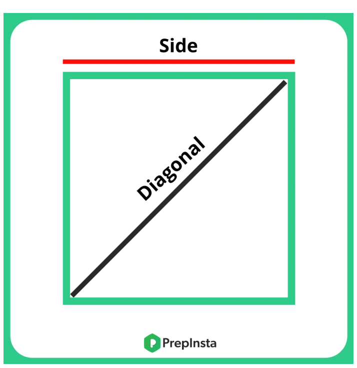
- **Geometry** is a branch of mathematics concerned with the study of various shapes and sizes. It can be categorized into two categories: Geometry of Planes and Solids
- Circles, triangles, rectangles, and squares are all examples of **plane geometry.**
- The length, perimeter, area, and volume of different geometric figures and shapes are calculated using solid geometry. The length, area, volume, and perimeter of different shapes and figures can be calculated using the following simple formulas.

16.1 Formulas to solve Perimeter, Area and Volume

- **Formulas for Square**

Here, s = side

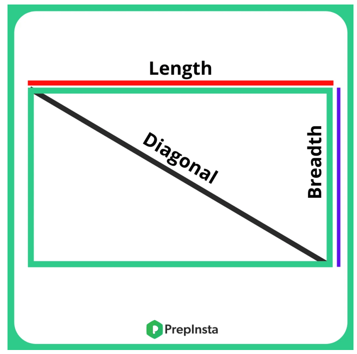
- Perimeter: 4 * s
- Area: S^2
- Diagonal: $s\sqrt{2}$
- Area of square when diagonal is given = $\frac{1}{2} \times d^2$



- **Formulas for Rectangle**

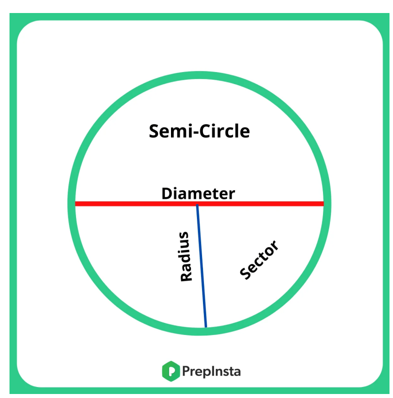
Here , l = length, b = breadth.

- Perimeter: 2 (l + b) (l = length, b = breadth)
- Area: l × b
- Diameter: $\sqrt{l^2 + b^2}$
- Area of 4 walls of a room = 2 (Length + Breadth) × Height

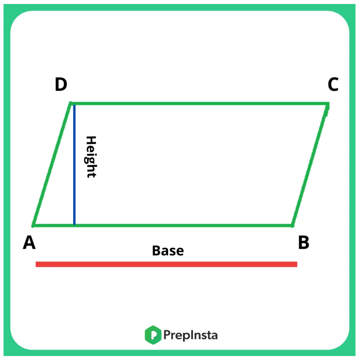


- **Formulas of Circle**

- Area of circle = πr^2
- Area of semicircle= $\frac{r^2}{2}$
- Circumference of a circle = $2\pi r$
- Circumference of a semicircle = πr
- Length of arc = $\frac{2r}{360}$
- Area of sector = 1/2 (arc*R) = $\pi r^2 \theta/360$



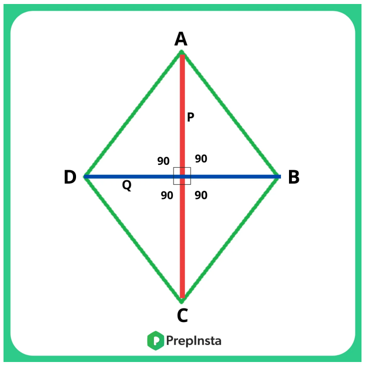
- **Parallelogram formulas**
  - Perimeter: 2(a + b)
  - Area: b × h
  - Height of parallelogram = $\frac{A}{b}$



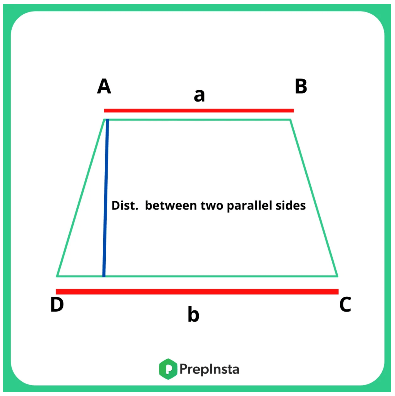
- **Rhombus formulas**
  - Perimeter: 4 × a
  - Area: $\frac{p \times q}{2}$
  - Diagonals

    p = $p = \sqrt{4a^2 - q^2}$

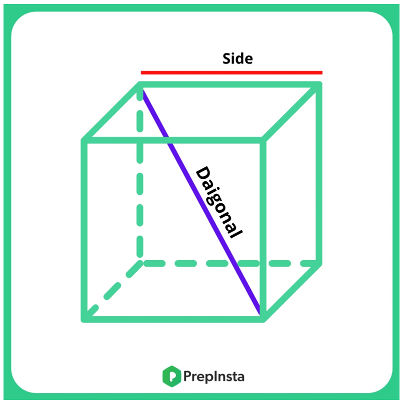
    q = $p = \sqrt{4a^2 - q^2}$



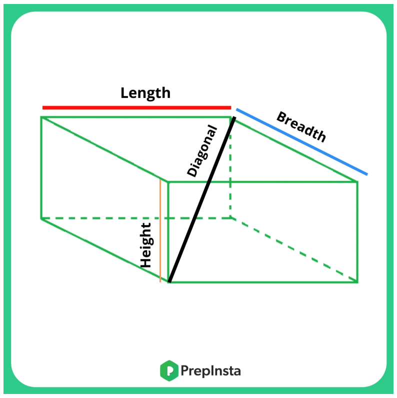
- **Trapezium formulas**
  - Perimeter: a + b + c + d
  - Area: $\frac{1}{2}$ × (sum of parallel sides) × distance between them
  - To find the distance between parallel sides you will have to convert trapezium to rectangle and then use **PYTHAGORAS THEOREM.**



- **Cube formulas**
  - Volume: $(\text{side})^3$
  - Surface area = $6s^2$
  - Partial Surface area = $4s^2$
  - Diagonal = $\sqrt{3}s$



- **Cuboid formulas**
  - Volume: l * b * h
  - Surface area = 2 (lb + bh + hl)
  - Curved Surface area = 2h(l+b)
  - Diagonal = $\sqrt{l^2 + b^2 + h^2}$

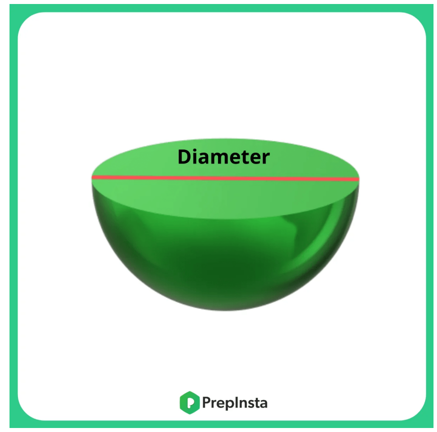


- **Sphere formulas**

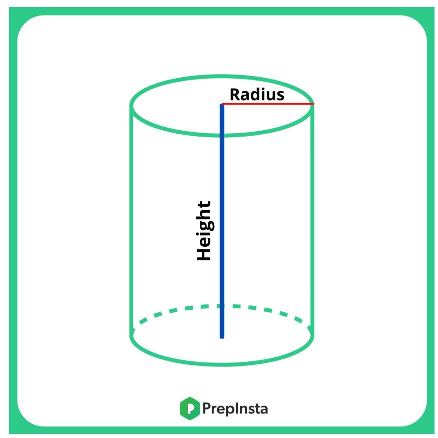
  Formulas for Sphere

  - Volume: $\frac{4}{3}\pi r^3$
  - Surface area = $4\pi r^2$

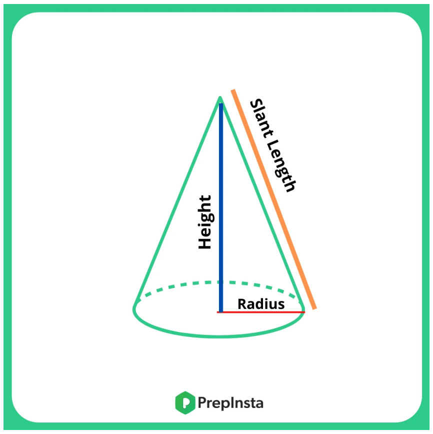
- **Hemisphere formulas**
  - Volume: $\frac{2}{3}\pi r^3$
  - Curved Surface area = $2\pi r^2$
  - Total Surface area = $3\pi r^2$



- **Cylinder formulas**
  - Volume: $\pi r^2 h$
  - Curved Surface area = $2\pi rh$
  - Total Surface area = $2\pi r\ (h + r)$



- **Cone formulas**
  - Volume: $\frac{1}{3}\pi r^2 h$
  - Slant height = l = $\sqrt{h^2 + r^2}$
  - Curved Surface area = πrl
  - Total Surface area = $\pi rl + \pi r$



16.2 How to Solve Perimeter, Area, and Volume questions:-

Geometry is concerned in calculating the length, perimeter, area and volume of various geometric figures and shapes

- Type 1: Find the area, perimeter, length, breadth and some other sides of the shapes
- By finding the volume and total surface area
- Percentage increase or decrease

Exercise:

Question 1:

From a rope a triangle is made of sides 21cm,24cm,28cm. from this a square is made. Find the area of the square.

Question 2:

From a square of side 2 cm, equal triangles are cut from its corners to form a regular octagon. We will get an octagon. What is the area of that octagon?

Question 3:

A regular polygon with 12 sides (dodecagon) is inscribed in a square of area 24 square units as shown in the figure where four of the vertices are mid points of the sides of the square. Then find the area of the dodecagon.

Question 4:

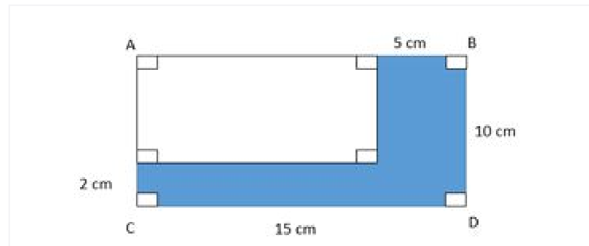
The cost of fencing a square field @ Rs. 20 per metre is Rs.10.080. How much will it cost to lay a three meter wide pavement along the fencing inside the field @ Rs. 50 per sq m .

Question 5:

An error 2% in excess is made while measuring the side of a square. Find the percentage of error in the calculated area of the square.

Question 6:

Find the area of the blue section.



Question 7:

The length and breadth of a rectangle are 10 cm and 7 cm respectively. What is the area of this rectangle?

Question 8:

If the breadth of a rectangle is decreased by 20% and the length increased by 10%, a square of area 1936 sq. m is obtained. Find the area of the rectangle in square meters

Question 9:

A swimming bath is 240 m long and 12.5 m wide. How many liters must be pumped into it to raise the water level by 1 cm?

Question 10:

8000 tiles of size 15 cm by 15 cm are required to cover the entire floor of a hall length 15m. What is the breadth (in m) of the hall ?

Question 11:

The length of a rectangle is halved, while its breadth is tripled. What is the percentage change in area?

Chapter 17.Pipes and Cisterns

A pipe connects a cistern or tank to the outside world.
The Inlet pipe is used to fill the cistern, and the Outlet pipe is used to empty the vessel.

Pipes are commonly divided into two types: inlet and outlet.

Inlet: The inlet pipe is the pipe that allows water to flow from the cistern or tank.

Outlet : The pipe that empties the cistern or tank is called a pipe.

Definition & Formula for Pipes & Cisterns

- A pipe is connected to a tank or cistern to fill or empty the tank or cistern
  - Inlet: A pipe which is connected to fill a tank is known as an inlet.
  - Outlet: A pipe which is connected to empty a tank is known as an outlet.
- In problems involving pipes and cisterns, we must first determine how much of the tank each of the pipes fills or drains in a unit of time (such as a minute, hour, or second), and then apply an arithmetic operation to this value.

17.1 Pipes and Cisterns Formulas :

Formula

1. If pipe can fill a tank in x hours , then part filled in one hour = $\frac{1}{x}$
2. If pipe can empty a tank in y hours , then part emptied in one hour = $\frac{1}{y}$
3. If pipe A can fill a tank in x hours, Pipe B can empty the full tank in y hours (where y > x). Then, on opening both the pipes, the net part filled in one hour= $\frac{1}{x} - \frac{1}{y}$ OR $\frac{xy}{y-x}$ hours
4. If pipe A can fill a tank in x hours. Pipe B can empty the full tank in y hours (where x > y). Then, on opening both the pipes, the net part filled in one hour = $\frac{1}{y} - \frac{1}{x}$ OR $\frac{yx}{x-y}$ hours.
5. If pipe A can empty a tank in X hours. Pipe B can empty the same tank in Y hours. Then part of the tank emptied in one hour when both the pipes start working together = $\frac{1}{x} + \frac{1}{y}$

17.2 How to solve Quickly Pipes and Cisterns Questions:

- Type 1: Calculate time taken to fill a tank by two or more pipes
- Type 2: Calculate time taken to fill a tank having a hole or leakage
- Type 3: Calculate Time Taken When Pipes Are Opened For Different Periods

17.3 Tips and tricks and shortcuts of Pipes and Cisterns questions

- Pipes and Cisterns questions are almost like Time and Work questions.
- The percentage of the tank that is filled or emptied is computed in pipes and cistern problems, and it is comparatively the same as the amount of work done in time and job problems.
- Similarly, the time it takes to entirely fill or empty a tank, or to a desired level, is the time it takes to complete a task.

Exercise:

1. **Two pipes A and B can fill a tank in 5 and 6 hours respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?**
2. **Tube T1 and T2 can fill a swimming pool respectively in 5 and 6 hours. Tube T3 can unfill the pool in half day. If all the 3 tubes are opened together, then in how much time will the swimming pool be filled?**

3. **Two pipes P1 and P2 are connected to a container. Pipe P1 can empty the container in 10 minutes less in half an hour and pipe P2 can unfill the container in 30 minutes. If both P1 and P2 are opened together. Calculate the time taken to unfill the container completely?**

4. **Suppose there are 3 pipes, A, B, and C. Pipe A can fill a tank in 7 hours and pipe B in 8 hours. Pipe C can completely unfill the tank in 10 hours. If all pipes are opened together, in how much time will the tank be filled?**

5. **One pipe is four times faster than another pipe. Calculate the time taken by the slower pipe to fill the cistern, if the faster pipe can fill it in 40 minutes.**

6. **There are three pipes: pipe1, pipe2 and pipe3. They can together fill a cistern in 9 hours. Pipe3 is closed after working for 3 hours. Then the remaining cistern is filled by pipe1 and pipe2 in 9 hours. Calculate the number of hours pipe3 alone will take to completely fill the cistern.**

7. **If two pipes X and Y separately can fill a tub in 3 and 9 minutes. Instead of using separately a man puts both the pipes together to fill a tub. Calculate in what time they both can fill the tub.**

8. **Suppose Pipe D takes 10 minutes and Pipe F takes 30 minutes to fill the tank.and Pipe H takes 15 minutes to empty the same. How much time will the 3 pipes take to fill the cistern if they are combined together?**

9. **Suppose the two-fifth part of a cistern is full. If pipe X can empty it in 8 minutes and pipe Y can fill it in 16 minutes. If both pipe X and Y are started together, how much time will it take to completely fill or empty the cistern?**

10. **How much time together pipe P, pipe Q, and Pipe R will take to fill a tanker? If separately they can fill it in 6 hours, 10 hours, and 20 hours respectively.**

11. **Two pipes can fill a tank in 20 hr. and 32 hr. respectively, A third pipe can empty the tank in 90 hr. if all the three pipes are opened and functioned simultaneously, then in how much time the tank will be full?**

Chapter 18: Permutation and Combination

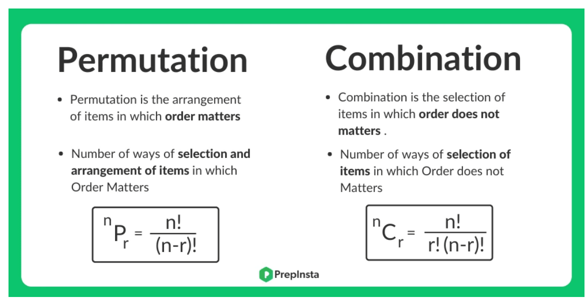
Permutation:

Permutations are the various configurations of a given number of things by taking some or all at once. ^nP_r is the symbol for this.

- Permutations are discussed in almost every branch of mathematics as well as a wide range of other scientific disciplines.They are used in computer science to analyze sorting algorithms.

Combination:

A combination is any of the various groups or choices that can be created by taking some or all of a number of objects. ^nc_r. is the symbol for this.



18.1 Formula for Permutation and Combination

- Number of all **permutations** of n things, taken r at a time, is given by

$$^nP_r = \frac{n!}{(n-r)!}$$

- Number of all **combinations** of n things, taken r at a time, is given by

$$^nC_r = \frac{n!}{(r)!(n-r)!}$$

Points to remember

- Factorial of any negative quantity is not valid.
- If a particular thing can be done in m ways and another thing can be done in n ways, then
  - Either one of the two can be done in ***m*** + ***n*** ways and

- Both of them can be done in $\boldsymbol{m} \times \boldsymbol{n}$ ways

- $0! = 1$
- $1! = 1$
- If from the total set of n objects and 'p_1' are of one kind and 'p_2' and 'p_3' and so on .... till p_r are others respectively then

$$^nP_r = \frac{n!}{p_1!q_2!.......p_r!}$$

- $^nP_n = n!$
- $^nc_n = 1$
- $^nc_0 = 1$
- $^nc_r = {}^nc_{(n-r)}$
- $^nc_0 + {}^nc_1 + {}^nc_2 + {}^nc_3 + \ldots {}^nc_n = 2^n$

Permutation and Combinations Formulas- Factorial

$n! = n(n-1)(n-2) \ldots\ldots 1$

Eg. – $5! = 5(5-1)(5-2)(5-3)(5-4) = 5(4)(3)(2)(1)$

Standard Truths

- $0! = 1$
- n! only exists of n >= 0 and doesn't exist for n < 0

n	n!
0	1
1	1
2	2
3	6
4	24
5	120
6	720
7	5040
8	40,320
9	362 880
10	3 628 800

Permutations Formulas

Number of ways in which Permutations out of n things r things can be SELECTED & ARRANGED (denoted by ^nP_r).

^nP_r = number of permutations (arrangements) of n things taken r at a time.

$$^nP_r = \frac{n!}{(n-r)!} \quad \mathbf{n \geq r}$$

Eg.

- Arrangement of Letters/Alphabets to form words with meaning or without meaning.
- Arrangements of balls on a table.

Formulas for Combinations

The number of ways in which r things at a time can be SELECTED from n things is Combinations (represented by ^nC_r)..

^nC_r = Number of combinations (selections) of n things taken r at a time.

- $$^nP_r = \frac{n!}{(r)!(n-r)!}$$ **; where n ≥ r (n is greater than or equal to r).**

Eg.

- Selections for people from total numbers who want to go out on a picnic.
- Filling posts with people
- Selection for a sports team out of available players
- Selection of balls from a bag

<u>18.2 Imp Properties:</u>

<u>Property 1</u>

Number of permutations (or arrangements) of n different things taken all at a time = n!

<u>Property 2</u>

For Objects in which P1 are alike and are of one type, P2 are alike or other different types and P3 are alike or another different type and the rest must be all different, Number of permutations = $\frac{n!}{(p1)!(p2)!(p3)!}$

<u>Property 3</u>

When repetition is allowed number of permutations of n different things taken r at a time = n × n× n ×… (r times) = n^r

Property 4

Here, we are counting the number of ways in which **k balls** can be distributed into ***n boxes*** under various conditions.

The conditions which are generally asked are

1. The balls are either distinct or identical.
2. No box can contain more than one ball or any box may contain more than one ball.
3. No box can be empty or any box can be empty.

Distribution of	How many balls boxes can contain				
k Balls	into n Boxes	No Restrictions	≤ 1 (At most one)	≥ 1 (At least one)	= 1 (Exactly one)
Distinct	Distinct	n^k (formula 1)	^nP_k (formula 2)	S(k,n) × n! (formula 3) (Not Imp)	^nP_n = n! if k = n; 0 if k ≠ n
Identical	Distinct	$^{(k+n-1)}C_{(n-1)}$ (formula 5)	^nC_k (formula 6)	$^{(k-1)}C_{(n-1)}$ (formula 7)	1 if k = n; 0 if k ≠ n (formula 8)

Other Properties

- $^nP_r = r! \times {^nC_r}$
- $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$
- $^nC_x = {^nC_y}$ when x = y or x + y = n
- $^nC_r = {^nP_{n-r}}$
- $r \cdot {^nC_r} = n\ {^{n-1}C_{r-1}}$
- $\dfrac{^nC_r}{r+1} = \dfrac{^{n+1}C_{r+1}}{n+1}$
- For ^nC_r to be greatest,
  - (a) if n is even, r = $\dfrac{n}{2}$
  - (b) if n is odd, r = $\dfrac{n+1}{2}$ or $\dfrac{n-1}{2}$

18.3 Tips and Tricks to solve Permutation and Combination:

1. In how many ways can a word be arranged.
2. In how many ways x objects out of a and y objects out of b can be arranged.
3. There are x objects and y objects, a form x has been selected and b from y. How many ways can it be done when N Number of objects from x should always be selected
4. Coloured Ball Questions
5. Circular Combinations Problems

18.4 How to Solve Permutation and Combination Questions Quickly.

- **Permutation** is an arrangement of objects in a definite order.
- Number of all permutations of n things, taken r at a time, is given by
  $^nP_r = \dfrac{\mathbf{n!}}{\mathbf{(n-r)!}}$
- **Combination** is selection of objects where order doesn't matter.
- Number of all **combinations** of n things, taken r at a time, is given by
  $^nC_r = \dfrac{n!}{(n-r)!r!}$
- Here we can easily understand how to solve permutation and combination easily.

Exercise:

1. **In a class there are 12 girls and 17 boys. The teacher wants to select 1 girl and 1 boy from the class as volunteers for school function. In how many ways is this selection possible?**

2. **In a class there are 20 students from which 9 boys and 11 girls. From the class 4 students are to be selected, such that at least one of them should be a boy. Find the number of ways in which the selection can be made.**

3. **There are 5 red boxes, 2 white boxes, and 4 pink boxes. Find the number of ways in which these boxes can be arranged such that the same color boxes always remain together.**

4. **A question paper contains 2 sections: A and B. Each of the sections contain 10 questions each. The student needs to answer only 15 questions such that at least 4 questions must be from part A and 6 questions must be from part B. In how many ways can he select the questions?**

5. **There are 10 boys and 12 girls in the school. In how many ways a teacher can select 1 student for the annual function?**

6. **In A Group Of 6 Boys And 4 Girls, Four Children Are To Be Selected. In How Many Different Ways Can They Be Selected Such That At Least One Boy Should Be There?**

7. **The Indian Cricket Team Consist Of 16 Players. It Includes 2 Wicket Keeper And 5 Bowlers. In How Many Ways Can A Cricket Eleven Be Selected If We Have To Select 1 Wicket Keeper And At Least 4 Bowlers?**

Chapter 19: Work and Time

Definition

- **Work:** Work is specified in mathematics as the amount of work allocated or the amount of work completed.
- **Time:** The number of days or hours needed to complete a task is referred to as time.

19.1 Formula for Work and Time

- Work from Days:

If A can do a piece of work in n days, then A’s one day work $= \frac{1}{n}$

- Days from work:

If A’s one day work = $\frac{1}{n}$, then A can finish the work in n days

- Work Done by A and B

A and B can do a piece of work in 'a' days and 'b' days respectively.

When working together they will take $\frac{ab}{a+b}$ days to finish the work

In one day, they will finish $(\frac{a+b}{ab})^{th}$ part of work.

- **Ratio:**

If A is thrice as good a workman as B, then:

Ratio of work done by A and B = 3: 1.

Ratio of times taken by A and B to finish a work = 1: 3

- **Efficiency:**

Efficiency is inversely proportional to the

Time taken when the amount of work done is constant.

Efficiency α = $\frac{1}{TimeTaken}$

19.2 Basic Rules for Work and Time

Rule 1:

If A completes a piece of work in x days. And B can complete the same piece of work in y days .

Then,

One day work of A = $\frac{1}{x}$ One day work of B = $\frac{1}{y}$

Work done by A + B = $\frac{1}{x}+\frac{1}{y}=\frac{x+y}{xy}$

Rule 2:

If A completes a piece of work in x days. B completes same piece of work in y days .C completes same piece of work in z days

Then,

One day work of A = $\frac{1}{x}$

One day work of B = $\frac{1}{y}$

One day work of C = $\frac{1}{z}$

Work done by A + B + C =

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{yz+xz+xy}{xyz}$$

Total time = $\frac{xyz}{xy+yz+zx}$

Rule 3:

If M_1 men can complete a work W_1 in D_1 days and M_2 men can complete a work W_2 in D_2 days then,

$$\frac{M_1D_1}{W_1}=\frac{M_2D_2}{W_2}$$

If Time required by Both M_1 and M_2 is T_1 and T_2 respectively, then relation is

$$\frac{M_1D_1T_1}{W_1}=\frac{M_2D_2T_2}{W_2}$$

Rule 4:

If A alone can complete a certain work in 'x' days and A and B together can do the same amount of work in 'y' days,
Work done by b =1/y - 1/x = x-y/xy
Then B alone can do the same work in xy/(x-y)

Rule 5:

If A and B can do work in 'x' days.
If B and C can do work in 'y' days.
If C and A can do work in 'z' days.
Work done by A,B and C = ½ *(1/x+1/y+1/z)
Total time taken when A, B, and C work together xyz/(xy + yz + zx)

Rule 6:

Work of one day =

$$\frac{\text{Total work}}{\text{Total number of working days}}$$

Total work = one day work × Total number of working days
Remaining work = 1 – work done
Work done by A = A's one day work × Total number of working days of A

Rule 7:

If A can finish $\frac{m}{n}$ part of the work in D days.
Then total time taken to finish the work by A =
$\frac{D}{\frac{m}{n}} = \frac{n}{m}$ × D days

Rule 8:

If A can do a work in 'x' days
B can do the same work in 'y' days
When they started working together, B left the work 'm' days before completion then total time taken to complete the work = (y+m)x/x+y

Rule 9: A and B finish work in a day.
They work together for 'b days and then A or B leave the work.
B or A finished the rest of the work in 'd' days.

Total time taken by A or B alone to complete the work =
$\frac{ad}{a-b} or \frac{bd}{a-b}$

19.3 Work and Time Formulas & Three Universal Rules

- If M_1 persons can do W_1 work in D_1 days and M_2 persons can do W_2 works in D_2 days then the formula $\frac{M_1D_1}{W_1} = \frac{M_2D_2}{W_2}$ .It can be written as $M_1D_1W_2 = M_2D_2W_1$.
- If the persons work T1 and T2 hours per day respectively $\frac{M_1D_1T_1}{W_1} = \frac{M_2D_2T_2}{W_2}$
- It can be written as, $M_1D_1\ T_1\ W_2 = M_2D_2\ T_2W_1$
- If the persons has efficiency of E_1 and E_2 respectively then
  $\frac{M_1D_1T_1E_1}{W_1} = \frac{M_2D_2T_2E_2}{W_2}$
  Therefore, $M_1D_1T_1E_1W_2 = M_2D_2T_2E_2W_1$.

In all the above formula,

M = Number of workers

D = Number of days

T = Time required

W = Units of work

E = Efficiency of workers

19.4 Tips and tricks of the Work and Time questions to Understand the relation between Man, work, and Time

1. More men can do more work. Similarly, less men will do less work
2. More work takes more time. Similarly, less work takes less time
3. More man can do work in less time, Similarly, less men can do work in more time

Formula 1 : Calculate time taken or work completed by one, two or more workers --- When working together they will take $\frac{ab}{a+b}$ **days to finish the work**

Question: Zack is three times as good as Cody. If they finish a piece of work together in 15 days, calculate the number of days taken by Zack alone to finish the work.
Sol: One day work by Zack : One day work by Cody = 3 :1
(Zack + Cody)'s one day work = 1/15
Now, when we divide 1/15 in the ratio of 3:1
Therefore, Zack's one day work = 1/15 \times ¾
= 1/20
Hence, Zack can alone finish the work in 20 days.

Question: Sam can complete a piece of work in 2 days. While Mat can do the same work in 5 days. Calculate the number of days taken by both of them to complete the work?
Sol: Time is taken by Sam = 2 days
Work done by Sam in one day = ½
Time taken by Mat to do the work = 5 days
Work done by Mat in one day = 1/5
Therefore, the time is taken by both to do the work = ½ + 1/5
= 7/10

Question: Ben and Alen can do a piece of work in 15 days together. Ben can finish this work alone in 20 days. Calculate the number of days taken by Alen to do the same work?
Sol: Ben and Alen's one day's work = 1/15
Ben's one day's work = 1/20
Therefore, Alen's one day's work = (Ben and Alen)'s one day's work- Ben's one day's work =1/15 – 1/20 =1/60
Hence, Alen alone can finish the work in 60 days

Question: Jack takes 12 days to complete a piece of work. Jamie takes 16 days to complete the same work. Calculate

the amount of time required to complete the same work if they both work together.
Sol: Work done by jack in 1 day = 1 /12.
Work done by Jamie in 1 day = 1/16.
Work done if they both work together in 1 day = 1/12 + 1/16

Hence, together they will complete the work in 48/7 = 6.8 days.

Question: P, Q, and R, together can finish a piece of work in 50 minutes. If P and Q together can finish the piece of work in 60 minutes, calculate the amount of time taken by R alone to finish that work.
Sol: Time taken by r alone to complete the work = (1/50) - (1/60) = 300 minutes

Question: Bryn, Jamie, and Terry can drive to reach the destination in 60 minutes. Bryn and Jamie alone can drive in 80 minutes. How much time will Terry take to drive alone?
Sol: Terry alone can take = 1/60 – 1/80 = 240 minutes

Question; Rakesh and Rajesh together can complete a piece of work in 4 days. If Rakesh alone can complete the work in 12 days, calculate the number of days needed by Rajesh to complete the same work.
Sol: One day work and time (Rakesh + Rajesh) does work in one day = ¼
Rakesh's one day's work = 1/12
Same work done by Rajesh = ¼ - 1/12
= 1/6
Hence, Rajesh can complete the piece of work in 6 days.

Question: 8 men can complete a piece of work in 40 hours. How many hours will 20 men take to complete the same piece of work?
Sol: Men x hour = Men x s
8 x 40 = 20 x s
S =320/20
=16 hours

Question: 30 laborers can complete a piece of work in 50 days. Calculate the number of days needed for 15 labors to complete the same piece of work.
Sol: Number of days =
=>30 x 15 = 15 x s
=>S = 100 days

Question: The ratio of work done by Alex and Kai is in the ratio 3:5. If both of them work together for 30 days to complete the work, in how many days will Kai alone complete the work?
Sol: It is given that the ratio of the capacity to do work of a and b is 3 : 5.
They together can complete the work in 30 days.
Let the efficiency of alex and kai are 3x and 5x respectively.
The sum of their efficiency is
3x + 5x=8x
Efficiency = work/time
8x= work/30
work =30*8= 240
Total work is 240x.
The efficiency of kai is 5x and the total work is 240x.
5x=240x/time
time=240x/5x= 48

Question: A, B and C can do a piece of work in 20, 30 and 60 days respectively. In how many days can A do the work if he is assisted by B and C on every third day?
Sol: One day work done by A=1/20
Work done by A in 2 days=1/10
Work done by A,B,C on 3rd day=1/20+1/30+1/60=1/10
Total work done in 3 days= 1/10+1/10= 1/5
1/5th of the work will be completed at the end of 3 days
The remaining work =1-1/5=4/5
4/5 of work will be completed 1/5*5/4*3= 12 days
Total work will be done in 3+12 =15 days

Formula 2 : How To Solve Quickly Work and Time Questions when efficiency is given ---- Efficiency is inversely proportional to the time taken when the amount of work done is constant.

Efficiency $= \dfrac{1}{TimeTaken}$

Question: Anthony, Philip, and Steven together can complete a work in 20 minutes. If Anthony and Philip together can complete the same task in 60 minutes, then calculate the time required for Steven to complete the work.
Sol: The time required to complete the work = (1/20) – (1/60)

= 1/30
30 minutes
Question: Trace can complete an assignment in 12 days. Bob can do an assignment in 4 days. Calculate the number of days to complete the work if they work together.
Sol: 1/12+¼ = 1/3
Hence, if they work together, they can complete the piece of work in 3 days.

Question: Lio, Jack, and Charlie can build a wall if they work together for 30 minutes. If Lio and Jack together build a wall in 80 minutes, then calculate the number of minutes to complete the work by Charlie alone.
Sol: Time taken by Lio alone to build the wall = (1/30) + (1/80)
= 5/240
=1/48
Hence, the required time is 48 minutes.

Question: 22 students take 32 days to complete a project. How many days will 16 boys take to complete the same project?
Sol: $22 \times 32 = 16 \times s$

$$S = 22 \times \frac{32}{16}$$

= 44
Hence, 16 students will complete a piece of work in 44 days.

Question: George can do some work in 8 hours. Paul can do the same work in 10 hours while Hari can do the same work in 12 hours. All the three of them start working at 9 AM. while George stops work at 11 am,the remaining two complete the work, approximately when will the work be finished?
Sol: Let the total work = 120 units.
As George completes this entire work in 8 hours, his capacity is 15 units /hour
Similarly, the capacity of paul is 12 units / hour
the capacity of Hari is 10 units / hour
All 3 started at 9 am and worked up to 11 am. So total work done upto 11 am = 2 x (15 + 12 + 10) = 74
Remaining work = 120 - 74 = 46
Now this work is to be done by paul and hari. 46 / (12 + 10) = 2 hours (approx)
So work gets completed at 1 pm

Question: 25 women can paint a house in 15 days. How many women can paint the house in 5 days?
Sol: Using formula,
M1 D1 W1 = M2 D2 W2
as the work is same therefore W1 = W2 = W1
M1 D1 W1 = M2 D2 W1
25 x 15 x W1 = M2 x 5 X W1
(25 x 15)/5 = M2
25 x 3 = M2
75 women = M2

Question: Each of A, B and C need a certain unique time to do certain work. C needs 1 hour less than A to complete the work. Working together they require 30 minutes to complete 50% of the work. The work also gets completed if A and B start working together and A leaves after 1 hour and B works further 3 hours. How much work does C do per hour?
Sol: 50% suppose A do work in x hrs,B in y, then c would do in x-1 hrs..then
1/x+1/x-1+1/y=1 in 1 hrs..also A&B 1 hrs work 1/x+1/y then work remaining
xy-x-y/xy which is done by B in 3 hrs so (xy-x-y)/xy=3/y
It results y=4x/x-1 putting value we get x=3 so y=6 so A CAN DO WORK in 3 hrs B in 6 hrs & C IN 2 hrs that is 50%

Question: Raju can do a piece of work in 10 days..vicky 12days,tinku 15 days.. day all start the work together,but raju leaves after 2 days,vicky leaves 3 days before the work is completed.Then in how many days work will be completed?
Sol: Raju can do a piece of work in 10 days.
Work Rate of Raju = 100/10 = 10% per day.
Vicky can do a piece of work in 12 days.
Work rate of Vicky = 100/12 = 8.33% per day.
Tinku can do a piece of work in 15 days.
Work rate of Tinku = 100/15 = 6.66% per day.
In first 2 days, all three together do the work
= 2*(10 + 8.33 + 6.66) = 50% work.
In last three days, Tinku completed,
= 3 * 6.66 = 20% work.
Rest work = 100 - 20 + 50 = 30%.
30% work has been completed by Vicky and Tinku with a combined work rate of (8.33 + 6.66 = 15% per day).
Thus, they take two days to complete the 30% of work with a work rate of 15% per day.
Hence, whole work completed in,
= 2 + 2 +3 = 7 days.

Question: Danny painted a house in 3 days working 12 hours each. Reed can paint the same house in 3 days working 8 hours each. How many days will it take for both of them to paint the house if they work for 12/5 hours each day?
Sol: Work done by Danny = 3 x 12 = 36 hours
Work done by Reed = 3 x 8 = 24 hours
Work done by Danny in one hour = 1/36
Work done by Reed in one hour = 1/24
Work done together = 1/36 + 1/ 24
= 5/72 hours
Or,
72 /5 x 5/12
= 6 days

Question: A garrison of 3300 men has provisions for 32 days, when given at a rate of 850 grams per head. At the end of 7 days reinforcement arrived and it was found that now the provisions will last 8 days less, when given at the rate of 825 grams per head. How many more men can it

feed?Late the number of days 15 children can complete the same work?
Sol: We have each man eating 850 gms of food for 3300 men for 32 days
Hence, Quantity of food = 3300 × 32 × 850
= 89760000 gms
= 89760 kgs.
Consumption of food in 7 days = 3300 × 7 × 850 gms
= 19635kgs
Hence, remaining food = 89760 – 19635
= 70125kgs
Now the number of days food will last = 32 – 7 – 8 = 17 days.
Hence, we have each soldier eating food = 825 gms or 0.825 kgs.
Hence number of soldiers now who will be able to get food = 70125/17 × 0.825
= 70125/14.025
= 5000 men
Hence number of extra men who will be able to get food = 5000- 3300
= 1700 men

Question: Bhumi can stuff advertising circulars into envelopes at the rate of 36 envelopes per minute and Ananaya Pandey requires a minute and a half to stuff the same number of envelopes. Working together, how long will it take Bhumi and Ananya to stuff 360 envelopes?
Sol: In 1.5 minutes Ananya Stuffs = 36 envelopes
⇒ In 1 minutes Ananya Stuffs = 36/1.5 = 24 envelopes
Ananya Stuffs 36 enveloped in time = 1 min
⇒ Working together no of envelopes stuffed together = 36 + 24 = 60
⇒ Hence time required to stuff 360 envelopes if working together = 360/60 = 6 minutes

Formula 3 : Calculate time/work when workers leave/join in between

Question: Lane and Rudy can complete a project in 20 and 12 days respectively. Lane started working alone, and after 4 days, Rudy joined him. Calculate the number of days to complete the work.
Sol: Work done by Lane in 4 days = 1 x 4 /20
= 1/5
Work done by both in one day = (1 /20 + 1/12)
= 2/15
When they both work together = 15 / 2 x ⅘
= 6 days
Therefore, total time taken = 6 +4 = 10 days.

Question:Ahmed and Kale can complete a project in 30 days. "Both of them worked together for 30 days, and then Kale left". 20 more days later, Ahmed finished the remaining work, calculate the number of days taken by Ahmed alone to finish the work.
Sol: One day work by together Ahmed and Kale = 1/ 30
20 day's work done by them = 1 x 20 /30
Remaining work done = 1 – 2/3
= 1/3
Now when this work is done by Ahmed alone in 20 days,
Therefore, the complete work done by Ahmed is,
20 / 1 / 3
=20 x 3
= 60 days

Question: Vera and Zuri can complete a piece of work in 45 days and 40 days respectively. Both of them started their work together, but after some time, Vera left after some time and Zuri finished the remaining work in 23 days. Calculate the number of days Vera worked for.
Sol: Zuri works alone for 23 days
Therefore, work done by Zuri in 23 days = 23/40
Vera and Zuri's work together = (1-23/40)
Work done by both in one day = 40 x 45 / 40 + 45
= 40 x 45 /85
Hence, work done by both in days = 40 x 45 / 85 x 17 /40

Question: 39 people can repair a road in 12 days, working 5 hours a day. In how many days will 30 persons, working 6 hours a day, complete the work?
Sol: We have 39 people who can repair a road in 12 days, working 5 hours a day.
Hence, number of men hours required to complete the task = 39 × 12 × 5 = 2340 hours
Also, now we have 30 persons, working 6 hours a day.
Hence number of days required to complete the task = 2340/(30 × 6)
= 2340/180
= 13 days

Question: 5 men or 8 women do an equal amount of work in a day. A job requires 3 men and 5 women to finish the job in 10 days. How many women are required to finish the job in 14 days?
Sol: Since 5 men or 8 women do equal amount of work in a day
Hence, efficiency of men : women = 8 : 5
Let a man do 8 units of work in 1 day and a woman does 5 units of work in 1 day.
Now, for the job, we need 3 men and 5 women for 10 days
Hence, total units of work = (3 × 8 + 5 × 5) × 10
= (25 + 24) × 10
= 49 × 10
= 490 units of work
Now we need to get the work done by women.
Number of women now required = 490 / (14 × 5)

= 490/70
= 7

Question: John, James, and Jhony started work after 1-day work John left, again after 1 day James left to determine the number of days, required by Jhony to complete the remaining work, if john, James and Johny individually can complete work in 8, 10, 15 days
Sol: LCM= 8,10,15= 120
John- 18 = 15 unit
James-10 = 12 unit
Jhony -15 =8 unit

Total unit = 35
1 day = 35 unit
=120-35 =85
2 day = john left = 20 unit
= 85-20 = 65
3 day Jhony left = 8 unit = 65
= 65/8.

Question: Harry can complete a project in 16 days. Matt completes the same project in 18 days. Matt leaves the project after 9 Harry picks the job from where Matt left. Calculate the number of days Harry will need to complete the project.
Sol: Matt's 9 day's project = 1/ 18 x 9
= ½
Now, when Harry completes the remaining project,
= 1 – ½
= ½
Time needed by Harry to complete the project
= 1/16 x 2
= 1/8
= 8 days

Question: John can complete a given task in 20 days. Jane will take only 12 days to complete the same task. John and Jane set out to complete the task by beginning to work together. However, Jane was indisposed 4 days before the work got over. In how many days did the work get over from the time John and Jane started to work on it together?
Sol: Let the job = the LCM of 20 and 12 = 60 units.
Since John can complete the job in 20 days, John's rate = w/t - 60/20 = 3 units per day.
Since Jane can complete the job in 12 days, Jane's rate = w/t = 60/12 = 5 units per day.

After Jane leaves, John works on his own for 4 days to complete the job.
Work produced by John in the last 4 days = r*t = 3*4 = 12 units.
Of the 60 units that must be produced, John produces the last 12.
Thus, the first 48 units are produced by John and Jane together.
Combined rate for John and Jane = 3+5 = 8 units per day.
Time for John and Jane to produce 48 units = w/r = 48/8 = 6 days.
Total time for the job = (4 days for John alone) + (6 days for John and Jane together) = 10 days.

Question: Jack and John can construct a wall in 20 days and 24 days respectively by working separately. They worked alternatively by 2 days, If Jack started the work, How long will it take to construct a wall?
Sol: Jack -> 20 days
John-> 24 days
LCM 120 => total work
Jack per unit -> 6
John per unit-> 5

If Jack started work for 2 days then
6,6=12 units
And after 2 days John turn then
5,5=10 units
And so on
Therefore 22 units in 4 days
Then 110 units in 20 days
So,
120-110= 10 units left
After 20 days Jack turn
He will done 6 units then 4 units are left
Again Jack turn so total days are
21.6 or 21 2/3

Question: There are two groups of workers A and B. Each group A worker is twice as efficient as each group B worker. There are 30 workers in group A and 20 workers in group B. Group A workers would take 120 days to complete a certain job. All the workers of both the groups started working on the job simultaneously. Determine the time they would take to complete it.

Sol: $$\frac{M_1 \times D_1}{E_2 \times W_1} = \frac{M_{2D}2}{E_1 \times W_2}$$ [A is twice as efficient as B] A = 2B
30*120× 2 /W1=20*D2/w2
D2=180*2=360

<u>**Formula 4 : Share of salary based on work**</u>

Question: John, James and Jerry can complete tasks in 6, 12 and 8 days respectively. If they get paid Rs 1350 for task completion, what is James share?
Sol:

John (6)	24	4
James (12)		2
Jerry (8)		3

1 unit = 150 Rs

James share = 2 unit = 300 Rs

Question: Peter completed the task of flooring in 6 days, whereas Donald completed the same task in 5 days. If they get paid Rs. 220, Determine the amount received by Donald when they worked together.
Sol: Total number of days = 6 days+ 5 days
= 11 days
11 days working charges= 220
Then charge per day= 20
Amount received by Donald=6x20 = 120 Rs

Question: A Sum of Rs 480 is paid to Mary, Michael and Marina. Determine Marina's earning if (1/4)th work is completed by Mary and Michael and rest by Marina.
Sol: Amount received by Mary and Michael = 480/4 = 120 Rs
Marina's earning = 480-120 = Rs 360

Question: Katrina does a work in 20 days .While Ranveer does the same work in 15 days. they worked together and earned Rs.56000 to complete the task, Find the share of Katrina.
Sol: LCM Method
Lcm - 60
K=20=3
R=15=4
=7

$$K = \frac{3}{7}, R = \frac{4}{7}$$

$$K = \frac{3}{7} \times 56000$$

K=24000

Question: Bablu alone can do a piece of work in 10 days Ashu alone can do it in 15 days. The total wages for the work in Rs.5000 . how much should bablu be paid of they work together for entire duration of the work
Sol: Bablu 10 day------ now 30/10=3 unit per day
L.C.M = 30(total work they have to perform)
Ashu 15 day------- 30/15=2 unit per day
So their ratios of work is 3:2
So babla will get 3/5*5000=3000

Question: The wages of 24 men and 16 women amounts to Rs.11600/day. Half the number of men and 37 women earns the same amount /day. What is the daily wage of a man?
Sol: Say M and W denote daily wage of Man and Woman respectively.
Given 24 × M + 16 × W = 11600 ……. (Eq.1)
Also 12 Men and 37 Women earn the same amount i.e. 11600.
⇒ 12 × M + 37 × W = 11600
⇒ 24 × M + 74 × W = 23200 ……. (Eq.2)
Eq.2 – Eq.1 ⇒ 58 × W = 11600
⇒ W = 11600/58 = 200
⇒ 24 × M = 11600 – 16 × 200 = 11600 – 3200 = 8400
⇒ M = 8400/24 = 350

Question: A sum of Rs.25 was paid for a work which A can do in 32 days, B in 20 days, B and C in 12 days and D in 24 days. How much did C receive if all the four work together ?
Sol: B+ C 1 day work = ½ and B\'s 1 day\'s work = 1/20
Therefore, C 1 day\'s work = (1/12) – (1/20) = 4/120 = 1/30
Monet will be distributed according to the ratio of work done i.e A: B: C: D
= 1/32 : 1/20 : 1/30 : 1/24 = 15 :24:16:20
Therefore, C\'s Share = 16/(15+24+16+20) = Rs 16/3

Question: Kulfi can do work in 10 days. Another girl joined and they completed the same work in 6 days. If they get Rs. 100 for the work, what is the share of another girl ?
Sol: Kulfi can do the work in = 10 days
Both can do the work in = 6 days
Another girl can do the work = $\frac{1}{6}\frac{1}{10}$

$$= \frac{1}{15}$$

Kulfi and another girl's share = 15: 10 = 3: 2
Therefore, another girl's share = $\frac{3}{5}100 = Rs.60$

Chapter 20. Problems on Ages

In the Problem on Ages questions, these kinds of Formulas are frequently used. These formulas are useful for quickly and efficiently answering questions in examinations.

20.1 Basic Formulas on Ages

1. If the present age is *x*, then *n* times the age is *nx*.
2. If the present age is *x*, then *age n* years later/hence = *x* + *n*.
3. If the present age is *x*, then age *n* years ago = *x* - *n*.
4. The ages in a ratio *a* : *b* will be *ax* and *bx*.
5. If the present age is *x,* then $\frac{1}{2}$ of age is $\frac{x}{n}$

Ages Formulas and Concept on Problems on Ages

1. x years ago the age of A was n1 times the age of B, and at present A's age is n2 times that of B, then;

A's current = $\frac{(n1-1)}{(n1-n2)} n2 \times x \text{ years}$.

and, B's current age = $\frac{(n1-1)}{(n1-n2)} x \text{ years}$.

2. t1 years ago, the age of A was X times the age of B and after t2 years age of A becomes Y times the age of B, then;

A's present age ={ X(t1+t2) (Y-1) }/ (X-Y) + t1 years

And B's present age = {t2(Y-1)+ t1(X-1)} / (X-Y) years

3. **The sum of present ages of A and B is X years, t years after, the age of A becomes Y times the age of B, then;**

A's present age = XY + t(Y-1)/ (Y+1) years

And B's present age= X-t (Y-1)/ (Y+1) years

4. **The ratio of the present ages of A and B is p: q and after t years, it becomes r: s, then;**

A's present age = pt(r-s)/(ps-qr) years.

And, B's present age = qt(r-s)/ (ps-qr) years

5. **The sum of present ages of A and B is X years, t years ago, the age of A was Y times the age of B, then;**

Present age of A = XY+t(Y-1)/(Y-1) years

And, the present age of B = X+t(Y-1)/(Y+1) years

20.2 How To Solve Problem on Ages Questions Quickly:

The subject of ages is one of the most prevalent and significant in the Quantitative Aptitude section. We'll show you how to solve age questions quickly in this article.

Some people avoid age-related issues because they find them too perplexing. So, let us try to clear up their misunderstandings and assist them in quickly answering the questions.

- The important thing in Age Problem is to decide which age – either it is present or past or future.
- After deciding the age, Consider it as X.
- In most cases, we take the present age as ‘X’, i.e., the base year works just fine.
- Past will be expressed as (x-5) years.
- The future can be expressed as (x+5).
- But sometimes, ‘present age’ is not directly given in words. Then, *take ‘x’ to be the age you are going to find*.
- Sometimes when nothing works then just look at the options and solve it through back calculations! It also works fine.

20.3 Tips and Tricks to solve Problem on Ages :

With a few straightforward tips and tricks, you can easily solve age-related issues. Here are some fast tips for answering these questions.

1. Assign variables to the key words and organize the information.
2. To solve the problem, choose only one variable.
3. Assume the current age as ‘x' if the information in the question includes ages at various points in time.
4. If the information in the question includes the ages of several people, use ‘x' as the youngest person's age.
5. Pay close attention to the issue.The word "n times more than" must be accurately interpreted.

The questions can be categorized into the following types:

Type 1: x years ago the age of A was n1 times the age of B, and at present A's age is n2 times that of B

Question: After 6 years Raju's fathers age was twice Raju's age, 2 years ago.His mothers age was twice that of Raju's age. What is the sum of the age of their parents?

Solution;
F+6=2(R+6)
F= 2R+6
M-2=2(R-2)
M= 2R-2

Therefore the sum of Raju's Parent's age is

F+M=2R+6+2R-2
F+M=4R+4
4 more than four times Raju's age

Type 2: The present age of A is n1 times the present age of B. After x years, age of A becomes n2 times the age of B

Question: In 10 years, A will be twice as old as B was 10 years ago. If A is now 9 years older than B, the present age of B is :

Solution:

Let B's present age = x years. Then, A's present age = (x + 9) years.

(x + 9) + 10 = 2(x - 10)

=> x + 19 = 2x - 20

=> x =39.

Type 3: t1 years ago, the age of A was X times the age of B and after t2 years age of A becomes Y times the age of B

Question: 10 years ago 10 people were 33 . After 3 years a person of age 40 dies. After another 3 years another person of 40 years dies. After another 3 years another person of 27 years dies. Find the present average age?

Solution:

10 year ago 10 people= 33

10 year ago total age=330

after 3 year 1 person with age 40 died = ie take his age as 37 before 3 years lly for next 2 persons ; consider as 34 (40-6) and as 18(27-9) in 6 and 9 years ago(ie 37+34+18=89) 10 years ago age of 7 people =330-89=241

now consider present age 7*10=70+241=311
now avg= 311/7=44.43(ans)

Type 4:The sum of present ages of A and B is X years, t years after, the age of A becomes Y times the age of B

Question: The sum of the present ages of Brett lee and Jasprit Bumrah is 60. If the age of Brett Lee is twice that of Jasprit Bumrah, find the sum of their ages 5 years hence?

Solution:

Let the age of brett lee be x and the age of Jasprit Bumrah be y.

Given,Sum of their ages=60

x+y=60-----(i)

And ,Brett Lee is twice that of Jasprit Bumrah

=>x=2y----(ii)

Substituting above value in (i),we get,

2y+y=60

y=20 years

Hence,the age of Jasprit=20 years

so,Age of Brett Lee=(60-20) yrs

=>40 yrs

After 5 years their ages will be,45 years and 25 years

Sum of their ages after 5 years=(45+25)years

=70 years.

Type 5:The ratio of the present ages of A and B is p: q and after t years, it becomes r: s

Question: The ratio of Radha and Shyam age is 7:4. On multiplying their ages we get the value as 448. What will be the ratio of Radha and Shyam's age after 3 years.

Solution:

Let Radha's age be 7x and Shyam's age be 4x

Given, product of their ages : 7x*4x = 448

28 x^2 = 448

x^2 = 16

x = 4

Radha's age after 3 years will be 7x + 3 = 7*4+3 = 31 years

Shyam's age after 3 years will be 4x+3 = 4*4+3 = 19 years

Question: The ages of Preeti and Sakshi are in the ratio of 8:7 and by adding their ages we get 60. Determine the ratio of their ages after 6 years.

Solution:

Let, the current age of Preeti and Sakshi be 8x and 7x.

Given, 8x+7x = 60

15x = 60

X = 4

After 6 years their age will be, 8x+6 and 7x+6

= 8*4+6 and 7*4+6

= 38 and 34

The ratio will be 19:17**T**
ype 6: The sum of present ages of A and B is X years, t years ago, the age of A was Y times the age of B

Question: The sum of the present ages of a father and his son is 60 years. five years ago, a father's age was four times the age of the son. so now the son's age will be:Solution:

Let the present ages of son and father be x and (60 -x) years respectively.

Then, (60 - x) - 5= 4(x - 5)

55 - x = 4x - 20

5x = 75 => x = 15

Exercises:

1. **The ratio of Sara's age 4 years ago and Vaishali's age after 4 years is 1: 1. Presently, the ratio of their ages is 5: 3. Find the ratio between Sara's age 4 years hence and Vaishali's age 4 years ago.**
2. **At the end of 1994 rohit was half as old as his grand mother.The sum of years in which they were born is 3844. How old rohit was at the end of 1999**
3. **In 4 years, raj father age twice as two years ago,raj mother age twice as raj.if raj is 32 years old in eight years from now ,what is the age of raj mother and father?**
4. **Saransh is 50 years old and Nazma is 40 years old. How long ago was the ratio of their ages 3:2?**
5. **Roy is now 4 years older than Erik and half of that amount older than Iris. If in 2 years, roy will be twice as old as Erik, then in 2 years what would be Roy's age multiplied by Iris's age?**
6. **8 year old Eesha visited her grandpa. He gave her this riddle. I started working at 13. I spent 1/6 of my working life in a factory. I spent 1/4 of my working life in an office, and I spent 1/4 of my working life as a school caretaker. For the last 32 years of my working life I've been doing social service. How old am I?**
7. **A father said to his son , " I was as old as you are at present at the time of your birth. " If the father age is 38 now, the son age 5 years back was :**
8. **The total age of A and B is 12 years more than the total age of B and C. C is how many years younger than A ?**
9. **The age of a man is 4 times that of his son. Five years ago, the man was nine times old as his son was at that time. The present age of man is?**
10. **Six years ago Anita was P times as old as Ben was. If Anita is now 17 years old, how old is Ben now in terms of P ?**
11. **The ratio of the ages of Maala and Kala is 4 : 3. The total of their ages is 2.8 decades. The proportion of their ages after 0.8 decades will be [1 Decade = 10 years]**
12. **The average age of a group of 10 students is 15 years. When 5 more students join the group, the**

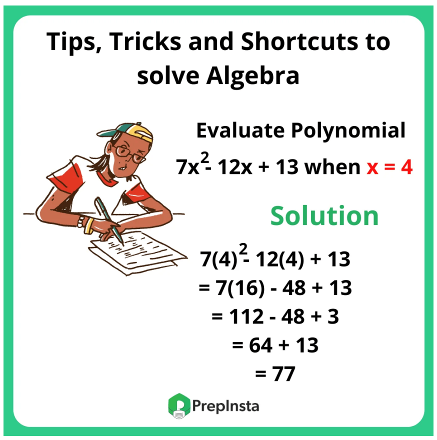
average age increases by 1 year. The average age of the new students is?

13. Rahul is 15 years older than Rohan. If 5 years ago, Rahul was 3 times as old as Rohan, then find Rahul's present age.
14. The sum of the ages of 5 children born at the intervals of 3 years each is 50 years. what is the age of the youngest child ?
15. The age of a person is thrice the total ages of his 2 daughters. 0.5 decades hence, his age will be twice of the total ages of his daughters. Then what is the father's current age? [0.5 Decades = 5 Years]
16. The sum of the ages of a father and son is 45 years. Five years ago, the product of their ages was four times the fathers age at that time. The present age of father and son
17. Rani is 4 years younger than Dipti, who is thrice as old as Deepak. Find the age of Dipti if the sum of their ages is 66.
18. The sum of Aditi and Arnav's age is 14 years more than the sum of Arnav and Aashay's age. Find the difference between Aditi and Aashay's age.
19. In 7 years, Z will be thrice the age of M 7 years ago. Find the present age of M, if he is 6 years younger than Z in current.
20. The sum of son's and daughter's present age of Rama is 50. Four years ago son's age was twice the age of my daughter. Find daughter's present age.
21. If Lakhan was two-fifth as old as Rashmi 3 years back. At present Lakhan's age is 15 years. Determine the present-day age of Lakhan.
22. Shivam got married to Anjali 6 years back. His age at present is 5/4 times his age when he got married. Shivam was 6 years elder to his brother when he got married. What is Shivam's brother's present age?
23. Ravi's present age is thrice the sum of his 2 son's present age. After 4 years, Ravi's age will be double of the sum of his son's age. What is Ravi's present age?
24. The addition of the ages of Asha, Reena, and Anu is 68. Reena is twice the age of Asha and Anu is 8 years older than Reena. Find the ages of Asha, Reena and Anu respectively.
25. Find Rano's present age if six years back her age was five-fourth times of Shano's age.
26. There are four friends named Ram, Mohan, Raj, Roy. The sum of their ages was 96. 6 years later the ratio will become 8:7:6:9. What is Raj's present age?
27. Radhika's present age is five seventh of her father's. After 7 years, she will be two thirds of her father. What is the existing age of Radhika's father?
28. There are 60 girls in a hostel whose average age is 12. The average age of 20 girls in the hostel is 13 and the average age of Other 15 is 11. Find the average age of remaining girls.
29. The product of Neha and Kunal's age is 578. If Neha is 17 years old at present. Find the remainder if the sum of their ages is divided by 3.
30. The sum of Aditi and Arnav's age is 14 years more than the sum of Arnav and Aashay's age. Find the difference between Aditi and Aashay's age.
31. If we add the ages of 4 children born at the intervals of 3.5 years each, we get 45. What is the age of the youngest child?
32. Six years ago, the ratio of the ages of Sachin tendulkar and virender Sehwag was 6 : 5. Four years hence, the ratio of their ages will be 11 : 10. What is Sehwag's age at present?
33. The present ages of Rohit Sharma,Virat Kohli and Karun Nair in proportions 4:7:9. Eight years ago, the sum of their ages was 56. Find their present ages (in years).
34. Sachin Tendulkar is younger than Rahul Dravid by 7 years. If their ages are in the respective ratio of 7 : 9, how old is Sachin?
35. If 13:11 is the ratio of the present age of Arijit Singh and Sonu Nigam respectively and 15:9 is the ratio between Arijit's age 4 years hence and Sonu's age 4 years ago. Then what will be the ratio of Arijit's age 4 years ago and Sonu's age 4 years hence?
36. The ages of Tapsee Pannu and Kangana ranaut are in the ratio of 7:3 respectively.After 6 years,the ratio of their ages will be 5:3.What is the difference in their ages?

Chapter 21. Algebra

Definition:

The study of mathematical symbols is known as algebra. It also includes the laws that govern the manipulation of these symbols. Almost everything is covered in this course, from basic equation solving to the study of abstractions like groups, rings, and fields.

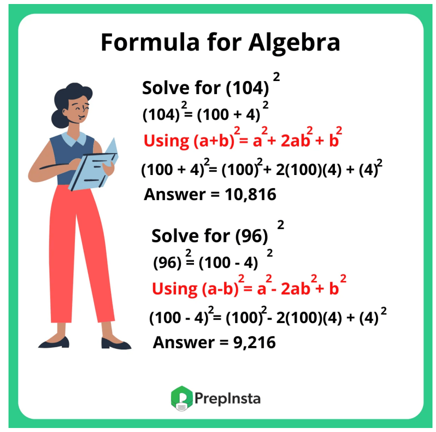


Use:

Algebra is an indispensable life skill that should be well-understood. It prepares us for statistics and calculus by going beyond fundamental math. It is useful for a variety of careers, including those that a student may pursue as a second career. Algebra comes in handy around the home and when reviewing news stories.

There are 6 main types of Algebraic Expressions. These are as under:

1. Monomial Expression
2. Binomial Expression
3. Trinomial Expression
4. Linear polynomial
5. Quadratic polynomial
6. Cubic polynomial



21.2 Formulas for Algebra:

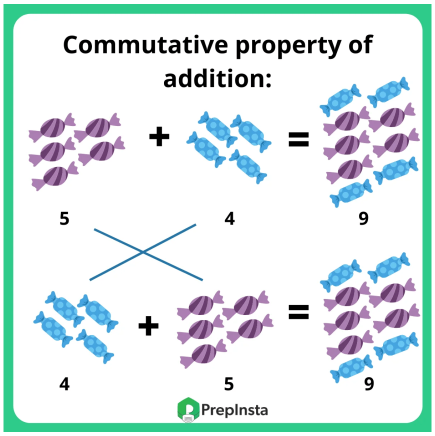
- $a^2 – b^2 = (a – b)(a + b)$
- $(a+b)^2 = a^2 + 2ab + b^2$
- $a^2 + b^2 = (a – b)^2 + 2ab$
- $(a – b)^2 = a^2 – 2ab + b^2$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$
- $(a – b – c)^2 = a^2 + b^2 + c^2 – 2ab – 2ac + 2bc$
- $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$; $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
- $(a – b)^3 = a^3 – 3a^2b + 3ab^2 – b^3$
- $a^3 – b^3 = (a – b)(a^2 + ab + b^2)$
- $a^3 + b^3 = (a + b)(a^2 – ab + b^2)$
- $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- $(a – b)^3 = a^3 – 3a^2b + 3ab^2 – b^3$
- $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4)$
- $(a – b)^4 = a^4 – 4a^3b + 6a^2b^2 – 4ab^3 + b^4)$
- $a^4 – b^4 = (a – b)(a + b)(a^2 + b^2)$
- $a^5 – b^5 = (a – b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$

21.2 Algebra Formulas & Properties of Algebra:

1.Commutative property of addition:

$a + b = b + a$

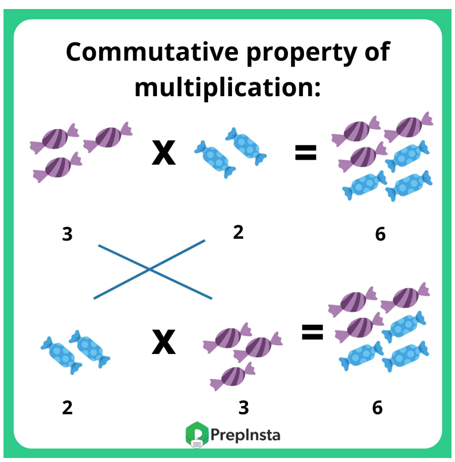
The sum of the expression does not change if the order of the elements is changed. The elements can be expressions or numbers.



2. Commutative Property of Multiplication:

a x b = b x a

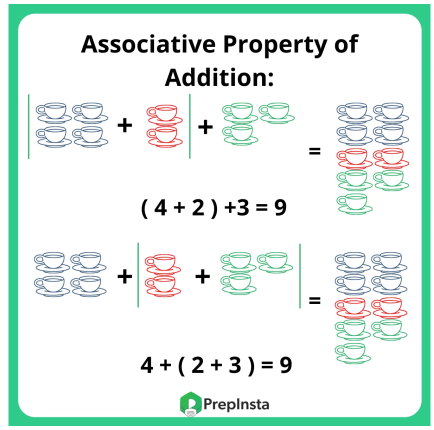
When the order of the factors are changed, the product does not change. These factors can be numbers or expressions



3. Associative Property of Addition:

(a + b)+ c = a + (b + c)

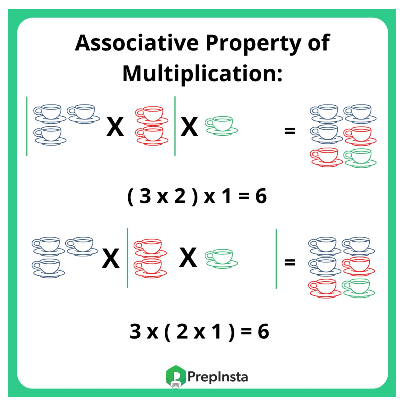
The property defines that when two or more numbers are grouped together which are performing basic arithmetic addition, their order does not play a significant role in the result.



4. Associative property of Multiplication:

(a x b) xc = a x (b x c)

Associative property states that when two or more elements are grouped together in the basic arithmetical multiplication, their order does not change the final result. Also, in this case, the grouping is usually done by parenthesis.



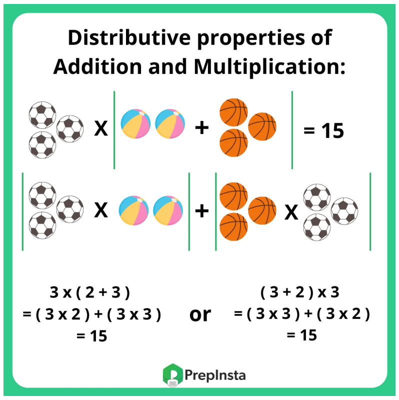
5. Distributive properties of Addition and Multiplication:

a × (b + c) = a × b + a × c

and

(a + b) × c = a × c + b × c.

The distributive property says that the product of a single term and a sum or a difference of two or more terms present in the bracket is the same as multiplying each of the elements by a single term and then adding and subtracting the products.



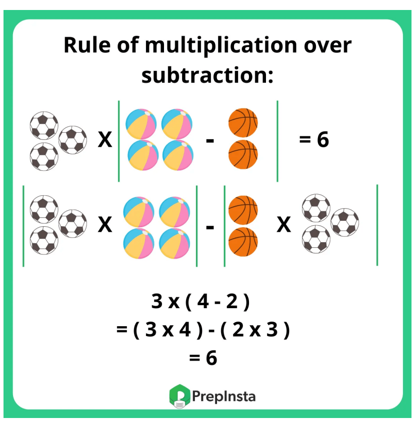
6. Rule of multiplication over subtraction:

If p, q, and r are any numbers, then, p (q-r) = p*q – p*r. Similarly, in the addition rule, the distribution for multiplication over subtraction can be done in left distribution and right distribution.

If p* (q-r) = (p * q) – (r*q)- Left distributive law

and

If (p-q)*r = (p*r) – (q*r)- Right distributive law



Formula 1 : How To Solve Algebra by Evaluation or finding the value of "a" variable

Question: Solve the below equation: If a-b = 6 and a2 + b2 = 58, then find the value of ab ---
Sol: Applying the formula = 2ab = (a2 + b2) – (a-b)2
=> 58- 36 = 2ab
ab = 22/2 = 11

Question: Solve the equation for a, where 8a – 30 + 2a = 5 +10 - a

Sol: 10a – 30 = 15 – a
9a = 15 + 30
= a = 45/9 = 5

Formula 2: How to Solve Algebra Questions Quickly by- Substitution Methods

Question: Grass in the lawn grows equally thick and at a uniform rate. It takes 12 days for 70 horses and 48 for 30 horses . How many horses can eat away the same in 60 days?

Sol: 12 days 70 horses==total work done=12*70=840
48 days 30 horses=== total work done= 48*30= 1440
Now see here that total work done must be the same but there is a difference. that difference corresponds to the increasing grass/day i.e. work done by grass
work done by grass =1440-840 =600 (this work is done in (48-12=36 days).
work done by grass per day =600/36= 50/3
now work done by grass in 12 days= (50/3)*`12=200
actual work = 840 -200 = 640
now work done by grass for 96 days = (50/3)*96== 1600
total work required to be done = 640 + 1600 = 2240
Horse required = 2240/96 == 23.33=24

Question: The subtraction between a two-digit number and the number obtained by interchanging the digits is 63. What is the difference between the sum and the difference of the digits of the number if the ratio between the digits of the number is 2 : 3 ?.

Sol: Since the number is greater than the number obtained on reversing the digits, so the ten's digit is greater than the unit's digit.
Let tens and units digits be 2x and x respectively.
Then, (10 x 2x + x) - (10x + 2x) = 63
9x = 63
x = 7
Required difference = (2x + x) - (2x - x) = 2x = 14

Exercise:

1. When the polynomial is x^5 - 3x^{3} + 10x - 20 is divided by (x - 2), calculate its remainder?

2. Solve the equation for a, where 20(a+b)+18 = 20(-a+b) -22

3. The cost of 4 matts and 6 blankets is Rs.1500. The cost of 6 matts and 4 blankets is Rs.1400. Calculate the cost of each blanket is more than that of each matt.

4. Ram has Rs. 589 in the values of one-rupee, ten-rupee, and twenty-rupee stardust notes. If the number of

stardust notes of each denomination is equivalent. Calculate the total number of notes that Ram has?

5. If a^{2} + 2 = 2a, then the value of a^{4} – a^{3} + a^{2} + 2 will be ---

6. The ages of Karan and Ritu differ by 45 years. If 10 year ago, Karan would be 10 times as old as Ritu, Calculate their current ages of the year?

7. A teacher asked its student to divide a number by 12 and add 24 to the quotient. He, however, first added 24 to the number and then divided it by 12, getting 224 as the answer.

8. To fill a Reservoir, 50 containers of water is required. How many containers of water will be required to fill the reservoir if the capacity of the container is reduced to fourth-tenth of its present?

Chapter 22. Calendar

Definition:

A calendar is a tool for keeping track of the days of a year. The year begins on January 1st and ends on December 31st. Ordinary years (365 days) and leap years (365 days) are the two kinds of years (366 days). There are 365 days in a year, which equals 52 x 7 + 1, or 52 weeks and one day.

22.1 Calendar Formulas & Definitions

- For religious, governmental, commercial, or administrative purposes, a calendar is a way of systematically grouping days. It is accomplished by naming time periods such as days, weeks, months, and years.

- A date is single, specific day within such a system

- The year is classified into two type:

1. Ordinary years (365 days)
2. Leap year (366 days.)z

- The calendar starts on 1st January and ends on 31st December.

Calendar Formulas for Odd days

- Odd days are the number of days that are more than the number of days in a complete week.
- **For example:**

Calculate odd days for 10 and 14 days

10 days = 1 week (7 days) + 3 days. Here, 3 days are odd days

14 days = 2 weeks (14 days) + 0 day (0 odd day).

Formulas of Calendar For Number of Odd Days

- 1 ordinary year has 1 odd day

Explanation : In an ordinary year, there are 365 days, which means 52 x 7 + 1, or 52 weeks and one day. This additional day is called an odd day.

- 1 leap year has 2 odd days

Explanation: A leap year has 366 days. There are 29 days in February in a leap year. There are 52 weeks and 2 odd days in a leap year.

- 100 years has 5 odd days

Explanation : Odd days in a leap year = (52 weeks +2) days .In 100 years , there will be 24 leap years and 76 non-leap years.

So odd days in 100 years will be (76 x 1 + 24 x 2) which is 124 odd days.

This can also be written as 17 weeks + 5 days.

So every 100 years will have 5 odd days.

- 200 years has 3 odd days

Explanation: 100 years give us 5 odd days as calculated above. 200 years give us 5 x 2 = 10 .Hence , 7 days (one week) = 3 odd days.

- 300 years has 1 odd day

Explanation: 300 years give us 5 x 3 = 15.Hence, 14 days (two weeks) = 1 odd day.

- 400 years has 0 odd day

Explanation: The number of odd days in 400 years will be (5 x 4 + 1) because 400 is itself a leap year and that is why it has one odd day extra. Thus odd days in 400 will be 0.

- Similarly, all the 4^{th} centuries 800 years, 1200 years, 1600 years, 2000 years etc. have 0 odd days.
- Mapping of the number of odd day to the day of the week

Days	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Number of Add Days	0	1	2	3	4	5	6

Reference Chart that gives odd days for the given months

- To determine whether a year is a leap year, follow these steps:

1. If the year is evenly divisible by 4, go to step 2. Otherwise, go to step 5.
2. If the year is evenly divisible by 100, go to step 3. Otherwise, go to step 4.
3. If the year is evenly divisible by 400, go to step 4. Otherwise, go to step 5.
4. The year is a leap year (it has 366 days).
5. The year is not a leap year (it has 365 days).

For example:

1. Check for years not ending with “00”.

Year 1997 is not a leap year because it is not divisible by 4.

Year 2016 is a leap year because it is divisible by 4.

2. Now, check for years ending with “00”.

Year 2000 is a leap year because it is divisible by 4,100 and 400.

Year 1900 is not a leap year because it is divisible by 4 and 100 but not 400.

Year 1600 is a leap year because it is divisible by 4, 100, and 400.

For this reason, the following years are not leap years:

1700, 1800, 1900, 2100, 2200, 2300, 2500, 2600

This is because they are evenly divisible by 100 but not by 400.

The following years are leap years:

1600, 2000, 2400

This is because they are evenly divisible by both 100 and 400.

Points to remember

- Last day of a century cannot be Tuesday or Thursday or Saturday because of the number of odd days.
- For the calendars of two different years to be the same, the following conditions must be satisfied.

a. Both years must be of the same type. i.e., both years must be ordinary years or both years must be leap years.

b. 1^{st}January of both the years must be the same day of the week.

<u>22.2 Solve Quickly Calendar Problems with Definitions</u>

The days in a year are organized through a system of calendar The year is classified into two types :

1. Ordinary Years (365 days)
2. Leap year (366 days)

The difference between leap and ordinary year and leap year is ONE extra day designated as 29^{th} February. A leap year occurs every four years, to keep the calendar in alignment with the earth's revolution around the sun. Earth takes 365 ¼ days to complete one revolution. Hence, to compensate for 1/4th day or 6 hours in the calendar we have a leap year.Therefore, a year cannot be a leap year if it is not divisible by 4 or 400.

For example – 1700, 1800, 1900 are not a leap year because it is not divisible by 400.

1600, 2000, 2400 are leap years because they are divisible by 400

- In an ordinary year, there are 365 days, which means 52 x 7 + 1, or 52 weeks and one day.
- Here are easy tricks and tips on Calendar problems Learn tricks on calendar.

<u>Exercise:</u>

1. **On 1st January 2005, it was a Monday. What will the day be on 1st January 2009?**
2. **On the 1st of January of a leap year the day is Monday. What will be the day on 1st May the same year?**
3. **I am 15 weeks and 24 days younger to Vishal, while Shashank is 25 weeks 19 days older to Vishal. If Shashank was born on Monday, on what day was I born?**
4. **The days in which the calendar will be similar to that of 1993?**
5. **Today is Friday. After 15 days, which day will it be ?**
6. **On what dates of April , 2001 did Wednesday fall ?**
7. **January 1 2013 is Tuesday .What day of the week lies on January 1 2014 ?**
8. **How many years have 29 days in February from 1900 to 2000?**
9. **What was the day of the week on 5 dec 2019 ?**
10. **January 1, 2013 was Tuesday. What day of the week lies on Jan. 1, 2014?**
11. **If the Gandhi jayanti in 2013 falls on Wednesday, then on which day will the Gandhi Jayanti fall in 2018 ?**
12. **March 1st is Wednesday. Which month of the same year starts with the same day?**
13. **Today is Wednesday. After 23 days, which day will it be ?**
14. **Today is 4th November 2020 .The day of the week is Wednesday. This is a leap year. What will be the day of the week on this date after 5 years?**

15. **February 03, 2020 is Monday. What day of the week lies on February 03, 2021 ?**
16. **Which of the following is not a leap year ?**
17. **What is the average number of days per month in the year 2004 ?**
18. **Which of the following is Ordinary days ?**
19. **Today is Friday. A person wants to meet a lawyer and as that lawyer is busy he asks him to come three days after the day of the day after tomorrow? On which day the lawyer asks the person to come?**
20. **What was the day on 14 April, 2000 ?**
21. **26th January, 1996 was a Friday. What day of the week lies on 26th January, 1997?**
22. **The calendar 2006 will be the same for which year ?**
23. **If 1st Jan 2005 is Saturday then what will be 1st Jan 2008?**
24. **What will be the day on week 4th April 2001?**
25. **The last day of the century cannot be ----**
26. **What day of the week was 31st July, 1993?**
27. **The calendar for the year 2001 is the same for which year?**
28. **What was the day on 16 August, 1947 ?**
29. **26th January, 1996 was a Friday. What day of the week lies on 26th January, 1997?**
30. **The calendar 2006 will be the same for which year?**
31. **If 1st Jan 2005 is Saturday then what will be 1st Jan 2008?**

Chapter 23. Clock

A clock is a circle with 360 degrees on it. It's divided into 12 equal parts with numbers ranging from 1 to 12. 360/12 = 30° for each part. One hour is when the minute hand completes a full circle, i.e. covers 360°. The hour hand and the minute hand are the two hands on a clock. Both hands go around the dial, which is circular.

23.1 Clocks Formulas & Properties

- The dial of the clock is circular in shape and is divided into 60 equal minute spaces.
- 60-minute spaces trace an angle of 360^0. Therefore, one minute space traverses an angle of 6^0.
- The hands of a clock are perpendicular when they are 15-minute space apart.
- Hands of a clock are in a straight line and are opposite to one another when they are 30 minutes apart.
- Hands of a clock are perpendicular to each other 22 times in 12 hours and 44 times in 24 hours.
- Hands of the clock are opposite to each other for 11 times in 12 hours and 22 times in a day.
- The minute hand gains 55 minutes over hour hand every hour.

23.2 Best Tips & Tricks on Clocks problems to solve easily

- Below are some easy tips and tricks for you on problems based on clocks which efficiently help in the competitive exam . It is a very important topic for recruitment drives.
- Speed of the minute hand= 6° per minute.
- Speed of the hour hand= 0.5° per minute.
- Relative Speed between hour and minute hand. Relative speed of a minute hand is 6 – 0.5 dpm = 5.5 dpm. Where dpm = dial per minute.

23.3 How to solve Quickly:

A) Angle between hands of a clock

Question: When the minute hand is behind the hour hand, the angle between the two hands at M minutes past H o’clock.

$= 30 [H - (M/5)] + M/2$ degree

$= 30H – (11M/2)$

Question: In the case where the minute hand is ahead of the hour hand, the angle between the two hands at M minutes past H ‘o clock will be calculated as

$= 30 (M/5 - H) - M/2$ degree

$= 11/2M – 30H$

B) To calculate x minute space gain by the minute hand over the hour hand,

$= x60/55$

$= x12/11$

C) Both the two hands of the clock will be at the right angles between H and (H+1) o’clock at (5H ± 15) minutes H<’o clock.

D) If the minute hand of a clock overtakes the hour hand at the interval of M minutes when the time is correct, the clock gains or loses a day by

$= [(720/11)-M] [(60×24)/M]$ minutes.

E) Between H and H+1 o’clock, the two hands of the clock are M minutes apart at (5H ± M)12/11 minutes past H o’ clock.

<u>Solved Examples:</u>

Question: What is the angle between the hour hand and the minute hand when the time on the clock is 3:10 PM

Answer: 30H - (11M/2)

=30*3 - (11*10/2)

=45 degree

Question: What is the angle between the hour hand and the minute hand when the time on the clock is 4:50 PM

Answer: 11/2 * M - 30H

= 11/2 * 50 - 30* 4

=155 degree

Question: The number of times the hands of a clock coincide in one day is:

Sol: The hands of the clock coincide 11 times in 12 hours. So in one day it coincides 22 times.

Question: Currently the clock shows 9 o’clock. What angle will be the angle traced by the hour hand when it shows 3 o’clock?

Sol: The time difference between 9 o’clock and 3 o’clock is 6 hours.

Angle traced by hour hand in 6 hours = 180°

Question: What is the measure of the angle formed by the hands of the clock when the time is 11:25 AM?

Sol: The angle traced by minute hand in one minute = 6°

Therefore, the angle traced by minute hand in 25 minutes = 25*6 = 150°

The angle traced by hour hand in one hour = 30°

Therefore, the angle traced by hour hand in 11 hour = 11*30 =330°

However, since the time is 11:45 the hour hand is not exactly at 11. Instead it lies somewhere between 11 and 12. Since the time is 11:45 and 45 minutes is ¾ of an hour; therefore, the exact angle it subtends:

330° + ¾ * 30°

330°+22.5°

352.5°

The angle between minute hand and the hour hand can be determined by:

352.5° – 150° = 202.5°

Question: How many times in a day do the hands of the clock subtend right angles?

Sol: There will be 2 times per hour when the angle between minute and hour hand is 90◦

Total of 22 times in 12 hours.

∴ In 24 hours, 22×2=44 times the angle between minute and hour hand is 90◦.

Question: What is the angle between hour hand and minute hand when the time is 10:10?

Sol: The angle subtended by hour hand 61/6 hours =

Angle subtended by minutes hand in 10 minutes =
The angle between the two hands of the clock = 305-180 = 125°

Question: What was the angle between two hands when time was 3:40 ?

Sol: We know that the formula find between the angel
x=(11/2min-30Hr)
(11/2 * 40 -30(3))
(220-90)
(130 deg).

Question: At what angle the hands of a clock are inclined in 22 minutes after 6 ?

Sol: We know that the formula find between the angel
6:22 min
x=(11/2min-30Hr)
(11/2 * (22) -30(6))
(121-150)
(29 deg).
So the answer is 29 deg.

Question: Find the reflex angle between the hands of a clock at 5:54 ?

Sol: We know that the formula find between the angel
10:25 min
x=(11/2min-30Hr)
(11/2 * (54) -30(5))
(297-150)
(147-150)
(3 deg)
So the angle of 5:54 min is 3 deg
Now,
The opposite angle is :- 360-3
= 327 deg

Question: What is the angle between when time is 5:30?

Sol: At 5:30 minute hand will be at position 6 and hour hand will be exactly between 5 and 6 and will cover 15°, so angle between two hands=30 -15 = 15°

Question: At what time between 2 '0' clock and 3'o' clock,both the needles will coincide with each other ?

Sol: Deg = 0 deg in coincide
So,
0=(11/2 min-30(2))
0=(11/2 min - 60)
60=11/2 min
120= 11min
min=120/11
min= 10.90 min
So the time between 2 and 3 'o' clock when needles are coincide with each other is
2:10.90 min

Question: At what time between 5 and 6 'o' clock will the hands of a watch be together ?

Sol: Deg = 0 deg in coincide
So,
0=(11/2 min-30(5))
0=(11/2 min - 150)
150=11/2 min
300= 11min
min=300/11
min=27.27 min
So the time between 5 and 6 'o' clock when needles are coincide with each other is
5:27.27 min

Question: What is the angle between the two hands of a clock when time is 7:30?

Sol: Angle traced by hour hand in 12 hours = 360 °
Angle traced by hour hand in 1 hour = 360°/12 =30°
Angle traced by hour hand in 8 hour = 7 × 30° = 210°
At 7:30, hour hand will be in middle of 7 & 8, so angle traced by hour hand at 7:30
= 210° + (30°/2) = 210° + 15° = 225° -(i)
Angle traced by minute hand in 60 minutes = 360 °
Angle traced by minute hand in 1 minute = (360°/60)= 6°
Angle traced by minute hand in 30 minutes = 30 × 6° = 180° -(ii)
Now, the angle between the two hands of a clock when time is 8:30 = 225° - 180° (from i & ii)
= 45°

Question: Between 9 a.m and 9 p.m of a particular day for how many times are the minute and hour hands together?

Sol: The hands will be together for once per hour..e.g 9.45,10.50..but between 11 and 12 the coming together will be counted as 1 for both hence in 12 hrs time they will be together for 12-1=11 times

Question: What is the angle between the two hands of a clock when the time shown by the clock is 9.30 p.m. ?

Sol: Formula used is ----

x = 11/2 m – 30h

= 11/2 *30 - 30 *9

= /165-270/ = 115 deg

Question: A clock shows 5 O'clock in the morning. By how much angle will the hours hand rotate when the clock shows 9 'O' Clock in the morning ?

Sol: In 12 hours ,the hand turns 360 deg.

Here the difference between time = 4 hours.

Then, Required Angle =360/12 *4 = 120 deg

Question: At what time 2and 3 'O' Clock will be the hands of a watch point in Opposite direction ?

Sol: x=(11/2 min - 30 H)

x=180 deg

180=(11/2 min- 30(2))

180=(11/2 min -60)

180+60=11/2 min

240=11/2 min

480=11min

min=43.63

So the time between 2 and 3 'o' clock when Clock will be the hands of watch point in Opposite direction is 2:43.63 min

Question: A watch gains 5 sec in 3 minutes and was set right at 8 am. What time will it show at 10 pm on the same day?

Sol: The watch gains 5 seconds in 3 minutes = 100 seconds in 1 hour.

From 8 AM to 10 PM on the same day, time passed is 14 hours.

In 14 hours, the watch would have gained 1400 seconds or 23 minutes 20 seconds.

So, when the correct time is 10 PM, the watch would show 10 : 23 : 20 PM

Question: A clock shows 4:00 in the morning. By how much angle will the hour hand rotate when the clock shows 7:00 in the noon ?

Sol: In 12 hours, the hand turns 360 deg .

Here, the difference between time = 3 hours

Then, Required angle 360/12 * 3= 90 deg

Question: The famous church in the city of Kumbakonam has a big clock tower and is said to be over 300 years old. Every Monday at 10.00 AM the clock is set by Antony, doing service in the church. The Clock loses 6 minutes every hour. What will be the actual time when the faulty clock shows 3 P.M on Friday?

Sol: Let us start from Monday 10 am to friday 3.00 pm.....total hours=

4 days 5 hrs.=4*24+5=101 hrs.

so total time loses=101*6 minutes=606 minutes=606/60=10 hrs 6 minutes

so actual time=friday 3.00 pm +10 hrs 06 minutes=saturday 1.06 am.

Now, in most of the places they are giving answers as of 4.54 AM, which is incorrect. As the clock loses 6 mins. Thus we have to add the time.

Question: How many times do the hands of a clock coincide in 5 hours?

Sol: The hour hand covers 2*360 degrees in 24 hours, i.e 1440 minutes.

So, in one minute, it covers 1440/720 = 1/2 degree.

Difference in angular distance travelled by the minute hand and the hour hand in one minute is thus 6-1/2 = 11/2 degrees.

So, on a full rotation (360^{o}), any similar event between them will be repeated every 360/(11/2) = 65 5/11 minutes.

The hands of a clock coincide 11 times in every 12 hours (Since between 11 and 1, they coincide only once, i.e., at 12 o\'clock).

The hands overlap about every 65 minutes, not every 60 minutes.

The hands coincide 22 times in a day.

Question: Between 9 a.m and 9 p.m of a particular day for how many times are the minute and hour hands together?

Sol: The hands will be together for once per hour..e.g 9.45,10.50..but between 11 and 12 the coming together will be counted as 1 for both hence in 12 hrs time they will be together for 12-1=11 times

Question: How many palindromes are there in a clock from noon to midnight (For Example 5.45 is a palindrome)?

Sol: After 12 oclock 12:21.......=1

from 1 to 9 it is 1.01, 1.11, 1.21, 1.31, 1.41, 1.51.= 6

similarly 2.02,2.12,.................

6*9=54+1=55

after 10 o'clock 10:01, 11:11

Ans: 55+2=57

Question: One-quarter of the time till now from midnight and half of the time remaining from now up to midnight adds to the present time. What is the present time?

Sol: x is present time

1/4(x)+1/2(24-x)=x

solve it then x = 9:36

Question: A clock loses 1% time during the first week and then gains 2% time during the next one week. If the clock was set right at 12noon on Sunday, what would be the exact time that the clock would show 14days from the time it was set, right?

Sol: The clock loses 1% time during the first week.

In a day there are 24 hours and in a week there are 7 days.

Therefore, there are 7 * 24 = 168 hours in a week.

If the clock loses 1% time during the first week, then it will show a time which is 1% of 168 hours less than 12 Noon at the end of the first week = 1.68 hours less.

Subsequently, the clock gains 2% during the next week. The second week has 168 hours and the clock gains 2% time = 2% of 168 hours = 3.36 hours more than the actual time.

As it lost 1.68 hours during the first week and then gained 3.36 hours during the next week, the net result will be a -1.68 + 3.36 = 1.68 hour net gain in time.

So the clock will show a time which is 1.68 hours more than 12 Noon two weeks from the time it was set right.

1.68 hours = 1 hour and 40.8 minutes = 1 hour + 40 minutes + 48 seconds.

i.e. 1 : 40 : 48 P.

Chapter 24. Number System

Number system: A number system is a way of describing a numerical value on a number line. A numeric system is a method of representing or writing integers.

Number System : Definition

A numerical system is a method of displaying numbers on a number line in writing. A numeric system is a method of representing or writing integers.

There are generally two type of Number

Whole Number
Natural Number.

Number system is a technique to represent number and present number in a discrete manner.

Whole Number, Rational Number, Natural Number, Odd Number, Even Number, Irrational Number etc are general types of Number System.

24.1 Number System Formulas & Definitions

Natural Numbers

All positive integers are called natural numbers. All counting numbers from 1 to infinity are natural numbers. **N = {1, 2, 3, 4, 5, 6………∞}**

Whole Numbers

The set of numbers that includes all natural numbers and the number zero are called whole numbers. They are also called Non-negative integers. **W = { 0,1,2,3,4,5,6,7,8,…………..∞}**

Integers

All numbers that do not have the decimal places in them are called integers. **Z = {∞…….-3, -2, -1, 0, 1, 2,** 3………∞}

Positive Integers: 1, 2, 3, 4….. is the set of all positive integers.

Negative Integers: −1, −2, −3….. is the set of all negative integers.

- c. Non-Positive and Non-Negative Integers: 0 is neither positive nor negative.

Real Numbers

All numbers that can be represented on the number line are called real numbers.

Rational Numbers

A rational number is defined as a number of the form a/b where ‘a’ and ‘b’ are integers and b ≠ 0. The rational numbers that are not integers will have decimal values. These values can be of two types

a. Terminating decimal fractions: For example:

$$\frac{1}{5} = 0.5, \frac{125}{4} = 31.25$$

b. Non-Terminating decimal fractions

$$= 3.1666666, \frac{21}{9} = 2.33333$$

Irrational Numbers: It is a number that cannot be written as a ratio $\frac{x}{y}$ form (or fraction). Irrational numbers are non-terminating and non-periodic fractions. For example: $\sqrt{2}$ = 1.414

- **Complex Numbers**

The complex numbers are the set {a+bi}, where, a and b are real numbers and 'i' is the imaginary unit.

- **Imaginary Numbers**

A number that does not exist on the number line is called an imaginary number. For example square roots of negative numbers are imaginary numbers. It is denoted by ‘i’ or ‘j.

- **Even Numbers**

A number divisible by 2 is called an even number.

For example: 2, 6, 8, 14, 18, 246, etc.

- **Odd Numbers**

A number not divisible by 2 is called an odd number.

For example: 3, 7, 9, 15, 17, 373, etc.

- **Prime numbers**

A number greater than 1 is called a prime number, if it has exactly two factors, namely 1 and the number itself.

For example: 2, 3, 5, 7, 11, 13, 17, etc.

- **Composite numbers**

Numbers greater than 1 which are not prime, are known as composite numbers. For example: 4, 6, 8, 10, etc.

24.2 Formulas for Number System and Basic Concept

1. $(a - b)(a + b) = (a^2 - b^2)$.
2. $(a + b)^2 = (a^2 + b^2 + 2ab)$
3. $(a - b)^2 = (a^2 + b^2 - 2ab)$
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
5. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
6. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
7. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$
8. When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

Formulas for finding the Squares of a number .

Squares of numbers 91-100:

- 97^2
  Step 1: 100-97 = 3

Step 2: 97-3 = 94

Step 3: 3^2= 09

Final result: From step 2 and

Step 3 => 97^2= 9409

- 91^2

Step 1: 100-9 = 91

Step 2: 91-9 = 82

Step 3: 9^2 = 81

Final Result: From step 2 and step 3 => 91^2 = 8281

- Squares of numbers 100-109:
- 102^2

Step 1: 102-100 = 2

Step 2: 102 +2 = 104

Step 3: 2^2 = 04 Final result:

From step 2 and step 3 => 102^2=10404

- 107^2

Step 1: 107-100 = 7

Step 2: 107+7 = 114

Step 3: 7^2 = 49

Final Result: From step 2 and step 3 => 107^2 = 11449

- **Squares of numbers 51-60**
- 53^2

Step 1: 53-50 = 3

Step 2: 25+3 = 28

Step 3: 3^2 = 09

Final result: From step 2 and step 3 => 53^2 = 2809.

- 42^2

Step 1: 50-42 = 8

Step 2: 25-8 = 17

Step 3: 8^2 = 1764

Final Result From step 2 and step 3 => 42^2 = 1764

24.3 Number System Tips and Tricks and Shortcuts

Tips and Tricks number system help to answer number system problems effectively.
Whole Number , Natural number and Integers are some common examples of number systems.

Here are easy tips and tricks on Number System problems easily, and efficiently in competitive exams.

For example : Question : π is rational number or irrational.

Solution: Yes, π is a rational number as it can be written in pq $\frac{p}{q}$ qp form.

Some solved examples for better understanding:

Question: How many different integers can be expressed as the sum of three distinct numbers from the set {3, 10, 17, 24, 31, 38, 45, 52}?

Solution:

If you think that 8C3 = 56 is correct then it is the wrong answer. We are not asked how many ways we can select 3 numbers out of 8.
But how many different numbers can be expressed as a sum of three numbers from the given set.

For example, 3 + 10 + 31

= 3 + 17 + 24 = 47.
So 47 can be expressed as a sum of 3 numbers in two different ways but 47 should be considered as only one number. Now the minimum number that can be expressed as a sum of 3 numbers = 30. The next number is 37. Similarly the largest number is 38 + 45 + 52 = 135. So there exist many numbers in between, with a common difference of 7.

Total numbers =(l−a/d)+1=(135−30/7)+1 = 16.

Question: How many prime numbers are there which are less than 100 and greater than 3 such that they are of the following forms

a. 4x + 1
b. 5y - 1

Solution:

Let the number be N. So N = 4x + 1 = 5y - 1

⇒x=5y−2/(4)

y = 2 satisfies the equation.

So minimum number satisfies both the equations is 9 and general format of the numbers which satisfies the equation

= k. LCM (4, 5) + 9 = 20k + 9.

Now by putting values 1, 2, 3 . . . . for k, we get 29, 49, 69, 89. Of which only 29, 89 are primes.

Question: All even numbers from 2 to 98 inclusive, except those ending 0, are multiplied together. What is the rightmost digit (the units digit) of the product?

Solution:

2 × 4 × 6 × 8 × 12 × 14 × . . . . . × 98

Now units digit of 2 × 4 × 6 × 8 = 4

Also 12 × 14 × 16 × 18 also 4. So on
Total 10 times 4 occurs in the units digit = 4^10=6

Question: What is the greatest power of 143 which can divide 125! Exactly

Solution:

Highest power of 11 in 125! Is 12 but the highest power of 13 is only 9. That means, 125!=11^12×13^9×....

So only nine 13\'s are available. So we can form only nine 143\'s in 125!. So the maximum power of 143 is 9.

Question: 5400 x ? = 7302

Solution;

5400 x s = 7302

s = 7302/5400

= 3651/2700

=1217/900

Question: A teacher teaching students of third standard gave a simple multiplication exercise to the kids. But one kid reversed the digits of both the numbers and carried out the multiplication and found that the product was exactly the same as the one expected by the teacher. Only one of the following pairs of numbers will fit in the description of the exercise. Which one is that?

a. 14,22
b. 19,33
c. 13,62
d. 42,28

Solution:

Trial and error will give option (b) as the correct answer, since 13 × 62 = 26 × 31

Question: What is the product of 2004 x 2004?

Solution:

$$2004 \times 2004 = 4016016$$

Question: Which among the following options is a prime number?

a. 21
b. 25
c. 56
d. 17

Solution:

As 17 cannot be divided by any of the numbers except 1 and itself, it is clearly the prime number.

Question: The value of $2^{73} - 2^{72} - 2^{71}$ is equal to

Solution:

We have,

$$2^{73} - 2^{72} - 2^{71}$$

Taking 2^{71} as common, we get

$$2^{71}(2^2 - 2^1 - 2^0)$$

$$= 2^{71}(4 - 2 - 1)$$

$$= 2^{71}(4 - 3)$$

$$= 2^{71}(1)$$

$$= 2^{71}$$

$$\therefore\ 2^{73} - 2^{72} - 2^{71} = 2^{71}$$

Question: A positive integer is said to be a prime if it is not divisible by any positive integer other than itself and one. Let P be a prime number strictly greater than 2. Then, when p^2+17 is divided by 12, the remainder is-

Solution:
The square of any prime greater than 3, when divided by 12 leaves a remainder 1 · p2 when divided by 12 leaves a remainder of 1 and 17 when divided by 12 leaves a remainder of 5. So p^2+17 when divided by 12 leaves a remainder of 6.

Exercises:

1. **101 x 101 + 99 x 99 =?**
2. **The money order commission is calculated as follows. From ` X to be sent by money order, subtract 0.01 and divide by 10. Get the quotient and add 1 to it, if the result is Y, the money order commission is ` 0.5Y. If a person sends two money orders to Aurangabad and Bhatinda for ` 71 and ` 48 respectively, the total commission will be?**
3. **123456789 x 62 =?**
4. **If the multiplication result of 4864 and 9P2 is completely divisible by 12. What is the value of P?**
5. **The number of integers n satisfying – n + 2 ≥ 0 and 2n ≥ 4 is**
6. **-20 x 19 + 200 =?**
7. **Three times the first of three consecutive odd integers is 3 more than twice the third. What is the third integer?**
8. **(31 + 32 + 33 +…….+100)=?**

9. If 8 + 12 = 2, 7 + 14 = 3 then 10 + 18 =?
10. (560 / 5) x (128 / 4) =?
11. What is the third prime number?
12. X and Y are playing a game. There are eleven 50 paise coins on the table and each player must pick up at least one coin but not more than five. The person picking up the last coin loses. X starts. How many should he pick up at the start to ensure a win no matter what strategy Y employs?
13. 297 x 297 + 102 + 102 + 2 x 297 x 102 = ?
14. 12565 + 45878 + 56891 – (215 x 79) = ?
15. The sum of two numbers is 15, and their product is 56. Find out the sum of their reciprocals of these numbers.
16. It is possible to pair up all the numbers from 1 to 70 so that the positive difference of the numbers in each pair is always the same. For example, one such pairing up is (1,2), (3,4), (5,6),….(69,70). Here the common difference is 1. What is the sum of all such common differences?
17. What is the number of positive integers less than or equal to 2017 that have at least one pair of adjacent digits that are both even. For example 24,564 are two examples of such numbers while 1276 does not satisfy the required property.
18. Find the number of prime factors for (3 x 5)6, (2 x 7)8, (9)4 is:
19. What could be the value of x to make a number 34567x divisible by 9?
20. The difference between the place value and the face value of 6 in the numeral 856973 is:
21. What would be (2/20) of 1440?

Chapter 25. Heights and Distance

Introduction:

Angle of Elevation occurs when an object is viewed by an observer and the horizontal line and the line from the object to the observer's eye form an angle. The angle between the horizontal line and the observer's line of sight is considered the angle of depression when the object is below the height of the observer.

Height is a term used to describe the vertical measurement of an object.

Distance: The measurement of an object in the horizontal direction from a certain point.

25.1 Height and Distance Formulas

There are basically two terms associated with heights and distances which are as follows :

- Angle of Elevation.
- Angle of Depression.

Formulas for Angle of Elevation

Assume a man is viewing an object from a distance, such as the top of a building. The line of sight is the line that connects a person's eye to the top of a building. Angle of elevation is the angle formed by the line of sight and the horizontal line.

Angle of elevation of P from O = $\angle$ AOP.

Formulas for Angle of Depression

Assume a man is standing at a certain height and looking down at an object. The line of sight is the line that connects the man's eye to the bottom of the object.The angle of depression is the angle formed by the line of sight and the horizontal line.

Height and Distance Formulas for Trigonometric ratio

- **ΔABC is a right angled triangle where AB is the perpendicular, AC is the hypotenuse, and BC is the base.**

Then,

- Sin θ = $\frac{AB}{AC}$
- Cos θ = $\frac{BC}{AC}$
- Tan θ = $\frac{AB}{BC}$
- Cosec θ = $\frac{AC}{AB}$
- Sec θ = $\frac{AC}{BC}$
- Cot θ = $\frac{BC}{AB}$

Trigonometric Identities:

- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

<u>25.2 How to Solve Height and Distance Question Quickly</u>

- **Angle of elevation:** The angle between the horizontal and the line from the object to the viewer's eye is the angle of elevation of an object as viewed by an observer.
- **Angle of Depression:** The angle of depression is the angle formed by the horizontal and the observer's line of sight when the target is below the observer's eye level.

How to solve word problems that involve angle of elevation or depression Shortcuts and Easy Tips & Tricks to solve questions

- **Step 1**: Draw a sketch of the situation given.
- **Step 2**: Mark in the given angle of elevation or depression and other information.
- **Step 3**: Use trigonometry to find the required missing length

<u>Learn the values of these trigonometric ratios.</u>

	0^o	30^o	45^o	60^o	90^o
Sin	0	1/2	$1/\sqrt{}$	$\sqrt{3}/2$	1
Cos	1	$\sqrt{3}/2$	$1/\sqrt{}$	1/2	0
Tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Not defined

Type 1 Solve Example

Find the distance/height/base/length when angle is given

Question 1:From the top of a 9 meters high building AB,the angle of elevation of the top of tower CD is 30 and the angle of depression of the foot of the tower is 60. What is the height of the tower?

let h be the height of tower,y be the distance betn tower & building
draw the diagram
tan60=9/y
or y=9/tan60
also, tan30=(h-9)/y
or y*tan30=h-9
or (9/tan60)*tan30=h-9
or 9/3=h-9
or 3=h-9
hence h=12

Question 2: From the top of a building 60 meters high the angles of depression of the top and bottom of a tower are 30Â° and 60Â° respectively. The height of the tower is?

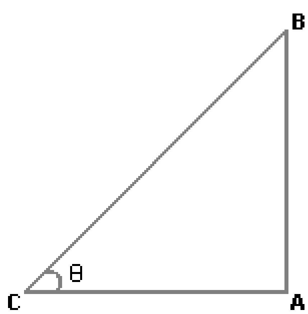
As given,
From the top of a building 60 metres high the angles of depression of the top and bottom of a tower are 30Â° and 60Â° respectively,
hence,
applying tan 60 = p/b =60/base i.e, base = 60/tan 60,
again,
tan 30 = p/b = p/(60/tan 60)
hence, p=20 meters
so,The height of the tower = (60-20)meters
= 40 meters

Type 2 Solved Example

Find the angle when distance/height/base/length is given

Question 1: The angle of elevation of the sun, when the length of the shadow of a tree $\sqrt{3}$ times the height of the tree, is:

Let AB be the tree and AC be its shadow.



Let $\angle$ ACB = θ.
Then AC/AB=$\sqrt{3}$
cot θ = 3
θ = 30

Exercise:

Question 1 Two ships are sailing in the sea on the two sides of a lighthouse. The angle of elevation of the top of the lighthouse observed from the ships are 30° and 45° respectively. If the lighthouse is 100 m high, the distance between the two ships is:

Question 2: A man is standing on the deck of a ship, which is 10m above water level. He observes the angle of elevation of the top of a lighthouse as 600 and the angle of depression of the base of the lighthouse as 300. Find the height of the light house.

Question 3: A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank is 450. When he moves 20m away from the bank, he finds the angle of elevation to be 300. Find the height of the tree

Question 4: From the top of a building 60m high, the angle of elevation and depression of the top and the foot of another building are α and β respectively. Find the height of the second building.

Question 5: 10 m long flagstaff is fixed on the top of a tower on the horizontal plane. From a point on the ground, the angles of elevation of the top and bottom of the flagstaff are 600 and 450 respectively. Find the height of the tower.

Question 6: The angles of elevation of the top of a tower from two points on the same side of the tower are α and β (α >β). If the distance between the two points is 40m, find the height of the tower.

Question 7: The angle of elevation of the top of a tower from point A on the ground is 300. On moving a distance of 40m towards the foot of the tower, the angle of elevation increases to 450. Find the height of the tower.

Question 8: An aeroplane, when 4000m high from the ground, passes vertically above another aeroplane at an instance when the angles of elevation of the two aeroplanes from the same point on the ground are 600 and 300 respectively. Find the vertical distance between the two aeroplanes.

Question 9: A car is moving at uniform speed towards a tower. It takes 15 minutes for the angle of depression from the top of the tower to the car to change from 300 to 600. What time after this, the car will reach the base of the tower?

Question 10: A man is watching from the top of a tower, a boat speeding away from the tower. The angle of depression from the top of the tower to the boat is 600 when the boat is 80m from the tower. After 10 seconds, the angle of depression becomes 300. What is the speed of the boat? (Assume that the boat is running in still water).

Question 11: A man standing at a point P is watching the top of a tower, which makes an angle of elevation of 30º with the man's eye. The man walks some distance towards the tower to watch its top and the angle of the elevation becomes 45º. What is the distance between the base of the tower and the point P?

Question 12: From a point P on a level ground, the angle of elevation of the top tower is 30º. If the tower is 200 m high, the distance of point P from the foot of the tower is:

Question 13: The angle of elevation of the sun, when the length of the shadow of a tree is equal to the height of the tree, is:

Question 14: The angle of elevation of a ladder leaning against a wall is 60º and the foot of the ladder is 12.4 m away from the wall. The length of the ladder is:

Question 15: A man on the top of a vertical observation tower observers a car moving at a uniform speed coming directly towards it. If it takes 8 minutes for the angle of depression to change from 30° to 45°, how soon after this will the car reach the observation tower?

Question 16: A man is watching from the top of a tower a boat speeding away from the tower. The boat makes an angle of depression of 45° with the man's eye when at a distance of 100 metres from the tower. After 10 seconds, the angle of depression becomes 30°. What is the approximate speed of the boat, assuming that it is running in still water?

Question 17: The top of a 15 metre high tower makes an angle of elevation of 60° with the bottom of an electronic pole and angle of elevation of 30° with the top of the pole. What is the height of the electric pole?

Question 18: The angle of elevation of the top of a tower from a certain point is 30°. If the observer moves 40 m towards the tower, the angle of elevation of the top of the tower increases by 15°. The height of the tower is:

Question 19: On the same side of a tower, two objects are located. Observed from the top of the tower, their angles of depression are 45° and 60°. If the height of the tower is 600 m, the distance between the objects is approximately equal to :

Question 20: From a tower of 80 m high, the angle of depression of a bus is 30°. How far is the bus from the tower?

Question 21: The angle of elevation of the top of a lighthouse 60 m high, from two points on the ground on its opposite sides, is 45° and 60°. What is the distance between these two points?

Question 22: Two persons are on either sides of a tower of height 50 m. The person observes the top of the tower at an angle of elevation of 30° and 60°. If a car crosses these two persons in 10 seconds, what is the speed of the car?

Chapter 26. Surds and Indices

Surds: Surds are the natural numbers which can be expressed in the form $\sqrt{p} + \sqrt{q}$

Indices: Indices refers to the power to which a number is raised. For example; 3²

26.1 Formulas for Surds and Indices & Definitions

Surds: Numbers which can be expressed in the form √p + √q , where p and q are natural numbers and not perfect squares. Irrational numbers which contain the radical sign (n√) are called as surds Hence, the numbers in the form of √3, 3√2, ……. n√x in other words

For example : $\sqrt{3},$ it can't be simplified.
$\sqrt{4},$ it can be simplified so it is not a surd.

Indices: Indices refers to the power to which a number is raised. For example; 3²

Surds and Indices formulas pages are very useful for solving the ques.. Prepinsta provides Surds and Indices Formulas and ques.

Types of Surds and Definitions

Pure Surds:- Those surds which do not have factors other than 1. For example 2√3, 3√7

Mixed Surds:- Those surds which do not have a factor of 1. For example √27 = 3√3, √50 = 5√2

Similar Surds:- When the radicands of two surds are the same. For example 5√2 and 7√2

Unlike Surds:- When the radicands are different. For example √2 and 2√5

26.2 Surds and Indices Formulas and Rule

Rule Name	**Surds Rule**	**Indices Rule**
Multiplication Rule	$a^n * b^n = (a * b)^n$	$a^n * a^m = a^(m$
Division Rule		
Power Rule	n√(a)n = a n√(a)/b = n√(a)/n√(b) m√n√(b) = mn√(b)	$a^0 = 1$ $[a/b]^n = (a^n)/($ $(ab)^n = a^n x b^n$

26.3 Basic Formula for Surds and Indices

- $(a + b)(a - b) = (a^2 - b^2)$
- $(a + b)^2 = (a^2 + b^2 + 2ab)$
- $(a - b)^2 = (a^2 + b^2 - 2ab)$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
- $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
- $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
- $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$
- When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

26.4 More Formulas:

Indices Multiplication rules:-

- **Multiplication rule with same base**

  $$a^n a * m = a^(m + n)$$

  Example:

  $$2^3 2^4 = 2^(3 + 4) = 2^7 = 2222222 = 128$$

Indices Division rules:-

Division rule with same indices

$$a^n / b^m = (a/b)^n$$

Example: $$\frac{93}{33} = (9/3)3 = 27$$

Surds Multiplication rules:-

- **Multiplication rule with same indices**
  $$a^n b^n = (ab)^n$$
  Example:
  $$3^2 2^2 = (32)^2 = 36$$

Surds Division rules:-

- **Division rule with same base**
  $$a^m/a^n = a^{(}mn)$$
  Example:
  $$3^5/3^3 = 3^{(}5-3) = 9$$

26.5 Surds and Indices Power rules

Power rule 1

- $$(a^n)^m = a^{(}n.m)$$
  Example:
  $$(2^3)^2 = 2^{(}32) = 2^6 = 222222 = 64$$

Power rule 2

- $$(a^n)^m = a^{(}n^m)$$
  Example:
  $$(2^3)^2 = 2^{(}3^2) = 2^{(}33) = 2^9 = 222222222 = 512$$

Power rule 3

- **n√a =a^1/n**
  Example:
  $$27^1/3 = 327 = 3$$

26.6 Tips And Tricks & Shortcuts for Surds and Indices

- **Surds**: Numbers which can be expressed in the form p+q
  $$\sqrt{p} + \sqrt{q}p$$
- +q , where p and q are natural numbers and not perfect squares.
- **Indices**: Indices refers to the power to which a number is raised. For example; 32
- Here are quick and easy tips and tricks for you to solve Surds and Indices Formulas with Power questions quickly, easily, and efficiently in competitive exams and other recruitment exams .
- Memorize all square and cube values from numbers 1 – 20 to solve the questions fast.

The questions can be categorized into the following types:

Type 1: Simplify the expression

Question: 36 * 36 * 36 * 36 = $6^?$

Solution:

36 * 36 * 36 * 36 = $6^2 * 6^2 * 6^2 * 6^2$

= $6^{(2+2+2+2)} = 6^8$

Question: Simplify $(256)^{¾}$

Solution:
$(256)^{3/4} = (4^4)^{3/4} = 4^3 = 64$

1. $(625)^{0.16} * (625)^{0.09} = ?$

Solution:

$(625)^{0.16} * (625)^{0.09} = (625)^{0.16\ +\ 0.09}$

= $(625)^{0.25}$

= $(625)^{(25/100)}$

= $(5^4)^{(1/4)}$

Question: (1331) ^– (2/3)

Solution:

1331 is the cube root of 11

(11 3) – (2/3)

We know, law of indices (x m) n = x mn

(11) -3 * (2/3)

(11) -2

We know that x -1 = 1/x

(11) -2 = 1/11 -2 = 1/121

Question: Find the value of $\frac{5^{250}}{5^{245}}$.

Solution;

$\frac{5^{250}}{5^{245}}$

$=5^{250-245} = 5^5 = 3125.$

2. $5^{6.5} \times 25^{4.5} \div 125^{4.5} = (5)^?$

Solution:

$5^{6.5} \times 25^{4.5} \div 125^{4.5} = (5)^?$

$5^{6.5} \times [(5)^2]^{4.5} \div [(5)^3]^{4.5}$

$5^{6.5} \times (5)^9 \div (5)^{13.5}$

$(5)^{(6.5+9-13.5)}$

$(5)^2$

3. $5^{150} \div 5^{144} =?$

Solution:

$5^{150} \div 5^{144}$

$5^{(150-144)} = 5^6 = 15625$

4. $256^{(0.16)} + 256^{(0.09)} =?$

Solution:

$256^{(0.16)} + 256^{(0.09)} = 256^{(0.16+0.09)} = 256^{(0.025)}$

$256^{(1/4)} = (2^8)^{(1/4)} = 2^{8/4} = 2^2 = 4$

5. $(4)^{1.25} \times (10)^{0.25} \times (40)^{0.75} =?$

Solution:

$(4)^{1.25} \times (10)^{0.25} \times (40)^{0.75}$

$= (4)^{1.25} \times (10)^{0.25} \times (10 \times 4)^{0.75}$

=

$(4)^{1.25} \times (10)^{0.25} \times (10)^{0.75} \times (4)^{0.75}$

$= (4)^{(1.25+0.75)} \times (10)^{0.25+0.75}$

$=(4)^2 \times (10)^1$

=16*10

=160

6. $(0.04)^{-1.5} =?$

Solution:

$$(0.04)^{-1.5} = \left(\frac{4}{100}\right)^{-1.5}$$

$$\left(\frac{1}{25}\right)^{-3/2}$$

$$(25)^{3/2}$$

$$\left(5^2\right)^{3/2}$$

$=5^3$

=125

7. $64 \times 64 \times 64 \times 64 \times 64 \times 64$
= $8^?$

Solution:

$64{\times}64{\times}64{\times}64{\times}64{\times}64 = 8^2{\times}8^2{\times}8^2{\times}8^2{\times}8^2{\times}8^2 = 8^{(2+2+2+2+2+2)} = 8^{12}$

8. $\frac{1}{1+x^{(b-a)}+x^{(c-a)}} + \frac{1}{1+x^{(a-b)}+x^{(c-b)}} + \frac{1}{1+x^{(b-c)}+x^{(a-c)}} =?$

Solution:

Given Exp=

$$\frac{1}{(1+\frac{x^b}{x^a}+\frac{x^c}{x^a})} + \frac{1}{(1+\frac{x^a}{x^b}+\frac{x^c}{x^b})} + \frac{1}{(1+\frac{x^b}{x^c}+\frac{x^a}{x^c})}$$

=

$$\frac{x^a}{(x^a+x^b+x^c)} + \frac{x^b}{(x^a+x^b+x^c)} + \frac{x^c}{(x^a+x^b+x^c)}$$

$= \dfrac{(x^a + x^b + x^c)}{(x^a + x^b + x^c)}$

=1.

9. If $x = 3 + 2\sqrt{2}$,then the value of $\left(\sqrt{x} - \dfrac{1}{\sqrt{x}}\right)$ is:

Solution:

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} - 2$$

$= (3 + 2\sqrt{2}) + \dfrac{1}{(3 + 2\sqrt{2})} - 2$

$= (3 + 2\sqrt{2}) + \dfrac{1}{(3 + 2\sqrt{2})} \times \dfrac{(3 - 2\sqrt{2})}{(3 - 2\sqrt{2})} - 2$

$= (3 + 2\sqrt{2}) + (3 - 2\sqrt{2}) - 2$

=4

Hence , $\left(\sqrt{x} - \dfrac{1}{\sqrt{x}}\right) = 2$

10. $\left(\dfrac{x^b}{x^c}\right)^{(b+c-a)} \cdot \left(\dfrac{x^c}{x^a}\right)^{(c+a-b)} \cdot \left(\dfrac{x^a}{x^b}\right)^{(a+b-c)}$

Solution:

$x^{(b-c)(b+c-a)} \cdot x^{(c-a)(c+a-b)} \cdot x^{(a-b)(a+b-c)}$

$= x^{(b-c)(b+c)-a(b-c)} \cdot x^{(c-a)(c+a)-b(c-a)} \cdot x^{(a-b)(a+b)-c(a-b)}$

$= x^{(b^2-c^2+c^2-a^2+a^2-b^2)} \cdot x^{-a(b-c)-b(c-a)-c(a-b)}$

$= (x^0 c \times x^0)$

=1.

11. $(1024)^{n/5} \times \dfrac{4^{2n+1}}{16^n \times 4^{4n-1}}$

Solution:

$\dfrac{(1024)^{n/5} \times 4^{2n+1}}{16^n \times 4^{4n-1}}$

$= \dfrac{(4^5)^{n/5} \times 4^{2n+1}}{(4^2)^n \times 4^{4n-1}}$

$= \dfrac{4^n \times 4^{2n+1}}{4^{2n} \times 4^{n-1}}$

$= 4^{(n+2n+1)-(2n)-(n-1)}$

$= 4^2$

=16.

12. If 6^m =46656,what is the value of $6^{(m-2)}$?

Solution:

We know that

$$\frac{a^m}{a^n} = a^{m-n}$$

Given that 6^m =46656

$$6^{m-2} = \frac{6^m}{6^2} = \frac{46656}{6^2} = \frac{46656}{36} = 1296$$

13. $2^{63} - 2^{62} - 2^{61}$ is same as

Solution:

2^{61} $(2^2 - 2^1 - 2)$

$= 2^{61} (4 – 2 – 2)$

$= 2^{61}$

14. Find the value of (0.000001) ⅓ ?

Solution;

3√0.000001

Question: If any cube has 6 decimals, then its cube root becomes 6/3 = 2 decimals

Therefore, the value is 1/100 = 0.01

Question: Find out the value of $2^{2(-2)}$?

Solution;

$2^{-2} = 1/2^2$ = ¼

$= 2^{1/4} = 4$

Question: Find the value of $(1000)^6/10^{15}$=?

Solution:

$=1000^6/10^{15}$

$=10^{(3+6)}/10^{15}$

$= 10^3$

Type 2: Find the value of x

Question: If 2^a = 3√32, then a is equal to:

Solution;

Given value 2^a = 3√32

$= 2^a = (32)^{1/3}$

$= 2^a = (2^5)^{1/3} = (2)^{5/3}$

a = 5/3

Question: If $3^x + 3^{x+1} = 36$, then the value of x^x is

Solution:

$3^x(1 + 3) = 36$

$= 3^x = 36/4$

$= 3^x = 9$

= x = 2

$x^x = 3^2 = 9$

Question: If p and q are whole numbers such that p^q = 144, then find the value of $(p – 1)^{q+1}$

Solution;

$144 = 12^2$

So, the value of p = 12 and q = 2

Now, $(p – 1)^{q+1}$

$= (12 – 1)^{2+1}$

$= (11)^3$

= 1331

1. $(16)^{2.5} \times (16)x =$ (16) ^{5} $(16)^5$ 6) ^{5} $6)^5$ ^{5} 5 {5} 5 } $

Solution:

(16) 2.5 * (16) x = (16) 5

(16) 2.5 + x = (16) 5

2.5 + x = 5

x = 5 – 2.5

x = 2.5

2. If (2)^x-1 = (2)^x-4 , then find the value of x?

Solution;

(2)^x-1 = (2)^x-4

(2)^x-1 = (2)^-(x-4)

x-1 = 4-x

x = 5/2

3. If x and y are whole numbers and $x^y=196$,what is the value of $(x-3)^{y+1}$?

Solution:

$x^y=196$

We know that, $14^2=196$.

Hence we can take x = 14 and y = 2.

$(x-3)^{y+1}=(14-3)^{2+1}.=11^3=1331$

4. $54^{2.3}\times54^{?}=54^{13.7}.$

Solution:

$$54^{2.3}\times54^{?}=54^{13.7}.$$

= 2.3+x=13.7

= x= 11.4

5. If $9^{(a-b)}=729$ and $9^{(a+b)}=59049$,what is the value of a ?

Solution:

$9^{(a-b)}=729=9^3$.

(a-b)=3

$9^{(a+b)}=59049=9^5$.

(a+b)=5

2a=8

a=4

6. $5^x\times2^3=36.5^{(x+1)}=?$

Solution:

$$5^x\times2^3=36$$

=> $5^x=\dfrac{36}{2^{(3)}}$

$$5^{(x+1)}=5^x\times5=\frac{36}{2^{(3)}}\times5=\frac{36\times5}{2\times2\times2}=\frac{9\times5}{2}=22.5$$

7. If $2^{(P+6)}-2^{(P+3)}=7$,what is the value of p ?

Solution:

$$2^{P+6}-2^{P+3}=7$$

$$2^{(P+3+3)}-2^{(P+3)}=7$$

$$2^{(P+3)}\times2^3-2^{(P+3)}=7$$

$$2^{(P+3)}[2^{(3)}-1]=7$$

$$2^{P+3}\times7=7$$

$$2^{(P+3)}=1.$$

P+3 =0

P=-3

8. If m and n are whole numbers such that $m^n=121$, then the value of $(m-1)^{(n+1)}$ is ?

Solution;

We know that $11^2=121$

Putting m=11 and n=2,we get:

$$(m-1)^{(n+1)}=(11-1)^{(2+1)}=10^3=1000$$

9. Given that $10^{0.48}=x,10^{0.70}=yandx^z=y^2$, then the value of z is close to:

Solution:

$$x^z=y^2$$

$$10^{(0.48z)}=10^{(2\times0.70)}=10^{1.40}$$

0.48z=1.40

$$z = \frac{140}{48} = \frac{35}{12} = 2.9$$ (approx)

10. If $3^{(n+4)} - 3^{(n+2)} = 8$,What is the value of n?

Solution:

$$3^{(n+4)} - 3^{(n+2)} = 8$$

$$3^{(n+2+2)} - 3^{(n+2)} = 8$$

$$3^{(n+2)} \times 3^2 - 3^{(n+2)} = 8$$

$$3^{(n+2)}\left[3^{(2)} - 1\right] = 8$$

$$3^{(n+2)} = 1$$

n=-2

11. If $6^x = 1296$, then the value of $6^{(x-2)}$ is:

Solution:

$6^x = 1296$

$6^x = 6^4$

x = 4

Therefore,$6^{(x-2)} = 6^{(4-2)} = 6^2 = 36$

12. If $3^{(a-b)} = 27$ and $3^{(a+b)} = 729$, then find the value of a?

Solution;

$3^{(a-b)} = 27$

$3^{(a-b)} = 3^3$…..(1)

a – b = 3

$3^{(a+b)}$ =243

$3^{(a+b)} = 3^6$

a + b = 6

On equating equation 1 and 2 we get, a = 4.5

13. If $x^a = y$, $y^b = z$ and $z^c = x$, then the value of abc is

Solution:

$x^1 = z^c$

$= (y^b)^c$

$= (y^{bc})$

$= (x^a)^{bc}$

So, abc = 1

14. If $2^x = 3\sqrt{64}$, then x is equal to

Solution:

$2^x = 3\sqrt{64}$

$= 64^{1/3}$

$= (2^6)^{1/3}$

$= 2^{6/3} = 2^2$

Chapter 27. Logarithms

When the power of a number must be raised in order to get some other number known as Logarithm.

Basic Concept:

- **Definition & Logarithm Formulas:**

Logarithms are the power to which a number is raised to achieve some other number.

- **Logarithms is of 2 types:-**
    - Common logarithm
    - Natural logarithm.
- **Common Logarithm-**

Logarithm with **base 10** is Common logarithm.

It is expressed as $\log_{10}$ X, and if any expression is not given with the base, then the **base 10** is considered.

- **Natural Logarithm-**

Logarithm with **base e** is Natural Logarithm.

It is expressed as $\log_e$ X.

- **Very Important:** If the base is not provided, then always remember to consider base as 10.

Formulas For Logarithm

Exponential Form	Logarithm Form
$a^x = y$	$x = \log_a y$
$2^3 = 8$	$3 = \log_5 8$
$3^2 = 9$	$2 = \log_3 9$

PrepInsta

27.1 Formulas for Logarithm

- $\log_x$ X= 1
- $\log_a$ 1= 0
- a $\log^{\log_a x}$ =X
- $\log_a x = \dfrac{1}{log_x a}$
- $\log_a(x^p) = p(\log_a x)$
- $\log_a x = \dfrac{log_{10} X}{log_{10} a}$
- $\log_a \frac{x}{y}$ = $\log_a$ X- $\log_b$ Y
- $\log_a$ (xy)= $\log_a$X+ $\log_b$Y

Value of log(2 to 10): Remember

- Log 2 = 0.301
- Log 3 = 0.477= 0.48
- Log 4 = 0.60
- Log 5 = 0.698 = 0.7
- Log 6 = 0.778 = 0.78
- Log 7 = 0.845 = 0.85
- Log 8 = 0.90
- Log 9 = 0.954= 0.96
- Log 10 = 1

27.2 Logarithm Formulas (Antilog):

- An antilog is the inverse function of a logarithm. log(b) x = y means that antilog (b) y = x.
- The best way to understand any problem is by having a look at the Solved Example.

Basic formulas used :

- $log_a 1 = 0$,
- $log_a x = \dfrac{1}{log_x a}$

Formulas 1 : $log_a(x^p) = p(log_a x)$

1. The value of log(.01)(0.0001) is :

Sol: log_{(.01)}(0.0001)
This can be written as
$$log_{(.01)}(0.01)^2$$
Using the Quotient Rule of Logarithm
$$2log_{(.01)}(0.01)$$
We know that loga a = 1
2(1) = 2
$$Thevalueoflog_{(.01)}(0.0001) = 2$$

2. Solve : log2√xy

Sol: log2√xy = log2 (xy)(1/2)

$$(\frac{1}{2})(log(4xy) = (\frac{1}{2})(log2x + log2y)$$

Formula 2: log_{x}x= 1

1. Prove: log_{10} 10 = 1

Sol: From the identity log_{a}a ---

$$=> log_{10}10 = 1$$

Hence Proved

2. Solve: log_{3}9

Sol: $log_3 9 = log_3 3^2$

$= 2log3_3$

=2

Formula 3 : $loga\frac{x}{y} =$ **ga X- logb Y**

log_{a} (xy)= log_{a}x+ log_{b}y

$og_a(xy) = log_a x + log_b y$ **_{a} (xy)=**

log_{a}x+ log_{b}y $a(xy) = log_a x + log_b y_{\}}$

(xy)= log_{a}x+ log_{b}y

$(xy) = log_a x + log_b y$ **xy)= log_{a}x+**

log_{b}y $y) = log_a x + log_b y$ **= log_{a}x+**

log_{b}y $log_a x + log_b y$ **og_{a}x+ log_{b}y**

$g_a x + log_b y$ **{a}x+ log_{b}y x+ log_{b}y**

$+log_b y$ **log_{b}y** $og_b y$ **_{b}y** by **}y** y **$**

1. If log xy - log yx = log (x+y), then, (x + y)= ?

Sol: log(xy) - log(yx)=(logxy/yx)=log 1

So x+y=1

2. If log10 (7) = 0.8450, then find the value of log7 (10).

Sol: Given log10 (7) = 0.8450,

From formula loga(b)=logc(b)/logc(a)

Here a=7 b=10 c=10

log7(10)=log10(10)/log10(7)=1/0.845=1.183

Exercise:

1. **Find the value of : log√6log 6 .**
2. **If log_{x}y=100 and log_{2} x=10, then the value of y is**
3. $If log8x + log8\frac{1}{6} = \frac{1}{3}$ **, then the value of x will be -----**
4. **Find the value of :** $log9^6$ **(value of log 9 = 0.942)**
5. **The value of** $log_2 log_2 log_3 log_3 27^3$ **is**
6. **If the value of log 3 = 0.477, then find the number of digits in 336**
7. **If logx 518 = 1/2 , then find the value of x:**
8. **The value of log3 27 is :**
9. **If log 5 = 0.698, find the number of digits in 525**
10. **Solve :** $p^x = q^y$ **,and find log(p-q)**
11. **Solve the given logarithmic equation: log_{7}x = 3**
12. **Solve: log √9 /log 9**
13. $log_x 3$ **= 1/2**
14. **Solve:** $log_6 x^3$ **= 18**
15. **Prove :** $log_6 36$ **= 3x**
16. **Find the roots of the equation:**
$$log_8(x^2) = 5$$
17. **Solve: 2 log_{4}(√ab)**
18. **if** $log\frac{a}{b} + log\frac{b}{a} = log(a + b)$ **the, the equations that can be derived are ----**
19. **Find the root of the following equation:**
$$log_2(2^{2x}2^x + 1) = x$$
20. **Find the value of x:**
log_{8} (2x +1) = 2

Chapter 28. Divisibility

A divisibility law determines whether or not a number is divisible by another number. As number an is divided by another number b, the remainder is equal to zero. As a result, the number a can be divided by b. It is a method for locating large-number variables.

Definitions:

- A divisibility rule is a quick way to see if a given number is divisible by a fixed divisor without having to divide it, normally by looking at its digits.
- A whole number is divisible by another if the remainder after division is zero.
- The second number is a factor of the first number if the entire number is divisible by another number.

28.1 Divisibility Formulas

- When we set up a division problem in an equation using our division algorithm, and $r = 0$, we have the following equation: a = bq
- When this is the case, we say that *a* is divisible by *b*. If this is a little too much technical jargon for you, don't worry! It's actually fairly simple. If a number *b* divides into a number *a* evenly, then we say that *a* is divisible by *b*.
- For example, 8 is divisible by 2, because $\frac{8}{2} = 4$.

  However, 8 is not divisible by 3, because of $\frac{8}{3} = 2$ with a remainder of 2. We can check to see if a number, *a* , is divisible by another number, *b* , by simply performing the division and checking to see if b divides into an evenly.

Divisibility Formulas & Divisibility Rules

- **Divisibility rule for 1**
  - Every number is divisible by 1.
    Example: 5 is divisible by 1
- **Divisibility rule for 2**
  - Any even number or number whose last digit is an even number (0, 2, 4, 6, 8) is divisible by
    Example: 220 is divisible by 2.
- **Divisibility rule for 3**

A number is divisible by 3 if the sum of its digits is divisible by 3.
Example: 315 is divisible by 3.
Here, 3 + 1 + 5 = 9
9 is divisible by 3. It means 315 is also divisible by 3.

- **Divisibility rule for 4**

A number is divisible by 4, if the number formed by the last two digits is divisible by 4.
Example: Example: 7568 is divisible by 4
Here, 68 is divisible by 4 (68÷4 = 17)

Therefore, 7568 is divisible by 4

- **Divisibility rule for 5**

A number is exactly divisible by 5 if it has the digits 0 or 5 at one's place.
Example: 5900, 57895, 4400, 1010 are divisible by 5.

- **Divisibility rule for 6**

A number is exactly divisible by 6 if that number is divisible by 2 and 3 both. It is because 2 and 3 are prime factors of 6. Example: 63894 is divisible by 6, the last digit is 4, so divisible by 2, and sum 6+3+8+9+4 = 30 is divisible by 3.

- **Divisibility rule for 7**

Double the last digit and subtract it from the remaining leading truncated number to check if the result is divisible by 7 until no further division is possible
Example: 1093 is divisible by 7

Remove 3 from the number and double it = 6

Remaining number is 109, now subtract 6 from 109 = 109 – 6 = 103.

Repeat the process, We have last digit as 3, double = 6

Remaining number is 10, now subtract 6 from 10 = 10 – 6 = 4.

As 4 is not divisible by 7, hence the number 1093 is not divisible by 7.

- **Divisibility rule for 8**

If the last three digits of a number are divisible by 8, then the number is completely divisible by 8.

Example: 215632 is divisible by 8, as the last three digits 632 is divisible by 8.

- **Divisibility rule for 9**

It is the same as divisibility of 3. Sum of digits in the given number must be divisible by 9.
Example: 312768 is divisible by 9, Sum of digits = 3+1+2+7+6+8 = 27 is divisible by 9.

- **Divisibility rule for 10**

Any number whose last digit is 0, is divisible by 10.
Example: 10, 60, 370, 1000, etc.

- **Divisibility rule for 11**

If the difference of the sum of alternative digits of a number is divisible by 11, then that number is divisible by 11.
Example: 737 is divisible by 11 as 7 + 7 = 14 and 14 – 3 = 11, 11 is divisible by 11.
416042 is divisible by 11 as 4 + 6 + 4 = 14 and 1 + 0+ 2 = 3, 14 – 3 = 11, 11 is divisible by 11.

- **Divisibility rule for 12**

A number is exactly divisible by 12 if that number is divisible by 3 and 4 both.
Example: 108 is divisible by 12. Sum of digit = 1 + 8 = 9, 9 is divisible by 3. And last two
digits 08 is divisible by 4. Therefore, 108 is divisible by 12.

- **Divisibility rule for 13**

Multiply the last digit with 4 and add it to the remaining number in a given number, the result must be divisible by 13.
Example: 208 is divisible by 13, 20 + (4*8) = 20 + 32 = 52, 52 is divisible by 13.

- **Divisibility rule for 14**

A number is exactly divisible by 14 if that number is divisible by 2 and 7 both. It is because 2 and 7 are prime factors of 14.
Example: 1246 is divisible by 14, as the last digit is even, so divisible by 2.
Now check for 7,

Remove 6 from the number and double it = 12

Remaining number is 124, now subtract 124 from 12 = 112.

Repeat the process, We have the last digit as 2, double = 4

The remaining number is, now subtract 11 from 4 = 7

As 7 is divisible by 7, hence the number 1246 is divisible by 7.

- **Divisibility rule for 15**

If the number is divisible by both 3 and 5, it is divisible by 15.

Example: 23505 is divisible by 15.

Check for 3: 2 + 3 + 5 + 0 +5 = 15, 15 is divisible by 3.
Check for 5: It has the 5 at one's place, therefore, divisible by 5.

- **Divisibility rule for 16**

The number formed by the last four digits in the given number must be divisible by 16.
Example: 152448 is divisible by 16 as the last four digits (2448) are divisible by 16.

- **Divisibility rule for 17**

Multiply the last digit with 5 and subtract it from remaining number in a given number, the
the result must be divisible by 17.
Example: 136 is divisible by 17. 13 - (5 * 6) = 13 - 30 = 17, 17 is divisible by 17.

- **Divisibility rule for 18**

If the number is divisible by both 2 and 9, it is divisible by 18.
Example: 92754 is divisible by 18.
Check for 2: the last digit is even, therefore, it is divisible by 2.
Check for 9: 9 + 2 + 7 + 5 + 4 = 27, 27 is divisible by 9.

- **Divisibility rule for 19**

Multiply the last digit with 2 and add it to the remaining number in a given number, the result must be divisible by 19.
Example: 285 is divisible by 19, 28 + (2 *5) = 28 + 10 = 38, 38 is divisible by 19.

- **Divisibility rule for 20**

The number formed by the last two digits in the given number must be divisible by 20.
Example: 245680 is divisible by 20.

28.2 How To Solve Divisibility Questions Quickly & Definition

- The capacity of a dividend to be exactly divided by a given number is termed as divisibility.
- One whole number is divisible by another if, after dividing, the remainder is zero.
- If the whole number is divisible by another number then the second number is a factor of the 1st number.

Formula : Divisibility by integers
Question 1:
If 5425X6 is divisible by 8, what is the least value that can be assigned to X?

Answer :
For a number being divisible by 8 the last three digits should be divisible by 8 therefore in place of X we will be replacing it by numbers from 0-9. by that method we will get 3 as the least no. as 536 is perfectly divisible by 8

Question 2:
$2^2 * 3^2 * 113$ **is divisible by ?**

Answer :
$2^2 * 3^2 * 113 = 4068$ which is divisible by 18.

Question 3:
What could be the value of x to make a number 34567x divisible by 9?

Answer :
34567x
= 3 + 4 + 5 + 6 + 7 + x
= 25+x
For the number divisible by 9 ,
x = 2 .

Formula : Type 1 : Find the largest or smallest number
Question 1:
Identify the nearest number of 16208 which is divided by 502 .

Answer :
As 16208 /502 gives 144 remainder. So rather than subtracting 144 from 16208 if we add (502–144 = 358) we will get a value closer to 16208 which is divisible by 502

when we Divide 16208 by 502, we get 144 as remainder.
502-144 = 358.

Either subtract 144 from 16208 or add 144 in 16208 to get a number divisible by 502.

The number nearest to 16208, which is divisible by 502
= 16208- 144 = 16064

Question 2:
Determine the smallest 4-digit number which is exactly divisible by 7, 11 and 13 .

Answer :
LCM of 7, 11, 13 =1001
Therefore ,
Smallest 4-digit number = 1001 .

Question 3:
Find the maximum value of n such that $15!$ **is perfectly divisible by** 2^n **.**

Answer:
15! = 1*2*3*4*5*6*7*8*9*10*11*12*13*14*15

Let us simplify this in powers of 2.

1*2*3*4*5*6*7*8*9*10*11*12*13*14*15 can be written as
=
1*2*3*(2*2)*5*(2*3)*7*(2*2*2)*9*(2*5)*11*(2*2*3)*13*(2*7)*15

Total number of 2's above are 11.

Formula : Type 2: Not Divisible
Formula : Type 3 : Remainder
Question 1:
Find the remainder when 446 is divided by 6 .

Answer:
Dividing 446 by 6 gives remainder 2 .

Hence, Remainder is 2 .

Question 2:
Find the remainder when 2567 is divided by 14.

Answer :
On dividing 2567 by 14 we will get the remainder as 5 .

Question 3:
Find the remainder when we divide $x^2 + 6x + 9$ **by** $x + 3$ **.**

Chapter 29. Set Theory

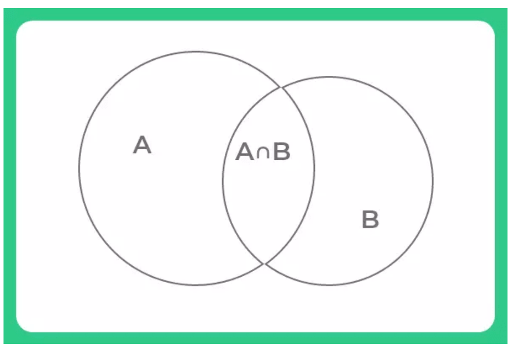
Set theory is a branch of mathematical logic concerned with the study of sets, which are loosely described as collections of elements.

Although any object can be compiled into a sequence, set theory is most often extended to objects that are mathematical in nature.

DEFINITION

Set Theory is a branch of Mathematics that deals with the properties of well- defined **collections of an object.**

In other words, its natural habit for all of us to classify similar things into groups. These groups are known as **Set**.



29.1 Formulas of Set Theory

Notation used in **set theory**
Notations used in ***set theory:***
***n(A)* -** Cardinal number of set A.
***n∘(A)* -** Cardinality of set A.

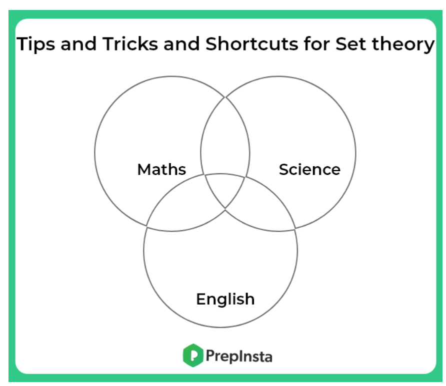
$\overline{A}$ = A^c - complement of set A.
U - Universal
A ⊂ ***B* -** Set A is proper subset of subset of B.
A ⊆ ***B* -** Set A is subset of set B.
∅ **-** Null set.
a ∈ ***A* -** element "a" belongs to set A.
A ∪ ***B* -** union of set A and set B.
A ∩ ***B* -** intersection of set A and set B.
Set Theory Formulas

1. If A and B are overlapping set, **n(A ∪ B) = n(A) + n(B) - n(A ∩ B).**
2. If A and B are disjoint set, **n(A ∪ B) = n(A) + n(B).**
3. **n(A) = n(A ∪ B) + n(A ∩ B) - n(B).**
4. **n(A ∩ B) = n(A) + n(B) - n(A ∪ B).**
5. **n(B) = n(A ∪ B) + n(A ∩ B) - n(A).**
6. **n(U) = n(A) + n(B) - n(A ∩ B) + n((A ∪ B)^c).**
7. **n((A ∪ B)^c) = n(U) + n(A ∩ B) - n(A) - n(B).**
8. **n(A ∪ B) = n(A - B) + n(B - A) + n(A ∩ B).**
9. **n(A - B) = n(A ∪ B) - n(B).**
10. **n(A - B) = n(A) - n(A ∩ B).**
11. **n(A^c) = n(U)- n(A)**

29.2 Tips and Tricks and Shortcuts for Set theory

A **set** is a collection of some items (elements). To define a set we can simply list all the elements in curly brackets,
For Ex- To define a set A that consists of the two elements x and y, we write A={x,y}.
To say that y belongs to A, we write y ∈ A, where "∈" is pronounced "belong to". To say that an element does not belong to a set, we use "∉". **For example –** B ∉ A

- The set of natural numbers N = {1,2,3……}.
  Z={⋯,−3,−2,−1,0,1,2,3,⋯}
  "> Q
  ">
- The set of integers Z={….., -3,-2,-1,0,1,2,3…..}.
- The set of Real number is **R**
- The set of Rational number is **Q**



29.3 Set Theory Tips and Tricks

Example of Subsets

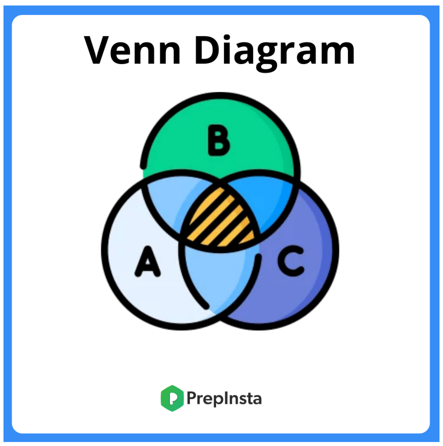
If E= {1,4} and C = {1,4,9}, then **E⊂C**

- **N⊂Z**
- **Q⊂**

Chapter 30. Venn Diagrams

Introduction:

A Venn diagram is a visual representation of common traits. Mathematical sets or words of categorical statements are expressed by overlapping circles inside a universal set boundary, such that all possible configurations of the corresponding properties are represented by different distinct areas in the image.



When a list of given sets is provided. We draw a Venn Diagram to find all possible relationships between sets, i.e. a Venn Diagram is a representation for finding all rational relationships between different sets.

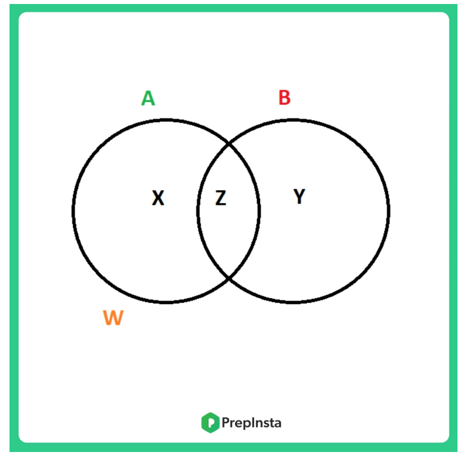
Definition and use of Venn Diagrams:-

- **Definitions: Venn diagram, also known as Euler-Venn diagram is a simple representation of sets by diagrams.**
- **Venn diagram representing mathematical or logical sets pictorially as circles or closed curves within a rectangle.**
- **The usual picture makes use of a rectangle as the universal set and circles for the sets under consideration.**

30. 1 Basic Formula for the Venn Diagram

- Some basic formulas for Venn diagrams of two and three elements.
- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$
- And so on, where n(A) = number of elements in set A.
- After understanding the concept of the venn diagram with diagrams, we don't have to remember the formulas.

30. 2 Venn Diagram for 2 sets



$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Where;

X = number of elements that belong to set A only

Y = number of elements that belong to set B only

Z = number of elements that belong to set A and B both $(A \cap B)$

W = number of elements that belong to none of the sets A or B

From the above figure, it is clear that

$n(A) = x + z$;

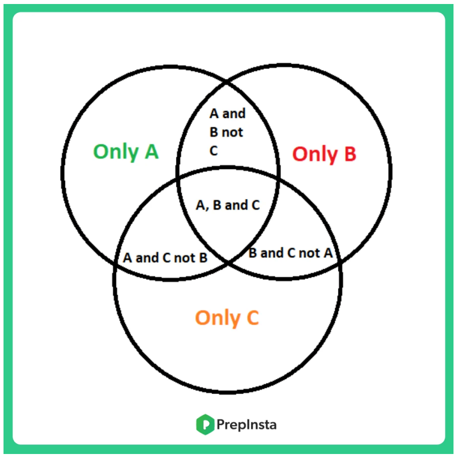
$n(B) = y + z$;

$n(A \cap B) = z$;

$n(A \cup B) = x + y + z$.

Total number of elements = x + y + z + w

30.3 Venn Diagram for 3 sets



n (A ∪ B ∪ C) = n(A) + n (B) + n (C) - n (A ∩ B) - n (B ∩ C) - n (C ∩ A) + n (A ∩ B ∩ C)

Where, W = number of elements that belong to none of the sets A, B or C

Chapter 31. Discounts

Introduction:

The term "discount" refers to a reduction in the normal sale price of a product by a certain number or percentage. If the item is broken, we might ask the manager for a discount.

The term "discount" refers to the price of a bond that is less than its face value. Formula for Successive discount is when the first discount is x% and 2nd discount is y% then,

Total discount = $(x + y - \frac{xy}{100})$**%**

31.1 Successive Discounts Formulas & Definitions

- Successive discount is discount on discount.
- Successive Discount is the reduction in price of a Goods or Service .
- Successive Discount is the discount applied on other discounts .

Case 1: If there are two discounts:

The formula for total discount in case of successive-discounts:

If the first discount is x% and 2nd discount is y% then,

Total discount = (x + y - $\frac{xy}{100}$)%

Case 2: If there are three discounts:
If there are three discounts as x%, y% and z% then find the total discount of x % and y% first and using it find the total discount with z%

1. Marked price is the price marked on the product. It is the same price on which you get discounts.
2. They revolve around Profit & Loss, Selling Price & Marked Price.

31.2 Formulas for Successive Discounts

- Single discount, which is equal to two successive discounts m % and n % is calculated as M + N - $\frac{\mathbf{MN}}{\mathbf{100}}$
- When the SP of x articles is equal to CP of y articles, then earned profit in percentage is calculated as Profit percent = $\frac{\text{difference in x and y}}{x}$ × 100
- Discount = D% of marked price, M
- Discount = Marked Price – Selling Price
- M (1-D%) = Selling Price
- Selling Price = Cost Price + Gain

But in exam,you can do it directly in your head.So just think that 10 percent discount means you've to pay 100 percent minus 10 percent=90 percent of the marked price which means,

$$(90100\frac{90}{100}10090) * 1000 = Rs.900$$

Chapter 32. Inverse

Inverse Formulas and Definitions for Inverse:-

Inverse functions give lots of troubles so here's a swift run down of what an inverse function is, and how to find it.

- For a function to have an inverse, the function has to be 1 to 1. That means every output only has one input.

Inverse functions give lots of troubles so here's a quick rundown of what an inverse function is, and how to find it.

For a function to have an inverse, the function has to be 1 to 1. That means every output only has one input.

Inverse questions are basically of 2 types.

- Algebraic Questions
- Geometric Questions

32.1 Inverse Formulas Example-

f(x) = 3x -2 and g(x) = $\frac{x}{3}$ + $\frac{2}{3}$

(f∘g)(x)= (g∘f)(x) = x

- The first case is really,

(g∘f)(−1)=g[f(−1)]=g[−5]=−1

- The second case is really,

(f∘g)(2) = f(2) = f[g(2)] = f [$\frac{4}{3}$ = 2

- A function is called one-to-one if no two values of x produce the same y. Mathematically this is the same as saying $f(x_1) \neq f(x2) whenever x1$\neq $x2$
- Inverse functions are one to one functions f(x) and g(x) if
(f∘g)(x) = x **AND** (g∘f)(x) = x, then we can say f(x) and g(x) are inverse to each other.
- g(x) is inverse of f(x) and denoted by
g(x)= f^{-1} (x)

- Likewise f(x) is the inverse of g(x) are denoted by **$f(x)= g^{-1}(x)$**

For the two functions that we started off this section with we could write either of the following two sets of notation.

$f(x) = 3x - 2$ $\quad$ **$f^{-1}(x) =$** $\frac{x}{3} + \frac{2}{3}$

$g(x) =$ $\frac{x}{3} + \frac{2}{3}$ $\quad$ **$g^{-1}(x) = 3x-2$**

32.2 Tips and Tricks and Shortcuts for Inverse:-

As clear by name Inverse means the opposite in position, directions, etc.

In mathematical language, it is defined as a reciprocal quantity.

In this page, we will discuss two types of INVERSE

- Trigonometric Inverse
- Algebraic Inverse

32.3 Trigonometric Inverse Tips and Tricks and Shortcuts:

They are the inverses of the sine, cosine, tangent, cotangent, secant, and cosecant functions.

Property of trigonometry inverse functions

PROPERTY 1

a) $\sin^{-1}(\frac{1}{x}) = \text{cosec}^{-1} x, x \geq 1$ or $x \leq -1$

b) $\cos^{-1}(\frac{1}{x}) = \sec^{-1} x, x \geq 1$ or $x \leq -1$

c) $\tan^{-1}(\frac{1}{x}) = \cot^{-1} x, x > 0$

PROPERTY 2

a) $\sin^{-1}(-x) = -\sin^{-1}(x), x \in (-1,1)$

b) $\tan^{-1}(-x) = \tan^{-1}(x), x \in R$

c) $\text{cosec}^{-1}(-x) = \text{cosec}^{-1}(x), |x| \geq 1$

PROPERTY 3

a) $\cos^{-1}(-x) = \pi - \cos^{-1} x, X \in |-1,1|$

b) $\sec^{-1}(-x) = \pi - \sec^{-1} x, |x| \geq 1$

c) $\cot^{-1}(-x) = \pi - \cot^{-1} x, X \in R$

PROPERTY 4

a) $\sin^{-1} x + \cos^{-1} x \; \frac{\$}{2}, X \in |-1,1|$

b) $\tan^{-1} x + \cot^{-1} x = \frac{\$}{2}, X \in R$

c) $\text{cosec}^{-1} + \sec^{-1} x = \frac{\$}{2}, |x| \geq 1$

PROPERTY 5

a) $\tan^{-1} x + \tan^{-1} Y = \tan^{-1}(\frac{(x+y)}{(1-xy)}), xy < 1$

b) $\tan^{-1} x - \tan^{-1} y = \tan^{-1}(\frac{(x-y)}{(1=xy)}), xy > -1$

PROPERTY 6

a) $2\tan^{-1} x = \sin^{-1}(\frac{(2x)}{(1+x^2)})), |x| \leq 1$

b) $2\tan^{-1} x = \cos^{-1}(\frac{(1-2)}{(1+x^2)})), x \geq 1$

c) $2\tan^{-1} x = \tan^{-1}(\frac{(2x)}{(1-x^2)})), |x| \leq 1$

32.4 Algebraic Inverse Tips and Tricks and Shortcuts:

Inverse is a reverse of any quantity.

Addition is the opposite of subtraction; division is the opposite of multiplication, and so on.

For Example-

If, **f** is the inverse of y,

Then, the inverse of f(x)= 2x+3 can be written as,

$f^{-1}(y) = \frac{(y-3)}{2}$

Chapter 33: Allegation and Mixture

In this chapter, we will go into the in-depth definition of mixture and alligation, as well as some main formulas for answering questions about it.
We will also go through some pointers that will help to simplify the solution and clarify them with the help of some sample instances.

Allegation
"Definition - Allegation is the rule which enables us
(i) to find the mean or average value of mixture when the prices of two or more ingredients which may be mixed together and the proportion in which they are mixed are given; and
(ii) to find the proportion in which the ingredients at given prices must be mixed to produce a mixture at a given price."

Mixture
"Definition - Combining two or more items and deciding any feature of either the components or the resultant combination is what a mixture dilemma involves. For example, we might like to know how much water to add to dilute a saline solution, or how much concentrate is in a jug of orange juice."

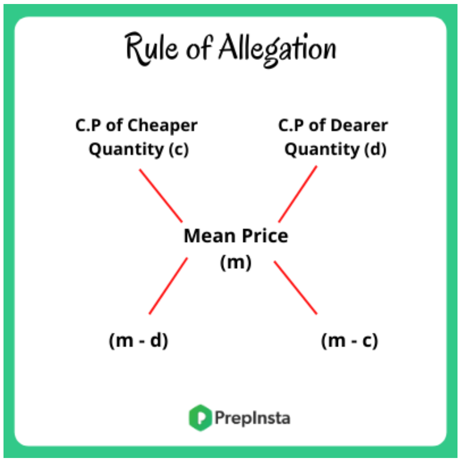
33.1 - Formulae for Allegation and Mixtures

To answer every numerical skill query, candidates must be familiar with a series of formulas for each subject, which makes answering the questions simpler and saves time. Here are a few calculations to assist candidates with this and make it easier for them to solve mixture and alligation questions:

Type - 1
The basic formula which is used to find the ratio in which the ingredients are mixed is

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{C.P or Dearer - Mean price}}{\text{Mean Price - C.P of Cheaper}}$$

This is also known as **Rule of Allegation** and can also be represented as



The above picture is also known as the Butterfly method.

Then, (cheaper quantity) : (dearer quantity) = (d - m) : (m - c)

Type 2:

This type is also known as a method of repeated dilutions. In this type consider a jar containing x units of liquid A, from which y units are removed and filled with water. If this operation is repeated n times, the amount of pure liquid will be given by the formula:

$$[x(1-\frac{y}{n})^n]$$

33.2 - How to Solve Mixture and Allegations

Type 1:

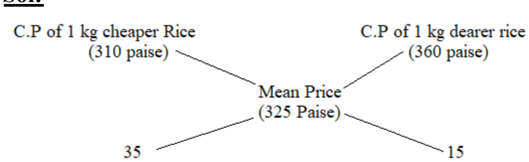
Questions below have been solved on the basis of Rule of allegation which is

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{C.P or Dearer - Mean price}}{\text{Mean Price - C.P of Cheaper}}$$

Example 1:

In what portion must rice at Rs 3.10 per kg be mixed with rice at Rs 3.60 per kg, so that the mixture be worth Rs 3.25 a kg?

Sol:



By the allegation rule:

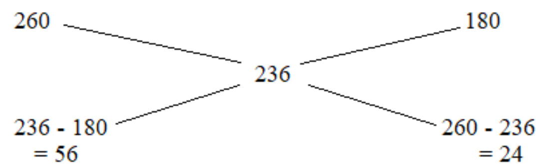
$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{35}{15} = \frac{7}{3}$$

Therefore, They must be mixed in the ratio 7:3

Example 2:

Sum of Rs.118 was shared among 50 boys and girls,each girl received Rs.2.60 and boy received Rs.1.80.Find the number of girls.

Sol:



56 : 24 = 7 : 3

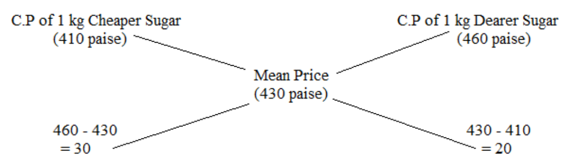
No. of Girls = $\frac{7}{10} \times 50 = 35$

Therefore, total number of girls is 35

Example 3:

In what proportion must sugar at Price 4.10 per kg be mixed with weat at Price 4.60 per kg, so that the mixture be worth Rs 4.30 a Kg ?

Sol:



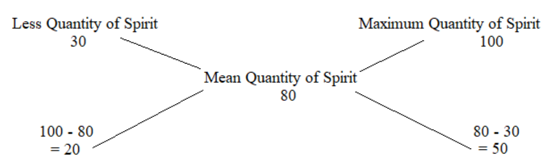
30: 20 = 3: 2

Therefore, both the sugars must be mixed in the ratio of 3:2

Example 4:

400 gm spirit solution has 30 % spirit in it , what is the ratio of spirit should be added to make it 80 % in the solution ?

Sol:



All quantity given in the above Diagram is in percentage

Hence the required ratio is 20 : 50 = 2 : 5

Therefore to make the ratio 80% in the solution the mixture should be in the ratio 2:5.

Example 5:

How much coffee of variety A, costing Rs. 5 a kg should be added to 20 kg of Type B coffee at Rs. 12 a kg so that the cost of the two coffee variety mixture be worth Rs. 7 a kg?

Sol: As per the rule of alligation,

Quantity of Dearer: Quantity of Cheaper = (12-7) : (7-5) = 5:2

Quantity of Variety A coffee that needs to be mixed ⇒ 5:2 = x:20

⇒ x =50 kg

Type 2:

Questions below have been solved on the basis of the following formula which is Method Of the Repeated Dilutions

$$[x(1-\frac{y}{n})^n]$$

Example 1:

A container contains 40 litres of milk. From this container, 4 litres of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by the container?

Sol:

The container contains x = 40 litres of milk.

The quantity of milk that is taken out and replaced by water = y = 4 litres.

Also, we have been given that number of times the process is repeated or

n = 3 (=1 +2 times).

Therefore from the method of repeated dilution, substituting the relevant values, we have:

Amount of milk left after three operations =

$[40(1-\frac{4}{40})^3]$ litres. Therefore we may write:

$[40 \times (\frac{9}{10}) \times (\frac{9}{10}) \times (\frac{9}{10})]$ = 29.16 litres.

Therefore, 29.16 liters of milk is now contained by the container.

Example 2:

A can contains a mixture of two liquids A and B in the ratio 7: 5. When 9 litres of the mixture is drawn off and the can is filled with B, the ratio of A and B becomes 7: 9. How many litres of liquid A was contained by the can initially?

Sol:

Suppose the can initially contain 7x and 5x litres of mixtures A and B respectively. The quantity of A in the mixture left =

$$[7x-(\frac{7}{12})\times 9] \text{ liters } = [7x-\frac{21}{4}] \text{ litres }.$$

The quantity of B in mixture left =

$$[5x-(\frac{5}{12})\times 9] \text{ litres } = [5x-\frac{15}{4}] \text{ litres}$$

.

Therefore we can write: the ratio of the two quantities as

$$\frac{[7x-\frac{21}{4}]}{[5x-\frac{15}{4}]=\frac{7}{9}}$$

In other words, we may say that 252x – 189 = 140x + 147

Hence, x = 3. Therefore the can had 7(3) = 21 litres of the quantity A.

Thus the correct answer is 21 liters.

33.3 - Tips and Tricks used for solving Mixture and Allegations

Type - 1: Rule of Allegation

It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of the desired price.

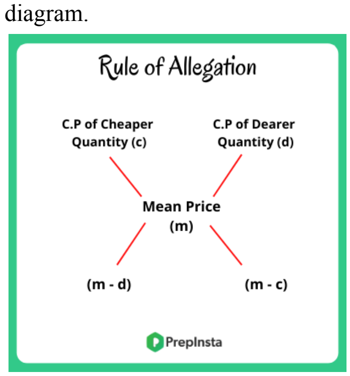
To find that , we have the best trick for you.

When two commodities are mixed then ,

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{C.P or Dearer - Mean price}}{\text{Mean Price - C.P of Cheaper}}$$

Also, can be written as : (cheaper quantity) : (dearer quantity) = (d - m) : (m - c)

This equation can also be developed using the pictorial diagram.



Type 2: Method of Repeated Dilutions

In this we will have to calculate quantity of pure Liquid after 'n' successive operations:

In other words **for example:** If a Container contains 'x' units of pure liquid , and we replace the liquid with 'y' units of water :

Then after 'n' successive operations, the units of pure liquid left is given by the following formula:

$$[x(1-\frac{y}{n})^n]$$

Exercise

Question 1:

A Man covered a distance of 50km in 5 hrs partly by bus at 12kmph and partly by foot at 7kmph.The distance covered by bus is?

Question 2:

How many Kg of sugar costing Rs. 10kg must be mixed with 36Kg of sugar costing Rs.9 per Kg ,so that there may be gain of 11% by selling the mixture of Rs.10.30 per Kg?

Question 3:

A jar contains 'x' liters of Milk, a seller withdraws 25 liter of it and sells it at Rs.20 per liter. He then replaces it with water. He repeated the process a total three times. Every time while selling he reduces the selling price by Rs.2. After this process Milk left in the mixture is only 108 liters so he decided to sell the entire Mixture at Rs. 15 per liter. Then how much profit did he earn if bought Milk at Rs.20 per liter?

Question 4:

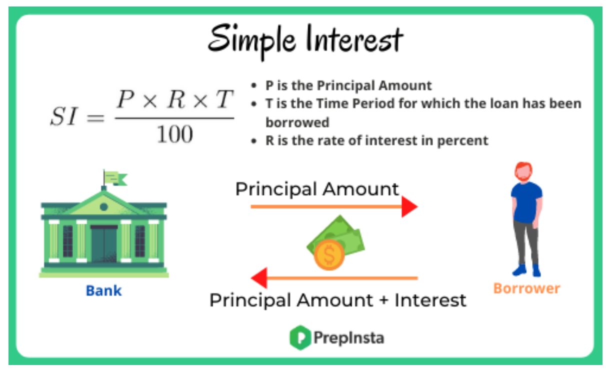
'X' Liters of the mixture contain Milk and Water in the ratio 4:3. If 13 liters of Water is added then the ratio becomes 1:1. Then what is the final quantity of water in the mixture?

Question 5:

A Jar contains a mixture of Milk and Water 18 and 12 Liters respectively. When the 'x' liter of the mixture is taken out and replaced with the same quantity of Water, then the ratio of Milk and Water becomes 2:3. Then what is the quantity of Water in the final Mixture?

Chapter 34: Simple Interest

In the following section, we will describe the key words and formulae that will assist us in solving and comprehending the basic interest questions. We will describe the principle of simple interest and extend these formulae and meanings to address questions that we anticipate will emerge from this section. Let's start with definitions!



Principal Amount or Principal (P):

"Definition - Money borrowed by a borrower or the money lent by a lender is called the principal (P)."

Time Period:

"Definition - The time for which it is borrowed or lent is called time period (T)."

Interest:

"Definition - The extra money paid by the borrower to the lender is called the interest. Interest is defined as rate (R) per cent per annum (p.c.p.a.)."

Simple interest:

"Definition - When interest is calculated on the original principle for any length of time then it is defined as simple interest."

34.1 - Formulae related to Simple Interest

Simple Interest:

Formula to find simple interest as the name suggests is very simple. The formula to find simple interest is as follows:

$$SI = \frac{P \times R \times T}{100}$$

Where P is the Principal Amount

T is the Time Period for which the loan has been borrowed

R is the rate of interest in percent

Total Amount of Money:
Total amount of money is calculated as sum of Interest and principal amount, which is given by
A = P + I
Where A is the total Amount of money
P is the Principal Amount borrowed
I is the interest paid on that amount

Principle:
Money borrowed by a borrower or the money lent by a lender is called the principal (P). The formula to find principle is given by:

$$P = \frac{100 \times I}{T \times R}$$

Where I is the Interest paid
T is the time period and
R is the rate of interest.

Similarly for

Rate Percent:

$$R = \frac{100 \times I}{P \times T}$$

Time:

$$T = \frac{100 \times I}{P \times R}$$

34.2 - How to Solve Questions of S.I?

Type 1: Questions on Simple Interest
In this type you will be able to understand that how a problem for finding simple interest is solved by using the formula

$$SI = \frac{P \times R \times T}{100}$$

Example:
Rishav takes a loan of Rs 10000 from a bank for a period of 1 year. The rate of interest is 10% per annum. Find the interest and the amount he has to the pay at the end of a year.
Sol: Here, the loan sum = P = Rs 10000
Rate of interest per year = R = 10%
Time for which it is borrowed = T = 1 year

Thus, simple interest for a year,

$$SI = \frac{(P \times R \times T)}{100} = \frac{(1000 \times 100 \times 1)}{100} = Rs1000$$

Amount that Rishav has to pay to the bank at the end of the year = Principal + Interest = 10000 + 1000 = Rs 11,000

Type 2: When Rates are different for different years
In this particular type you will get to understand how to solve the problems when two different R.O.I s are given for different years.

Example:
Nisha borrowed some money at the rate of 5% p.a. for the first two years. She again borrowed at the rate of 10% p.a. for the next three years. Later at the rate of 15% p.a. for the rest of the years. Total interest paid by her was Rs. 15000 at the end of 10 years. Calculate the amount of money she borrowed?
Sol: According to the question,
r1 = 5% , T1 = 2 years
r2 = 10% , T2 = 3 years
r3 = 15% , T3 = 5 years
(since, beyond 5 years rate is 14%)
Simple interest = 15000

Therefore, P = $\frac{1500 * 100}{5 * 2 + 10 * 3 + 15 * 4}$

= $\frac{1500000}{10 + 30 + 60}$

= $\frac{1500000}{100}$

= Rs. 15000

Type 3: When you have to find Rate, Time or Principal
In this type of questions with the help of solved examples you will be able to understand that how the questions in which you have to find R, T or P when either two are given is found out using the following formulae:

$$P = \frac{100 \times I}{T \times R},$$

$$R = \frac{100 \times I}{P \times T},$$

$$T = \frac{100 \times I}{P \times R}$$

Example 1:
A sum of money becomes six times in 30 years. Calculate the rate of interest.

Sol: We know that, if sum of money becomes x times in n years at some rate of interest, then rate of interest is calculated as,

$$R = 100(\frac{x-1}{n})\%$$

$$R = 100(\frac{6-1}{30})$$

$$R = 100([\frac{5}{30})$$

$$R = \frac{500}{30}$$

R = 16.66%

Example 2:

How much time will it take for an amount of Rs. 630 to yield Rs. 72 as interest at 5.4 % p.a. of simple interest?

Sol: We know that,

Time = $\frac{100 \times SI}{R \times P}$

T = $\frac{100 \times 72}{630 \times 5.4}$

T = $\frac{7200}{3402} = \frac{7200}{3402}$

T = 2 years and 11 months

Example 3:

Mr. Tata invested Rs. 13,900 in two different schemes I and II. The rate of interest for both the schemes were 14% and 11% p.a. respectively. If the total amount of simple interest earned in 2 years be Rs. 3508, what was the amount invested in Scheme II?

Sol: Let the amount invested in schemes I = x

Therefore, in schemes II = 13900 – x

SI (scheme I) = $\frac{P \times R \times T}{100}$

Then For scheme I,

SI = $\frac{x \times 14 \times 2}{100}$

Then For scheme II,

SI (scheme II) = $\frac{13900 - x \times 14 \times 2}{100}$

SI (scheme I) + SI (scheme II) = 3508

$$\frac{(x * 14 * 2)}{100} + \frac{(13900 - x \times 14 \times 2)}{100} = 3508$$

$$\frac{28x}{100} + \frac{(13900x) * 22}{100} = 3508$$

6x = 45000

x = 45000/6

x = 7500

Hence, the sum invested in Scheme II = 13900 – 7500 = Rs. 6400

34.3 - Tips and Tricks for Questions on S.I

Basic tips and tricks points:

- If the interest on a sum of money is $\frac{1}{x}$ of the principal, and the number of years is equal to the rate of interest then rate can be calculated using the formula: $\sqrt{\frac{100}{x}}$
- The rate of interest for t_1 years is r_1%, t_2 years is r_1%, t_3 years is r_3 %. If a man gets interest of Rs x for (t_1+ t_2 + t_3 = n) years, then principal is given by: $\frac{x * 100}{r_1 t_1}$ **+ r_2t_2 + r_3t_3**
- If sum of money becomes x times in t years at simple interest, then the rate is calculated as $R = \frac{100(x-1)}{t}\%$
- If a sum of money becomes x times in t years at simple rate of interest, then the time is calculated as $t = \frac{100(x-1)}{R}$
- If an amount P_1 is lent out at simple interest of R_1% p.a. and another amount of P_2 at simple interest of R_2% p.a, then the rate of interest of the whole sum is given by: $R = \frac{P_1R_1 + P_2R_2}{P_1 + P_2}$

34.4 - Exercise

Question - 1:

Find the simple interest on Rs 3000 at 25/4% per annum for the period from 4th Feb, 2005 to 18th April, 2005.

Question - 2:

A sum of Rs, 800 amounts to Rs. 920 in 3 years at simple interest. If the interest rate is increased by 3%, it would amount to how much?

Question - 3:

Geeta borrowed some money at the rate of 6% p.a for the first two years, at the rate of 9% p.a for the next three years, and at the rate of 14% p.a for the period beyond five years. If she pays a total interest of Rs.11400 at the end of nine years, how much did she borrow?

Question - 4:

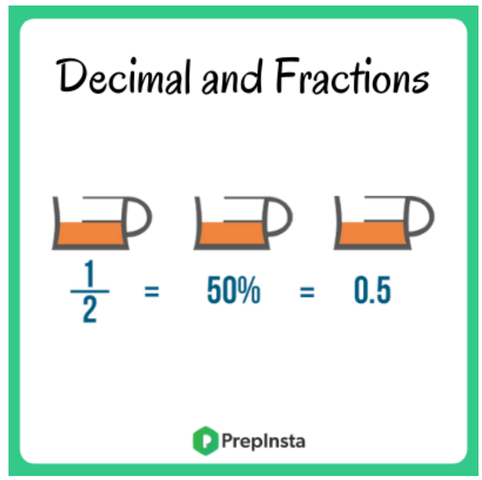
Kamla took a loan of Rs. 2400 with simple interest for as many years as the rate of interest. If she paid Rs. 864 as interest at the end of the loan period, what was the rate of interest?

Question - 5:

In how many years will a sum of money triple itself in 24% per annum ?

Chapter 35: Decimals and Fractions

Decimals and fractions have long been a source of consternation for pupils. Many people just do not appreciate the utility and importance of our research. But, like anything else you learn, it has a need and value of its own. So, before we get into decimals, you should know why they were invented in the first place. Let's get started.



Decimals:
"Definition - Decimals are numbers that have a whole number and a fractional component divided by a decimal point. The decimal point is the dot that appears between the entire number and the fractions section. 34.5, for example, is a decimal integer."

Fractions:
"Definition - Fractions are numbers that represent a part of the whole. When an object or a group of objects is divided into equal parts, then each individual part is a fraction. A fraction is usually written as 1/2 or 5/12 or 7/18 and so on. It is divided into a numerator and denominator where the denominator represents the total number of equal parts into which the whole is divided. The numerator is the number of equal parts that are taken out. For e.g. in the fraction 3/4, 3 is the numerator and 4 is the denominator."

35.1 - Formulae for Decimals and Fractions

Decimals:
There are two types of decimal

- **Non-recurring decimal or terminating decimal**: A non-recurring decimal is a non-repeating decimal. It is terminated at some point. For example: $\frac{1}{5}$ = 0.5, $\frac{125}{4}$ = 31.25
- **Recurring decimal or non-terminating decimal**: A recurring decimal is a repeating decimal. For example: $\frac{19}{6}$ = 3.1666666, $\frac{21}{9}$ = 2.33333

Fractions:

There are two types of fraction

- **Common fraction:** A common fraction is a fraction in which both numerator and denominator are integers. For example, $\frac{2}{7}$, $\frac{3}{2}$….etc.
- **Decimal fraction:** A decimal fraction is a fraction in which denominator is an integer power of ten. For example, $\frac{1}{10}$, $\frac{4}{100}$…etc.

Proper Fraction : **A proper fraction is a fraction in which the numerator is less than the denominator.Proper Fraction = Numerator < Denominator**
Example : 1/2, 6/7, 15/200

Improper fraction : **An Improper fraction is a fraction in which the numerator is greater/larger than the denominator.**
Improper Fraction = Numerator > Denominator
Example : 3/2, 60/7, 150/2

Mixed fraction : **A mixed fraction is a mixture of whole number and fraction. A mixed fraction is always greater than 1.**
Mixed Fraction = Whole number + Proper Fraction
Example : 1 3/4, 2 3/4, 1 50/260

Like fraction :

Like fractions are fractions which have the exactly same denominators.
Example : 1/5, 3/5, 2/5, 4/5, 6/5

Unlike fraction :

Unlike fractions are fractions which have different denominators.
Example : 2/5, 1/6, 2/3, 7/7, 8/9

Equivalent fraction :

Equivalent fractions are the fractions in which numerator and denominators have different values but are equal to the same value.

Example : 2/12, 3/18, 4/24, 5/30

35.2 - How to Solve Questions related to Decimals and Fractions

Every Decimal number can also be written as Fraction number . And every Fraction number can also be written as a Decimal number.

Type 1: Find Greatest or Smallest Fraction

As the name suggests in this particular type you will be learning how to calculate a greater fraction between the given lot.

Example 1:

Which among $\frac{5}{7}$ and $\frac{9}{2}$ is greater?

Sol: $\frac{5}{7}$ = 0.71

$\frac{9}{2}$ = 4.5

The decimal values of the above fractions clearly indicate that the value of $\frac{9}{2}$ is more than $\frac{5}{7}$.

Therefore, $\frac{9}{2}$ is the greater number among the two.

Example 2:

Which among the given numbers is greater $\frac{5}{8}$ or $\frac{1}{2}$?

Sol: $\frac{5}{8}$ = 0.62

$\frac{1}{2}$ = 0.5

The decimal value of $\frac{5}{8}$ is greater than that of $\frac{1}{2}$.

Therefore, it is greater than $\frac{1}{2}$.

Example 3: Which among $\frac{4}{7}$ and $\frac{9}{7}$ is greater?

Sol: $\frac{4}{7}$ = 0.57

$\frac{9}{7}$ = 1.28

Since the decimal value of $\frac{9}{7}$ is greater, it is the greater number between the given two numbers.

Type 2: Simplification Questions of Decimals and Fractions

In this particular type with the help of solved examples you can simplify the questions related to both decimal and fractions.

Example 1: Simplify and calculate the result of 56.25 + 36.56+ 12.45.
Sol: The result can be calculated simply by adding the given numbers before and after the decimal sign.
Therefore, 56.25 + 36.56+ 12.45 = 105.26

Example 2: Simplify and calculate the result of 300.45 + 1223.89+ 1210.12.
Sol: The result can be calculated simply by adding the given numbers before and after the decimal sign.
Therefore, 300.45 + 1223.89+ 1210.12= 2734.46

Example 3: Simplify for the following problem

$$12\frac{1}{3} \text{ of } 321-? = 18.5 \times 14$$

Sol: $12\frac{1}{3} \text{ of } 321-? = 18.5 \times 14$

(37/3) of 321 – ? = 37 × 7
? = 37 × 107 – 37 × 7 = 37 × 100
? = 3700

35.3 - Tips and Tricks of Decimals and Fractions

Here are quick and easy tips and tricks for you to solve Decimals and Fractions questions quickly, easily, and efficiently in competitive exams.

Convert Decimal to Fraction:

- Step 1: Write down the decimal divided by 1, in the form $\frac{decimal}{1}$.
- Step 2: Multiply both top and bottom by 10 for every number after the decimal point. (For example, if there are two numbers after the decimal point, then use 100, if there are three then use 1000)
- Step 3: Simplify the fraction

For example, Convert 0.75 to a fraction.

- Step 1: $\frac{0.75}{1}$
- Step 2: Multiply both top and bottom by 100 as there are two digits after the decimal point. We get $\frac{75}{100}$
- Step 3: $\frac{75}{100} = \frac{3}{4}$

Exercise

Simplify the following questions.

Question 1: 15/21 of 45/3 – 1/15 ÷ 3/7

Question 2: 11/5 + 4/21 of 42 - 18/5 ÷ 27/45

Question 3: (6.4 × 6.4 - 3.3 × 3.3)/(6.4 + 3.3)

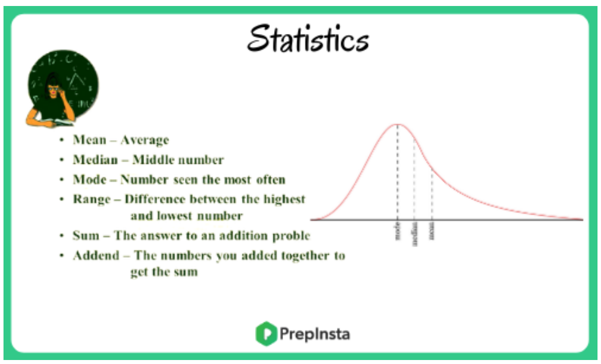
Question 4: 6.4 ÷ 1.6 of 5 + 1.3 × 3.1 - 0.07

Question 5: 27.08 - [5.6 + 3 of (6.5 - 0.5 × 2.01)]

Chapter 36: Statistics

Statistics can be seen all over you. Did you know that numbers simplify and enliven so many aspects of your everyday life? Without data, you wouldn't have all the stats on your favorite football star, you wouldn't be able to watch your exam success, you wouldn't have weather predictions, and so on.

Statistics is both a science and an art that involves processing, arranging, displaying, and predicting numerical data in order to make an educated decision. So, let's get started with those exciting numbers ideas.



36.1 - Formulae and Definitions important for Statistics

There are many formulae which are present in statistics but here we are going to tell you the most important ones with respect to most exams point of view.

Mean:

Mean is equal to the sum of all the values of a collection of data divided by the number of values in the data. In simpler terms, you add up all the values in a data set and divide it by the number of values you added. This can be found out by using a very simple formula:

$$\bar{x} = \frac{\sum x_i}{N}$$

$$x_i = \text{terms given}$$

N=Total number of terms

Median:

The median is the value in the center of a data set. That is the value that distinguishes the data set's upper and lower halves. It is referred to as the "middle value" of any given population. To measure the Median, organize the data in ascending or descending order, and take the middle number from that arrangement. It is calculated by the given formula:

$$M = \left(\frac{n+1}{(2)}\right)^{th} \textbf{ term}$$

To measure the Median of an odd number of terms, organize the data in ascending or descending order, and the Median is the middlemost expression. To measure the Median of an even number of terms, arrange the data in ascending or descending order and average the two middle terms. It is calculated by the given formula:

$$M = \left[\left(\frac{n}{2}+1\right)^{th} + \left(\frac{n}{2}\right)^{th}\right] \div 2$$

Mode:

The mode is the value that appears the most often in a series of observations. Simply put, it is the number that is most often replicated, i.e. the number with the greatest frequency. It is an essential part in the field of statistics for interpreting data in a relevant manner.

Example: Let us find the Mode of the following data 4, 89, 65, 11, 54, 11, 90, 56
Sol: Here in these varied observations the most occurring number is 11, hence the Mode = 11

Mean Deviation:

The mean deviation is a mathematical measure used to quantify the average deviation from the mean value of a given data set. The mean variance of the data values can be conveniently determined using the formula given below:

$$\frac{\sum |x_i - M|}{N}$$

Here,
Σ represents the addition of values
X represents each value in the data set
M represents the mean value of the data set
N represents the number of data values

Standard Deviation:

Standard deviation is the most important tool for dispersion measurement in a distribution. Technically, the standard deviation is the square root of the arithmetic mean of the squares of deviations of observations from their mean value. It is generally denoted by sigma i.e. **σ.** It is given by the following formula:

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N-1}}$$

Variance:

The variance is an indicator of the degree of uncertainty. It is determined by averaging the squared deviations from the norm. The degree of distribution in your data set is implied by variance. It is generally denoted by sigma i.e. **σ^2** and is given by the following formula:

$$\sigma = \frac{\sum (x_i - \bar{x})^2}{N-1}$$

36.2 - How to solve Statistics Questions?

There are few things to keep in mind while solving statistics problem:

- **Stay focused and don't be panic:**

When students get the question usually they need to be more focused and calm at the same time.This will help them to make a right approach to solve the question.

- **Analyze the Problem:**

Instead of attempting the question immediately give some time on the question.Analyze the question properly and then attempt the question.This will eliminate the chances of making mistakes.

- **Go for the right strategy:**

If you are going for the right strategy your chances for making mistakes will be eliminated and time taken to solve the problem will be minimized so it will allow you to solve the problem within a limited time.

Type 1: To find the mean of the given ungrouped data.

In this following type you will learn to find mean of a given ungrouped data by using the formula $\bar{x} = \frac{\sum x_i}{N}$

Example:

Find the mean of the following set of integers.
8, 11, –6, 22, –3

Sol:

$$\text{Mean} = \frac{8+11-6+22-3}{5} = 6.4$$

Type 2: To find the mean of the given grouped data.

In this following type you will learn to find mean of a given ungrouped data by using the formula

$$x_{group} = \frac{\sum (f_i \times x_i)}{\sum f_i}$$

Example: Find the mean of the following frequency distribution by a suitable method.

C.I.	50-70	70-90	90-110	110-130	130-150	150-170
Frequency	8	12	13	27	18	22

Sol:
By using direct method we are going find mean :

C.I	Frequency fi	Class−mark xi	Frequency x Class Mark fixi
50−70	8	60	480
70−90	12	80	960
90−110	13	100	1300
110−130	27	120	3240
130−150	18	140	2520
150−170	22	160	3520
Total	∑fi=100		∑fixi=12020

$$x_{group} = \frac{\sum (f_i \times x_i)}{\sum f_i} = \frac{12020}{100}$$

Therefore, Mean is 120.2

Type 3: Median of the Ungrouped Data

In this type you will learn how to find the median of the given ungrouped data. As the definition of the median says that “Median is the middlemost expression”. So taking the account of even ungrouped data.

Example:

Find the median of the following data:
3,1,5,6,3,4,5,6

Sol: Given the data 3,1,5,6,3,4,5,6.
Now arranging the given data in ascending order we get,
1,3,3,4,5,5,6,6.
Number of entries is equal to 8.

So median will be $\frac{8}{2}$ term i.e. 4 th term.
And it is 4. So the median is 4.

So taking the account of even ungrouped data.

Example:

Find the median of the following data:
2,5,6,3,8,9,10,1,4

Sol: Given data: 2,5,6,3,8,9,10,1,4
Now arranging the given data in ascending order we get,
1,2,3,4,5,6,8,9,10
Number of entries is equal to 9.

So median will be $\frac{9+1}{2}$ term i.e 5th term
And the 5th term is 5. So the median is 5.

Type 4: Median of the Grouped data

In this type with the help of the solved example you will be able to understand how the median of the grouped data is actually calculated.

Example: At a shooting competition, the scores of a competitor were as given below : What is his median score?

Score	0	1	2	3	4	5
No. of shots	0	3	6	4	7	5

Sol:
We write the marks in the cumulative frequency table.

Score (x)	No. of shots (f)	fx	Cumulative frequency
0	0	0	0
1	3	3	3
2	6	12	9
3	4	12	13
4	7	28	20
5	5	25	25
Total	Σf=25	Σfx=80	

Here the number of observations, n=25 which is odd.

$$\text{Median} = \left(\frac{(n+1)}{2}\right) \text{th term}$$

$= \frac{(25+1)}{2}$

$= \frac{26}{2}$

=13th term
=3
Hence his median score is 3.

Type 5: Mode of the given Ungrouped Data

In this type you will get to know how mode in an ungrouped data is calculated. As you know that mode is the most repeated entity in the given data.

Example: The following table represents the number of wickets taken by a bowler in 10 matches. Find the mode of the given set of data.

Age	10	11	12	14	15	16	18	20
Number of boys	3	6	4	7	12	9	8	11

Sol: Mode = Mode is the value which occurs the most frequently.
Since the number of boys of age group 15 is maximum .
So, the modal age of boys is 15 .

Type 6: Question on Variance and Standard Deviation

In this particular type you will try to learn to solve questions related to the variance and standard deviation by using the following formulae:

Variance: $$\sigma = \frac{\sum(x_i - \bar{x})^2}{N-1}$$

Standard Deviations: $$\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{N-1}}$$

Example: **Find the variance and standard deviation of the provided data**.

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	27	10	7	5	4	2

Sol:

Class Interval	Frequency (fx)	Mid Value x_i	fx_i	fx_i^2
0-10	27	5	135	675
10-20	10	15	150	2250
20-30	7	25	175	4375
30-40	5	35	175	6125
40-50	4	45	180	8100
50-60	2	55	110	6050
Total	$\sum f =$		$\sum fx_i =$	$\sum fx_i^2 =$

$$N = \sum f = 55$$

$$Mean = \frac{\sum fx_i}{N}$$

$$= \frac{925}{55} = 16.818$$

Variance =
$$\frac{1}{N-1}[\sum fx_i^2 - \frac{1}{N}(\sum fx_i^2)]$$

$$= \frac{1}{55-1}[27575 - \frac{1}{55}(925^2)]$$

$$= \frac{1}{54}[27575 - 15556.8182]$$

=222.559

Standard deviation = $\sqrt{variance}$

$= \sqrt{222.559}$

= 14.918

36.3 - Solved Examples of the questions related

Example 1: **Consider the data set containing the values 20, 24, 25, 36, 25, 22, 23.Find the mean of the data.**

Sol: We know that ,

$$Mean = \frac{\text{sum of all the elements}}{\text{number of elements}}$$

Mean=
$$\frac{20+24+25+36+25+22+23}{7}$$

=25

Example 2: **For the above given data calculate the median.**

Sol: Rearranging the above data in ascending order we get 20, 22, 23, 24, 25, 25, 36.
The middle value will be the median.
So,the middle value is 24.Hence,the median is 24.

Example 3: Calculate the mode for the above data.

Sol: The mode can be defined as the most frequent value in the given set of data.In the data given the most frequent value is 25 which occurred more than any other value.Hence the mode is 25.

Example 4: Calculate the range for the above set of data.

Sol: Range=higher value-lower value
=36-20
=16

Example 5: **Calculate the variance of the set of data given.**

Sol: $$\sigma^2 = \frac{\sum(x_i - \bar{x})^2}{N-1}$$

Mean of the data=25

now,using $\sum(x_i - \bar{x})^2$

20-25=-5 and square of -5=25
24-25=-1 and square of -1=1
25-25=0 and square of 0=0
36-25=11 and square of 11=121
25-25=0 and square of 0=0
22-25=-3 and square of -3=9
23-25=-2 and square of -2=4
Adding the squared difference,

25+1+0+121+0+9+4=160
and number of elements=7

Hence,Variance= $\frac{160}{6} = 26.667$ (approximately)

Exercise

Question 1: **Find the range of the following data**

143,148,135,150,128,139,149,146,151,132

Question 2: **The median for the data 2, 4, 6, 8, 10, 12, 14 is**

Question 3: **The mean of a set of data is 5. What will be the mean if ten is subtracted from each data ?**

Question 4: **What is the mean deviation of the data 8,9,12,15,16,20,24,30,32,34? and Find the distance of each value from that mean.**

Question 5: **The mean and the standard deviation of a data which consists of a set of ten positive numbers are 8 and 2 respectively. If the sum of squares of 9 among the 10 members is 599,what is the 10th number?**

Question 6:

With what value should the highest quantity in the data : 65, 52, 14, 26, 18, 35, 32, 38 be replaced so that the mean and medium become equal?

Question 7: **If the mean of the data 5,7,10,14,x,25,32,19 is x,then what is the value of x?**

Question 8: **What is the standard Deviation (correct upto two decimal places) of the numbers: 21334, 21335, 21336, 21337 and 21338?**

Question 9: **If mean of 29 observations is 33 and on adding one more observation the new mean becomes 34. What is the value of 30th observation?**

Question 10: **The mean of the median, the mode and the range of the data:**

15, 10, 17, 13, 25, 17, 11, 18, 14, 19, 12, 20 is:

Question 11: **If the mean of 26 observations is 29, and on adding four more observations, the new mean becomes 32. What is the mean of the last four observations?**

Question 12: **Find the mode of the given set of numbers:-**

6,11,5,8,6,8,5,11,8,9,7,11,5,6,8,9,11,8,7,6,9.

Question 13: **What is the difference between the mean and the median of the given data ?**

5,9,8,15,12,9,2,19,21,11

Question 14: **If the mode of a data is 12 and the arithmetic mean is 9,the median is______________?**

Question 15: **In 50 numbers 10 are threes,15 are fours,18 are fives and remaining are sixes.if a,b and c respectively represent the mean,mode and median of data,what is the value of a+2b-c?**

Question 16: **If mean of a distribution is 10 and the standard deviation is 4. What is the value of the coefficient of variation?**

Question 17: **The salary in rupees of 10 employees in a company per day is 50, 55, 60, 65, 70, 72, 75, 80, 84, 89. What is the standard deviation in the above data?**

Question 18: **A particular distribution is represented by two data points. If the Range and Standard deviation of the distribution are R and S respectively, what is the relation between them?**

Question 19: **Marks obtained by a student in 5 subjects are given as**

History - 23
Civics - 35
Economics - 46
Hindi - 30 and
English 32.
If the weightage given to these subjects be 1, 1, 2, 3, 5 respectively, then find the weighted mean.