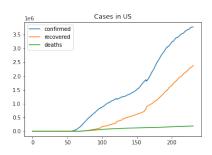
First-principles machine learning modelling of COVID-19

Che-Chia Chang

The COVID-19 Data

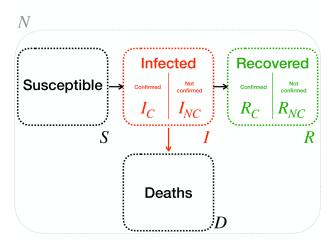
- Used data: COVID-19 Data Repository by the Center for Systems Science and Engineering (CSSE) at Johns Hopkins University (https://github.com/CSSEGISandData/COVID-19)
- Data includes number of:
 - confirmed cases
 - recovered cases
 - deaths cases

of each day for 188 countries.



The SIRD model

To describe the data, we may consider the SIRD model to fit it.



• Note that here N = S + I + R + D.

The SIRD model

The original ODE system of the SIRD model is written as:

The SIRD model

$$\begin{split} \frac{dS}{dt} &= -\frac{\beta IS}{N}, \\ \frac{dI}{dt} &= \frac{\beta IS}{N} - (\mu + \gamma)I, \\ \frac{dR}{dt} &= \gamma I, \\ \frac{dD}{dt} &= \mu I, \end{split}$$

where β , γ , μ , are rates of infection, recovery, and mortality, respectively.

• Note that here β , γ , μ may be dependent of time t.

The discrete SIRD model

We solve the ode numerically using Euler's method with step size 1, which results in the discrete SIRD model:

The discrete SIRD model

$$S(t+1) = S(t) - \frac{\beta(t)I(t)S(t)}{N},$$

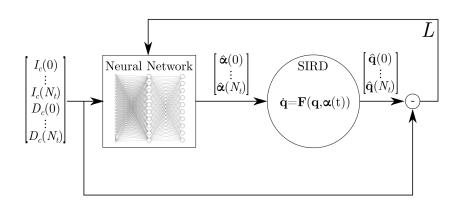
$$I(t+1) = I(t) + \frac{\beta(t)I(t)S(t)}{N} - (\mu(t) + \gamma(t))I(t),$$

$$R(t+1) = R(t) + \gamma(t)I(t),$$

$$D(t+1) = D(t) + \mu(t)I(t).$$

with I(0) and D(0) being the number of confirmed and deaths of the first confirmed day respectively, S(0) = N - I(0) - D(0), R(0) = 0.

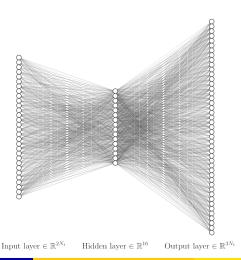
First-principles machine learning model



- $I_c(t)$: infected people = confirmed recovered deaths
- $D_c(t)$: deaths
- $\hat{\alpha}(t) = [\hat{\beta}(t), \hat{\gamma}(t), \hat{\mu}(t)], \quad \hat{q}(t) = [\hat{S}(t), \hat{I}(t), \hat{R}(t), \hat{D}(t)]$

The neural network

• A Fully connected neural network with 1 hidden layer and sigmoid activation. Input size : $2(N_t + 1)$, Output size: $3(N_t + 1)$.



Initialization

- We pre-train the neural network to output a constant guess of β_0, γ_0, μ_0 .
- β_0, γ_0, μ_0 is found by minimizing

$$\sum_{t=0}^{\tilde{N}_t} (I_c(t) - \hat{I}(t))^2 + 100 \sum_{t=0}^{\tilde{N}_t} (D_c(t) - \hat{D}(t))^2$$

where \tilde{N}_t is the time of initial exponential growth, using optimization package. (For USA, take $\tilde{N}_t=70$)

• Pre-training uses mean-squared-error with Adam optimizer and step size 0.012, 2500 iterations.

Loss function and training

• The loss function L of the neural network is given by

$$L = \sum_{t=0}^{N_t} \left((\log(I_c(t) - \log(\hat{I}(t)))^2 + (\log(D_c(t) - \log(\hat{D}(t)))^2) + \right.$$

$$0.01 \frac{\log(\max(I_c))}{\max(I_c)} \sum_{t=0}^{N_t} \left((I_c(t)) - \hat{I}(t))^2 + (D_c(t) - \hat{D}(t))^2 \right) +$$

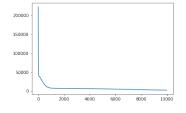
$$100 \frac{\log(\max(I_c))}{\max(\alpha_0)} \sum_{t=0}^{N_t - 1} (\hat{\beta}(t) - \hat{\beta}(t+1))^2 + (\hat{\gamma}(t) - \hat{\gamma}(t+1))^2 +$$

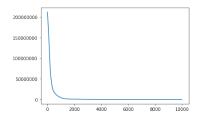
$$+ 100(\hat{\mu}(t) - \hat{\mu}(t+1))^2 +$$

$$100 \frac{\log(\max(I_c))}{\max(\alpha_0)} \left((\hat{\beta}(0) - \beta_0)^2 + (\hat{\gamma}(0) - \gamma_0)^2 + 100(\hat{\mu}(0) - \mu_0)^2 \right)$$

Loss function and training

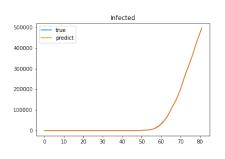
- We'll train networks of the USA data. The training only involves a single data.
- We train the network using Adam and learning rate 0.00005 with 10000 iterations for $N_t=81$ and $N_t=231$

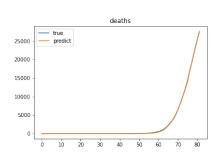


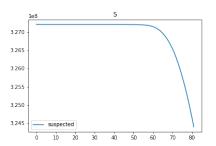


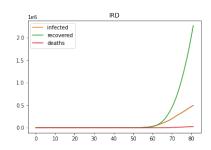
- left: $N_t = 81$, right: $N_t = 231$
- Final loss value: 2680, 5920

Results of model $(N_t = 81)$

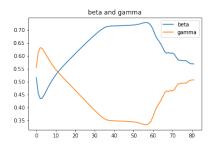


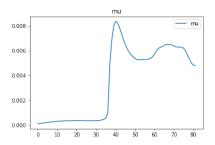




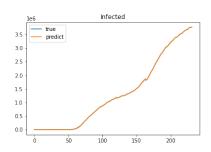


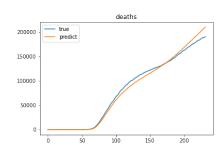
Results of model ($N_t = 81$)

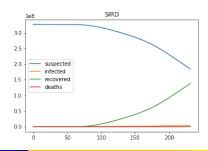




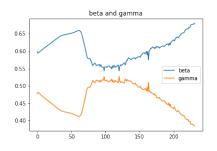
Results of model $(N_t = 231)$

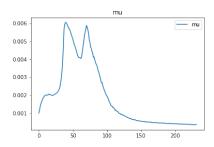






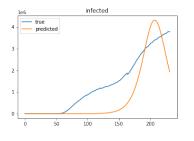
Results of model($N_t = 231$)

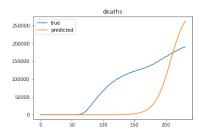




Non-Machine Learning methods

• constant parameters optimization:





- time dependent parameters: still need work.
- May try to separate time into some phases to reduce number of variables.

Future works

- Train networks for other countries and compare the results.
- Try non-machine learning methods.
- Different approach such as predict future states, or use data of several countries to estimate a country with missing data, etc.