Probability Homework #8

(Coverage: 7.1, 7.2, 7.3)

1. Suppose that 15 points are selected at random and independently from the interval (0, 1). In average, how many of them will be greater than 3/4?

2. A line segment of length k is cut randomly into two segments. What is the probability that none of the two segments is smaller than $k/3$?

3. Let X be a uniform random number from (0, 1). Find the density functions of $Y = -\ln(1-X)$.

4. Let Z be a standard normal random variable and α be a given positive constant. Find the real number x that maximizes $P(x < Z < x + \alpha)$.

5. The grades for a certain exam are normally distributed with mean 67 and variance 64. What percent of students get A(≥ 90), B(80 − 90), C(70 − 80), D(60 − 70), and F(< 60)?

6. Let $X \sim N(\mu, \sigma^2)$. Prove that $P(|X-\mu| > k\sigma)$ does not depend on μ or σ.

7. Let $X \sim N(0,1)$. Calculate the probability density function of $Y = \sqrt{|X|}$.

8. Let X be an exponential random variable with parameter λ, mean $E(X)$ and standard deviation σ_X. Find $P(|X - E(X)| > 2\sigma_X)$.

9. Let X, the lifetime (in years) of a radio tube, be exponentially distributed with mean 1/λ. Find the probability mass function (p.m.f.) of $\lfloor X+1 \rfloor$. (It is noted that $\lfloor X+1 \rfloor$ is the integer part of $X+1$, i.e. the greatest integer less than or equal to $X+1$. Actually, $\lfloor X+1 \rfloor$ can be proved to be a Geometric random variable.)