Probability Homework#9
(Coverage: 8.1, 8.2, 8.3)

(1)

Let the joint probability mass function of discrete random variables X and Y be given by

$$p(x, y) = \begin{cases} k(x^2 + y^2) & \text{if } (x, y) = (1, 1), (1, 3), (2, 3) \\ 0 & \text{otherwise.} \end{cases}$$

Determine (a) the value of the constant k, (b) the marginal probability mass function of X and Y, and (c) $E(X)$ and $E(Y)$.

(2)

Two fair dice are rolled. The sum of the outcome is denoted by X and the absolute value of the difference by Y. Calculate the joint probability mass function of X and Y and the marginal probability mass functions of X and Y.

(3)

Let the joint probability density function of random variables X and Y be given by

$$f(x, y) = \begin{cases} 2 & \text{if } 0 \le y \le x \le 1 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Calculate the marginal probability density functions of X and Y, respectively.

(b) Find $E(X)$ and $E(Y)$.

(c) Calculate $P(X < 1/2)$, $P(X < 2Y)$, and $P(X = Y)$.

(d) Are X and Y independent? Why or why not?

(e) $f_{X|Y}(x \mid y) = ?$

(4)

Let the joint probability mass function of random variables X and Y be given by

$$p(x, y) = \begin{cases} \frac{1}{7}x^2 y & \text{if } (x, y) = (1, 1), (1, 2), (2, 1) \\ \\ 0 & \text{elsewhere.} \end{cases}$$

Are X and Y independent? Why or why not?

(5)

Let the joint probability density function of X and Y be given by

$$f(x, y) = \begin{cases} 2e^{-(x+2y)} & \text{if } x \geq 0, \quad y \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Find $E(X^2Y)$.

(6)

Let the joint probability mass function of discrete random variables X and Y be given by

$$p(x, y) = \begin{cases} \frac{1}{25}(x^2 + y^2) & \text{if } x = 1, 2, \quad y = 0, 1, 2 \\ 0 & \text{otherwise.} \end{cases}$$

Find $p_{X|Y}(x|y)$, $P(X = 2 \mid Y = 1)$, and $E(X \mid Y = 1)$.

(7)

A point (X, Y) is selected randomly from the triangle with vertices $(0, 0)$, $(0, 1)$, and $(1, 0)$.

(a) Find the joint probability density function of X and Y.

(b) Calculate $f_{X|Y}(x|y)$.

(c) Evaluate $E(X \mid Y = y)$.

(8)

Let X and Y be two independent (continuous) uniform random variables chosen from the interval [-1, 1], respectively. Let Z= max (X,Y). Then (a) find the cumulative distribution function of Z, $F_z(t)$ = ? (b) compute the expected value of Z, E(Z).