Probability Homework #1

1. The waiting time (in seconds) between one song's end and the next song's beginning on a certain radio station is a random number between 3 and 6.8. Find the probability that the time between one song and another is at least 5.0 seconds.

2. Two fair 20-sided dice are rolled. How many total possible outcomes are there? What is the probability that the second number rolled is exactly 4 more than the first?

3. At a certain convenience store, 40% of patrons buy hot cocoa, 45% do not buy dairy product, and 25% of patrons get both hot cocoa and dairy product. How many patrons (in percentage) get hot cocoa or dairy product?

4. Let $S = \{w_1, w_2, \ldots\}$ be the sample space of some random experiment, containing an infinite number of samples w_n, $n = 1,\ldots,\infty$. Can we define a probability distribution P on S so that $P(\{w_n\}) = \frac{1}{3^n}$? (**Hint**: You may check whether such an assignment can satisfy the Axioms of Probability.)

5. A stick of length one is broken into two pieces at a random point. What is the probability that the length of the longer piece will be at least four times the length of the shorter piece?

6. Let $S = \{a,b,c\}$ be the sample space of a random experiment. If $P(\{a , b\})=0.6$ and $P(\{a , c\})=0.8$, find $P(\{a\}), P(\{b\}), P(\{c\})$.

7. Two numbers are successively selected at random and with replacement from the set $\{1, 2, \ldots , 100\}$. What is the probability that the first one is greater than or equal to the second?

8. From a small town 120 persons were selected at random and asked the following question: Which of the three pens, *A*, *B*, or *C*, do you use? The following results were obtained: 20 use *A* and *C*, 10 use *A* and *B* but not *C*, 15 use all three, 30 use only *C*, 35 use *B* but not *C*, 25 use *B* and *C*, and 10 use none of the three. If a person is selected at random from this group, what is the probability that he or she uses (a) only *A*; (b) only *B*; (c) *A* and *B*? (Hint: You may draw a Venn diagram.)

9. Use the probability axioms to prove that $P(A \cup B) = P(A) + P(B) - P(AB)$

10. Let A and B be two events. Prove that $P(AB) \geq P(A) + P(B) - 1$