On Extraction of TMDs from SIDIS

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Motivation and objective

- Importance of TMDs

✓ information about the nucleon structure, i.e the 3-D imaging of the nucleon in momentum space

✓ help to understand some important aspects of QCD such as gauge invariance and universality properties

✓ help to Learn about confinement and hadronization

- Extraction of TMDs such as transversity function and Collins fragmentation function from SIDIS with global fits

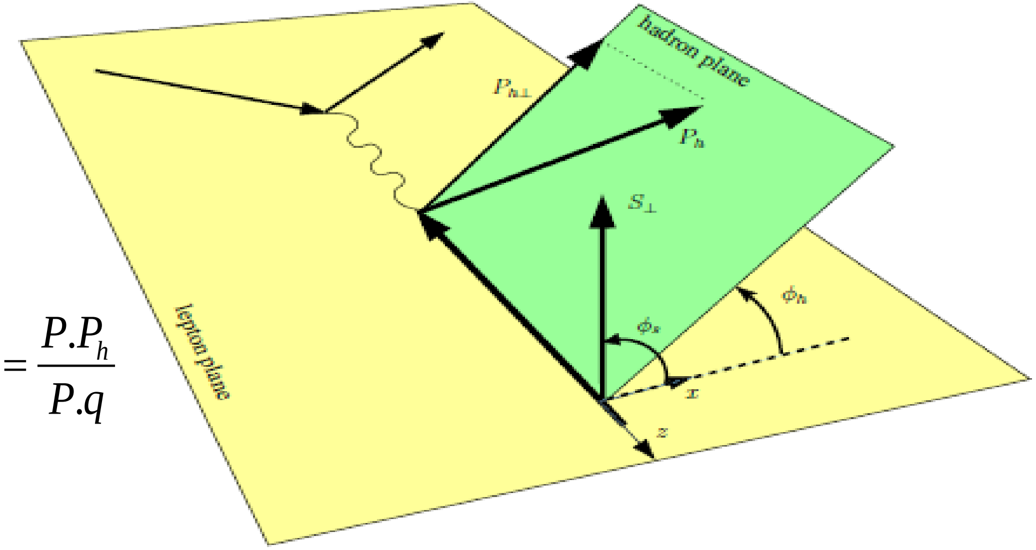
Extraction of TMDs

- Semi-inclusive DIS process:

$$e(\ell)+p(P)\rightarrow e(\ell')+h(P_h)+X$$

$$q=\ell-\ell',\quad Q^2=-q^2$$

$$S_{ep}=(P+\ell)^2,\ x_B=\frac{Q^2}{2P.q},\ y=\frac{P.q}{P.\ell}=\frac{Q^2}{x_B S_{ep}},\ z_h=\frac{P.P_h}{P.q}$$



- Factorization : a tool for separating effects from short-distance (s.d) scales and long-distance (l.d) effects and parametrizing (l.d) effects in universal quantities

$$\frac{d^5\sigma(S_\perp)}{dx_B dy dz_h d^2P_{h\perp}}=\sigma_0(x_B,y,Q^2)\left[F_{UU}+\sin(\phi_h+\phi_s)\frac{2(1-y)}{1+(1-y)^2}F_{UT}^{\sin(\phi_h+\phi_s)}+\ldots\right]$$

Extraction of TMDs

Structure functions:

$$F_{UU}\left(Q;P_{h\perp}\right)=\frac{1}{z_h^2}\int\frac{d^2b}{(2\pi)^2}e^{i\vec{P}_{h\perp}.\vec{b}/z_h}\tilde{F}_{UU}\left(Q;b\right)+Y_{UU}\left(Q;P_{h\perp}\right),$$

$$F_{\text{collins}}^{\alpha}\left(Q;P_{h\perp}\right)=\frac{1}{z_h^2}\int\frac{d^2b}{(2\pi)^2}e^{i\vec{P}_{h\perp}.\vec{b}/z_h}\tilde{F}_{\text{collins}}^{\alpha}\left(Q;b\right)+Y_{\text{collins}}^{\alpha}\left(Q;P_{h\perp}\right)$$

The Y- term dominates If $Ph_{\perp}\geq Q$. In the region of low $q_{\perp}=Ph_{\perp}/z_h$

$$F_{UU}\left(Q;P_{h\perp}\right)=\frac{1}{z_h^2}\int\frac{d^2b}{(2\pi)^2}e^{i\vec{P}_{h\perp}.\vec{b}/z_h}\tilde{F}_{UU}\left(Q;b\right),$$

$$F_{\text{collins}}^{\alpha}\left(Q;P_{h\perp}\right)=\frac{1}{z_h^2}\int\frac{d^2b}{(2\pi)^2}e^{i\vec{P}_{h\perp}.\vec{b}/z_h}\tilde{F}_{\text{collins}}^{\alpha}\left(Q;b\right),\quad \alpha=1,2$$

Collins azimuthal asymmetry :

$$\mathrm{A}_{UT}^{\sin(\phi_s+\phi_h)}=\frac{\sigma_0\left(x_B,y,Q^2\right)}{\sigma_0\left(x_B,y,Q^2\right)}\frac{2(1-y)}{1+(1-y)^2}\frac{F_{UT}^{\sin(\phi_s+\phi_h)}}{F_{UU}},$$

$$\sin\left(\phi_h+\phi_s\right)F_{UT}^{\sin(\phi_h+\phi_s)}=\varepsilon^{\alpha\beta}S_{\perp}^{\alpha}\left(g_{\perp}^{\beta\rho}-2\hat{e}_x^{\beta}\hat{e}_x^{\rho}\right)F_{\text{collins}}^{\rho}$$

Extraction of TMDs

Structure functions are expressed in b-space, e.g.

$$\tilde{F}_{UU}(Q;b) = \sum_q e_q^2 \tilde{f}_1^q(x_B, b; \rho, \varsigma, \mu) \tilde{D}_q(z_h, b; \rho, \hat{\varsigma}, \mu) H_{UU}(Q/\mu, \rho) S(b, \rho, \mu)$$

$$\varsigma^2 = 2(\upsilon.P)^2/\upsilon^2, \quad \hat{\varsigma}^2 = 2(\tilde{\upsilon}.P_h)^2/\tilde{\upsilon}^2, \quad \rho^2 = (2\upsilon.\tilde{\upsilon})^2/\upsilon^2\tilde{\upsilon}^2$$

$$\upsilon = (\upsilon^-, \upsilon^+, \upsilon_\perp), \ \tilde{\upsilon} = (\tilde{\upsilon}^-, \tilde{\upsilon}^+, \tilde{\upsilon}_\perp)$$

- Light-cone singularity if the gauge link is along the light-cone direction and energy divergence due to the soft factor
- A choice of regularization defines a TMD scheme, for e.g in Ji-Ma-Yuan scheme:

$$n = (1^-, 0^+, 0_\perp) \to \upsilon = (\upsilon^-, \upsilon^+, 0_\perp) \ \text{with} \ \upsilon^- \gg \upsilon^+$$

so the gauge link is slightly off-light cone

Extraction of TMDs

Unpolarized structure function in b-space

$$F_{UU}\left(Q;b\right)=\sum_q e_q^2\,\tilde{f}_{1\mathrm{JMY}}^{\,q(sub)}\left(x_B,b;\rho,\varsigma,\mu\right)\tilde{D}_{q\mathrm{JMY}}^{(sub)}\left(z_h,b;\rho,\hat{\varsigma},\mu\right)H_{UU}^{JMY}\left(Q/\mu,\rho\right)$$

where

$$\tilde{f}_{1\mathrm{JMY}}^{\,q(sub)}\left(x_B,b;\rho,\varsigma,\mu\right)=\frac{\tilde{f}_1^{\,q}\left(x_B,b;\rho,\varsigma,\mu\right)}{\sqrt{S\left(b,\rho;\mu\right)}}$$

$$\tilde{D}_{q\mathrm{JMY}}^{(sub)}\left(z_h,b;\rho,\hat{\varsigma},\mu\right)=\frac{\tilde{D}_q\left(z_h,b;\rho,\hat{\varsigma},\mu\right)}{\sqrt{S\left(b,\rho;\mu\right)}}$$

After solving evolution eq., the final expressions for TMDs are obtained by considering

$$\varsigma^2=\hat{\varsigma}^2=\rho Q^2$$

In the new Collins-11 scheme, the subtraction of the soft factor leads to the absence of both light-cone singularity and energy divergences, e. g:

$$\tilde{f}_1^{\,q\,JCC}\left(x,b;\varsigma_F,\mu\right)=\tilde{f}_1^{\,q}\left(x,b;\varsigma_F,\mu\right)\sqrt{S^{\tilde{n},\upsilon}\left(b\right)/S^{n,\tilde{n}}\left(b\right)S^{n,\upsilon}\left(b\right)}$$

Extraction of TMDs

Divergence and evolution

❖ Divergences lead to evolution:

- Ultraviolet divergence : renormalization group equation, e.g. running of coupling constant
- Collinear divergence : DGLAP evolution of collinear parton distribution function (PDF), fragmentation function (FF)

  DGLAP evolution = resummation of single logs in the higher -order corrections
- Rapidity divergence (light-cone singularity): TMD evolution

  TMD evolution = resummation of double logs in the higher -order corrections

single logs: $\left(\alpha_s \ln\left(Q/\mu\right)^2\right)^n$ double logs: $\left(\alpha_s \ln^2\left(Q/q_\perp\right)^2\right)^n$

Extraction of TMDs

How to make sense of divergences:

- Ultraviolet (UV) divergence : renormalization (redefine coupling constant)
- Collinear divergence : redefine the PDFs and FFs
- Soft divergence : usually cancel between real and virtual diagrams for collinear PDFs/FFs; do not cancel for TMDs, leads to new evolution equations

Extraction of TMDs

Collins-Soper (CS) equation gives the rapidity evolution with resp. to ς

$$\frac{\partial \ln \tilde{f}_q\left(x_B, b; \varsigma_F, \mu\right)}{\partial \ln \sqrt{\varsigma_F}} = \frac{\partial \ln \tilde{D}_q\left(z_h, b; \varsigma_D, \mu\right)}{\partial \ln \sqrt{\varsigma_D}} = \tilde{K}\left(b, \mu\right)$$

where $\tilde{K}(b,\mu)$ is CS kernel. The dependence on μ scale is given by RGE for $\tilde{f}, \tilde{D}$ and $\tilde{\mathrm{K}}$

$$\frac{d\tilde{K}\left(b, \mu\right)}{d \ln \mu} = -\gamma_K\left(\alpha_s\left(\mu\right)\right), \quad \frac{d \ln \tilde{f}\left(x_B, b; \varsigma_F, \mu\right)}{d \ln \mu} = \gamma_F\left(\alpha_s\left(\mu\right), \varsigma_F / \mu^2\right),$$

$$\frac{d \ln \tilde{D}_q\left(z_h, b; \varsigma_D, \mu\right)}{d \ln \mu} = \gamma_D\left(\alpha_s\left(\mu\right), \varsigma_D / \mu^2\right)$$

At low values of $\mathrm{b} \ll 1/\Lambda_{QCD}$, $1/b$ becomes a hard scale, then one defines an new scale, $\mu_b = c_0/b,$ with $c_0 = 2e^{-\gamma_E}$ and $\gamma_E \approx 0.57$

The b-dependence of TMDs can be computed in terms of collinear PDF and FF in the region $1/Q \ll b \ll 1/\Lambda_{QCD}$

Extraction of TMDs

The energy evolution of TMDs from μ_b to Q is encoded in the Sudakov factor:

$$S_{\text{pert}}(Q,b) = \int_{\mu_b^2}^{Q^2} \frac{d\tilde{\mu}^2}{\tilde{\mu}^2}\left[A(\alpha_s(\tilde{\mu}))\ln\frac{Q^2}{\tilde{\mu}^2} + B(\alpha_s(\tilde{\mu}))\right]$$

The perturbation breaks down for large value of b, and $\alpha_s(\mu_b)$ reaches the Landau pole which indicates non-perturbative Physics. In the CSS approach, the b* prescription introduces a cutoff $b_{\max}$

$$b \Rightarrow b_* = b\Big/\sqrt{1+b^2/b_{\max}^2}\,,\quad b_{\max} < 1/\Lambda_{QCD}$$

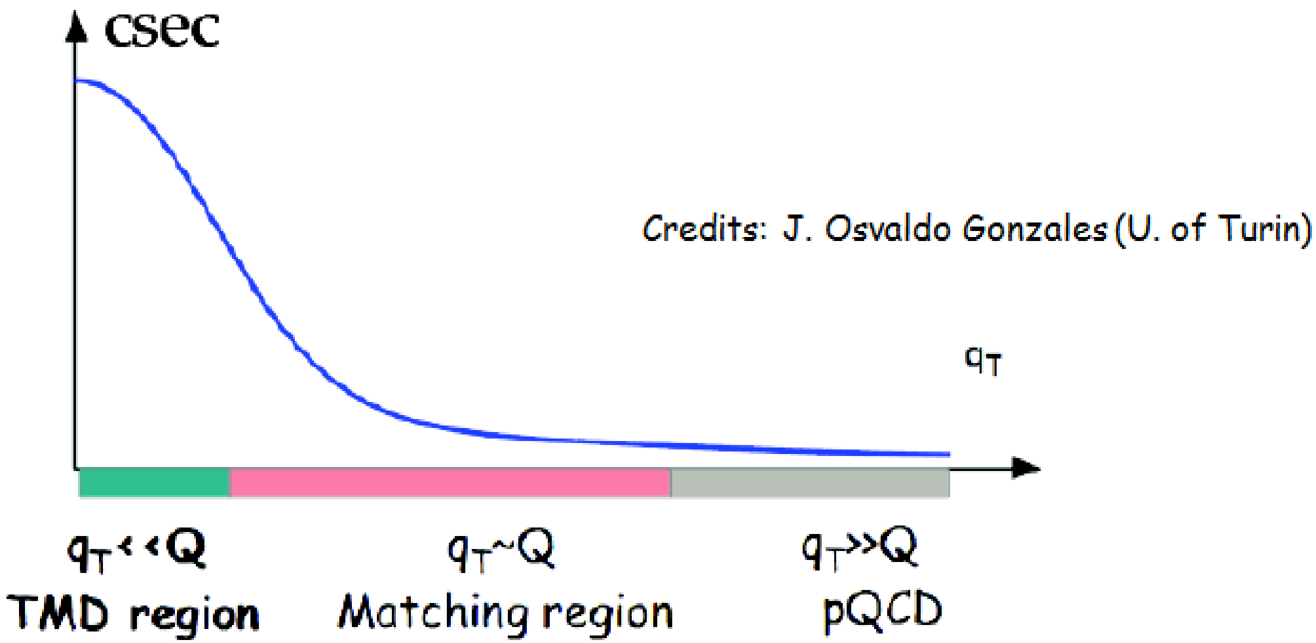
$$S_{(sud)}(Q;b) \Rightarrow S_{\text{pert}}(Q;b_*) + S_{NP}(Q;b)$$

$$S_{NP}(Q;b) = g_2(b)\ln Q/Q_0 + g_1(b),\quad g_2(b) = g_2 b^2,\quad g_2(b) = g_2\ln(b/b_*)$$

The b* prescription allows a smooth transition from the perturbative region to the non-perturbative region

Extraction of TMDs

Matching in SIDIS



- In the perturbative region, TMDs are matched onto their collinear counterparts with perturbatively calculable *C*-functions

Extraction of TMDs

Un-polarized PDF and FF with TM evolution:

$$\tilde{f}_1^{q(sub)}\left(x_B,b;Q^2,Q\right)=\exp\left\{-\frac{1}{2}S_{\text{pert}}\left(Q,b_*\right)-S_{NP}^{f_1}\left(Q,b\right)\right\}\tilde{\mathcal{F}}_q\left(\alpha\left(Q\right)\right)C_{q\leftarrow i}\otimes f_1^i\left(x_B,\mu_b\right)$$

$$\tilde{D}_q^{(sub)}\left(z_h,b;Q^2,Q\right)=\exp\left\{-\frac{1}{2}S_{\text{pert}}\left(Q,b_*\right)-S_{NP}^{D_1}\left(Q,b\right)\right\}\tilde{\mathcal{D}}_q\left(\alpha\left(Q\right)\right)\hat{C}_{j\leftarrow q}\otimes D_{h/j}\left(z_h,\mu_b\right)$$

$\tilde{\mathcal{F}}_q$, and $\tilde{\mathcal{D}}_q$ are scheme-dependent. In the standard CSS formalism, they are absorbed in the C-functions

$$F_{UU}\left(Q;b\right)=F_{UU}\left(b_*\right)\exp\left\{-S_{\text{pert}}\left(Q,b_*\right)-S_{NP}^{(\text{SIDIS})}\left(Q,b\right)\right\}$$

$$=\sum_q e_q^2\left(C_{q\leftarrow i}\otimes f_1^i\left(x_B,\mu_b\right)\right)\left(\hat{C}_{j\leftarrow q}\otimes D_{h/j}\left(z_h,\mu_b\right)\right)\exp\left\{-S_{\text{pert}}\left(Q,b_*\right)-S_{NP}^{(\text{SIDIS})}\left(Q,b\right)\right\}$$

Convolution integrals are evaluated using the Mellin transformation

Extraction of TMDs

Collins structure functions:

$$\tilde{F}^{\alpha}_{\text{collins}}\left(Q;b\right)=\sum_q e_q^2 \tilde{h}_1^{q(sub)}\left(x_B,b;\rho,\varsigma,\mu\right)H_{1h/q}^{\perp\alpha(sub)}\left(z_h,b;\rho,\hat{\varsigma},\mu\right)H\left(Q/\mu,\rho\right)$$

With TMD evolution

$$\tilde{h}_1^{q(sub)}\left(x_B,b,\rho;Q^2,Q\right)=\exp\left\{-\frac{1}{2}S_{pert}\left(Q,b_*\right)-S_{NP}^{h_1}\left(Q,b\right)\right\}\tilde{\mathcal{H}}_{1q}\left(\alpha_s\left(Q\right)\right)\delta C_{q\leftarrow q'}\otimes h_1^{q'}\left(x_B,\mu_b\right)$$

$$\tilde{H}_{1h/q}^{(sub)\perp\alpha}\left(z_h,b,\rho;Q^2,Q\right)=\frac{-ib^{\alpha}}{2z_h}\exp\left\{-\frac{1}{2}S_{pert}\left(Q,b_*\right)-S_{NP}^{D_1}\left(Q,b\right)\right\}\tilde{\mathcal{H}}_c\left(\alpha_s\left(Q\right)\right)\delta C_{q'\leftarrow q}\otimes\hat{H}_{h/q'}^{(3)}\left(z_h,\mu_b\right)$$

$\tilde{\mathcal{H}}_{1q}$, and $\tilde{\mathcal{H}}_c$ are absorbed in C-functions

$$\tilde{F}^{\alpha}_{\text{collins}}\left(Q;b\right)=\frac{-ib^{\alpha}}{2z_h}\exp\left\{-S_{pert}\left(Q,b_*\right)-S_{NP\,\text{collins}}^{SIDIS}\left(Q,b\right)\right\}\tilde{F}_{\text{collins}}\left(b_*\right)$$

$$=\frac{-ib^{\alpha}}{2z_h}\sum_q e_q^2\left(\delta C_{q\leftarrow i}\otimes h_1^i\left(x_B,\mu_b\right)\right)\left(\delta\hat{C}_{j\leftarrow q}^{SIDIS}\otimes H_{h/j}^{(3)}\left(z_h,\mu_b\right)\right)\exp\left\{-S_{pert}\left(Q,b_*\right)-S_{NP\,\text{collins}}^{SIDIS}\left(Q,b\right)\right\}$$

Extraction of TMDs

Twist-3:

$$H_{1h/q}^{\perp\alpha}\left(z_h,b\right)=\int d^2p_\perp e^{-ip_\perp.b}p_\perp^\alpha H_{1h/q}^{\perp}\left(z_h,p_\perp\right)$$

$$H_{h/j}^{(3)}\left(z_h\right)=\int d^2p_\perp \frac{\left|p_\perp^2\right|}{M_h}H_{1h/j}^{\perp}\left(z_h,p_\perp\right)$$

The evolution equations for h_1^q and $H_{h/j}^{(3)}$ are

$$\frac{\partial h_1^q\left(x_B,\mu\right)}{\partial\ln\mu^2}=\frac{\alpha_s}{2\pi}\int_{x_B}^1\frac{d\hat{x}}{\hat{x}}P_{q\leftarrow q}^{h_1}\left(\hat{x}\right)h_1^q\left(x_B/\hat{x},\mu\right),\ P_{q\leftarrow q}^{h_1}\left(\hat{x}\right)=C_F\left(\frac{2\hat{x}}{\left(1-\hat{x}\right)_+}+\frac{3}{2}\delta\left(1-\hat{x}\right)\right)$$

$$\frac{\partial\hat{H}_{h/q}^{(3)}\left(z_h,\mu\right)}{\partial\ln\mu^2}=\frac{\alpha_s}{2\pi}\int_{z_h}^1\frac{d\hat{z}}{\hat{z}}\hat{P}_{q\leftarrow q}^{c}\left(\hat{z}\right)H_{h/q}^{(3)}\left(z_h/\hat{z},\mu\right),\ \hat{P}_{q\leftarrow q}^{c}\left(\hat{z}\right)=C_F\left(\frac{2\hat{z}}{\left(1-\hat{z}\right)_+}+\frac{3}{2}\delta\left(1-\hat{z}\right)+\dots\right)$$

Extraction of TMDs

Parametrization of transversity:

$$h_1^q(x,Q_0) = N_q^h x^{a_q}(1-x)^{b_q}\left\{(a_q+b_q)^{a_q+b_q} \big/ a_q^{a_q} b_q^{b_q}\right\}\frac{1}{2}\left(f_1^q(x,Q_0)+g_1^q(x,Q_0)\right)$$

$f_1^q(x,Q_0)$ is unpolarized PDF and $g_1^q(x,Q_0)$ is helicity PDF

Parametrization of Collins function:

$$\hat{H}_{fav}^{(3)}(z,Q_0) = N_u^c z^{\alpha_u}(1-z)^{\beta_u} D_{\pi^+/u}(z,Q_0)$$

$$\hat{H}_{unf}^{(3)}(z,Q_0) = N_d^c z^{\alpha_d}(1-z)^{\beta_d} D_{\pi^+/d}(z,Q_0)$$

$D(z,Q_0)$ is unpolarized FF, $Q_0^2 = 2.4\mathrm{GeV}^2$

and then one solves DGLAP equations both for h1 and Collins FF to the scale $\mu_b = c_0/b_*$

Free parameters: $N_q, a_q, b_q, \alpha_q, \beta_q$

Extraction of TMDs

- Parametrization of non-pertubative Sodakov factors for all TMDs:

$$S_{NP}^{f_1}(Q,b) = S_{NP}^{h_1}(Q,b)$$

$$S_{NP}^{h_1}(Q,b) = \frac{g_2}{2}\ln\left(\frac{b}{b_*}\right)\ln\left(\frac{Q}{Q_0}\right) + g_q b^2$$

$$S_{NP}^{D_1}(Q,b) = \frac{g_2}{2}\ln\left(\frac{b}{b_*}\right)\ln\left(\frac{Q}{Q_0}\right) + \frac{g_h}{z^2} b^2$$

$$S_{NP}^{\mathrm{collins}}(Q,b) = \frac{g_2}{2}\ln\left(\frac{b}{b_*}\right)\ln\left(\frac{Q}{Q_0}\right) + \frac{g_h - g_c}{z^2} b^2$$

- Some parameters:
- fixed parameters: $\left\{g_2 = 0.84,\ g_q = g_1/2 = 0.106,\ g_h = 0.042\left(\mathrm{GeV}^2\right)\right\}$
- free parameters: $\left\{N_u^h, N_d^h, a_u, a_d, b_u, b_d, N_u^c, N_d^c, \alpha_u, \alpha_d, \beta_u, \beta_d, g_c\right\}$

Extraction of TMDs

Fitting procedure:

One minimize χ^2

$$\chi^2(\{a\}) = \sum_{i=1}^{N} \sum_{j=1}^{N_i} \frac{\left(T_j(\{a\}) - E_j\right)^2}{\Delta E_j^2}$$

$T_j(\{a\})$ is the theoretical estimate for a set of free parameters $\{a\} \subseteq \{N_u^h, N_d^h, a_u, a_d, b_u, b_d, N_u^c, N_d^c, \beta_d, \beta_u, g_c\}$

$i = 1, \ldots, N$ data sets each containing N_i data points

E_j is the experimental measurement of each data point

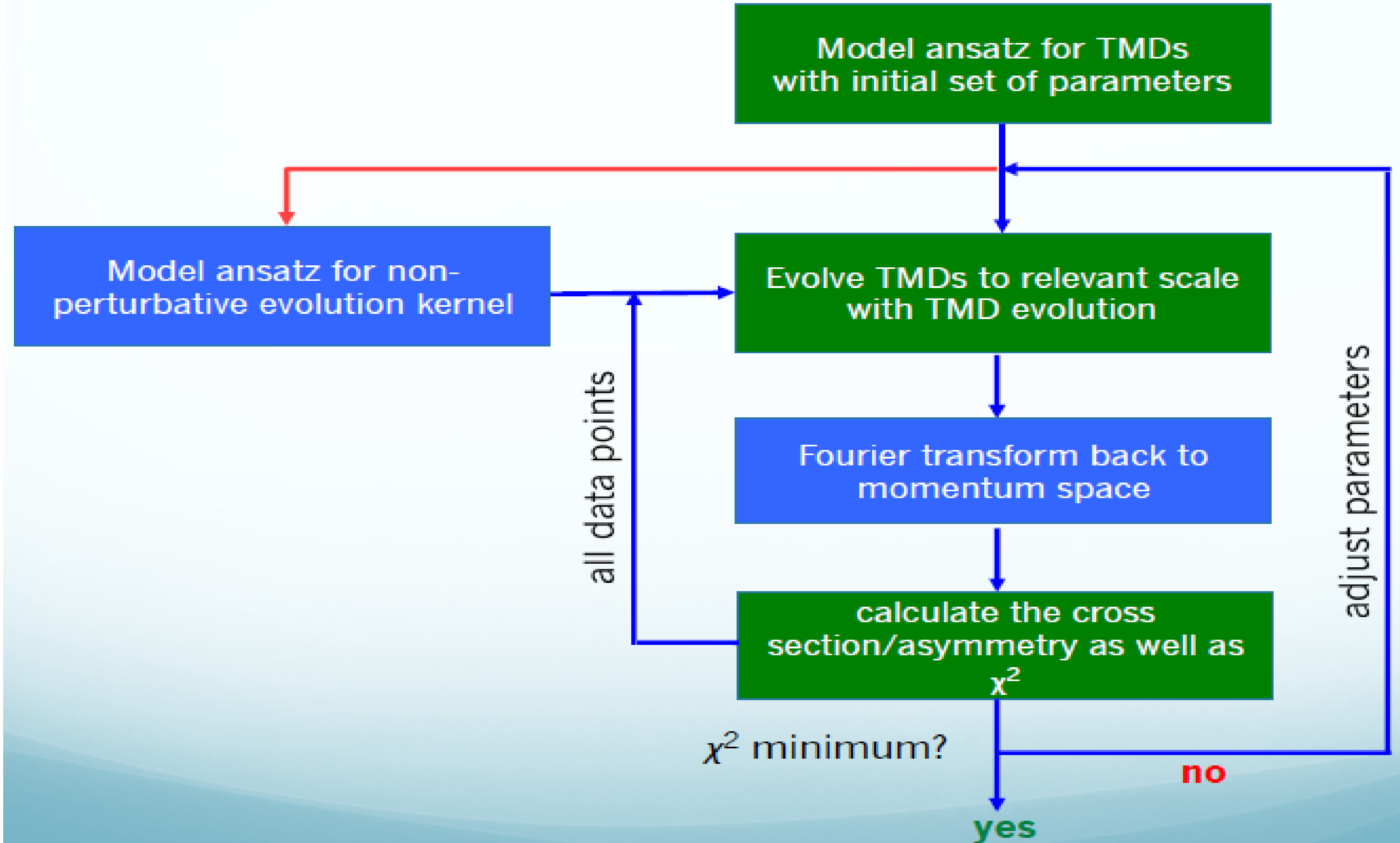
ΔE_j is the experimental uncertainty

Rough idea of a good fit, for N points: $N - \sqrt{2N} < \chi^2(\{a\}) < N + \sqrt{2N}$

In principal, the model describes the data: $\chi^2 / n_{d.o.f} \simeq 1$

Noise: $\chi^2 / n_{d.o.f} \ll 1$

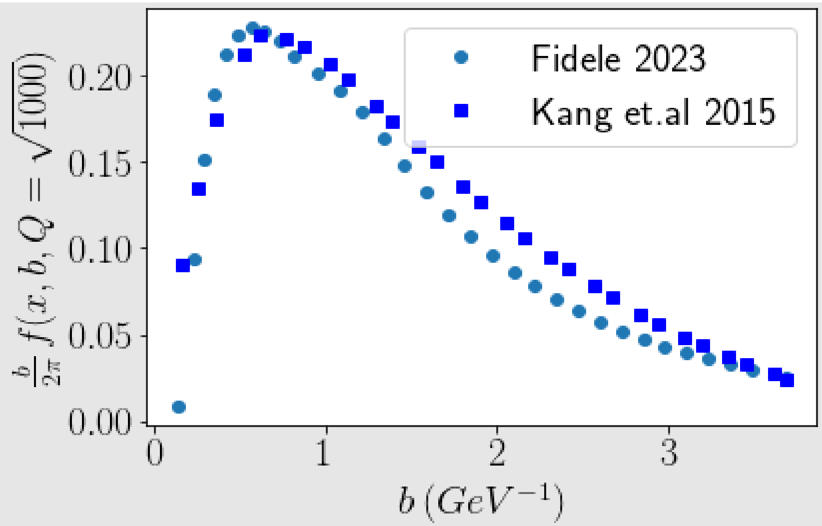
Extraction of TMDs

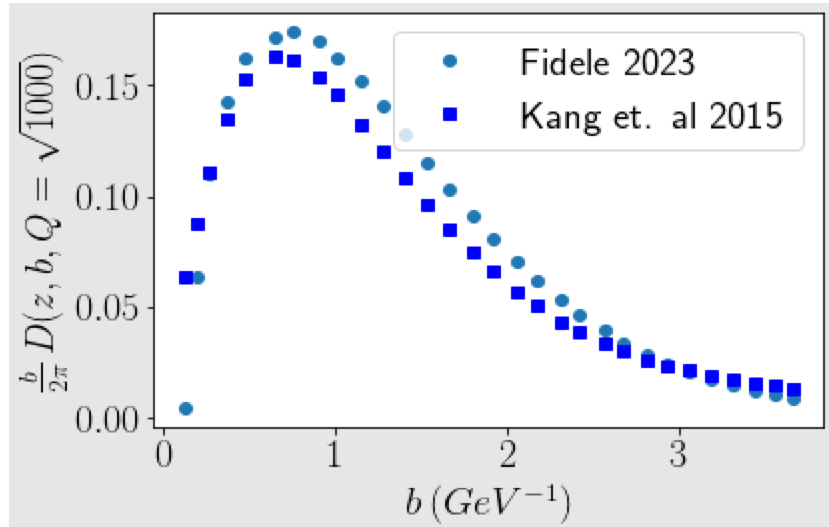


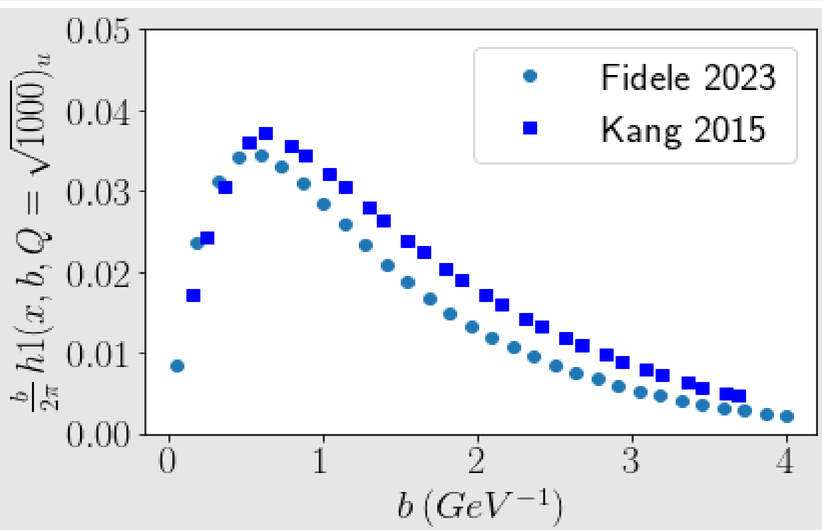
Credits: Prof. Zhongbo Kang (UCLA)

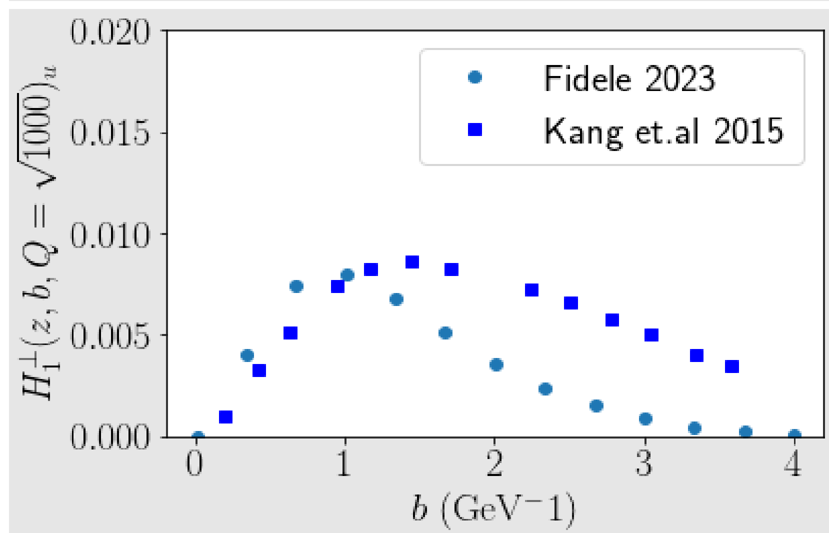
Extraction of TMDs

Benchmark with Kang et. al 2015









Extraction of TMDs

Summary and future work:

- TMDs are important tools to investigate structure of the nucleon and QCD dynamics, and much more
- The benchmark of Fidele 2023 with Kang et al. 2015 is done for the scale $Q^2 = 1000$, and there are some minor discrepancies
- The future work will consist of finding the source and physical meaning of these discrepancies. Also, there is a need for benchmarking for different scales, and finally the extraction of h1 and Collins FF will follow

Thank you for your hospitality